Global Journal of Researches in Engineering: J
General Engineering
Volume 20 Issue 3 Version 1.0 Year 2020
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4596 & Print ISSN: 0975-5861

Development of Analytical Model for a Vertical Single U-Tube Ground-Coupled Heat Pump System

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GJRE-J Classification: *FOR Code: 091399*

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Strictly as per the compliance and regulations of:

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Keywords: borehole thermal resistance, analytical modeling, thermal resistance correlation, DX heat exchangers, R-410A.

I. Introduction

The ground has been utilized since the forties of the last century as an energy source, an energy sink, or for energy storage. This was done in a parallel effort of developing efficient heat pumps to raise this heat source to a higher level of temperatures for heating purposes or heat rejection for cooling purposes. Augmentation of efficiency led to tremendous research to improve the performance of the ground part of the heat pump system. Hence qualitative and quantitative work was focused on the thermal design of the ground heat exchanger, vertical and horizontal orientations.

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Naili et al. [1] studied experimentally a horizontal ground source heat pump system in the cooling mode. The heat pump COP and the system COP were found to be (4.25) and (2.88), respectively. Bakirci [2] evaluated the performance of a ground-source heat pump system in a cold climate region. The experimental results indicated that the average heat-pump COP values are approximately (3) and (2.6) in the coldest months of a heating season. Fan et al. [3] conducted a theoretical study on the performance of an integrated ground-source heat pump system. The results refer to various factors affecting the performance of the vertical heat exchangers and hence the performance of the heat pumps system. Esen et al. [4] studied experimentally the transient temperature distribution inside a borehole for a vertical U-tube heat exchanger at (30, 60, and 90) m depth and (150) mm borehole diameter. A two-dimensional finite element model was built, and ANSYS code was implemented for the numerical analysis to predict the temperature distribution. They concluded that the numerical analysis appears to be most promising for predicting the response of GHEs to thermal loading.

Wood et al. [5] studied the heat pump performance and ground temperature of a ground heat exchanger system for a residential building. The seasonal coefficient of performance of the heat pump was found to be (3.62), and the temperature at (5) m was undisturbed. Florides et al. [6] investigated the thermal performance of a double U-tube GHE and the assessment of its efficiency with regard to its building cost. A numerical model was also developed for energy flows and temperature changes in and around a borehole. It was validated upon comparing its results with established experimental results for a single GHE.

Liao et al. [7] studied numerically the effective borehole thermal resistance of a vertical, single U-tube ground heat exchanger for a range of shank spacing. They claimed that their study produced a correlation that showed better accuracy than available correlations. Sharqawy et al. [8] postulated a 2-dimensional numerical model for the steady-state heat conduction within the borehole. He developed a correlation for the effective borehole thermal resistance and was also concluded that his correlation predicted the thermal resistance better than other available formulas. The analytical models for (GHE) utilize mainly a line heat source [9, 10] and cylinder heat source theory [11, 12] to predict the heat transfer rate between the ground and the heat carrier fluid flowing in the (GHE).

The equivalent diameter of U-tube can be presented in the form of:

$$d_e = \beta d_o \quad (1)$$

Where (β) is an equivalency coefficient greater than (1.0). Claesson and Dunand [13] postulated the value of (β) for two buried horizontal pipes to be ($\sqrt{2}$).

Shonder and Beck [14] implemented a one-dimensional heat transfer model for the U-tube and arrived at the same value as that of Claesson and Dunand [13] for a single vertical U-tube heat exchanger in the form:

$$R_{f=}\frac{ln\left(\frac{D_B}{\sqrt{n}\,d_o}\right)}{2\,\pi\,k_g} \quad (2.a)$$

In which the equivalent diameter corresponds to:

$$d_e = \sqrt{n}\,d_o \quad (2.b)$$

Where (n) is equal to (2) for a single U-tube system and is corresponding to (2 do) for double U-tube ground heat exchanger. Gu and O'Neal [15] utilized a steady-state heat transfer simulation based on the cylindrical source model to produce a correlation for the grout resistance for a vertical U-tube ground heat exchanger in the form:

$$R_{f=}\frac{ln\left(\frac{D_B}{d_o}\sqrt{\frac{d_o}{S_p}}\right)}{2\,\pi\,k_g} \quad (3.a)$$

This form of equation reveals that the equivalent diameter was expressed as:

$$d_e = \sqrt{S_p\,d_o} \quad (3.b)$$

Koenig [16] has analyzed the heat transfer problem in a borehole with single and multi-vertical U-tube loops. He has arrived at an analytical solution to the borehole thermal resistance for different U-tube geometry configuration and presented a validation for the model with acceptable accuracy limits. Tarrad [17] reported a simple correlation for the prediction of a borehole thermal resistance in a vertical single U-tube ground heat exchanger incorporates the following expression of the equivalent diameter:

$$d_e = \frac{D_B}{\left(x+\sqrt{x^2-1}\right)} \quad (4.a)$$

$$x = \frac{{D_B}^2 + {d_o}^2 - {S_p}^2}{2\,D_B\,d_o} \quad (4.b)$$

The correlation showed an acceptable agreement with previously available ones in the open literature. More recently, Tarrad [18] developed a correlation to predict the borehole thermal resistance in which the equivalent tube was derived as:

$$d_e = \frac{\sqrt{2}\,d_o + 2\,d_o}{2} \approx \sqrt{3}\,d_o \quad (5)$$

The correlation showed excellent agreement with previously published expressions in the open literature.

In this study, the thermal resistance of the grout was coupled with the tube resistance to accomplish a model for the assessment of effective borehole resistance. The obstruction to heat conduction inside the borehole due to the presence of the second tube leg was also studied, and a correlation was addressed for this purpose. The shank spacing was ranged between (2) and (4) times the U-tube outside diameter producing a geometry factor (ζ) defined as the ratio of tube spacing to borehole diameter occupies the range of (0.29-0.59).

II. Present Model

Model Derivitive

The model suggests that there is a single U-tube installed in the borehole to compose a ground heat exchanger for heating or cooling heat pump system, Figure 1.

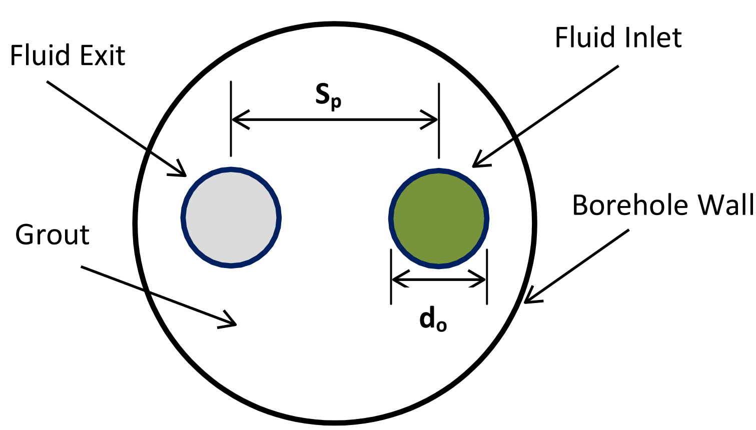

Figure 1: A schematic diagram for the proposed present model

The presentation of the thermal resistance circuit may be illustrated for the U-tube geometry, as depicted in Figure 2. A similar thermal resistance circuit was also postulated by Koenig [16] for a single U-tube ground heat exchanger.

