Global Journal of Researches in Engineering: E
Civil And Structural Engineering
Volume 19 Issue 1 Version 1.0 Year 2019
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4596 & Print ISSN: 0975-5861

Fatigue Strain based Approach for Damage Evolution Model of Concrete

By Indra Narayan Yadav & Dr. Kamal Bahadur Thapa
Tribhuvan University

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GJRE-E Classification: FOR Code: 090599

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Fatigue Strain based Approach for Damage Evolution Model of Concrete

Indra Narayan Yadav [α] & Dr. Kamal Bahadur Thapa [σ]

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Keywords: fatigue strain based approach; damage; evolution model; concrete fatigue modulus; thermodynamics; fatigue modulus; inelastic, strength reduction.

I. Introduction

In recent times, concrete has become the bedrock of infrastructural civilization in the world. Statistics have shown that over 75% of the infrastructures in the world have to do with concrete. Therefore, it is necessary to study regarding the behavior of concrete in every aspect from the production, transportation, placing and eventually maintenance of concrete.

Concrete today has a very wide range of applications. Virtually every civil engineering work in Nepal today is directly or indirectly involving the use of concrete. The use of concrete in civil engineering works includes: construction of residential houses, industrial warehouses, roads pavement construction, Shore Protection works, piles, domes, bridges, culverts, drainages, canals, dams etc. (Shetty, 2005; Neville, 2011; Edward and David, 2009; Duggal, 2009; Gambhir, 2005). In recent practice, the cases of failure of structures and roads (concretely related failure) occur on a yearly basis.

Variation of material internal as well as external deformation of concrete materials due to fatigue loading to the failure is reflected by fatigue strain. For, qualitative understanding of the failure fatigue strain, the detailed study of the evolution curve is essential. Longitudinal and residual deformations in three stage namely rapid, stable and ultimate growth stage which is generally used in all types of concrete as well as all types of fatigue failure i.e. compression, tension, bending, uniaxial, biaxial or multiaxial fatigue (Chen. Et. Al), which is in the form of cubical polynomial fitting curve, resulting in the correlation coefficients is more than 0.937. According to Cachim et. Al., in a constant order of magnitude, the stress in the different level of concrete have different coefficients used in logarithmic form regarding the curve obtained from the maximum strain versus the number of cycles graph at the second phases of concrete. The linear nature of curve obtained from graphs regarding maximum strain versus the number of cycles to the failure according to Xie. Et. Al. who had also given the well-developed experienced formula for fatigue strain in second phases of the concrete matrix. Data regarding fatigue strain in a similar stage was nonlinear in nature given by Wang et. Al.

At the low accuracy, three staged fatigue evolution equations are described in a simpler way in different literature. Strictly speaking, it became complicated to develop nonlinear equations of high precision based on the relation between fatigue strain and the number of cycles at different amplitude. At low fatigue stress with the comparison to the ultimate stress of concrete material but greater than ultimate value, very few research has been done yet. Without considering the initial strain, for three-stage fatigue strain and curve regarding strain to the number of cycles to the failure is obtained which caused alter fittings of curves coefficient fittings parameters. Therefore endurance limit for concrete is not guaranteed according to Miner's hypothesis [1].

Author α: Ph.D. Research Scholar, Department of Civil Engineering, Pulchowk Campus, Institute of Engineering, Tribhuvan University, Nepal. e-mail: eecconsultant@ gmail.com
Author σ: Associate Professor, Department of Civil Engineering, Pulchowk Campus, Institute of Engineering, Tribhuvan University, Nepal. e-mail: kamal.thapa@ ioe.edu.np

For the production of concrete, except cement, all materials are locally available i.e. sand, aggregate, and water. So, it is very much popular in the list of construction material is construction engineering. Concrete is a heterogeneous matrix related to the composition i.e. cement, sand, aggregate and water among them cement is the weakest part compared to the remaining ingredients. At the initial stage of production, water and air are inside the matrix of the composition of structure slowly released from that matrix during an initial setting time to final setting time creating microvoids at the original place of air and water made alteration of the chemistry of the matrix. When the cyclic load which is lower than ultimate load but higher than threshold limit is applied to the concrete then due to alteration i.e. separation of the matrix in composition, alters the ingredient from each other by creating microvoids continuously increasing up to microvoids and finally break up which is called fracture. Force applied until fracture appears is usually lesser than ultimate monotonic loads phenomenon which deals about the chemistry of fracture is called fatigue mainly caused by progressive cyclic loading tends to change the [2] permanent internal structure resulting microcracks until macrocracks creating the permanent damage in the concrete matrix.

Based on the concept of dual nature of fatigue damage, the model for ordinary concrete has been documented through the number of investigations presented in the different researches. It is very much essential to predict the progressive creep damage model based on cyclic dependent and time-dependent damage at constant and variable amplitude. [3] Damage in the concrete pavement was carried out through the accelerated pavement testing results. As per Minor hypothesis, one cannot predict the cumulative fatigue damage in concrete accurately. The theoretical model for the prediction of cumulative fatigue model in compression, compression-tension, tension-tension, flexural, torsional, uniaxial, bi-axial, tri-axial under monotonic and cyclic loading using different approaches such as bounding surface approach with using the energy released rate by constructing damage effective tensor poorly described in different past research papers and articles also. The need for validation of such models in inelastic flow and microcracking related to plasticity theories and voids caused degradation of elastic moduli through energy dissipations. The experimental work of [4] described that the increase of damage in the concrete material takes place is about last 20% of its probable fatigue life. [5] Presented a theoretical model to describe the fatigue process of concrete material in alternate tension-compression fatigue loading utilizing double bounding surface approach with strain-energy release rate by evaluating damage-effective tensor. A number of damage constitutive models regarding failure fatigue life of concrete have been published for capturing the model regarding mechanical behavior of concrete under monotonic and cyclic loading ([6], [7], [8], [9], and [10]), which have done in the past.

This paper presents the physical meanings, the ranges, and the impact on the shape of the curve of parameters in the nonlinear strain evolution model are all discussed. The evolution model of fatigue modulus was established under constant amplitude bending fatigue loading based on the fatigue strain evolution model and the hypothesis of fatigue modulus inversely related fatigue strain amplitude. A class of damage mechanics theory to model the fatigue damage and failure of concrete caused by the multitude of cracks and microcracks whereby anisotropic damaging behavior is captured through the use of proper response function involving damage parameter in material stiffness tensor is also developed. The increment of damage parameter is obtained from consistency equation in cycle dependent damage surface in strain space. The model is also capable of capturing the inelastic deformations that may arise due to misfits of crack surfaces and development of sizable crack tip process zone. Moreover, the whole process is validated by the experimental data

II. Formulation

According to the continuum damage mechanics approach to describe the constitutive relation for the concrete matrix relate to fatigue loading at low frequency by neglecting thermal effects. Considering, the isothermal process, small deformations and rate independent behavior, the Helmholtz Free Energy (HFE) per unit volume can be written from [1] is given below :

$$\mathrm{A}(\boldsymbol{\varepsilon},\mathrm{k}) = \tfrac{1}{2}\boldsymbol{\varepsilon} : \mathbf{E}(k) : \boldsymbol{\varepsilon} - \dot{\boldsymbol{\sigma}}^{\mathbf{i}} : \boldsymbol{\varepsilon} + \mathrm{A}^{\mathrm{i}}(k) \tag{1}$$

Where, E (k) = fourth-order elastic stiffness tensor, $\boldsymbol{\varepsilon}$ = strain tensor, $\boldsymbol{\sigma}^{\mathbf{i}}$ = stress tensor. $A^{i}(k)$= surface energy of microcracks [2], and k = cumulative fatigue damage parameter. The colon (:) indicates the tensor contraction operation.

For inelastic fatigue damage, a constitutive relation between the fatigue stress and fatigue strain tensors shall be established by fourth order material's stiffness tensor such as

$$\boldsymbol{\sigma} = \frac{\partial A}{\partial \boldsymbol{\varepsilon}} = \mathbf{E}(k) : \boldsymbol{\varepsilon} - \boldsymbol{\sigma}^{i}(k) \tag{2}$$

The rate of change of Eqn (2) with respect to cyclic number N is given by

$$\dot{\boldsymbol{\sigma}} = \mathbf{E}(k) : \dot{\boldsymbol{\varepsilon}} + \dot{\mathbf{E}}(k) : \boldsymbol{\varepsilon} - \dot{\boldsymbol{\sigma}}^{i}(k) \\ = \dot{\boldsymbol{\sigma}}^{\mathbf{e}} + \dot{\boldsymbol{\sigma}}^{\mathbf{D}}(k) + \dot{\boldsymbol{\sigma}}^{\mathbf{i}}(k) \quad (3)$$

Where $\dot{\boldsymbol{\sigma}}^{\mathbf{e}}$, = stress increment, $\dot{\boldsymbol{\sigma}}^{\mathbf{D}}$ = rate of stress-relaxation, and $\dot{\boldsymbol{\sigma}}^{\mathbf{i}}(k)$ = rate of stress tensor

For small deformation, the following matrix of the fourth-order stiffness tensor, E, when adopted

$$\frac{\partial^2 A}{\partial \boldsymbol{\varepsilon} \partial \boldsymbol{\varepsilon}} = \mathbf{E}(k) = \mathbf{E}^{\mathbf{0}} + \mathbf{E}^{\mathbf{D}}(k) \quad (4)$$

Where $\mathbf{E}^{\mathbf{0}}$ = Initial stiffness before fatigue loading and $\mathbf{E}^{\mathbf{D}}(k)$ = overall stiffness degradation during fatigue loadings. Further, $\dot{\mathbf{E}}(k)$ and $\dot{\boldsymbol{\sigma}}^{\mathbf{i}}(k)$ = fluxes in the thermodynamic state sense and are expressed in terms of fatigue evolutionary equations as

$$\dot{\mathbf{E}}^{\mathbf{D}} = -\dot{k}\mathbf{L} \quad \text{and} \quad \dot{\boldsymbol{\sigma}}^{i} = \dot{k}\mathbf{M} \quad (5)$$

Where L and M are, fourth and second-order response tensors which determine the directions of the elastic and inelastic fatigue damage processes. Following the Clausius- Duhem inequality equations, applying the standard thermodynamic discussions [13] and a potential function by assuming unloading is in an elastic process

$$\Psi(\boldsymbol{\varepsilon}, k) = \tfrac{1}{2}\boldsymbol{\varepsilon} : \mathbf{L} : \boldsymbol{\varepsilon} - \mathbf{M} : \boldsymbol{\varepsilon} - \tfrac{1}{2} p^2(\boldsymbol{\varepsilon}, k) = 0 \quad (6)$$

In Eqn (6), $p(\boldsymbol{\varepsilon}, k)$ = damage function which is given as

$$p^2(\boldsymbol{\varepsilon}, k) = 2\left[h^2(\boldsymbol{\varepsilon}, k) + \frac{\partial A^i}{\partial k}\right] \quad (7)$$

Which is for some scalar-valued function $h^2(\boldsymbol{\varepsilon}, k)$. It should be noted that as long as the function $p^2(\boldsymbol{\varepsilon}, k)$ is well defined, the right-hand side of Eqn (7) need not be identified.

