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Mathematical Model of Fluid Flow in Rocket Fuel System

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Abstract- The article reviews mathematical model of liquid flow in metering system of fuel tank of rocket. The control system contains one horizontal and two vertical channels. Vertical channel has sensors for fixing free surface level of fluid in the channel. When the level of fuel reaches the sensor, it is activated, and the signal comes to the control system. As a result, fuel consumption is changing. Fuel level in the tank is determined on the basis of the fuel level in the channel. It is known that in the course of fuel consumption, surface free levels in the channel and in the tank do not match. The task is described by unsteady-state equation of motion. Viscous incompressible liquid model is used. The solution of the differential equation was performed numerically. Measurement error of liquid level in the fuel tank has been determined. The study proposes engineering solution to avoid the measurement error.

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Mathematical Model of Fluid Flow in Rocket Fuel System

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Abstract- The article reviews mathematical model of liquid flow in metering system of fuel tank of rocket. The control system contains one horizontal and two vertical channels. Vertical channel has sensors for fixing free surface level of fluid in the channel. When the level of fuel reaches the sensor, it is activated, and the signal comes to the control system. As a result, fuel consumption is changing. Fuel level in the tank is determined on the basis of the fuel level in the channel. It is known that in the course of fuel consumption, surface free levels in the channel and in the tank do not match. The task is described by unsteady-state equation of motion. Viscous incompressible liquid model is used. The solution of the differential equation was performed numerically. Measurement error of liquid level in the fuel tank has been determined. The study proposes engineering solution to avoid the measurement error.

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I. INTRODUCTION

The problem of mathematical simulation of fluid flow in the fuel consumption control system is reviewed. In the course of the rocket travel the fuel from the oxidizer tank and fuel tank enters to the rocket combustion chamber. Synchronous fuel entry provides efficient operation. In real conditions this requirement is violated due to various reasons [1], resulting in inefficient fuel consumption. Residual fuel should have a minimum volume. Accomplishment of this objective depends on accurate measurement of fuel level in the tank. The problem is non-stationary, and is described by parabolic equation of motion. Solution of unsteady-state equation of motion for one-dimensional problem was found by a number of researchers.

Solutions reviewed in [2, 3] may be considered as classical. Paper [2] investigates laminar flow development from the rest state, work [3] reviews the pulsating flow. In [4] calculation results are compared with experimental records. Operational calculus methods are used to resolve parabolic equations in [2-4]. Research paper [5] presents oscillatory flow mathematical model. The solution is obtained using numerical method, obtained results are compared with experimental data. The authors [6] review non-Newtonian fluid throbbing stream in cylindrical channel with immediate valve closing. Method of Runge-Kutta was used to resolve the motion equation. Paper [7] contains the results of incompressible liquid flow in micro-tube at pressure jump research. The problem solution was obtained analytically, using Laplace transformation, and numerically, using Boltzmann method. Stationary flows and pulsating streams in slightly bent tube for a wide range of Reynolds numbers are reviewed in [8]. Numeric methods were used to resolve the problem.

Work [9] presents pulsed incompressible flow through the pipeline. The flow is generated by periodical pressure gradient. The results show good compliance between analytical and numerical solutions. The study [10] represents method of characteristics for fluctuating streams simulation in the pipeline. It provides convergence estimate and method accuracy. Article [11] contains analysis of dynamical interference between the pipe and non-stationary flow on the basis of experiments and numerical models. Method of characteristics for determination of one-dimensional model of fluctuating fluid stream in the pipeline is used in [12]. Paper [13] provides experimental study of characteristics of non-stationary oscillatory flow in cylindrical channel. Obtained results comparison with known experimental results confirms good compliance. Work [14] reviews incompressible liquid non-steady laminar flow in expanding (convergent) channel with porous walls. Analytical solutions are compared with numerical solutions. In [15] the authors study non-stationary fluctuation problems related to non-viscous and low viscosity fluid in extensive network.

II. PHYSICAL STATEMENT OF THE PROBLEM

Liquid level metering system is provided in the tank to control propellant consumption. For this purpose, vertical cylindrical channel, with fuel surface level indicators, is installed in the tank. Due to tank design features, the vertical channel may not match the tank centre line. Besides, short-period oscillations may occur at liquid free surface. In order that liquid level in the vertical channel reflects the liquid level in the tank, the metering system is supplemented by two horizontal channels located at the tank bottom. Horizontal channels outlets are located at one tank diameter. Horizontal channels overall length may exceed the tank diameter (Fig.1).