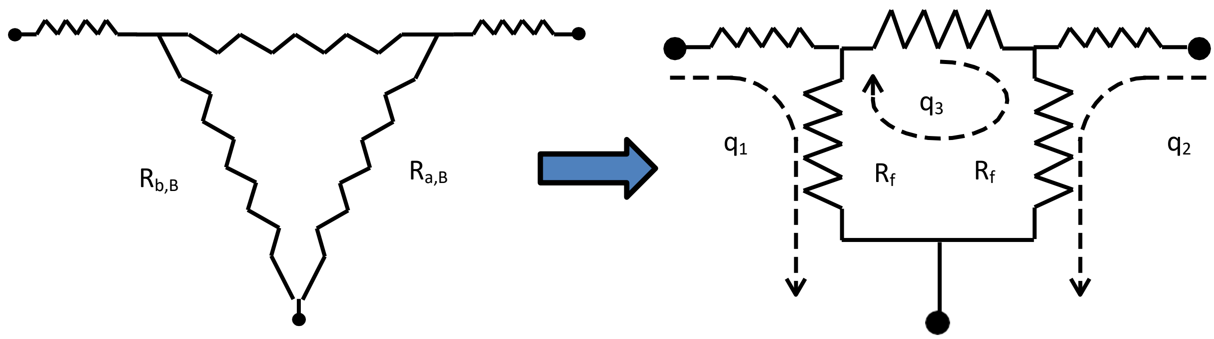

Figure 2: Analogy to electrical resistance circuit presentation of the present work

The U-tube geometry in the borehole is usually chosen to be identical, and parallel loop circuits are utilized. Hence the same fluid flows in both U-tube legs. This leads to equal tube thermal resistances on the fluid side and conduction through the tube wall for both tubes. Further for identical tube geometries, the grout thermal resistance is the same between the tube wall and the grout boundary, as illustrated on the right side of Figure 2. Therefore the following conditions hold for the present work:

$$R_{a,B} = R_{b,B} = R_f \tag{6}$$

$$R_{p,a} = R_{p,b} = R_p \tag{7}$$

$$R_p = \frac{1}{\pi\, d_i\, h} + \frac{\ln\left(\frac{d_o}{d_i}\right)}{2\,\pi\, k_p} \tag{8}$$

The following mathematical expressions are to be solved simultaneously for (q_1, q_2, and q_3).

$$\Delta T_{b,B} - (q_1 - q_3)R_f - q_1\, R_p = 0 \tag{9}$$

$$\Delta T_{a,B} - (q_2 + q_3)R_f - q_2\, R_p = 0 \tag{10}$$

$$\Delta T_{b,a} - q_3\, R_s - (q_1 - q_2)R_p = 0 \tag{11}$$

In these expressions $\Delta T_{b,B}$, $\Delta T_{a,B}$, and $\Delta T_{b,a}$ represents a temperature difference as follows:

$$\Delta T_{a,B} = T_a - T_B \tag{12.a}$$

$$\Delta T_{b,B} = T_b - T_B \tag{12.b}$$

$$\Delta T_{b,a} = T_b - T_a \tag{12.c}$$

Equations (9-11) were solved simultaneously to yield the following relations for each of heat transfer rate inside the borehole and its mutual exchange with ground and tubing systems:

$$q_1 = \frac{\Delta T_{b,B} + \varepsilon}{\beta\, R_p} \tag{13}$$

$$q_2 = \frac{\Delta T_{a,B} - \varepsilon}{\beta\, R_p} \tag{14}$$

$$q_3 = \frac{\Delta T_{b,a}\, \beta - \Delta T_{b,B} + \Delta T_{a,B}}{R_s\, \beta + 2\, R_f} \tag{15.a}$$

This expression of (q_3) can be further simplified to obtain:

$$q_3 = \left\{\frac{\Delta T_{b,a}\,(\beta - 1)}{\beta\, R_s + 2\, R_f}\right\} \tag{15.b}$$

In these expressions, the following definitions were implemented:

$$\varepsilon = R_f \left\{\frac{\Delta T_{b,a}\, \beta - \Delta T_{b,B} + \Delta T_{a,B}}{\beta\, R_s + 2\, R_f}\right\} = R_f \left\{\frac{\Delta T_{b,a}\,(\beta - 1)}{\beta\, R_s + 2\, R_f}\right\} \tag{16.a}$$

and

$$\beta = \frac{R_f + R_p}{R_p} \tag{16.b}$$

Equations (13-15) reveal that the individual values of the heat transfer rate have complex relation criteria with the parameters related to the thermal resistance of different parts of the system. Hence it is usually treated as a semi-analytical problem or a complete analytical solution with several assumptions to simplify the problems having a margin of error in their applications. The total heat transfer rate between the two legs of the U-tubing walls and borehole boundary is represented by the algebraic summation of (q_1 and q_2) as:

$$q_1 + q_2 = \frac{\Delta T_{b,B} + \Delta T_{a,B}}{\beta\, R_p} \tag{17}$$

Shunt Thermal Resistance

The thermal shunt resistance (R_s) can be modeled as an isothermal pipe to pipe conduction shape factor per unit length in an infinite medium per unit length Holman [19].

$$S = \frac{2\,\pi}{\cosh^{-1}\left\{2\left(\frac{S_p}{d_o}\right)^2 - 1\right\}} \tag{18.a}$$

$$R_s = \frac{1}{S\, k_g} = \frac{\cosh^{-1}\left\{2\left(\frac{S_p}{d_o}\right)^2 - 1\right\}}{2\,\pi\, k_g} \tag{18.b}$$

Grout Thermal Resistance

The thermal resistance of an offset tube inside a cylindrical geometry with a length to be much bigger than the radius of the tube can be deduced from the shape factor cited in Holman [19] as:

$$S_f = \frac{2\,\pi\, L}{\cosh^{-1}\left\{\frac{{D_B}^2 + {d_o}^2 - 4\,{l_p}^2}{2\, D_B\, d_o}\right\}} \tag{19.a}$$

$$R_{off} = \frac{1}{S_f\; k_g} \tag{19.b}$$

This relation possesses the same volume of tubes, grout volume, mass flow rate of fluid inside the U-tube, and the same borehole geometry. Further, the same temperature conditions around the borehole exist.

Obstruction Factor

There is a conductive borehole obstruction due to the presence of the other U-tube leg of the loop in the radial direction of heat flow. This interference or obstruction is addressed by including the factor (σ). Hence the thermal resistance between the U-tubing wall and the borehole boundary is defined as:

$$R_f = \frac{R_{off}}{\sigma} \tag{20}$$

The obstruction factor to heat transfer is effectively represented by the surface area that is shadowed by the thermal beam of one leg at an angle of (α), Figure 3.

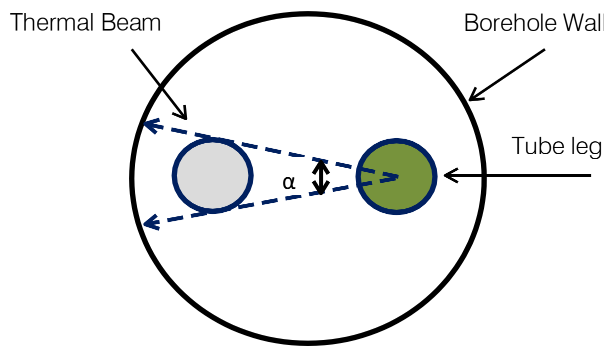

Figure 3: Thermal representation of the obstruction factor

Here, it is assumed that the heat source, the tube leg represents a line heat transfer source at the center of the tube. This is the case where heat is lost from the ground heat exchanger, and it works as a heat sink for heating purposes. The heat transfer mechanism occurs in the radial direction, and circumferential heat conduction at the borehole surface is neglected. The latter assumption implicitly states an isothermal condition at the borehole surface. The envelope behind the tube, which is projected at the borehole surface, is calculated per unit length from:

$$A_{env} = (D_B + S_p)\ \tan^{-1}\left(\frac{d_o}{2\sqrt{{S_p}^2 - \left(\frac{d_o}{2}\right)^2}}\right) \tag{21.a}$$

Simultaneously, the second leg performs the same obstruction for heat transfer, which is reflected in the total heat transfer rate in the borehole configuration. Hence, the obstruction factor can be expressed as:

$$\sigma = 1 - \frac{2\,(D_B + S_p)\,\tan^{-1}\left(\frac{d_o}{2\sqrt{{S_p}^2 - \left(\frac{d_o}{2}\right)^2}}\right)}{\pi\, D_B} \tag{21.b}$$

Equation (21) shows that the obstruction factor is only a geometry dependence parameter; its value lies in the range of ($0<\sigma\leq1$) and depends on the borehole and U-tube configurations. The obstruction factor is equal to unity when there is no obstruction object since the thermal beam diverging angle (α) tends to zero. Hence it is always greater than zero and less or equal to (1).

This expression experiences a decrease as the two legs become closer and approaches a minimum when these tubes touch each other with ($S_p = d_e$); hence its value shows a weak dependence on the tube diameter. In other words, it produces the highest expected borehole thermal resistance for a given configuration. The obstruction factor shows a rise as the leg spacing increases and approaches the maximum as the tubes are touching the borehole surface. Hence, it showed the lowest borehole thermal resistance. However, the extreme case where the tube legs are situated along the outer surface of the borehole violates the principle assumption for heat transfer in a 1-dimensional radial direction. Accommodating the tubes at the borehole wall will create a large circumferential temperature maldistribution; the uniform borehole surface temperature assumption will be demolished.

The expression of the obstruction factor could also be confirmed by the work of Remund [20], who has reported the borehole thermal resistance for three different cases of the two tube legs spacing. These conditions were described according to the U-tube leg spacing as, close, average, and along the outer wall of the borehole in the form:

$$R_f = \frac{1}{C_1\, k_g \left(\frac{D_B}{d_o}\right)^{C_2}} \tag{22}$$

Where the values of the coefficient (C_1) and the index (C_2) were stated for three cases, as illustrated in Table 1.

Table 1: Coefficients of equation (22), [20]

Configuration	C_1	C_2
Close together	20.10	-0.9447
Average	17.44	-0.6052
Along outer wall	21.91	-0.3796

The results of equation (22) revealed that for a given borehole configuration as the tube legs get closer, the borehole thermal resistance showed a rise and approached maximum as they touch each other. It approaches a minimum value as the tube spacing reaches a maximum as the tubes touch the borehole surface. Further, Gu and O'Neal [14] in their work for replacement of the U-tube by equivalent concentric tube at the borehole had arrived at the same conclusion. Increasing the two legs spacing results in an increase of the equivalent diameter and in turn, reduces the grout and borehole thermal resistance.

These results are consistent with the present work outcomes for the obstruction factor (σ), and its numerical value is a geometrical parameter only regardless of the operating conditions.

Borehole Thermal Resistance

The thermal resistances of different sources inside the borehole to heat transfer between the fluid flow in the tubes as a heat source or sink, and the borehole boundary are presented in eq. (8) and eq. (20). Therefore, the total borehole resistance is expressed by:

$$R_B = R_p + R_f \tag{23}$$

Equation (14) can be simplified further to give:

$$q_1 + q_2 = \frac{T_b + T_a - 2\,T_B}{\beta\, R_p} \tag{24.a}$$

The net total heat transfer rate that crosses the boundary of the borehole is expressed as:

$$q_1 + q_2 - q_{env} = \frac{\Delta T_m}{R_B} = \frac{(T_m - T_B)}{R_B} \tag{24.b}$$

In eq. (24.b) the mean temperature of the fluid inside the tubes is considered as:

$$T_m = \frac{(T_a + T_b)}{2} \tag{24.c}$$

and

$$q_{env} = (q_1 + q_2)\,(1 - \sigma) \tag{24.d}$$

Combining eq. (24.a) and eq. (24.b) yields to:

$$R_B = \frac{\beta\, R_p}{2\,\sigma} \tag{25.a}$$

Hence the borehole thermal resistance corresponds to:

$$R_B = \frac{R_p + R_f}{2\,\sigma} \tag{25.b}$$

When the obstruction factor is dismissed, then (σ) is equal to unity, and the same expression will be obtained as that of Koenig [16].

An interesting result may be deduced from the present analysis when it is applied for geothermal DX evaporators and condensers. The change of phase usually takes place in an isothermal (T_b = T_a= T) process for pure refrigerants, nonazeotrop mixtures and azeotropic mixtures of the negligible boiling range such as R-410A, then:

$$q_1 = q_2 \tag{26}$$

$$q_1 + q_2 - q_{env} = \frac{2\,(T - T_B)}{\beta\, R_p} \tag{27}$$

Further, the heat transfer rate (q_3) as presented in eq. (15.b) approaches zero and eq. (25) is still applicable.

Ground Thermal resistance

The ground thermal resistance is important for the assessment of heat transfer rate and temperature distribution of the ground heat exchanger. Garbai and Méhes [21] have included the effect of the ground as a resistance to heat transfer from or to the U-tube fluid for a region extended to infinity. They have concluded that after (1) year of operation, the heat transfer process approaches steady-state conditions and the value of (0.053) m.K/W for a ground thermal conductivity of (2.42) W/m.K was estimated. Hence, it was decided to implement this value at the present work.

Total Thermal Resistance

The total thermal resistance per unit length of the borehole is estimated by:

$$R_t = R_B + R_{ground} \tag{28}$$

This expression represents the thermal resistance of the double U-tube GHE to heat transfer.

