Non-Dominated Sorting Whale Optimization Algorithm (NSWOA): A Multi-Objective Optimization Algorithm for Solving Engineering Design Problems

By Pradeep Jangir & Narottam Jangir

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NONDOMINATEDSORTINGWHALEOPTIMIZATIONALGORITHMNSWOAAMULTIOBJECTIVEOPTIMIZATIONALGORITHMFORSOLVINGENGINEERINGDESIGNPROBLEMS

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Keyword: *non-dominated; crowing distance; nswoa algorithm; multi-objective algorithm;economic constrained emission dispatch.*

I. Introduction

Optimization is a work of achieving the best result under given limitation or constraints. Now a day, optimization is used in all the fields like construction, manufacturing, controlling, decision making, prediction etc. The final target is always to get feasible solution with minimum use of resources. In this field computers make a revolutionary impact on every field as it provides the facility of virtual testing of all para-meters that are involved in a particular design with less involvement of human efforts, benefits in less time consuming, human efforts and wealth as well.

Today we use computer-aided design where a designer designs a virtual system on computer and gives only command to test all parameters involved in that design without even the need for a single prototype.

A designer only to design and simulate a system and set all the parameter limitation for the computer.

Computer-aided design technique becomes more effective with the additional feature of auto-generation of solutions after it's mathematically formulation of any system or design problem. Auto generation of solution, this feature is come into nature with the development of algorithms. In past years, real world designing problems are solved by gradient descent optimization algorithms. In gradient descent optimization algorithm, the solution of mathematically formulated problem is achieved by obtaining its derivative. This technique is suffered from local minima stagnation[1, 2] more time consuming and their solution is highly dependent on their initial solution.

The next stage of development of optimization algorithms is population basedstochastic algorithms.These algorithms had number of solutions at a time so embedded with a unique feature of local minima avoidance. Later population based algorithms are developed to solve single objective at a time either it may be maximization or minimization on accordance the problems objective function. Some popular algorithms for single objective problems are Moth-Flame optimizer (MFO) [3], Bat algorithm (BA) [4], Particle swarm optimization (PSO) [5], Ant colony optimization (ACO) [6], Genetic algorithm (GA) [7], Cuckoo search (CS)[8], Mine blast algorithm (MBA) [9], Krill Herd (KH) [10], Interior search algorithm (ISA) [11] etc. These algorithms have capabilities to handle uncertainties [12], local minima [13], misleading global solutions [14], better constraints handling [15] etc. To overcome these difficulties different algorithms are enabled with different powerful operators. As mention above here is only objective then it is easy to measure the performance in

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terms of speed, accuracy, efficiency etc. with the simple operational operators.

In general, real world problems are nonlinear and multi-objective in nature.In multi-objective problem there may be some objectives are consisting of maximization function while some are minimization function. So now a day, multi-objective algorithms are in firm attention.

Let's take an example of buying a car, so we have many objectives in mind like speed, cost, comfort level, space for number of people riding, average fuel consumption, pick up time required to gain particular speed, type of fuel requirement either it is diesel driven, petrol driven or both etc. To simply understand multi-objective problem, from Fig. 1, we considertwo objectives, first cost and second comfort level. So we go for sole objective of minimum cost possible then we have to deny comfort level objective and vice-versa. It means real word problems are with conflicting objectives. So as, we are disabled to find an optimal solution like single objective problems. About multi-objective algorithm and its working is detailed described in next portion of the article.
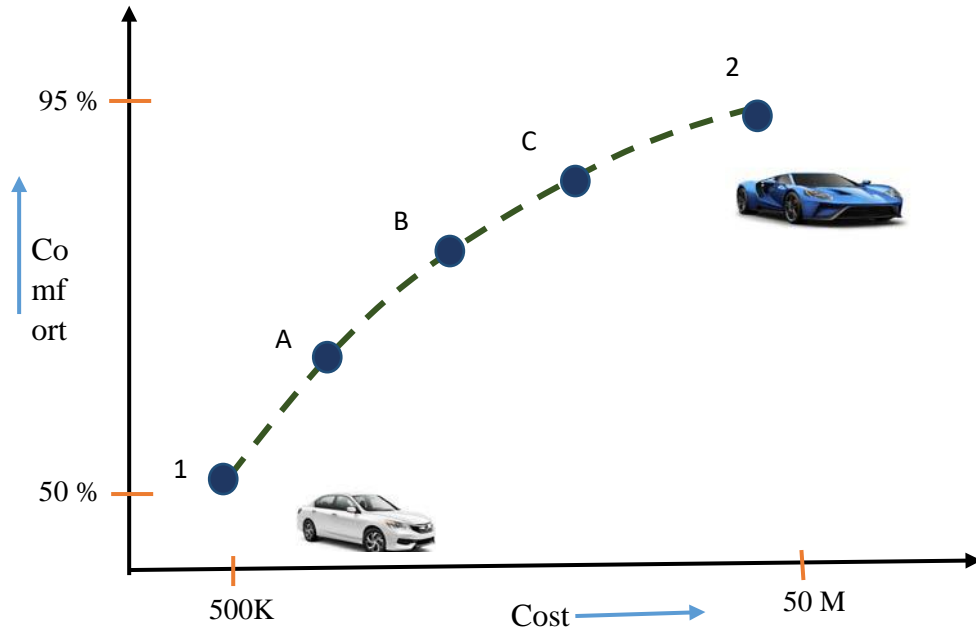

Fig. 1: Car-buying decision-making problem (Hypothetical multi-objective problem solutions)

The No free launch [16] theorem that logically proves that none of the only algorithm exists equally efficient for all engineering problem. This is the main reason that it allows all researcher either to propose new algorithm or improve the existing ones. This paper proposed the multi-objective version of the well-known whale optimization algorithm(WOA) [17]. In this paper non-sorted WOA (NSWOA) is tested on the standard un-constraint and constrainttest function along with some well-known engineering design problem, their results are also compared with contemporary multi-objective algorithms Multi objective Colliding Bodies Optimizer (MOCBO) [18], Multi objective Particle Swarm Optimizer (MOPSO)[19-20], Non-dominated Sorting Genetic Algorithm (NSGA) [21-23], non-dominated sorting genetic algorithm II (NSGA-II)[24] and Multi objective Symbiotic Organism Search (MOSOS)[25] that are widely accepted due to their ability to solve real world problem.

The structure of the paper can be given as follows: - Section 2 consists of literature; Section 3 includes the proposed novel NSWOAalgorithm; Section 4 consists of competitive results analysis of standard test functions as well as engineering design problem and section 5 includes real world application, finally conclusion based on results and future scope of work is drawn.

II. Literature Review

As the name describes, multi-objective optimization handlessimultaneously multiple objectives. Mathematically minimize/maximize optimization problem can be written as follows:

$$Minimize/maximize: \quad Fn \quad (\vec{x}) = \{fn_1(\vec{x}), fn_2(\vec{x}), \dots, f_{no}(\vec{x})\} \tag{2.1}$$

$$Subject\ to: \quad p_i(\vec{x}) \geq 0, \quad i = 1,2,\dots,q \tag{2.2}$$

$$t_i(\vec{x}) = 0, \quad i = 1,2,\dots,r \tag{2.3}$$

$$L_i^{lb} \leq x_i \leq U_i^{ub} \ , \qquad i = 1,2,\ldots,k \tag{2.4}$$

Where q is the number of inequality constraints, r is the number of equality constraints, k is the number of variables, p_i is the ith inequality constraints, no is the number of objective functions, t_i indicates the ith equality constraints, and $[L_i^{lb}, U_i^{ub}]$ are the boundaries of ith variable.

$$\forall n \in \{1,2,\ldots,k\}: f\_n(\vec{x}) \leq f\_n(\vec{y}) \quad \wedge \quad \exists n \in \{1,2,\ldots,k\}: f\_n(\vec{x}) < f\_n(\vec{y}) \tag{2.5}$$

where $\vec{x} = (x_1, x_2, \ldots, x_k)$ and $\vec{y} = (y_1, y_2, \ldots, y_k)$.

For maximization problems, Pareto optimal dominance is defined as follows:

$$\forall n \in \{1,2,\ldots,k\}: f\_n(\vec{x}) \geq f\_n(\vec{y}) \quad \wedge \quad \exists n \in \{1,2,\ldots,k\}: f\_n(\vec{x}) > f\_n(\vec{y}) \tag{2.6}$$

where $\vec{x} = (x_1, x_2, \ldots, x_k)$ and $\vec{y} = (y_1, y_2, \ldots, y_k)$.

These equationsshow that a solution is better than another in a multi-objective search space if it is equal in all objective and better in at least one of the objectives. Pareto optimal dominance is denoted with ≺ and ≻. With these two operator's solutions can be easily compared and differentiated.

Population based multi-objective algorithm's solution consists of multiple solution. But with multi-objective algorithm we cannot exactly determine the optimal solution because each solution is bounded by other objectives or we can say there is always conflict between other objectives. So the main function of stochastic/population based multi-objective algorithm is to find out best trade-offs between the objectives, so called Pareto optimally set [26-28].

Obviously, relational operators are ineffective in comparing solutions with respect to multiple objectives. The most common operator in the literate is Pareto optimal dominances, which is defined as follows for minimization problems:

The principle of working for an ideal multi-objective optimization algorithm is as shown in Fig. 2.

Step No. -1 Find maximum number of non-dominated solution according to objective, it expresses the number of Pareto optimal set so as shows higher coverage

Step No. -2 Choose one of the Pareto optimal solution using crowding distance mechanism that fulfills the objectives.
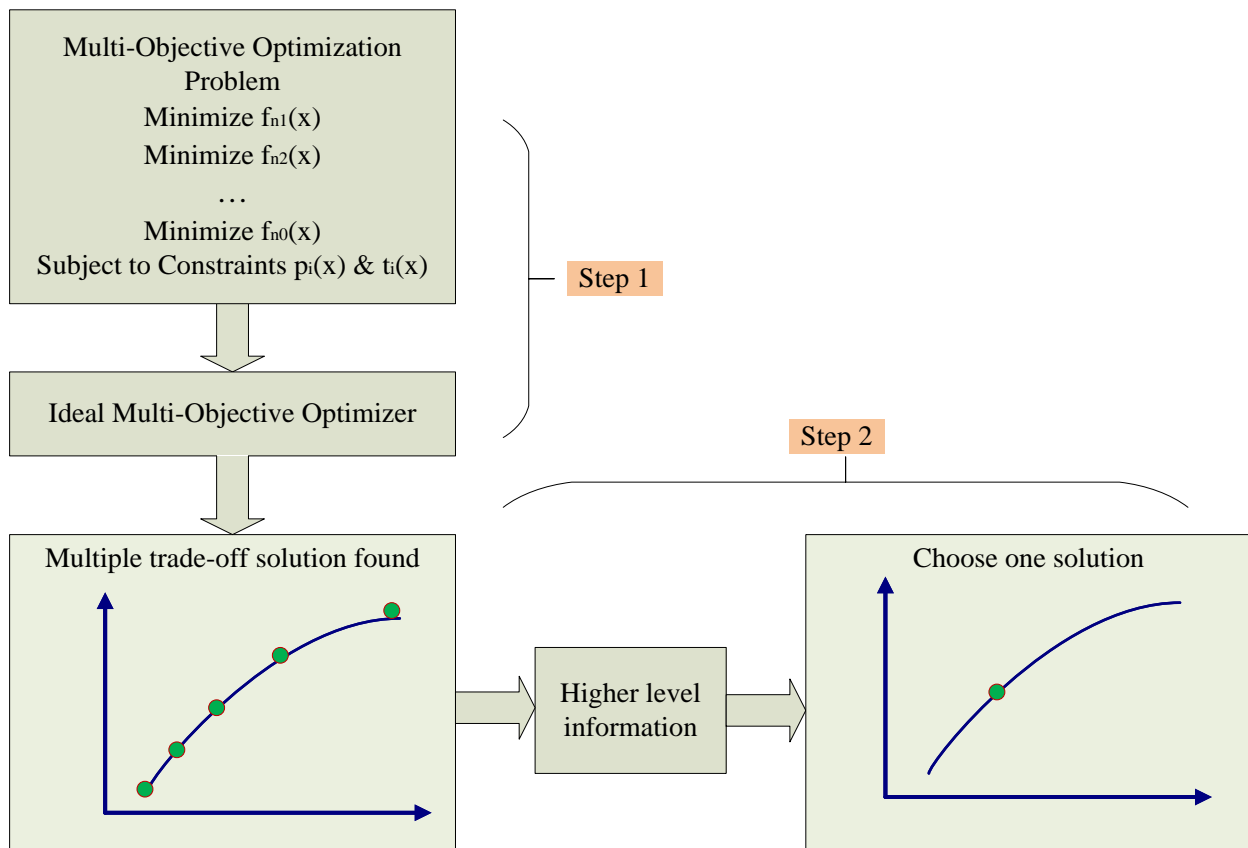

Fig. 2: Multi-objective optimization (Ideal) procedure

Now a day recently proposed sole objective algorithms are equipped with powerful operators to provide them a capability to solve multi-objective problems as well. In the same manner we proposed NSWOA algorithm in a hope that it will perform efficiently for multi-objective problems. These are: Multi-objective GWO [29], Multi-objective Bat Algorithm [30], Multi-objective Bee Algorithm [31],Pareto Archived Evolution Strategy (PAES) [32], Pareto-frontier Differential Evolution (PDE) [33], Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) [34], Strength-Pareto Evolutionary Algorithm (SPEA) [35, 36]andMulti-objective water cycle algorithm with unconstraint and constraint standard test functions [37][38].Performance measurement for approximate robustness to Pareto front of multi-objective optimization algorithms in terms of coverage, convergence and success metrics.

