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Reliability Analysis of Timber Roof Truss Systems using Genetic Algorithm

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Abstract- Structural reliability analysis was carried out on the *Mansonia altissima* timber, to ascertain its structural performance in timber roof truss system. Structural analysis of the timber truss was in accordance with Eurocode 5 (2004) and was carried out under the Ultimate Limit State of loading. A developed MATLAB (2010) programme was employed for reliability analysis of the timber roof truss of *Mansonia altissima* timber so designed, to ascertain its level of safety using GA-based First-Order Reliability Method. The uncertainties in the strength and load variables were accommodated in the reliability analysis. The result of the analysis revealed that the Joint failure mode is the critical safety index that is minimum safety index among the failure modes of the truss under the design conditions. The *Mansonia altissima* timber was found to be a satisfactory structural element for timber roof truss at depth of 75mm, breadth of 50mm and under the ultimate limit state of loading with the corresponding of 2.58. Sensitivity analysis proves that the degree of reliability of the timber roof truss can be improved if cross-sections of species, diameter of nail at joint, pitch of truss and loadings are suitable selected.

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I. Introduction

The traditional way of dealing with uncertainties in design process is to use conservative values of the uncertain quantities and/or safety factors in a deterministic approach. The shortcomings of this approach may become more obvious when designing for loads with very high variability. It is not easy to account for all factors that affect assessment of loads consistent with acceptable risk (Anthony, 1991; Afolayan, 1999 and Abubakar and Mohammed, 2011). However, since no structure may be free from the possibility of failure, loads must be designed to fit the risk. A deterministic design approach does not an explicit consideration for this. A more meaningful treatment of uncertainties in structural timber can be through a probability-based design philosophy, which has received considerable attention (Afolayan, 2005; Abejide, 2006; Ahmed *et al.*, 2010,; Kachalla and Kolo, 2012; Aguwa, 2013; Ditlevsen and Madsen, 2005).

The tensile and compressive properties of the timber are particularly important when applying timber as structural components such as roof trusses (Ahmad *et al.*, 2010). The tensile strength of the lower chord of a truss is considered the critical design parameter (Bostrom *et al.*, 1999). It had been identified that joints in timber structures are the most critical components that need special extensive research (Racher, 1995; Smith and Foliente, 2002; Riley and Sadek, 2003). According to Frank and Philip (1997), bottom chord joints are located in areas such that they experience a small bending moment, and are stressed primarily in tension. He determined the steel net section capacity of bottom chord joints of wood trusses subjected to tension and moment loading.

Genetic algorithm is intelligent search and optimization method that work very similar to the principles of natural evolution called Darwin's survival-of-the fittest principles. If GA is incorporated in to reliability methods such as FORM, population of limit functions with different combination design variables are considered, and safety index is obtained for each set. The sets of safety index are assembled and the minimum that is the globally best and fittest is considered. Several generations are further considered through crossover, mutation and elitism operation in GA until a convergent is achieved. This widen the search space for the global minimum (critical) safety index (Mohammed and Abubakar, 2011; Cheng, 2007; Wang and Ghosn, 2005).

II. Limit State Functions

The Eurocode 5 design criteria of roof truss members subjected to combination of varying design actions are briefly reviewed. Identification of the significant failure modes was deterministically analysed and of failure modes (tension, compression, bending of the top and bottom chord) were established.

a) Structural model

The analysed structural model of the truss system is shown in Fig. 1. It was assumed that the truss had a roof pitch of 35^0, spacing between the trusses of 1.2 m, Length of 7.2 m, dead load of 0.55 kN/m^2, fixed nailed length of 90 mm, nail diameter of 4.0mm and dead-to-live load ratio of 0.275. The roofing material used was aluminium-roofing sheets. The connections between the members were assumed to be pinned joints as stipulated in Eurocode 5 (2004).

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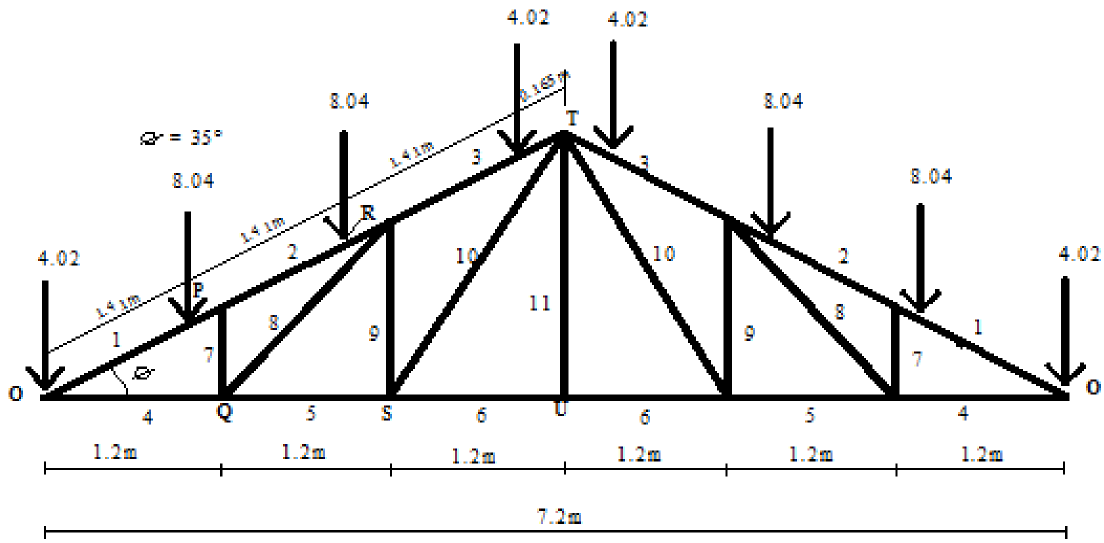

Fig. 1 : Typical truss subjected to nodal design loadings

The following limit state functions were established from the structural analysis of the model.

i. *Compression failure criterion*

The limit state function for compression is given as:

$$G(x) = \frac{(k_{mod} f_{c,0,k})}{\gamma_m} - \sigma_{ci,d} \tag{1}$$

where k_{mod} is modification factor for variation in density and moisture content. $f_{c,0,k}$ is the characteristic compressive strength parallel to grain. γ_m is the timber material partial safety factor for strength, $\sigma_{ci,d}$ is the design compressive stress for members under compression that are members 4, 5, 6, 8 and 10.

ii. *Tension failure criterion*

The limit state function for tension is given as:

$$G(x) = \frac{(k_{mod} f_{t,0,k})}{\gamma_m} - \sigma_{ti,d} \tag{2}$$

where $f_{t,0,k}$ is the characteristic tensile strength parallel to grain, $\sigma^{ti,d}$ is the design tensile stress for members under tension that are members 1, 2, 3, 7, 9 and 11.

iii. *Bending failure criterion*

The following is the limit state function for bending is as following:

$$G(x) = \frac{(k_{mod} f_{m,k})}{\gamma_m} - \sigma_{mi.d} \tag{3}$$

$f_{m,k}$ is the characteristic bending strength, $\sigma_{mi.d}$ is the design bending stress for members under bending that include member 1, 2, 3, 4, 5, and 6.

iv. *Connection failure criterion*

The EC5 (2004), defines the characteristic load-carrying capacity for nailed joints per shear plane per fastener (R_k), at the specified minimum spacing should be the minimum value from the following expressions:

$$R_k = \min \begin{cases} f_{h..1.k} t_1 d \\ f_{h..2.k} t_2 d \\ \dfrac{f_{h.1.k} t_1 d}{1+\theta}\left[\sqrt{\theta + 2\theta^2 \vartheta + \theta^3 T^2} - \theta(1+T)\right] \\ \dfrac{f_{h.1.k} t_1 d}{2+\theta}\left[\sqrt{2\theta(1+\theta) + \dfrac{5\theta(1+2\theta)M_{yk}}{f_{h.1.k} d t_1^2}} - \theta\right] \\ \dfrac{f_{h.1.k} t_2 d}{1+2\theta}\left[\sqrt{2\theta^2(1+\theta) + \dfrac{5\theta(1+2\theta)M_{yk}}{f_{h..1.k} d t_2^2}} - \theta\right] \\ 1.15 k_{cal} \sqrt{\dfrac{2\theta}{1+\theta}} \sqrt{2 M_{yk} f_{h.1.k} d} \end{cases} \tag{4}$$

where $\theta = \frac{f_{h.2.k}}{f_{h.1.k}}$, $T = \frac{t_2}{t_1}$, M_{yRk} , $f_{h.1.k}$, and $f_{h.2.k}$ are the yield moment and embedding strength corresponding to head side (t_1) and point side (t_2) timber thickness respectively, d being the nail diameter.

Eurocode 5 (2004) recommends that the value of k_{cal} should be 1.3 and 1.5 for smooth nails of round, square or groove cross sections, and threaded nails respectively.

The characteristic values for high yield moment using round wire nail can be deduced from the following expression:

$$M_{y.k} = \frac{f_u}{600} 180 d^{2.6} \tag{5}$$

The limit state formulation for the nail joint only is given as

$$G(x) = \frac{n(K_{mod})R_k t_i}{\gamma_m} - S \tag{6}$$

where S is the load effect in member; K_{mod} is the composite modification factor taking into account deviations from normal load and climate conditions during the service life; γ_m, is partial safety factor for the material (1.3); n is number of fasteners; t_i is depth of the timber species .

The statistics of the design variables employed in the study suitable for targeted performance levels are shown in Table 1.

Table 1 : Stochastic model of design variables

Variable	Coefficient of Variation	Distribution Model
Bending strength (N/mm^2)	15	Lognormal
Modulus of Elasticity (N/mm^2)	13	Lognormal
Density (kg/m^3)	10	Normal
Dead load, G_k	10	Normal
Imposed load, Q_k	25	Gumbel
Load duration factor, k_{mod}	15	Lognormal
Model uncertainty (load), Θ_S	10	Lognormal
Model uncertainty (strength), Θ_R	10	Lognormal
Diameter of nail	10	Normal
Depth of timber species	6	Normal

(Source: Ellingwood et al, 1980; Bartlett et al, 2003; Ranta-Maunus, 2004; Afolayan, 2005; Andre and Antonio, 2010; Aguwa,2013)

III. Structural Reliability Analysis

Analysis is aimed at a systematic consideration of the variability in the design variables. Assuming u is an independent, standard normal vector containing the parameters of the stress-strength interference and g(u) the state function representing the interference then according to Afolayan (2005) a measure of violation of such a state is

$$P_f = P(u \in F) = P(g(u) \le 0) \tag{7}$$

where F is the failure domain. Equation (1) can be approximated to give (Gollwitzer et al., 1988; Padmanabhan, 2003):

$$P_f \approx \Phi(\beta) \tag{8}$$

The GA for reliability analysis can be formulated in the following form (Cheng, 2007):

$$Minimize\ \beta = \|\mu\|^2 = \mu^T.\mu \tag{9}$$

The convergence is achieved using the following condition;

$$\beta_{average}^{(k+1)generati\ on} > \gamma_{average}^{kgeneration} \tag{10}$$

where γ can be set to 0.95 (Wang and Ghosn, 2005).

IV. Results and Discussion

The force and stress in each member due to action loads was determined using resolution of forces. The critical load at each joint was used in the analysis of the joints. The member-force, member-stress and formulated model function for each member as presented in Table 2 were used in the reliability analysis.

Table 2 : Member-Force and Member-Stress Model Function

Member	Model Function	
	F(x)	σ_i
1 (T)	$7.56l_i(0.9\alpha + 1)$	$\frac{7.56l_i(0.9\alpha + 1)}{bt}$
2 (T)	$7.56l_i(0.9\alpha + 1)$	$\frac{7.56l_i(0.9\alpha + 1)}{bt}$
3 (T)	$5.46l_i(0.9\alpha + 1)$	$\frac{5.46l_i(0.9\alpha + 1)}{bt}$

4 (C)	$6.24l_i(0.9\alpha + 1)$	$\frac{6.24l_i(0.9\alpha + 1)}{bt}$
5 (C)	$4.92l_i(0.9\alpha + 1)$	$\frac{4.92l_i(0.9\alpha + 1)}{bt}$
6 (C)	$2.84l_i(0.9\alpha + 1)$	$\frac{2.84l_i(0.9\alpha + 1)}{bt}$
7 (T)	$1.52l_i(0.9\alpha + 1)$	$\frac{1.52l_i(0.9\alpha + 1)}{bt}$
8 (C)	$2.14l_i(0.9\alpha + 1)$	$\frac{2.14l_i(0.9\alpha + 1)}{bt}$
9 (T)	$4.12l_i(0.9\alpha + 1)$	$\frac{4.12l_i(0.9\alpha + 1)}{bt}$
10 (C)	$5.64l_i(0.9\alpha + 1)$	$\frac{5.64l_i(0.9\alpha + 1)}{bt}$
11 (T)	$3.64l_i(0.9\alpha + 1)$	$\frac{3.64l_i(0.9\alpha + 1)}{bt}$

where (T) and (C) represent tension and compression members respectively, l_i is the length of member, α is the dead-to-live load ratio, b is breadth and t is depth of timber species.

The result of the reliability analysis of the roof truss for *Mansonia altissima* at the ultimate state of loading was presented in Table 3. The safety indices for the bending, tension, compression and joint failure modes are 3.94, 2.92, 3.62 and 2.58 respectively. Joint failure mode is the least failure mode hence predetermines the safety of the truss. The computed critical safety index of 2.58 agrees with Melchers (1987) who stated that target reliability index (β_T) for timber members ranges from 2.0 to 3.0 with strong mean of 2.5. This implies that at this depth of section the timber roof truss is reliable under specified conditions of loadings and geometric properties. However, the degree of reliability of the roof truss can be improved if suitable cross-section is chosen (Benu and Sule, 2012). The sensitivity analysis was conducted to ascertain the effect of some of design variables on the reliability of the truss.

Table 3 : Safety Indices for the timber roof truss of *Mansonia altissima*

Failure mode	Safety index
Bending	3.94
Tension	2.92
Compression	3.62
Joint	2.58

Fig. 2 shows the relationship between safety index (β) and depth of section (t) for the timber roof truss of Mansonia *altissima*. An increase in safety index (β) from 1.96 to 4.47 was recorded for joint failure mode as the depth was increased from 50mm to 250mm respectively. The Joint failure mode has the least safety index among all the failure criteria for the *Mansonia altissima* then followed by tension failure mode as shown in Fig. 2 The increase in safety index (β) could be attributed to the increase in EI values, which increased the rigidity of the section (Aguwa, 2013). It is worthy to note that at a larger depth, the structure may be very reliable but not economical because drying and lifting will be a problem. Since structural safety must recognize financial burden involved in project execution and general utility, the derived factors of safety are improved to balance conflicting aims of safety and economy (Afolayan, 1999).

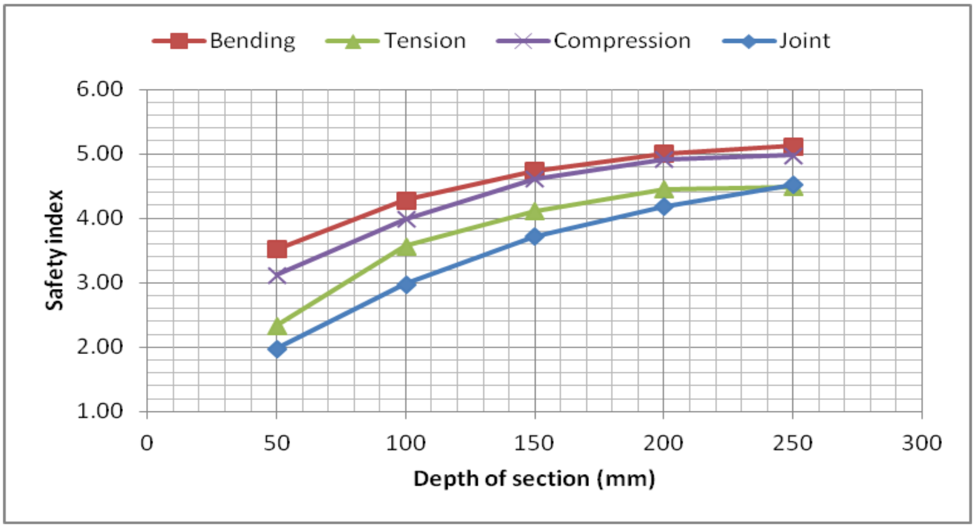

Figure 2 : Variation of safety index against depth of section for *Mansonia altissima*

Fig. 3 shows the relationship between depth of section and live load for the timber roof truss of Mansonia *altissima* at the ultimate limit state of loading and at variable live load. An increase in the depth of section was recorded for all the failure modes as the live increases. The result revealed that Joint failure mode is predominant which recorded an increase in the depth of section from 105mm to 157mm as the live increases from 1.0kN/m to 7.0kN/m respectively. This implies that live load has significant effect on the design depth of the roof truss members of *Mansonia altissima*.

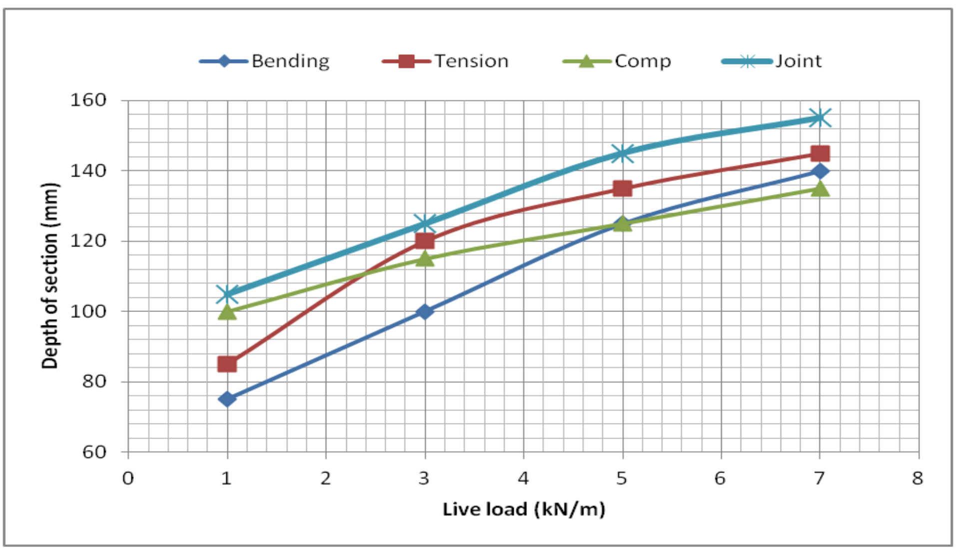

Figure 3 : Variation of depth of section with live load for *Mansonia altissima*

At live load of 1.0kN/m, the bending and compression failure modes recorded the depth of sections of 75mm and 100mm respectively. As the live load increased to 5.0kN/m, the depth of sections for bending and compression converged to an approximate depth of section of 125mm. This implies that there are overlaps of behaviours among the truss members at different live loads.

Fig. 4 shows the relationship between safety index and live load for timber roof truss of *Mansonia altissima* at the ultimate limit state of loading and at variable live load. A decrease in safety index (β) was recorded for all the failure modes with joint failure mode been predominant then followed by tension failure mode. A general consistent decrease in safety index was recorded for joint failure mode from 4.12 to 1.23 as the live load was increased from 1.0kN/m to 7.0kN/m respectively. This could be attributed to the increase in EI values, which increased the rigidity of the beam (Aguwa and Sadiku, 2012). The members of the roof truss for *Mansonia altissima* is safe at a minimum breadth of 50mm under the specified design conditions.

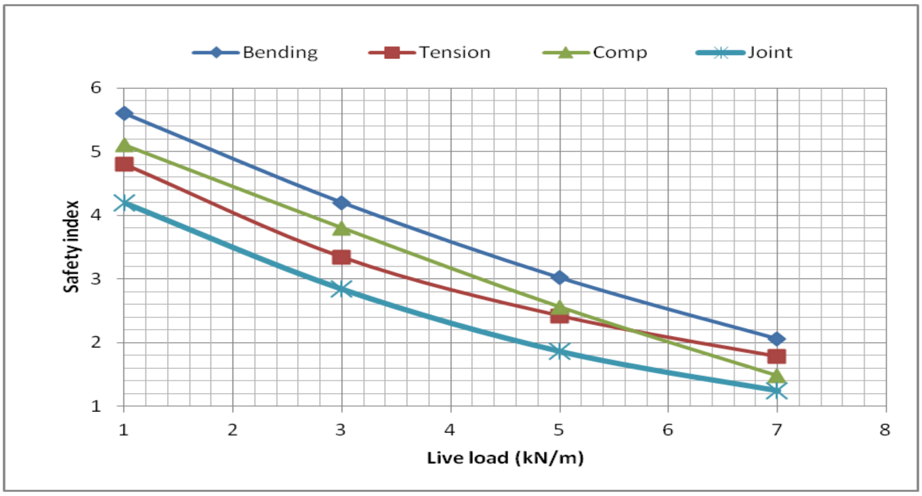

Figure 4 : Variation of depth of section with live load for *Mansonia altissima*

Fig. 5 shows the relationship between safety index and diameter of nail at joint for the timber roof truss of *Mansonia altissima* at the ultimate limit state of loading. An increase of safety index was recorded from 3.98 to 4.4 as the diameter of nail increases from 3mm to 5mm. The safety index then declined to 4.08 at 7 mm. This indicates that at the peak value of safety index the timber species reached its highest capacity to resist the effect of diameter of nails and thus hold the timber pieces firmly together, but beyond this critical diameter of nail the timber species have less resistant capacity to withstand any increase in stresses due to increase in diameter of nails. It therefore tends to split. To avoid this split of timber piece EC 5 (2004) recommends pre-drilling of holes for large diameter of nails.

