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Pressure and Temperature Response of Pneumatic System with Thermal Consideration

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Abstract- The temperature and pressure response within control volume of a pneumatic system with thermal consideration is presented in this paper. The non-linear modeling equations of temperature and pressure are derived in a systematic way based on realistic estimation of heat transfer to the system, energy equation, ideal gas law and compressibility of fluid. The pressure and temperature response is compared analytically with the adiabatic condition and found that the system responds differently with thermal consideration.

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Keywords: pneumatic system, thermal effect, temperature response, pressure response.

Nomenclature

ρ	Density of fluid
h	Specific enthalpy of fluid
h_c	Heat transfer coefficient or film coefficient
k	Ratio of specific heat
P_{cv}	Pressure inside control volume
R_{th}	Thermal resistance of system's wall
W	Work done at constant pressure
R	Gas constant
q	Rate of heat transfer
T_{cv}	Temperature inside control volume
V_{cv}	Control volume of the system
U	Internal energy
$\dot{}$	Rate of change with time
$\dot{m}_{in}$	Mass flow rate at inlet
$\dot{m}_{out}$	Mass flow rate at outlet
m_{net}	Mass inside the control volume
ρ_{cv}	Density of fluid
A_{th}	Area of thermal resistance
P_u	Pressure at upstream
P_d	Pressure at downstream

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I. Introduction

Pneumatic systems are an important part of the industrial world as compressed air can be easily and readily obtained. Pneumatic systems are widely used in industrial automation, such as drilling, gripping, spraying and other applications due to their special advantages, e.g. low cost, high power-to-weight ratio, cleanliness and ease of maintenance. It has long been promoted as low cost alternatives to hydraulic and electric servo motor in automated material handling tasks [1]. In spite of these advantages, Pneumatic systems are more complicated due to high compressibility and nonlinearities of the flow characteristics. In addition, when air is compressed, the density changes significantly which even further complicates the analytical considerations. Due to complexity of the models, which realistically describe the fluid power components and systems, the designers have elected to use only steady-state conditions in the process of developing of Pneumatic systems. However, in reality operation of Pneumatic systems are not steady-state condition [1-2]. The performance and reliability of the Pneumatic systems are depend on the pressure & temperature response within the control volume of the system. The mathematical models of pressure and temperature response are non-linear differential equations which are correlated to each other.

In general, the temperature variation within a control volume of a Pneumatic system is ignored in modeling and simultaneously declaring adiabatic condition in gas capacitance modeling. This approach disregard both temperature variations associated with gas compression as well as effect of heat transfer from the system's wall to the control volume or surrounding. It is observed that thermal consideration has significant effect on system response because of heat transfer takes place in between control volume and system's wall as well as effect of compressibility of pneumatic fluid [3].

The thermal effect of Pneumatic system, i.e. heat transfer in between control volume and the system's wall is not included in Fernandez and Woods [4]. They suggested that an accurate thermodynamic model will, furthermore, require inclusion of heat transfer effects and it is left for an expanded discussion [4]. To evaluate the thermal effect of Pneumatic system, the non-linear modeling equations of pressure and temperature response are developed (Appendix-A) and applied to a rigid pneumatic accumulator. Before modeling of non-linear differential equations, the theory of fluid power control, compressibility and technique of simulation have been studied well [3], [5-8].

The thermo-physical properties of fluid are considered homogenous within the control volume. Kinetic energy, potential energy and viscous friction of the fluid are neglected. Assuming, there is no frictional heating in the system and fluid follow the laws of perfect gases. The heat is transferred in between control volume and the system's wall by conduction only and other mechanisms of heat transfer are neglected.

The non-linear modeling equations are derived (Appendix-A) from the conservation of energy, first principles of pressure and temperature state equations for an ideal gas; which includes the rate of change of pressure, temperature and control volume. These equations are used to evaluate the temperature and pressure response within the control volume of a pneumatic accumulator.

