Improvement of Forecasting Method of Recession Characteristics of River Flow Rate into a Dam by using Estimation of Steady State

By Tomonari Kawai, Katsuhiro Ichiyanagi, Takuo Koyasu, