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Analytical Study on Motion Behavior of Non-Spherical Particles in Incompressible Fluid with Presence of Electrostatic Force

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Abstract - In this paper, the accelerated motion of non-spherical particles in an incompressible fluid in both the presence and the absence of electrostatic force was investigated. Differential transformation method (DTM) and a FORTRAN code was used to calculate the instantaneous velocity of particles. Regarding particles' instantaneous velocity in the absence of electrostatic force, DTM approach was resulted in a proper accordance with previous studies which utilized variational iteration method (VIM). In addition, a good agreement between DTM and VMT was seen as sphericity of particles was varied from 0.5 to 0.9. The results showed that falling velocity increased with increasing sphericity. Moreover, the presence of electrostatic force (by assuming the electrical load equal to 1 micro colon) was compared to the one with no electrostatic force. The results showed that the falling velocity was decreased by 23.33% under the effect of an additional resistant force.

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Abstract- In this paper, the accelerated motion of non-spherical particles in an incompressible fluid in both the presence and the absence of electrostatic force was investigated. Differential transformation method (DTM) and a FORTRAN code was used to calculate the instantaneous velocity of particles. Regarding particles' instantaneous velocity in the absence of electrostatic force, DTM approach was resulted in a proper accordance with previous studies which utilized variational iteration method (VIM). In addition, a good agreement between DTM and VMT was seen as sphericity of particles was varied from 0.5 to 0.9. The results showed that falling velocity increased with increasing sphericity. Moreover, the presence of electrostatic force (by assuming the electrical load equal to 1 micro colon) was compared to the one with no electrostatic force. The results showed that the falling velocity was decreased by 23.33% under the effect of an additional resistant force.

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I. Introduction

Sedimentation of solid or liquid particles in fluids occurs in the different natural and artificial phenomena. Different researches were studied the behavior of dispersed particles in an incompressible media, in which most of the mutilized Eulerian-Eulerian or Eulerian-Lagrangian approaches. When using an Eulerian-Lagrangian approach for two-phases fluids, the continuity and momentum balance equations must be derived from the hydrodynamics of the continuous phase. However, each encapsulated particle is considered as a main mass for which velocity and state are derived from Newton's second law as follows

$$m_i \ \frac{du_i}{dt} = F_i \ , \ \frac{dx_i}{dt} = u_i \qquad (1)$$

wherem_i is particle mass,$u$is velocity, $t$is time, x is original analytical function, andF_i is the resultant of forces that are appliedtothe particle, including gravity, buoyancy, drag, virtual mass force, the Basset force, and lift force.

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Equation (1) has been solved numerically in different studies by various methods, for example the finite difference method [1]. Some analytical methods which were applied for analysis of the acceleration of spherical and non-spherical particles motion in Newtonian fluids was addressed in[2-10]. Jalaal and Ganji[3]studied spherical and non-spherical particles motion in unsteady state at Newtonian media. They used friction coefficient governed by Chhabra and Ferreira's equations[11]for a range of Reynolds numbers by using the homotopy perturbation method (HPM). Jalaal et al. [6]studied non-spherical particles motion at the Newtonian media using the VIM and friction equations derived by Chien[12]. In another researches, Jalaal et al.[4] and Jalaal and Ganji[5]studied non-spherical particles by using the HPM.

Stokes [13]assigned following equation for drag coefficient of a sedimenting particle. The equation is derived for a flow field that is totally dominated by viscous diffusion as below

$$C_D = \frac{24}{Re} \qquad (2)$$

where C_D is drag coefficient and Re is Reynolds number. This equation denied the effect of inertia and is accepted for$Re < 0.4$. Therefore, Oseen[14], assuming the effect of inertia, completed the Stokes's equation as follow

$$C_D = \frac{24}{Re}\left(1 + \frac{3}{16} Re\right) \qquad (3)$$

Most of previous researches on spherical and non-spherical particles carried out experimentally and only a few of them were analytically investigated the solution of motion equations. In this regard, Proudman and Pearson [15]proposed$Re^2 \times \ln(Re)$ parameter to consider the behavior of drag coefficient for spherical particles and then Sano [16]completed aforementioned equation. Lovalenti and Brady [17] used the Kim and Karilla's equations [18]to solve directly the behavior of applied forces on particles and derived different parameters of time reduction, Includingt^{-1}, $t^{-5/2}$,e^{-t}, $t^{-2}.e^{-t}$,$t^{-1/2}$, and t^{-2}, depending on initial conditions. Ferreira and Chhabra[11]achieved the following equation for $0 \leq Re \leq 10^5$

$$C_D = \frac{24}{Re}\left(1+\frac{1}{48}Re\right) \quad (4)$$

Equation (4) showed a suitable accordance with results of experimental researches. Also, in many cases, a linear equation can describe the drag force very well. In some cases, Reynolds number has an average value and the liner and exponential terms are used in[11, 12, 19-22].

