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Ship Handling when the Environmental Parameters Varied as the Function of the Way

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SHIPHANDLINGWHENTHEENVIRONMENTALPARAMETERSVARIEDASTHEFUNCTIONOFTHEWAY

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Abstract- The paper devotes the algorithm of ship handling when the environmental parameters varied as the function of the way. In nautical practice, when the ships sail in the channel, they often arrange as the convoy with the leader ship. In order to ensure the maritime safety, the mariner should establish the algorithm for control of ship engine system and steering gear complex. In this research, the author uses the maximum principle of Pontryagin L.S to establish the similar control. However, in order to obtain these ranges of numerical solutions like this, sometimes it's difficult to use the maximum principle. Because, there is not enough the initial condition for using of the auxiliary vector that is the quantity to define the time of control variation. These obstacles shall be cleared by the selection of the transversal conditions. The problems are solved under Maier's and G. Kelly's condition as well as the Hamiltonian operator and Cardano's formula.

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I. Introduction

In order to control the navigation of ship in following the object in the sub-system of higher order, example in the coastal system, there should be the unique complex of the programs (or algorithm) for controlling of power system and steering complex in normal and emergency situation. Basing on these programs, the mariner evaluates the situation and the context around the ship and obtains the objective information that he can make the good decision and give the proper solution. In necessary case, it can be transferred the program control to the diesel system. These programs like this can help the mariner estimate the controlled movement of ship as the time.

Those algorithms should be considered as the evaluation and auxiliary. Their objective is to help obtaining the proper controls that are not compulsory to use directly on board the vessel. Also, they can be used as the initial information for maneuvering of the specified vessel to escape from the emergency and critical situation.

These algorithm creations are carried out by the way how the classes of limited condition are used for imposing on the control action and phase co-ordinate. Also, it may be easily extended for the limitations that applied for the speed of variable control or acceleration, where the general form of obtained algorithm is fully preserved. That property of them is to help the mariner using the given algorithms to synthesize the systems of engine complex control and ship steering gear that is for purpose of safety and economical navigation.

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II. Literature Review

In this subject, there are researches of authors such as Krasovsky A.A. (1999), Peshekhonov V.G (2000), Kolesnikov A.A. (2002), Astana Y.M (2002), V.S Medvedev et al. (2005), Hecht-Nielsen r. (2007), Stone M. (2009), Weierstrass K. (2010). Their works are based on the classical methods of construction of automatic control systems and in particular the ship's course allows classifying the type of techniques used by the mathematical model of the vessel, processed information, methods of adaptation, design features. In some cases, the sufficient condition purely is the evaluation of the proposal algorithm that is to create the exactly controls. But, it is often required the more detail solutions that means the numerical ones.

In this research, the author uses the maximum principle of Pontryagin L.S to establish the similar control. However, in order to obtain these ranges of numerical solutions like this, sometimes it's difficult to use the maximum principle. Because, there is not enough the initial condition for using of the auxiliary vector that is the quantity to define the time of control variation. These obstacles shall be cleared by the selection of the transversal conditions.

III. Method of Research

It's assumed that the external environment is changing its characteristics as the function of the way. This happens when the parameters are considered as characteristic of the depth, width, and tortuosity of the channel[1, 2, 3 and 4], then

$$\psi = \psi(s) \qquad (1)$$

Equations of the ship complex in this case will be:

$$\left.\begin{aligned} \frac{dv}{dt} &= -\frac{1}{T_c}v + \frac{K_c^{\omega}}{T_c}\omega - \frac{K_c^{\psi}}{T_c}\psi(s), \\ \frac{d\omega}{dt} &= -\frac{1}{T_g}\omega + \frac{K_g^{h}}{T_g}h + \frac{K_g^{v}}{T_g}v, \\ \frac{dG_m}{dt} &= K_g^{\omega}K_g^{h}\omega h, \quad \frac{ds}{dt} = v. \end{aligned}\right\} \qquad (2)$$

And the restricted conditions that applied on the control action hare

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$$0 \le h \le h_{max} \tag{3}$$

It's necessary to find the control law for the given complex in the dynamic condition:

$$h = h(t)$$

Which the function can be minimized in the sailing time T:

$$\Delta G_m = G_m(v,s) \tag{4}$$

Basing on the principle of maximum, it's developed and solved the problem of optimal control in the form of Mayer [6, 7, 8 and 11]. The problem relates to the problem of fixed right and left ends.

The boundary conditions are written:

$$\left.\begin{array}{l}\text{at the left end } t=0, v_0=\omega_0=G_{T_0}=s_0=0;\\ \text{at the right end } t=T, v_T,\ \omega_T-\text{free quantities}, s=s_T\end{array}\right\}$$

The Hamiltonian of equation (2) is:

$$H=\left[-\frac{1}{T_c}V+\frac{K_c^{\omega}}{T_c}\omega-\frac{K_c^{\psi}}{T_c}\psi(S)\right]\alpha_1+ \\ +\left[-\frac{1}{T_g}\omega+\frac{K_g^h}{T_g}h+\frac{K_g^h}{T_g}V\right]\alpha_2+\left[K_g^{\omega}\omega K_g^h h\right]\alpha_3+V\alpha_4 \tag{5}$$