For specific forms of response tensors, L and M shall be specified. Since fatigue damage is highly directional, so, directionality response tensors should be developed. For the development of response tensor, the strain tensor is divided into positive and negative cones. The positive and negative cones of the fatigue strain tensor completely hold the corresponding positive and negative eigenvalue of the system, i.e., $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon} \quad + \boldsymbol{\varepsilon}^{-}$ as positive and negative cones of the strain tensor, respectively. Based on the fact of experimental observations for concrete materials, the damage is assumed to arrive in the cleavage mode of cracking as per Figure1.

Damage function p(k) is obtained from an experimental test of uniaxial tensile loading, then the equation can be written as

$$\mathrm{p}(k) = \varepsilon_{\mathrm{u}} ln\left(\frac{\mathrm{E}^0}{\mathrm{E}^0 - \mathrm{k}}\right) \quad (11)$$

When, $\beta = 0$ in the inelastic damage surface, the limit damage surface reduces to

$$\mathrm{p}(k) = \varepsilon_{\mathrm{u}} \quad (12)$$

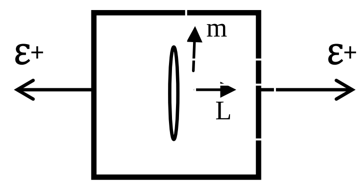

Figure 1: Crack Opening and Tensile Mode I damage

For the mode of cleavage cracking, the terms of response tensors are postulated for L and M

$$\mathbf{L} = \frac{\boldsymbol{\varepsilon}^{+} \otimes \boldsymbol{\varepsilon}^{+}}{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}} \quad (8)$$

$$\mathbf{M} = \beta \boldsymbol{\varepsilon}^{+} \quad (9)$$

Substituting the response tensors L and M from Eqns (8) and (9) into Eqn (6) gives the final form of the fatigue cracked damaged surface

$$\Psi(\ , k) = \tfrac{1}{2} \quad : \frac{\boldsymbol{\varepsilon}^{+} \otimes \boldsymbol{\varepsilon}^{+}}{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}} : \boldsymbol{\varepsilon} - \quad \boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon} \\ - \tfrac{1}{2} p^2(\boldsymbol{\varepsilon}, k) = 0 \quad ....(10a)$$

The equation of damage surface for uniaxial tensile loading Eqns (10a) is rewritten as

$$\Psi(\boldsymbol{\varepsilon}, k) = \tfrac{1}{2}\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+} - \beta \boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+} - \tfrac{1}{2} p^2 \ (\boldsymbol{\varepsilon} k,) \ = 0$$

$$= \tfrac{1}{2}\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}(1 - 2\beta) - \tfrac{1}{2} p^2(\boldsymbol{\varepsilon}, k) = 0 \quad (10b)$$

Where ε_{u} = strain corresponding to the uniaxial tensile strength of concrete,

For describing the three-stage fatigue damage law, we have

$$\varepsilon^n = \varepsilon^0 + \propto \left(\frac{\beta}{\beta - \frac{n}{Nf}} - 1\right)^{1/p} \quad (13)$$

Where, $\boldsymbol{\varepsilon}^0$ = initial strain and $\boldsymbol{\varepsilon}^n$ = fatigue strain, $\boldsymbol{n}$ = cycle times of fatigue loads. $\boldsymbol{N_f}$ = fatigue in life. $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{p}$ were the parameter regarding fatigue.

III. Fatigue Damage Model

In fact, progressive permanent structural changes in the form of cracks due to fatigue loading flows material fails at lower stress than the ultimate tensile strength of the material which has a higher value than the threshold limit. Damage surface of the material within the given prescribed strain, fatigue loading (reloading and unloading process) increases the growth of microcracks which leads inelastic deformation tends to reduce the ultimate overall strength of the concrete material. Therefore, for modified damage surface, fatigue damage with respect to the number of cycles i.e. $\Psi(\boldsymbol{\varepsilon}, k)$ is obtained from

$$\tfrac{1}{2}\boldsymbol{\varepsilon}^{+}:\boldsymbol{\varepsilon}^{+}(1-2\beta)X(N)-\tfrac{1}{2}p^{2}(\boldsymbol{\varepsilon},k)=0 \qquad (14)$$

Where, X (N) = function that depends on the number of loading cycles. Propose a power function for X (N) as

$$X(N)=N^{A} \qquad (15)$$

Here, N = number of loading cycles, and A = material parameter. From, Eqns (11) and (14), we can obtain the cumulative fatigue parameter k as under

$$k=E^{0}\left[1-\frac{1}{exp\left(\frac{\sqrt{(1-2\beta)N^{A}\boldsymbol{\varepsilon}^{+}:\boldsymbol{\varepsilon}^{+}}}{\varepsilon_{u}}\right)}\right] \qquad (16)$$

Differentiating Eqns (15) with respect to N, an increment of damage in one cycle can be obtained as

$$\dot{k}=\frac{dk}{dN}$$

$$=\frac{AN^{\frac{A}{2}-1}E^{0}\sqrt{\boldsymbol{\varepsilon}^{+}:\boldsymbol{\varepsilon}^{+}(1-2\beta)}}{2\varepsilon_{u}exp(-\sqrt{\boldsymbol{\varepsilon}^{+}:\boldsymbol{\varepsilon}^{+}(1-2\beta)N^{A}/\varepsilon_{u}^{2}})} \qquad (17)$$

Finally, the rate of damage parameter $\dot{k}$ can be used in the simple constitutive relation in Eqn (14) for uniaxial tensile stress state to get inelastic deformation, stiffness reduction and strength reduction due to fatigue cycles to the failure. Substituting all related parameters, we can get,

$$\dot{\boldsymbol{\sigma}}=\mathbf{E}(k):\dot{\boldsymbol{\varepsilon}}-\dot{k}\left(\frac{\boldsymbol{\varepsilon}^{+}\otimes\boldsymbol{\varepsilon}^{+}}{\boldsymbol{\varepsilon}^{+}:\boldsymbol{\varepsilon}^{+}}:\boldsymbol{\varepsilon}+\beta\boldsymbol{\varepsilon}^{+}\right) \qquad (18)$$

When $\beta = 0$Eqn (17) can be treated for uniaxial tension-tension fatigue loading then the process is classified as elastic-damaging, in which stress-strain curve returns to original conditions upon unloading of the material. In fact, damage incurred in concrete shall not be considered perfectly elastic. The tired unloaded material shows some residual strains due to the development of sizable crack tip process zone at the surface and misfits of the crack surfaces.

At the condition of uniaxial tension, Eqn (18) can be written as

$$\dot{\boldsymbol{\sigma}}=\mathbf{E}:\dot{\boldsymbol{\varepsilon}}-\left[\frac{AN^{\frac{A}{2}-1}E^{0}\sqrt{\boldsymbol{\varepsilon}^{+}:\boldsymbol{\varepsilon}^{+}\eta}\left((1+\beta)\right)}{2\varepsilon_{u}exp\left(-\sqrt{\frac{\boldsymbol{\varepsilon}^{+}:\boldsymbol{\varepsilon}^{+}\eta N^{A}}{\varepsilon_{u}^{2}}}\right)}\right]\boldsymbol{\varepsilon}^{+} \qquad (19)$$

Where, $\eta = 1-2\beta$

IV. Fatigue Strain Evolution Model

Depending upon the different stress types, three-stage variation law of fatigue evolution model was proposed. Moreover, some valuable physical parameters like initial strain, instability speed of the third stage as a form of acceleration directly proportional to the total fatigue life of concrete. Mathematically, the model could be obtained as below.

$$\boldsymbol{\varepsilon}^{\boldsymbol{n}}=\boldsymbol{\varepsilon}^{\mathbf{0}}+\propto\left(\frac{\boldsymbol{\beta}}{\boldsymbol{\beta}-\frac{\boldsymbol{n}}{\boldsymbol{N_f}}}-\mathbf{1}\right)^{\mathbf{1}/\boldsymbol{p}} \qquad (20)$$

In formula (20), $\boldsymbol{\varepsilon}^{0}$ = initial strain and $\boldsymbol{\varepsilon}^{n}$ = atigue strain, $\boldsymbol{n}$ = cycle times of fatigue loads, $\boldsymbol{N_f}$ = fatigue life. $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{p}$ were damage parameters.

If $\boldsymbol{\varepsilon}^{n}{}_{max}$ or $\boldsymbol{\varepsilon}^{n}{}_{res}$ was interpreted in the form of $\boldsymbol{\varepsilon}^{n}$, formula (20) can be modified. if the initial maximum strain $\boldsymbol{\varepsilon}^{0}{}_{max}$ or initial residual strain $\boldsymbol{\varepsilon}^{0}{}_{res}$ is regarded as the value of $\boldsymbol{\varepsilon}^{0}$, formula (21 and 22) should be obtained.

$$\boldsymbol{\varepsilon}_{\boldsymbol{max}}^{\boldsymbol{n}}=\boldsymbol{\varepsilon}_{\boldsymbol{max}}^{\mathbf{0}}+\propto\left(\frac{\boldsymbol{\beta}}{\boldsymbol{\beta}-\frac{\boldsymbol{n}}{\boldsymbol{N_f}}}-\mathbf{1}\right)^{\mathbf{1}/\boldsymbol{p}} \qquad (21)$$

$$\boldsymbol{\varepsilon}_{\boldsymbol{res}}^{\boldsymbol{n}}=\boldsymbol{\varepsilon}_{\boldsymbol{res}}^{\mathbf{0}}+\propto\left(\frac{\boldsymbol{\beta}}{\boldsymbol{\beta}-\frac{\boldsymbol{n}}{\boldsymbol{N_f}}}-\mathbf{1}\right)^{\mathbf{1}/\boldsymbol{p}} \qquad (22)$$

Equation (21) is a formula for maximum strain and equation (22) is the formula for the residual strain.

On the basis of the elastic proportional limit, if the upper limit of fatigue stress is large then fatigue strain increases fastly. The slope of the curve regarding this increment will be large and became vertical that causes the degeneration of the three-stage curve. When the upper limit of fatigue does not exceed the threshold value, the elastic strain should be added to the initial strain and value became unchanged, shows similarity in curve formulation. By the experiment, it can be shown that the value of most stresses falls in between the value of threshold and upper limit.