III. Verification Methodology

Ground Heat Exchanger Specifications

The verification of the model was accomplished by the comparison with previously published correlations in the open literature. The following conditions were utilized for a hypothetical heat pump system utilizes the U-tube GHE:

1. A heat pump coupled ground heat exchanger is utilized for cooling purpose having the following operating conditions:
   - Cooling load of (3.5) kW to be extracted from the space throughout the circulation of chilled water in fan coils installed at the required points.
   - Chilled water to be produced by circulating through the chiller at a temperature range of (7-12) °C.
   - Rejected load to the ground by the copper tubing of the condenser was estimated in the range of (4.4) kW with COP of (3.57) for cooling.
2. A single copper U-tube and borehole dimensions are shown in Table 2.

Table 2: Selected geometrical configurations for a single U-tube

Geom.	d_o (mm)	D_B (mm)	S_p/d_o (-)	S_p/D_B (-)	Gref (kg/m^2 s)	V_{ref} (m/s)		AU-tube (m^2/m)
						Vap	Liq.	
1	9.525	65	2-4	0.29-0.59	371.43	5.03	0.364	0.05985
2	12.7	75	2-3.3	0.34-0.56	199.27	2.7	0.196	0.0798

3. The borehole is filled with grout, having a thermal conductivity range between (0.73) W/m.K and (1.9) W/m.K, [22].
4. R-410A is circulated through the heat pump in the DX system. It has a typical condensation heat transfer coefficient of (3000) W/m^2 K, Huang et al. [23] and Kim and Shin (2005) [24]

The mass flux density and fluid flow velocity were calculated from:

$$G_{ref} = \frac{\dot{m}_{ref}}{A_{c,i}} \tag{29.a}$$

$$V_{ref} = \frac{G_{ref}}{\rho_{ref}} \tag{29.b}$$

Maximum U-tube Spacing

The tube spacing (Sp) was selected according to the relation given by Koenig [16] for practical applications of the ground U-tube heat exchanger as follows:

$$D_s + 2\, d_o \leq 0.75\, D_B \tag{30}$$

Rearranging this relation in terms of the tube spacing (Sp) gives:

$$S_p + d_o \leq 0.75\, D_B \tag{31}$$

This expression shows that the maximum tube spacing inside the borehole is controlled by:

$$S_{p,max} = 0.75\, D_B - d_o \tag{32}$$

IV. Results and Discussion

Grout Thermal Resistance

The present work results of the thermal resistance of grout are compared to other available correlations in the open literature. The single loop borehole specific thermal resistance for the WF of (14.29) geometry is compared in Figure 4.

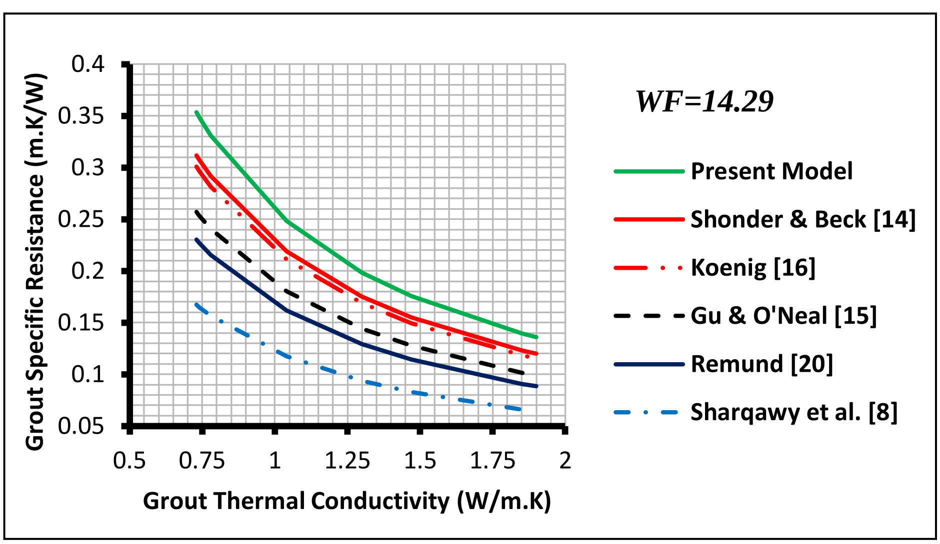

Figure 4: Comparison of different grout thermal resistance expressions for the test WF=14.29 geometries at (S_p/ d_o=3.3)

The data revealed that the present model predicted higher grout thermal resistance than other investigators. It was higher than that of the closest data of Shonder and Beck [14] by about (15) %. Sharqawy et al. [8] prediction was the lowest among other correlations and the rest occupies the zone in between the two mentioned models. The thermal resistance predicted by Koenig [16] and Shonder and beck were close to each other and were higher than those of Gu and O'Neal [15] and Remund [20]. As the grout thermal conductivity increases, the predicted results are getting closer and approaching a minimum discrepancy at the highest tested thermal conductivity of (1.9) W/m.K. This is due to the decrease of thermal resistance of the grout as the thermal conductivity increases, eq. (19).

At a constant value of (S_p / D_B), the thermal resistance showed dependence on the grout thermal conductivity in the form:

$$R_B = \beta_0 \, k_g{}^{\beta_1} \tag{33}$$

The index (β_1) has a negative value. A linear behavior for the borehole thermal resistance with the ratio (S_p / D_B) at constant grout thermal conductivity is obvious in Figure 5 for both geometries. It was a steeper for the bigger tube size (12.7) mm outside diameter than that of the smaller one of (9.52) mm. This, of course, is related directly to the thermal resistance of the grout layer that covers these tubes; the lower thermal resistance corresponds to the steeper line variation in Figure 5.

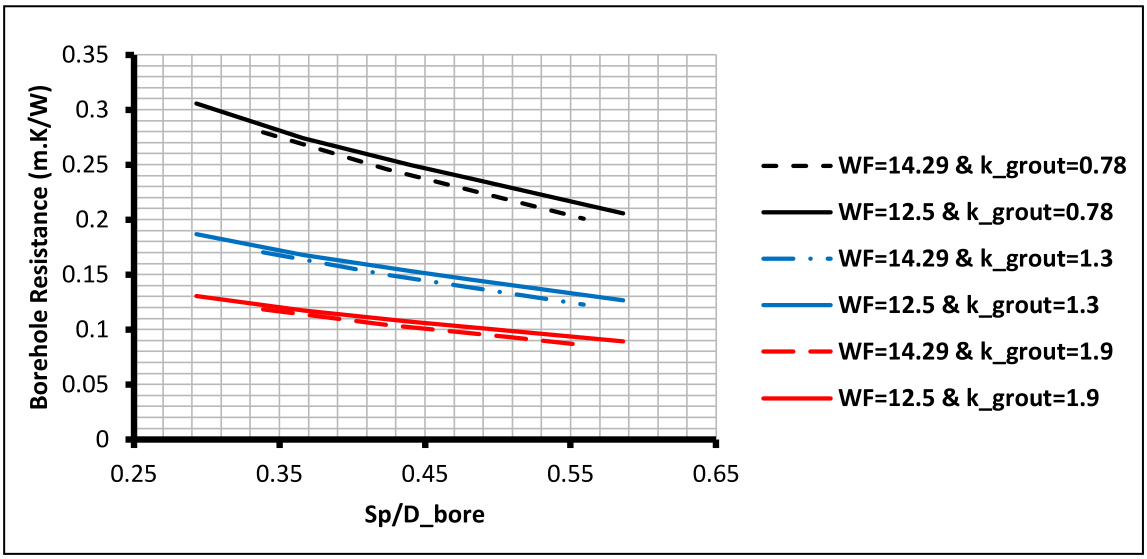

Figure 5: A comparison for the variation of borehole thermal resistance with (ζ) at different grout thermal conductivity

