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Strictly as per the compliance and regulations of :

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I. Introduction

Shell [1] is applied to bodies bounded by two curved surfaces, where, the distance between the surfaces is small in comparison with other body dimensions. The centre of points lying at equal distances from these two curved surfaces defines the middle surface of the shell. The lengths of the segment, which is perpendicular to the curved surfaces, is known as the thickness of the shell and is denoted by h. Shells have all the characteristics of plates, along with an additional one, which is curvature. Mindful of intrinsic, functional essence of shells, [2] presents shells as skin structures by virtue of their geometry and shell action, is essentially more towards transmitting the load by direct stresses with relatively small bending stresses. In line with this functional essence of the shell, [3], [15], shells are spatially curved surface structures which support applied external loads or forces. Shell structures [3], [5], [7] can be referred to as "form resistant structures". This implies a surface structure whose strength is derived from this shape, and which resists loads by developing stresses in its own plane [3], [5], [7].

An early form of shell construction [6] was the dome known to Romans thousands of years ago. With shell concrete construction, [6] it becomes quite possible to produce satisfactory domes which weigh only a fraction of the weights of the much earlier massive domes.

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Properties of shells [1], [5] which are of particular importance in structural usage and which also earn wide application of shell structures in engineering are the following: (i) Efficiency of Load carrying behavior; (ii) High degree of reserved strength and structural integrity; (iii) High strength versus weight ratio; (iv) Very high stiffness; (v) Containment of space.

Areas where shell structures [1], [4] are used in Building and Civil Engineering are:- (i) Large-span roofs; (ii) Liquid retaining structures and water tanks; (iii) Containment shells of nuclear power plants; and (iv) Concrete arch domes. Shell forms in Mechanical Engineering [1], [4] are used in: (i) Piping systems; (ii) turbine disks; (iii) Pressure vessels technology. The use of shells [1], [4] in aeronautical and marine engineering are in the following forms:- (i) aircrafts; (ii) missiles; (iii) rockets; (iv) ships; and (v) submarines. Shells [8] found in various biological forms such as the eye, the skull and the egg, represent another application of shell engineering, this time, in the field of biomechanics. An egg, as a natural thin-walled structure, can be considered as one of the most beautiful structural shapes. It combines extreme fitness for its purpose with an economy of material and cleanliness of design. This account depicts only a small list of shell forms in engineering and nature.

There are [1], [8] two different classes of shells: thin and thick shells. Shells are said to be thin when the ratio of their thickness, h, to the radius of curvature R of the middle surface is less than or equal to 1/20, i.e. $h/R \leq 1/20$. For a large number of practical applications [1], the thickness of thin shells lies within the range: $1/100 \leq h/R \leq 1/20$. Hence, shells for which these h/R stipulations do not lie within the stated range, belong to thick shells [1].

Thin plates and thin shells belong to a category of structures known as Thin-Walled Structures. Thin-walled structures [9] possess the following three characteristics: (i) two dimensions are much longer than the third, its thickness; (ii) They have a great strength as a result of their spatial character of working under the action of external loads; (iii) They make use of a minimal quantity of material.

Depending on the curvature of the surface [1], [3], shells are divided into (i) cylindrical, comprising of noncircular and circular; (ii) conical; (iii) spherical (iv) ellipsoidal; (v) paraboloidal; (vi) toroidal; and (vii) hyperbolic paraboloidal shells.

The usual theory of thin shells utilizes the main suppositions of the theory of thin plates. Nonetheless, thin plate and thin shell have a substantial difference in behaviour under external loadings. The static equilibrium of a plate element under lateral load [2], [8] is only possible by the action of bending and torsional or twisting moments, usually accompanied by shearing forces. Conversely, a shell, in general, [10] is capable of transmitting the surface load by “membrane” stresses uniformly distributed over its thickness. This property of shells [8], [10] makes them to be not only more economical but also more rigid than plates and other types of construction under the same conditions. Apart from these obvious advantages over other systems [1], [8], shell structures are very well known and used for their performance, strength against accidental damage, resistance to fire and low upkeep cost as well as their aesthetic appearance.

The economy and/or feasibility of many modern constructions necessitate light weight, a property which thin-walled structures are replete with. Strictly speaking, the aim in structural engineering has always been to lower, as much as possible, the cost and thus the quantity of material used without, compromising the structural integrity of the system. Thin-walled structures meet this requirement.