Being the maximum and minimum value of stress and strain in fatigue test, two types of the curve regarding maximum strain i.e. ε^0_{max} and residual strain i.e. ε^0_{res} with respect to the cyclic number are obtained. The main causes for obtaining these two types of the curve are due to defects in materials and preloading conditions also. It is very much difficult to differentiate these two maximum and residual value, so experiment regarding fatigue test is essential.

Therefore, at that condition of fatigue loading reaches to the upper limit then, the corresponding maximum strain ε^1_{max} and residual strain ε^1_{res} are obtained and adopted in this paper. For comparison, strain obtained the formula of ε^1_{max} and ε^1_{res} compared to the actual experimental data i.e. $\varepsilon^1_{res} = 0.25 (\varepsilon^1_{max}/\varepsilon_{unstable})^2$. In this formula, $\varepsilon_{unstable}$ is a total strain of concrete in an unstable state.

For the study of fatigue strain parameters α, β and p, on the basis of evolution law of fatigue strain curves, divided by fatigue strain in both side of formulas (21) and (22), we get

$$\frac{\varepsilon^n_{max}}{\varepsilon^f_{max}} = \frac{\varepsilon^0_{max}}{\varepsilon^f_{max}} + \frac{\propto}{\varepsilon^f_{max}} \cdot \left(\frac{\beta}{\beta - \frac{n}{N_f}} - 1\right)^{1/p} \tag{23}$$

$$\frac{\varepsilon^n_{res}}{\varepsilon^f_{res}} = \frac{\varepsilon^0_{res}}{\varepsilon^f_{res}} + \frac{\propto}{\varepsilon^f_{res}} \cdot \left(\frac{\beta}{\beta - \frac{n}{N_f}} - 1\right)^{1/p} \tag{24}$$

Formula (23) and (24) are the normalized fatigue strain evolution model. Where, εf_{max} = limited maximum fatigue strain and εf_{res} = limited fatigue residual strain.

β = destabilizing factor the value of which depends on p and α. If n/N_f (Circulation ratio) is equal to 1, the coordinate point (1, 1) will be adopted in formulas (23) and (24), thus obtained the values of β as formula (25) and 26, which is the maximum fatigue strain and the residual fatigue strain.

$$\beta_1 = \left(\frac{\left(1 - \frac{\varepsilon^0_{max}}{\varepsilon^f_{max}}\right)}{\left(\frac{\propto}{\varepsilon^f_{max}}\right)}\right)^{-p} + 1 \tag{25}$$

$$\beta_2 = \left(\frac{\left(1 - \frac{\varepsilon^0_{res}}{\varepsilon^f_{res}}\right)}{\left(\frac{\propto}{\varepsilon^f_{res}}\right)}\right)^{-p} + 1 \tag{26}$$

From equation (23) impacts of p and α on the fatigue, strain evolution curve can be calculated. Firstly, the impact of p was analyzed i.e. $\varepsilon^0_{max}/\varepsilon f_{max}$ and $\alpha/\varepsilon f_{max}$. After that, combined with p and $\varepsilon^0_{max}/\varepsilon f_{max}$, the impact of α was further calculated. The curve regarding the impact of p and α were shown in Figures.

It is obviously shown that according to the rate of convergence speed of p, influences the convergence speed of curve in S nonlinear model. The third stage of the curve will grow faster when the faster increment of P which is also called instability speed factor. Therefore the factor p should be located in the curve.

The parameter α values on the curve shall also affect the curve in the sense of total fatigue life of the material which shall be shown in the third stage of the nonlinear curve. After increasing of α, the part of acceleration shall become shorter. $\alpha/\varepsilon f$max is located corresponding to (0, $1 - \varepsilon^0_{max}/\varepsilon f_{max}$), whereas, α was placed in the comparison of (0, $\varepsilon f_{max} - \varepsilon^0_{max}$). The obtained value of the parameters α, β and p are mainly aimed which is found in b-type curves having three stages of evolutions. Therefore, it can be imagined that the values for both type curve are not limited by the literature. By modeling, S-shaped curves contents various parameters including different kinds of fatigue strain evolutions at the different stages for the concrete material.

V. Numerical Examples

The proposed model contains two material parameters, first is A which is a factor related to materials intermolecular microcracks and the second one is β which is called damage factor related to kinematic phenomena of the particle i.e. crack surface close perfectly after unloading. Damage parameter i.e. k, indicates the reduction in stiffness, is obtained by measuring stiffness at different three stages of the fatigue loading cycle. The kinematic parameter, β, is obtained by obtaining the permanent deformation during one of the fatigue cyclic loadings. Due to the scarcity of reliable experimental data from the different researches for obtaining the fatigue damage parameters in performing numerical simulation, analyst's judgments to obtain numerical results. Table (1) analysis for fatigue curve regarding stiffness, cumulative damage parameter versus the number of cycles, Table (2) analysis for fatigue curve regarding maximum stress versus the number of cycles, Table (3) Maximum Stress predicted by Peiyin Lu. Et. Al 2004 through experiment, Table (4) Fatigue Damage predicted by Peiyin Lu. Et. Al 2004 through experiment, Table (5) Fatigue Strain Evolution Model, Influence of Strain parameter "p", Table (6) Fatigue Strain Evolution Model, Influence of Strain parameter "α", Table (7) Iteration calculation table for finding out the best value of strain parameter "β", Table (8) Analysis for S-shaped family of fatigue strain curve, all Table from (1) to (8) are prescribed in this paper for sample calculation which gives the clear idea of fatigue strain behaviour.

The model formulation for obtaining modulus reduction with an increment at t the number of fatigue and (7) shows the decrease of maximum stress level (S-N curve) in cyclic tension-tension loading, Figure (8) model prediction for maximum stress level regarding fatigue damage parameter i.e. A=0.10 and β=0.00, Figure (9) model prediction for maximum stress level regarding fatigue damage parameter i.e. A=0.10 and β=0.15, Figure (10) and (11) on other hand, shows corresponding experimental result regarding decrease in stiffness is shown in Figure (2) and (3). Figure (6) of materials stress and increase of cumulative fatigue damage parameter with respect to increase of number of cyclic loading, Figure (12) Concrete Fatigue Strain Evolution, Influence of Fatigue Strain Parameter "ρ", Figure (13) Concrete Fatigue Strain Evolution, Influence of Fatigue Strain Parameter "∝", Figure (14) Concrete Fatigue Strain Evolution, "Family Strain Curve" . Finally, the model captures the relevant features of the cyclic response.

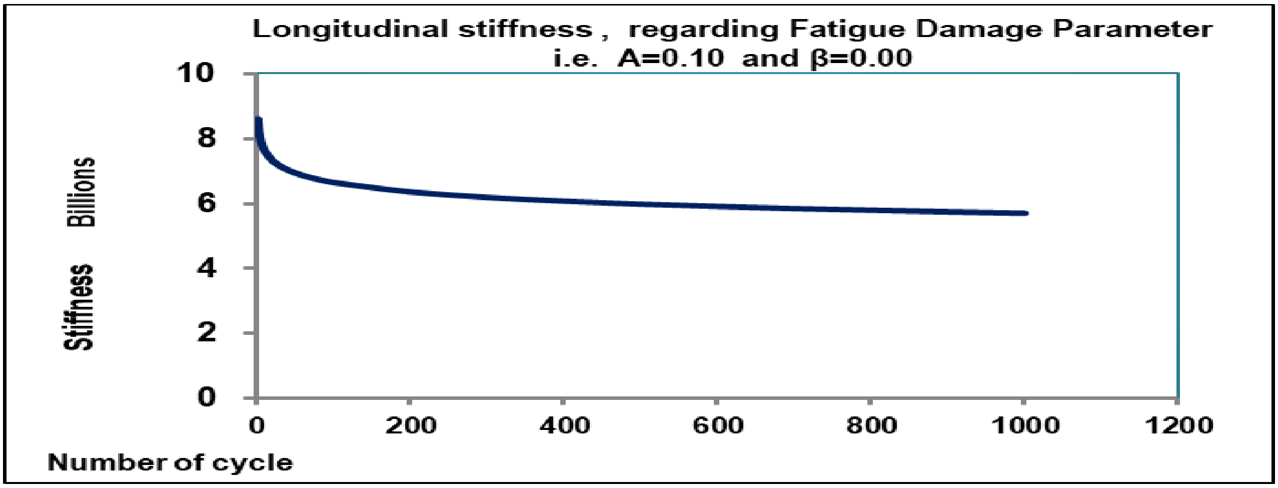

Figure 2: Formulation of Model against stiffness reduction with the number of cyclic loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.00

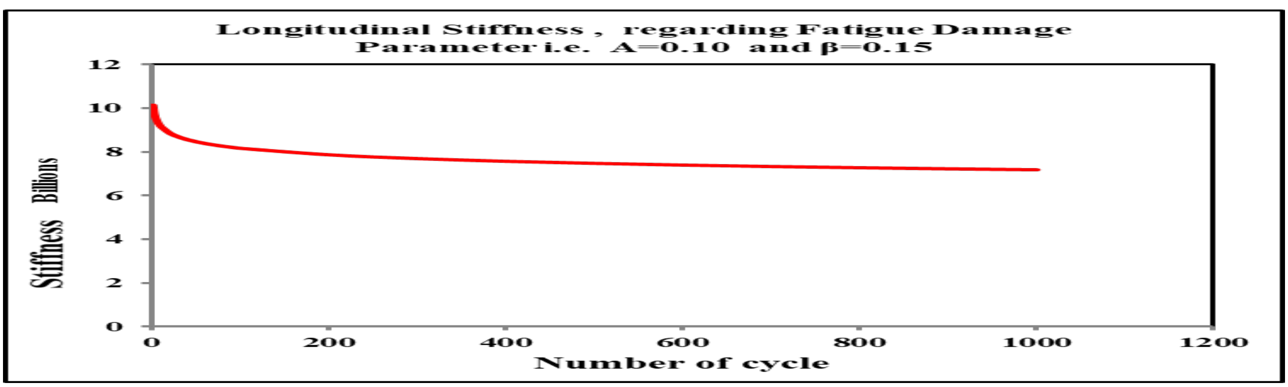

Figure 3. Formulation of Model against stiffness reduction with the number of cyclic loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.15