Grout Thermal Conductivity

The thermal resistance showed a reduction with thermal conductivity increase regardless of the (ζ) ratio value and exhibited the lower at the higher test grout thermal conductivity. The trend of the data may be presented in a linear formula as:

$$R_B = a_0 + a_1 \left(\frac{S_p}{D_B}\right) \tag{34}$$

The coefficient (a_0) is a negative value. These results were also confirmed by Koenig [16] and Gu and O'Neal [15] work. The borehole resistance revealed a declination with the geometry factor (ζ) increase. Increasing of the geometry factor refers to the increase of the U-tube spacing (S_p) of the tube legs. Hence, the thermal resistance decreases as the distance of the two tubes increases. The lower grout thermal conductivity of (0.78) W/m.K showed a higher value for (R_B) and exhibited a steeper gradient with the geometry factor (S_p/D_B). As the grout thermal conductivity increases, it produces a lower (R_B) level and flatter curve representation. The latter is mainly due to the decrease of the temperature gradient with (k_g) increase and hence improves the heat transfer inside the borehole body.

Borehole Thermal Resistance

Figure 6 depicts the comparison of the borehole thermal resistance of various models for two configurations.

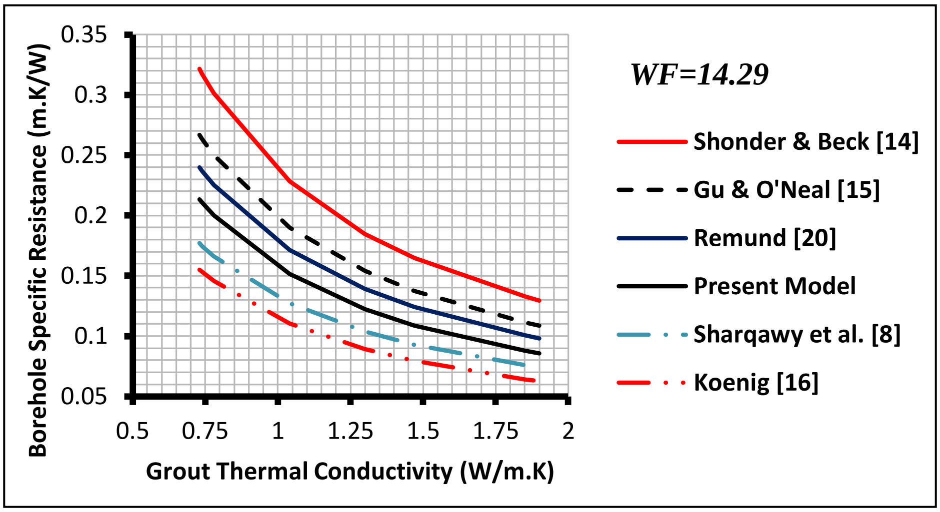

Figure 6.a: Borehole thermal resistance at WF=14.29

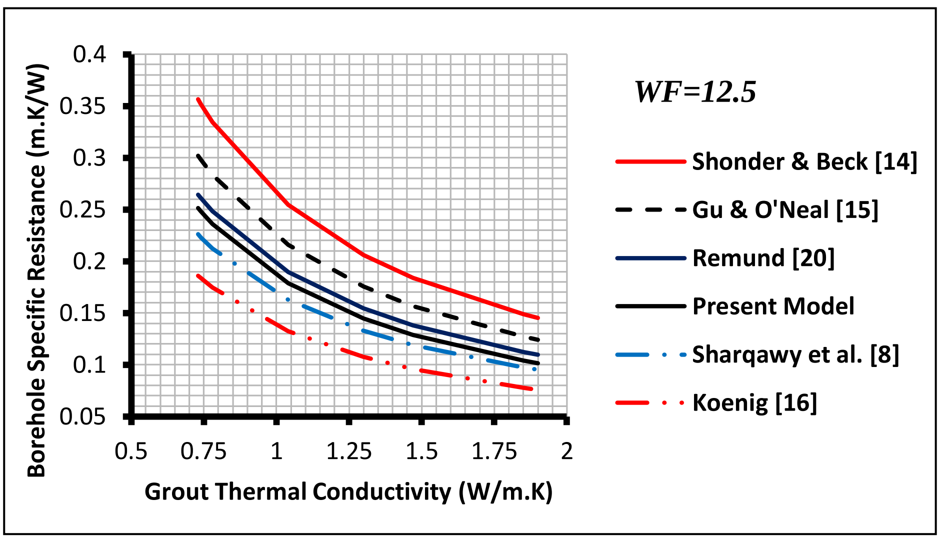

Figure 6.b: Borehole thermal resistance at WF=12.5

Figure 6: Comparison of different borehole thermal resistance expressions for the test single loop geometries at ($S_p/d_o=3.3$)

The present model showed a similar trend of data to those of other investigators, and it is located closer to that of Remund [20] and Sharqawy et al. [8] ones for both tested configurations. Koenig [16] expression revealed the lowest resistance value and Shonder and Beck [14] showed the highest values among other correlations; it was about double of that of the earlier investigator. Shonder and Beck [14] have replaced the two legs of the U-tube by an equivalent tube diameter concentric at the borehole; this procedure loses a surface area for the U-tube by about (30) % per unit length. Hence, it will exhibit higher thermal resistance, or it needs more surface area to accomplish the same heat load. The same dialogue is true for Gu and O'Neal [15] correlation because a similar technique was implemented to build up their model. The response of the present work for a single loop GHE to the variation of the geometrical configuration is present, as shown above. It is also predicted reasonable values for the borehole thermal resistance when compared with other available correlations.

U-Tube Size

A comparison of the predicted borehole thermal resistance by the present model for the single loop of the four geometries is illustrated in Figure 7. The thermal resistance showed a decrease as the U-tube diameter increases, and it is also showed a declination as the grout thermal conductivity increases.