The computational complexity of NSWOA algorithm is order of $O\left(mn^2\right)$where N is the number of individuals in the population and M is the number of objectives. The complexity for other good algorithms in this field: NSGA-II, MOPSO, SPEA2 and PAES are $O\left(mn^2\right)$. However, the computational complexity is much better than some of the algorithms such as NSGA and SPEA which are of $O\left(mn^3\right)$.

III. Non-Dominated Sorting Whale Optimization Algorithm (nswoa)

The Whale Optimization Algorithm (WOA) with sole objective was proposed by Mirjalili Seyedali and Andrew Lewis in 2016 [17]. It is basically a stochastic population based, nature inspired algorithm. In this algorithm the basic strategy based on special hunting nature of humpback whales.Some fact about humpback whales that they are: fancy creatures, biggest mammals in the world and have power of think, learn, judge, communicate, being emotional etc. They are also considered as predators as they never sleep, only half of the brain sleeps, as they have to breathe from surface of the ocean. One more interesting fact about whales is their hunting or foraging behavior to hunt small fishes. Such type of foraging behavior is known as bubble-net feeding strategy where whales went down in water approximate 10-15 meter and then after start to produce bubbles in a spiral shape encircles prey and then follows the bubbles and moves upward the surface. This foraging behavior is done by making distinct bubbles along with a circle or '9-shaped path' represented in Fig. 3 and clearly explained in [54-56].
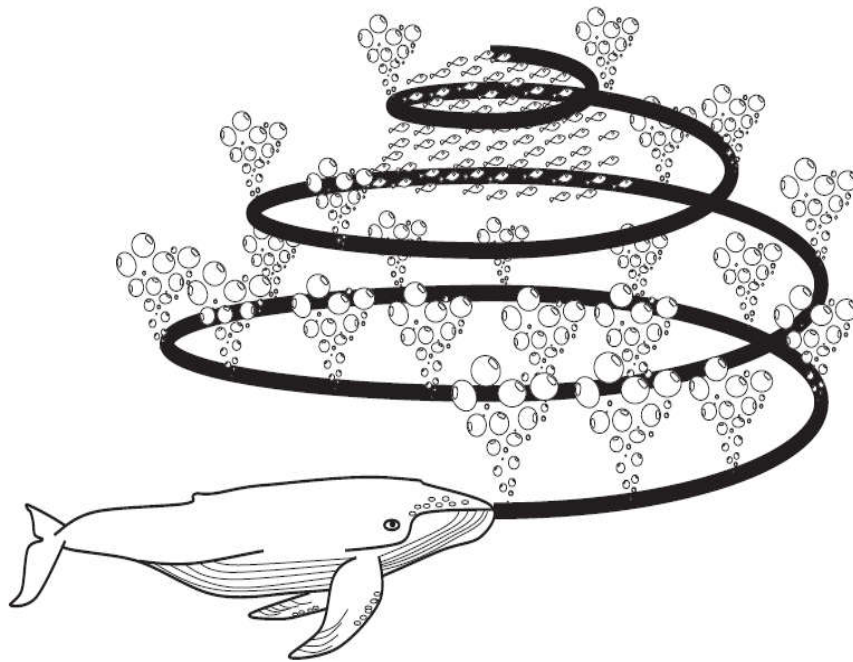

Fig. 3: Representation of foraging behavior or '9-shaped path' of the humpback whales

Mathematical modelling of The Whale Optimization Algorithm:

a) *Encircling Prey Equation*

Humpback whale encircles the prey (small fishes) then updates its position towards the optimum solution over the course of increasing number of iteration from start to maximum number of iteration.

$$\vec{D} = \left| C \cdot \vec{X} * (t) - X(t) \right| \tag{1}$$

$$\vec{X}(t+1) = \vec{X} * (t) - \vec{A} \cdot \vec{D} \tag{2}$$

Where: $\vec{A}, \vec{D}$ are coefficient vectors, t is current iteration, $\overrightarrow{X*}(t)$ is position vector of optimum solution so far and $X(t)$ is position vector.

Coefficient vectors $\vec{A}$, $\vec{D}$ are calculated as follows:

$$\vec{A} = 2\vec{a}*r - \vec{a} \qquad (3)$$

$$\vec{C} = 2*r \qquad (4)$$

Where: $\vec{a}$ is a variable linearly decrease from 2 to 0 over the course of iteration and r is a random number [0, 1].

b) Bubble-net attacking method

In order to mathematical equation for bubble-net behaviour of humpback whales, two methods are modelled as:

i. *Shrinking Encircling Mechanism*

This technique is employed by decreasing linearly the value of $\vec{a}$ from 2 to 0. Random value for vector $\vec{A}$ in rang between [-1, 1].

$$\overrightarrow{X}(t+1) = \begin{cases} \overrightarrow{X*}(t) - \vec{A}.\vec{D} & if \quad p < 0.5 \\ \overrightarrow{D'}.e^{bl}.\cos(2\pi l) + \overrightarrow{X*}(t) & if \quad p \geq 0.5 \end{cases} \qquad (6)$$

Where: p expresses random number between [0, 1].

iii. *Search for prey*

$\vec{A}$ Vector can be used for exploration to search for prey; vector $\vec{A}$ also takes the values greater than one or less than -1. Exploration follows two conditions

$$\vec{D} = \left| \vec{C}.\overrightarrow{X_{rand}} - \vec{X} \right| \qquad (7)$$

$$\overrightarrow{X}(t+1) = \overrightarrow{X_{rand}} - \vec{A}.\vec{D} \qquad (8)$$

Finally follows these conditions:

➤ $\left| \vec{A} \right| > 1$ enforces exploration to WOA algorithm to find out global optimum avoid local optima

➤ $\left| \vec{A} \right| < 1$ for updating the position of current search agent/best solution is selected

Basic working of NSWOA algorithm is as follows:

➤ *Stage 1*
- First of all, initialize the population of the whales
- Randomly generated sets of whales and prey and their position vectorsare represented in matrix for convenience to understand
- Then fitness of each whale's positionis calculated on an according as objective function

ii. *Spiral Updating Position*

Mathematical spiral equation for position update between humpback whale and prey that was helix-shaped movement given as follows:

$$\overrightarrow{X}(t+1) = \overrightarrow{D'}* e^{bt} * \cos(2\pi l) + \overrightarrow{X*}(t) \qquad (5)$$

Where: l is a random number [-1, 1], b is constant defines logarithmic shape, $\overrightarrow{D'} = \left| \overrightarrow{X*}(t) - X(t) \right|$ expresses the distance between i-*th* whale to the prey mean best solution so far.

Note: We assume that there is 50-50% probability that whale either follow the shrinking encircling or logarithmic path during optimization. Mathematically we modelled as follows:

➤ *Stage 2*
- Position of whales are updated as a spiral or helix shaped movement function and so as value of next position of whales is decided
- The value of absolute distance is achieved which is basically a distance between the current best solution (whales current position) to the final (prey position) optimal solution
- We assume that there is 50-50% probability that whale either follow the shrinking encircling or logarithmic path during optimizationmuch needed to update their position towards optimal one

➤ *Stage 3*
- Termination counter in integrated to limit/forcefully stop the search in uncertain search space (max. iteration counter to forcefully converge the search to optimal one)
- Size of the position vector matrix is continuously reduced over the course of iteration due to directed search to find global best solution
- Continuously position of the whalesis updated towards the optimal one viaeither follow the shrinking encircling or logarithmic path during optimizationequation for each iteration

➤ *Stage4*
- Likewise, multi-objective optimization the NSWOA algorithm is made to capable to store the pareto

optimal solutions in a collectionset and make it as flexible to change solution over the course of iteration

- Solution is assigned a rank according to their ability as if a solution is not dominated by other solution is assigned rank1, dominated by only solution assigned rank 2 and so on & if collection set is full (archive size) over predefined size then some solutions that are less non-dominated (according to fitness value) in natureare directed to be out from the collection set according to the crowding distance mechanism.

This collection set is similar to the term achieve used in MOSOS and NSGA-II. It is a repository to store the best non-dominated solutions obtained so far. The search mechanism in NSWOA is very similar to that of WOA algorithm, in which solutions are improved using position vectors. Due to the existence of multiple best solutions, however, the best whales position should be chosen from the collection set.

In order to select solutions from the archive to establish tunnels between solutions, we employ a leader selection mechanism. In this approach, the crowding distance between each solution in the archive is first selection and the number of solutions in the neighbourhood is counted as the measure of coverage or diversity. We require the NSWOAalgorithm to select solutions from the less populated regions of the archive using the following equation to improve the distribution of solutions in the archive across all objectives.

This section proposes multi-objective version of the WOA algorithm called NSWOA algorithm. The non-dominated sorting has been of the most popular and efficient techniques in the literature of multi-objective optimization. As its name implies, non-dominated sorting sort Pareto optimal solutions based on the domination level and give them a rank. This means that the solutions that are not dominated by any solutions is assigned with rank 1, the solutions that are dominated by only one solution are assigned rank 2, the solutions that are dominated by only two solutions are assigned rank 3, and so on. Afterwards, solutions are chosen to improve the quality of the population base on their rank. The better rank, the higher probability to be chosen. The main drawback of non-dominated sorting is its computational cost, which has been resolved in NSGA-II.

The success of the NSGA-II algorithm is an evidence of the merits of non-dominated sorting in the field of multi-objective optimization. This motivated our attempts to employ this outstanding operator to design another multi-objective version of the WOA algorithm. In the NSWOA algorithm, solutions are updated with the same equations presented in equation 3.9. In every iteration, however, the solutions to have optimal position of whalesare chosen using the following equation:

$$\llbracket P \rrbracket\_i = c \diagup \llbracket Rank \rrbracket\_i \qquad (3.9)$$

where c is a constant and should be greater than 1 and $Rank_i$ is the rank number of solutions after doing the non-dominated sorting.

This mechanism allows better solutions to contribute in improving the solutions in the population. It should be noted that non-dominated sorting gives a probability to dominated solutions to be selected as well, which improves the exploration of the NSWOA algorithm. Flow chart of NSWOA algorithm is represented as Fig. 4.

Constraint Handling Approach

With the extended literature survey we find that the population based algorithms are the common way to solve the multi-objective problems as they are more commonly provides the global solution and capable of handling both continuous and combinational optimization problem with a very high coverage and convergence. Multi-objective problems are subjected to various type of constraints like linear, non-linear, equality, inequality etc. So with these problems embedded it is very difficult to find simple and good strategy to achieve considerable solutions in the acceptable criterion. So in this paper NSWOA algorithm uses a very simple approach to get feasible solutions. In this mechanism, after generating number of solutions at each generation, all the desirable constraint checked and then some solution that fulfills the criterion of acceptable solution are selected and collected them in achieve. Afterward non dominated solutions added in archive as we find more suitable solution to get acceptable solution. So as if achieve is full then less dominated solutions are removed. Finally, according to crowing distance mechanism all these solutions (more suitable position of the whales) from archive is selected to get desired solution.
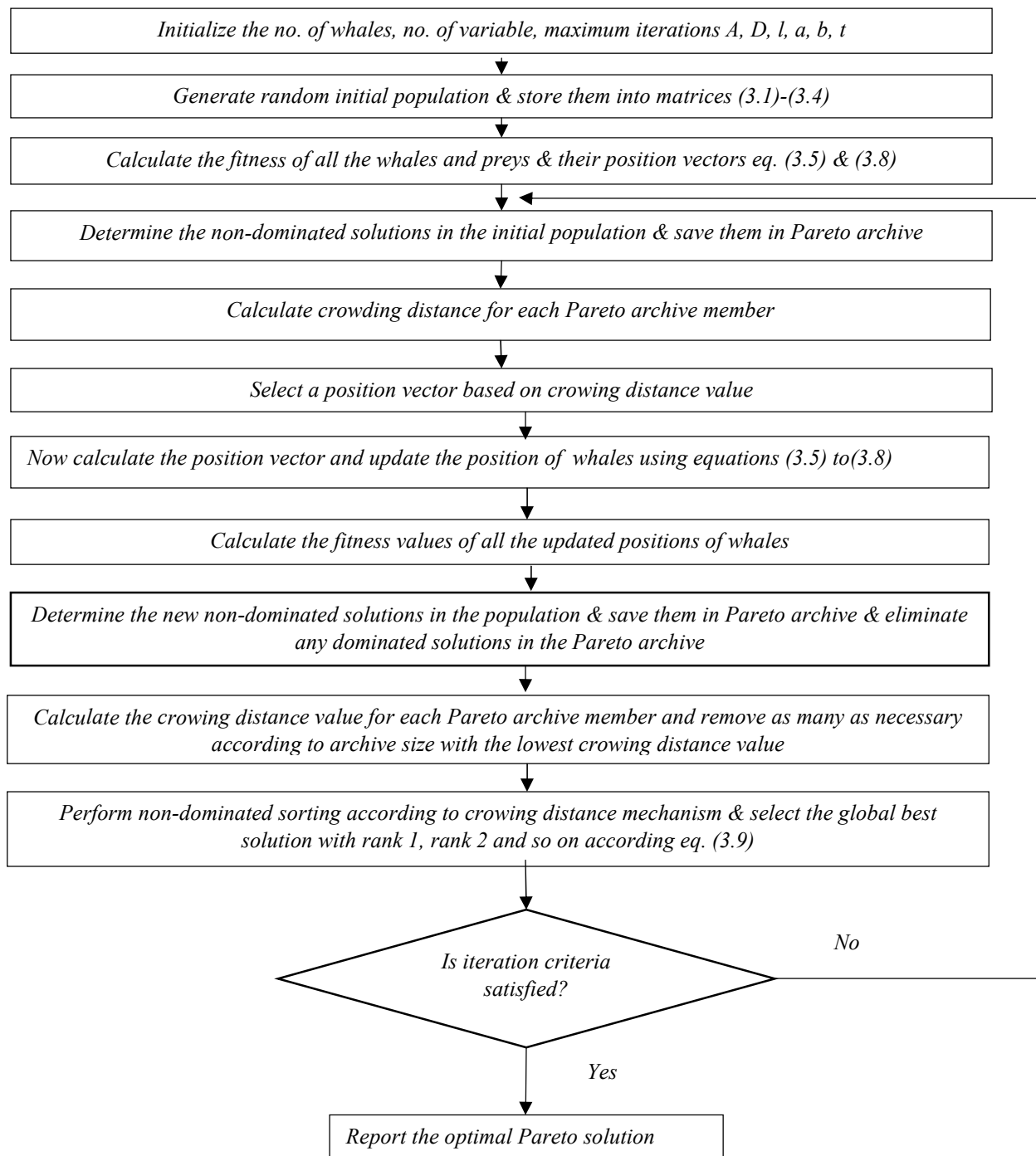
Fig. 4: Flow chart of NSWOA algorithm