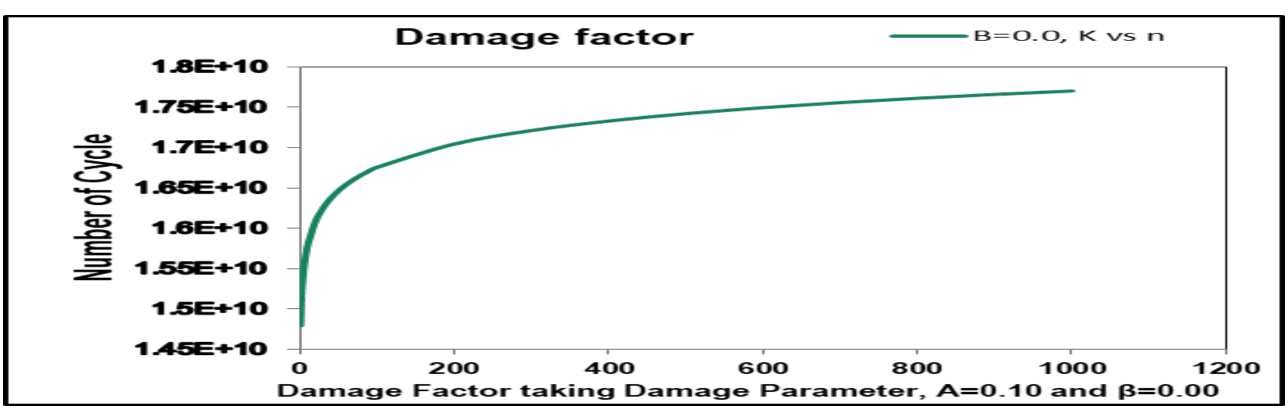

Figure 4: Model prediction of Damage Factor with the number of cyclic loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.00

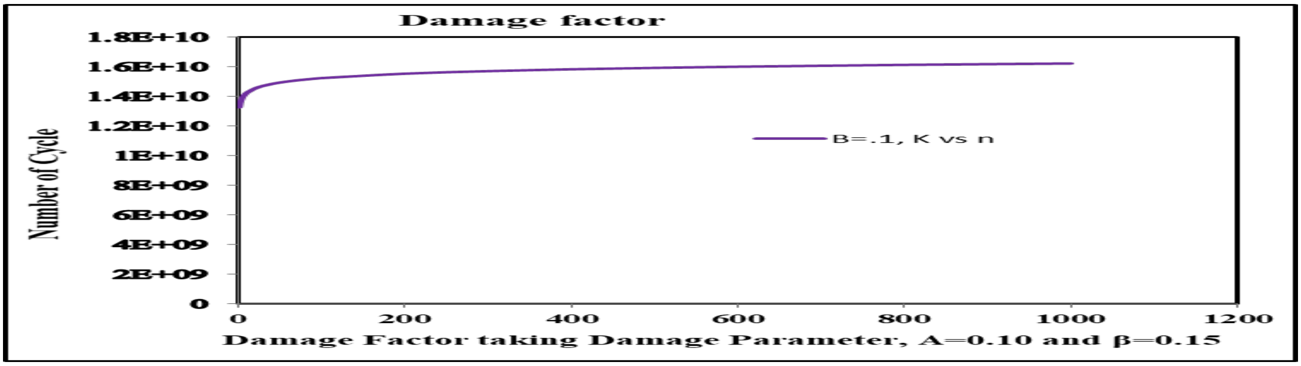

Figure 5: Model prediction of Damage Factor with the number of cyclic loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and $\beta = 0.15$

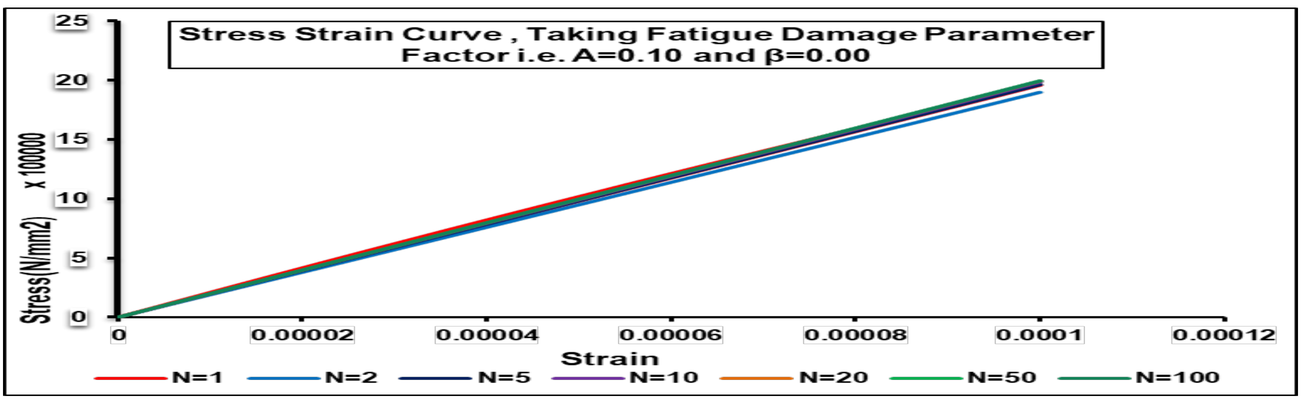

Figure 6: Cyclic stress-strain behavior of concrete during elastic damaging process theoretically under uniaxial fatigue loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and $\beta = 0.00$

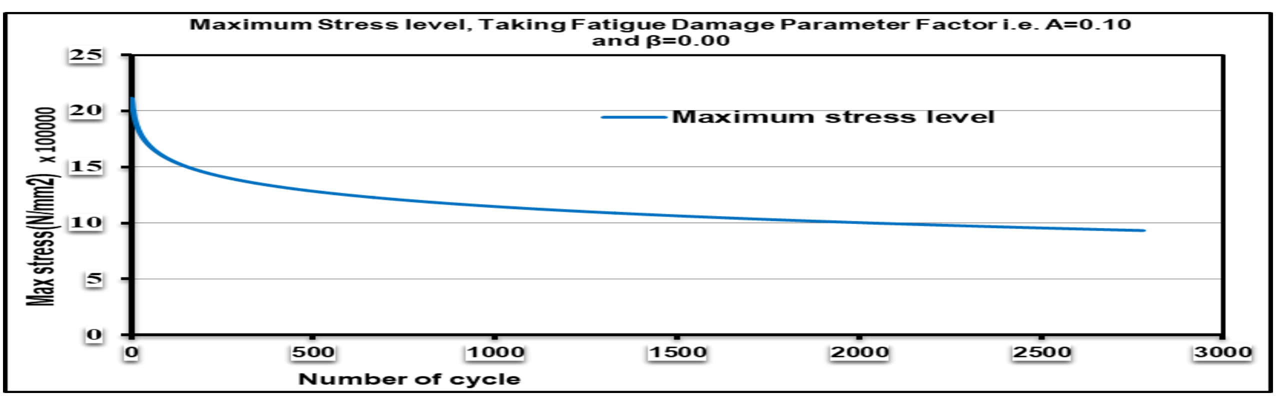

Figure 7: Theoretical cyclic stress-strain behavior of concrete during elastic damaging process under uniaxial fatigue loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and $\beta = 0.15$

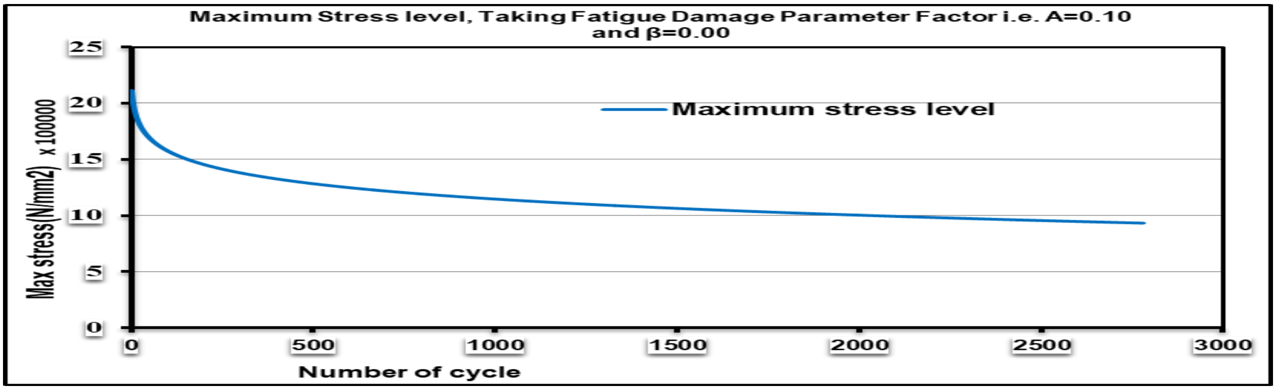

Figure 8: Model prediction of Maximum Stress Level versus Number of Cycle under uniaxial fatigue loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.00

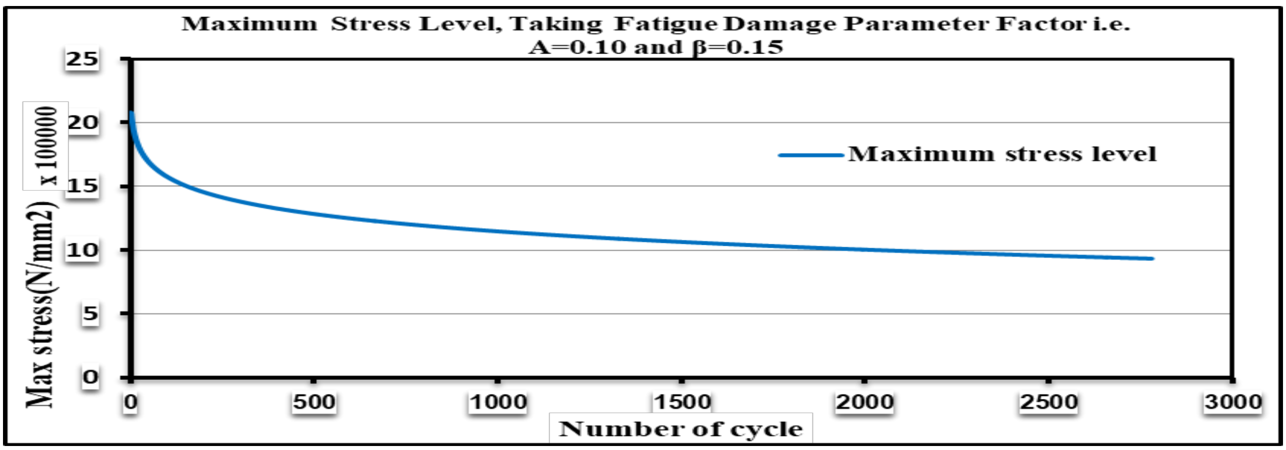

Figure 9: Model prediction of Maximum Stress Level versus Number of Cycle under uniaxial fatigue loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.15