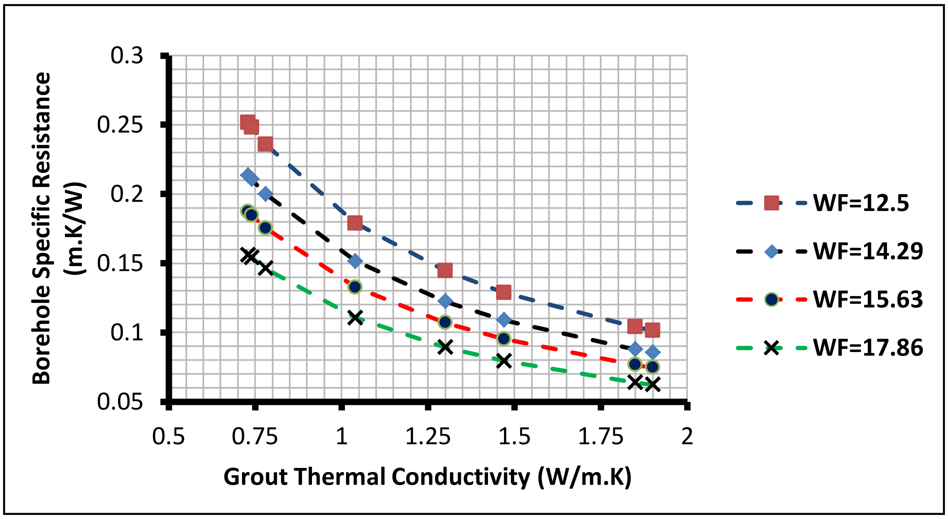

Figure 7: A comparison for the borehole thermal resistance of different configurations at ($S_p / d_o = 3.3$)

The bigger tube size with WF of (17.86) possesses the lowest borehole thermal resistance, among other tested sizes. Whereas, the smaller tube size having a WF of (12.5) revealed the highest thermal resistance. This is related directly to the thickness of the grout layer, which covers these tubes. The bigger thickness reveals higher thermal resistance and vice versa. Also, the bigger tube size possesses a larger surface area per unit length and hence increases the heat transfer rate in the borehole. The smaller tube size of (9.52) mm outside diameter showed a higher thermal resistance by (66.7) % and (39) % than those of (19.05) mm diameter at grout thermal conductivity of (0.73) W/m.K and (1.9) W/m.K respectively. The other tested U-tube sizes occupied the zone bounded by these two tube configurations. This phenomenon was also confirmed by other investigators presented in this work.

U-Tube Legs Spacing

The center to center U-tube legs spacing is of vital importance in the process of heat transfer inside the borehole and U-tube configuration system. It has been found that the ground heat exchanger depth is proportional to the total borehole thermal resistance and hence the spacing of its tube legs [15, 16, and 20]. The present model was investigated for the verification of the ratio of tube spacing to the borehole diameter (S_p / D_B). Figure 8 illustrates a comparison for the borehole thermal resistance obtained at different values of the ratio of (S_p / D_B) in the range of (0.29) and (0.59) for both geometries.

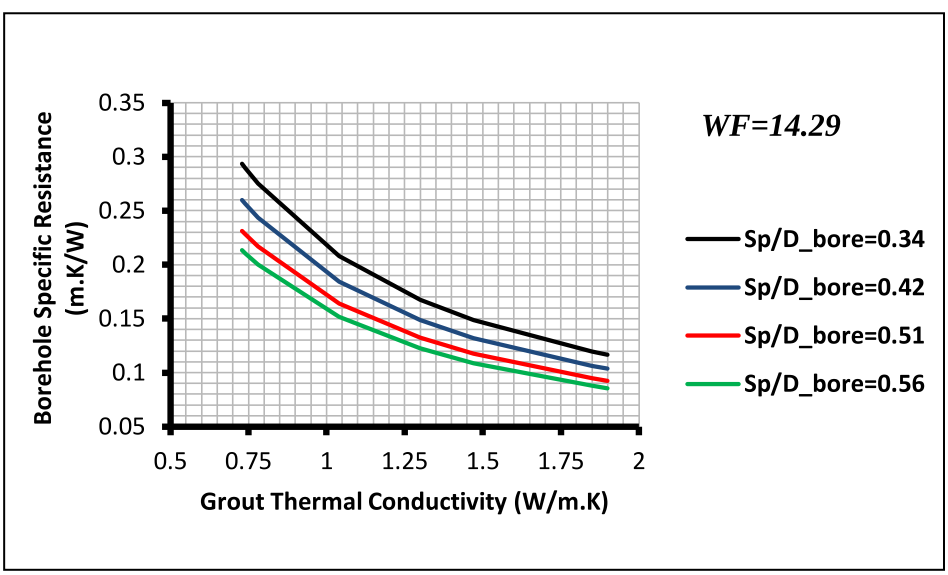

Figure 8.a: A comparison at WF=14.29

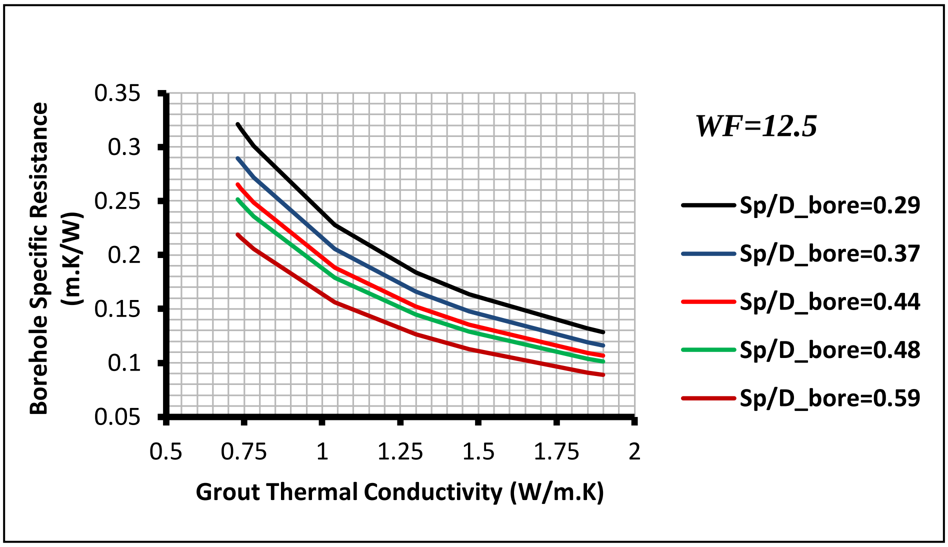

Figure 8.b: A comparison at WF=12.5

Figure 8: A comparison for the present model prediction of borehole thermal resistance for different configurations at various (S_p / D_B)

As the geometry factor (ζ) increases, the spacing between the U-tube legs is also increasing and hence revealed a lower borehole thermal resistance. Similar behavior is noticed for both geometries regardless of the U-tube borehole configurations. These results revealed consistency with other investigators who have studied this factor, Gu and O'Neal [15], Koenig [16], Garbai and Méhes [21], and Remund [20]. It is clear that the borehole thermal resistance of a GHE is a function of the ratio (S_p / D_B) and grout thermal conductivity, Figure 8. The present work showed that when increasing (S_p /d_o) ratio from (2) to (3.3) for both tube wall factors showed a decrease in the borehole thermal resistance. The predicted borehole thermal resistance at (S_p /d_o) of (2) was higher than that of (3.3) by the range of (22-54) % and (26.5-28) % at wall factor of (12.5) and (14.29) respectively for the examined range of grout thermal conductivity.