The function for finding the vector **α** will be:

$$\left.\begin{array}{l}\dfrac{d\alpha_1}{dt}=\dfrac{1}{T_c}\alpha_1-\dfrac{K_g^v}{T_g}-\alpha_2-\alpha_4,\\ \dfrac{d\alpha_2}{dt}=-\dfrac{K_c^{\omega}}{T_c}\alpha_1+\dfrac{1}{T_g}\alpha_2-K_g^{\omega}K_g^h h\alpha_3,\\ \dfrac{d\alpha_3}{dt}=0,\ \dfrac{d\alpha_4}{dt}=\dfrac{K_c^{\psi}}{T_c}\dfrac{d\psi(s)}{ds}\alpha_1.\end{array}\right\} \tag{6}$$

The transversal conditions are:

$$[(1+\alpha_3)\delta G_m-H\delta t+\alpha_1\delta v+ \\ +\alpha_2\delta\omega+\alpha_4\delta s]_0^T=0 \tag{7}$$

Then the above problem has the 1st order integral form:

$$\left[-\frac{1}{T_c}v+\frac{K_c^{\omega}}{T_c}\omega-\frac{K_c^{\psi}}{T_c}\psi(s)\right]\alpha_1+ \\ +\left[-\frac{1}{T_g}\omega+\frac{K_g^h}{T_g}h+\frac{K_g^v}{T_g}v\right]\alpha_2+ \\ +\left[K_g^{\omega}\omega K_g^h h\right]\alpha_3+v\alpha_4=c \tag{8}$$

The equality (8) is only relied on the contingent selection of variations δG_m, δv, $\delta\omega$ when:

$$\left.\begin{array}{l}\alpha_{3T}=-1, H=c=0,\\ \alpha_{1T}=0, \alpha_{2T}=0, \alpha_{4T}=0\end{array}\right\} \tag{9}$$

The structure of resulted control action is investigated:

$$\frac{\partial H}{\partial h}=K_g^h/T_g\ \alpha_2+K_g^h K_g^{\omega}\omega\alpha_3 \tag{10}$$

Thence, there is the variable law of the control action as:

$$\left.\begin{array}{l}h=h_{max}\ \text{at}\ K_g^h/T_g\ \alpha_2+K_g^h K_g^{\omega}\omega\alpha_3>0,\\ h=0\ \text{at}\ K_g^h/T_g\ \alpha_2+K_g^h K_g^{\omega}\omega\alpha_3<0\end{array}\right\} \tag{11}$$

From the equation (6), it is obtained the solution of component vector:

$$\alpha \quad \alpha_3=c_3^{\alpha}$$

Because of the transversal condition, it implies that

$$\alpha_{3T}=-1,$$

Consequently

$$c_3^{\alpha}=-1 \text{ and } \alpha_3=-1$$

Basing on this, the control will be varied as following law:

$$\left.\begin{array}{l}h=h_{max}\ \text{at}\ K_g^h/T_g\ \alpha_2-K_g^h K_g^{\omega}\omega>0,\\ h=0\ \text{at}\ K_g^h/T_g\ \alpha_2-K_g^h K_g^{\omega}\omega<0\end{array}\right\} \tag{12}$$

Now, it is going to integrate the equations of the problem. From the equation:

$$v=\frac{ds}{dt},$$

it's found:

$$ds = vdt$$

and

$$s=\int_0^T vdt.$$

Therefore the equations of α_4 (equation 6) can be rewritten as following:

$$\frac{d\alpha_4}{dt}=\frac{K_c^{\psi}}{T_c}\frac{1}{v}\frac{d\psi\left(\int_0^T vdt\right)}{dt}\alpha_1,$$

Or

$$\frac{d\alpha_4}{dt}=\frac{K_c^{\psi}}{T_c}\frac{1}{v}\frac{d\psi(t)}{dt}\alpha_1 \tag{13}$$

So the given task can be converted to the problem of variable external conditions that change as the function of time [14, 15, 16 and 19].

On the basis of the equation (13), it should be integrated the following differential equations with the control action $h = h_{max}$ on the interval time $0 \div t^*$, it means the time where:

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$$\left.\begin{aligned}
&\frac{K_g^h}{T_g} - \alpha_2 + K_g^h K_g^\omega \omega \alpha_3 : \\
&\frac{dv}{dt} = -\frac{1}{T_c}v + \frac{K_c^\omega}{T_c}\omega - \frac{K_c^\psi}{T_c}\psi(v,t), \\
&\frac{d\omega}{dt} = -\frac{1}{T_g}\omega + \frac{K_g^h}{T_g}h_{max} + \frac{K_g^v}{T_g}v, \\
&\frac{dG_m}{dt} = K_g^\omega K_g^h \omega h_{max}, \ \frac{ds}{dt} = v, \\
&\frac{d\alpha_1}{dt} = \frac{1}{T_c}\alpha_1 - \frac{K_g^v}{T_g}\alpha_2 - \alpha_4, \\
&\frac{d\alpha_2}{dt} = -\frac{K_c^\omega}{T_c}\alpha_1 + \frac{1}{T_g}\alpha_2 - K_g^\omega K_g^h h_{max}\alpha_3, \\
&\frac{d\alpha_3}{dt} = 0, \ \frac{d\alpha_4}{dt} = \frac{K_c^\psi}{T_c}\frac{1}{v}\frac{d\psi(t)}{dt}\alpha_1.
\end{aligned}\right\} \quad (14)$$