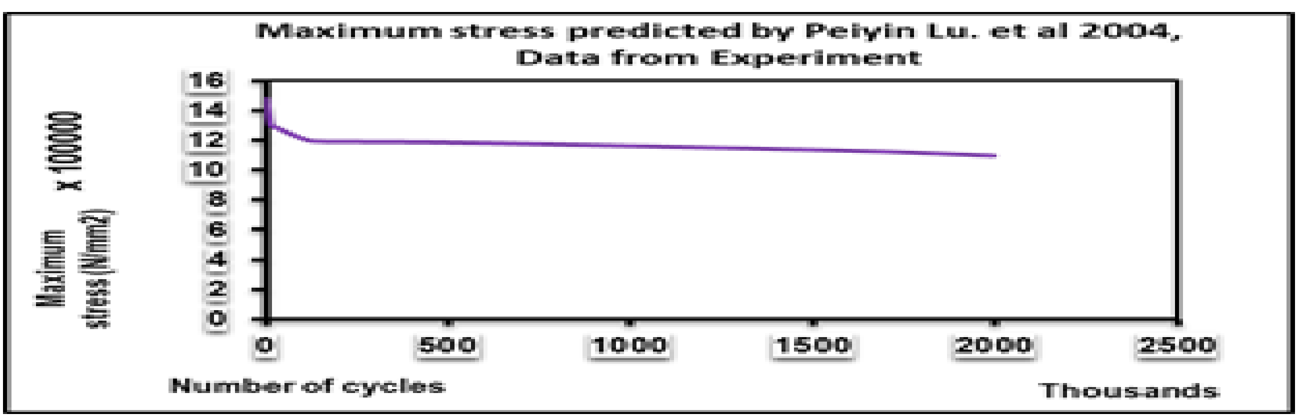

Figure 10: Model of maximum stress level during cyclic tension. Enhancement of the theory, *Figure 8 and 9 by Peiyin Lu. Et. Al 2004*

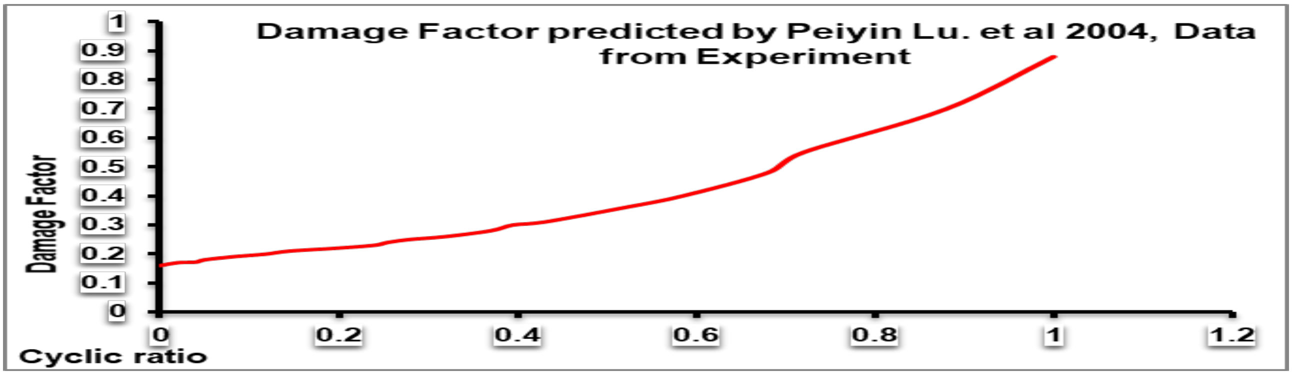

Figure 11: Damage variation with the number of cyclic loading. Prediction of the theory, *Figure (4) and Figure (5). Experimental Figure [11], by Peiyin Lu. Et. Al 2004*

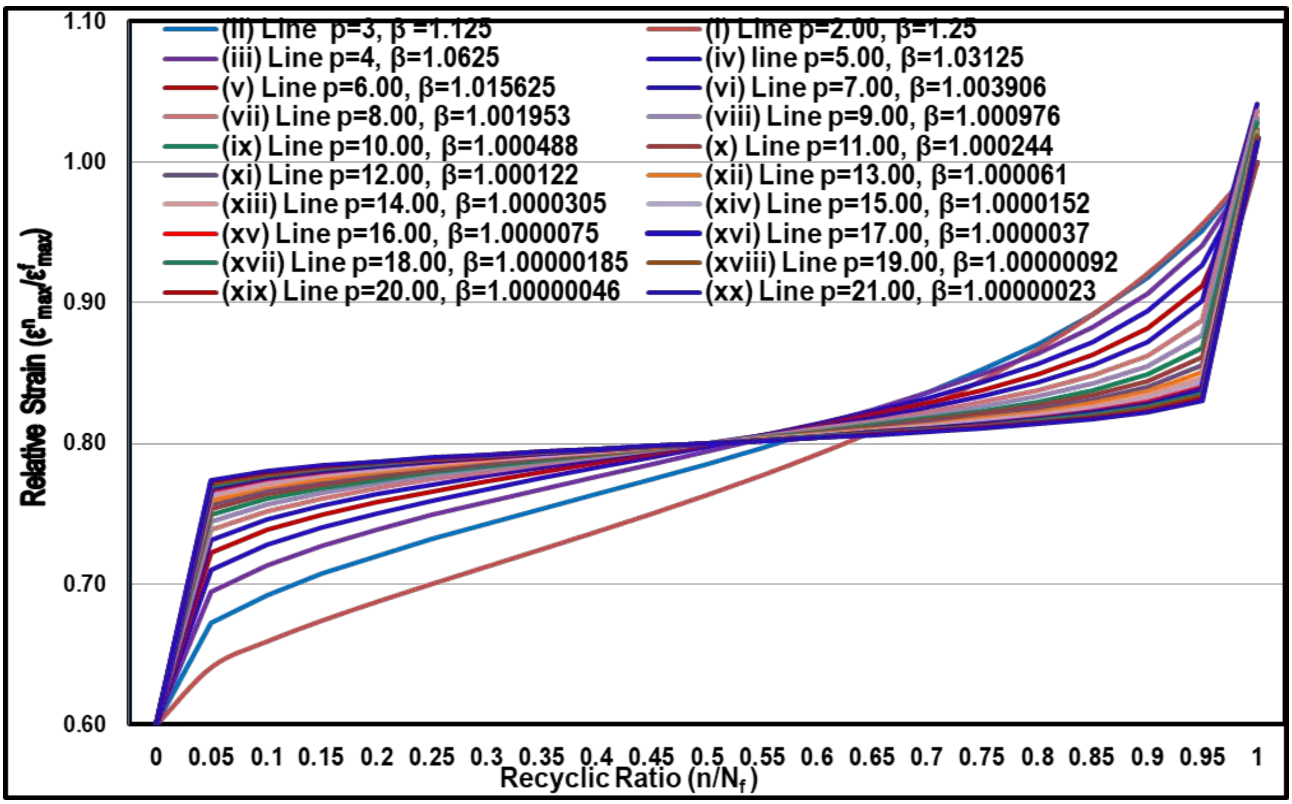

Figure 12: Concrete Fatigue Strain Evolution, Influence of Fatigue Strain Parameter "*p*" on Fatigue Strain Curve by Putting the value of (i) p=2.00, β=1.25 (ii) p=3.00, β=1.125 (iii) p=4.00, β=1.0625 (iv) p=5.00, β=1.0315 (v) p=6.00, β=1.015625 (vi) p=7.00, β=1.003906 (vii) p=8.00, β=1.001953 (viii) p=9.00, β=1.000976 (ix) p=10.00, β=1.000488 (x) p=11.00, β=1.000244 (xi) p=12.00, β=1.000122 (xii) p=13.00, β=1.000061 (xiii) p=14.00, β=1.0000305 (xiv) p=15.00, β=1.0000152 (xv) p=16.00, β=1.0000075 (xvi) p=17.00, β=1.0000037 (xvii) p=18.00, β=1.00000185 (xviii) p=19.00, β=1.00000092 (xix) p=20.00, β=1.00000046 (xx) p=21.00, β=1.00000023 and $\varepsilon^0_{max}/\varepsilon^f_{max}$ =0.60 and $\propto/\varepsilon^f_{max}$=0.20

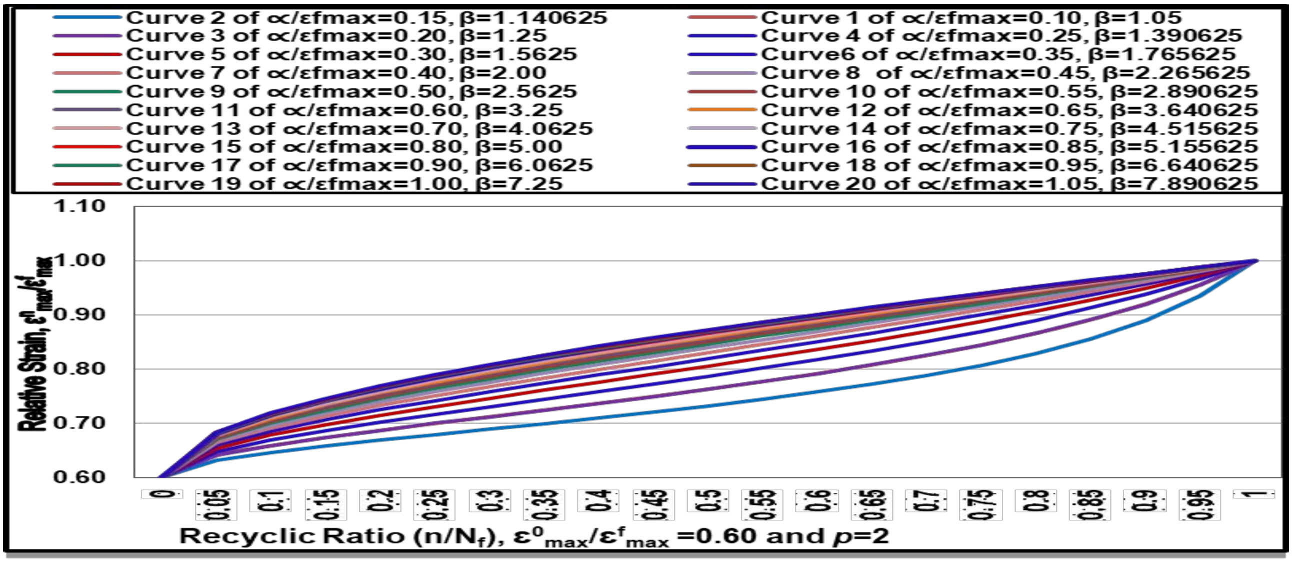

Figure 13: Concrete Fatigue Strain Evolution, Influence of Fatigue Strain Parameter "∝" on Fatigue Strain Curve, Putting the value of (i) **$\propto/\varepsilon^f_{max}$**=0.10 to 1.05, β=1.05 to 7.890625 and **$\varepsilon^0_{max}/\varepsilon^f_{max}$ =0.60, p=2 constant in all cases.**