Total Borehole Thermal resistance

The total thermal resistance between the fluid and soil region is compared for various correlations is illustrated in Figure 9.

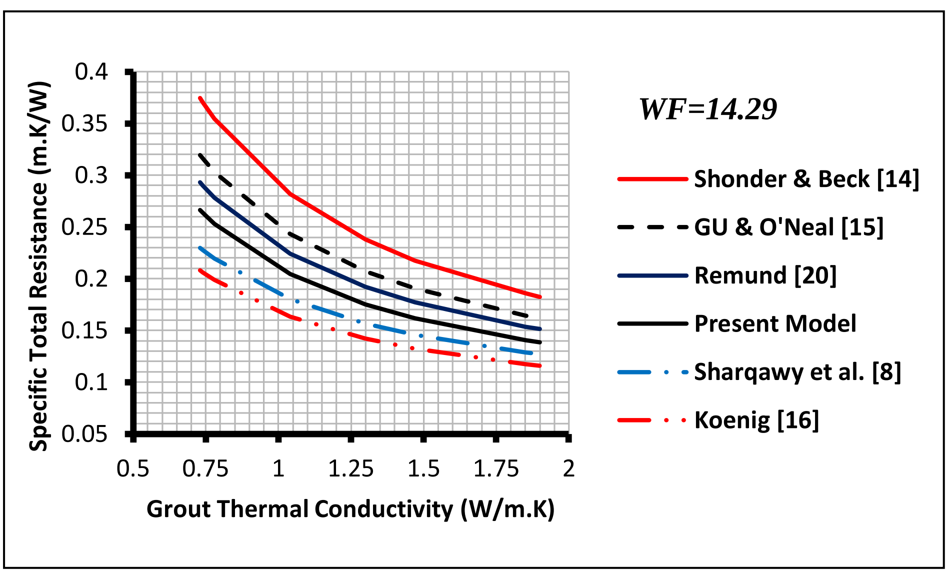

Figure 9.a: A comparison at WF=14.29

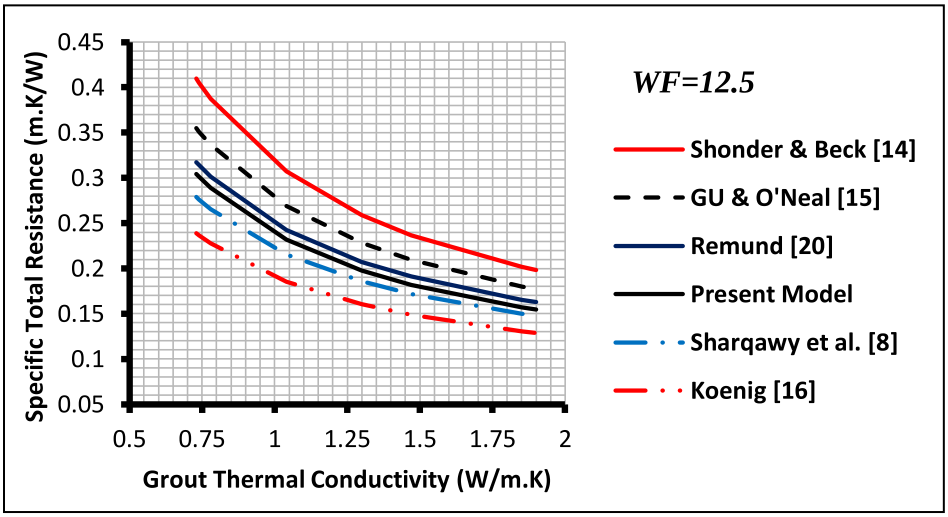

Figure 9.b: A comparison at WF=12.5

Figure 9: Comparison of different total borehole thermal resistance expressions for the test single loop geometries at (S_p / d_o=3.3)

The same trend can be inferred from all of the examined correlations for both borehole configurations. The trend of the curves and predicted values are similar to those of borehole thermal resistance in their distributions for all of the correlations presented in this graph. The Shonder and Beck [14] correlation showed the highest level among other models, and the Koenig [16] model revealed the lowest. As the grout thermal conductivity increases, the curves are getting closer to each other due to the decrease in the temperature gradient and reduced borehole thermal resistance. The present model predicted a moderate values and are closer to Remund [20] predictions than other test correlations.

Heat Loading

The comparison of predict heat loading per unit length of the U-tube heat exchanger is presented in figure 10 for both investigated configuration.