Regarding non-spherical particles, Haider and Levenspiel[23]proposed an accurate equation in order to contemplate drag force taking into account of Reynolds number and sphericity. Rewriting the force balance and assuming $\rho << \rho_s$, the equation (1) can be implemented as

$$\mathrm{m}\frac{du}{dt} = mg\left(1-\frac{\rho}{\rho_s}\right) - \frac{1}{8}\pi D^2 \rho C_D u^2 - \frac{1}{12}\pi D^3 \rho \frac{du}{dt} \quad (5)$$

where$g$ is acceleration due to gravity, ρ is fluid density, ρ_s is particle density and D is particle's equivalent diameterThe main difficulty in solving equation (5) is the non-linear part appeared to express drag force.By insertingequation (4) in equation (5) and rewriting the equation (5), the equation (6) is derived as follow

$$a\frac{du}{dt} + bu + cu^2 - d = 0 \ , u(0) = 0 \quad (6)$$

Where a, b, c, and d are constants and depend on physical condition of the system.

Different methods were proposed in previous studies to solve equation (6). Among those, VIM, a technique based on repeated integration proposed by He [24], was used successfully to solve linear and non-linear equations by different authors. This method is based on the true function by a general Lagrangian multiplier. DTM is another method to solve the linear and non-linear terms proposed by Yaghoobi and Torabi[8]. Noorpoor and Nazari [25] used DTM to calculate the falling velocity of a particle in the acoustic field.

II. Materials and Methods

In this paper, equation (5) was solved by using the Yaghoobi and Torabi's equations[8]. The solving procedure included applying DTM by using a FORTRAN code.

By combining equation (5), Reynolds number ($Re = \frac{\rho u D}{\mu}$, μ is dynamic viscosity), and drag coefficient equation, the final equation of a falling acceleration of non-spherical particle motion in an incompressible fluid was written as follow

$$\left(m+\frac{1}{12}\pi D^3\rho\right)\frac{du}{dt} + 3.75\pi D\mu u + \frac{67.289e^{(-5.03\emptyset)}}{8}\pi D^2\rho u^2 - mg\left(1-\frac{\rho}{\rho_s}\right) = 0 \quad (7)$$

where $\emptyset$ is sphericity of particle. For simplicity, this equation's coefficients were assigned as a, b, c and d in further equations. First term of equation (7)is to describe the added mass to a falling non-spherical particle in an incompressible fluid. The second and third term show the resistant force of particle's linear and non-linear motion respectively, and the fourth shows the applied gravity and buoyancy forces on the falling non-spherical particle in an incompressible fluid.

$$a = (m + \frac{1}{12}\pi D^3 \rho) \quad (8)$$

$$b = 3.75\pi D\mu \quad (9)$$

$$c = \frac{67.289e^{(-5.03\emptyset)}}{8}\pi D^2 \rho \quad (10)$$

$$d = mg\left(1-\frac{\rho}{\rho_s}\right) \quad (11)$$

So, equation (7) was rewritten as follow

$$a\frac{du}{dt} + bu + cu^2 - d = 0, u(0) = 0 \quad (12)$$

According to DTM conversion functions, equation (12) is provided as following transformed equation

$$a(k+1)U(k+1) + bU(k) + c\left(\sum_{l=0}^{k} U(l)U(k-1)\right) - d \times \delta(k) = 0 \quad (13)$$

The value of $u(0)$ is equal to zero. Other value of $u(k)$ for k=1, 2, 3 … are governed by above equation and can be calculated as bellow.