The deliberate effort in the analysis and design of shells [3] is to make the shell as thin as practical requirements would permit, so the dead weight is reduced and the structure functions as a membrane free from the large bending stresses.

Thin shell concrete structures [3] are pure compression structures formed from inverse catenary shapes. The inverse catenary is a pure compression scenario. Pure compression is ideal for concrete, as concrete has high compressive strength and very low tensile strength. These shapes maximize the effectiveness of concrete, allowing it to form thin light spans [3].

This paper presents the application of initial value model in analysis of underground circular cylindrical shell structure subject to axi-symmetrical loads of hydrostatic pressure but considered under two conditions: (i) when the tank is empty; (ii) when the tank is full. In addition, the paper seeks validity through the classical model.

Aim in this study is to investigate stress effect on the underground cylindrical shell structure when full of water as well as when empty. The study intends to achieve the aim through the following objectives:

1) To solve the governing differential equation of equilibrium of cylindrical shells subjected to radially symmetrical loads using both initial value and classical models.
2) To evaluate the five internal stresses of the underground cylindrical shell structure when full of water, using the two models named above.
3) To determine the five internal stresses of the underground cylindrical shell when empty, also using the same two models.

II. Previous Works

The analysis and design of shells attracted many researchers. Among them, perhaps the best known, are: Love, U. F., Pasternak, P. L. and Timoshenko, S. P. [11].

a) The Membrane Theory

Modern shell construction [11] has its origin in the work of Lame and Clopeyron who, in 1826, proposed the membrane analogy. This theory suggests that a shell is capable of resisting external loads by direct stresses called membrane stresses without bending. Hence, when membrane theory is applied for shell design, torsional or twisting, bending moments, and shear forces in the cross-section are neglected. This is only possible if torsion and bending stresses were small compared to stresses of normal or axial, and shear forces. Membrane theory fails to represent the true stresses in those portions close to the edges, since the edge conditions usually cannot be completely satisfied by considering only membrane stresses. It is expected [12] that membrane theory gives an approximate picture of stress distribution in the case of shells not long, say $L \leq 2R$, where R is radius of the shell and L is its length. For longer shells a satisfactory solution can be obtained only by considering bending as well as membrane stresses [12].

b) The Moment Theory

Mathematical conceptions [6], developed during the 19th century, made possible a more accurate analysis than could be achieved using any membrane theory. Aron, H., who derived an expression for potential energy of a shell as well as equations for shell equilibrium and strains, was the first to consider the new theory, evolved from those mathematical conceptions, which made room for both membrane and bending stresses [11].

Love [13] developed a detailed derivation of equilibrium equations and equations of motion of shells with correction to a number of slips in Aron’s original treatment as analogous to the theory of plates of Kirchhoff [14], and was based on identical assumptions. In each particular case, the moment theory involves the solution of a system of three differential equations, which is very complicated.

c) The Semi-Moment Theory

In the 20th Century, [15] suggested a simplified method of analyzing and designing cylindrical shells using the theory of semi-moment. This theory, on the basis of experimental data concerning medium length cylindrical shells, length/diameter = 2 – 8, neglects the effect of longitudinal bending moment, shear forces and torques, and introduces geometrical hypothesis. This method has the advantage to be simpler than the moment theory and gives accurate solution when a distributed load is applied. This paper admires this method for the simple reason it appears to be the most appropriate for circular cylindrical shell loaded symmetrically with respect to its axis – the axi-symmetric loading scenario.

Detailed information on general thin-walled structure theory were given in many general treatises such as: [9], [8] and [16], as well as monographs: [10], [17], [18], and [19]. These consider, in a great extent, the mathematical theory of thin-walled structures and the derivation of differential equations pertaining to them. Furthermore, the possibility of solving various shell-theory problems using analytic methods, has also been discussed in: [25], [26], [27], [28], [29], [30], [31], [32], and [33].

d) Methods of Solution

As far as techniques of solving the derived differential equations are concerned, difficulties involved in realizing mathematically rigorous methods, led to approximate techniques of integrating equilibrium equations. One of such methods, known as asymptotic integration, consists in replacing a given differential equation by another with specially selected coefficients different from those in the exact equation, and whose solution can be obtained by strict method and expressed in elementary functions. Blumenthal [20] suggested the method in 1912. In 1913, Timoshenko [21] applied it to shell equations. Shtayerman, Novozhilov [8], and Gol'denveiser [22] perfected the same asymptotic integration method.

