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A Study on the Eigen-Property of the Cylindrical Coaxial Cavity by FEM

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Abstract- The eigen-properties of the cylindrical coaxial cavity have been investigated by FEM. The eigen-equation has been constructed basing on tangential edge vectors of the tetrahedral element. It was retreated with the shift-invert strategy to maintain the calculation stability. Krylov-Schur iteration method has been applied to it in order to obtain the eigen-pairs of TM and TE modes. Eigen-modes were calculated from the unitary similar transforming matrices of this iteration loop. Eigen-values have been determined from diagonal components of the Schur matrix. The eigen-pairs have been revealed as a result in the schematic representations for each modes. The eigen-modes were so complex that the surface features also have been shown in accompanying with them to identify their characteristics.

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I. Introduction

It has been well known that the knowledge about eigen-mode is one of the most important thing designing the resonant cavity. Acquiring an information about the eigen-property is indispensable in the process of developing more valuable product. There are several factors influencing the eigen-property in the cavity. Among others, the geometrical structure has been considered as the most dominant factor influencing on the eigen-property. Its structure determines the eigen-mode which characterizes the resonant electromagnetic field. The cavity would be taken a variety of form in accordance with its applying purpose. Previously, we have studied the eigen-properties of cylindrical and rectangular resonant cavities using FEM (Finite Element method) [1] [2]. These studies have revealed the several prominent eigen-modes and corresponding eigen-values for each TM and TE modes. The spectra have been shown visually with the 3-Dim (Dimensional) schematic representation. These results have suggested that the similar method may be carried out on varied 3-dimensional cavities and give valuable information understanding the physical property of more complicating system. In this study, FEM has been performed on the cylindrical coaxial cavity as like the previous study. The mesh element was a simple tetrahedron and the shape functions were constructed with constant tangential edge vectors. The matrix eigen-equation was established basing on the vector Helmholtz equation. For a three-dimensional problem, the number of variables increases drastically comparing to a two-dimensional problem. It may be very difficult problem to calculate the huge dimensional matrix equation by common eigen-solving method. Krylov-Schur iteration method has been known as one of the most important and actively developing algorithms for calculating the large dimensional eigen-equation [3] [4]. This method compresses and transforms similarly the eigen-matrix into the Shur form. Even using personal computer, this method was easily carried out on the calculation obtaining the several prominent eigen-pairs. Accompanying with it, the shift-invert strategy add more helpful benefit to obtain the specific eigen-mode. So, Krylov-Schur iteration method has applied to the matrix eigen-equation in this study. As the results, the spectra for each eigen-pairs have been visualized with the schematic representations as like the previous study. The spectra were so complex that surface components of the field vector separated and presented side by side to each spectra.

II. Finite Element Formulation

The calculation for the eigen-mode is the same as describing in previous studies. The formulation can be followed by using either $\vec{E}$(electric field strength) or $\vec{B}$(magnetic field strength) field. For a convenience of calculation, only $\vec{E}$ would be considered in the following discussion. The vector Helmholtz equation would be used in determining the wave property of the resonant cavity. It is described as following equation [5] [6]

$$\vec{\nabla} \times \left(\frac{1}{\mu_r}\vec{\nabla} \times \vec{E}\right) - k^2\varepsilon_r\vec{E} = 0 \quad (1)$$

where k, μ_r and ε_r is the wave number, relative permeability μ/μ_o and relative permittivity $\varepsilon/\varepsilon_o$ respectively. The eigen-equation is constructed from FEM basing on the tetrahedral elemental mesh. The cylindrical coaxial resonant cavity and the tetrahedral mesh is shown in the Fig.1. In the calculation, the lateral surface of the cavity has been assumed to be PEC (perfect electric conductor). This boundary condition makes TM and normal derivative for TE components to be vanished at the lateral surface. The Galerkin method of weighted residual has been used to construct a linear equation [7]. The equation resulting from this method is given as following

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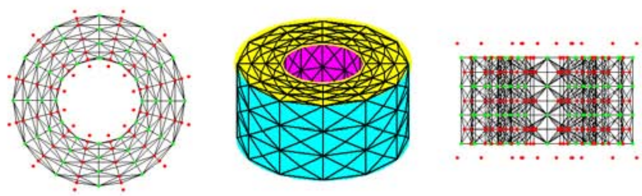


Fig. 1: Schematic representation of the 3-Dim mesh of the coaxial cylindrical cavity

$$\iiint \frac{1}{\mu_r}(\vec{\nabla}\times\vec{T})\cdot(\vec{\nabla}\times\vec{E})dV$$
$$= k_0^2\varepsilon_r \iiint \vec{T}\cdot\vec{E}dV \quad (2)$$