And, it's continuously integrated that equation in range of $t^* \div T$ until:

$$\frac{K_g^h}{T_g}\alpha_2 + K_g^h K_g^\omega \omega \alpha_3 < 0 .$$

When the actions control h = 0:

$$\left.\begin{aligned}
&\frac{dv}{dt} = -\frac{1}{T_c}v + \frac{K_c^\omega}{T_c}\omega - \frac{K_c^\psi}{T_c}\psi(v,t), \\
&\frac{d\omega}{dt} = -\frac{1}{T_g}\omega + \frac{K_g^v}{T_g}v, \ \frac{ds}{dt} = v, \\
&\frac{d\alpha_1}{dt} = \frac{1}{T_c}\alpha_1 - \frac{K_g^v}{T_g}\alpha_2 - \alpha_4, \\
&\frac{d\alpha_2}{dt} = -\frac{K_c^\omega}{T_c}\alpha_1 + \frac{1}{T_g}\alpha_2, \ \frac{d\alpha_3}{dt} = 0, \\
&\frac{d\alpha_4}{dt} = \frac{K_c^\psi}{T_c}\frac{1}{v}\frac{d\psi(t)}{dt}\alpha_1.
\end{aligned}\right\} \quad (15)$$

The initial conditions about the solution of the equations (15) will be the values ofthe phase coordinate calculated by the solution of the equation (14) at time t*.

The solutions of the equations (14) and (15)can be numerical. In order to consider the structure of the control action and the switching functions (11) and (12), it should be rewritten the equations (6):

$$\left.\begin{aligned}
&\frac{d\alpha_1}{dt} = a_{11}\alpha_1 - a_{12}\alpha_2 - a_{14}\alpha_4, \\
&\frac{d\alpha_2}{dt} = -a_{21}\alpha_1 + a_{22}\alpha_2 - a_{23}\alpha_3, \\
&\frac{d\alpha_3}{dt} = 0, \ \frac{d\alpha_4}{dt} = a_{41}\alpha_1.
\end{aligned}\right\} \quad (16)$$

Where:

$$\left.\begin{aligned}
&a_{11} = \frac{1}{T_c}, \ a_{12} = \frac{K_g^v}{T_g}, \ a_{21} = \frac{K_c^\omega}{T_c}, \ a_{14} = -1, \\
&a_{22} = \frac{1}{T_g}, \ a_{23} = K_g^h K_g^\omega h, \ a_{41} = \frac{K_c^\psi}{T_c}\frac{d\psi(s)}{ds}.
\end{aligned}\right\} \quad (17)$$

The typical determinant of the given equations is:

$$\begin{vmatrix} a_{11}-p & a_{12} & 0 & -a_{14} \\ -a_{21} & a_{22}-p & -a_{23} & 0 \\ 0 & 0 & -p & 0 \\ a_{41} & 0 & 0 & -p \end{vmatrix} = 0$$

Based on the above determinant, it can be written the typical equations as following:

$$p^4 - q_1p^3 + q_2p^2 - q_3p = 0 \quad (18)$$

That typical equation is invariable for the action control, because of:

$$\left.\begin{aligned}
&q_1 = a_{22} + a_{11}, q_2 = a_{14}a_{41} + a_{11}a_{22} - a_{12}a_{21}, \\
&q_3 = a_{14}a_{41}a_{22}
\end{aligned}\right\} \quad (19)$$

Now, it is continuously found the solutions of equation (16) in the form:

$$\left.\begin{aligned}
&\alpha_1 = c_1\gamma_1^{(1)}l^{p_1t} + c_2\gamma_1^{(2)}l^{p_2t} + c_3\gamma_1^{(3)}l^{p_3t} + \\
&+c_4\gamma_1^{(4)}l^{p_4t}, \\
&\alpha_2 = c_1\gamma_2^{(1)}l^{p_1t} + c_2\gamma_2^{(2)}l^{p_2t} + c_3\gamma_2^{(3)}l^{p_3t} + \\
&+c_4\gamma_2^{(4)}l^{p_4t}, \\
&\alpha_3 = c_1\gamma_3^{(1)}l^{p_1t} + c_2\gamma_3^{(2)}l^{p_2t} + c_3\gamma_3^{(3)}l^{p_3t} + \\
&+c_4\gamma_3^{(4)}l^{p_4t}, \\
&\alpha_4 = c_1\gamma_4^{(1)}l^{p_1t} + c_2\gamma_4^{(2)}l^{p_2t} + c_3\gamma_4^{(3)}l^{p_3t} + \\
&+c_4\gamma_4^{(4)}l^{p_4t}.
\end{aligned}\right\} \quad (20)$$

Wherein γ_i^k is constant factor that specified for each solution k of the typical equation (18) in the following systems:

1. $P_1 = 0$

$$\left.\begin{aligned}
&a_{11}\gamma_1^{(1)} - a_{12}\gamma_2^{(1)} - a_{14}\gamma_4^{(1)} = 0, \\
&-a_{21}\gamma_1^{(1)} + a_{22}\gamma_2^{(1)} - a_{23}\gamma_3^{(1)} = 0, \ a_{41}\gamma_1^{(1)} = 0.
\end{aligned}\right\} \quad (21)$$

