Simulation of the Stochastic Resonance Effect in a Nonlinear Device

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Abstract- The possibility to separate a useful signal from the realization of the random process representing the sum of the harmonic signal and Gaussian noise, using the method of stochastic resonance, is shown. The results of calculation of the signal-to-noise ratio at the output of a nonlinear device, creating an effect of stochastic resonance, are given. It is shown that the nonlinear device, described by the equation of stochastic resonance, operates as a stochastic low-pass filter. A simulation model of a non-linear device possessing effect of SR is constructed.

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I. Introduction

It is considered that noise in a system is a negative factor and the fight against noise is one of actual problems of radio engineering systems. Low-noise devices and methods of noise reduction are developed, noiseproof codes, digital communication, signals with the necessary correlation properties are created.

However, research conducted recently in the field of theoretical and experimental physics has shown that in some cases an input weak signal can be amplified and optimized with the assistance of noise (Anishchenko et al., 1999; Geraschenko, 2003). The integral characteristics of the process at the system output, such as the spectral power amplification (SPA), the signal-to-noise ratio (SNR) have a well-marked maximum at a certain optimal noise level.

The notion of stochastic resonance (SR) determines a group of phenomena wherein the response of a nonlinear system to a weak input signal can be significantly increased by appropriate tuning of the noise intensity. SR refers to a generic physical phenomenon typical for nonlinear systems.

This article discusses the simulation of the effect of SR in the case of additive sum of a harmonic signal and Gaussian noise at the nonlinear device input.

II. Characterization of Stochastic Resonance

A weak input signal significantly increases with increasing intensity of noise and reaches its maximum at a certain noise level in nonlinear systems in which SR occurs.

Consider the equation describing the model of nonlinear systems in which SR occurs (Anishchenko et al., 1999; Kharchenko et al., 2015):

\[ \frac{d\eta}{dt} = \eta(t) - \eta^3(t) + x(t), \]

where \( x(t) \) – input signal; \( \eta(t) \) – output signal.

This formula is Abel’s equation of the first kind and has no analytical solution (Kamke, 1961). It is also impossible to find two-dimensional probability density of the output signal by using the exact solution of the Fokker–Planck equation even in the absence of an input harmonic signal (Middleton, 1996).

Therefore, the signal correlation functions and spectral density at the nonlinear system output can’t be defined exactly. Naturally, there are additional difficulties in the analytical description of actual effects, if there is an additive sum of harmonic signal and Gaussian noise.

III. Signal-to-Noise Ratio at the Nonlinear System Output

Unlike linear systems, in which the energy spectrum at the output follows input energy, output spectrum of the nonlinear system has a more complicated structure (Levin, 1969, Volochuk, 2005). Signal and noise are independent in all near system. The output of the nonlinear device forms new spectral components due to the interaction of the components of the input process.

Moreover, the type of non-linear transformation and the statistical characteristics of the input signal determine the type and intensity of the additional component.

If the input process is an additive sum of the unmodulated carrier and noise, there are three main parts in the output power spectrum of the nonlinear device:

\[ F_{SxS}(\omega) \] – corresponds to the beats between the components of the signal and its harmonics (a discrete part of the spectrum);

\[ F_{NxN}(\omega) \] – is formed by beats of noise components (continuous component of the spectrum);
$F_{SxN}(\omega)$ – is formed by mutual beats of signal components and noise (continuous component of the spectrum).

The discrete part of the spectrum is complemented by the spectral lines at zero frequency, representing the DC component at the output, which is also determined by the beats of the signal components and noise. Consequently, the energy spectrum of the output of the nonlinear device is determined as (Levin, 1969, Voloshchuk, 2005):

$$F(\omega) = F_{SxS}(\omega) + F_{SxN}(\omega) + F_{NxN}(\omega).$$

Practically the most convenient power indicator of the output signal is the signal–to–noise ratio (SNR). Since the output process is an inseparable mixture of an input signal and noise, it is impossible to specify components, which would depend only on the signal and, accordingly, only on noise.

In order to evaluate the SNR at the output of the nonlinear system, it is necessary to determine the portion of the spectrum $F_{SxN}(\omega)$.