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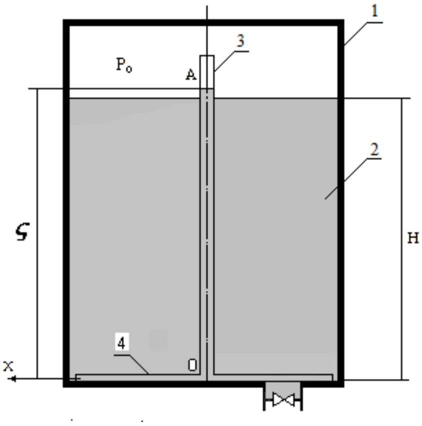

Figure 1: Second diagram of a fuel tank. 1- tank, 2- fuel, 3- measurement vertical channel, 4- horizontal channels, p_0 - gas pressure, ξ - liquid level in the channel, H - liquid level in the tank, x - coordinate axis

In case of fuel level reduction in the tank, the fuel level in the vertical channel is also reduced. When the propellant level in the channel reaches the indicator, the indicator is activated. The signal comes to the fuel consumption control system. As a result, fuel consumption may by changing. Thus, fuel level in the tank is determined on the basis of the fuel level in the channel. The channel and the propellant tank are communicating vessels. The problem is that in case of fuel consumption free surface levels in the channel and in the tank do not match. The error in the fuel level measurement results to inefficient fuel consumption. As a result, rocket motor is operated not with the optimum performance, and "excessive" fuel volume is left in the tanks.

At the initial moment the tank and the channel are filled with the fuel with level H_0. Free upper end of the cylindrical channel is above the fuel level in the tank, therefore the fuel overflow from the tank to the channel at this point is excluded. Fuel is free communicating between the tank and the channel. Constant pressure p_0 is maintained above free fuel surface in the tank and in the channel. From the time point $t > 0$ fuel is taken from the tank, so that the liquid level in it is reduced in linear fashion $H(t) = H_0 - V_0 t$, where V_0 – fuel level depression rate in the tank. Therefore, liquid level in the channel is changing.

III. Mathematical Model Of Liquid Flow

The equation of viscous incompressible liquid non-steady motion in horizontal cylindrical channel is used as the flow model

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\nu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right), \qquad (1)$$

where $u = (r,t)$ - liquid velocity in the horizontal channel, p - pressure, ρ - density, t - time, ν - kinematic viscosity.

Find approximate solution of equation (1). Enter average channel section longitudinal flow velocity - $<u> = \frac{2}{R^2}\int_0^R rudr$. Multiply left and right sections of equation (1) by r and integrate each additive component from 0 to R, where R - horizontal channel radius. Taking into account, that in the process motion the acceleration of rocket increases in linear fashion $(g = 9{,}8 + 0{,}07t)$, we obtain equation (the oblique brackets are omitted in the following).

$$\frac{du}{dt} = -\frac{\rho(9{,}8 + 0{,}07t)(H_0 - V_0 t - \xi(t))}{\ell} - \frac{\lambda u^2}{4R}, \qquad t = 0, u = 0, \xi = H_0, \qquad (2)$$

where λ - friction coefficient, ℓ - horizontal channel length, $\xi(t)$ – liquid level in a vertical channel.

Using volumetric flow rate conservation law, we write down: $u_1(t) = \frac{2u(t)R^2}{R_1^2}$, where R_1 - vertical channel radius. Then

$$\xi(t) = H_0 - \frac{2R^2}{R_1^2}\int_0^t u(t)dt, \qquad (3)$$

and the problem will be determined by system of equations (2) and (3).

As a result, we obtain Cauchy problem. For numerical solution of set problem, formulate system of equations (2), (3) as standard form. For that purpose take derivative with time from equation (3)

$$\frac{d\xi}{dt} = -\frac{2R^2 u}{R_1^2} \quad . \qquad (4)$$

Now the problem will be determined by the system (2) and (4).