Another widely accepted method was a version of the membrane theory which takes boundary effects into cognizance. Geckeler's equations [23] replaced exact equations owing to the presence of boundary effect. Generally speaking, methods for obtaining solutions for the shell differential equations [8] can be classified as follows: (i) Exact analytical methods, also known as classical solution; (ii) methods using variational calculus; (iii) numerical methods such as: finite difference, finite element, finite strip, to mention but a few; (iv) approximate methods based upon exact equilibrium of the problem.

Among others [1], [8], the classical solution, the finite difference, the finite element, the finite strip, the boundary element, the boundary collocation, and the boundary value methods are some of the well-known models used for solution of shell problems.

The classical solution gives accurate results but fails to capture the boundary conditions. The finite difference model considers the boundary conditions but does not give room to further optimization for the simple reason solutions are obtained only for some selected nodes. Finite element generates very large matrices for considerable accuracy, handling of those large matrices being only suitable for use of computer. Besides, they are numerical models. Numerical models are more inclined to approximate solutions than exact solutions. Those models connoting boundary in their names, sound though they may be, involve a level of mathematics beyond the scope of an average engineer!

The semi-moment theory is as suitable for circular cylindrical shells under hydrostatic pressure and uniform gas pressure as it is amenable to application of initial value model. This study adopts the initial value model for among the suggested analytical solutions. Initial value model gives the least number of unknowns enabling such manual handling as to imbue the analyst with the ability and privilege to solve a problem in so systemic a manner that represents a mandatorily profound understanding in the-every-step-of-the-way of the problem for the analyst.

III. Assumptions

a) Soil Characteristics

a. The filled earth outside the cylindrical shell structure is that of pure cohesionless soil.
b. The angle of internal friction, $\phi = 29^0$
c. $C = 0$
d. Unit weight of soil, $\gamma_s = 17.3 kN/m^3$
e. Condition of soil is dry.
f. The earth-filled top level is horizontal

b) The Cylindrical shell structure is fixed at its base or floor. It has a cover at the top. A cast insitu reinforced concrete slab serves as cover at the top, simply supported. The soil covers the walls of the tank from outside. The earth pressure is calculated without surcharge.

c) Parameters of the cylindrical shell structure

a. Height of tank = 4m
b. Radius of tank, R, = 5m
c. Thickness of tank, h = 0.2m
d. Unit weight of water, $\gamma_w = 9.81 KN/m^3$
e. Young's modulus of elasticity of concrete, Ec, 25 x 10^6 KN/m^2
f. Poisson's ratio, $\nu = 0.25$

d) When the tank is empty

i. Soil pressure at any level Z, is given by Ka γ_s Z
ii. When the tank is full:
iii. The net pressure on tank's wall becomes γ_w Z - Ka γ_s Z

Ring installation is neglected since the cylindrical shell reserviour is completely underground.

IV. Analysis

a) The Governing Equation of Equilibrium

[1] and [24] derived the governing differential equation of equilibrium as:

$$\frac{d^4W}{dx^4} + 4\beta^4 W = \frac{\gamma_x}{D} \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

Using, for the derivation, the following:

i. Forces acting on the shell;
ii. Moments acting on the shell;
iii. Equilibrium equations of stress;
iv. Equations of forces and moments displacements.

Equation (1) is applicable only to circular cylindrical shell structure subjected to axi-symmetric loading. This means the loading or forces and moments do not vary along the circumferential section. In other words, the loads are radially symmetrical loads.

For radially symmetrical loads, the governing differential equation of circular cylindrical shell structure, the equation of equilibrium, is equivalent to that of Beam on an elastic Winkler Foundation [34].

b) Initial Value Solution of The Governing Differential Equation of Equilibrium for Circular Cylindrical Shell Structure