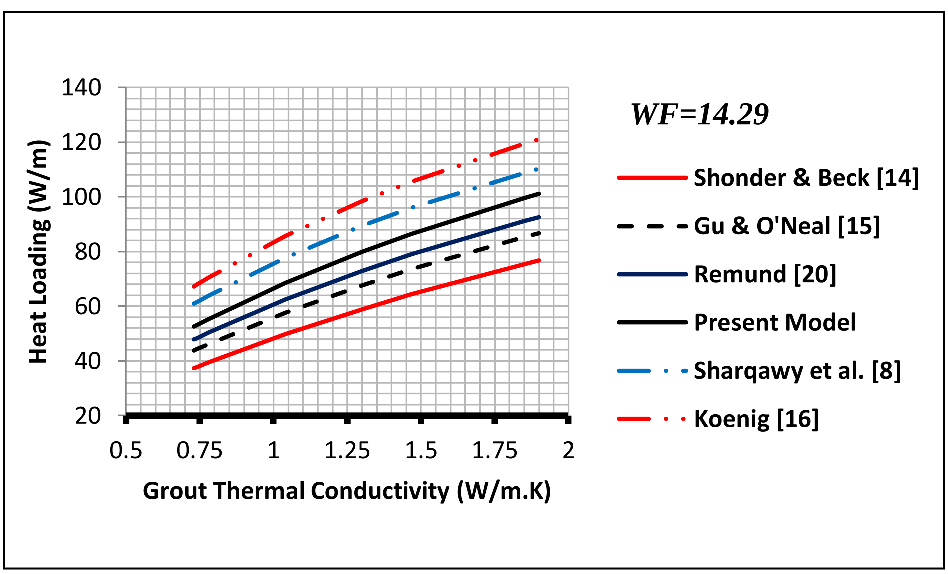

Figure 10.a: A comparison at WF=14.29

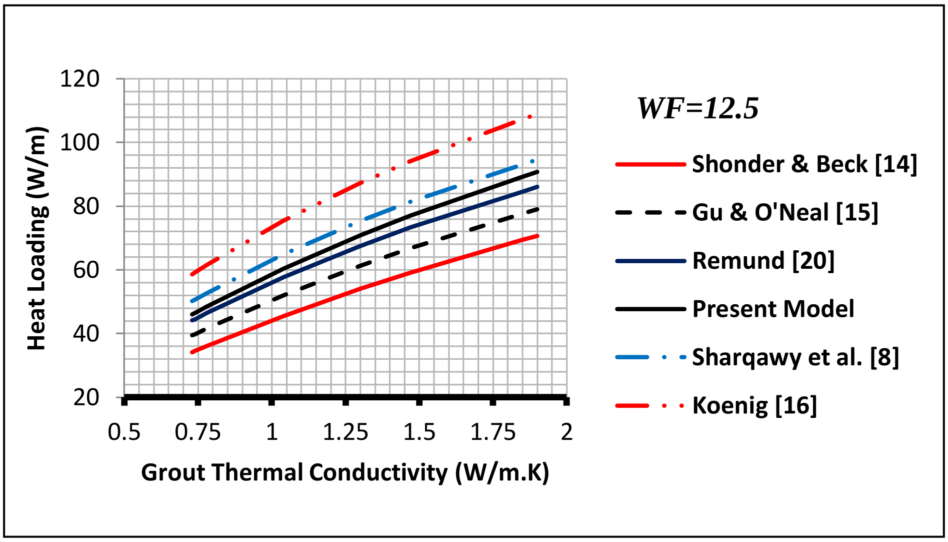

Figure 10.b: A comparison at WF=12.5

Figure 10: Comparison of different borehole heat loading expressions for the test single loop geometries at (S_p / d_o=3.3)

Although the underground temperature is almost stable over the year-round it is also a dependent measure on the altitude. The temperature in the ground below (6) m is roughly equal to the mean annular air temperature at that altitude. It is at the range of (10-16) °C. Seasonal variation decreases with depth and disappears below (7 to 12) m, Tarrad [25]. Hence to construct the objectives of performing the heat loading calculation, a temperature difference between the refrigerant and ground of (14) °C was utilized. It was calculated from the conduction heat transfer relation in the form:

$$q = \frac{\Delta T_m}{R_t} \tag{35}$$

The predicted specific heat loading for the bigger tube size (12.7) mm outside diameter was higher than that of the smaller tube size (9.52) mm for all of the examined correlations and had similar data trends. Koenig [16] model predicted the higher heat loading of the GHE; it was in the range of (68-120) and (60-110) W/m for WF values of (14.29) and (12.5) respectively. Shonder and Beck [14] correlation produced the lower level for both geometries; the numerical values were in the range of (38-77) W/m and (35-70) W/m at WF of (14.29) and (12.5) respectively. The present model revealed a moderate heat loading, among other investigated correlations, regardless of the U-tube size. The respective numerical values at WF of (14.29) and (12.5) were (53-101) W/m and (47-90) W/m. These values of heat loading are almost occupying the midway zone between Koenig [16] and Shonder and Beck [14] correlations. Sharqawy et al. [8] and Remund [20] predicted heat loadings higher and lower than those of the present work respectively.

Koenig [16] model predicts the lowest borehole depth, whereas the Shonder and Beck [14] correlation produces the deepest borehole. This is because the predicted total thermal resistance of the borehole/ground for Shonder and Beck was higher than those of Koenig [16] and other models, including the present work, as illustrated in Figure 9. The correlations presented by Shonder and Beck [14] and that of Remund [20] have no response to the ratio (S_p / d_o). Hence they are expected to predict constant values of the depth regardless of the tube spacing for a given configuration. The present model showed an interaction response to the tube spacing variation as those of Koenig [16], Sharqawy et al. [8], and Gu and O'Neal [15] models. Further, the present model predicted almost a mean value for those of Shonder and Beck [14] and Koenig [16].

V. Conclusion

An analytical model was performed to formulate a thermal analysis for a single U-tube ground source coupled heat pump. An obstruction factor to heat transfer in the borehole configuration was addressed and implemented in the present model. Consistence of the borehole thermal resistance behavior with different geometry factors existed with previously published work in the open literature. The borehole thermal resistance of the U-tube showed a decrease with tube diameter and grout thermal conductivity increase. The borehole thermal resistance decreases with the geometry factor (ζ) increase and approaching a minimum as the two legs of the U-tube are located close to the borehole surface. Increasing of (S_p / d_o) ratio from (2) to (3.3) for both tube wall factors showed a decrease in the borehole thermal resistance. The predicted borehole thermal resistance at (S_p / d_o) of (2) was higher than that of (3.3) by the range of (22-54) % and (26.5-28) % at wall factor of (12.5) and (14.29) respectively for the examined range of grout thermal conductivity. The results showed that the predicted heat loading of the heat exchanger at (S_p / d_o) of (3.3) and (ΔT_m) of (14) °C, was ranged between (46-53) W/m and (91-101) W/m as predicted at (0.73) W/m.K and (1.9) W/m.K grout thermal conductivity respectively. The model could be improved to allow the implementation of different tube sizes for the U-tube legs, which is the usual case for DX ground condensers and evaporators.

Nomenclature

Parameter	Definition
a_0	Coefficient in eq. (34)
a_1	Coefficient in eq. (34)
A	Tube area, m^2
COP	Coefficient of performance
d	Tube diameter, m
D	Borehole diameter, m
G	Mass flux density, kg/m^2 s
GHE	Ground heat exchanger
h	Convection heat transfer coefficient, W/m^2 K
k	Thermal conductivity, W/m.K
L	Length, m
l_p	Tube offset length, m
$\dot{m}$	Mass flow rate, kg/s
q	Heat transfer rate per unit length, W/m
$\dot{Q}$	Heat transfer rate, W
R	Thermal resistance per unit length, m.K/W
S	Geometry shape factor, m
S_p	Tube legs spacing, m
t	Thickness, m
T	Fluid or wall temperature, K
ΔT	Temperature difference, K
V	Fluid velocity, m/s
WF	Wall factor=d_o/t

Subscribes

a	Tube (a)
b	Tube (b)
B	Borehole
c	Cross sectional
cond	Condenser
e	Equivalent
env	Envelop
f	Filling
g	Grout
i	Inside
m	Mean
max	Maximum value
o	Outside
off	Offset tube value
p	Pipe
ref	Refrigerant
s	Shunt
t	Total

Greek Letters

α	Thermal beam diverging angle
β	Equivalency coefficient
β_0	Coefficient in eq. (33)
β_1	Index in eq. (33)
ε	Parameter defined in eq. (16.a)
ζ	Geometry factor
ρ	Refrigerant density, kg/m^3
σ	Obstruction factor defined in eq. (21.b)

Acknowledgement

The author sincerely thanks the administration of (PAUSE) program and the Laboratory of Energy and, Theoretical and Applied Mechanics (LEMTA) of the University of Lorraine for their valuable support and giving the opportunity to complete the publication of the presented article.

Funding

This is a noncommercial research. It is based on the author's own initiative and scientific interests.

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