IV. Numerical Solution And Results

Problem solution is obtained numerically for $R_1 = 0{,}039\ m$, $R = 0{,}02\,m$, $H_0 = 8{,}2\ m$, $\lambda = 5\cdot 10^{-2}$, $V_0 = 0{,}039\ m/s$, $\ell = 2m$. Using Mathcad application software package, solution results are given at diagrams (Fig.2-Fig.5). Fig. 2 illustrates liquid levels in the tank and in the vertical channel, Fig.3 illustrate under damping oscillations of liquid average velocity in vertical channels.

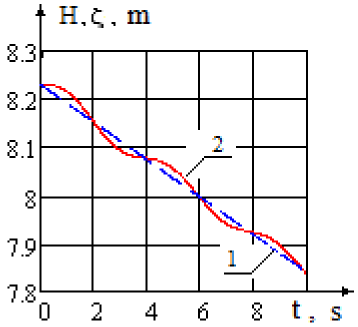

Figure 2: Level of liquid: 1- in the tank, 2- in the vertical channel

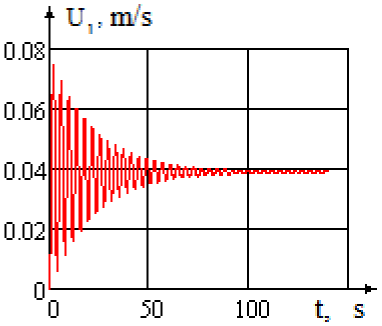

Figure 3: The average velocity of the fluid in the vertical channel

The oscillations are damping with constant period - $T = 3{,}9\ s$, maximum amplitude - $a = 0{,}024\ m$.

Fig. 4 and Fig.5 illustrate the error $\Delta(t) = H(t) - \varsigma(t)$ of liquid level measurement in the tank for friction coefficient $\lambda = 5\cdot 10^{-2}$ and $\lambda = 10^{-1}$

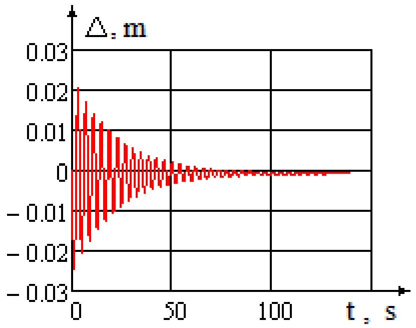

Figure 4: Error in the measurement the liquid level, $\lambda = 5\cdot 10^{-2}$

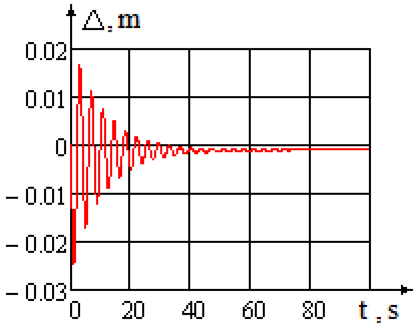

Figure 5: Error in the measurement the liquid level, $\lambda = 10^{-1}$

For the friction coefficient $\lambda = 5\cdot 10^{-2}$ oscillations period - $T = 3\,s$, maximum error $\Delta = 0{,}024\ m$ is observed at the beginning of the process; approximately in $130\ s$ from the beginning of motion the error is becoming small to negligible. For the friction coefficient $\lambda = 10^{-1}$, oscillations damp in 73 seconds, at that, oscillations period $T = 4\,s$, maximum error do not changed $\Delta = 0{,}024\ m$.

V. Discussion

Can be seen (Fig.3), that the average velocity of the fluid in the vertical channel has synchronous damped oscillations. Fluctuations in a vertical channel are attenuated through 100 seconds. We can see (Fig.4), that the magnitude of the error is a periodic function, in which the amplitude of oscillations decreases with time. The maximum error in determining the level of fuel in the tank is observed in the beginning of the flight of a rocket $\Delta = 0{,}015\ m$, after about 130 seconds error becomes negligible. Mathematical experiment demonstrates, that increasing of the horizontal channel length from $\ell = 2m$ to $\ell = 4m$ results in the increase of oscillation period $T = 5{,}4\,s$ and maximum measurement error $\Delta = 0{,}034\ m$.

VI. Conclusion

Executed study proves that it is impossible to completely exclude liquid oscillations. Measurement error reduction may be expected in case of changing fuel consumption measurement system design features (introduction of holes on the vertical channel or dampers installation in the horizontal channels). To ensure zero error the indicators should be located at the points, corresponding to functions intersection nodes $H(t)$ and $\xi(t)$.

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