



GLOBAL JOURNAL OF RESEARCHES IN ENGINEERING : F  
ELECTRICAL AND ELECTRONICS ENGINEERING  
Volume 15 Issue 6 Version 1.0 Year 2015  
Type: Double Blind Peer Reviewed International Research Journal  
Publisher: Global Journals Inc. (USA)  
Online ISSN: 2249-4596 & Print ISSN: 0975-5861

## Spectral Kurtosis Theory: A Review through Simulations

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**GJRE-F Classification :** FOR Code: 090699



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# Spectral Kurtosis Theory: A Review through Simulations

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## I. INTRODUCTION

Characterization of a given signal as noise like or tone like finds several applications in music-speech classification [1,2,3], perceptual audio coding [4], Multi Band Excitation (MBE) model based perceptual coding of speech [5] and voice activity detection [6]. Within each category, the signals may be stationary/nonstationary/transient signal or gaussian/nongaussian. The Nongaussianity and/or nonstationary signal also occur when a radar signal is reflected by a fluctuating target or clutter [7] or when a communication signal passes through a fading wireless channel [8,9]. The fourth-order cumulant based kurtosis of the time signal was traditionally used for nongaussianity detection [10], Harmonic Retrieval from nongaussian processes[11], nonminimum phase system identification [12]. Recently the frequency dependent kurtosis defined in spectral domain was proposed and successfully used in bearing fault detection [13,14,15] and vibratory surveillance and diagnostics of rotating machines [16]. Spectral Kurtosis

(SK) was originally introduced by Dwyer [17] where it was defined on the real part of the STFT filter bank output to overcome the deficiency of the power spectral density to detect and characterize the signal transients. Vrabie [18] justified the theoretical definition of SK and proposed an unbiased estimator of SK. Antoni [19,20] formulated SK differently by using Wold-Cramér decomposition with a theoretical basis for the SK estimation of non-stationary processes. He also practically used it for machine surveillance and diagnostics [16,21]. Other applications of spectral kurtosis reported in the literature include SNR estimation in speech signals [22], denoising [23] and subterranean termite detection [24].

In this paper, the important aspects of original spectral kurtosis theory is reviewed from fundamentals. The spectral kurtosis properties of both stationary and nonstationary signals as well as the stochastic mixtures are analytically obtained. Extensive Monte Carlo simulations are carried out to support the theory reviewed and the spectral kurtosis of several processes at different signal-to-noise ratios (SNRs) is estimated. The results are in perfect match with the previous analytical findings.

The paper is organized section wise as follows. The mathematical basics of spectral kurtosis are introduced in the section II. The Test Signal Set comprising several popular signals are analytically described in section III. Short time fourier transform (STFT) for dynamically estimating the magnitude spectrum and the expression for estimation of SK from STFT is given in section IV. In Section V the details of Monte Carlo simulations of the Test Signal Set and the derived mixture processes, and the simulation results are presented. Finally the review summary and future work is given in Section VI.

## II. SPECTRAL KURTOSIS

### a) Background

Wold-Cramer's decomposition uniquely describes a non-stationary signal  $y(t)$  as the response of a causal linear system with time varying impulse response  $h(t, s)$  excited by a signal  $X(t)$  i.e.

$$Y(t) = \int_{-\infty}^t h(t, t - \tau)X(\tau)d\tau \quad (1)$$

Here  $h(t,s)$  means the linear causal impulse response of the system at time instant  $t$  when excited by an impulse

at time instant  $t$ -s. The frequency counterpart of eq.(1) is given by

$$Y(t) = \int_{-\infty}^{\infty} H(t, f) \exp(j2\pi ft) dX(f) \quad (2)$$

where  $H(t, f)$  is the time varying transfer function of the system, which can be interpreted as the complex envelope of signal  $Y(t)$  at frequency  $f$  and  $dX(f)$  is an ortho-normal spectral process associated with input driving process  $X(t)$ . In many cases,  $H(t, f)$  is stochastic and can be represented with  $H(t, f, \xi)$  where  $\xi$  is a representative random parameter of filter's time varying transfer function. Let  $H(t, f)$  be conditioned to  $\xi$  i.e. the shape of  $H(t, f)$  depends on the outcome of the random variable  $\xi$ .  $H(t, f, \xi)$  can be assumed to be time stationary, stochastic and independent of the spectral process  $dX(f)$ . Thus the signal  $X(t)$  is stationary in general but non-stationary for a particular outcome  $\xi$ . Such a process was designated as conditionally nonstationary (CNS) process in [19]. It may be noted down that the simplest way to convert a nonstationary process to a CNS process is the time datum randomization. Any CNS process driven by a white process  $X(t)$  of order  $p \geq 4$  is likely to be leptokurtic i.e. its probability density function having tails flatter than those of its generating gaussian process and hence non-gaussian. *In fact, this connectivity between the CNS and nongaussianity makes the kurtosis, originally defined on time processes to characterize the nongaussianity, a very useful in analyzing the nonstationary processes through kurtosis defined in spectral domain.*

For the stationary white driving process  $X(t)$  of order  $p \geq 2n$ , the spectral kurtosis of the nonstationary signal  $Y(t)$  is defined as the normalized fourth-order spectral cumulant [19] as

$$\begin{aligned} K_Y(f) &= \frac{S_{4Y}(f)}{S_{2Y}(f)^2} - 2 \\ &= \frac{E\{|H(n, f)|^4\}}{E\{|H(n, f)|^2\}^2} (2 + \kappa_X) - 2 \geq \kappa_X \\ &= \gamma_{4H}(f) (2 + \kappa_X) - 2 \geq \kappa_X \quad f \neq 0 \quad (3) \end{aligned}$$

where the factor 2 in place of 3 as in usual definition of cumulants comes from the fact that  $dX(f)$  is a circular random variable,  $\gamma_{4H}(f)$  is the kurtosis of the (stochastic) frequency response of the time varying filter and  $\kappa_X$  is the time kurtosis of the input process.

If  $Y(t)$  is a purely stationary process, then  $\gamma_{4H}(f)$  is independent of frequency and is unity, then spectral kurtosis of  $Y(t)$  is given by

$$K_Y(f) = \kappa_X \quad (4)$$

which is a constant and independent of the frequency. In particular, the spectral kurtosis of a stationary gaussian process is zero (i.e.  $\kappa_X = 0$ ).

#### b) Mixing Processes

Let  $Z(t)$  be the mixture of two processes (i) a non-stationary process  $Y(t)$  and (ii). a stationary additive noise  $N(t)$ .

$$Z(t) = Y(t) + N(t) \quad (5)$$

The spectral kurtosis of this mixture is given by [19]

$$K_z(f) = \frac{K_Y(f)}{[1 + \rho(f)]^2} + \frac{\rho(f)^2 K_N(f)}{[1 + \rho(f)]^2} \quad f \neq 0 \quad (6)$$

where  $\rho(f)$  is the local Noise-to-Signal power ratio at the frequency  $f$  given by

$$\rho(f) = \frac{S_{2N}(f)}{S_{2Y}(f)} \quad (7)$$

If the mixing process  $N(t)$  is stationary (white or colored) gaussian process, then  $K_N(f)$  vanishes at all frequencies except at  $f \neq 0$ . Thus eq.(6) becomes

$$K_z(f) = \frac{K_Y(f)}{[1 + \rho(f)]^2} \quad f \neq 0 \quad (8)$$

If the noise power is zero i.e. signal is clean, then  $\rho(f) = 0$  and hence

$$K_z(f) = K_Y(f) \quad f \neq 0 \quad (9)$$

When the process  $Y(t)$  is gaussian, eq.(6) becomes

$$K_z(f) = \frac{\rho(f)^2 K_N(f)}{[1 + \rho(f)]^2} \quad f \neq 0 \quad (10)$$

where  $\rho(f)$  is finite. If the noise power becomes larger and larger,  $\rho(f) \rightarrow \infty$ , the process  $Z(t)$  becomes purely the mixing noise process, the spectral kurtosis becomes

$$K_z(f) = K_N(f) \quad f \neq 0 \quad (11)$$

### III. TEST SIGNAL SET

In what follows, some commonly found signals are considered for analytically computing the spectral kurtosis. The spectral kurtosis of some other signals not considered earlier is also obtained analytically.