$$U(1) = \frac{d}{a} \quad (14)$$

$$U(2) = -\frac{1}{2}\times\frac{bd}{a^2} \quad (15)$$

$$U(3) = \frac{1}{3!}\times\frac{d(b^2-2cd)}{a^3} \quad (16)$$

$$U(4) = -\frac{1}{4!}\times\frac{bd(b^2-8cd)}{a^4} \quad (17)$$

$$U(5) = \frac{1}{5!}\times\frac{d(b^4-22b^2cd+16c^2d^2)}{a^5} \quad (18)$$

$$U(6) = -\frac{1}{6!}\times\frac{bd(b^4-52b^2cd+136c^2d^2)}{a^6} \quad (19)$$

$$U(7) = \frac{1}{7!}\times\frac{d(b^6-114b^4cd+720b^2c^2d^2-272c^3d^3)}{a^7} \quad (20)$$

$$U(8) = -\frac{1}{8!}\times\frac{bd(b^6-240b^4cd+3072b^2c^2d^2-3968c^3d^3)}{a^8} \quad (21)$$

To calculate the velocity at each moment following equation is using

$$U_i(t) = \sum_{k=0}^{n}\left(\frac{t}{H_i}\right)^k U_i(k),\ 0 \le t \le H_i \quad (22)$$

By inserting equations (14) and (21) in equation (22), final equation of estimating the fall velocity of non-spherical particles in an incompressible fluid is derived as

$$U_i(t) = U(1)t + U(2)t^2 + U(3)t^3 + U(4)t^4 + \cdots \quad (23)$$

In order to study the impact of electrostatic force, Coulomb's lawis used as follow

$$F = \frac{Q^2}{4\times\pi\times\varepsilon_0\times r^2}, \varepsilon_0 = 8.85\times 10^{-12} \quad (24)$$

Where F is electrostatic force, Q is thesigned magnitude of the charges, r is the distance between the charges, and ε_0 is vacuum permittivity coefficient.

By inserting the above equation in equation (5), the equation (25) is derived.

$$\mathrm{m}\frac{du}{dt} = mg\left(1-\frac{\rho}{\rho_s}\right) - \frac{1}{8}\pi D^2\rho C_D u^2 - \frac{1}{12}\pi D^3\rho\frac{du}{dt} + \frac{Q^2}{4\times\pi\times\varepsilon_0\times r^2} \quad (25)$$

Assuming that all varieties of equation (24) are independent of time, following equation can be written by rewriting terms of equation (25) and inserting equation (7) in (11), and a non-linear deferential equation is formed:

$$a\frac{du}{dt} + bu + cu^2 - d - e = 0 \ ,u(0) = 0 \quad (26)$$

By applying the DTM one quation (26) the following equation is derived.

$$a(k+1)U(k+1) + bU(k) + c\left(\sum_{l=0}^{k} U(l)U(k-1)\right) - (d+e)\times\delta(k) = 0 \quad (27)$$

By considering equations numbers from (13) to (27),the term (d+e) was replaced to (d) to calculate U(1) to U(8). As a result:

$$U(1) = \frac{(d+e)}{a} \quad (28)$$

$$U(2) = -\frac{1}{2}\times\frac{b(d+e)}{a^2} \quad (29)$$

$$U(3) = \frac{1}{3!}\times\frac{(d+e)(b^2-2c(d+e))}{a^3} \quad (30)$$

$$U(4) = -\frac{1}{4!}\times\frac{b(d+e)(b^2-8c(d+e))}{a^4} \quad (31)$$

$$U(5) = \frac{1}{5!}\times\frac{(d+e)(b^4-22b^2c(d+e)+16c^2(d+e)^2)}{a^5} \quad (32)$$

$$U(6) = -\frac{1}{6!}\times\frac{b(d+e)(b^4-52b^2c(d+e)+136c^2(d+e)^2)}{a^6} \quad (33)$$

$$U(7) = \frac{1}{7!}\times\frac{(d+e)(b^6-114b^4c(d+e)+720b^2c^2(d+e)^2-272c^3(d+e)^3)}{a^7} \quad (34)$$

$$U(8) = -\frac{1}{8!}\times\frac{b(d+e)(b^6-240b^4c(d+e)+3072b^2c^2(d+e)^2-3968c^3(d+e)^3)}{a^8} \quad (35)$$

Finally, using final derived equations described above, the equation (23)was solved as the general equation using a FORTRAN code.

III. Results and Discussion

The FORTRAN code is used for calculating instantaneous velocity,$U_i(t)$, by assuming sphericity, density and equivalent diameterof particle equal to 0.9, 2100$\frac{kg}{m^3}$ and 3mm respectively.