IV. RESULTS ANALYSIS ON TEST FUNCTIONS

For determine the performance of proposed NSWOA algorithm is applied to:

- A set of unconstraint and constraint standard multi-objective test functions
- Tested on well-known engineering design problems
- Non-linear, highly complex practical application known as economic constrained emission dispatch (ECED)

- With and Without stochastic integration of wind power (WP) in the next section
- A simple six-operating generational unit with power demand 1200 MW.

NSWOA algorithm is tested on seventeen different multi-objective case studies, including eight unconstrained test functions, five constrained test functions, and four real world engineering design problem, later algorithm is applied to the main application economic constrained emission dispatch

with wind power (ECEDWP). These can be classified into four groups given below:

- Standard multi-objective unconstrained test functions (KUR, FON, ZDT1, ZDT2, ZDT3, ZDT4, SCHN1, and SCHN2)
- Standard multi-objective constrained test functions (TNK, OSY, BNH, SRN, and CONST)
- Real world engineering multi-objective design problem (Four bar truss design, welded beam design, speed reducer and disk brake design problem)
- Modeling of ECEDWP problem

Mathematical representation of these standard test functions is given in Appendix 1. (Multi-objective unconstrained test functions), 2. (Multi-objective constrained test functions), 3. (Engineering multi-objective design problem) with distinct characteristics like non-linear, non-convex, discrete pareto fronts and convex etc. are selected to measure the performance of proposed NSWOA algorithm. To deal with real world engineering design problem is really a typical task with unknown search space, in this article we include four

different engineering problems are considered and performance is compared with various well known algorithms like MOWCA, NSGA-II, MOPSO, PAES and μ-GA multi-objective algorithms. Each algorithm is separately runs fifteen times and numeric results are listed in tables below. To measure the quality of obtained results we match their coverage of obtained true pare to front with respect to their original or true pare to fronts.

For numeric as well as qualitative performance of purposed NSWOA algorithm on various case studies we consider Generational Distance (GD) given by Veldhuizen in 1998 [39]for measuring the deviation of the distance between true pare to front and obtained pare to front, Diversity matric (Δ) also known as matrix of spread to measure the uniformly distribution of non-dominated solution given by Deb [24]and Metric of spacing (S) to represent the distribution of non-dominated distribution of obtained solutions by purposed algorithm given by Schott [40].

The mathematical representation of these performance indicating metric are as follows:

$$GD = \sqrt{(\int_{(i=1)}^{n_{PFs}} \llbracket \llbracket (d) \_i \rrbracket ^2)/n} \tag{4.1}$$

where d_i shows the Euclidean distance (calculated in the objective space) between the ith Pareto optimal solution achieved and the nearest true Pareto optimal solution in the reference set, n_{PFs} is the total number of achieved Pareto optimal solutions.

$$\Delta = \frac{\left| d_l + d_m + \int_{i=1}^{n_{PFs}} |d_i - d| \right|}{d_l + d_m + (n-1)d} \tag{4.2}$$

where, d_l, d_m are Euclidean distances between extreme solutions in true pareto front and obtained pareto front. d_i shows the Euclidean distance between each point in true pare to front and obtained pare to front. n_{PFs} and 'd' are the total number of achieved Pareto optimal solutions and averaged distance of all solutions.

$$S = \sqrt{\frac{1}{n_{PFs}-1} \int_{i=1}^{n_{PFs}} (d_i - d)^2} \tag{4.3}$$

where "d" is the average of all d_i, n_{PFs} is the total number of achieved Pareto optimal solutions, and $d_i = \min_j \left(|f_1^i(\vec{x}) - f_1^j(\vec{x})| + |f_2^i(\vec{x}) - f_2^j(\vec{x})| \right)$ for all $i,j=1,2,…,n$. Smallest value of "S" metric gives the global best non-dominated solutions are uniformly distributed, thus if numeric value of d_i and d are same then value of "S" metric is equal to zero.

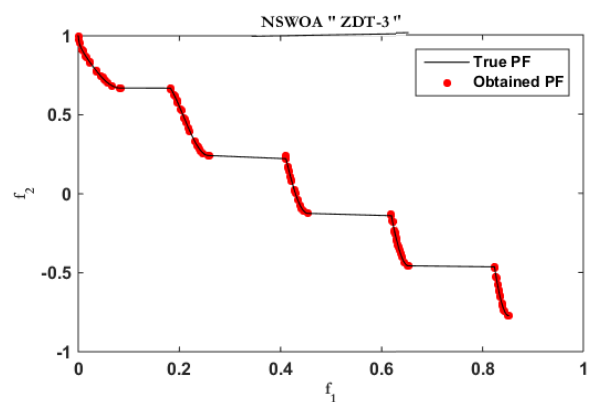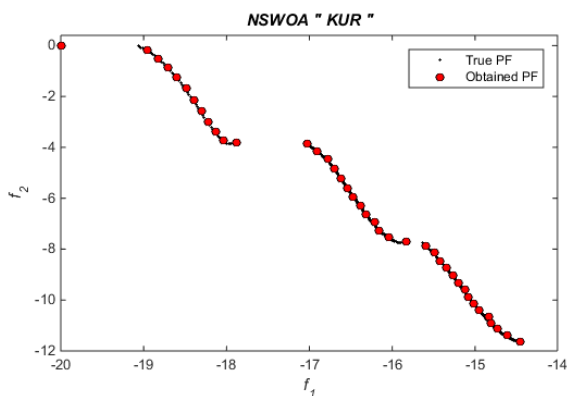a) *Results on unconstrained test problems*

Like as above mentioned, the first set of test problems consist of unconstrained standard test

functions. All the standard unconstrained test functions mathematical formulation is shown in Appendix A. Later, the numeric results are represented in Table 1 and best optimal pare to front is shown in Fig. 5.

All the statistical results are shown Table 1 suggests that the NSWOA algorithm effectively outperforms with most of the unconstraint test functions compare to the MOSOS, MOCBO, MOPSO and NSGA-II algorithm. The effectiveness of proposed non-dominated version of WOA (NSWOA algorithm) can be seen in the Table 1, represents a greater robustness and accuracy of NSWOA algorithm in terms of mean and standard deviation with the help of GD, diversity matrix along with computational time. However, proposed NSWOA algorithm shows very competitive results in comparison with the MOPSO, MOCBO and MOSOS algorithms and in some cases these algorithms perform better than proposed one. Pare to front obtained by proposed NSWOA algorithm shows almost complete coverage with respect to true pare to front.

Table 1: Results of the multi-objective NSWOA algorithms (using GD, Δ, CT) on the unconstrained test functions employed

| Algorithm → Function ↓	PFs	NSWOA MEAN±SD	MOSOS MEAN±SD	MOCBO MEAN±SD	MOPSO MEAN±SD	NSGA II MEAN±SD
	GD	**0.00722±0.00211**	0.0075±0.0042	0.0083±0.0062	0.015±0.0075	0.0301±0.0043
KUR	Δ	**0.02699±0.00025**	0.0295±0.0122	0.0357±0.0236	0.0991±0.031	0.0362±0.0240
	CT	**7.55752±0.43359**	10.7413±0.822	7.9531±0.5823	8.0532±0.621	20.4368±3.102
	GD	**0.00163±0.00022**	0.0019±0.0002	0.0022±0.0003	0.0042±0.000	0.0026±0.0003
FON	Δ	**0.28815±0.03648**	0.3875±0.0062	0.3955±0.0068	0.4158±0.008	0.3987±0.0082
	CT	09.6571±0.54537	11.4013±1.140	**8.6606±0.8862**	8.732±0.9134	22.0323±4.522
	GD	0.36659±0.06618	**0.3325±0.0256**	0.3337±0.0319	0.3348±0.035	0.3352±0.038
ZDT-1	Δ	**0.34579±0.00775**	0.3803±0.0122	0.3825±0.0125	0.3876±0.024	0.3905±0.0220
	CT	6.59899±0.00371	8.2351±0.0204	**3.1435±0.0193**	3.7533±0.006	11.2681±0.364
	GD	**0˙07001±0.00066**	0.0731±0.0010	0.0729±0.0005	0.0733±0.001	0.0725±0.0004
ZDT-2	Δ	**0.04133±0.06577**	0.4307±0.0007	0.4316±0.0007	0.4321±0.001	0.431±0.00075
	CT	4.65825±0.02000	8.2345±0.0457	**3.1502±0.0130**	3.6113±0.014	11.2811±0.024
	GD	**0.07132±0.03917**	0.1022±0.5187	0.0982±0.5007	0.1235±0.009	0.1147±0.0039
ZDT-3	Δ	0.69774±0.23268	0.6537±0.0052	**0.65325±0.002**	0.8234±0.108	0.7386±0.0474
	CT	8.77756±0.34789	13.4567±0.129	6.2846±0.1059	**8.3764±0.231**	14.3406±0.144
	GD	**0.49888±0.00022**	0.5015±0.0006	0.5078±0.0013	0.5146±0.001	0.5204±0.0019
ZDT-4	Δ	**0.35779±0.01477**	0.4585±0.0073	0.4795±0.0079	0.6543±0.024	0.7003±0.0089
	CT	7.87855±0.12275	13.9022±0.121	**6.6922±0.1440**	8.8203±0.218	14.8102±0.170
	GD	0.00999±0.00075	**0.0028±0.0024**	0.0031±0.0032	0.0032±0.003	0.0034±0.0042
SCHN-1	Δ	**0.50066±0.01477**	0.5295±0.1312	0.5302±0.1356	0.8582±0.164	0.5502±0.1360
	CT	11.7600±1.23165	8.2135±1.121	**5.4845±1.1320**	5.5721±1.133	17.9121±2.162
	GD	**0.04977±0.00188**	0.0705±0.0215	0.0932±0.0228	0.1497±0.022	0.3096±0.0217
SCHN-2	Δ	**0.65698±0.02888**	0.7821±0.0512	0.801±0.08326	0.8652±0.060	0.9562±0.0921
	CT	5.79912±0.14008	8.7015±0.4532	5.9751±0.2821	6.0272±0.582	18.421±2.1802
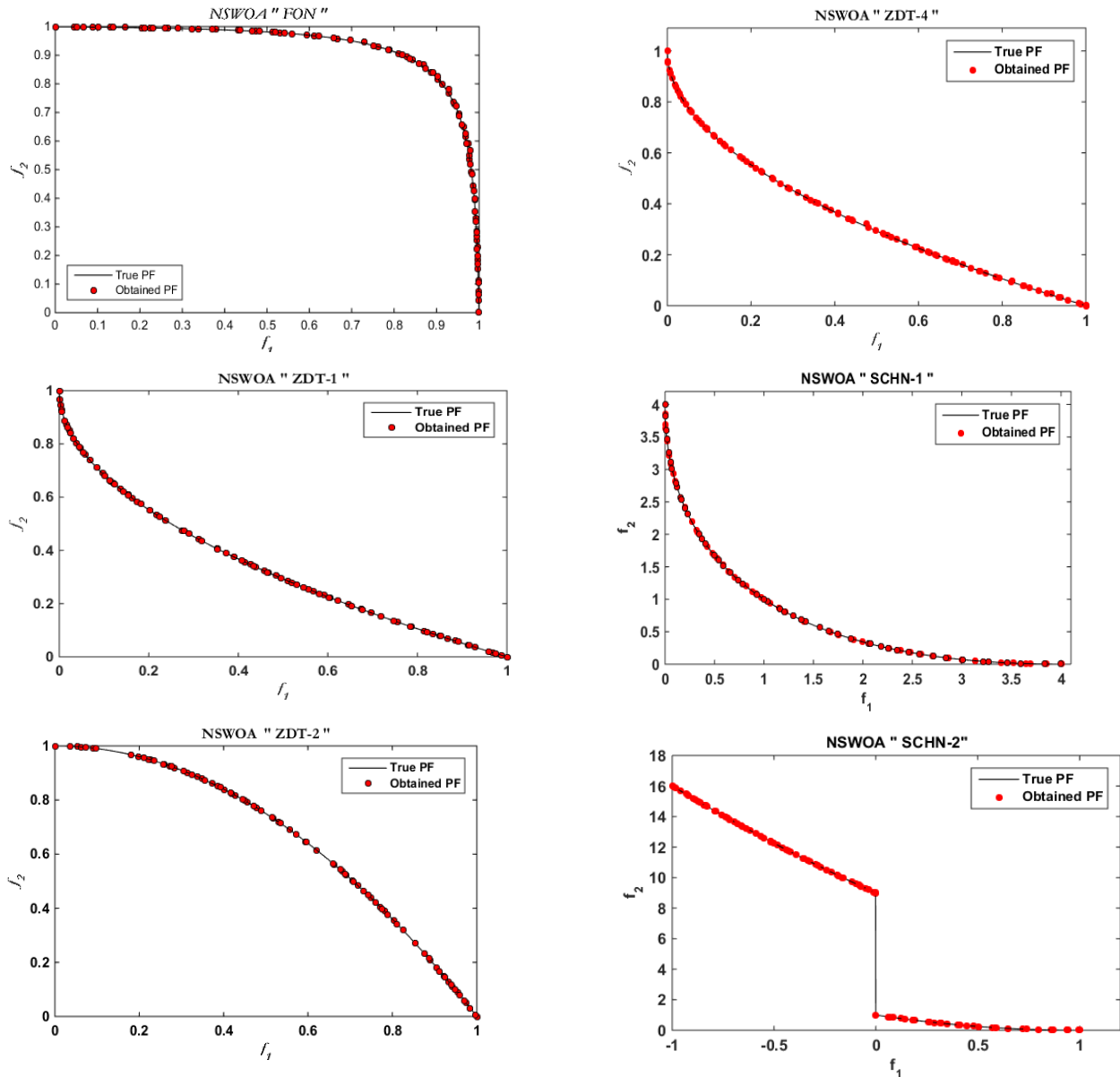
Fig. 5: Best Pareto optimal front of KUR, FON, ZDT1, ZDT2, ZDT3, ZDT4, SCHN1 and SCHN2 obtained by the NSWOA algorithm