#### a) Real Sinusoids

A real sinusoid of constant amplitude and constant frequency is given by

$$y(t) = A \cos(2\pi f_0 t + \varphi) \quad (12)$$

where  $\varphi$  is a constant initial phase from  $U(-\pi, \pi)$ .

$$K_Y(f_0) = \frac{S_{4Y}(f_0)}{S_{2Y}(f_0)^2} - 2 = \frac{E\{|A|^4\}}{E\{|A|^2\}^2} - 2 = -1 \quad (13)$$

A real sinusoid of time varying amplitude and constant frequency is given by

$$y(t) = A(t) \cos(2\pi f_0 t + \varphi) \quad (14)$$

If the amplitude decreases exponentially, the signal is called a damped sinusoid and is given by

$$y(t) = A e^{-kt} \cos(2\pi f_0 t + \varphi) \quad (15)$$

where  $k$  is the damping (bandwidth) factor,  $e^{-kt}$  is the decaying real envelope and  $\varphi$  is the constant initial phase from  $U(-\pi, \pi)$ .

#### b) Random Amplitude Sinusoid

When a radar transmitted carrier signal is reflected by a fluctuating target, the carrier amplitude of the echo signal undergoes random fluctuations [7]. Similarly, a communication signal passing through a fading wireless channel, the amplitude of the signal at the receiver input undergoes random fluctuations [8,9]. In these cases, the time varying amplitude  $A(t)$  in eq.(14) can be represented by

$$A(t) = A_c + A_r(t) \quad (16)$$

where  $A_c$  is the constant part and the random part  $A_r(t)$  which is also called as the multiplicative noise. In case of deep fluctuations the constant part becomes zero and then  $A(t) = A_r(t)$ . The spectral kurtosis of such a process is totally dependent on the probability density function of  $A_r(t)$ . In [25] the spectral kurtosis of such a random amplitude sinusoid was shown to be

$$K_Y(f) = \kappa_{A_r} + 1 \quad (17)$$

where  $\kappa_{A_r}$  is the coefficient of kurtosis or time kurtosis of the random amplitude. The coefficient of kurtosis  $\kappa_{A_r}$  based on cumulants can be obtained from the following expressions [26, 27].

$$\begin{aligned} C_2 &= M_2 - M_1^2 \\ C_4 &= M_4 - 4M_1M_3 + 6M_1^2M_2 - 3M_1^4 \\ \gamma_2 &= C_2 \\ \gamma_4 &= C_4 - 3\gamma_2^2 = C_4 - 3C_2^2 \\ \kappa_{A_r} &= \frac{\gamma_4}{\gamma_2} = \frac{C_4}{C_2^2} - 3 \end{aligned} \quad (18)$$

where  $M_k$  are the k-th standard moments,  $C_k$  are the k-th central moments and  $\gamma_k$  are the cumulants.

a) If the amplitude of the random sinusoid in eq.(16) is totally random with a gaussian density function, then from eq.(17) and eq.(18), the spectral kurtosis is given by

$$K_Y(f) = \kappa_{A_r} + 1 = \frac{3\sigma^4}{(\sigma^2)^2} - 3 + 1 = 1 \quad (19)$$

b) If the amplitude of varies uniformly in the range (a,

b), then the spectral kurtosis from eq.(17) and eq.(18) is given by

$$K_Y(f) = \frac{(b-a)^4/80}{((b-a)^2/12)^2} - 3 + 1 = \frac{144}{80} = -0.2 \quad (20)$$

c) For a rayleigh distributed random amplitude, the standard moments are given by

$$M_{2k} = 2^k b^{2k} k! \quad k = 1, 2$$

Then from eq.(17) and eq.(18), the spectral kurtosis can be obtained as

$$K_Y(f) = 0 \quad (21)$$

Similarly the spectral kurtosis for other density functions can be obtained. The focus of this paper is not to derive such expressions, but to show that  $K_Y(f)$  is a positive constant depending on the probability density function. However for some more density functions, the simulations are carried out as described in the section V.

#### c) Harmonic and Inharmonic Sinusoids

A harmonic sinusoid comprises a sine wave of fundamental frequency  $f_0$  and its finite number of harmonics  $f_m = mf_0$  as denoted by

$$y(t) = \sum_{m=0}^{M-1} A_m \cos(2\pi m f_0 t + \varphi) \quad (22)$$

where  $M$  is the number of harmonics. The amplitudes of the harmonics decay at different rates depending on the instrument or the note played. A typical profile of the harmonic amplitudes is given by  $A_m = 1/m$ . If the frequencies are independent and no harmonic relation among them i.e.  $f_m \neq mf_0$ , then the sinusoids are called inharmonic sinusoids. Both harmonic and inharmonic sinusoids appear frequently in music produced by several instruments.

#### d) Additive Gaussian Noise

Additive White Gaussian Noise (AWGN) signal is a noise commonly found in communication channel which adds to the transmitted signal. The spectrum of this signal is flat with a constant one-sided power spectral density  $\eta$  within the channel bandwidth.

If power spectral density of the noise within the channel bandwidth is function of frequency, then the noise is called colored noise and is characterized by  $\eta(f)$ .

#### e) Modulated Signals

An amplitude modulated signal with a carrier frequency  $f_c$  modulated by a sine wave of a low frequency  $f_m$  and is given by

$$y(t) = A_c(1 + m_A \text{mes}(t) \cos(2\pi f_c t + \varphi) \quad (23a)$$

where  $A_c$  is the amplitude of the carrier and  $m_A$  is the modulation index,  $\text{mes}(t)$  is the low frequency message

signal. The Double Side Band (DSB) signal, the Lower Side Band (LSB) signal and the Upper Side Band (USB) signal can be formed by appropriate processing the AM signal.

The Frequency Modulated (FM) signal can be obtained as

$$y(t) = A_c \cos \left( 2\pi f_c t + 2\pi k_f \int_0^t m e s(t) dt \right) \quad (23b)$$

#### f) Chirp Signal

A chirp signal is a frequency modulated signal in which the frequency of the carrier is linearly or hyperbolically varied. This kind of chirp signal is extensively used in pulse radars for achieving higher range resolution using a longer transmitted pulse, which is otherwise possible with a shorter transmitted pulse [14]. A phase modulated signal is given by

$$y(t) = A \cos \varphi(t) = A \cos 2\pi f(t)t \quad (24)$$

where  $\varphi(t)$  is the time-varying phase of the carrier. The frequency profile  $f(t)$  can be linear, quadratic or logarithmic.

In a *linear chirp signal*, the carrier frequency  $f$  is varied as  $f_c + \alpha t$ , where  $\alpha = df/dt$  is the chirp rate. Then time-varying phase of the carrier is given by

$$\begin{aligned} \varphi(t) &= 2\pi \int_0^t f(t) dt = 2\pi \int_0^t (f_c + \alpha t) dt \\ &= 2\pi \left( f_c t + \frac{1}{2} \alpha t^2 \right) \end{aligned} \quad (25)$$

Substituting eq.(23) in eq.(22), we get

$$y(t) = A \cos 2\pi \left( f_c t + \frac{1}{2} \alpha t^2 \right) \quad (26)$$

If the start frequency at  $t = 0$  is  $f_c$  and the end frequency at  $t = t_1$  is  $f_1$ , then the chirp rate is given by  $\alpha = (f_1 - f_c)/t_1$ .

In a *quadratic chirp signal*, the instantaneous frequency is given by

$$f(t) = f_c + \alpha t^2 \quad (27)$$

where  $\alpha = (f_1 - f_0)/t_1^2$ .

In a *logarithmic chirp signal*, the instantaneous frequency is given by

$$f(t) = f_c \alpha^{t_1} \quad (28)$$

where  $\alpha = (f_1/f_0)^{1/t_1}$ .