II. Governing Equations

The governing equations are conservation of energy equation, ideal gas law, rate of change of internal energy, rate of work done, heat transfer rate, specific enthalpy, specific heat and mass flow rate; which are readily available in [3], [9].

The conservation of energy equation,

$$q_{in} - q_{out} + \dot{W} + h_{in}\,\dot{m}_{in} - h_{out}\,\dot{m}_{out} = \dot{U} \tag{1}$$

Ideal gas law, $P_{cv}\,V_{cv} = m_{cv}\,R\,T_{cv}$ (2)

Rate of change of internal energy,

$$\dot{U} = \frac{d}{dt}\left(C_v\,m_{cv}\,T_{cv}\right) \tag{3}$$

Rate of work done, $\dot{W} = -P_{cv}\dot{V}_{cv}$ (4)

Heat transfer rate,

$$q_{net} = q_{in} - q_{out} \tag{5.1}$$

$$q_{net} = \frac{T_w - T_{cv}}{R_{th}} \tag{5.2}$$

$$q_{net} = A_{th}\,h_c(T_w - T_{cv}) \tag{5.3}$$

Specific enthalpy,

$$h = C_p T \tag{6.1}$$

$$h_{in} = C_p T_{in} \tag{6.2}$$

$$h_{out} = C_p T_{out} \tag{6.3}$$

Specific heat,

$$k = \frac{C_p}{C_v} \tag{7.1}$$

$$C_v = \frac{R}{k-1} \tag{7.2}$$

Mass flow rate,

$$\dot{m}_{cv} = \dot{m}_{in} - \dot{m}_{out} \tag{8}$$

III. Modeling of Rate of Change of Pressure within Control Volume ($\dot{P}_{cv}$)

From governing equation (2), (3) and (7),

$$\dot{U} = \frac{d}{dt}\left(\frac{C_v}{R}\,P_{cv}\,V_{cv}\right) = \frac{C_v}{R}\left(P_{cv}\dot{V}_{cv} + V_{cv}\,\dot{P}_{cv}\right)$$

$$\dot{U} = \frac{1}{k-1}\left(P_{cv}\,\dot{V}_{cv} + V_{cv}\,\dot{P}_{cv}\right) \tag{9}$$

Substituting the value from equations (4) to (6) and (9) to the governing equation (1) and rearranging the variables yield,

$$\dot{P}_{cv} = \frac{kP_{cv}}{V_{cv}}\left(\frac{T_{in}\,\dot{m}_{in} - T_{out}\,\dot{m}_{out}}{\rho_{cv}\,T_{cv}} - \dot{V}_{cv}\right) + (k-1)h_c A_{th}\left(\frac{T_w - T_{cv}}{V_{cv}}\right) \tag{10}$$

The equation (10) is found consisted with [4] except thermal part or second right side part of the equation (10)

IV. Modeling of Rate of Change of Temperature within Control Volume ($\dot{T}_{cv}$)

From governing equation (3),

$$\dot{U} = C_v\left(m_{cv}\,\dot{T}_{cv} + T_{cv}\,\dot{m}_{cv}\right) \tag{11}$$

Substituting the value from equations (4) to (5), (8) and (11) to the governing equation (1) and rearranging the variables yield,

$$\dot{T}_{cv} = \frac{R}{V_{cv}}\left(\frac{T_{cv}}{P_{cv}}\right)\left[\dot{m}_{in}\,(k\,T_{in} - T_{cv}) - \dot{m}_{out}\;T_{cv}\,(k-1) - \frac{P_{cv}\,\dot{V}_{cv}}{C_v} + \frac{h_c\,A_{th}\,(T_w - T_{cv})}{C_v}\right] \tag{12}$$

The equation (12) is found consisted with [4] except thermal part or second right side part of the equation (12).

V. Modeling of Rate of Change of Control Volume ($\dot{V}_{cv}$)

The rate of change of control volume ($\dot{V}_{cv}$) depends on the system's physical characteristics, configuration and arrangement of the piston & cylinder of actuator. In case of rigid container, rate of change of control volume is equal to zero and other Pneumatic system like an actuator, rate of change of control volume can be determined from the physical characteristic of the actuator [3].