b) Results on constrained test problems

The next set of standard test functions consisting of constrained functions. For constrained test function it should be necessary that NSWOA algorithm has a capability of handling constraints so algorithm is equipped with a death penalty function to search agents that violate any of the constraints at any level [41]. For comparing the results of different algorithms, we have utilized GD and Δ metrics.

Table 2: Results of the multi-objective NSWOA algorithms on constrained test problems

| Algorithm→ Function ↓	PFs	NSWOA MEAN±SD	MOSOS MEAN±SD	MOCBO MEAN±SD	MOPSO MEAN±SD	NSGA-II MEAN±SD
	GD	0.14566±0.00216	0.1508±0.0040	0.1528±0.0051	0.1576±0.0062	0.1542±0.0072
TNK	Δ	0.09996±0.05027	0.1206±0.0423	0.1242±0.0512	0.1286±0.0522	0.126±0.06242
	CT	10.7775±0.04668	15.1286±0.063	11.0104±0.052	12.0212±0.054	17.4204±0.055
	GD	0.10004±0.00029	0.1196±0.0031	0.1210±0.0041	0.1282±0.0042	0.1242±0.0043
OSY	Δ	0.54798±0.06679	0.5354±0.0616	0.5422±0.0712	0.5931±0.0721	0.5682±0.0751
	CT	15.4470±0.02008	20.2124±0.032	12.2104±0.030	14.6420±0.042	24.2204±0.039
	GD	0.14447±0.00488	0.1436±0.0062	0.1498±0.0076	0.1644±0.0078	0.1566±0.0042
BNH	Δ	0.44477±0.03786	0.4288±0.0625	0.4798±0.0721	0.4975±0.0632	0.4892±0.0832
	CT	07.5524±0.04587	16.2664±0.054	9.1544±0.0420	9.7452±0.0464	19.652±0.0511
	GD	0.05881±0.01499	0.0988±0.0014	0.1018±0.0015	0.1125±0.0026	0.1024±0.0032
SRN	Δ	0.20444±0.00098	0.2295±0.0017	0.2352±0.0019	0.2730±0.0023	0.^{2}468±0.0018
	CT	7.24456±0.00122	12.3254±0.012	7.3251±0.0082	9.2134±0.0083	17.0231±0.023
	GD	0.42115±0.02998	0.5162±0.0021	0.5202±0.0034	0.5854±0.0036	0.5532±0.0041
CONST	Δ	0.7865±0.000666	0.7122±0.0072	0.7235±0.0083	0.7344±0.0084	0.8126±0.0087
	CT	16.7555±0.00050	10.0112±0.003	5.2252±0.0028	6.4766±0.0035	14.0892±0.003
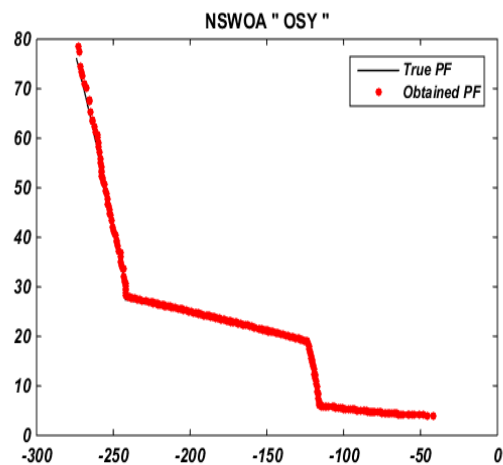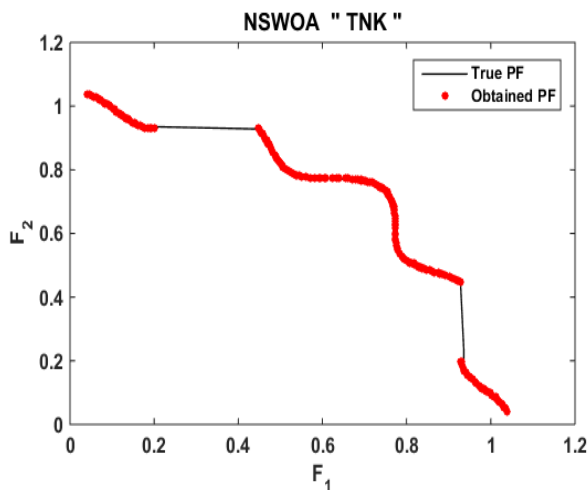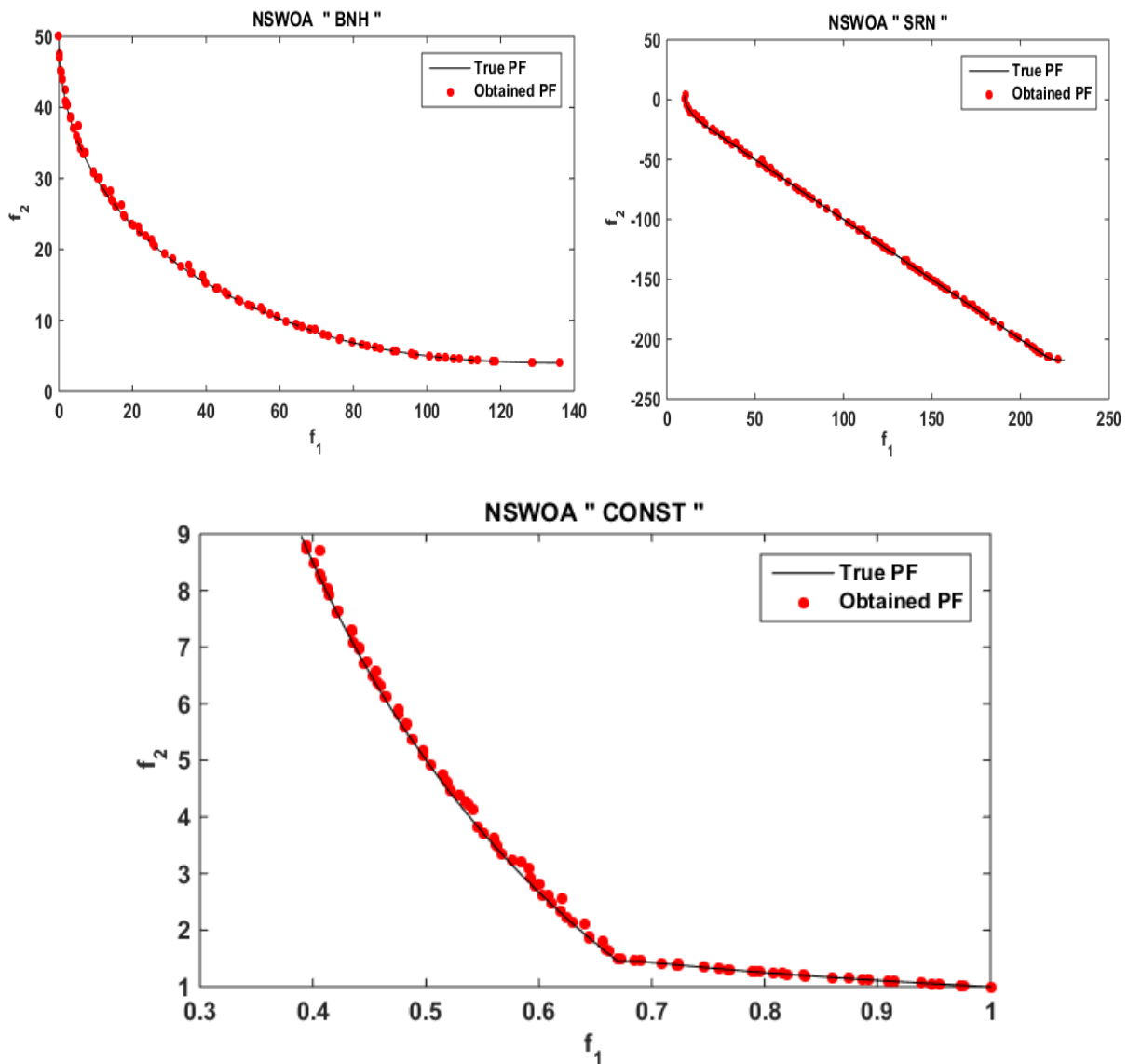
Fig. 6: Best Pareto optimal front TNK, OSY, BNH, SRN and CONST obtained by NSWOA algorithm

Table 2 suggests that the NSWOA algorithm comparatively performs better than other four algorithms for most of the standard constrained test functions employed. The best Pareto optimal fronts in Fig. 6 also helps in proving since all the Pareto optimal solutions exactly follow the true pare to fronts obtained from by NSWOA algorithm.

CONST function consists of concave front with linear front, OSY is similar to CONST but consists of many linear regions with different slops while TNK almost similar to wave shaped. These also suggests that NSWOA algorithm has a capability to solve various type of constraint problem. All the constraint test functions are mathematically given in Appendix B.

c) Results On Constrained Engineering Design Problems

The third set of test functions is the most complicated one and consists of four real engineering design problems. Mathematical model of all the four engineering design problem are given in Appendix C. Same as before both GD and diversity matrix is employed to measure the performance of NSWOA algorithm with respect to other algorithms to solve them, numeric results are given in Tables and Figure respectively shows the best optimal front obtained by NSWOA algorithm.

i. *Four-bar truss design problem*

The statistical results of four bar truss design problem [42] in given in Table 3 and best optimal front is given in Fig. 7. It consists of two minimization objectives displacement and volume with four design control variable mathematically given in Appendix C.