i. *In Summary, initial value homogenous solution becomes:*

$$\mathrm{W_h(Z) = y_1(Z)W_0} + \frac{y_2(Z)}{\beta}\theta_0 - \frac{4\beta^2R^2}{Eh}y_3(Z)\mathrm{M_0}$$
$$-\frac{4\ \ R^2}{Eh}y_4(Z)Q_0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\theta_h(Z) = -4\beta\ y_4(Z)Wh + y_1(Z)\theta_0 - \frac{4\beta^3R^2}{Eh}y_4(Z)M_0$$
$$-\frac{4\beta^2R^2}{Eh}y_4(Z)Q_0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$M_h(Z) = \frac{Eh}{\beta^2R^2}y_3(Z)W_0 + \frac{Eh}{\beta^3R^3}Y_4(Z)\theta_0 + y_1(Z)M_0$$
$$+\frac{y_2Z}{\beta}Q_0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$.\ Q_h(Z) = \frac{Eh}{\beta R^2}y_2(Z)W_0 + \frac{Eh}{\beta^2R^2}Y_3(Z)\theta_0$$
$$-4\quad y_4(Z)M_0 + y_1(Z)Q_0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$N_h(Z) = \left[\frac{Eh}{R}y_1(Z)\right]W_0 + \left[\frac{Eh}{\beta R}y_2(Z)\right]\theta_0$$
$$4\beta^2\ Ry_3(Z)M_0 - 4\beta y_4(Z)Q_0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

ii. *The Initial Value Particular Solution Consider the following:*

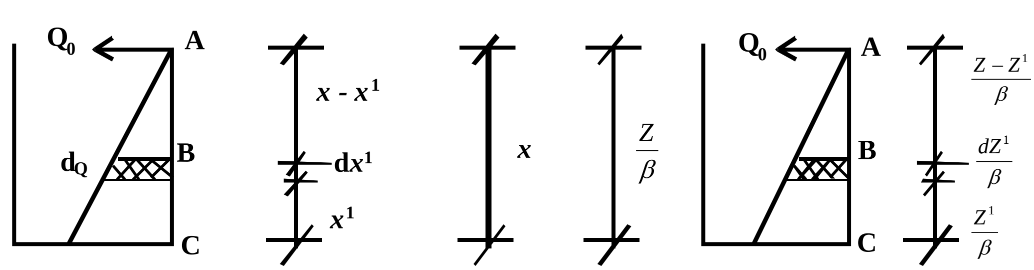

Fig. 1 : Origin Transformation

Understanding an origin transformation leads to finding the initial value particular solution. The origin that was previously at A is shifted to B, while introducing, at the same time, a new variable x^1.

$$\text{Let } Z^1 = \beta x^1 \Rightarrow dx^1 = \frac{dZ^1}{\beta}$$

He distributed load at the new origin, B, is given by:

$$q(Z^1)_w = \frac{Z - Z^1}{\beta}\gamma$$

∴ The elemental force can be expressed as:

$$dQ = q(Z^1)\frac{dZ^1}{\beta} = \frac{\gamma(Z - Z^1)}{\beta^2}dZ^1 \qquad \text{………… (7)}$$

In summary, the initial value particular solution would be:

$$W_p = \frac{\gamma R^2}{Eh\beta}[Z - y_2(Z)] \qquad \text{……………… (8)}$$

$$\theta_p = \frac{R^2\gamma}{Eh}[1 - y_1(Z)] \qquad \text{………. (9)}$$

$$M_p = \frac{-\gamma}{\beta^3}y_4(Z) \qquad \text{………. (10)}$$

$$\theta_p \quad \frac{\gamma}{\beta^2}y_3(Z) \qquad \text{……………. (11)}$$

$$N_p = \frac{\gamma R}{\beta}[Z - y_2(Z)] \qquad \text{………. (12)}$$

Obviously, general solution = homogeneous solution + particular solution.

c) Classical Model Homogeneous Solution

Basically, in summary, the homogenous solution would be as follows:

$$W_h(x) = c_1e^{\beta x} * e^{i\beta x} + c_2e^{\beta x} * e^{-\beta x} + c_3e^{-i\beta x} * e^{-i\beta x} + c_4e^{-\beta x} * e^{i\beta x} \qquad \text{……………………(13)}$$

Where: C_1, C_2, C_3 and C_4 are constants.
Noting that:

$$e^{\beta x} = Cosh\beta x + Sin\beta x \qquad \text{……………………… (14)}$$

$$e^{-\beta x} = Cosh\beta x - Sinh\beta x \qquad \text{……………………… (15)}$$

$$e^{i\beta x} = Cos\beta x - iSin\beta x \qquad \text{……………………… (16)}$$

$$e^{-i\beta x} = Cos\beta x - iSin\beta x \qquad \text{……………………… (17)}$$

Equation [13] can be expressed as:

$$\begin{aligned} W_h(x) &= C_1(Cosh\beta x + Sinh\beta x)(Cos\beta x + iSin\beta x) \\ &+ C_2(Cosh\beta x + Sinh\beta x)(Cos\beta x - iSin\beta x) \\ &+ C_3(Cosh\beta x - Sinh\beta x)(Cos\beta x - iSin\beta x) \\ &+ C_4(Cosh\beta x - Sinh\beta x)(Cos\beta x + iSin\beta x) \end{aligned}$$

