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Using Fuzzy Goal Programming Technique to Obtain the Optimum Production of Vehicle Spare Parts, A Case Study

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Abstract- This paper studies the vehicle spare parts production problem to obtain the optimum production rate under fuzzy capital budget. We applied the integer goal programming technique to determine the best compromise solution. There are two goals in our case study. These goals are minimization of the uncertain capital budget and maximization of the uncertain expected profits. The case study is a factory which produces different types of vehicle heat exchangers. The results indicate that the problem solution depends on the membership function and the **α**-cut. The optimum quantities of heat exchangers' production are found to be biased to the lower limit of production.

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Using Fuzzy Goal Programming Technique to Obtain the Optimum Production of Vehicle Spare Parts, A Case Study

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Abstract- This paper studies the vehicle spare parts production problem to obtain the optimum production rate under fuzzy capital budget. We applied the integer goal programming technique to determine the best compromise solution. There are two goals in our case study. These goals are minimization of the uncertain capital budget and maximization of the uncertain expected profits. The case study is a factory which produces different types of vehicle heat exchangers. The results indicate that the problem solution depends on the membership function and the α-cut. The optimum quantities of heat exchangers' production are found to be biased to the lower limit of production.

I. Introduction

Due to huge changes in vehicle prices, fuels, oils, and spare parts' prices, it is important for transportation companies to study their fleet of vehicles from the opinion of vehicle operations economics. The vehicle operations economics field studies the maximization of profit and/or minimization of costs. One of the topics that is needed to be studied and applied to the real world applications is finding the optimum production rate of vehicle spare parts.

The capital budgeting in automobile firms is introduced by AbouelNour [1]. The optimal distribution of a certain amount of capital budgeting in production of vehicle spare parts had been obtained using integer programming technique in formulation and solution. Mohan A. et al [2] developed a decision support system for budget allocation of an R&D organization.

Multiple criteria decision making (MCDM) refers to making decisions in the presence of multiple, usually, conflicting objectives. These problems can be solved either directly [10] or using different secularization forms (SOP) [11]. Most investigators in the general area of multiobjective mathematical programming agree that goal programming technique represents the work horse of multiobjective mathematical programming. Lee S.M. et al [3] introduced capital budgeting for multiple objectives. Zamfirescu, L. et al [4] prepared a goal programming as a decision model for performance-based budgeting.

Goal programming is found to be useful in real life situations, for many problems it may not be possible to satisfy certain specified goals within given constraints. Then problem then becomes one of maximizing the degree of attainment of these goals. Using goal programming for marketing decisions with a case study is illustrated by Lee, S. et al [5]. Lee, S. [6] introduced goal programming for decision analysis. Dauer, J.P. et al [7] introduce a finite iteration algorithm for solving general goal programming problems. The approach enables one to solve linear, no linear, integer and other goal programming problems using the corresponding optimization technique in an iterative manner.

In the real life systems, the uncertainties and vagueness accomplishing the determination of cost of production, the expected selling price and upper and lower pounds of the production of the spare parts make the problem fuzzy or stochastic rather than deterministic one.

Hu, C.F. et al [8] introduced a fuzzy goal programming approach to multi-objective optimization problem with priorities. Khalili, K. et al [9] presented a paper about solving multi-period project selection problems with fuzzy goal programming beased on TOPSIS and fuzzy preference relation. Eid, M.H. [12] gives methods of solving integer multi criteria decision making problems with fuzzy parameters.

This paper introduces the analysis of the optimum vehicle spare parts production, where the problem of distribution of fuzzy capital budget is applied.

The technique is introduced in Sakawa [13] for transforming fuzzy problems to non-fuzzy form is combined with the interactive approach to goal programming [12] to develop the method of solution of such problem, where the method is through two goals is used. The first goal is the fuzzy capital budgeting minimization, while the second one is fuzzy profit maximization.

Our study is applied on a company which produces different types of vehicles' heat exchangers. The aim of this study is to obtain the optimum number of

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heat exchangers which maximizes profits and minimizes the production cost under fuzzy budget.

II. Concept of Goal Programming

a) General

Goal programming is a modification and extension of linear programming technique. The goal programming approach allows a simultaneous solution of a system of complex objectives rather than a single objective. In other words, goal programming is a technique that is capable of handling decision problems that deal with a single goal and multiple sub-goals, as well as problems with multiple goals and multiple sub-goals. In addition, the objective function of a goal programming model may be composed of non-homogeneous units of measure, such as pounds and dollars, rather than one type of units. Often, multiple goals of management are in conflict or are achievable only at the expense of other goals. Furthermore, these goals are incommensurable. Thus, the solution of the problem requires an establishment of a hierarchy of importance among these incompatible goals so that the low-order goals are considered only after the higher-order goals are satisfied or have reached the point beyond which no further improvements are desired. In goal programming, instead of trying to maximize or minimize the objective criterion directly as in linear programming, deviations between goals and what can be achieved within the given set of constraints are to be minimized [6].

b) Formulation of the Goal Programming Proble

$$min. p_1(\varphi_1(w_1, d_1^+, d_1^-)) + p_2(\varphi_2(w_2, d_2^+, d_2^-)) + \cdots + p_n(\varphi_n(w_n, d_n^+, d_n^-))$$

Subject to:

$F_i(x) + d_i^- - d_i^+ = b_i, \qquad i = 1, 2, \dots, n ,$

$x \in M ,$

$x, d_i^+, d_i^- \geq 0, \qquad i = 1, 2, \dots, n$

Where:

$P_1 >>> p_2 >> \dots >>> p_n$: are the priority structure;

$\varphi_i(w_i, d_i^+, d_i^-)$: are linear functions of the concerned variables;

$w_i > 0$: are different weights;

d_i^+: are under achievement deviational variables;

d_i^-: are over achievement deviational variables;

M: is the set representing the system constraints.

c) Methods of Solving Goal Programming Problems

There are more than one method for solving goal programming problems. From these methods, the most common are:

1. The graphical method of goal programming.
2. The simplex method of goal programming.
3. The interactive approach of goal programming.

These methods are illustrated in [6, 7].

In this work the interactive approach of goal programming was used.

III. Vehicle Spare Parts Production Formulation

The mathematical model of vehicle spare parts production in fuzzy environment which minimizes the total fuzzy capital budgeting and maximizes the total expected fuzzy profit is formulated as an integer goal programming problem with fuzzy parameters. There are two goals here, fuzzy capital budgeting goal and fuzzy profit goal. Also, there are a set of constraints represent upper and lower bounds of the quantities which should be produced from the different types of spare parts.

Let:

x_i: is the quantity of production from spare parts i, i= 1, 2, …, n.

c_i: is the production cost of the spare part I, i= 1, 2, …, n.

$a + \lambda_1^{\sim} \bar{a}$: is the fuzzy budget which allocated to the production of spare parts.

b_i: is the profit associated with the spare part i, i= 1, 2, …, n.

$b + \lambda_2^{\sim} \bar{b}$: is the total fuzzy profit.

$L_i + \mu_i^{\sim} \bar{L}_i$: is fuzzy lower bound of the production of spare parts i, i= 1, 2, …, n.

$U_i + v_i^{\sim}\bar{U}_i$: is fuzzy upper bound of the production of spare parts i, i= 1, 2, …, n.

$p_1 >>> p_2$: is the priority structure.

d_1^-, d_2^-: are the under achievements of the deviational variables.

d_1^+, d_2^+: are the upper achievements of the deviational variables.

Then the problem takes the following form:

$$min.\ z = p_1 d_1^+ + p_2 d_2^-$$

Subject to:

$$\sum_{i=1}^{n} c_i x_i + d_1^- - d_1^+ = a + \tilde{\lambda}_1 \bar{a}$$

$$\sum_{i=1}^{n} b_i x_i + d_2^- - d_2^+ = b + \tilde{\lambda}_2 \bar{b}$$

$L_i + \tilde{\mu}_i \bar{L}_i \le x_i \le U_i + \tilde{v}_i \bar{U}_i$, i= 1. 2, …, n

$x_i, d_1^-, d_1^+, d_2^-, d_2^+ \ge 0, x_i\ integers$, i= 1. 2, …, n

IV. Deterministic (Crisp) Form of the Problem

The mathematical formulation of spare parts production in fuzzy environment problem can be transformed to the following non-fuzzy form (α – fuzzy integer goal programming problem):

$$min.\ z = p_1 d_1^+ + p_2 d_2^-$$

Subject to:

$$\sum_{i=1}^{n} c_i x_i + d_1^- - d_1^+ = a + \lambda_1 \bar{a}$$

$$\sum_{i=1}^{n} b_i x_i + d_2^- - d_2^+ = b + \lambda_2 \bar{b}$$

$S_1 \le \lambda_1 \le T_1$

$S_2 \le \lambda_2 \le T_2$

$L_i + \mu_i \bar{L}_i \le x_i \le U_i + v_i \bar{U}_i$, i= 1. 2, …, n

$Q_i \le \mu_i \le R_i$, i= 1. 2, …, n

$M_i \le v_i \le N_i$, i= 1. 2, …, n

$x_i, d_1^-, d_1^+, d_2^-, d_2^+, \lambda_1, \lambda_2, \mu_i, v_i \ge 0, x_i\ integers$, i= 1. 2, …, n

V. Vehicle Spare Parts Production Application

a) *Data Collection*

The application was carried out in a vehicle heat exchangers production factory which produce different types of vehicle heat exchangers, for a group of ten different types of heat exchangers used for different types of vehicles which operates with diesel engines.

Table (1) illustrates the variable name, company part number, minimum limit, maximum limit of production, production cost and selling price.

Table (1) : Collected Data

Variable Name	Company Part Number	Minimum Limit of Production	Maximum Limit of Production	Production Cost (in $)	Selling Price (in $)
x_1	450 501 22 01	30	100	277	366
x_2	450 501 70 01	500	12000	175	255
x_3	450 501 24 01	4600	15000	144	203.2
x_4	450 501 19 01	600	1500	168	240
x_5	620 50591 04	700	1500	100	160
x_6	620 50594 04	50	1200	41.4	90

x_7	630 501 01 01	600	2750	80.3	130
x_8	655 501 86 01	1000	5500	71.7	150
x_9	350 501 19 01	600	2100	359	400
x_{10}	560 501 22 01	40	450	149.5	200

Where:

x_1 : is the number of heat exchangers for 3.5t transporter.

x_2: is the number of heat exchangers for 2.8t transporter.

x_3: is the number of heat exchangers for 2t transporter.

x_4: is the number of heat exchangers for 2.5t transporter.

x_5: is the number of heat exchangers for 5 cylinder microbus.

x_6: is the number of heat exchangers for 4 cylinder microbus.

x_7: is the number of heat exchangers for 5 cylinder microbus.

x_8: is the number of heat exchangers for 5 cylinder microbus.

x_9: is the number of heat exchangers for 6t transporter.

x_{10}: is the number of heat exchangers for 2.5t transporter.

The fuzzy budget allocated to the production $(a + \tilde{\lambda}_1 \bar{a})$ = LE 4000000 + 100000 $\tilde{\lambda}_1$

The fuzzy expected profit (b + $\tilde{\lambda}_2 \bar{b}$) = LE 200000 + 500000 $\tilde{\lambda}_2$

b) Mathematical Formulation of Applied Proble

The applied problem takes the following form:

$min.\ z = p_1 d_1^- + p_2 d_2^-$

Subject to:

$$277\,x_1 + 175\,x_2 + 144\,x_3 + 168\,x_4 + 100\,x_5 + 41.4\,x_6 + 80.3\,x_7 + 71.7\,x_8 + 359\,x_9 + 149.5\,x_{10} + d_1^- - d_1^+ \le 4000000 + 100000\,\tilde{\lambda}_1$$

$$89\,x_1 + 80\,x_2 + 59.2\,x_3 + 72\,x_4 + 60\,x_5 + 48.6\,x_6 + 49.7\,x_7 + 78.3\,x_8 + 41\,x_9 + 50.5\,x_{10} + d_2^- - d_2^+ \ge 200000 + 50000\,\tilde{\lambda}_2$$

$30 + 4\,\tilde{\mu}_1 \le x_1 \le 60 + 10\,\tilde{v}_1$

$500 + 200\,\tilde{\mu}_2 \le x_2 \le 800 + 500\,\tilde{v}_2$

$4600 + 150\,\tilde{\mu}_3 \le x_3 \le 9000 + 750\,\tilde{v}_3$

$600 + 100\,\tilde{\mu}_4 \le x_4 \le 700 + 100\,\tilde{v}_4$

$700 + 70\,\tilde{\mu}_5 \le x_5 \le 800 + 140\,\tilde{v}_5$

$50 + 10\,\tilde{\mu}_6 \le x_6 \le 600 + 100\,\tilde{v}_6$

$600 + 100\,\tilde{\mu}_7 \le x_7 \le 1250 + 200\,\tilde{v}_7$

$1000 + 200\,\tilde{\mu}_8 \le x_8 \le 2500 + 500\,\tilde{v}_8$

$600 + 100\,\tilde{\mu}_9 \le x_9 \le 1100 + 200\,\tilde{v}_9$

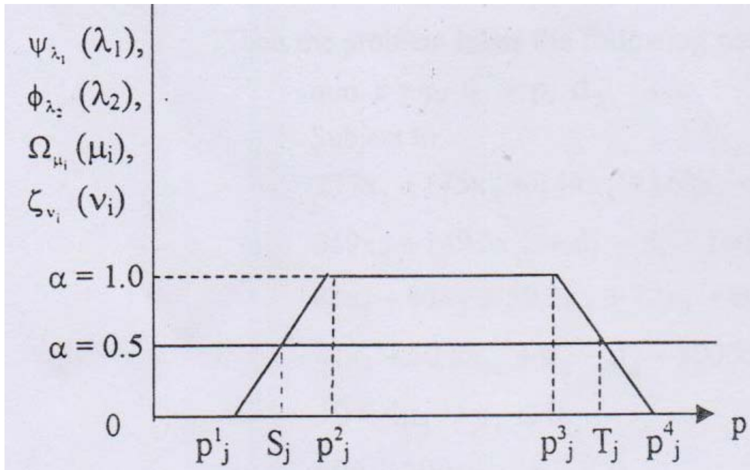
$40 + 10\,\tilde{\mu}_{10} \le x_{10} \le 250 + 50\,\tilde{v}_{10}$

$x_i, d_1^-, d_1^+, d_2^-, d_2^+, \tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}_i, \tilde{v}_i \ge 0, x_i$ *integers* , i= 1. 2, …, n

Where:

$\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}_i, \tilde{v}_i$, i= 1. 2, …, n are fuzzy parameters.

The fuzzy parameters are represented by the following membership function.



Where:

$$\begin{matrix}\psi_{\lambda 1}(\tilde{\lambda}_1),\\ \phi_{\lambda 2}(\tilde{\lambda}_2),\\ \Omega_{\mu 1}(\tilde{\mu}_i),\\ \zeta_{v1}(\tilde{v}_i)\end{matrix} = \begin{cases} 0 & -\infty \le \lambda_i \le p_j^1 \\ \dfrac{\lambda_i - p_j^1}{p_j^2 - p_j^1} & p_j^1 \le \lambda_i \le p_j^2 \\ 1 & p_j^2 \le \lambda_i \le p_j^3 \\ \dfrac{\lambda_i - p_j^3}{p_j^4 - p_j^3} & p_j^3 \le \lambda_i \le p_j^4 \\ 0 & p_j^4 \le \lambda_i \le \infty \end{cases}$$

By taking the cut α = 0.5, then table 2 illustrates values of S_j, T_j, Q_j, R_j, M_j, N_j.

Table (2) : Values of S_j, T_j, Q_j, R_j, M_j, N_j

	p_j^1	p_j^2	p_j^3	p_j^4	S_j	T_j	Q_j	R_j	M_j	N_j
λ_1	3	4	7	10	3.5	8.5				
λ_2	1	2	3	4	1.5	3.5				
μ_1	1	2	4	6			1.5	5		
μ_2	1	3	4	6			2	5		
μ_3	1	3	6	8			2	7		
μ_4	1	2	3	5			1.5	4		
μ_5	1	2	3	5			1.5	4		
μ_6	1	2	4	6			1.5	5		
μ_7	1	2	3	5			1.5	4		
μ_8	2	4	7	8			3	7.5		
μ_9	1	2	3	5			1.5	4		
μ_{10}	1	2	5	7			1.5	6		
v_1	1	2	3	5					1.5	4
v_2	3	5	7	9					4	8
v_3	2	4	7	9					3	8
v_4	3	5	7	9					4	8
v_5	2	4	4	6					3	5
v_6	2	4	5	7					3	6
v_7	1	3	7	8					2	7.5
v_8	3	4	5	7					3.5	6
v_9	2	4	4	6					3	5
v_{10}	1	2	3	5					1.5	4

Then the problem takes the following non-fuzzy (Crisp) form:

$$min.\ z = p_1 d_1^+ + p_2 d_2^-$$

Subject to:

$$277\,x_1 + 175\,x_2 + 144\,x_3 + 168\,x_4 + 100\,x_5 + 41.4\,x_6 + 80.3\,x_7 + 71.7\,x_8 + 359\,x_9 + 149.5\,x_{10} + d_1^- - d_1^+ - 100000\,\lambda_1 \le 4000000$$

$$89\,x_1 + 80\,x_2 + 59.2\,x_3 + 72\,x_4 + 60\,x_5 + 48.6\,x_6 + 49.7\,x_7 + 78.3\,x_8 + 41\,x_9 + 50.5\,x_{10} + d_2^- - d_2^+ - 50000\,\lambda_2 \ge 200000$$

$30 + 4\,\mu_1 - x_1 \le 0$

$x_1 - 10\,v_1 - 60 \le 0$

$500 + 200\,\mu_2 - x_2 \le 0$

$x_2 - 500\,v_2 - 8000 \le 0$

$4600 + 150\,\mu_3 - x_3 \le 0$

$x_3 - 750\,v_3 - 9000 \le 0$

$600 + 100\,\mu_4 - x_4 \le 0$

$x_4 - 100\,v_4 - 700 \le 0$

$700 + 70\,\mu_5 - x_5 \le 0$

$x_5 - 140\,v_5 - 800 \le 0$

$50 + 10\,\mu_6 - x_6 \le 0$

$x_6 - 100\,v_6 - 600 \le 0$

$600 + 100\,\mu_7 - x_7 \le 0$

$x_7 - 200\,v_7 - 1250 \le 0$

$1000 + 200\,\mu_8 - x_8 \le 0$

$x_8 - 500\,v_8 - 2500 \le 0$

$600 + 100\,\mu_9 - x_9 \le 0$

$x_9 - 200\,v_9 - 1100 \le 0$

$40 + 10\,\mu_{10} - x_{10} \le 0$

$x_{10} - 50\,v_{10} - 250 \le 0$

$3.5 - \lambda_1 \le 0 \lambda_1 - 8.5 \le 0$

$1.5 - \lambda_2 \le 0 \lambda_2 - 3.5 \le 0$

$1.5 - \mu_1 \le 0 \mu_1 - 5 \le 0$

$2 - \mu_2 \le 0 \mu_2 - 5 \le 0$

$2 - \mu_3 \le 0 \mu_3 - 7 \le 0$

$1.5 - \mu_4 \le 0 \mu_4 - 4 \le 0$

$1.5 - \mu_5 \le 0 \mu_5 - 4 \le 0$

$1.5 - \mu_6 \le 0 \mu_6 - 5 \le 0$

$1.5 - \mu_7 \le 0 \mu_7 - 4 \le 0$

$3 - \mu_8 \le 0 \mu_8 - 7.5 \le 0$

$1.5 - \mu_9 \le 0 \mu_9 - 4 \le 0$

$1.5 - \mu_{10} \le 0 \mu_{10} - 6 \le 0$

$1.5 - v_1 \le 0 v_1 - 4 \le 0$

$4 - v_2 \le 0 v_2 - 8 \le 0$

$3 - v_3 \le 0 v_3 - 8 \le 0$

$4 - v_4 \le 0 v_4 - 8 \le 0$

$3 - v_5 \le 0 v_5 - 5 \le 0$

$3 - v_6 \le 0 v_6 - 6 \le 0$

$2 - v_7 \le 0 v_7 - 7.5 \le 0$

$3.5 - v_8 \le 0 v_8 - 6 \le 0$

$3 - v_9 \le 0 v_9 - 5 \le 0$

$1.5 - v_{10} \le 0$

$v_{10} - 4 \le 0$

$x_i, d_1^-, d_1^+, d_2^-, d_2^+, \lambda_1, \lambda_2, \mu_i, v_i \ge 0, x_i$ *integers* , i= 1. 2, …, n

c) *Application Results*

The problem is solved by a mixed integer linear programming package using the iterative approach of goal programming. The optimum solution which minimizes the allocated capital budget and maximizes the profit is given in Table (3).

Table (3) : 0.5 Application Optimum Quantity of Production

Variable Name	Company Part Number	0.5 Optimum Quantity
x_1	450 501 22 01	36
x_2	450 501 70 01	900
x_3	450 501 24 01	4900
x_4	450 501 19 01	750
x_5	620 50591 04	805
x_6	620 50594 04	65
x_7	630 501 01 01	750
x_8	655 501 86 01	1600
x_9	350 501 19 01	750
x_{10}	560 501 22 01	55

VI. Conclusion

1. A method of obtaining the optimum production of spare parts in fuzzy environment is presented. This method is based on the formulation of the problem in the form of integer goal programming problem with fuzzy parameters in the right hand side of its constraints. The problem is solved by a mixed integer linear programming (MILP) package using the interactive approach of goal programming. It should be mentioned that the solution depends on

the membership functions determination and the α–cut.

2. From the application of the vehicle heat exchangers production factory, it can be stated that:

- The minimum production cost is $1544680.5
- The maximum profit is $666825.5
- The optimum quantities of heat exchangers are biased to the lower limits of the production as a result of solution of the problem with the minimization of total capital budget goal with a higher priority than the maximization of the expected profit goal.

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