Table 3: Results of the multi-objective NSWO Aalgorithm on four-bar truss design problem in terms mean and standard deviation

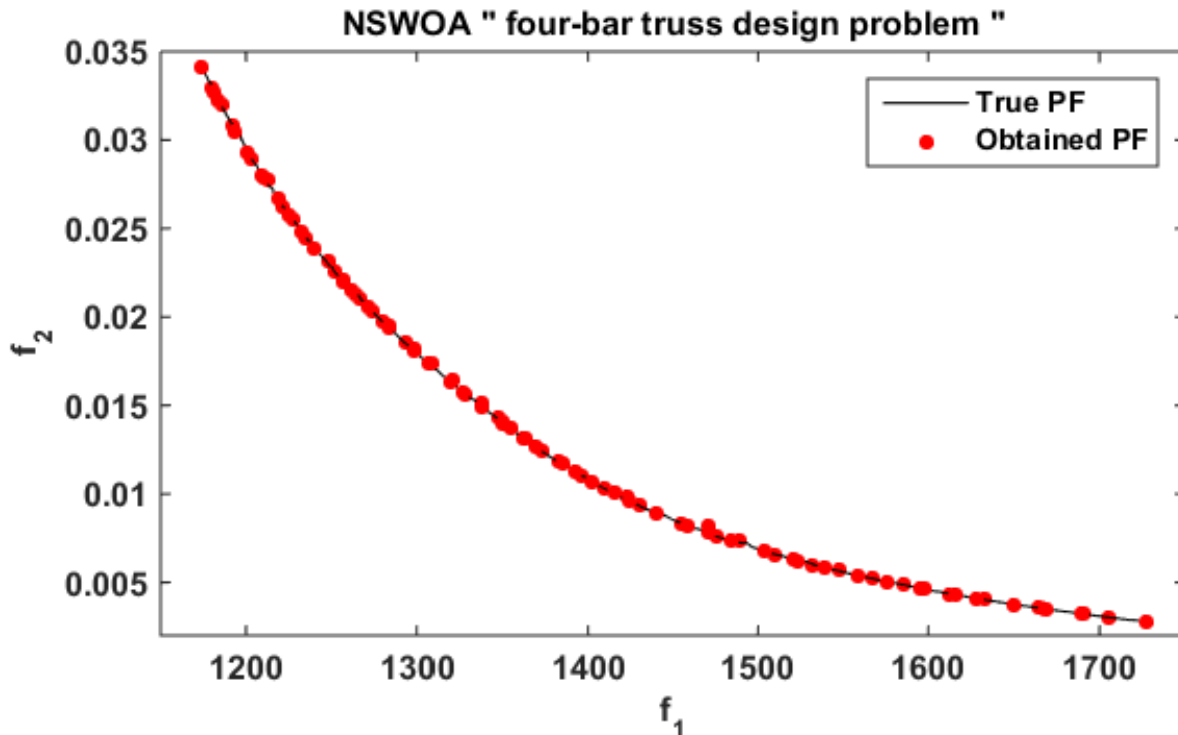| PFs→ Methods ↓	GD	S
	MEAN±SD	MEAN±SD
NSWOA	0.1875±0.0414	1.9829±0.1102
MOWCA	0.2076±0.0055	2.5816±0.0298
NSGA-II	0.3601±0.0470	2.3635±0.2551
MOPSO	0.3741±0.0422	2.5303±0.2275
μ- GA	0.9102±1.7053	8.2742±16.831
PAES	0.9733±1.8211	3.2314±5.9555

Fig.7: Pareto optimal front obtained by the NSWOA Algorithm for "Four –bus truss design problem"

ii   *Speed-reducer design problem*

The statistical results of speed reducer design problem [43] is given in Table 4 and best optimal front is given in Fig. 8. It is a well-known mechanical design problem consists of two minimization objectives stress and weight with seven design control variable mathematically given in Appendix C.

Table 4: Results of the multi-objective NSWOA algorithm on speed-reducer design problem in terms mean and standard deviation

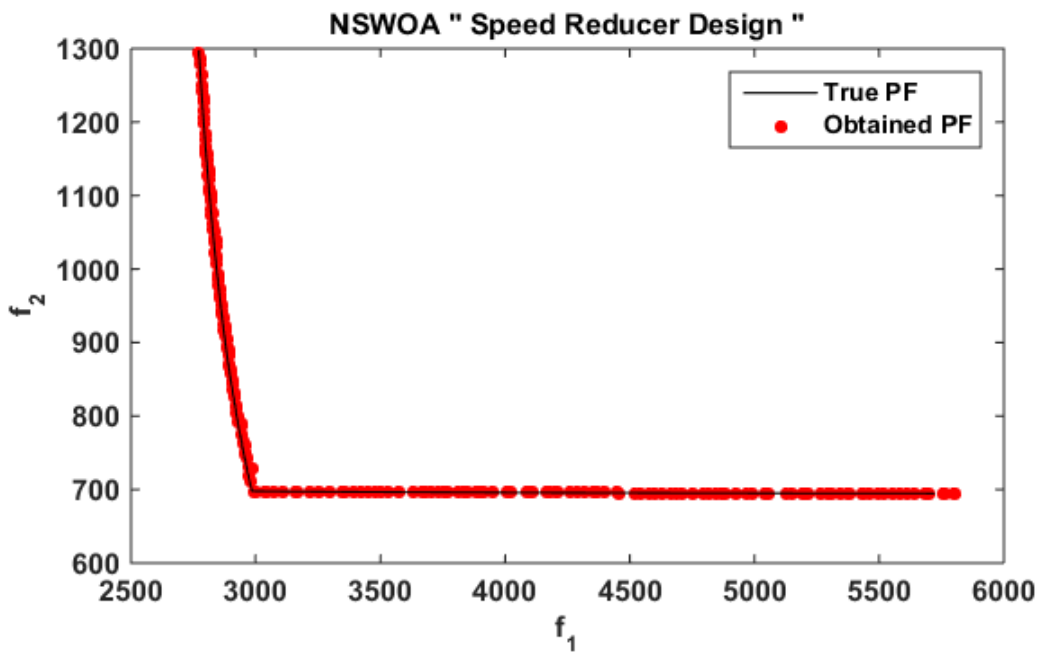| PFs → Methods ↓	GD	S
	MEAN±SD	MEAN±SD
NSWOA	0.96469±0.41014800	1.778124±04.943415
MOWCA	0.98831±0.17894217	16. 68520±2.6969443
NSGA-II	9.843702±7.0810303	02.7654494±3.534978
μ- GA	3.117536±1.6781086	47.80098±32.8015157
PAES	77.99834±4.2102608	16.20129±4.26842769

Fig.8: Pareto optimal front obtained by the NSWOA Algorithm for "Speed Reducer design problem"

iii. *Welded-beam design problem*

The statistical results of welded beam design problem [44] is given in Table 5 and best optimal front is given in Fig. 9. It is a well-known mechanical design problem consists of two minimization objectives fabrication cost and deflection of beam with four design control variable mathematically given in Appendix C.

Table 5: Results of the multi-objective NSWOA algorithms on welded-beam design problem in terms mean and standard deviation

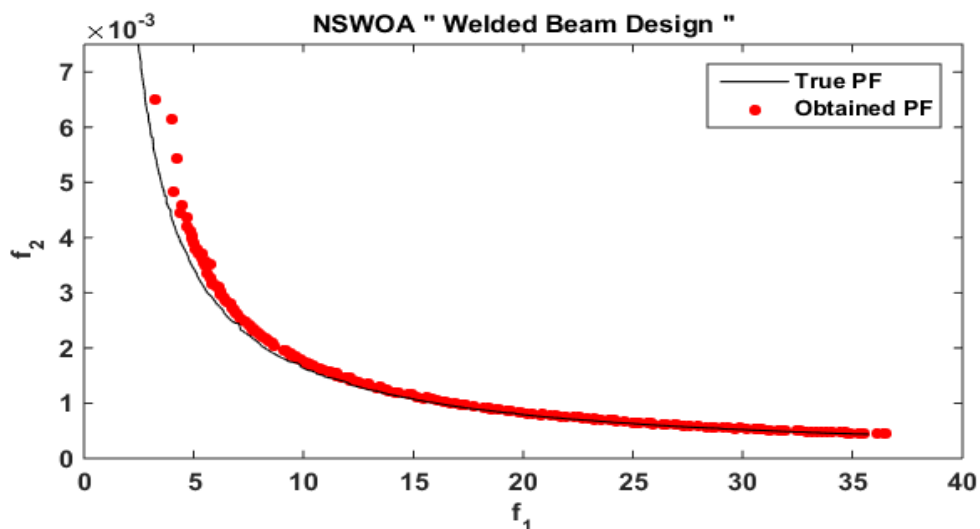| PFs→ Methods ↓	GD	Δ
	MEAN±SD	MEAN±SD
NSWOA	0.03641±0.02588	0.75543±0.02777
MOWCA	0.04909±0.02821	0.22478±0.09280
NSGA-II	0.16875±0.08030	0.88987±0.11976
paε-ODEMO	0.09169±0.00733	0.58607±0.04366

Fig.9: Paretooptimal front obtained by the NSWOA Algorithm for "Welded Beam Design problem"

iv. *Disk brake design problem*

The statistical results of welded beam design problem [44] is given in Table 6 and best optimal front is given in Fig. 10. It is a well-known mechanical design problem consists of two minimization objectives stopping time and mass of brake of a disk brake with four design control variable mathematically given in Appendix C.

Table 6: Results of the multi-objective NSWOA algorithms on the Disk brake design problem in terms mean and standard deviation

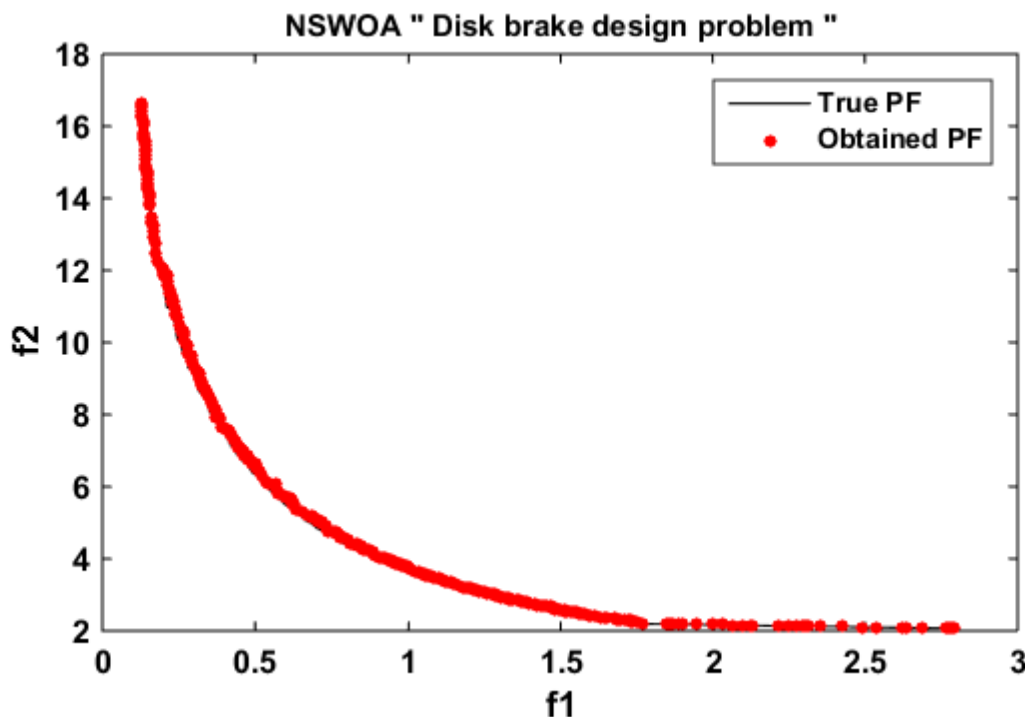| PFS➔ Methods ↓	GD	Δ
	MEAN±SD	MEAN±SD
NSWOA	0.0577±0.27030	0.23341±0.05226
ραε-ODEMO	2.6928±0.24051	0.84041±0.20085
NSGA-II	3.0771±0.10782	0.79717±0.06608
MOWCA	0.0244±0.12314	0.46041±0.10961

Fig.10: Paretooptimal front obtained by the NSWOA Algorithm for "Disk brake design problem"

Due to high complexity of engineering design problem it is really hard to gain results alike true pare to front but we can clearly see that optimal pare to obtained by NSWOA algorithm is covers almost whole solutions that are the actual/true solutions of an engineering design problem. From all above tested function we can conclude that problem either it consists of constraints or unconstraint problem NSWOA algorithm shows its capability to solve any kind of linear, non-linear and complex real world problem. So in the next section we attached a highly non-linear complex real problem to show its effectiveness regarding the real world complex application with many objectives.

d) Formulation Of Economic Constrained Emission Dispatch (ECED) With Integration Of Wind Power (WP)

i. *Mathematical Formulation Of Wind Power*

In case of wind power generation, the output power of wind generator is calculated with the help of a stochastic variable wind speed υ (meter/seconds). Wind speed is a variable function so their probability distribution plays a very important role. Wind speed mathematically formulated as two-parametric Weibull distribution function, probability density function (PDF) and cumulative distribution function (CDF) as follows:

$$s(v) = (k/c)\,(v/c)^{k-1} * \exp[-(v/c)^k],\ v \geq 0 \qquad (4.1)$$

$$S(v) = 1 - \exp[-(v/c)^k],\ v \geq 0 \qquad (4.2)$$

where, S(v) and s(v) are CDF and PDF respectively. Shape factor and scale factor are k and c respectively. The wind speed and output wind power are related as:

$$P_{wind} = \begin{cases} 0, & v < v_{in} \ or \ v \geq v_{out} \\ P^{rated} \frac{v-v_{in}}{v_{rated}-v_{in}} v_{in} \leq v < v_{rated} & (4.3) \\ P_{rated} v_{in} \leq v < v_{out} \end{cases}$$

where, v_{rated} and P_{rated} are the rated speed of wind and rated power output. v_{out} and v_{in} are cut-out and cut-in speed of wind respectively. The CDF of P_{wind} in the boundary of [0, P_{rated}] on an accordance with the speed range of wind can be formulated as:

$$S(P_{wind}) = 1 - exp\left\{-\left[\left(1 + \frac{v_{rated}-v_{in}}{v_{in}*P_{rated}} P_{wind}\right)\frac{v_{in}}{c}\right]^k\right\} + exp[-(v_{out}/c)^k]$$
$$0 \leq P_{wind} < P_{rated}$$
(4.4)

Above equation is very meaningful to calculate the ECED problems with speculative wind power with variable speed.

ii. *Modeling of ECEDWP problem*

As wind power is formulated as system constraint, so the objective function of economic emission dispatch problem (EEDP) stays on unchanged as classical EEDP:

Fuel cost objective is given by:

$$Minimization \quad S(P_i) = \sum_i^N (a_i + b_i P_i + c_i P_i^2) \quad (4.5)$$

where, the thermal power generators cost coefficients are a_i, b_i, c_i for i-th generator, Sum of the total fuel cost of the system and N is the total number of generators. Total Emission is calculated by:

$$Minimization \ E(P_i) = \sum_i^N[\{(\alpha_i + \beta_i P_i + \gamma_i P_i^2)*10^{-2}\} + \delta_i * exp(\varphi_i * P_i)] \quad (4.6)$$

where, $\alpha_i, \beta_i, \gamma_i, \delta_i$ and φ_i are emission coefficients with valve point effect taking into consideration for i-th thermal generator.