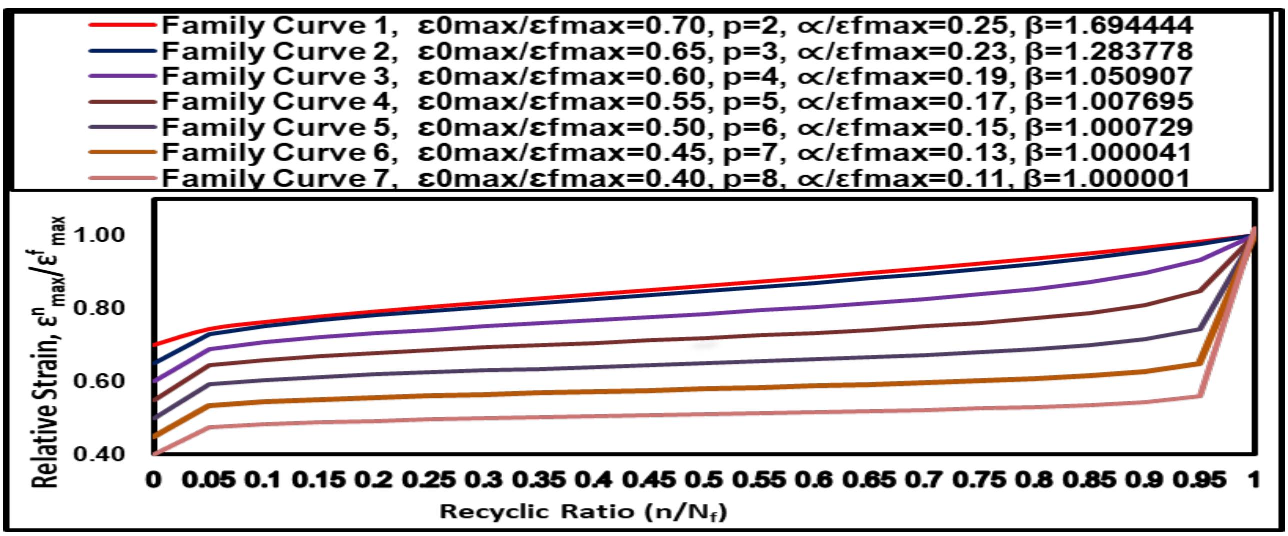

Figure 14: Concrete Fatigue Strain Evolution, Family Strain Curve (i) $\varepsilon^0_{max}/\varepsilon^f_{max}$ =0.70, p=2.00, $\propto/\varepsilon^f_{max}$=0.25, β=1.694444 (ii) $\varepsilon^0_{max}/\varepsilon^f_{max}$ =0.65, p=3.00, $\propto/\varepsilon^f_{max}$=0.23, β=1.283778 (iii) $\varepsilon^0_{max}/\varepsilon^f_{max}$ =0.60, p=4.00, $\propto/\varepsilon^f_{max}$=0.19, β=1.050907 (iv) $\varepsilon^0_{max}/\varepsilon^f_{max}$ =0.55, p=5.00, $\propto/\varepsilon^f_{max}$=0.17, β=1.007695 (v) $\varepsilon^0_{max}/\varepsilon^f_{max}$ =0.50, p=6.00, $\propto/\varepsilon^f_{max}$=0.15, β=1.000729 (vi) $\varepsilon^0_{max}/\varepsilon^f_{max}$ =0.45, p=7.00, $\propto/\varepsilon^f_{max}$=0.13, β=1.000041 (vii) $\varepsilon^0_{max}/\varepsilon^f_{max}$ =0.40, p=8.00, $\propto/\varepsilon^f_{max}$=0.11, β=1.000001

Figure (4) and (5) shows the increase in damage with increasing loading cycles. The experimental work of Figure [11] is also shown for comparison. Theoretical model which is also shown well captures the similar nature of increment of damage with respect to fatigue cyclic loading as observed in the experiment [11]. For numerical simulation, the following constant were used, A = 0.10 and β = 0.15 and 0.00 in two cases, Parameter A is estimated by comparing predicted results and experimental results over a range of applied strains.

Figures (6) and (7) depict the theoretical cyclical stress-strain behavior of concrete material in tension. In Figure (6), no permanent deformations are found on the condition of fatigue unloading of concrete material but progressive damage is accumulated in each fatigue loading cycle due to the reduction of elastic modulus. In fact, it is an ideal case for elastic-perfectly damaging behavior in damage mechanics which can be obtained by letting $\beta = 0$with assuming that crack surfaces i.e. microcracks, macrocracks, etc. shall close perfectly upon unloading. As the concrete material is heterogeneous, therefore it falls on permanent deformations after fatigue loading and unloading. Figure 7 shows the versatile behavior of the model where the stiffness degradation and permanent deformation are illustrated simultaneously.

VI. Conclusion

a) Concrete Fatigue Strain-based evolution Model by utilizing continuum thermodynamics Approach

Fatigue Strain Based Approach for Damage Evolution Model of Concrete materials during low frequency is presented by utilizing the framework of continuum thermodynamics of Continuum Mechanics by taking two material fatigue damage parameter i.e. A=fatigue damage Parameter regarding energy microcracks of the material particle and another is β=kinematic damage Parameter (phenomena of material crack surface close perfectly after unloading). For the production of concrete, except cement, all materials are locally available i.e. sand, aggregate, and water. So, it is very much popular in the list of construction material is construction engineering. Concrete is a heterogeneous matrix related to the composition i.e. cement, sand, aggregate and water among them cement is the weakest part compared to the remaining ingredients. So, fatigue damage in concrete in the fatigue process is obviously due to the development of internal micro-cracks, microvoids, macrocracks, a cycle-dependent damage surface is obtained in the formulation of the model. Fatigue damage evolution law regarding functions of damage response were obtained and used in the developing the constitutive relation to demonstrating the capacity for validation of the model for further diagnosis of concrete material, relate to stiffness degradation including inelastic deformations, under tension-tension, tension-compression fatigue loading by finding out the cumulative fatigue damage parameter i.e. K. The curve regarding fatigue response at A =0.10 and β =0.15 and 0.00 is calculated firstly by the modeling and after that this generated model curve is compared to the Curve obtained from the experimental data of Peiyin Lu. Et al (2004) which shows similar tread of generation of fatigue curve. This shows the good relationship between results obtained from modeling and experiments also. Lower value in the experimental curve is due to 0.85 times maximum stress level whereas, modeling takes 100% value.

b) Ordinary Concrete Fatigue Strain Evolution Model

The model curve regarding maximum fatigue strain and fatigue residual strain under different strain and stress levels using the model formulas (21) to (22) are described in Figures. Coefficients of different damage parameters regarding the evolutionary model are shown in Table. The data in the figure for the Strain Family Curve are the average of each group. From Figures (12), (13), (14) and Table of Fatigue Strain evolutionary Model, fatigue strain evolution equations (21) and (22) can be a good fit to the experimental data. Correlation coefficients are above 0.98. The evolution in the sense of fatigue damage parameter regarding maximum fatigue strain and fatigue residual strain has been plotted which clearly shows the similar three-phase variation at the different intermediate stage close to the linear change in their behavior. When the cycle ratio is exceeded by 0.90 then the curve converged rapidly. The level-S shaped curve of strain evolution is from the lower left corner to the upper right corner in the plotting of graph. This is due to experimenting measured of initial maximum strain and lacking measurement of initial residual strain, the strain evolution curve regarding maximum strain starts from the initial value, but the strain evolution curve of fatigue residual strain starts from zero. This is due to the defect in the material structure and de-orientation of molecules of the concrete. Based on the Model formation on the basis of $(0, \varepsilon^{f}_{max} - \varepsilon^{0}_{max})$. α fall in these the prescribed ranges while fitting of the curve is done surrounding its prescribed boundary conditions.

Authors' contributions

All authors read and approved the final manuscript.

Acknowledgments

The authors gratefully acknowledge the Tribhuvan University, Institute of Engineering, Department of Civil Engineering, Pulchowk Campus, IOE, Dean Office, CARD Section and NTNU Norway for their invaluable contributions and financial support as a scholarship to this research. The authors also acknowledge the assistance received from NTNU, Norway through CARD Section of IOE, Pulchowk Campus, Tribhuvan University.

Competing interests

The authors declare that they have no competing interests.

Availability of data and materials

Not applicable.

Consent for publication

Not applicable.

Ethics approval and consent to participate

Not applicable.

Funding

Not applicable.

References

1. European Union – BriteEURamIII (2000) Fatigue of Normal Weight Concrete and Lightweight Concrete, *Document BE 96-3942/R34.*
2. Szerszen, M.M. (2013) Fatigue damage model for ordinary concrete. *Open Journal of Functional Material Research (OJFMR)* 1(2), 17-20

3. Rao, S. and Roesler, J. (2004) Cumulative Fatigue Damage Analysis of Concrete Pavement using Accelerated Pavement Testing Results. *In: Proceedings of the 2nd International Conference on Accelerated Pavement Testing, Minneapolis, Sep*.
4. Dyduch, K., Szerszen, M.M. and Destrebecq, J-F (1994) Experimental Investigation of the Fatigue Strength of Plain Concrete under High Compressive Loading. *Structures and Materials*, 27 (173), 505-509.
5. Lü p., Li, Q., Song, Y. (2004) Damage Constitutive of Concrete under Uniaxial Alternate Tension-Compression Fatigue Loading Based on Double Bounding Surfaces. *International Journal of Solids and Structures*, 41, 3152-3166.
6. Fardis, M.N., Alibi, B. and Tassoulas, J.L. (1983) Monotonic and Cyclic Constitutive Law for Concrete. *J. Engrg, ASCE 108EM2*, 516-536.
7. Yang, B.L., Dafalias, Y.F., Herrmann, L.R. (1985) A Bounding Surface Plasticity Model for Concrete, Proc. Paper 19539. *J. Engg. Mech., ASCE 111/EM3*, 359-380.
8. Sauris, W., Ouyang, C., Fernando, V.M. (1990) Damage Model for Cyclic Loading for Concrete. *J. Engg. Mech., ASCE* 116(5), 1020-1035.
9. Papa, E. (1993) A Damage Model for Concrete Subjected to Fatigue Loading. *Eur. J. Mech.-A/solids*, 12(3), 449-440.
10. Khan, K.R., Al- Gandhi, A.H., Baluch, M.H (1998) An Elasto-Damage Constitutive Model for High Strength Concrete. *In: Proceeding of the EURO-C 1998 Conference on the computational model of the concrete structure at Austria, March*, 133-142
11. Thapa, K.B., Yazdani, S. (2013) Combined Damage and Plasticity Approach for Modeling Brittle Materials with Application to Concrete. *International Journal of Civil and Structural Engineering*, 3(3), 513-525.
12. Chaboche, J.L. (1992) Damage Induced Anisotropy: On the Difficulties Associated with the Active/Passive Unilateral Conditions. *Int. J. Damage Mechanics*, 1, 148-171.
13. Coleman, B.D. & Gurtin, M.E. (1967) Thermodynamics with International State Variables. *J. Chem. Phys.*, 47(2), 597-613.
14. Smith, G.M. & Young, L.E. (1955) Ultimate Theory in Flexure by Exponential Function. *Proceedings of the American Concrete Institute*, 52(3), 349-359.
15. Thapa, K.B., Yazdani, S. (2014) A Strain-Based Damage Mechanics Model for Plain Concrete. *International Journal of Civil Engineering Research*, 5(1), 27-40.
16. P. B. Cachim, J. A. Figueiras, and P. A. A. Pereira, "Fatigue behavior of fiber-reinforced concrete in compression," Cement &Concrete Composites, vol.24, no.2 ,pp.211–217,2002.
17. Duggal, S.K. (2009). "Building materials". New Delhi, India. New Age International
18. Edward, S. H. and David P. G., (2009). "Concrete construction". Edward S. Hoffman, Ltd., Chicago.
19. Gambhir, M .L. (2005) "Concrete Technology", Third edition Tata McGraw- Hill Publishing Company Limited, New Delhi.
20. Neville A. M., (2011). "Properties of concrete". Fifth edition. Pearson Publications, Harlow, England. The UK.
21. Shetty M. S., (2005). "Concrete technology: Theory and practice". S. Chand & Company, Ram
22. J. B. Xie, T. C. He, and H. R. Cheng, "Investigation flexural fatigue behavior of steel fiber reinforced concrete for pavement surface stratum under cyclic load," Journal of Lanzhou University of Technology, vol.30, no.2, pp.104–109, 2004.
23. R.M. Wang, G.F. Zhao, and Y.P. Song, "Research on compression fatigue performance of concrete," China Civil Engineering Journal, vol. 24, no.4, pp.38–47, 1991.
24. Y. B. Chen, Z. N. Lu, and D. Huang, "Fatigue defect of layer steel fiber reinforced concrete," Journal of Wuhan University of Technology (Natural Science Edition), vol. 18, no. 1, pp. 65–68, 2003.