## IV. STFT BASED SPECTRAL KURTOSIS ESTIMATION

In this section a means of estimating the spectral kurtosis from the short time fourier transform (STFT) is presented. Here the input signal  $y(n)$  is divided into overlapping or non overlapping frames each of size  $N$ , multiplied by a window function  $w(k)$  like a hamming window of same size and analyzed by using the Fourier Transform. A matrix popularly known as a spectrogram is formed by arranging STFT coefficients as columns as given by

$$S(k, l) = \frac{1}{MW_n N} \left| \sum_{n=0}^{N-1} y(n + lM) w(n) e^{-j \frac{2\pi n k}{N}} \right|^2 \quad (29)$$

$$0 \leq k \leq K - 1, 0 \leq l \leq L - 1$$

where  $k$  is the frequency index,  $l$  is the time frame index,  $M$  is the hop size,  $K$  is the total number of frequency bins of one-sided STFT and  $L$  is the total number of frames contained in the signal.

An un-biased estimator of the spectral kurtosis is proposed in [19] based on  $L$  realizations of  $K$ -sample signal. The discrete fourier transform (DFT) on a  $K$ -sample signal computes the signal spectrum at  $K$ -number of discrete frequencies. If the  $L$  number of nonoverlapping frames used in the STFT analysis are considered as  $L$  number of the independent stochastic signal realizations, the spectral kurtosis  $K_S(k_0)$  at the frequency index  $k_0$  can be computed from the  $k_0$ -th row of the spectrogram matrix  $S(k_0, \cdot)$ . The spectral kurtosis at all frequency indices  $0 \leq k \leq K - 1$  is given by

$$K_S(k) = \frac{L}{L-1} \left[ \frac{(L+1) \sum_{l=0}^{L-1} |S(k, l)|^4}{\left\{ \sum_{l=0}^{L-1} |S(k, l)|^2 \right\}^2} - 2 \right] \quad (30)$$

$$0 \leq k \leq K - 1$$

## V. SIMULATIONS AND RESULTS

The following signals are simulated using eq.(12), eq.(14), eq.(15) and eq.(20) through eq.(28).

1. A Constant Amplitude Sinusoid signal
2. A Random Amplitude Sinusoid signal
  - a. A Uniform Amplitude Sinusoid signal
  - b. A Gaussian Amplitude Sinusoid signal
  - c. A Exponential Amplitude Sinusoid signal
  - d. A Rayleigh Amplitude Sinusoid signal
  - e. A Lognormal Amplitude Sinusoid signal
  - f. A Gamma Amplitude Sinusoid signal
  - g. A Weibull Amplitude Sinusoid signal
  - h. A Chisquare Amplitude Sinusoid signal
3. A Damped Sinusoid signal

- a.  $\exp(-kt)$  envelope Sinusoid signal
- b.  $\exp(-kt^2)$  envelope Sinusoid signal
4. A Harmonic Sinusoid signal
5. An Additive Gaussian Noise
  - a. White Gaussian Noise
  - b. Colored Gaussian Noise
6. An Analog Modulated signals
  - a. An Amplitude Modulated signal
  - b. A Double Side Band (DSB) signal
  - c. A Lower Side Band (LSB) signal
  - d. A Upper Side Band (USB) signal
  - e. A Wideband Frequency Modulated Signal
7. Chirp signals
  - a. A Linear Chirp signal
  - b. A Quadratic Chirp (Convex) signal
  - c. A Quadratic Chirp (Concave) signal
  - d. A Logarithmic Chirp Signal

Different mixture processes are formed by adding two or more of the above simulated signals. The spectral kurtosis is estimated for each mixture process. In all simulations a sampling frequency of 44100Hz is used. Customized Matlab code is developed for generating the test signals, for computing STFT and for estimating the spectral kurtosis.

#### a) Mixture-1

A composite signal is formed by summing four sinusoids of frequencies 1800Hz, 4000Hz, 9000Hz and

18000Hz with respective amplitudes: 1.0, 0.25, 0.7 and 2.0 is formed. An additive white gaussian noise (AWGN) is added to the composite signal to form the first mixture process. Thus the mixture comprises a total five signals: four constant amplitude sinusoids and gaussian noise. The variance of AWGN is adjusted so as to obtain signal-to-noise ratios of 30dB, 20dB, 10dB, 0dB, -5dB and -10dB.

The mixture signal is generated for a duration of 1.1610 seconds. STFT is computed for frames or window size of 256 samples with 50% (i.e. 128 samples) overlap. Each frame is multiplied by a hamming window of 256 samples and a 256-point FFT is computed thus giving a spectrogram matrix of size  $256 \times 399$

$$S(k, l); \quad 0 \leq k \leq 255, \quad 0 \leq l \leq 398$$

each of 256 frequency bins of spectrogram matrix is averaged over 399 frames to obtain a mean STFT spectrum  $S_M(k) = \langle S(k, \cdot) \rangle; \quad 0 \leq k \leq 255$  which is called here as *Averaged STFT Spectrum*. Fig.1a gives the Averaged STFT Spectrum of Mixture-1 for different SNRs. Four peaks of different amplitudes observed in the spectrum correspond to four sine waves in the mixture. The low amplitude second peak is submerged in the noise floor at low SNRs below 0dB. As the SNR decreases, the noise floor also increases.

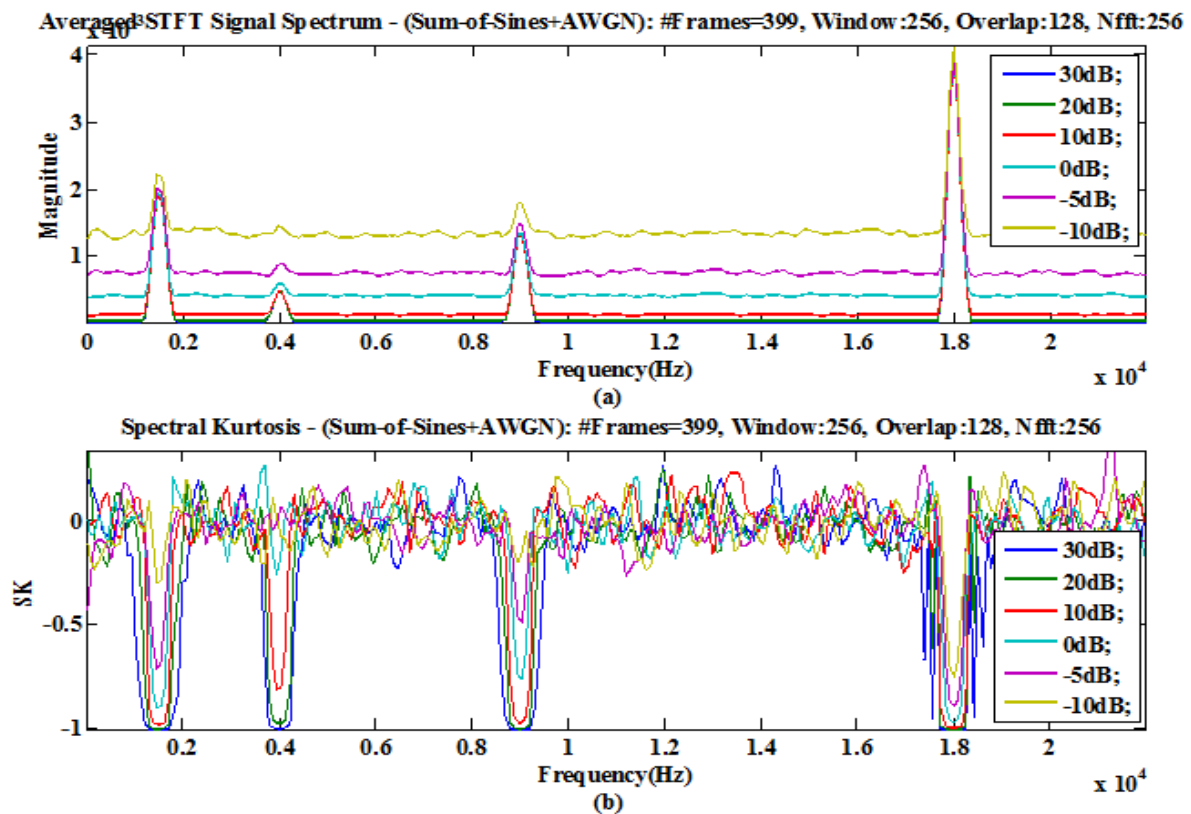


Fig. 1 : Sum of sinusoids (a). Averaged SFT spectrum of (b). Estimated Spectral Kurtosis for SNRs 30dB to -10dB

The spectral kurtosis  $K_S(k); 0 \leq k \leq 255$  is also computed from the spectrogram matrix using eq.(28) as explained in section IV. It may be noted that  $K_S(0)$  is to be ignored, as the kurtosis is not defined at  $f = 0$ . The Fig.1b gives the spectral kurtosis (SK) of Mixture-1 for different SNRs.