Consider a double acting pneumatic actuator (Fig. 1) with the following physical characteristics.

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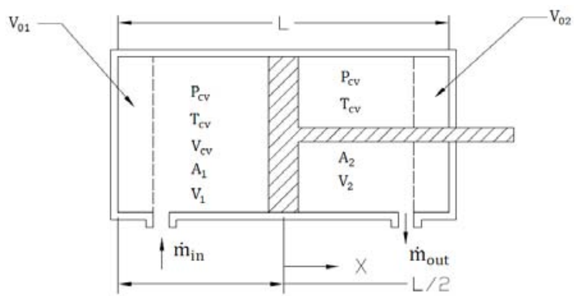

Fig.1. Configuration of double acting actuator.

Where,

V_1 = Volume of chamber[1] of the cylinder

V_2 = Volume of chamber[2] of the cylinder

A_1 = Area of chamber[1] of the cylinder

A_2 = Area of chamber[2] of the cylinder

V_{01} = Dead volume of chamber[1] of the cylinder

V_{02} = Dead volume of chamber[2] of the cylinder

L = Length of the cylinder

x = Position of the piston from neutral point

Assuming actuator's piston position at the middle point of the cylinder,

$$V_1 = V_{01} + A_1\left(\frac{L}{2} + x\right) \text{ and } V_2 = V_{02} + A_1\left(\frac{L}{2} - x\right)$$

Generalizing above equation,

$$V_i = V_{0i} + A_i\left(\frac{L}{2} + x\right) \text{ or } V_i = V_{0i} + A_i\left(\frac{L}{2} - x\right)$$

$$\frac{d}{dt}(V_i) = A_i\frac{d}{dt}(x) \quad \text{or} \quad \frac{d}{dt}(V_i) = A_i\frac{d}{dt}(-x)$$

$$\dot{V}_{cv} = A_i\dot{x} \text{ or } \dot{V}_{cv} = -A_i\dot{x} \tag{13}$$

VI. Modeling of Mass Flow Rate to Control Volume ($\dot{m}_{in}$ or $\dot{m}_{out}$)

The mass flow rate of compressible fluid flow through a restriction or through inlet and outlet valve port is given by [3],

$$\dot{m} = C_d\, A_s\, P_u \left(\frac{2}{R\,T_u}\right)^{\frac{1}{2}} \left(\frac{k}{k-1}\right)^{\frac{1}{2}} \left[\left(\frac{P_d}{P_u}\right)^{\frac{2}{k}} - \left(\frac{P_d}{P_u}\right)^{\frac{k+1}{k}}\right]^{\frac{1}{2}} \tag{14}$$

Where, C_d is the coefficient of discharge of Orifice meter. The mass flow rate depends on the ratio of downstream and upstream pressure. Equation (14) is subject to a phenomenon known as choking, which is unique to compressible flow. In choked flow, the mass flow rate will not increase with further decreasing in downstream pressure while upstream pressure is fixed, because sonic velocity is achieved at the throat at critical pressure ratio.

$$\left(\frac{P_d}{P_u}\right)_{critical} = \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}} \tag{15}$$

The critical pressure ratio is limiting the mass flow rate through the orifice meter.

VII. Application of Modeling Equation to a Pneumatic System

The non-linear modeling equations can be applied to any Pneumatic system particularly in pneumatic actuator, accumulator and servo system to evaluate the system's pressure and temperature characteristics within control volume. All pneumatic power systems are synonymous with compressed air in the vicinity of 7 bar (100 psi). The four modeling equations [10], [12], [13] and [14] are non-linear differential equations and interrelated to each other. These non-linear modeling equations are applied to a pneumatic accumulator to evaluate the pressure and temperature response within the control volume. Working principles of most pneumatic systems are close to the vicinity of working principle of pneumatic accumulator since all of them work with high pressure compressed air [1], [8].