Where $\vec{T}$ is a weighting function. To avoid the spurious solution attributed to the lack of enforcement of divergence condition for $\vec{E}$, basis functions have been constructed with constant tangential edge vectors $\vec{W}_m$ of the tetrahedral element

$$\vec{W}_m = l_m\left(N_{m1}\vec{\nabla}N_{m2} - N_{m2}\vec{\nabla}N_{m1}\right) \quad m = 1,2,3,4,5,6. \quad (3)$$

In this representation, N_{m1} and N_{m2} are the simplex coordinates associated with the 1st and 2nd nodes connected by edges m, and l_m is the length of edge m. The simplex coordinates for a given elementary mesh are

$$N_n = a_n + b_n x + c_n y + d_n z, \qquad n = 1,2,3,4 \quad (4)$$

And the gradient of any coordinate is

$$\vec{\nabla}N_n = b_n\hat{x} + c_n\hat{y} + d_n\hat{z} \quad (5)$$

The simplex coefficients are calculated by inverting the coordinate matrix

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{bmatrix}^{-1} \quad (6)$$

Where (x_n, y_n, z_n) is a rectangular coordinate of the node n of the tetrahedral mesh. Each edge and node for element mesh are related with each other as illustrated in Fig.2. The electric field strength in a single tetrahedral element is calculated with the tangential edge vector as

$$\vec{E} = \sum_{m=1}^{m=6} e_m \vec{W}_m \quad (7)$$

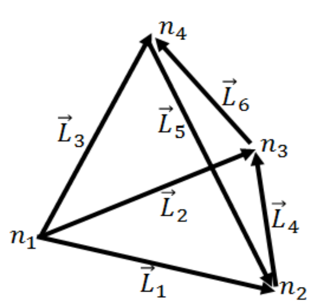


Fig.2 : The tetrahedral element mesh

The six unknown parameters $e_1, \ldots, e_6$ are associate with tangential edges of the tetrahedral elemental mesh. Substituting equation (7) into equation (2), the eigen-equation of one tetrahedral element can be written in matrix form

$$[S_{el}][e] = k^2[T_{el}][e] \quad (8)$$

Where the element matrices are given by

$$[S_{el}] = \iiint \frac{1}{\mu_r}(\vec{\nabla}\times\vec{W})\cdot(\vec{\nabla}\times\vec{W})dV \quad (9)$$

$$[T_{el}] = \varepsilon_r \iiint \vec{W}\cdot\vec{W}dV \quad (10)$$

The evaluation of the element matrix requires the curl product for each basis function $\vec{W}_m$

$$\begin{aligned} \vec{\nabla}\times\vec{W}_m &= \vec{\nabla}\times l_m(N_{m1}\nabla N_{m2} - N_{m2}\nabla N_{m1}) \\ &= 2l_m\vec{\nabla}N_{m1}\times\vec{\nabla}N_{m2} \\ &= 2l_m\big((c_{m1}d_{m2} - c_{m2}d_{m1})\hat{x} + (b_{m2}d_{m1} - b_{m1}d_{m2})\hat{y} \\ &\quad + (b_{m1}c_{m2} - b_{m2}c_{m1})\hat{z}\big) \\ &\equiv 2l_m\vec{w}_m \end{aligned} \quad (11)$$

And from it

$$[S_{el}]_{mn} = 4l_m l_n V(\vec{w}_m\cdot\vec{w}_n) \quad (12)$$

To obtain the element matrix $[T_{el}]$, the scalar product between $\vec{W}_m$ and $\vec{W}_n$ may be calculated as

$$\vec{W}_m\cdot\vec{W}_n = l_m\left(N_{m1}\vec{\nabla}N_{m2} - N_{m2}\vec{\nabla}N_{m1}\right)\cdot l_n\left(N_{n1}\vec{\nabla}N_{n2} - N_{n2}\vec{\nabla}N_{n1}\right) \quad (13)$$

$$= l_m l_n\left[N_{m1}N_{n1}\varphi_{m2,n2} - N_{m1}N_{n2}\varphi_{m2,n1} - N_{m2}N_{n1}\varphi_{m1,n2} + N_{m2}N_{n2}\varphi_{m1,n1}\right] \quad (14)$$

Where $\varphi_{mi,nj} = \vec{\nabla}N_{mi}\cdot\vec{\nabla}N_{nj} = b_{mi}b_{nj} + c_{mi}c_{nj} + d_{mi}d_{nj}$

In the process of $[T_{el}]$ calculation, following volume integration for 3-Dim simplex coordinates may be used

$$\iiint (N_1)^i(N_2)^j(N_3)^k(N_4)^l dV = \frac{3!\,i!\,j!\,k!\,l!}{(3+i+j+k+l)!}V \quad (15)$$

These integrals can be simply summarized in the following matrix form [8]

$$[M_{ij}] = \frac{1}{V}\iiint N_i N_j\, dV = \frac{1}{20}\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \quad (16)$$

From the equations (13), (14) and (16), the element matrix can be written as following

$$[T_{el}]_{mn} = Vl_m l_n\left[\varphi_{m2,n2}M_{m1,n1} - \varphi_{m2,n1M_{m1,n2}} - \varphi_{m1,n2M_{m2,n1}} + \varphi_{m1,n1}M_{m2,n2}\right] \quad (17)$$

These element matrices are assembled over all tetrahedral elements in the 3-Dim cavity to obtain a global eigen-matrix equation.

$$[S][e] = k^2[T][e] \quad (18)$$

III. Results and Discussion

The following discussion is similar to the previous studies. The same FEM formulation was applied to the cylindrical coaxial cavity. But it is confirm that the mesh structure was differently constructed from these studies and the results sufficiently reflected on the characteristics of the present cavity.

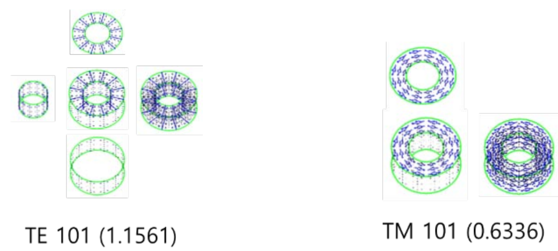
In this study,FEM has been used to construct the eigen-equation. The variable of vector Helmholtz equation was the vector edge of the tetrahedral mesh. The vertices of the tetrahedron were arranged following the right hand rule to obtain the positively determinant value of the element mesh. The dimension of the eigen-matrix equation was so large that the Krylov-Schur iteration method has been used to obtain several prominent eigen-modes. The calculation was more efficiently promoted in finding specific eigen-pairs by imploring the shift-invert strategy as following [9]

$$\lambda[e] = \frac{[T]}{[S]-\sigma[T]}[e] = [M][e] \qquad (19)$$

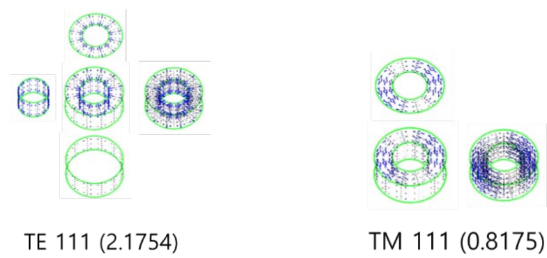
where $\lambda = \frac{1}{k^2-\sigma}$. As mentioned in the previous study, the sparsity and symmetry of the eigen-equation would be lost. But by this strategy, the convergent rate was further increased at the specific value σ. The Krylov-Schur iteration method has been performed on this square matrix $[M]$. By this iteration method, the matrix $[M]$ has been transformed into a Schur matrix. The eigen-modes were the column vectors of the similar transforming matrix which convert the square matrix $[M]$ to the Shure form. The wave numbers were calculated by converting each diagonal component of the Schur matrix into values$k^2 = \frac{1}{\lambda} - \sigma$. As a result, the eigen-pairs have been schematically represented in Fig. 3. The wave numbers were written in the blanket under each spectrum. As can be seen in the spectra, eigen-mode has shown the complicating distribution of electromagnetic fields. So, the surface components were separated from each spectra and positioned side by side to them. From these spectra, it could be identified that the field strength components were oriented to a specific direction. The mode type could be determined readily by investigating the direction of these field strength. These mode type are shown under each spectrum accompanying with a wave numbers. The lateral surface component of TM modes was not depicted definitely in the figure. The reason for it has been on PEC boundary condition which did not permitted tangential magnetic fields on the lateral surface.

IV. Conclusion

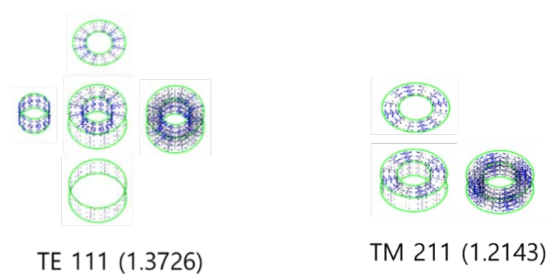
The 3-Dim eigen-equation of the cylindrical coaxial cavity has been constructed by FEM. Eigen-pairs have been calculated by applying the Krylov-Schur iteration method to the shift inverted matrix. As a result, the spectra have been represented schematically in the figure. To identify the mode type, the surface component were separated from the 3-Dim spectra. The mode type and wave numbers have been written under



each spectra. From these results, it could be identified that the spectra reflect the characteristics of the coaxial



cylindrical cavity.



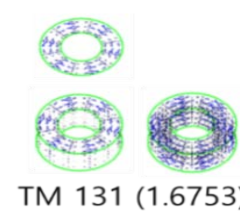


Fig.3 : The schematic representation of the eigen-modes and corresponding wave numbers

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