The spectral kurtosis has four negative peaks corresponding to four sinusoids. For higher SNR (30dB) the peaks have a value of -1 irrespective of the sinusoid amplitudes. As the SNR decreases, the peaks values increase from -1 towards zero. Local SNR computed at peak locations vary depending upon the amplitude of the sinusoid. Here it is maximum at 18000Hz and minimum at 4000Hz. It may be noted down that the SNR referred in the legend of the figures1(a) and (b) is the global SNR, which is computed based on the aggregate of all sinusoids. For global SNRs below 0dB, where the AWGN power dominates the aggregate power of all

sinusoids, the mixture becomes more and more gaussian, the spectral kurtosis tends to zero, relatively faster at 4000Hz where the local SNR is minimum.

#### b) Mixture-2

The second Mixture process comprises six components: constant amplitude sinusoid(CAS), two damped sinusoids(DS1 and DS2) with different damping factors, colored gaussian noise(CGN), colored uniform(i.e. nongaussian) noise(CnGN) and AWGN. Global SNR is computed with respect to AWGN considering the other five components as composite signal. The CGN is obtained by passing white gaussian noise through a 6-order butterworth band pass filter having passband between 10KHz and 13KHz. The CnGN obtained by passing white uniform noise through a 6-order butterworth band pass filter having passband edges at 15KHz and 17KHz.

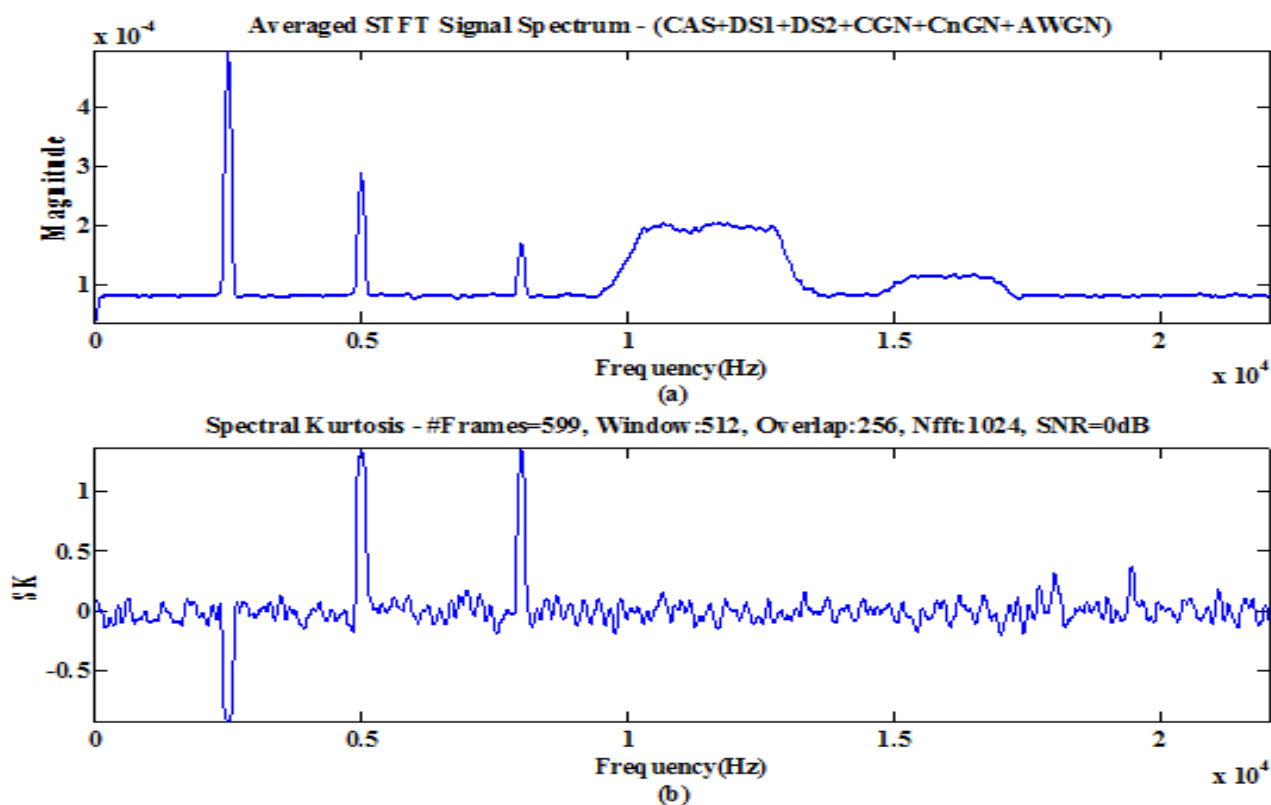


Fig. 2 : Mixture-2 (a). Averaged SFT spectrum of (b). Estimated Spectral Kurtosis for SNRs 0dB

Fig.2 gives the Averaged STFT Spectrum and SK of Mixture-2 for 0dB SNR. The first peak at 2500Hz having the spectral kurtosis of -1 the constant amplitude sinusoid. The peaks at 5000Hz and 8000Hz have the SK greater than zero indicate the nonstationary nature of the signals. In fact these signals correspond to damping sinusoids which amplitude is changing with time as  $e^{-kt}$  and  $e^{-kt^2}$ . The other two components: CGN and CnGN being stationary noise processes take a zero SK. The AWGN also takes a zero SK as expected.

#### c) Mixture-3

The third Mixture process is made up eight sinusoids, each having a random amplitude following different probability density functions: gaussian, uniform, exponential, Rayleigh, etc. The frequencies of these sinusoids are 1KHz, 2.5KHz, 4KHz, 5.5KHz, 7KHz, 8.5KHz, 10KHz, 11.5KHz, 13KHz, 14.5KHz and 16KHz. These sinusoids are generated one by one and processed separately to obtain the SK estimate separately. Then the SK functions of all sinusoids are

overlapped and shown in single figure. Fig.(3) gives the STFT spectrum and the SK of these its random amplitude sinusoids for SNR=30dB. The eight peaks in STFT spectrum correspond to eight sinusoids. As shown in Fig3(b), the SK of first sinusoid at 1000Hz is -0.2, corresponding to uniformly distributed amplitude variations (see eq.20). The SK of the second sinusoid at 2500Hz is +1.0, corresponding to gaussian distributed

amplitude variations (see eq.19). The SK of fourth sinusoid at 5500Hz is 0.0, corresponding to rayleigh distributed amplitude variations (see eq.21).

Fig.(4) gives the STFT spectrum and the SK of these random amplitude sinusoids for SNR=10dB. The SK values of the sinusoids at 10dB compared to the respective SK values at 30dB are different, but the trend is same.

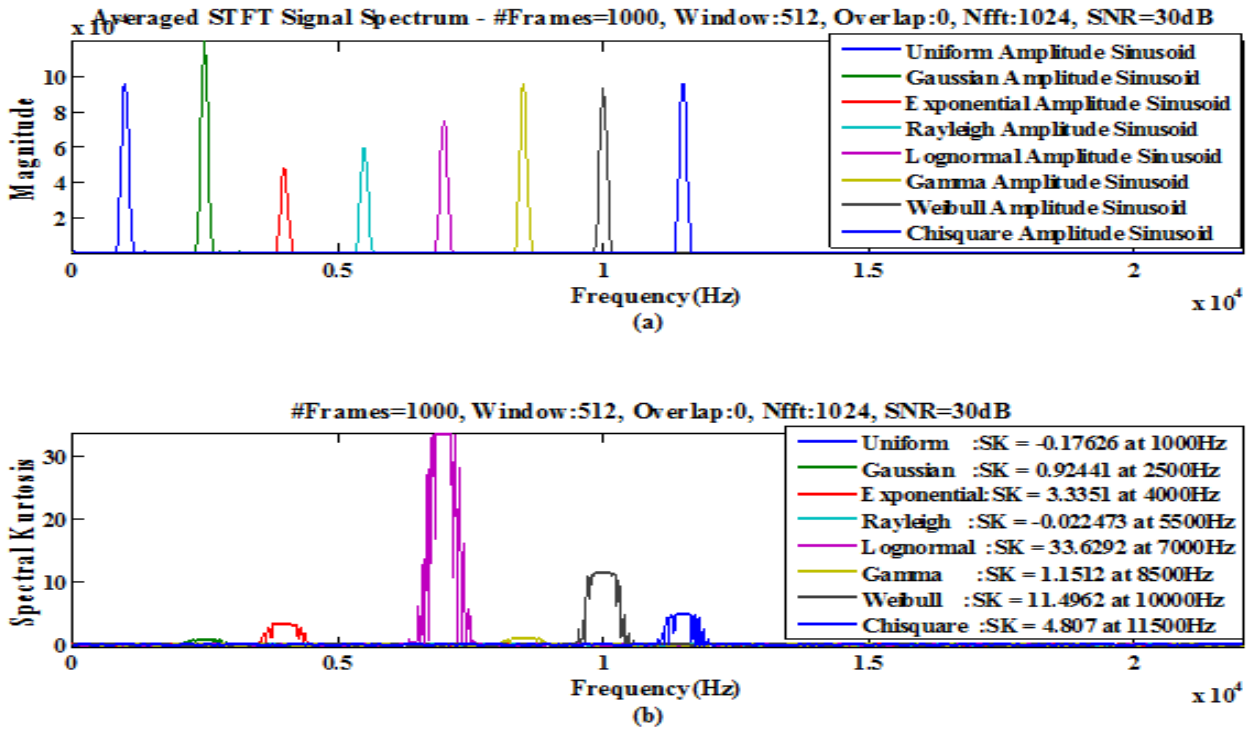


Fig. 3 : Mixture-3 (a). Averaged SFT spectrum of (b). Estimated Spectral Kurtosis for SNRs 30dB

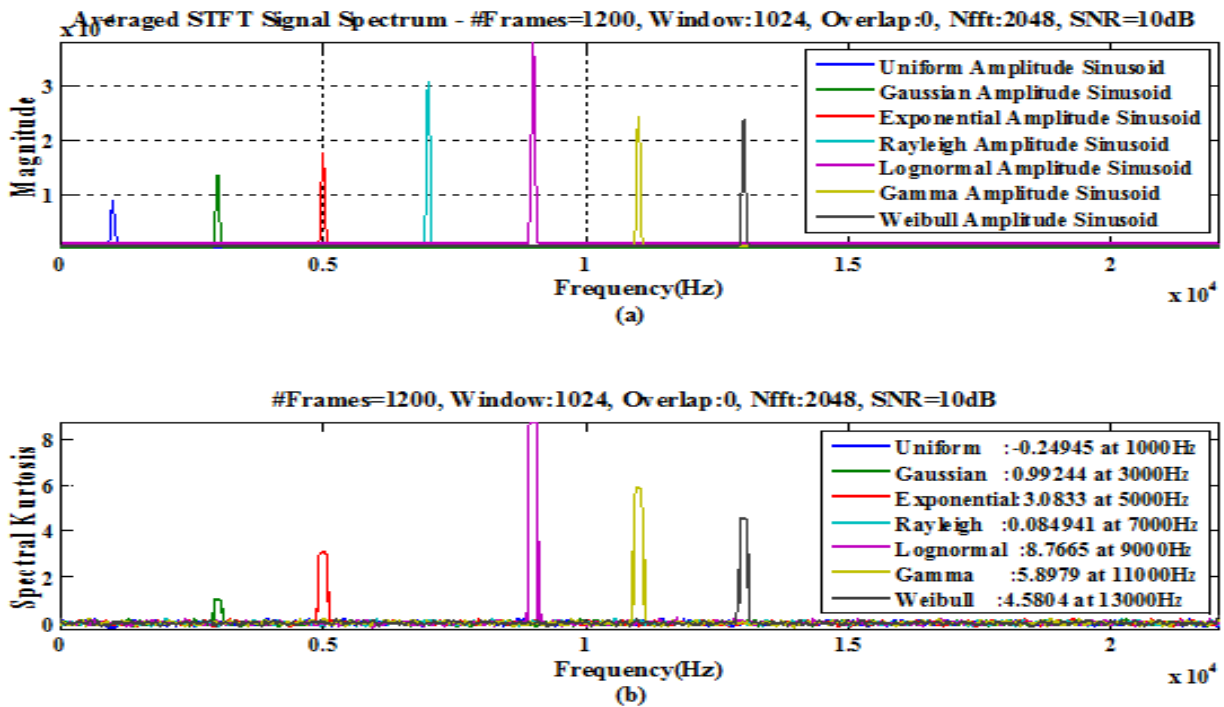


Fig. 4 : Mixture-3 (a). Averaged SFT spectrum of (b). Estimated Spectral Kurtosis for SNRs 10dB



d) *Mixture-4*

The fourth Mixture is basically a harmonic sinusoid with a fundamental at 800Hz and having 10 harmonics contaminated by an AWGN. The amplitude of n-th harmonic is  $1/n$ , but this amplitude remains constant with time. The STFT spectrum of this mixture is

shown in Figure 5(a) for SNRs of 20dB, 10dB and 0dB. The ten negative peaks in SK plot of Figure 5(b) correspond to seven sinusoids. Please note that each peak is -1 irrespective of the harmonic number for higher SNRs. As SNR decreases, the negative peaks move from -1 towards zero.

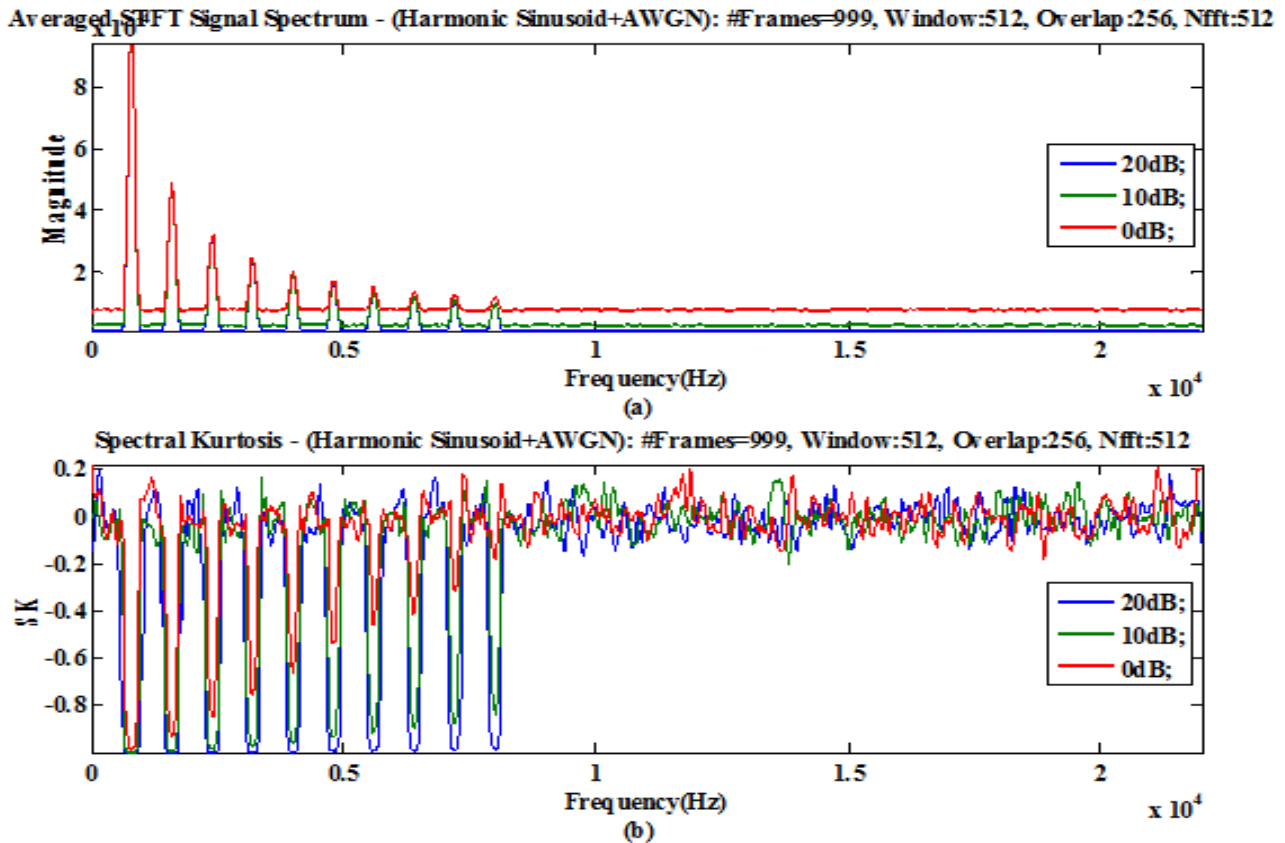


Fig. 5 : Mixture-4 (a). Averaged SFT spectrum of (b). Estimated Spectral Kurtosis for SNRs 20dB, 10dB and 0dB

e) *Mixtures-5*

In this category basically five analog modulation signals are considered; corresponding mixtures are AM + AWGN, AM-SC (DSB) + AWGN, LSB+ AGWN, USB+ AGWN and FM+AGWN. Fig.(6) through fig.(10) give the STFT spectra and SKs of these mixtures.

The SK of AM signal is -1.0 since the carrier is strong due to low modulation index and resembles a constant amplitude at carrier frequency of 12KHz.. The SK of DSB is positive at carrier frequency of 12KHz showing its nonstationary nature. The SK of the next two mixtures LSB and USB is over the signal bandwidth. However, at the band edges, the SK takes exceptionally large values.

f) *Mixture-6*

The sixth Mixture is made up of different chirp signals: linear, quadratic and logarithmic chirps generated using the eq.(24) through eq.(28). Each chirp is shown in different colors in Fig 11. The STFT spectrum of this mixture is shown in Figure 11(a) for

SNR of 10dB. It may be noted down that the SK of a chirp signal is nonzero positive, over the chirp bandwidth. However, at the band edges, the SK takes exceptionally large values.

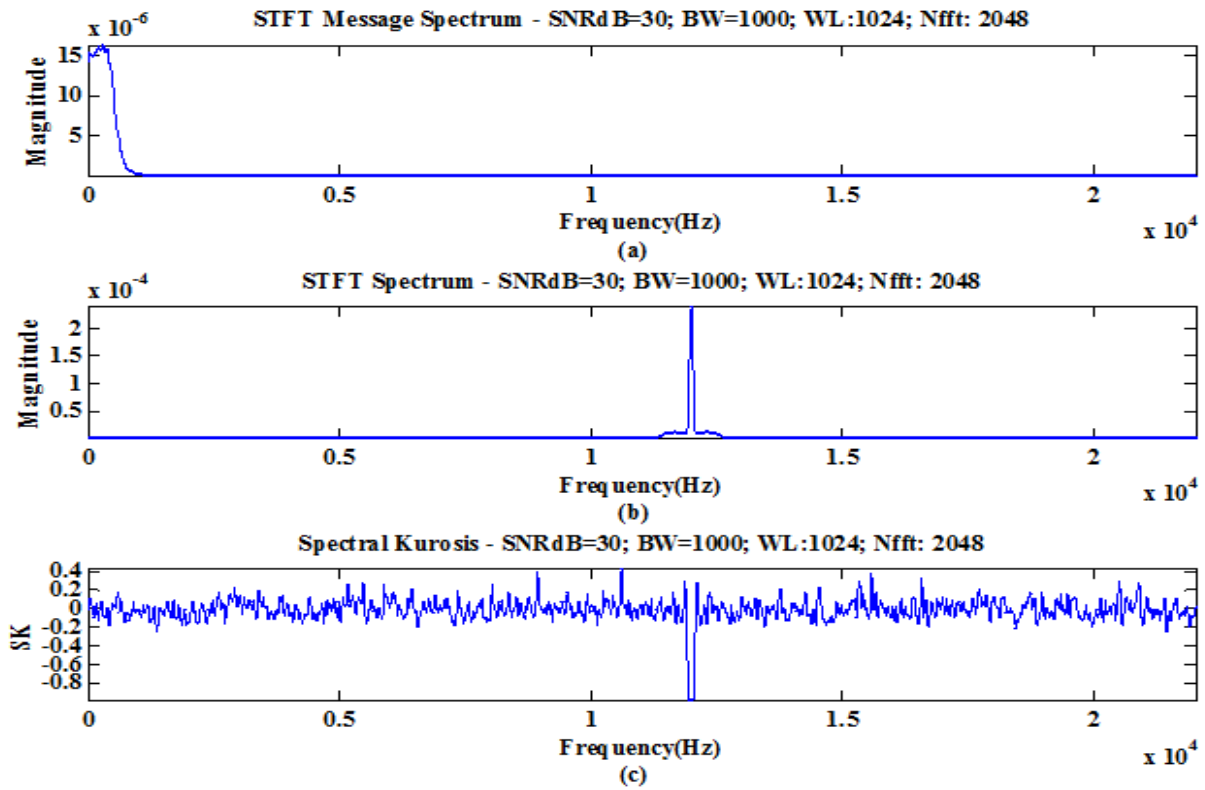


Fig. 6 (a) : Averaged SFT spectrum of AM+AWGN (b).Estimated Spectral Kurtosis for SNR=30dB

