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Mathematic Model and Kinematic Analysis for Robotic Arm

By Surachai Panich
Srinakharinwirot University, Thailand

Abstract- In this paper the mathematic model and kinematics of robotic arm are mainly analyzed. The robotic arm uses the Denavit Hartenberg (D-H) method to determine the parameters with transformation matrices. The direct kinematic analysis was conducted to determine the parameter of robotic arm by using Denavit Hartenberg (D-H) method. The calculated parameters of robotic arm were implemented by direct kinematics and compared with the measured parameter by rotary encoder to determine the accuracy of each parameter.

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Mathematic Model and Kinematic Analysis for Robotic Arm

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I. Introduction

A robotic arm is a type of mechanical arm, which can usually be programmed with similar functions to a human arm. The links of such a manipulator are connected by joints allowing either rotational motion translational displacement. The links of the manipulator can be considered to form a kinematic chain. The terminus of the kinematic chain of the manipulator is called the end effector and it is analogous to the human hand. The end effector or robot hand can be designed to perform any desired depended on the application such that robotic arms in automotive assembly lines perform a variety of tasks such as welding and parts rotation and placement during assembly. Huang and et al. [1] introduces method for solving the inverse kinematics equations with the D-H notation. The geometric analysis is use to calculate the motion trajectory of a robotic arm. They used Matlab software to verify and compare the results of the inverse kinematics equations analysis with the experimental results. Jie-Tong Zou and Des-Hun Tu [2] purposed a six D.O.F robotic arm for an intelligent robot. The used kinematic equations to verify the robotic arm by using the Denavit-Hartenberg (D-H) coordinate transformation method. The Inverse Kinematics analysis is used to determine six axes data and computed the forward or inverse kinematics by the Simulink function of Matlab software. Yoshimi, T and et al. [3] introduced robotic arm to execute a beverage can opening task. They control position trajectories for single robotic arm and dual robotic arm to open beverage can.

Author: Srinakharinwirot University, 114, Sukhumvit 23, Bangkok 10110, Thailand. e-mail: surachap@swu.ac.th

II. System Architecture of Robotic Arm

a) Mechanical Part

This research has main purpose to construct six degree of freedom robotic arm that it is firstly designed by graphic software and implemented by real hardware. The robotic arm is constructed onto a base that the drive and the part of first degree of freedom are mounted. In this work, the joints of the manipulator are driven through a worm drive gear arrangement. This provides great strength with relatively small motors, a zero backlash, the self locking properties of this arrangement, and zero power consumption while not moving. The disadvantage of this solution is the relatively low speed of the system. The robotic arm consists of six joints, which are two joints rotated about parallel axis with ground and four joints rotated about vertical axis with ground as shown in figure 1.

b) Electronic Components

In term of electronic control system, ET-BASE LPT is designed to be connected with motor drive board to perform desired robotic arm movements. Based on figure 1, ET-BASE LPT is connected with PC through parallel port. The interface circuit of control system is shown in figure 2.

Fig.1: Model of robotic arm.

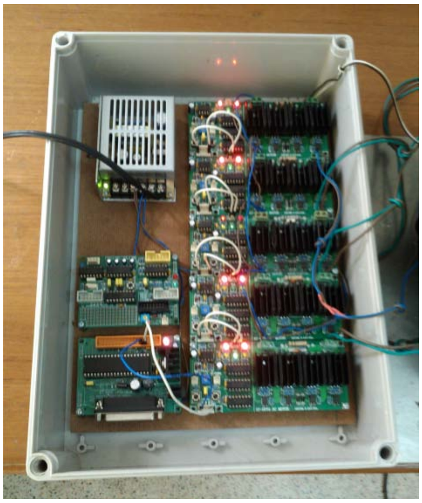

Fig.2 : Interface circuit of control system.

III. Kinematics of Robotic Arm

The relationships between the position, velocities and accelerations of a manipulator mainly are discussed in robotic arm research. Mathematical methods are developed to explain these relationships especially about position. The transformation between coordinate frames located in the base of the robot. The transformation specified position and orientation of manipulator in space respect to the base of the robot. To achieve kinematic analysis, the manipulator transformation must be defined in term of joint space related to the Cartesian coordinate. In Cartesian coordinate, the manipulator transform is a function of the position and orientation. The direct kinematic model describes Cartesian coordinates and orientation angles of the manipulator the joint variable. Conversely, the indirect kinematic model explains joints variable in term of Cartesian coordinates and orientation angles of the manipulator that it inverses of the direct kinematic model. The angles at the joints are the joint coordinates in a n dimensional joint space. The direct transform of a n link manipulator is equation 1.

$$^{R}T_{H} = A_1 \times A_2 \times A_3 \times \ldots A_n \tag{1}$$

The transformation from link $n-1$ to link n is completely defined by four link parameters below.

- A rotation about the z_{n-1} axis by the angle between the link (q_n)
- A translation along the z_{n-1} axis by the distance between the links (d_n)
- A translation along the x_{n-1} axis by the distance between the links (l_n)
- A rotation about the x_{n-1} axis by the angle between the link (a_n)

The transformation matrix of the coordinate could be abbreviated as D-H transformation matrix [5].

$$^{n-1}_{\ \ n}T = \begin{bmatrix} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & l_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & l_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Reference coordinate must be located on every joint and links between the joints. In this work, the link relations between the links and the joints are shown in figure 3. A joint's reference coordinate will change along with the rotation of the links. We need to consider the transformation relations between joint from link $n-1$ to link n that are in the two adjacent coordinate systems, while defining the four important parameters by the D-H transformation matrix is shown in table 1. The D-H transformation matrix can be formulated from two joints as below. General manipulators can move in three dimensions, so the orientation angles are complex to calculate, because the matrix elements contain terms with multiple angles. The method of orientation decomposition into three distinct rotations is not easy. The general roll-pitch-yaw orientation transform to obtain the orientation angles to the elements of the general transform matrix.

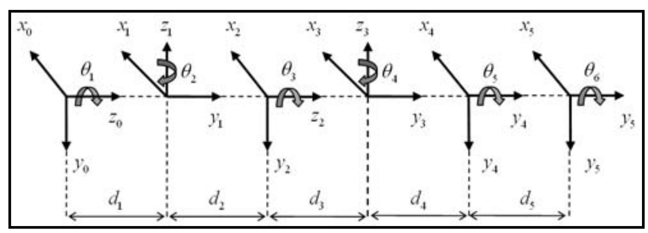

Fig.3 : Coordinate frames of robotic arm.

Table 1 : Parameter of robotic arm

Link	Variable	d_n	l_n	α_n
1	θ_1	d_1	0	90
2	θ_2	d_2	0	-90
3	θ_3	d_3	0	90
4	θ_4	d_4	0	-90
5	θ_5	d_5	0	90
6	θ_6	0	0	0

To obtain the Cartesian coordinates in term of joint coordinates and end effector, the elements of manipulator transformation matrix must be compared with general transformation matrix. The orientation elements of the general transform are dimensionless and translation elements have the dimension of length. The general transformation matrix can be given as

$$^{R}T_{H} = \begin{bmatrix} X_x & Y_x & Z_x & P_x \\ X_y & Y_y & Z_y & P_y \\ X_z & Y_z & Z_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$= Translation\left(P_x,\ P_y,\ P_z\right) \times Rotation\left(Roll(f),\ Pitch(q),\ Yaw(y)\right)$$

The compared manipulator transformation matrix with general transformation matrix is formulated as

$$ {}^{R}T_H = \begin{bmatrix} X_x & Y_x & Z_x & P_x \\ X_y & Y_y & Z_y & P_y \\ X_z & Y_z & Z_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4) $$

The element parameter can be determined by equating terms in these matrices as shown below.

$$ X_x = b_{11} = \begin{pmatrix} C_{123456} - C_{1456}S_{23} - C_{56}S_{14} - C_{126}S_{35} \\ - C_{36}S_{125} + C_1S_{2346} - C_{123}S_{46} - C_4S_{16} \end{pmatrix} $$

$$ X_y = b_{21} = \begin{pmatrix} C_{23456}S_1 - C_{456}S_{123} - C_{156}S_4 - C_{26}S_{135} \\ - C_{36}S_{235} - C_{23}S_{146} + S_{12346} + C_{14} \end{pmatrix} $$

$$ X_z = b_{31} = \left(C_{3456}S_2 - C_6S_{235} - S_{2346}\right) $$

$$ Y_x = b_{12} = \left(C_{35}S_{12} - C_{14}S_{235} + C_{1234}S_5 - S_{145} + C_{125}S_3\right) $$

$$ Y_y = b_{22} = \begin{pmatrix} C_{35}S_{23} - C_4S_{1235} + C_{234}S_{15} - C_1S_{45} \\ + C_{25}S_{13} \end{pmatrix} $$

$$ Y_z = b_{32} = \left(C_{34}S_{25} + C_5S_{23}\right) $$

$$ Z_x = b_{13} = \begin{pmatrix} C_3S_{1256} - C_{12345}S_6 + C_{145}S_{236} + C_5S_{146} \\ + C_{12}S_{356} + C_{16}S_{234} - C_{1236}S_4 - C_{46}S_1 \end{pmatrix} $$

$$ Z_y = b_{23} = \begin{pmatrix} C_{45}S_{1236} - C_{2345}S_{16} + C_{15}S_{46} + C_2S_{1356} \\ + C_3S_{2356} - C_{236}S_{14} + C_6S_{1234} + C_{146} \end{pmatrix} $$

$$ Z_z = b_{33} = \left(S_{2356} - C_{345}S_{26} - C_6S_{234}\right) $$

$$ P_x = b_{31} = \left(C_{3456}S_2 - C_6S_{235} - S_{2346}\right) $$

$$ P_y = b_{24} = \begin{pmatrix} C_4S_{1235}d_6 - C_{234}S_{15}d_6 + C_1S_{45}d_6 \\ - C_{25}S_{13}d_6 - C_{35}S_{23}d_6 + C_{23}S_{14}d_5 \\ - S_{1234}d_5 - C_{14}d_5 - C_2S_{13}d_4 \\ - C_3S_{12}d_4 - 2C_1d_2 \end{pmatrix} $$

$$ P_z = b_{34} = \left(S_{234}d_5 - C_{34}S_{25}d_6 - C_5S_{23}d_6 + d_1 - S_{23}d_4\right) $$

The orientations angles of the end effector can be determined by Roll-Pitch-Yaw orientation transform [5].

$$ \tan\phi = \frac{X_y}{X_x} = \frac{\begin{pmatrix} C_{23456}S_1 - C_{456}S_{123} - C_{156}S_4 \\ - C_{26}S_{135} - C_{36}S_{235} - C_{23}S_{146} \\ + S_{12346} + C_{14} \end{pmatrix}}{\begin{pmatrix} C_{123456} - C_{1456}S_{23} - C_{56}S_{14} \\ - C_{126}S_{35} - C_{36}S_{125} + C_1S_{2346} \\ - C_{123}S_{46} - C_4S_{16} \end{pmatrix}} \quad (5) $$

$$ \tan\theta = \frac{-X_z}{X_xC\phi + X_yS\phi} $$

$$ = \frac{-\left(C_{3456}S_2 - C_6S_{235} - S_{2346}\right)}{\begin{pmatrix} C\phi \begin{pmatrix} C_{123456} - C_{1456}S_{23} - C_{56}S_{14} \\ - C_{126}S_{35} - C_{36}S_{125} + C_1S_{2346} \\ - C_{123}S_{46} - C_4S_{16} \end{pmatrix} \\ + S\phi \begin{pmatrix} C_{23456}S_1 - C_{456}S_{123} - C_{156}S_4 \\ - C_{26}S_{135} - C_{36}S_{235} - C_{23}S_{146} \\ + S_{12346} + C_{14} \end{pmatrix} \end{pmatrix}} \quad (6) $$

$$ \tan\psi = \frac{Y_z}{Z_z} = \frac{\left(C_{34}S_{25} + C_5S_{23}\right)}{\left(S_{2356} - C_{345}S_{26} - C_6S_{234}\right)} \quad (7) $$

IV. Experiments and Results

The parameters of robotic arm were developed and tested by mathematic modeling and kinematic analysis. The robotic arm uses the Denavit Hartenberg (D-H) method to determine the parameters with transformation matrices. In experiment, we tested the accuracy of parameter from real hardware and the parameter is calculated from Denavit Hartenberg (D-H) method. The parameter of each joint is used in experiments to determine the rotation angle error of each joint. The angle error is difference angle between measured angle from rotary encoder compared with calculated angle. We selected the angle at 30 degree as desired angle and the results of angle error of each joint are shown in figure 4.

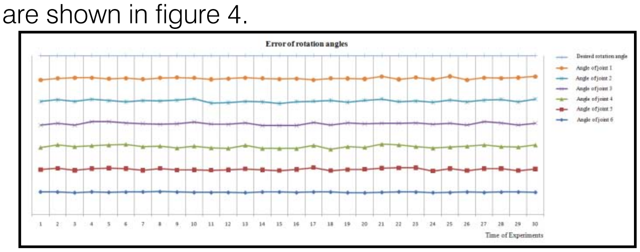

Fig.4 : Error angle of joint rotation.

V. Conclusion

A kinematic analysis of the real robotic arm was designed, analyzed and constructed. The direct kinematic analysis was conducted to determine the

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parameter of robotic arm by using Denavit-Hartenberg (D-H) method. The calculated parameters of robotic arm were implemented by direct kinematics and compared with the measured parameter by rotary encoder. The future work is to make the robotic arm that it can grab objects by using machine vision system.

VI. Acknowledgment

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