Table 1: Data analysis for fatigue curve regarding stiffness, cumulative Damage Parameter verses Number of Cycle of Concrete

Assuming , Fatigue Damage Factor related to Surface Energy Microcracks i.e. A= 0.10 and Fatigue Kinematic Damage Factor (Crack Surface closed perfectly after unloading) i.e. β=0.00										
N (Number of Cycle)	Fatigue Damage Parameter (A)	Stiffness factor (β)	E0	εu	Exp(sqrt((1-2xβ)xN^Axεuxεu)/εu)	(E0-k)/E0	(Eo-k)	1-((E0-k)/E0)	Fatigue Damage Parameter, K
1	0.1	0.00	2.34E+10	1E-04	2.718281828	0.367879	8608378923	0.632120559	14791621077
2	0.1	0.00	2.34E+10	1E-04	2.815852123	0.355132	8310095480	0.644867715	15089904520
3	0.1	0.00	2.34E+10	1E-04	2.876192321	0.347682	8135756372	0.652318104	15264243628
4	0.1	0.00	2.34E+10	1E-04	2.920554404	0.342401	8012177404	0.657599256	15387822596
5	0.1	0.00	2.34E+10	1E-04	2.955885852	0.338308	7916408539	0.661691943	15483591461
6	0.1	0.00	2.34E+10	1E-04	2.985369569	0.334967	7838225541	0.665033097	15561774459
7	0.1	0.00	2.34E+10	1E-04	3.010740403	0.332144	7772174570	0.667855788	15627825430
8	0.1	0.00	2.34E+10	1E-04	3.0330523	0.329701	7715000497	0.670299124	15684999503
9	0.1	0.00	2.34E+10	1E-04	3.052995299	0.327547	7664604006	0.67245282	15735395994
10	0.1	0.00	2.34E+10	1E-04	3.071046719	0.325622	7619551945	0.674378122	15780448055
20	0.1	0.00	2.34E+10	1E-04	3.194997641	0.312989	7323949068	0.687010724	16076050932
30	0.1	0.00	2.34E+10	1E-04	3.271916231	0.305631	7151772341	0.694368703	16248227659
40	0.1	0.00	2.34E+10	1E-04	3.328592706	0.300427	7029997981	0.699572736	16370002019
50	0.1	0.00	2.34E+10	1E-04	3.373807035	0.296401	6935784933	0.703598934	16464215067
60	0.1	0.00	2.34E+10	1E-04	3.411588452	0.293119	6858975029	0.706881409	16541024971
70	0.1	0.00	2.34E+10	1E-04	3.444135968	0.290349	6794156857	0.709651416	16605843143
80	0.1	0.00	2.34E+10	1E-04	3.472786939	0.287953	6738104126	0.712046832	16661895874
90	0.1	0.00	2.34E+10	1E-04	3.498417763	0.285844	6688738048	0.714156494	16711261952
100	0.1	0.00	2.34E+10	1E-04	3.521635146	0.283959	6644640638	0.716040998	16755359362
200	0.1	0.00	2.34E+10	1E-04	3.681503839	0.271628	6356098221	0.728371871	17043901779
300	0.1	0.00	2.34E+10	1E-04	3.781094589	0.264474	6188684110	0.73552632	17211315890
400	0.1	0.00	2.34E+10	1E-04	3.854660161	0.259426	6070574065	0.740573758	17329425935
500	0.1	0.00	2.34E+10	1E-04	3.913457797	0.255528	5979366896	0.7444715	17420633104
600	0.1	0.00	2.34E+10	1E-04	3.962663371	0.252356	5905119312	0.747644474	17494880688
700	0.1	0.00	2.34E+10	1E-04	4.005105817	0.249681	5842542263	0.750318707	17557457737
800	0.1	0.00	2.34E+10	1E-04	4.042507649	0.247371	5788486265	0.752628792	17611513735
900	0.1	0.00	2.34E+10	1E-04	4.075998823	0.245339	5740924131	0.754661362	17659075869
1000	0.1	0.00	2.34E+10	1E-04	4.106362272	0.243525	5698474331	0.756475456	17701525669

Table 2: Data Analysis for Fatigue Curve regarding Maximum Stress Verses Number of Cycle and Damage Factor Parameter Verses Number of Cycle

Assuming , Fatigue Damage Factor related to Surface Energy Microcracks i.e. A= 0.10 and Fatigue Kinematic Damage Factor (Crack Surface closed perfectly after unloading) i.e. β=0.00 at different Number of Cycle							
N	$K	K	E	$stress	maximim stress	resudial strain	Log(N)
1	1282173402	0	23400000000	2114008.7	2114008.703	0.000204421	0
2	670041551	1282173402	22117826598	2091299.33	2091299.328	0.000202493	0.30103
3	458458692	1952214953	21447785047	2061344.27	2061344.267	0.000201791	0.477121
4	350269622	2410673645	20989326355	2034632.17	2034632.172	0.000201418	0.60206
5	284280732	2760943267	20639056733	2011362.18	2011362.182	0.000201185	0.69897
6	239711345	3045223999	20354776001	1990921.92	1990921.916	0.000201023	0.778151
7	207530075	3284935345	20115064655	1972746.93	1972746.93	0.000200905	0.845098
8	183169905	3492465419	19907534581	1956400.11	1956400.112	0.000200813	0.90309
9	164069326	3675635324	19724364676	1941550.91	1941550.908	0.00020074	0.954243
10	148678707	3839704650	19560295350	1927947.54	1927947.54	0.000200681	1
11	136004711	3988383357	19411616643	1915395.34	1915395.344	0.000200631	1.041393
12	125380779	4124388068	19275611932	1903741.33	1903741.327	0.000200589	1.079181
13	116342712	4249768847	19150231153	1892863.35	1892863.347	0.000200553	1.113943
14	108557141	4366111560	19033888440	1882662.48	1882662.476	0.000200522	1.146128
15	101778373	4474668701	18925331299	1873057.53	1873057.529	0.000200494	1.176091
16	95821285.7	4576447074	18823552926	1863981.1	1863981.099	0.000200469	1.20412
17	90543702.5	4672268359	18727731641	1855376.62	1855376.622	0.000200448	1.230449
18	85834594.3	4762812062	18637187938	1847196.18	1847196.182	0.000200428	1.255273
19	81605972.8	4848646656	18551353344	1839398.83	1839398.832	0.00020041	1.278754
20	77787194.3	4930252629	18469747371	1831949.31	1831949.314	0.000200394	1.30103
21	74320876.4	5008039823	18391960177	1824817.05	1824817.047	0.000200379	1.322219
22	71159917.7	5082360700	18317639300	1817975.34	1817975.34	0.000200366	1.342423
23	68265288.9	5153520617	18246479383	1811400.75	1811400.754	0.000200353	1.361728
24	65604369.8	5221785906	18178214094	1805072.6	1805072.596	0.000200342	1.380211
25	63149682.3	5287390276	18112609724	1798972.51	1798972.506	0.000200331	1.39794
26	60877913.1	5350539958	18049460042	1793084.12	1793084.116	0.000200321	1.414973
27	58769151.4	5411417871	17988582129	1787392.77	1787392.775	0.000200312	1.431364
28	56806288.3	5470187023	17929812977	1781885.31	1781885.312	0.000200303	1.447158
29	54974539.3	5526993311	17873006689	1776549.85	1776549.846	0.000200295	1.462398
30	53261061.7	5581967850	17818032150	1771375.62	1771375.621	0.000200287	1.477121
31	51654644.9	5635228912	17764771088	1766352.87	1766352.869	0.00020028	1.491362
32	50145458.4	5686883557	17713116443	1761472.69	1761472.69	0.000200273	1.50515
33	48724845.3	5737029016	17662970984	1756726.95	1756726.954	0.000200267	1.518514
34	47385151.8	5785753861	17614246139	1752108.21	1752108.211	0.000200261	1.531479
35	46119585.2	5833139013	17566860987	1747609.62	1747609.624	0.000200255	1.544068
36	44922096.4	5879258598	17520741402	1743224.89	1743224.894	0.000200249	1.556303
37	43787280.1	5924180694	17475819306	1738948.21	1738948.212	0.000200244	1.568202

Table 3: Maximum Stress Observed by Peiyin Lu.Et al 2004 through Experiment

Smax	No. of Cycle	Factor	Max. Stress
0.85	1479000	2.2	158.4893192
0.84	1461600	2.3	199.5262315
0.75	1305000	4.1	12589.25412
0.74	1287600	4.4	25118.86432
0.69	1200600	5.05	112201.8454
0.685	1191900	5.2	158489.3192
0.68	1183200	5.75	562341.3252
0.65	1131000	6.2	1584893.192
0.63	1096200	6.3	1995262.315