Figure 1 illustrated the changes in the instantaneous velocity for sphericity of 0.9 when the fluid was water and no electrical load was applied. The falling velocity increased with increasing time. This increasing approach followed the linear behavior at the beginning, until the velocity passed 0.1 m/sec point. However, as it can be seen, the value of velocity reached to a constant value of 0.16 m/sec after 0.07 sec.

Figure 1

Figure 2 and 3 compares instantaneous velocities in water and ethylene glycol as their densities differ. The same approach can be seen regarding the increasing of velocity with time. However, the rate of increase in values of velocity was higher for water as its density is lower than that of ethylene glycol. In addition, the rate in ethylene glycol's representative diagram was increased in nonlinear approach in contrast with the one of water's.

Figure 2

Figure 3

Figure 4 represents the changes of particle's velocity when its sphericity changes. As it can be seen, the increase in sphericity resulted in an increase in falling velocity. The trend of increasing velocity was approximately the same before the velocity hit 0.04 m/sec independent of particles' sphericity. After this velocity, however, this rate was higher for particles with higher sphericity.

Figure 4

Falling velocity of particles with lower sphericity reached a constant value in shorter times comparing the ones with lower sphericity. These results are consistent with Yaghoobi's and Torabi's [8] and Jalaal's and Ganji's [3]. An error analyzing was carried out in order to compare the results obtained in this study and Yaghoobi and Torabi's[8]; for particles with sphericity of 0.5 and in absence of electrostatic force assuming the water as the incompressible fluid used for both studies, the value of falling velocity differ 6.7%. Table 1 lists important points which can be taken form Figure 4 for falling velocities of particles with different sphericities and the trend they increased.

Table 1

In order to investigate the effects an electrostatic force on particles' sedimentation instantaneous velocity, an electrical load equal to 1 micro colon applied to a non-spherical particle with sphericity of 0.9 in water. The effects of electrostatic force in the motion behavior of particles in different distances between electric charges (0.5 m and 1.0 m) are shows in Figure 5. It can be found from this figure that applying electrostatic force reduced sedimentation velocity by 23.33% due to the formation of a resistant force between particles. This reduction was more pronounced for shorter distance between the charges compared to the one with longer distance. Also, it can be seen from Figure 5 that the effect of electrostatic force was approximately constant with time especially after 0.04 sec.

IV. Conclusion

The differential transformation method (DTM) was used for analytical investigation of non-spherical particles' falling velocity in an incompressible fluid in both the presence and the absence of electrostatic force. Results of this study indicated that, in general, the increasing curve of the sedimentation of non-spherical particles in incompressible fluids mostly behave nonlinear until it reaches the constant value. In addition, the effects of changing fluid's density in sedimentation behavior of particles was investigated. The results showed the falling velocity in water (the fluid with lower density) was higher than the one for ethylene glycol (the fluid with lower density).Moreover, when the sphericity increased, the sedimentation rate increased even though this rate differed for each sphericity. As results showed, particles with sphericity of 0.9 experienced the highest falling velocity and increasing rate followed by those with sphericity of 0.7 and 0.5 respectively. After applying an electric load of 1 micro colon in with distant between charges of 0.5 m and 1.0 m, the falling velocity decreased as a resistant force was created. This decrease was more noticeable for the distance of 0.5 with decreasing about 23% in value of falling velocity.

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Tables

Table 1 : Important points in increasing behavior of falling velocity of particles with different sphericities

Sphericity	Time of linear velocity sec	Maximum linear velocity m/sec	Start of constant velocity sec	Instantaneous velocity m/sec
$\emptyset=0.5$	0.01	0.0306	0.03	0.06
$\emptyset=0.7$	0.03	0.079	0.04	0.096
$\emptyset=0.9$	0.06	0.146	0.08	0.16

Figures

Figure (1) – The changes of instantaneous velocity for sphericity of 0.9 with time in water and in absence of electrostatic force.

Figure (2) - The changes of instantaneous velocity for sphericity of 0.9 with time in water and ethylene glycol and in absence of electrostatic force.

Figure (3) - Positions of falling particle for sphericity of 0.9 in in water and ethylene glycol and in absence of electrostatic force.

Figure (4) - The changes of instantaneous velocity for sphericity of 0.9, 0.7 and 0.5 with time in water and in absence of electrostatic force.

Figure (5) – The changes of instantaneous velocity for sphericity of 0.9with time in water and in presence of electrostatic force with distance of charges of 1 m and 0.5 m.

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