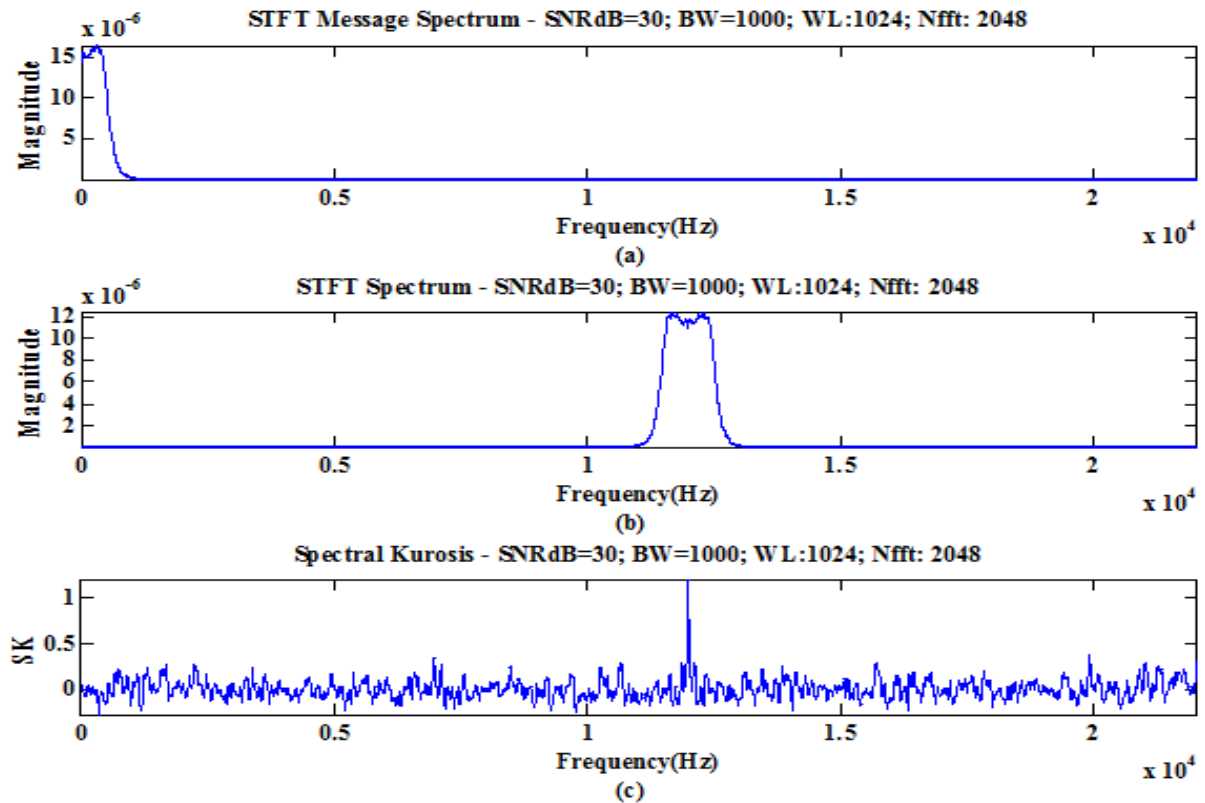


Fig. 7(a) : Averaged SFT spectrum of DSB+AWGN (b).Estimated Spectral Kurtosis for SNR=30dB

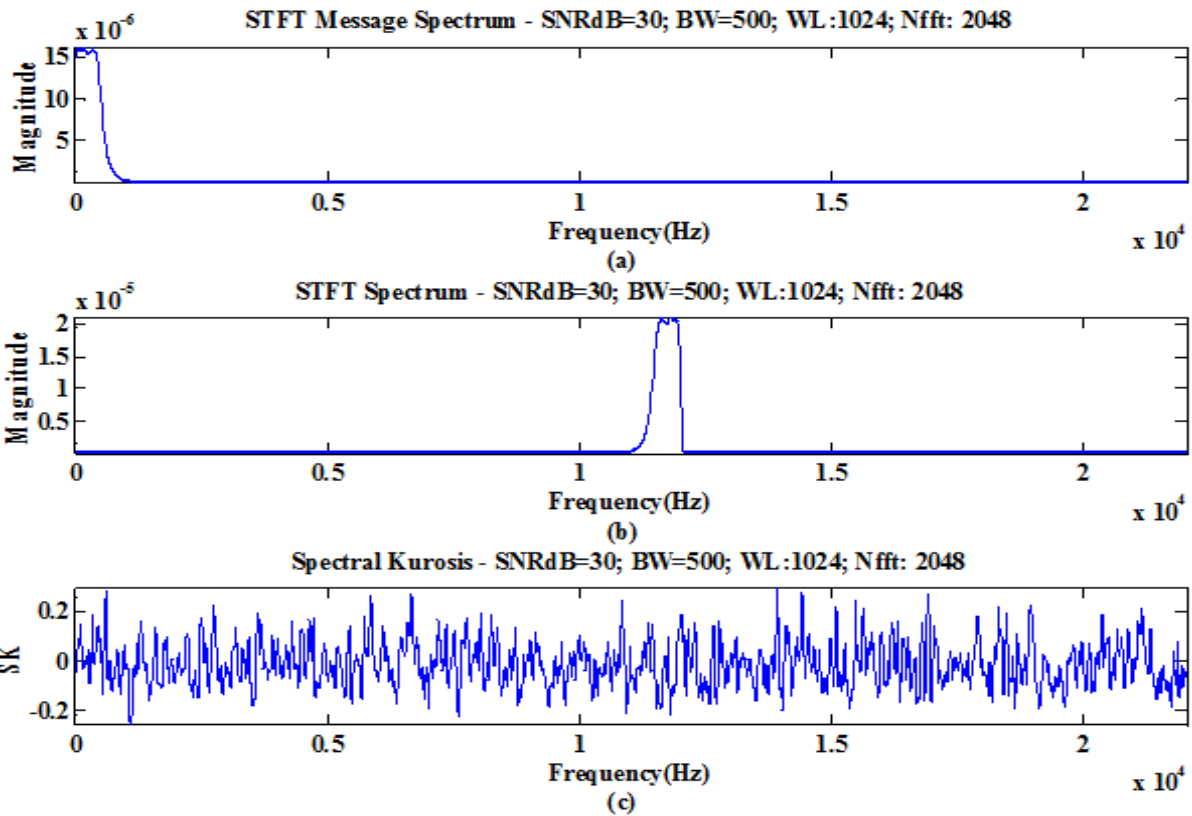


Fig. 8(a) : Averaged SFT spectrum of LSB+AWGN (b).Estimated Spectral Kurtosis for SNR=30dB

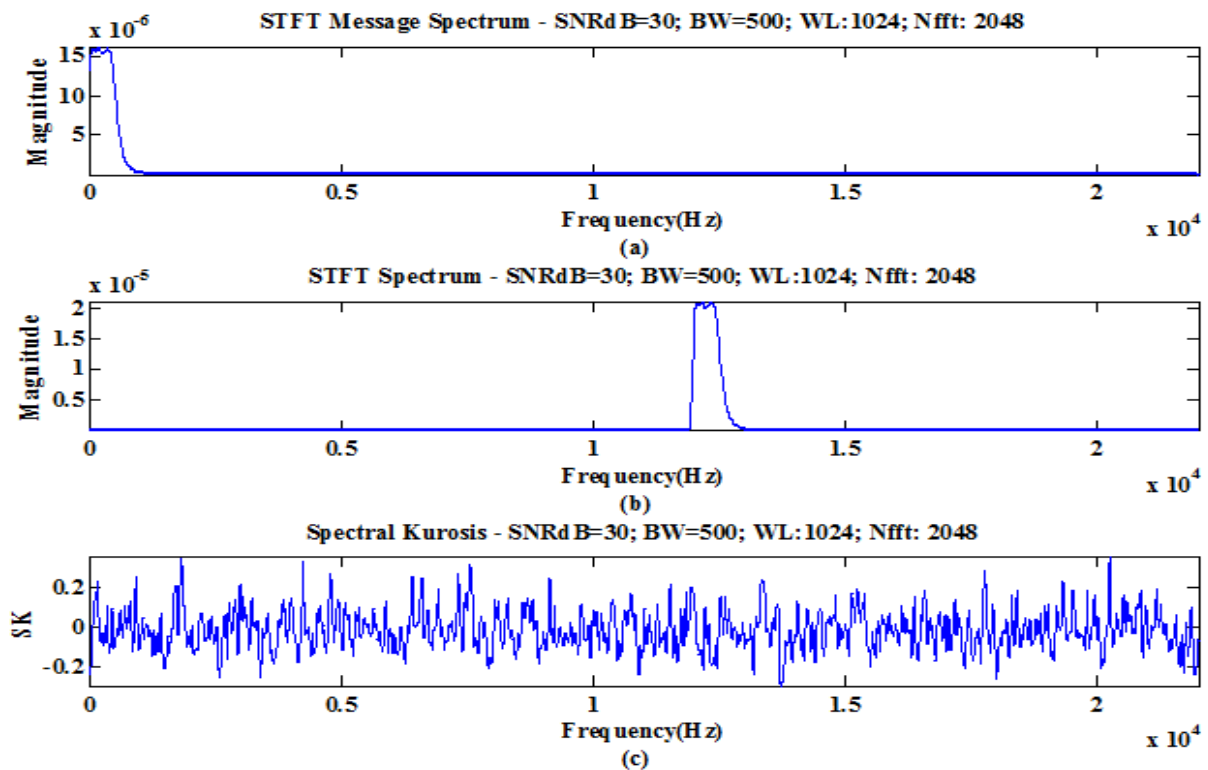


Fig. 9(a) : Averaged SFT spectrum of USB+AWGN (b).Estimated Spectral Kurtosis for SNR=30dB