Table 4: Damage Predicted by Peiyin Lu.Et al 2004 through Experirment

Damage	Cyclic Ratio
0.16	0
0.17	0.02
0.172	0.04
0.18	0.05
0.19	0.08
0.195	0.1
0.2	0.12
0.21	0.145
0.22	0.198
0.23	0.24
0.24	0.255
0.25	0.28
0.26	0.32
0.28	0.37
0.3	0.395
0.31	0.43
0.36	0.52
0.4	0.585
0.48	0.68
0.55	0.72
0.7	0.88
0.88	1

Fatigue Evolution Model

Table 5: Influence of fatigue strain Parameter "P" on Fatigue strain Curve by Putting the value of (i) P=2.00, β=1.25 (ii) P=3.00, β=.1.125 (iii) P=4.00. β=1.0625 (iv) P=5.00, β=1.0315 (v) P=6.00, β=1.015625 (vi) P=7.00, β=1.003906 and $\varepsilon^0{}_{max}/\varepsilon^f$ =0.60 and $\alpha/\varepsilon^f{}_{max}$=0.20

n/N_f	$\varepsilon^0{}_{max}/\varepsilon^f{}_{max}$	$\propto/\varepsilon^f{}_{max}$	β	p	β/(β-n/Nf)	$(\beta/(\beta-n/N_f)-1)^{(1/p)}$	$\varepsilon^n{}_{max}/\varepsilon^f{}_{max}$	$\varepsilon^n{}_{max}$
0	0.6	0.20	1.25	2	1	0	0.6	0.60
0.05	0.6	0.20	1.25	2	1.041666667	0.204124145	0.640824829	0.64
0.1	0.6	0.20	1.25	2	1.086956522	0.294883912	0.658976782	0.66
0.15	0.6	0.20	1.25	2	1.136363636	0.369274473	0.673854895	0.67
0.2	0.6	0.20	1.25	2	1.19047619	0.43643578	0.687287156	0.69
0.25	0.6	0.20	1.25	2	1.25	0.5	0.7	0.70
0.3	0.6	0.20	1.25	2	1.315789474	0.561951487	0.712390297	0.71
0.35	0.6	0.20	1.25	2	1.388888889	0.623609564	0.724721913	0.72
0.4	0.6	0.20	1.25	2	1.470588235	0.685994341	0.737198868	0.74
0.45	0.6	0.20	1.25	2	1.5625	0.75	0.75	0.75
0.5	0.6	0.20	1.25	2	1.666666667	0.816496581	0.763299316	0.76
0.55	0.6	0.20	1.25	2	1.785714286	0.88640526	0.777281052	0.78
0.6	0.6	0.20	1.25	2	1.923076923	0.960768923	0.792153785	0.79
0.65	0.6	0.20	1.25	2	2.083333333	1.040833	0.8081666	0.81
0.7	0.6	0.20	1.25	2	2.272727273	1.12815215	0.82563043	0.83
0.75	0.6	0.20	1.25	2	2.5	1.224744871	0.844948974	0.84
0.8	0.6	0.20	1.25	2	2.777777778	1.333333333	0.866666667	0.87
0.85	0.6	0.20	1.25	2	3.125	1.457737974	0.891547595	0.89
0.9	0.6	0.20	1.25	2	3.571428571	1.603567451	0.92071349	0.92
0.95	0.6	0.20	1.25	2	4.166666667	1.779513042	0.955902608	0.96
1	0.6	0.20	1.25	2	5	2	1	1.00

Fatigue Evoluation Model

Table 6: Infiuence of Fatigue Strin Parameter "α" on fatigue Strain Curve , putting the value of (i) $\alpha/\varepsilon^{f}_{max}$=0.10, β= 1.05 and $\varepsilon^{0}_{max}/\varepsilon^{f}_{max}$=0.60 and P=2

n/N_f	$\varepsilon^0_{max}/\varepsilon^f_{max}$	$\propto/\varepsilon^f_{max}$	β	p	β/(β-n/Nf)	$(\beta/(\beta-n/N_f)-1)^{(1/p)}$	$\varepsilon^n_{max}/\varepsilon^f_{max}$	$\varepsilon^n_{max}/\varepsilon_{max}$
0	0.6	0.10	1.0625	2	1	0	0.6	0.60
0.05	0.6	0.10	1.0625	2	1.049382716	0.222222222	0.622222222	0.62
0.1	0.6	0.10	1.0625	2	1.103896104	0.322329186	0.632232919	0.63
0.15	0.6	0.10	1.0625	2	1.164383562	0.405442427	0.640544243	0.64
0.2	0.6	0.10	1.0625	2	1.231884058	0.481543412	0.648154341	0.65
0.25	0.6	0.10	1.0625	2	1.307692308	0.554700196	0.65547002	0.66
0.3	0.6	0.10	1.0625	2	1.393442623	0.627250048	0.662725005	0.66
0.35	0.6	0.10	1.0625	2	1.49122807	0.700876644	0.670087664	0.67
0.4	0.6	0.10	1.0625	2	1.603773585	0.77702869	0.677702869	0.68
0.45	0.6	0.10	1.0625	2	1.734693878	0.857142857	0.685714286	0.69
0.5	0.6	0.10	1.0625	2	1.888888889	0.942809042	0.694280904	0.69
0.55	0.6	0.10	1.0625	2	2.073170732	1.035939541	0.703593954	0.70
0.6	0.6	0.10	1.0625	2	2.297297297	1.138989595	0.713898959	0.71
0.65	0.6	0.10	1.0625	2	2.575757576	1.255291829	0.725529183	0.73
0.7	0.6	0.10	1.0625	2	2.931034483	1.389616668	0.738961667	0.74
0.75	0.6	0.10	1.0625	2	3.4	1.549193338	0.754919334	0.75
0.8	0.6	0.10	1.0625	2	4.047619048	1.745743122	0.774574312	0.77
0.85	0.6	0.10	1.0625	2	5	2	0.8	0.80
0.9	0.6	0.10	1.0625	2	6.538461538	2.353393622	0.835339362	0.84
0.95	0.6	0.10	1.0625	2	9.444444444	2.905932629	0.890593263	0.89
1	0.6	0.10	1.0625	2	17	4	1	1.00

Calculation of fatigue Evolution Factor i.e. β

Table 7: Calculation of Fatigue Strain Parameter "β" on Fatigue Strain Curve, putting the value of (i) $\alpha/\varepsilon^f_{max}$=0.10 to1.05, β=1.05 and $\varepsilon^0_{max}/\varepsilon^f_{max}$=0.60 and P=2

p	$\varepsilon^0_{max}/\varepsilon^f_{max}$	$\propto/\varepsilon^f_{max}$	$(1-\varepsilon^0_{max}/\varepsilon^f_{max})$	$(1-\varepsilon^0 max/\varepsilon f max)/(\propto/\varepsilon^f_{max})^{\wedge -p}$	$\beta=((1-\varepsilon 0 max/\varepsilon f max)/(\propto/\varepsilon^f_{max}))+1$
2	0.6	0.10	0.4	0.0625	1.0625
2	0.6	0.15	0.4	0.140625	1.140625
2	0.6	0.20	0.4	0.25	1.25
2	0.6	0.25	0.4	0.390625	1.390625
2	0.6	0.30	0.4	0.5625	1.5625
2	0.6	0.35	0.4	0.765625	1.765625
2	0.6	0.40	0.4	1	2
2	0.6	0.45	0.4	1.265625	2.265625
2	0.6	0.50	0.4	1.5625	2.5625
2	0.6	0.55	0.4	1.890625	2.890625
2	0.6	0.60	0.4	2.25	3.25
2	0.6	0.65	0.4	2.640625	3.640625
2	0.6	0.70	0.4	3.0625	4.0625
2	0.6	0.75	0.4	3.515625	4.515625
2	0.6	0.80	0.4	4	5
2	0.6	0.85	0.4	4.515625	5.515625
2	0.6	0.90	0.4	5.0625	6.0625
2	0.6	0.95	0.4	5.640625	6.640625
2	0.6	1.00	0.4	6.25	7.25
2	0.6	1.05	0.4	6.890625	7.890625

Fatigue Evolution Model

Table 8: S-shaped curve family of fatigue strain, $\varepsilon^{0}_{max}/\varepsilon^{f}_{max}$=0.70,P=2.00, $\alpha/\varepsilon^{f}_{max}$ =0.25, β=1.694444

n/N_f	$\varepsilon^0{}_{max}/\varepsilon^f{}_{max}$	$\propto/\varepsilon^f{}_{max}$	β	p	β/(β-n/Nf)	$(\beta/(\beta-n/N_f)-1)^{(1/p)}$	$\varepsilon^n{}_{max}/\varepsilon^f{}_{max}$	$\varepsilon^n{}_{max}/\varepsilon^f{}_{max}$
0	0.7	0.25	1.69444	2	1	0	0.7	0.70
0.05	0.7	0.25	1.69444	2	1.0304055	0.174371694	0.743592923	0.74
0.1	0.7	0.25	1.69444	2	1.0627179	0.25043551	0.762608878	0.76
0.15	0.7	0.25	1.69444	2	1.0971226	0.311644961	0.77791124	0.78
0.2	0.7	0.25	1.69444	2	1.1338294	0.365827	0.79145675	0.79
0.25	0.7	0.25	1.69444	2	1.1730775	0.416025787	0.804006447	0.80
0.3	0.7	0.25	1.69444	2	1.2151401	0.463832004	0.815958001	0.82
0.35	0.7	0.25	1.69444	2	1.2603314	0.510226851	0.827556713	0.83
0.4	0.7	0.25	1.69444	2	1.3090139	0.55589022	0.838972555	0.84
0.45	0.7	0.25	1.69444	2	1.3616084	0.601338868	0.850334717	0.85
0.5	0.7	0.25	1.69444	2	1.4186062	0.646997843	0.861749461	0.86
0.55	0.7	0.25	1.69444	2	1.4805844	0.693241942	0.873310486	0.87
0.6	0.7	0.25	1.69444	2	1.5482256	0.740422566	0.885105642	0.89
0.65	0.7	0.25	1.69444	2	1.6223431	0.788887238	0.897221809	0.90
0.7	0.7	0.25	1.69444	2	1.7039138	0.838995686	0.909748921	0.91
0.75	0.7	0.25	1.69444	2	1.7941214	0.891134885	0.922783721	0.92
0.8	0.7	0.25	1.69444	2	1.8944144	0.945734837	0.936433709	0.94
0.85	0.7	0.25	1.69444	2	2.0065842	1.003286721	0.95082168	0.95
0.9	0.7	0.25	1.69444	2	2.1328735	1.06436529	0.966091323	0.97
0.95	0.7	0.25	1.69444	2	2.276127	1.129657922	0.982414481	0.98
1	0.7	0.25	1.69444	2	2.4400092	1.20000384	1.00000096	1.00