Expanding the above equation gives:

$$\begin{aligned} W_h(x) &= A_1 \quad Cosh\beta xCos\beta x + A_2Cosh\beta xSin\beta x \\ &+ A_3 \quad Sinh\beta xCos\beta x + A_4Sinh\beta xSin\beta x \end{aligned} \qquad \text{……………………………… (18)}$$

Where:

A_1 = $C_1 + C_2 + C_3 + C_4$

A_2 = $(C_1 - C_2 - C_3 + C_4)\, i$

A_3 = $C_1 + C_2 - C_3 - C_4$

A_4 = $(C_1 - C_2 + C_3 - C_4)\, i$

d) Classical Model Particular Solution

Recall the static equation of equilibrium:

$$W^{IV} + 4\beta^4W = \frac{\gamma}{D}x \qquad \text{……………………………………………………….(19)}$$

Assuming the particular solution is of the form:

$$W_p(x) = ax \qquad \text{…………………………………………………………. (20)}$$

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It follows that:

$$W_p^{IV}(x) = 0 \quad \text{.......} (21)$$

Making use of equations [20] and [21] in eqn [19] gives:

$$4\beta^4 ax + 4\beta^4 b = \frac{\gamma}{D}x \quad \text{.......} (22)$$

A comparison of co-efficient in equation [22] yields:

$$\left.\begin{aligned} b &= 0 \\ a &= \frac{\gamma}{4D\beta^4} \end{aligned}\right\} \quad \text{.......} (23)$$

Therefore, the classical model particular solution would be:

$$W_p(x) = \frac{\gamma}{4D\beta^4}x \quad \text{.......} (24)$$

Table 1: Summary of Internal Stresses When the Tank is Empty-Using the Initial Value Model

X(m)	Z	Deflection W(z)(mm)	Slope Θ(z) radians	Bending Moment M(z) (KNM)	Shear Force Q(z) (KN)	Hoop Tension N(z)(KN)
0	0	0	0.000122536	0	0.743859755	0
0.8	1.036008026	0.000052788	0.000123922	-0.4857128716	-1.183174309	91.03938151
1.6	2.072016051	0.000165917	0.000129602	-1.516548554	-3.019258682	170.0885245
2.4	3.108024077	0.000473429	0.000145863	-4.247252695	-8.416752018	226.0594408
2.67	3.453360086	0.000670475	0.000156135	-5.962376315	-11.88682999	284.9352786
3.2	4.144032102	0.000134648	0.000190643	-11.73091094	-23.73505572	211.3360398
4.0	5.180040128	3.55×10^{-11}	-4.4×10^{-11}	-32.31896214	-67.01017443	2.7×10^{-7}

Table 2 : Summary of Values of Internal Stresses When the Tank is Empty – Using the Classical Model

X(m)	Z	Deflection W(Z)(mm)	Slope $\theta(Z)$ radians	Bending Moment M(Z) (KNM)	Shear Force Q(Z) (KN)	Hoop Tension N(Z)(KN)
0	0	0	0.000122486	0	-0.743859731	0
0.8	1.036008026	0.000052764	0.000123834	-0.48571286i8	-1.183174342	91.03938184
1.6	2.072016051	0.000165855	0.000129585	-1.516548484	-3.019258654	170.0885258
2.4	3.108024077	0.000473396	0.000145792	-4.247252615	-8.416752001	226.0594463
2.67	3.453360086	0.000670493	0.000156193	-5.962376293	-11.88682675	284.9352765
3.2	4.144032102	0.00134592	0.000190589	-11.73091124	-23.73505498	211.33360358
4.0	5.180040128	4.27×10^{-12}	3.84×10^{-13}	-32.31896187	-67.01017413	2.3×10^{-8}

Table 3 : Summary of Values of Internal Stresses When the Tank is Full – Using the Initial Value Model