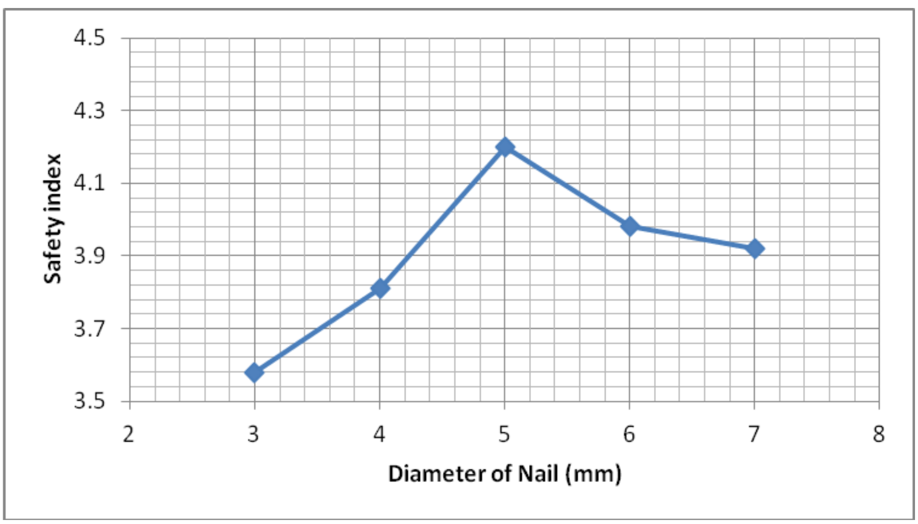

Figure 5 : Variation of safety index with diameter of nail

Fig. 6 shows the relationship between safety index and depth of section at various pitches of the timber roof truss of *Mansonia altissima* at the ultimate limit state of loading. An increase in safety index (β) was recorded for all the failure modes at various pitches of the truss with joint failure mode been predominant then followed by tension failure mode. It was observed that the pitch of the truss has significant effect on safety of the timber roof truss. Considering joint failure mode an increase in safety index (β) was recorded from 1.25 to 3.81 as the depth of section of timber members increases 50mm to 250mm at the pitch of 10^0 respectively. However, as the pitch increases to 20^0, the safety index significantly increases from 1.94 to 4.32 at the same ranges of the depth of section. This implied that for a pitched roof truss large rafter slope lead to high reliability.

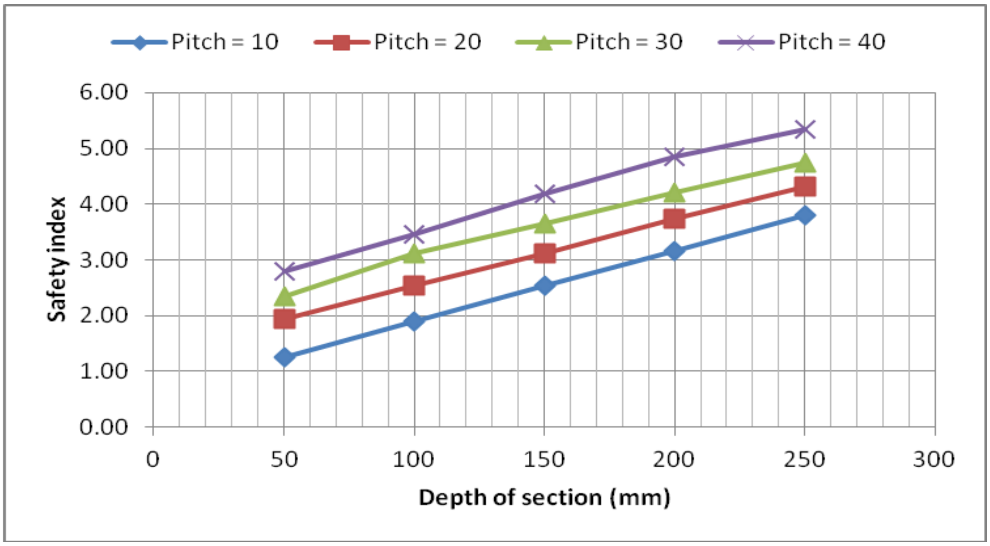

Figure 6 : Safety index against depth of section for various pitches

V. Conclusion

This paper has presented a reliability analysis of the timber roof truss using GA-based FORM, which searches for the globally best and fittest solution. The failure modes of truss were checked and the uncertainties in the strength and load variables were accommodated in the reliability analysis. It is shown that the *Mansonia altissima* timber species is a reliable structural material and economical for the roof truss system at the specified ultimate state of loading and geometrical parameters. The sensitivity analysis revealed that the safety index (β) is highly sensitive to the depth of section, dead-to-live load ratio, diameter of nail and pitch of truss; hence, they are the critical factors to be considered in design of timber roof truss.

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