We can calculate SNR at the output of the nonlinear system in two ways as:

a) if the beats between signal components and noise are attributed to noise:

$$\begin{align*}
SNR &= \frac{\int_{0}^{\infty} F_{SxS}(\omega)d\omega}{\int_{0}^{\infty} \left[ F_{SxN0}(\omega) + F_{NxN}(\omega) \right]d\omega} \\
\end{align*}$$

b) if the beats between signal components and noise are attributed to signal:

$$\begin{align*}
SNR &= \frac{\int_{0}^{\infty} \left[ F_{SxS}(\omega) + F_{SxN}(\omega) \right]d\omega}{\int_{0}^{\infty} F_{NxN}(\omega)d\omega} \\
\end{align*}$$

SNR is calculated using the last formula in case of the SR, as a high value of this parameter is predetermined by the component $F_{SxN}(\omega)$, i.e. by the interaction between signal and noise.

IV. SNR at the Output of the Nonlinear Device has the Effect of SR

Consider the case where the input signal of the nonlinear device is an additive sum of the sinusoidal signal and Gaussian noise

$$\frac{d\eta}{dt} = \eta(t) - \eta^3(t) + A\cos(2\pi f t + \varphi) + \xi(t) \quad (2)$$

where $\xi(t)$ – Gaussian noise with a zero average and variance $D$,

$\eta(t)$ – the process at the nonlinear device output.

Having solved equation (2) numerically, let's define the SNR at the output of the nonlinear device ($SNR_{output}$) as a function of frequency and SNR at the input ($SNR_{input}$). We can calculate $SNR_{input}$ as

$$SNR_{input} = \frac{A^2}{2D}.$$ 

The frequency is set in the range of 0.05÷10Hz. Power SNR at the input is considered equal, respectively: 0.005; 0.02; 0.5. The calculation results are shown in Fig. 1a and 1b. Fig. 1b shows the low frequency in more detail.
Figure 1: The SNR dependence of the output of the nonlinear device ($SNR_{output}$) on the frequency of the periodic input signal for various values of the SNR at the input ($SNR_{input}$).

The figures show that the phenomenon of SR is best expressed at low frequencies, thus a nonlinear device having the effect of SR, is a stochastic low-pass filter. In addition, there is a minimum SNR at the output at a frequency of $f = 0.2$ Hz, and this effect is observed at any SNR at the input. SNR at the output is a nonlinear function of the external noise and the input harmonic signal. You can then make three-dimensional SNR graphs of the input noise power and harmonic signal amplitude. It should be noted that the numerical simulations were performed by summing the data on limited time intervals (up to 50 periods of frequency). Naturally, the time delay affects the results of (Middleton, 1996).
V. Model of the Nonlinear Device Having the Effect of CR

Let’s create a simulation model of the nonlinear device according to the graphical programming environment SIMULINK (integrated with MATLAB), described by equation (2). This system has the SR effect. An additive mixture of the harmonic signal and Gaussian noise is sent to the input. Output signal stakes from the oscilloscope. This scheme can be the basis for practical implementation of the nonlinear filter (fig.2).

![Diagram of the simulation model of the nonlinear device.](image)

**Figure 2:** The simulation model of the nonlinear device.

Fig. 3 shows the signals taken from the oscilloscope. It can be seen clearly that the dispersion in Fig. 3a is much less than the dispersion in Fig. 3b. Thus, this model shows the increase in the SNR at the output of the nonlinear device having the effect of SR. In addition, the harmonic nature of the output signal is retained.

![Graph showing the input signal over time.](image)
The response of the nonlinear system on a weak external signal in case of SR noticeably increases with the height of the noise intensity in the system and arrives at a certain maximum at some level.

VI. Conclusions

In this paper we discuss the work of the nonlinear device having the effect of SR at the input of the additive mixture of the harmonic signal and the white noise of short duration. The results make it clear that the device works as a stochastic low-pass filter. In addition, numerical analysis of the equation describing the effect of SR showed that the SNR at the output of the nonlinear device under certain conditions exceeds the SNR at the input. Hence, the nonlinear device operates as an amplifier.

In this paper we can build a simulation model of the nonlinear device, described by equation SR. We have used an additive mixture of the harmonic signal and noise with a duration of 10 s at the input of the nonlinear device. We have used the graphical programming environment SIMULINK (integrated with MATLAB) for building a simulation model.

These results demonstrate that the signal obtained at the output of the nonlinear device has a lower noise level as compared with the input signal.

Prospects of development schemes of nonlinear filter based on the designed model are indicated.

References Références Referencias