X(m)	Z	Deflection W(Z)(m)	Slope $\theta(Z)$ (radians)	Bending Moment M(Z) (KNM)	Shear Force Q(Z) (KN)	Hoop Tension N(Z)(KN)
0	0	0	0.000077722	0	-0.471813873	0
0.8	1.036008026	0.000033482	0.000078601	-0.3085154309	-0.750461426	57.7444589
1.6	2.072016051	0.000105238	0.000082204	-0.9619133775	-1.915049339	107.8828632
2.4	3.108024077	0.000300286	0.000092518	-2.693938928	-5.338560581	143.3830003
2.67	3.453360086	0.000425269	0.000099033	-3.781804103	-7.539554672	180.7287485
3.2	4.144032102	0.000854047	0.000012092	-7.44065868	-15.05462351	134.0462233
4.0	5.180040128	-2.53×10^{-11}	-3.4×10^{-11}	-20.49920654	-42.50307896	-1.14×10^{-7}

Table 4 : Summary of Values of Internal Stresses when the Tank is Full – using the Classical Model

X(m)	Z	Deflection W(Z)(m)	Slope $\theta(Z)$ (radians)	Bending Moment M(Z) (KNM)	Shear Force Q(Z) (KN)	Hoop Tension N(Z)(KN)
0	0	0	0.000177694	0	-0.471813792	0
0.8	1.036008026	0.000033471	0.000078595	-0.3085154293	-0.750461386	57.7444536
1.6	2.072016051	0.000105168	0.000082193	-0.9619133724	-1.915049295	107.8828584
2.4	3.108024077	0.000300245	0.000092512	-2.693938892	-5.338560528	143.3830001
2.67	3.453360086	0.000425218	0.000099021	-3.781804084	-7.539554622	180.7287423
3.2	4.144032102	0.000854014	0.000012024	-7.44065818	-15.05462316	134.0462194
4.0	5.180040128	-3.62×10^{-12}	2.94×10^{-13}	-20.49920612	-42.50307854	1.21×10^{-8}

Table 5 : Summary of Values of Percentage Difference for Internal Stresses: When the Tank is Empty Versus When the Tank is Full

X(m)	Z	Deflection %Δ	Rotation %Δ	Bending Moment %Δ	Shear Force %Δ	Hoop Tension %Δ
0	0	0	36.572	0	36.572	0
0.8	1.036008026	36.572	36.572	36.572	36.572	34.572
1.6	2.072016051	36.572	36.572	36.572	36.572	36.572
2.4	3.108024077	36.572	36.572	36.572	36.572	36.572
2.67	3.453360086	36.572	36.572	36.572	36.572	36.572
3.2	4.144032102	36.572	36.572	36.572	36.572	36.572
4.0	5.180040128	0	0	36.572	36.572	0
TOTAL		182.86	219.432	219.432	256.004	182.86
Average Value		36.572	36.572	36.572	36.572	36.572
Total Average =		182.86				
Overall Average =		36.572				

* Percentage difference, %Δ, for applied pressure on tank wall: when the tank is empty versus when the tank is Full, = 36.572.
* Overall average percentage difference, %Δ, value for results of internal stresses for: when the tank is empty versus when the tank is full = 36.572