Basing on (21), it's obtained $\gamma_1^{(1)} = 0$ and $\gamma_2^{(2)} = 1$, there are:

$$\gamma_4^{(1)} = -\frac{a_{12}}{a_{14}}, \quad \gamma_3^{(1)} = \frac{a_{22}}{a_{23}}.$$

2. $P = P_2$

$$\left.\begin{aligned}
&(a_{11}-p_2)\gamma_1^{(2)} - a_{12}\gamma_2^{(2)} - a_{14}\gamma_4^{(2)} = 0, \\
&-a_{21}\gamma_1^{(2)} + (a_{22}-p_2)\gamma_2^{(2)} - a_{23}\gamma_3^{(2)} = 0, \\
&-p_2\gamma_3^{(2)} = 0, \ a_{41}\gamma_1^{(2)} - p_2\gamma_4^{(2)} = 0.
\end{aligned}\right\} \quad (22)$$

Basing on the equation (22), it's obtained:

$$\left.\begin{aligned} \gamma_1^{(2)} = 1,\ \gamma_2^{(2)} = \frac{a_{11}-p_2}{a_{12}} - \frac{a_{14}a_{41}}{a_{12}p_2}, \\ \gamma_3^{(2)} = 0,\ \gamma_4^{(2)} = \frac{a_{41}}{p_2}. \end{aligned}\right\} \quad (23)$$

3. P = P_3

$$\left.\begin{aligned} (a_{11}-p_3)\gamma_1^{(3)} - a_{12}\gamma_2^{(3)} - a_{14}\gamma_4^{(3)} = 0, \\ -a_{21}\gamma_1^{(3)} + (a_{22}-p_3)\gamma_2^{(3)} - a_{23}\gamma_3^{(2)} = 0, \\ -p_3\gamma_3^{(3)} = 0,\ a_{41}\gamma_1^{(3)} - p_3\gamma_4^{(3)} = 0. \end{aligned}\right\} \quad (24)$$

From that, there'll be:

$$\left.\begin{aligned} \gamma_1^{(3)} = 1,\ \gamma_2^{(3)} = \frac{a_{11}-p_3}{a_{12}} - \frac{a_{14}a_{41}}{a_{12}p_3}, \\ \gamma_3^{(3)} = 0,\ \gamma_4^{(3)} = a_{41}/p_3. \end{aligned}\right\} \quad (25)$$

4. P = P_4

$$\left.\begin{aligned} (a_{11}-p_4)\gamma_1^{(4)} - a_{12}\gamma_{12}^{(4)} - a_{14}\gamma_4^{(4)} = 0, \\ -a_{21}\gamma_1^{(4)} + (a_{22}-p_4)\gamma_3^{(4)} - a_{23}\gamma_3^{(4)} = 0, \\ -p_4\gamma_3^{(4)} = 0,\ a_{41}\gamma_1^{(4)} - p_4\gamma_4^{(4)} = 0. \end{aligned}\right\} \quad (26)$$

Basing on equation (26), it's obtained:

$$\left.\begin{aligned} \gamma_1^{(4)} = 1,\ \gamma_2^{(4)} = \frac{a_{11}-p_4}{a_{12}} - \frac{a_{14}a_{41}}{a_{12}p_4}, \\ \gamma_3^{(4)} = 0,\ \gamma_4^{(4)} = a_{41}/p_4. \end{aligned}\right\} \quad (27)$$

Analyzing of solutions of typical equation (18), there is:

$$p(p^3 - q_1p^2 + q_2p - q_3) = 0$$

Substituting the below - mentioned into the given equation:

$$p = y + q_1/3$$

It's obtained the following equation:

$$y^3 + by + c = 0 \quad (28)$$

Where:

$$b = q_2 - \frac{q_1^2}{3},\ c = \frac{9q_2q_1 - 2q_1^3}{27} - q_3$$

The equation (28) will be solved by Cardano formula:

$$y = \sqrt{-\frac{c}{2} + \sqrt{\left(\frac{c}{2}\right)^2 + \left(\frac{b}{3}\right)^3}} + \sqrt{-\frac{c}{2} - \sqrt{\left(\frac{c}{2}\right)^2 + \left(\frac{b}{3}\right)^3}}. \quad (29)$$

Understanding that each of three roots:

$$\delta = \sqrt{-\frac{c}{2} + \sqrt{\left(\frac{c}{2}\right)^2 + \left(\frac{b}{3}\right)^3}}$$

It should be chosen one value of solution:

$$\eta = \sqrt{-\frac{c}{2} - \sqrt{\left(\frac{c}{2}\right)^2 + \left(\frac{b}{3}\right)^3}}$$

In which, the following condition should be carried out:

$$\delta\eta = -\frac{b}{3}$$

On the basis of that condition, it can be written the root of the typical equation as following:

$$\left.\begin{aligned} p_2 &= (\delta_1 + \eta_1) - \frac{q_1}{3}, \\ p_3 &= -\frac{1}{2}(\delta_1 + \eta_1) + i\frac{\sqrt{3}}{2}(\delta_1 - \eta_1) - \frac{q_1}{3}, \\ p_4 &= -\frac{1}{2}(\delta_1 + \eta_1) - i\frac{\sqrt{3}}{2}(\delta_1 - \eta_1) - \frac{q_1}{3} \end{aligned}\right\} \quad (30)$$