The characteristics of the pneumatic system is a rigid cylindrical accumulator of length 360 mm, outer diameter 22 mm and inner diameter 19 mm, which is charging from a static chamber of 0.70 MPa pressure and 15^{0}C air temperature through an Orifice meter (C_d = 0.65) of diameter 0.03 inch. The initial temperature at the wall of the accumulator is 20^{0}C and pressure one atmospheric pressure. Over all heat transfer coefficient or film coefficient at the wall of the accumulator is 50 Watt/m^2 K.

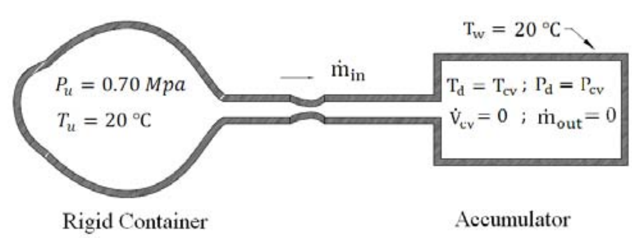

Fig.2. Charging of accumulator

The non-linear modeling equations are interrelated and solved with the following boundary conditions.

For rigid accumulator $\dot{V}_{cv} = 0$; $\dot{m}_{out} = 0$ and initial condition at time, t = 0, $\dot{m}_{in} = 0$; $P_{cv} = 1$ atmp and T_{cv} = 20^0 C = 293 K and T_u = 15^0 C = 288 K.

In adiabatic condition, $q_{net} = 0$ or no heat transfer in between control volume and the system's wall, which ultimately leads to $T_w = T_{cv}$ or isothermal condition.

VIII. Results and Discussion

The pressure and temperature increase significantly in thermal consideration than the adiabatic condition within control volume of the accumulator and it depends on initial temperature and pressure of the accumulator as well as of the charging system.

Temperature increases rapidly within short period of time than adiabatic condition because of instant compressibility, viscous friction, low heat capacitance or low specific heat of air. Usually, the polytropic equation $T_{cv} = T_{in}\left(\frac{P_{cv}}{P_{in}}\right)^{\frac{k-1}{k}}$ is used to predict the temperature but this equation is applied under closed or steady state condition. In Pneumatic system, mass added gradually and arbitrary to the system. So the thermal model of pressure and temperature response varies from the adiabatic model.

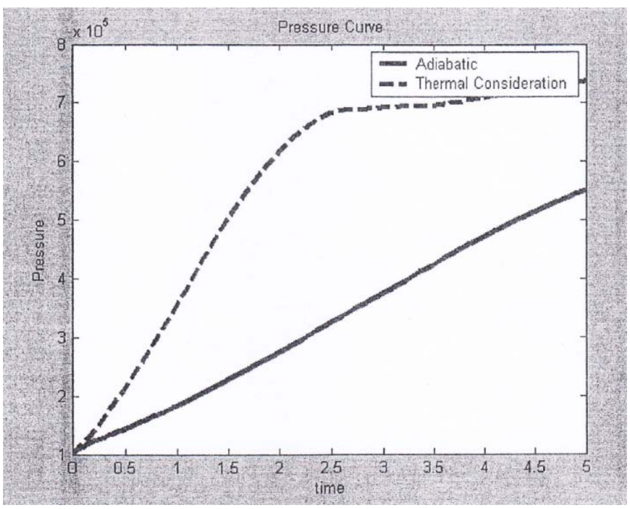

Fig.3. Pressure (N/m²) response within accumulator (time in second).

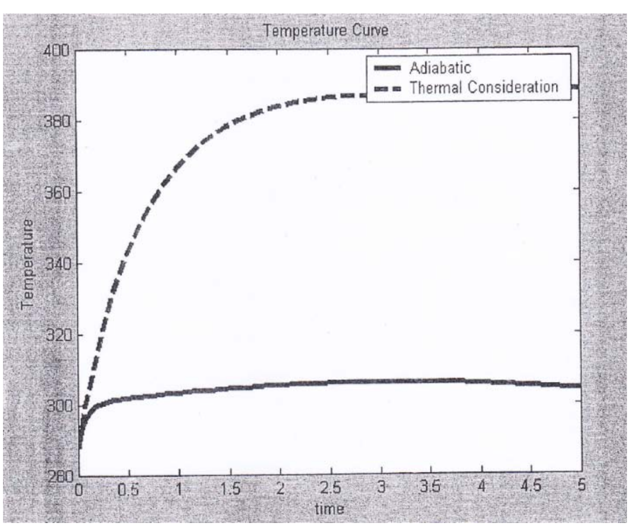

Fig.4. Temperature (K) response within accumulator (time in second).

IX. Conclusion

Thermal consideration in the modeling of pressure and temperature response of Pneumatic system has significant effect than adiabatic modeling especially to the system working in a high pressure and temperature environment. In thermal consideration, pressure and temperature increase exponentially within short period of time than adiabatic condition. Since performance of Pneumatic systems depend on the response of pressure, the thermal consideration in pneumatic system will improve the system's performance, accuracy, reliability as well as response time.

Acknowledgement

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Appendix-A

Modeling of Rate of Change of Pressure ($\dot{P}_{cv}$):

From governing equation (2), (3) and (7),

$$\dot{U} = \frac{d}{dt}\left(\frac{C_v}{R} P_{cv} V_{cv}\right) = \frac{C_v}{R}\left(P_{cv}\dot{V}_{cv} + V_{cv}\dot{P}_{cv}\right)$$

$$\dot{U} = \frac{1}{k-1}\left(P_{cv}\dot{V}_{cv} + V_{cv}\dot{P}_{cv}\right) \quad (9)$$

Substituting the value from equations (4) to (6) and (9) in the governing equation (1)

$$q_{net} - P_{cv}\dot{V}_{cv} + C_p(T_{in}\dot{m}_{in} - T_{out}\dot{m}_{out}) = \frac{1}{k-1}\left(P_{cv}\dot{V}_{cv} + V_{cv}\dot{P}_{cv}\right)$$

$$q_{net} - P_{cv}\dot{V}_{cv}\left(1 + \frac{1}{k-1}\right) + C_p(T_{in}\dot{m}_{in} - T_{out}\dot{m}_{out}) = \frac{V_{cv}\dot{P}_{cv}}{k-1}$$

Rearranging above equation,

$$\dot{P}_{cv} = \left(\frac{k-1}{V_{cv}}\right)q_{net} - \left(\frac{P_{cv}\dot{V}_{cv}}{V_{cv}}\right)k + \left(\frac{k-1}{V_{cv}}\right)C_p(T_{in}\dot{m}_{in} - T_{out}\dot{m}_{out})$$

$$\dot{P}_{cv} = \frac{kR}{V_{cv}}(T_{in}\dot{m}_{in} - T_{out}\dot{m}_{out}) - \left(\frac{P_{cv}\dot{V}_{cv}}{V_{cv}}\right)k + \left(\frac{k-1}{V_{cv}}\right)q_{net}$$

Substituting value of q_{net} from equation (5),

$$\dot{P}_{cv} = \frac{kP_{cv}}{V_{cv}}\left(\frac{T_{in}\dot{m}_{in} - T_{out}\dot{m}_{out}}{\frac{P_{cv}}{R}} - \dot{V}_{cv}\right) + \left(\frac{k-1}{V_{cv}}\right)\left(\frac{T_w - T_{cv}}{R_{th}}\right)$$

$$\dot{P}_{cv} = \frac{kP_{cv}}{V_{cv}}\left(\frac{T_{in}\ \dot{m}_{in} - T_{out}\ \dot{m}_{out}}{\rho_{cv} T_{cv}} - \dot{V}_{cv}\right) + \frac{k-1}{V_{cv}}\left(\frac{T_w - T_{cv}}{R_{th}}\right)$$

$$\dot{P}_{cv} = \frac{kP_{cv}}{V_{cv}}\left(\frac{T_{in}\ \dot{m}_{in} - T_{out}\ \dot{m}_{out}}{\rho_{cv} T_{cv}} - \dot{V}_{cv}\right) + (k-1)h_c A_{th}\left(\frac{T_w - T_{cv}}{V_{cv}}\right) \quad (10)$$