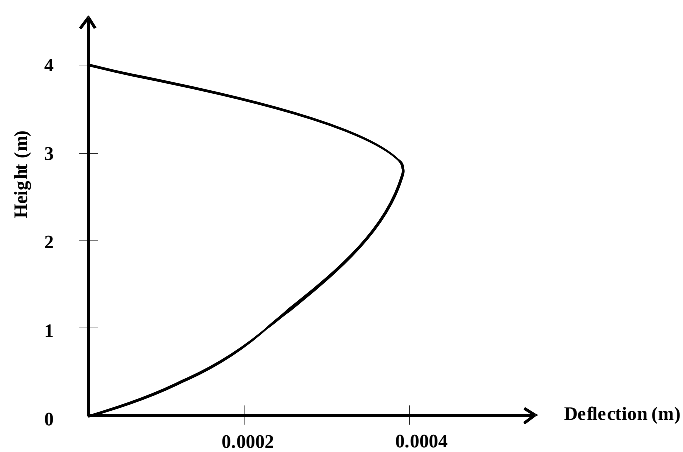

Fig. 2 : Deflection Diagram

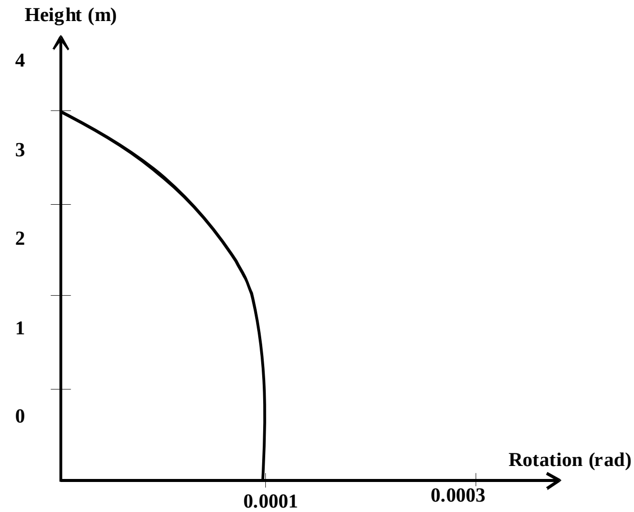

Fig. 3 : Rotation (Slope) Diagram

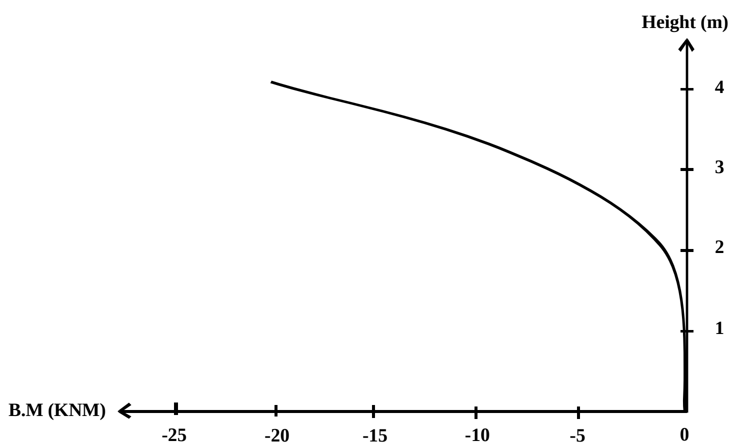

Fig. 4 : Bending Moment Diagram

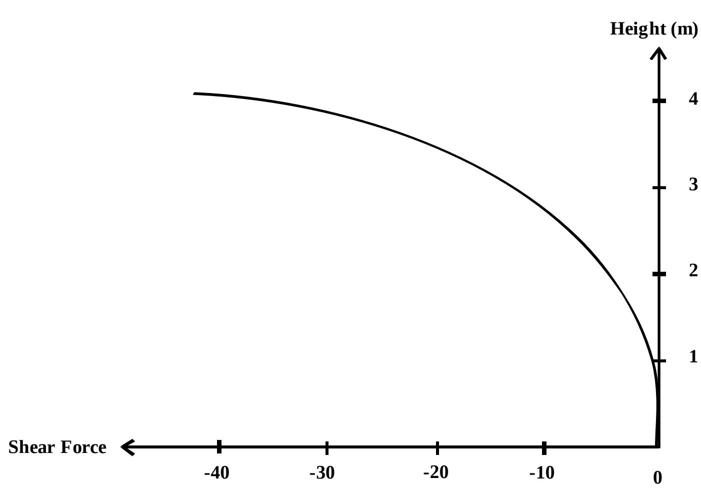

Fig. 5 : Shear Force Diagram

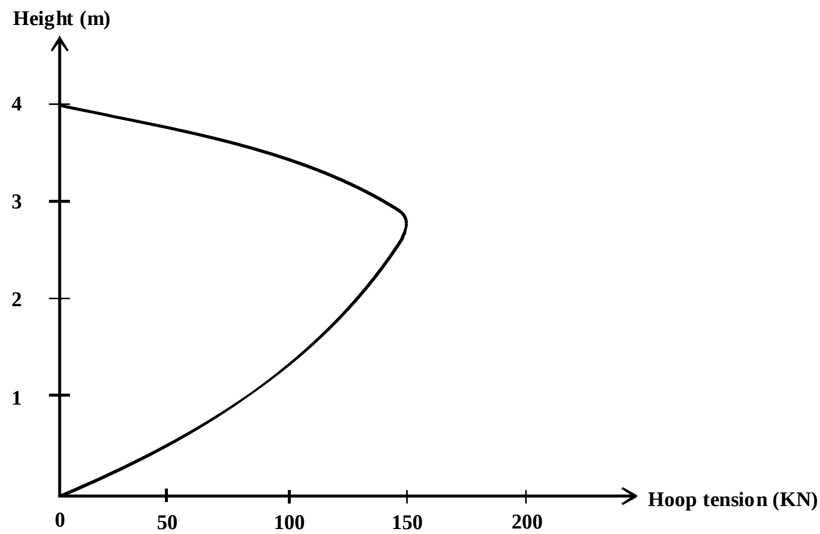

Fig. 6 : Hoop Tension Diagram

V. Discussion of Results

a) Solution of the Governing Differential Equation of Equilibrium

Two approaches were used in the analysis: the classical and initial value models. Two totally different approaches though they are, they led to identical results.