It is clarified a matter of the obtained structure of control.

$$\frac{\partial H}{\partial h} = \frac{K_g^h}{T_g}\alpha_2 + K_g^h K_g^\omega \omega \alpha_3$$

From there, there will be:

$$\left.\begin{aligned} h = h_{max}\ \text{khi}\ \frac{K_g^h}{T_g}\alpha_2 + K_g^h K_g^\omega \omega\alpha_3 > 0, \\ h = 0\ \text{khi}\ \frac{K_g^h}{T_g}\alpha_2 + K_g^h K_g^\omega \omega\alpha_3 < 0. \end{aligned}\right\} \quad (31)$$

On the basis of symbol (27), roots of equations (20), (21), (22), (23), (24), (25), (26), (27) and the expression solutions of the typical equation, it can be affirmed that:

$$\left.\begin{aligned} & p_2 = p_2[A, \frac{d\psi}{ds}];\quad p_3 = p_3[A, \frac{d\psi}{ds}]; \\ & p_4 = p_4[A, \frac{d\psi}{ds}];\quad \gamma_2^{(2)} = \gamma_2^{(2)}[A, \frac{d\psi}{ds}]; \\ & \gamma_4^{(2)} = \gamma_4^{(2)}[A, \frac{d\psi}{ds}];\quad \gamma_2^{(3)} = \gamma_2^{(3)}[A, \frac{d\psi}{ds}]; \\ & \gamma_4^{(3)} = \gamma_4^{(3)}[A, \frac{d\psi}{ds}];\quad \gamma_2^{(4)} = \gamma_2^{(4)}[A, \frac{d\psi}{ds}]; \\ & \gamma_4^{(4)} = \gamma_4^{(4)}[A, \frac{d\psi}{ds}]. \end{aligned}\right\} \quad (32)$$

In the function (32), A is vector of parameters of equation (16). The given expressions will be right for the constant case of value A and $d\psi/ds$ (method of freezing factor) [16, 17, 18 and 21].

It is necessary to find p_j and γ_i in correspondence with the new values of A and $d\psi/ds$, and define the subsequent roots of the differential equation.

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On the basis of (20÷27) and (32),it can be written:

$$\left.\begin{aligned} \alpha_2 &= c_1 + c_2\left\{\gamma_2^{(2)}\left[A,\frac{d\psi}{ds}\right]\right\}\exp\left\{p_2\left[A,\frac{d\psi}{ds}\right]\right\}+\\ &+c_3\left\{\gamma_2^{(3)}\left[A,\frac{d\psi}{ds}\right]\right\}\exp\left\{p_3\left[A,\frac{d\psi}{ds}\right]\right\}t+\\ &+c_4\left\{\gamma_2^{(4)}\left[A,\frac{d\psi}{ds}\right]\right\}\exp\left\{p_4\left[A,\frac{d\psi}{ds}\right]\right\}t,\\ \alpha_3 &= c_2\frac{1}{T_g K_g^{\omega}} \end{aligned}\right\} \qquad (33)$$

The integral constants of the given case are defined in correspondence with the obtained edge conditions (9).

The above problem of the optimal control when the external environment is function of way is required for the practical implementation of the algorithm found by measuring the magnitude $\frac{d\psi}{ds}$ and hence the value $\psi=\psi(s)$. As a rule, this information is available especially on canals and rivers. The main difficulty is to find the ways of formalizing this information. Such methods must be simple in structure, and at the same time provide a minimum amount of information loss in finding the controls. For these purposes, it may be proposed a method described in previously.

It's analyzed the possibility of existing of special control in the systems (2) and (6) and transformed the Hamiltonian (5):

$$H=\left[-\frac{1}{T_c}v\alpha_1+\frac{K_c^{\omega}}{T_c}\omega\alpha_1-\frac{K_c^{\psi}}{T_c}\psi(s)\alpha_1+ +\frac{K_g^{v}}{T_g}\alpha_2+v\alpha_4\right]+\left[\frac{K_g^{h}}{T_g}\alpha_2+K_g^{\omega}K_g^{h}\omega\alpha_3\right]h \qquad (34)$$

It's marked:

$$\left.\begin{aligned} H_0 &= -\frac{1}{T_c}v\alpha_1+\frac{K_c^{\omega}}{T_c}\omega\alpha_1- -\frac{K_c^{\psi}}{T_c}\psi(s)\alpha_1+\frac{K_g^{v}}{T_g}\alpha_2+v\alpha_4,\\ H_1 &= \frac{K_g^{h}}{T_g}\alpha_2+K_g^{\omega}K_g^{h}\omega\alpha_3. \end{aligned}\right\} \qquad (35)$$

In correspondence with the result in [5, 20], it is found:

$$\frac{d}{dt}H_1=\frac{K_g^{h}}{T_g}\frac{d\alpha_2}{dt}+K_g^{\omega}K_g^{h}\frac{d\omega}{dt}\alpha_3. \qquad (36)$$

The given expression is obtained from the condition $\alpha_3=\text{const}$. The special control may be existed in (35) only if the derivative H_1 is even order, so it can be found:

$$\frac{d^2}{dt^2}H_1=\frac{K_g^{h}}{T_g}\frac{d^2\alpha_2}{dt^2}+K_g^{\omega}K_g^{h}\alpha_3\frac{d^2\omega}{dt^2}=0. \qquad (37)$$