iii. *System Constraints*

As wind power generation is considered as system constraint with the summation of stochastic variables the classical power balance constraint changes to fulfill the predefined confidence level.

$$P_r \sum_{i=1}^N (P_i + P_{Wind} \geq P_D + P_{Loss}) \geq \eta_{pbc} \quad (4.7)$$

where, η_{pbc} is confidence level that a power system must follow the load demand and so as it is selected nearer to unity as values lesser than unity represents high operational risk. P_{loss} represents system losses can be calculated by B-coefficient method given below:

$$P_{Loss} = \sum_{i=1}^N \sum_{j=1}^N P_i B_{ij} P_j + \sum_{i=1}^N P_i B_{i0} + B_{00} \quad (4.8)$$

So as to change above described power balance constrained equation into deterministic form can be solved as:

$$P_r\{P_{Wind} < P_D + P_{Loss} - \sum_{i=1}^N P_i\} = F(P_D + P_{Loss} - \sum_{i=1}^N P_i) \leq 1 - \eta_{pbc} \quad (4.9)$$

Assume that the wind turbine has same speed and same direction and combination of Eqs. (4) and (9), the power balance constraint is represented as:

$$P_D + P_{Loss} - \sum_{i=1}^N P_i \leq \frac{cP_{rated}}{v_{rated}-v_{in}} \left| ln\left[\eta^{pbc} + exp*\left(\frac{v_{out}^k}{c^k}\right)\right]\right|^{\frac{1}{k}} - \frac{v_{in}*P_{rated}}{v_{rated}-v_{in}} \quad (4.10)$$

iv. *Reserve capacity system constraint*

So as to reduce the impact of stochastic wind power on system, up and down spinning reserve needs

to be maintained [22]. Such reserve constraints formulated as [15] and [16] respectively:

$$P_r\{\sum_{i=1}^N (P_i^{max} - P_i) \geq P_{sr} + t_u * P_{Wind}\} \geq \eta_{urc} \quad (4.11)$$

$$P_r\{\sum_{i=1}^N (P_i - P_i^{min}) \geq t_d * (P_{rated} - P_{Wind})\} \geq \eta_{drc} \quad (4.12)$$

where, P_{sr} represents the reserve demand of conventional thermal power plant system and it generally keeps the maximum value of thermal unit, P_i^{max} and P_i^{min} are maximum and minimum output level of operational generators of i-th unit, η_{drc} and η_{urc} are predefined down and upper confidence level parameter respectively, t_u and t_d are the demand coefficients of up and down spinning reserves.

v. *Generational capacity constraint*

The real output power is bounded by each generators upper and lower bounds given as:

$$P_i^{Minimum} \leq P_i \leq P_i^{Maximum} \quad (4.13)$$

V. Test System For Economic Emission Dispatch Problem

a) 40-Operational Thermal Generating Unit

i. *Case Study I- 40 Thermal-Generator Lossless System Without Wind Power*

In this case forty operational generating unit is consider without integration of wind power means all the generating units are coal fired. Input parameters like generators operating limit, fuel cost coefficients and emission coefficients are given in Appendix D and in

Table 11. extracted from [45]. System is considered lossless and its solution is compared with three well known multi-objective algorithms like SMODE [45], NSGA-II [45]and MBFA [46] in terms of various objectives such as best cost, best emission and best compromise between both objectives. Best compromise solution is then obtained by the fuzzy based method [47]. Total power demand for this system is 10500 MW. Results obtained by NSWOA algorithm is added to table 7 and best pare to front obtained by NSWOA algorithm is represented in Fig. 12.

Table 7 : Results of the multi-objective NSWOA algorithms for case study I- 40 thermal-generator lossless system without wind power

Case Study I	SMODE [45]			NSGAII [45]			MBFA [46]			NSWOA		
	Best emission	Best cost	Best compromise	Best emission	Best Cost	Best compromise	Best emission	Best cost	Best compromise	Best emission	Best cost	Best compromise
Cost ($/h)	156,700	119,650	124,230	128,490	124,380	126,180	129,995	121,415	123,638	127,555	**119,310**	**124,831**
Emission (tons/h)	**66,799**	377,560	96,578	93,002	153,560	99,671	176,682	356,424	188,963	87,123	408,020	**94,450**

Fig. 12: Pareto optimal front obtained by the NSWOA Algorithm for "40 thermal-generator lossless system without wind power"

ii. *Case study II- 40 thermal-generator lossless system with wind power*

All the conditions are remaining same as case study I like input parameters and power demand. While

integrating with wind power plant, the total rated output power of wind farm is set to 1000 MW [45, 47].Statistical results obtained by NSWOA algorithm is reported in Table 8 and best optimal front is represented in Fig. 13.

Table. 8: Results of Wilcox on test and simulation/computational time or speed

Case Study-II	SMODE[45]			NSGAII [45]			MOEA/D[51]			NSWOA		
	Best emission	Best cost	Best Compromise - Point	Best emission	Best cost	Best Compromise - Point	Best emission	Best cost	Best Compromise - Point	Best emission	Best cost	Best Compromise - Point
$\sum P_G$ P_w Cost Emission	10,245.76 254.24 153,830 54,055	10,177.55 322.45 116,430 385,770	10,225.71 274.29 123,590 68,855	10,241.72 258.28 132,410 73,894	10,242.09 257.91 122,610 121,850	10,241.63 258.37 126,240 78,860	10,244.43 255.568 154,0 0 0 55,754	10,242.71 257.294 115,770 440,240	10,242.8 257.156 120,950 79,485	10,241.6 255.321 145,636 56,508	10,224.18 276.81 118,789 179,098	10,236.58 263.75 123,449 68,804

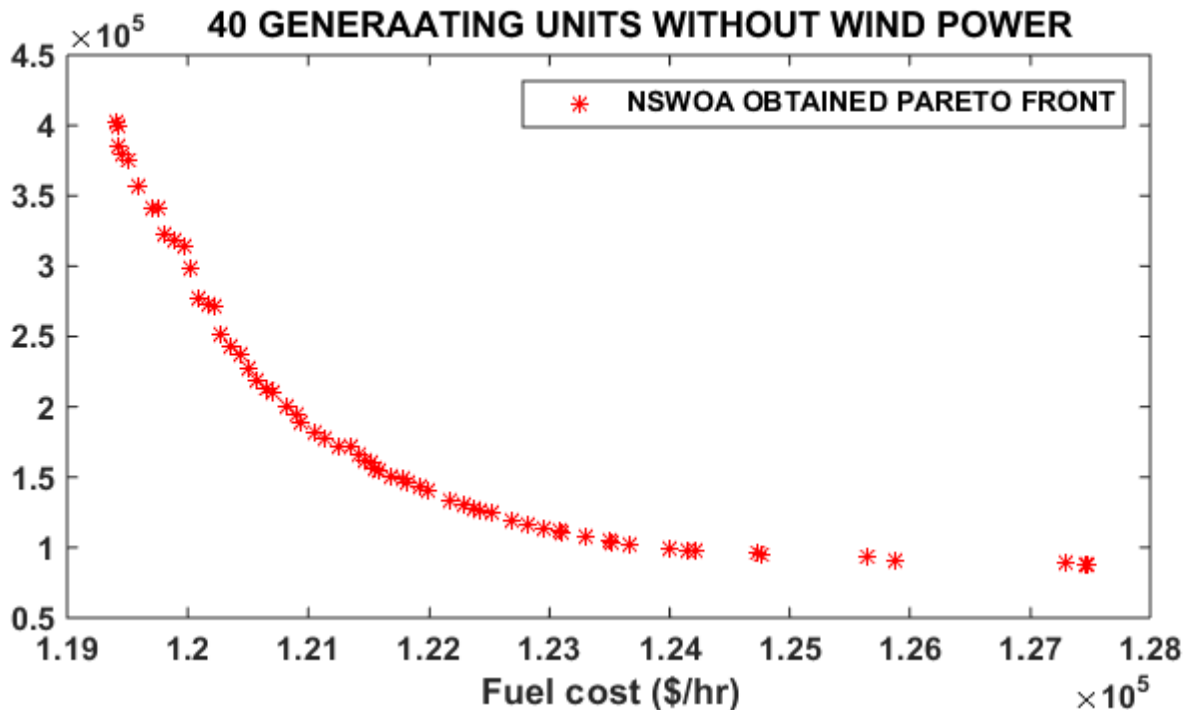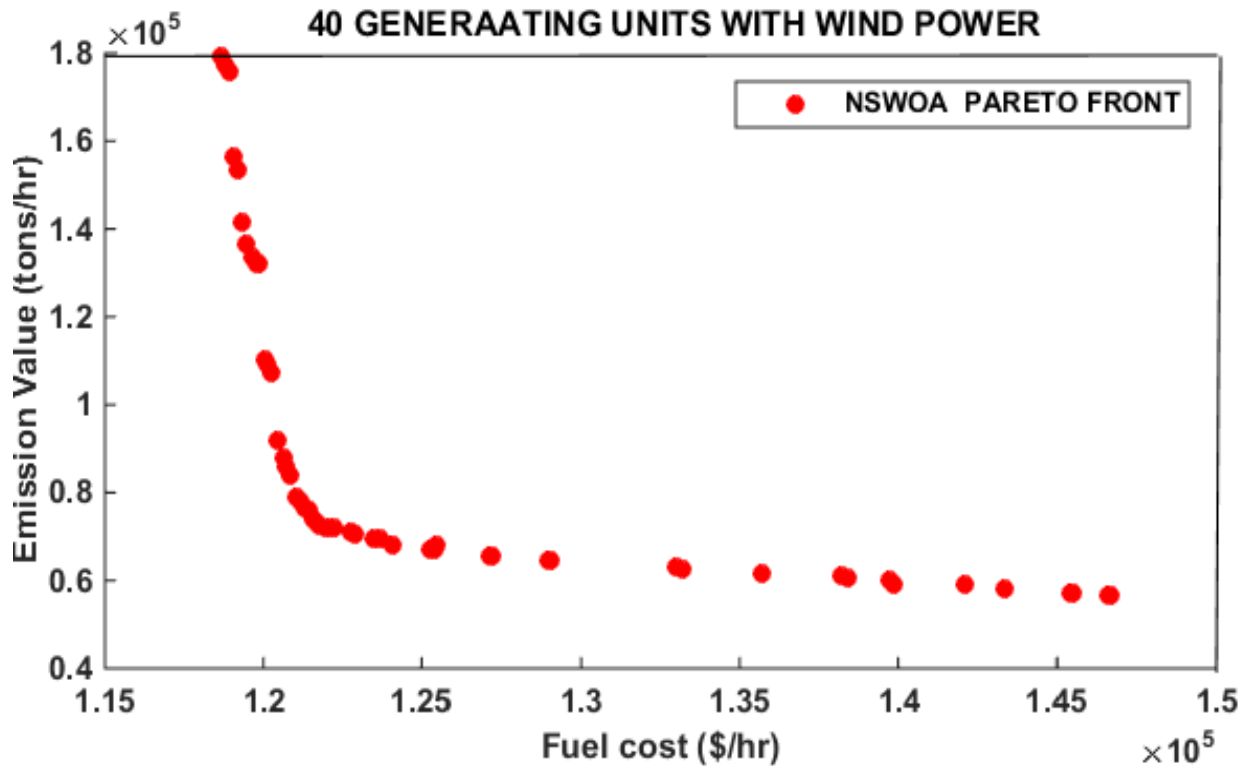*Fig. 13:* Pareto optimal front obtained by the NSWOA Algorithm for "40 thermal-generator lossless system with wind power"

Table 9: Results of Wilcox on test and simulation/computational time or speed

		NSWOA	NSGAII[45]			NSWOA	NSGAII[45]
Case Study I Cost	Best Worst Mean Wilcox on test (H/P) Simulation speed (s)	119310 127555 124831 1/5.38e−10 14.98	124,380 147,760 131,710	Case Study II Cost	Best Worst Mean Wilcox on test (H/P) Simulation speed (s)	118,789 145,636 123,449−10 1/5.65e 19.876	122,610 173,060 134,880
Case Study I Emission	Best Worst Mean Wilcoxon test (H/P) Simulation speed (s)	87,123 408.020 189,284 1/5.54e−10 40.57	93,002 194,830 141,800 154.78	Case Study II Emission	Best Worst Mean Wilcox on test (H/P) Simulation speed (s)	56,508 179,098 104,185 1/5.65e−10 45.67	73,894 158,250 102,120 127.57

b)   Test system with six operational generating unit

This test system consists of six operational generating unit with simply a quadratic fuel and emission objective function for a power demand of 1200 MW. Input data for operational generating unit loading limits and loss parameters are given in Table 12 of Appendix D extracted from [52, 53].