For rigid accumulator, $\dot{V}_{cv} = 0$; $\dot{m}_{out} = 0$. Substituting in the value in equation (10)

$$\dot{P}_{cv} = \frac{k\,R}{V_{cv}}(T_{in}\,\dot{m}_{in}) + \frac{(k-1)h_c A_{th}}{V_{cv}}(T_w - T_{cv})$$

Modeling of Rate of Change of Temperature ($\dot{T}_{cv}$):

From governing equation (3),

$$\dot{U} = C_v(m_{cv}\dot{T}_{cv} + T_{cv}\dot{m}_{cv}) \quad (11)$$

Substituting the value from equation (4) to (5) and (11) to the equation (1),

$$q_{net} - P_{cv}\dot{V}_{cv} + C_p(T_{in}\dot{m}_{in} - T_{out}\ \dot{m}_{out}) = C_v\left(m_{cv}\dot{T}_{cv} + \dot{m}_{cv}\ T_{cv}\right)$$

Substituting $\dot{m}_{cv}$ from equation (8),

$$q_{net} - P_{cv}\dot{V}_{cv} + C_p(T_{in}\dot{m}_{in} - T_{out}\ \dot{m}_{out}) = C_v\left[m_{cv}\dot{T}_{cv} + T_{cv}\ (\dot{m}_{in} - \dot{m}_{out})\right]$$

$$C_v m_{cv}\dot{T}_{cv} = q_{net} - P_{cv}\ \dot{V}_{cv} + \dot{m}_{in}(C_p\ T_{in} - C_v\ T_{cv}) - \dot{m}_{out}(C_p\ T_{cv} - C_v\ T_{cv})$$

$$\dot{T}_{cv} = \frac{q_{net}}{m_{cv}\,C_v} - \frac{P_{cv}\ \dot{V}_{cv}}{m_{cv}\ C_v} + \dot{m}_{in}\left(\frac{C_p\ T_{in} - C_v\ T_{cv}}{m_{cv}\ C_v}\right) - \dot{m}_{out}\left(\frac{C_p T_{cv} - C_v T_{cv}}{m_{cv}\ C_v}\right)$$

$$\dot{T}_{cv} = \frac{\dot{m}_{in}}{m_{cv}}(k\ T_{in} - T_{cv}) - \frac{\dot{m}_{out}}{m_{cv}}T_{cv}(k-1) + \frac{q_{net}}{m_{cv}\ C_v} - \frac{P_{cv}\ \dot{V}_{cv}}{m_{cv}\ C_v}$$

$$\dot{T}_{cv} = \frac{1}{m_{cv}}\left[\dot{m}_{in}(k\ T_{in} - T_{cv}) - \dot{m}_{out}\ T_{cv}(k-1) - \frac{P_{cv}\ \dot{V}_{cv}}{C_v} + \frac{h_c\ A_{th}\ (T_w\ - T_{cv})}{C_v}\right]$$

$$\dot{T}_{cv} = \frac{R}{V_{cv}}\left(\frac{T_{cv}}{P_{cv}}\right)\left[\dot{m}_{in}(k\ T_{in} - T_{cv}) - \dot{m}_{out}\ T_{cv}(k-1) - \frac{P_{cv}\ \dot{V}_{cv}}{C_v} + \frac{h_c\ A_{th}\ (T_w\ - T_{cv})}{C_v}\right] \quad (12)$$

For rigid accumulator, $\dot{V}_{cv} = 0$; $\dot{m}_{out} = 0$ Substituting in the value in equation (12)

$$\dot{T}_{cv} = \frac{R}{V_{cv}}\left(\frac{T_{cv}}{P_{cv}}\right)\left[\dot{m}_{in}(k\ T_{in} - T_{cv}) + \frac{h_c\ A_{th}\ (T_w\ - T_{cv})}{C_v}\right]$$

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