In which:

$$\left.\begin{aligned} \frac{d^2\alpha_2}{dt^2} &= -\frac{K_c^{\omega}}{T_c}\frac{d\alpha_1}{dt}+\frac{1}{T_g}\frac{d\alpha_2}{dt}-K_g^{\omega}K_g^{h}\alpha_3\frac{dh}{dt},\\ \frac{d^2\omega}{dt^2} &= -\frac{1}{T_g}\frac{d\omega}{dt}+\frac{K_g^{h}}{T_g}\frac{dh}{dt}+\frac{K_g^{v}}{T_g}v. \end{aligned}\right\} \qquad (38)$$

Substituting (38), respectively, (2) and (6), it's obtained:

$$\left.\begin{aligned} \frac{d^2\alpha_2}{dt^2} &= -\frac{K_c^{\omega}}{T_c}\left(\frac{1}{T_c}\alpha_1-\frac{K_g^{v}}{T_g}\alpha_2-\alpha_4\right)+ +\frac{1}{T_g}\left(\frac{K_c^{\omega}}{T_c}\alpha_1+\frac{1}{T_g}\alpha_2-K_g^{\omega}K_g^{h}h\alpha_3\right)- -K_g^{\omega}K_g^{h}\alpha_3\frac{dh}{dt},\\ \frac{d^2\omega}{dt^2} &= -\frac{1}{T_g}\left(-\frac{1}{T_g}\omega+\frac{K_g^{h}}{T_g}h+\frac{K_g^{v}}{T_g}v\right)+ +\frac{K_g^{h}}{T_g}\frac{dh}{dt}+\frac{K_g^{v}}{T_g}v. \end{aligned}\right\} \qquad (39)$$

On the basis of (39), it can be rewritten (37) in the developed form, as following:

$$\begin{aligned} \frac{d^2}{dt^2}H_1 &= b_1^H\alpha_1+b_2^H\alpha_2+b_3^H\alpha_4- \\ &-b_4^H h+c_1^H v+c_2^H\omega-c_3^H h=0. \end{aligned} \qquad (40)$$

In which:

$$\left.\begin{aligned} & b_1^H=\frac{K_c^{\omega}K_g^{h}}{T_g T_c^2}-\frac{K_c^{\omega}K_g^{h}}{T_g^2 T_c},\quad b_2^H=\frac{K_g^{h}}{T_g^3}+ +\frac{K_g^{v}K_c^{\omega}K_g^{h}}{T_g^2 T_c},\; b_3^H=\frac{K_c^{\omega}K_g^{h}}{T_g T_c},\\ & b_4^H=K_g^{\omega}K_g^{h}\alpha_3\frac{K_g^{h}}{T_g^2},\\ & c_1^H=-K_g^{\omega}K_g^{h}\alpha_3\frac{K_g^{h}}{T_g^2}+K_g^{\omega}K_g^{h}\alpha_3\frac{K_g^{v}}{T_g},\\ & c_2^H=K_g^{\omega}K_g^{h}\alpha_3\frac{1}{T_g^2},\quad c_3^H=K_g^{\omega}K_g^{h}\alpha_3\frac{K_g^{h}}{T_g^2}. \end{aligned}\right\} \qquad (41)$$

From the expression (40), it is found the special control:

$$h=\frac{b_1^H\alpha_1+b_2^H\alpha_2+b_3^H\alpha_4+c_1^Hv+c_2^H}{b_4^H+c_3^H} \quad (42)$$

It's rechecked the optimum of the special control (42) under the G. Kelly's condition [5, 20] as following:

$$\frac{\partial}{\partial h}\frac{d^2}{dt^2}H_1=-(b_4^H+c_3^H)$$

In correspondence with the conditions of transversal action (7), (9), and equation (6), as well as the signs inserted into (41), it's obtained:

$$\alpha_3=-1$$

and

$$b_4^H<0, c_3^H<0,$$

therefore:

$$\frac{\partial}{\partial h}\frac{d^2}{dt^2}H_1>0 \quad (43)$$

The G. Kelly's condition is satisfied and the special controls are optimal.

Now, it's re-examined the answer of the given problem with the less dimension of the model of the mobile system. This less dimension is carried out by excluding of the diesel equation from the equations (32). The problem setting is done as same as [9, 10, 12 and 13], the differential equations are following:

$$\left.\begin{aligned}&\frac{dv}{dt}=-\frac{1}{T_c}v+\frac{K_c^\omega}{T_c}\omega-\frac{K_c^\psi}{T_c}\psi(s),\\&\frac{dG_m}{dt}=K_g^\omega\omega^2,\\&\frac{ds}{dt}=v.\end{aligned}\right\} \quad (44)$$

The limitation that is necessarily imposed for the control action (Frequency of diesel rotation) will be:

$$0\le\omega\le\omega_{max}$$

It's necessarily found the control law in dynamics $\omega=\omega(t)$ to ensure that at the interval T, the given movement time is reached to minimum for the function:

$$\Delta G_m=G_m(v,s) \quad (45)$$

The problem is defined under Maier's condition and solved by the maximum principle [18, 22 and 23]. The edge condition is rewritten as following:

At the left end:

$$t=0, v_0=\omega_0=G_{T_0}=s_0=0$$

At the rights end:

$$t=T, v_T, \omega_T \text{ at free } s=s_T$$

The Hamiltonian of the equations (44) is:

$$H=\left[-\frac{1}{T_c}v+\frac{K_c^\omega}{T_c}\omega-\frac{K_c^\psi}{T_c}\psi(s)\right]\alpha_1+ +\left[K_g^\omega\omega^2\right]\alpha_2+v\alpha_3. \quad (46)$$

The transversal conditions are:

$$[(1+\alpha_2)\delta G_m-c\delta t+\alpha_1\delta v+\delta s\alpha_3]_0^T=0 \quad (47)$$

The considered problem is 1st order integral:

$$\left[-\frac{1}{T_c}v+\frac{K_c^\omega}{T_c}\omega-\frac{K_c^\psi}{T_c}\psi(s)\right]\alpha_1+ +K_g^\omega\omega^2\alpha_2+\alpha_3v=K=0.$$

The equality (47) can be only existed at the arbitrary selection of variation of $\delta G_m,\delta_v$,when

$$\alpha_{1T}=0, \alpha_{2T}=-1, \alpha_{3T}=0.$$

The structure of control is obtained as following:

$$\frac{\partial H}{\partial\omega}=\frac{K_c^\omega}{T_c}\alpha_1+2K_g^\omega\omega\alpha_2=0 \quad (48)$$

$$\left.\begin{aligned}&\omega=\omega_{max} \text{ when } \frac{K_c^\omega}{T_c}\alpha_1+2K_g^\omega\omega\alpha_2>0,\\&\omega=0 \text{ when } \frac{K_c^\omega}{T_c}\alpha_1+2K_g^\omega\omega\alpha_2<0.\end{aligned}\right\} \quad (49)$$

The equations for finding the vector α are:

$$\left.\begin{aligned}&\frac{d\alpha_1}{dt}=\frac{1}{T_c}\alpha_1-\alpha_3,\\&\frac{d\alpha_2}{dt}=0,\\&\frac{d\alpha_3}{dt}=\frac{K_c^\psi}{T_c}\frac{d\psi(s)}{ds}\alpha_1\end{aligned}\right\} \quad (50)$$

Understanding that $\alpha_2=c_c^\alpha$, in correspondence with the transversal condition, there's $c_2^\alpha=-1$.The 2nd equation doesn't relate to the remained tasks, thence it's found the solution of equation (50). Excluding α_3 from equation (50), it's obtained:

$$\frac{d^2\alpha_1}{dt^2}-a_1^\alpha\frac{d\alpha_1}{dt}+a_2^\alpha\alpha_1=0 \quad (51)$$

In which:

$$a_1^\alpha=\frac{1}{T_c}, \quad a_2^\alpha=\frac{K_c^\psi}{T_c}\frac{d\psi(s)}{ds}.$$

The typical equation will be:

$$p^2-a_1^\alpha p+a_2^\alpha=0 \quad (52)$$

And it's obtained the solution as form:

$$p_{1,2}={a_1^\alpha}/{2}\pm\sqrt{(a_1^\alpha)^2-a_2^\alpha}$$

For the ship complex, the below-expression is always right [24, 25 and 26]:

$$\left(a_1^{\alpha}\right)^2 << a_2^{\alpha}$$

Therefore the solutions of the equation (52) will be synchronization with the real positive part. On that basis, the quantity $\alpha_1 = \alpha_1(t)$ will be changed the sign for one more time. In order to find the analytic expression for the commutative function (49):

$$\alpha_1 = c_1^{\alpha} e^{p_1 t} + c_2^{\alpha} e^{p_2 t} \quad (53)$$

It's defined c_1^{α} and c_2^{α} from the transversal condition $\alpha_{1T} = 0$ and from the 1st order integral of the problem. The 1st order integral at all the control interval 0 ÷ T when t = T is defined that is equal 0.Applying the edge condition at the left end and $\omega = \omega_{max}$, it can be written the integral as following:

$$-\frac{K_c^{\omega}}{T_c}\psi(s)\alpha_{10} + \alpha_{10}\frac{K_c^{\omega}}{T_c}\omega_{max} + K_g^{\omega}\omega_{max}^2\alpha_{20} = 0 \quad (54)$$

Or with the condition at the interval 0 ÷ T, α_2 = constant = -1, it's obtained:

$$\alpha_{10}\left(\frac{K_c^{\omega}}{T_c}\omega_{max} - \frac{K_c^{\psi}}{T_c}\psi(s)\right) = K_g^{\omega}\omega_{max}^2 \quad (55)$$

$$\alpha_{10} = \frac{T_c K_g^{\omega}\omega_{max}^2}{K_c^{\omega}\omega_{max} - K_c^{\psi}\psi(s)} \quad (56)$$

At time t = 0, there is the algebraic equation as:

$$c_1^{\alpha} + c_2^{\alpha} = 0$$

And at time t = T, there is:

$$c_1^{\alpha} e^{p_1 t} + c_2^{\alpha} e^{p_2 t} = 0$$

Therefore, in order to define c_1^{α} and c_2^{α}, it should be necessarily used the following set of equations:

$$\left.\begin{array}{l} c_1^{\alpha} + c_2^{\alpha} = \alpha_{10} \\ c_1^{\alpha} e^{p_1 t} + c_2^{\alpha} e^{p_2 t} = 0. \end{array}\right\} \quad (57)$$

Those constant quantities are:

$$\left.\begin{array}{l} c_1^{\alpha} = \left(1 - \dfrac{e^{p_1 T}}{e^{p_1 T} - e^{p_2 T}}\right)\alpha_{10}, \\ c_2^{\alpha} = \dfrac{e^{p_1 T}}{e^{p_1 T} - e^{p_2 T}}\alpha_{10}. \end{array}\right\} \quad (58)$$

The given problem has the analytic solution. In order to analyze the particular navigational condition, it should be known the function $\psi = \psi(s)$.