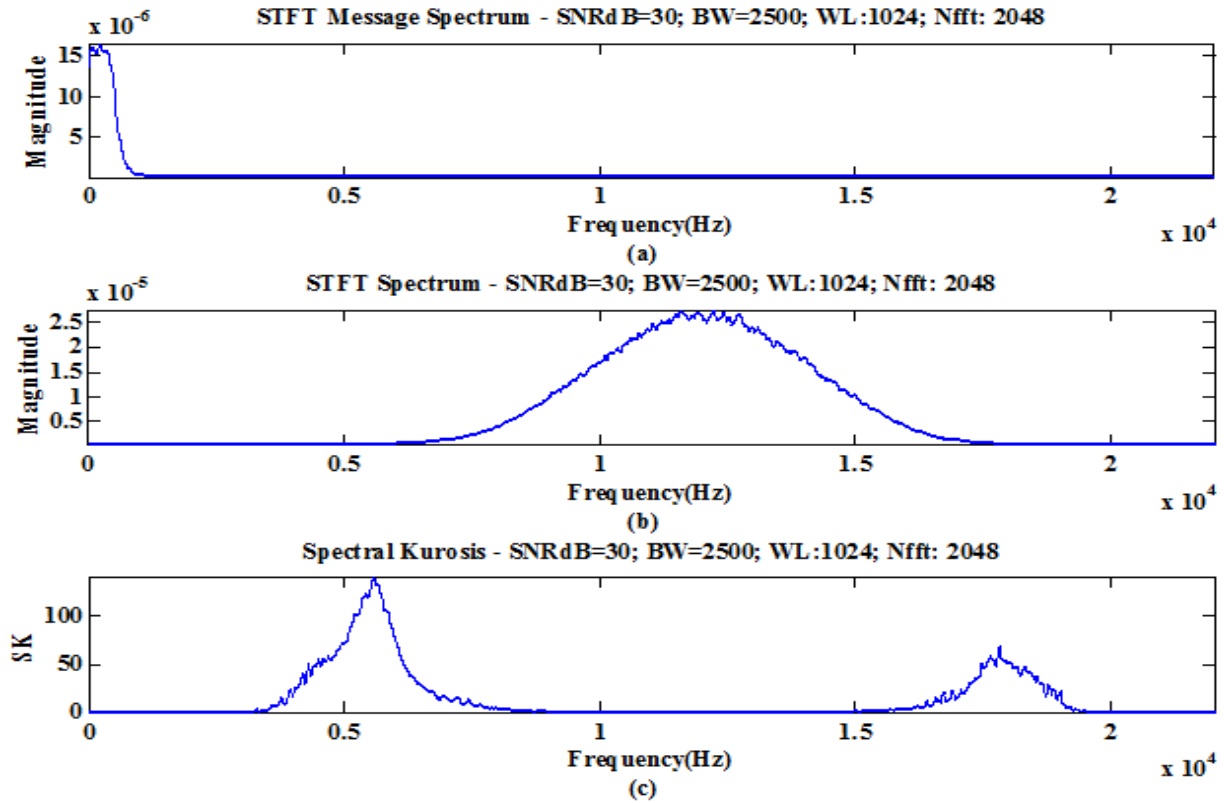


Fig. 10(a) : Averaged SFT spectrum of FM+AWGN (b).Estimated Spectral Kurtosis for SNR=30dB

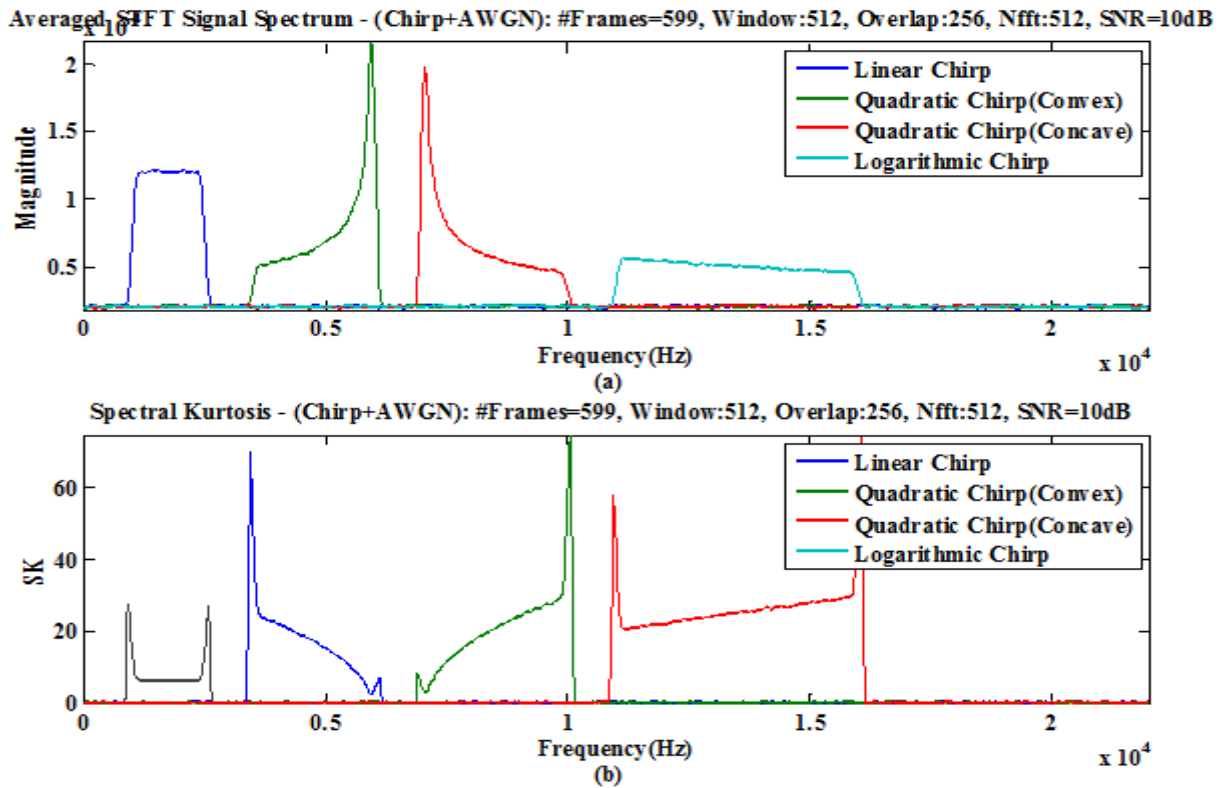


Fig. 11 : Mixture-6 (a). Averaged SFT spectrum of (b).Estimated Spectral Kurtosis for SNRs 10dB,

## VI. CONCLUSIONS AND FUTURE WORK

The cumulant based spectral kurtosis defined in frequency domain originally proposed for bearing fault detection and monitoring of electrical machines, is a promising tool for analyzing nonstationary signals. It complements the traditional power spectrum based on second order statistics. In this paper, the theory of spectral kurtosis is briefly reviewed from the fundamentals. The properties of spectral kurtosis of popular stationary signals, nonstationary signals and mixed processes are analytically given. Extensive Monte Carlo simulations were carried out to support the theory. The spectral kurtosis of the simulated stationary, nonstationary and mixed processes at different signal-to-noise ratios (SNRs) is estimated and the results are in good match with the previous analytical findings. The review highlights the usage of spectral kurtosis for other areas of signal processing like communications and radar signal processing. Future work could be (i). Obtaining closed-form expressions for spectral kurtosis of communication and radar signals (analog or digital modulated) (ii). classification of communication signals using spectral kurtosis at different SNRs. (iii). radar signal classification using spectral kurtosis.

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