b) Internal Stresses

i. *Direct Deflection*

The deflection curve, for the hydrostatic loading, fig. 2, has a parabolic like shape with a speak of 0.854 mm at x = 3.2m from the top. Zero deflection is obtained at both top and bottom ends of the reservoir, justifying zero deflection at the supports. Also, tables 3 and 4 refer.

ii. *Rotation*

Under hydrostatic pressure, fig. 2, tables 3 and 4, the reservoir presented a maximum rotation of 0.078 x 10^{-3} radians at its top end.

iii. *Bending Moment*

For the hydrostatic loading, fig. 3, tables 3 and 4, the bending moment varies from zero, at the top, to 20.5KMN at the bottom, describing a concave parabolic-like curve due to the cantilever action.

iv. *Shear Force*

In the case of hydrostatic loading, the shearing force varies from 0.472KN, at the top, through 42.503KN, at the bottom, fig. 5, tables 3 and 4.

v. *Hoop Tension*

The graphs, fig. 6, have the same shape as those obtained for direct deflection. For real, deflection and hoop tension are directly proportional. For the hydrostatic pressure, a peak of 180.729 KN is reached at x = 2.67m, fig. 6, tables 3 and 4.

The tables 3 and 4 show values for deflection and rotation at the bottom, x = 4m, not exactly equal to zero, but very close to zero in the extent they can be taken as zero with sufficient accuracy.

vi. *Stress effect at: when empty versus when full of water*

Table 5 reveals: (i) percentage difference for applied pressure on tank wall, when the tank is empty versus when the tank is full of water, is equal to: 36.572; (ii) Overall average percentage difference value for results of internal stresses for, when the tank is empty versus when the tank is full of water, is equal to: 36.572.

c) Conclusion and Recommendation

i. *Conclusion*

The half-moment theory, otherwise known as semi-moment theory appears to have proven to be one of the most accurate theories in shell analysis. It has the advantage to be more realistic than the membrane theory which does not consider the bending effect. The semi-moment theory is simpler than the moment theory which generates heavy equations.

In terms of capturing the boundary conditions of systems, the initial value model is more equipped than the classical model.

Sequel to results revealing that: percentage difference for applied pressure on tank wall: when the tank is empty versus when the tank is full of water is equal to 36.572%; again the overall average percentage difference value for results of internal stresses with respect to: when the tank is empty versus when the tank is full of water came to 36.572% as well, stress effect on the water reservoir which resulted from the two conditions would not diminish the fundamental structural integrity of the underground circular cylindrical shell structure, if all other sources of stress such as from weather elements are under control.

ii. *Recommendation*

i) The shell analyst could well feel free to adopt either the initial value model or the classical model for analysis. ii) Structural analysis of cylindrical shells and other types of shells should attract more interests of structural analysts. Further studies could be undertaken in: i) The furtherance of efforts towards making the analysis of shells simpler than this has ever been. ii) Study of stress effects on this facility with respect to other stress sources such as weather elements.

Notation

$$y_1(Z) = \Psi(Z) = CoshZ\ CosZ$$

$$y_2(Z) = \frac{1}{2}\left[\Psi_2(Z) + \Psi_3(Z)\right] = \frac{1}{2}\left[CoshZ\ SinZ + SinhZ\ CosZ\right]$$

$$y_3(Z) = \frac{1}{2}\Psi_4(Z) = \frac{1}{2}\left(SinhZ\ SinZ\right)$$

$$y_4(Z) = \frac{1}{4}\left[\Psi_2(Z) - \Psi_3(Z)\right] = \frac{1}{4}\left[CoshZ\ SinZ - SinhZ\ CosZ\right]$$

$$\beta^4 = \frac{3(1-V^2)}{R^2h^2}$$

L = Overall height of Reservoir

Ka = $\frac{1-\sin\phi}{1+\sin\phi}$ or $\tan^2\left(45-\frac{\phi}{2}\right)$

Φ = Angle of internal friction of soil.

Z = βL

D = Flexural Rigidity of shell element.

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