Now, it will be examined the appearance possibility of the special control in the set of equation (44), it is shown the Hamiltonian as following [27, 28 and 29]:

$$\begin{array}{l} H = \left[-\dfrac{1}{T_c}v\alpha_1 - \dfrac{K_c^{\psi}}{T_c}\psi(s)\alpha_1 + v\alpha_3\right] + \\ +\omega\left[\dfrac{K_c^{\omega}}{T_c}\alpha_1 + K_g^{\omega}\omega\alpha_2\right] \end{array} \quad (59)$$

It's marked:

$$\left.\begin{array}{l} H_0 = -\dfrac{1}{T_c}v\alpha_1 - \dfrac{K_c^{\psi}}{T_c}\psi(s)\alpha_1 + v\alpha_3, \\ H_1 = \dfrac{K_c^{\omega}}{T_c}\alpha_1 + K_g^{\omega}\omega\alpha_2. \end{array}\right\} \quad (60)$$

Because of $\alpha_2 = -1$ at the interval $0 \div T$, so there is:

$$H_1 = \frac{K_c^{\omega}}{T_c}\alpha_1 - K_g^{\omega}\omega. \quad (61)$$

It's found:

$$\frac{d}{dt}H_1 = \frac{K_c^{\omega}}{T_c}\frac{d\alpha_1}{dt} - K_g^{\omega}\frac{d\omega}{dt} \quad (62)$$

And

$$\frac{d^2}{dt^2}H_1 = \frac{K_c^{\omega}}{T_c}\frac{d^2\alpha_1}{dt^2} - K_g^{\omega}\frac{d^2\omega}{dt^2} = 0 \quad (63)$$

Applying the equation (53), it's found:

$$\frac{d^2\alpha_1}{dt^2} = c_1^{\alpha}p_1^2 e^{p_1 t} + c_2^{\alpha}p_2^2 e^{p_2 t}$$

From the equation (63), there is:

$$\frac{d^2\omega}{dt^2} = \frac{K_c^{\omega}c_1^{\alpha}p_1^2}{T_c K_g^{\omega}}e^{p_1 t} + \frac{K_c^{\omega}c_2^{\alpha}p_2^2}{T_c K_g^{\omega}}e^{p_2 t} \quad (64)$$

It's integrated respectively the equations (64), it's obtained the special controls:

$$\left.\begin{array}{l} \dfrac{d\omega}{dt} = \dfrac{K_c^{\omega}c_1^{\alpha}p_1}{T_c K_g^{\omega}}e^{p_1 t} + \dfrac{K_c^{\omega}c_2^{\alpha}p_2}{T_c K_g^{\omega}}e^{p_2 t} + c_3, \\ \omega = \dfrac{K_c^{\omega}c_1^{\alpha}}{T_c K_g^{\omega}}e^{p_1 t} + \dfrac{K_c^{\omega}c_2^{\alpha}p_2}{T_c K_g^{\omega}}e^{p_2 t} + c_3 t + c_4 \end{array}\right\} \quad (65)$$

Now, it is re-examined the optimum of the special control, particularly:

$$\frac{d^2}{dt^2}\frac{\partial H}{\partial\omega} = \frac{K_c^{\omega}}{T_c}\frac{d^2\alpha_1}{dt^2} - K_g^{\omega}\frac{d^2\omega}{dt^2} = 0$$

And the G. Kelly's condition is:

$$\frac{\partial}{\partial\omega}\frac{d^2}{dt^2}H_1 = 0 \quad (66)$$

And it's seen that the special control is optimal.

IV. Discussion

The above problem of the optimal control when the external environment is function of way is required for the practical implementation of the algorithm found by measuring the magnitude $\frac{d\psi}{ds}$ and hence the value $\psi = \psi(s)$. As a rule, this information is available especially on canals and rivers. The main difficulty is to find the ways of formalizing this information. Such methods must be simple in structure, and at the same time provide a minimum amount of information loss in finding the controls. For these purposes, it may be proposed a method described in previously.

V. Conclusion

The research is obtained the results:

It's proposed the establishing method of extremum principle control on the basis of the selection of the transversal condition that helps us to obtain not only the quality solutions but also the quantitative solution.

It's obtained the control algorithms of engine system that allow the following vessel approaching to the leader ship.

It's researched the programs of control for the steering complex that ensures the meeting movement of ships.

It's obtained the programs of control for engine system and steering complex that is solved the problem of head-on navigation in the confined water.

It's established the programs of control for engine system when the parameters of external environment are varied as function of time, way, and the parameters of ship's sailing are nonlinear variation.

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