It is represented in Table 10 that with the objective of least cost objective minimum fuel cost is 6.4197e+04 $ and emission value is 1345.9 lb. But fuel cost increases to 6.992e+04 $ and emission value reduced to a numeric value 1242.7 lb with the objective of emission minimization. Compromise point or true operating point obtained by NSWOA algorithm for multi-objective combined economic emission dispatch (MOCEED) problem is as fuel cost is 6.4830e+04 $ that is higher than minimum fuel cost 6.4197e+04$ and lower than 6.992e+04 $ obtained during least cost and emission value objectives respectively. So as with emission value for true operating point is 1285 lb that is lower than 1345.9 lb and higher than 1242.7 lb obtained during least cost and emission value objectives respectively. Statistical value obtained for compromise point is compared with other techniques solves same MOCEED problem like SPEA2, NSGA-II and PDE in Table 10. Fig. 14 shows 100 non-dominated solutions as true pare to front for 6-opertaional generating for PD=1200 MW.

Table 10: Statistical performance comparison of NSWOA Algorithm for 6-operational generating unit system.

| Parameters	NSWOA			MODE[53]	PDE [53]	NSGAII[53]	SPEA2[53]
	Economic dispatch	Emission dispatch	EED	EED	EED	EED	EED
P_1 (MW)	84.7275	125	107.9932	108.6284	107.3965	113.1259	104.1573
P_2 (MW)	93.4118	150	118.3631	115.9456	122.1418	116.4488	122.9807
P_3 (MW)	210	201.4824	210	206.7969	206.7536	217.4191	214.9553
P_4 (MW)	211.8607	198.8723	204.65	210.0000	203.7047	207.9492	203.1387
P_5 (MW)	315	288.5129	306.6592	301.8884	308.1045	304.6641	316.0302
P_6 (MW)	325	286.2913	303.8712	308.4127	303.3797	291.5969	289.9396
Cost ($)	64,197	65,992	64,830	64,843	64,920	64,962	64,884
Emission (lb)	1345.9	1242.7	1285	1286.0	1281.0	1281.0	1285
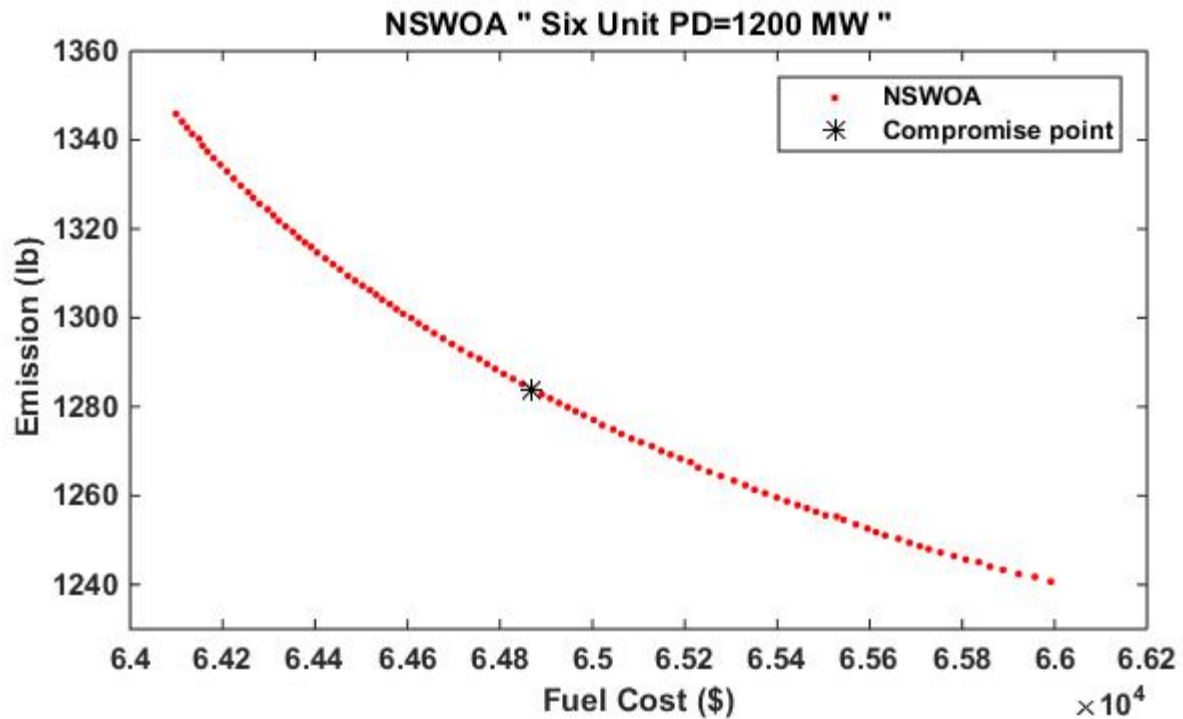
Fig. 14: Pare to optimal front obtained by the NSWOA Algorithm for "six operational generating unit system"

VI.   Result Discussion

In almost all the cases that we consider in this article where NSWOA algorithm proves its effectiveness in both prospective quantitative and qualitative. From plots also evident that NSWOA algorithm follows the exact pare to front similar to the true pare to front for all constrained, unconstrained and complex engineering design problem. So as for real world application of economic emission dispatch problem and its integration with stochastic wind power generation. So for this application Wilcox on test (statistical test)is performed.

In Table 9 the signed rank test is presented in third row of each results whereas the calculation time is represented in forth row. For this test null hypothesis cannot be rejected at 5% level for numeric value '0' while null hypothesis is rejected at 5% level with the value of '1'. Where NSWOA algorithm performs superior to other algorithms that are considered for comparative purpose.

NSWOA algorithm shows good performance in both coverage and convergence as main mechanism that guarantee convergence in WOA and NSWO Aalgorithms are continuously shrink its virtual limitation using helix shaped or 9-shaped path strategy in the movement of whales for their random walk. Both mechanism emphasizes convergence and exploitation proportional to maximum number of generation (iteration). Since this complex task might degrade its performance compare to without limitation or free movement should be a concern. However, the numerical results expresses that NSWOA algorithm has a little effect of slow convergence at all.

NSWOA algorithm has an advantage of high coverage, which is the result of the selection of position of whales and archive selection procedure. All the position is updated according to their fitness value that enable the algorithm to direct the search space in right direction to find the best solution without trapped in local solution. Archive selection criteria follow all the rules of the entrance and exhaust of any value in it for each iteration and updated when its size full. Solutions of higher fitness in archive have higher probability to thrown away first to improve the coverage of the pare to optimal front obtained during the optimization process.

VII.   Conclusion

In this paper the non-dominated sorting whale optimization algorithm-multi-objective version of recently proposed whale optimizationalgorithm (WOA) is proposed known as NSWOA algorithm. This paper also utilizes the bubble-net swarming strategy for exploration purpose used in its parent WOA version. The NSWOA algorithm is developed with equipping whale optimization algorithm with crowding distance criterion, an archive and whales position (accordance to ranking) selection method based on Pareto optimal dominance nature. The NSWOA algorithm is first applied on 17 standard test functions (including eight unconstraint, five constraint and four engineering design multi-objective problems) to prove its capability in terms of qualities and quantities showing numerical as well as convergence and coverage of pare to optimal front with respect to true pare to front. Then after NSWOA algorithm is applied to real world complex ECEDWP problem where

algorithm proves its dominance over other well recognized contemporary algorithms. The numeric results are stored and represented in performance indices: GD, metric of diversity, metric of spacing and computational time. The qualitative results are reported as convergence and coverage in best pare to optimal front found in 15 independent runs. To check effectiveness of proposed version of algorithm the results are verified with SMODE, MOSOS, MOCBO, MOPSO, NSGA-II and other well recognize algorithms in the field of multi-objective algorithms. We can also conclude from the standard test functions results that NSWOA algorithm is able to find pare to optimal front of any kind of shape. Finally, the result of complex real world ECEDWP problem validates that NSWOA algorithm is capable of solving any kind of non-linear and complex problem with many constraint and unknown search space. Therefore, we conclude that proposed non-dominated version of WOA algorithm has various merits among the contemporary multi-objective algorithms as well as provides an alternative for solving multi or many objective problems.

For future works, it is suggested to test NSWOA algorithm on other real world complex problems. Also, it is worth to investigate and find the best constrained handling technique for this algorithm.

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Appendix A: Unconstrained multi-objective test problems utilized in this work.

KUR:

Minimize:
$$f_1(x) = \sum_{i=1}^{2}\left[-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right]$$
$$f_2(x) = \sum_{i=1}^{2}[|x_i|^{0.8} + 5\sin(x_i^3)]$$
$$-5 \le x_i \le 5$$
$$1 \le i \le 3$$

FON:

$$\min imize = \begin{cases} f_1(x) = 1 - \exp\left[-\sum_{i=1}^{n}(x_i - \frac{1}{\sqrt{n}})^2\right] & -4 \le x_i \le 4 \\ f_2(x) = 1 - \exp\left[-\sum_{i=1}^{n}(x_i + \frac{1}{\sqrt{n}})^2\right] & 1 \le i \le n \end{cases}$$

ZDT1:

Minimise: $\quad f_1(x) = x_1$

Minimise: $\quad f_2(x) = g(x) \times h(f_1(x), g(x))$

Where:

$$G(x) = 1 + \frac{9}{N-1}\sum_{i=2}^{N} x_i \quad h(f_1(x), g(x)) = 1 - \sqrt{\frac{f_1(x)}{g(x)}} \quad 0 \le x_i \le 1, 1 \le i \le 30$$

ZDT2:

Minimise: $\quad f_1(x) = x_1$

Minimise: $\quad f_2(x) = g(x) \times h(f_1(x), g(x))$

Where: $\quad G(x) = 1 + \frac{9}{N-1}\sum_{i=2}^{N} x_i \quad h(f_1(x), g(x)) = 1 - \left(\frac{f_1(x)}{g(x)}\right)^2 \quad 0 \le x_i \le 1, 1 \le i \le 30$

ZDT3:

Minimise: $\quad f_1(x) = x_1$

Minimise: $\quad f_2(x) = g(x) \times h(f_1(x), g(x))$

Where: $\quad G(x) = 1 + \frac{9}{29}\sum_{i=2}^{N} x_i \quad h(f_1(x), g(x)) = 1 - \sqrt{\frac{f_1(x)}{g(x)}} - \left(\frac{f_1(x)}{g(x)}\right)\sin(10\pi f_1(x)) \quad 0 \le x_i \le 1, 1 \le i \le 30$

ZDT4:

Minimise: $\quad f_1(x) = x_1$

Minimise: $\quad f_2(x) = g(x) \times h(f_1(x), g(x))$

$$h(f_1(x), g(x)) = 1 - \sqrt{\frac{f_1(x)}{g(x)}} \qquad g(x) = 91 + \sum_{i=2}^{10}(x_i^2 - 10 * \cos(4\pi x_i))$$

SCHN-1 :

Minimize: $f_1(x) = x_i^2$

$f_2(x) = (x - 2)^2 \quad -A \le x \le A$

Where: value of can be from 10 to 10^5.

SCHN-2 :

Minimize:
$$\begin{cases} f_1(x) = \begin{cases} -x, & if \quad x \le 1 \\ x - 2, & if \quad 1 < x \le 3 \\ 4 - x, & if \quad 3 < x \le 4 \\ x - 4, & if \quad x > 4 \end{cases} \quad -5 \le x \le 10 \\ f_2(x) = (x - 5)^2 \end{cases}$$

Appendix B: Constrained multi-objective test problems utilised in this work.

TNK:

Minimise: $\quad f_1(x) = x_1$

Minimise: $\quad f_2(x) = x_2$

Where: $\quad g_1(x) = -x_1^2 - x_2^2 + 1 + 0.1Cos(16arctan\left(\frac{x_1}{x_2}\right))$

$$g_2(x) = 0.5 - (x_1 - 0.5)^2 - (x_2 - 0.5)^2 \quad 0.1 \le x_1 \le \pi, 0 \le x_2 \le \pi$$

BNH:

This problem was first proposed by Binh and Korn [48]:

Minimise: $\quad \boldsymbol{f_1(x) = 4x_1^2 + 4x_2^2}$

Minimise: $\quad f_2(x) = (x_1 - 5)^2 + (x_2 - 5)^2$

Where: $\quad g_1(x) = (x_1 - 5)^2 + x_2^2 - 25$

$$g_2(x) = 7.7 - (x_1 - 8)^2 - (x_2 + 3)^2 \quad 0 \le x_1 \le 5, 0 \le x_2 \le 3$$

OSY:

The OSY test problem has five separated regions proposed by Osyczka and Kundu [49]. Also, there are six constraints and six design variables.

Minimise: $\quad f_1(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2$

Minimise: $f_2(x) - [25(x_1 - 2)^2 + (x_2 - 1)^2 + (x_3 - 1)^2 + (x_4 - 4)^2 + (x_5 - 1)^2]$

Where:
$g_1(x) = 2 - x_1 - x_2$
$g_2(x) = -6 + x_1 + x_2$
$g_3(x) = -2 - x_1 + x_2$
$g_4(x) = -2 + x_1 - 3x_2$
$g_5(x) = -4 + x_4 + (x_3 - 3)^2$
$g_6(x) = 4 - x_6 - (x_5 - 3)^2 0 \leq x_1 \leq 10, 0 \leq x_2 \leq 10, 1 \leq x_3 \leq 5, 0 \leq x_4 \leq 6, 1 \leq x_5 \leq 5, 0 \leq x_6 \leq 10$

SRN:

The third problem has a continuous Pareto optimal front proposed by Srinivas and Deb [50].

Minimise: $f_1(x) = 2 + (x_1 - 2)^2 + (x_2 - 1)^2$

Minimise: $f_2(x) = 9x_1 - (x_2 - 1)^2$

Where: $g_1(x) = x_1^2 + x_2^2 - 255$

$g_2(x) = x_1 - 3x_2 + 10 -20 \leq x_1 \leq 20, -20 \leq x_2 \leq 20$

CONSTR:

This problem has a convex Pareto front, and there are two constraints and two design variables.

Minimise: $f_1(x) = x_1$

Minimise: $f_2(x) = (1 + x_2)/(x_1)$

Where: $g_1(x) = 6 - (x_2 + 9x_1), g_2(x) = 1 + x_2 - 9x_1 \ 0.1 \leq x_1 \leq 1, 0 \leq x_2 \leq 5$

Appendix C: Constrained multi-objective engineering problems used in this work.

Four-bar truss design problem:

The 4-bar truss design problem is a well-known problem in the structural optimisation field [42], in which structural volume (*f1*) and displacement (*f2*) of a 4-bar truss should be minimized. As can be seen in the following equations, there are four design variables (*x1-x4*) related to cross sectional area of members 1, 2, 3, and 4.

Minimise: $f_1(x) = 200 * (2 * x(1) + sqrt(2 * x(2)) + sqrt(x(3)) + x(4))$

Minimise: $f_2(x) = 0.01 * \left(\left(\frac{2}{x(1)}\right) + \left(\frac{2*sqrt(2)}{x(2)}\right) - ((2 * sqrt(2))/x(3)) + (2/x(1)))\right)$
$1 \leq x_1 \leq 3, 1.4142 \leq x_2 \leq 3, 1.4142 \leq x_3 \leq 3, 1 \leq x_4 \leq 3$

Speed reducer design problem:

The speed reducer design problem is a well-known problem in the area of mechanical engineering [43], in which the weight (*f1*) and stress (*f2*) of a speed reducer should be minimized. There are seven design variables: gear face width (*x1*), teeth module (*x2*), number of teeth of pinion (*x3* integer variable), distance between bearings 1 (*x4*), distance between bearings 2 (*x5*), diameter of shaft 1 (*x6*), and diameter of shaft 2 (*x7*) as well as eleven constraints.

Minimise: $f_1(x) = 0.7854 * x(1) * x(2)^2 * (3.3333 * x(3)^2 + 14.9334 * x(3) - 43.0934) - 1.508 * x(1) * (x(6)\wedge2 + x(7)\wedge2) + 7.4777 * (x(6)\wedge3 + x(7)\wedge3) + 0.7854 * (x(4) * x(6)\wedge2 + x(5) * x(7)\wedge2)$

Minimise: $f_2(x) = ((sqrt(((745 * x(4))/(x(2) * x(3)))\wedge2 + 16.9e6))/(0.1 * ... x(6)\wedge3))$

Where: $g_1(x) = 27/(x(1) * x(2)\wedge2 * x(3)) - 1$

$g_2(x) = 397.5/(x(1) * x(2)\wedge2 * x(3)\wedge2) - 1$

$g_3(x) = (1.93 * x(4)\wedge3)/(x(2) * x(3) * x(6)\wedge4) - 1$

$g_4(x) = (1.93 * x(5)\wedge3)/(x(2) * x(3) * x(7)\wedge4) - 1$

$g_5(x) = ((sqrt(((745 * x(4))/(x(2) * x(3)))\wedge2 + 16.9e6))/(110 * x(6)\wedge3)) - 1$

$$g_6(x) = ((sqrt(((745 * x(5))/(x(2) * x(3)))^2 + 157.5e6))/(85 * x(7)^3)) - 1$$

$$g_7(x) = ((x(2) * x(3))/40) - 1$$

$$g_8(x) = (5 * x(2)/x(1)) - 1$$

$$g_9(x) = (x(1)/12 * x(2)) - 1$$

$$g_{10}(x) = ((1.5 * x(6) + 1.9)/x(4)) - 1$$

$$g_{11}(x) = ((1.1 * x(7) + 1.9)/x(5)) - 1$$

$$2.6 \leq x_1 \leq 3.6, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 7.3 \leq x_4 \leq 8.3, 7.3 \leq x_5 \leq 8.3, 2.9 \leq x_6 \leq 3.9, 5 \leq x_7 \leq 5.5$$

Welded beam design problem:

The welded beam design problem has four constraints first proposed by Ray and Liew [44]. The fabrication cost (*f1*) and deflection of the beam (*f2*) of a welded beam should be minimized in this problem. There are four design variables: the thickness of the weld (*x1*), the length of the clamped bar (*x2*), the height of the bar (*x3*) and the thickness of the bar (*x4*).

Minimise: $\quad f_1(x) = 1.10471 * x(1)^2 * x(2) + 0.04811 * x(3) * x(4) * (14.0 + x(2))$

Minimise: $\quad f_2(x) = 65856000/(30 * 10^6 * x(4) * x(3)^3)$

Where: $\quad g_1(x) = tau - 13600$

$$g_2(x) = sigma - 30000$$

$$g_3(x) = x(1) - x(4)$$

$$g_4(x) = 6000 - P$$

$$0.125 \leq x_1 \leq 5, 0.1 \leq x_2 \leq 10, 0.1 \leq x_3 \leq 10, 0.125 \leq x_4 \leq 5$$

Where

$$Q = 6000 * \left(14 + \frac{x(2)}{2}\right); D = sqrt\left(\frac{x(2)^2}{4} + \frac{(x(1) + x(3))^2}{4}\right)$$

$$J = 2 * \left(x(1) * x(2) * sqrt(2) * \left(\frac{x(2)^2}{12} + \frac{(x(1) + x(3))^2}{4}\right)\right)$$

$$alpha = \frac{6000}{sqrt(2) * x(1) * x(2)}$$

$$beta = Q * \frac{D}{J}$$

$$tau = sqrt\left(alpha^2 + 2 * alpha * beta * \frac{x(2)}{2 * D} + beta^2\right)$$

$$sigma = \frac{504000}{x(4) * x(3)^2}$$

$$tmpf = 4.013 * \frac{30 * 10^6}{196}$$

$$P = tmpf * sqrt\left(x(3)^2 * \frac{x(4)^6}{36}\right) * \left(1 - x(3) * \frac{sqrt\left(\frac{30}{48}\right)}{28}\right)$$

Disk Brake Design Problem:

The disk brake design problem has mixed constraints and was proposed by Ray and Liew [44]. The objectives to be minimized are: stopping time (*f1*) and mass of a brake (*f2*) of a disk brake. As can be seen in

following equations, there are four design variables: the inner radius of the disk ($x1$), the outer radius of the disk ($x2$), the engaging force ($x3$), and the number of friction surfaces ($x4$) as well as five constraints.

Minimise: $f_1(x) = 4.9 * (10^{-5}) * (x(2)^2 - x(1)^2) * (x(4) - 1)$

Minimise: $f_2(x) = (9.82 * (10^6) * (x(2)^2 - x(1)^2))/((x(2)^3 - x(1)^3) * \dots x(4) * x(3))$

Where: $g_1(x) = 20 + x(1) - x(2)$

$g_2(x) = 2.5 * (x(4) + 1) - 30$

$g_3(x) = (x(3))/(3.14 * (x(2)^2 - x(1)^2)^2) - 0.4$

$g_4(x) = (2.22 * 10^{-3} * x(3) * (x(2)^3 - x(1)^3))/((x(2)^2 - x(1)^2)^2) - 1$

$g_5(x) = 900 - (2.66 * 10^{-2} * x(3) * x(4) * (x(2)^3 - x(1)^3))/((x(2)^2 - x(1)^2))$

$55 \le x_1 \le 80, 75 \le x_2 \le 110, 1000 \le x_3 \le 3000, 2 \le x_4 \le 20$

Appendix D:

Test system 1: 40-operational thermal generating unit

Unit	Pmin	Pmax	a_i	b_i	c_i	α_i	β_i	γ_i	ζ_i	λ_i
1	36	114	0.00690	6.73	94.705	0.048	-2.22	60	1.31	0.0569
2	36	114	0.00690	6.73	94.705	0.048	-2.22	60	1.31	0.0569
3	60	120	0.02028	7.07	309.54	0.0762	-2.36	100	1.31	0.0569
4	80	190	0.00942	8.18	369.03	0.054	-3.14	120	0.9142	0.0454
5	47	97	0.01140	5.35	148.89	0.085	-1.89	50	0.9936	0.0406
6	68	140	0.01142	8.05	222.33	0.0854	-3.08	80	1.31	0.0569
7	110	300	0.00357	8.03	287.71	0.0242	-3.06	100	0.655	0.02846
8	135	300	0.00492	6.99	391.98	0.0335	-2.32	130	0.655	0.02846
9	135	300	0.00573	6.6	455.76	0.425	-2.11	150	0.655	0.02846
10	130	300	0.00605	12.9	722.82	0.0322	-4.34	280	0.655	0.02846
11	94	375	0.00515	12.9	635.20	0.0338	-4.34	220	0.655	0.02846
12	94	375	0.00569	12.8	654.69	0.0296	-4.28	225	0.655	0.02846
13	125	500	0000421	12.5	913.40	0.0512	-4.18	300	0.5035	0.02075
14	125	500	0.00752	8.84	1760.4	0.0496	-3.34	520	0.5035	0.02075
15	125	500	0.00708	9.15	1728.3	0.0496	-3.55	510	0.5035	0.02075
16	125	500	0.00708	9.15	1728.3	0.0151	-3.55	510	0.5035	0.02075
17	220	500	0.00313	7.97	647.85	0.0151	-2.68	220	0.5035	0.02075
18	220	500	0.00313	7.95	649.69	0.0151	-2.66	222	0.5035	0.02075
19	242	550	0.00313	7.97	647.83	0.0151	-2.68	220	0.5035	0.02075
20	242	550	0.00313	7.97	647.81	0.0145	-2.68	220	0.5035	0.02075
21	254	550	0.00298	6.63	785.96	0.0145	-2.22	290	0.5035	0.02075
22	254	550	0.00298	6.63	785.96	0.0138	-2.22	285	0.5035	0.02075
23	254	550	0.00284	6.66	794.53	0.0138	-2.26	295	0.5035	0.02075
24	254	550	0.00284	6.66	794.53	0.0132	-2.26	295	0.5035	0.02075
25	254	550	0.00277	7.10	801.32	0.0132	-2.42	310	0.5035	0.02075
26	254	550	0.00277	7.10	801.32	1.842	-2.42	310	0.5035	0.02075
27	10	150	0.52124	3.33	1055.1	1.842	-1.11	360	0.9936	0.0406
28	10	150	0.52124	3.33	1055.1	1.842	-1.11	360	0.9936	0.0406
29	10	150	0.52124	3.33	1055.1	1.842	-1.11	360	0.9936	0.0406
30	47	97	0.01140	5.35	148.89	0.085	-1.89	50	0.9936	0.0406
31	60	190	0.00160	6.43	222.92	0.0121	-2.08	80	0.9142	0.0454
32	60	190	0.00160	6.43	222.92	0.0121	-2.08	80	0.9142	0.0454
33	60	190	0.00160	6.43	222.92	0.0121	-2.08	80	0.9142	0.0454
34	90	200	0.00010	8.95	107.87	0.0012	-3.48	65	0.655	0.02846
35	90	200	0.00010	8.62	116.58	0.0012	-3.24	70	0.655	0.02846
36	90	200	0.00010	8.62	116.58	0.0012	-3.24	70	0.655	0.02846
37	25	110	0.01610	5.88	307.45	0.095	-1.98	100	1.42	0.0677
38	25	110	0.01610	5.88	307.45	0.095	-1.98	100	1.42	0.0677

| 39 | 25 | 110 | 0.01610 | 5.88 | 307.45 | 0.095 | -1.98 | 100 | 1.42 | 0.0677 |
| 40 | 242 | 550 | 0.00313 | 7.97 | 647.83 | 0.0151 | -2.68 | 220 | 0.5035 | 0.02075 |

Test system 2: 6-operational thermal generating unit

Table 12: Input data for operational generating unit like loading limits and loss parameters of 6-unit system.

Unit	P_i^{min} (MW)	P_i^{max} (MW)	a_i ($/hr)	b ($/MWhr)	c_i ($/MW^2hr)	α_i (lb/hr)	β_i (lb/MWhr)	γ_i (lb/MW^2hr)
1	10	125	756.7988	38.539	0.15247	13.8593	0.32767	0.00419
2	10	150	451.3251	46.1591	0.10587	13.8593	0.32767	0.00419
3	35	210	1243.531	38.3055	0.03546	40.2669	−0.54551	0.00683
4	35	225	1049.998	40.3965	0.02803	40.2669	−0.54551	0.00683
5	125	315	1356.659	38.2704	0.01799	42.8955	−0.51116	0.00461
6	130	325	1658.57	36.3278	0.02111	42.8955	−0.51116	0.00461

Loss parameters:

$$A = \begin{bmatrix} 0.000140 & 0.000017 & 0.000015 & 0.000019 & 0.000026 & 0.000022 \\ 0.000017 & 0.000060 & 0.000013 & 0.000016 & 0.000015 & 0.000020 \\ 0.000015 & 0.000013 & 0.000065 & 0.000017 & 0.000024 & 0.000019 \\ 0.000019 & 0.000016 & 0.000017 & 0.000071 & 0.000030 & 0.000025 \\ 0.000026 & 0.000015 & 0.000024 & 0.000030 & 0.000069 & 0.000032 \\ 0.000022 & 0.000020 & 0.000019 & 0.000025 & 0.000032 & 0.000085 \end{bmatrix}$$

A1 = [0 0 0 0 0 0]

A3 = 0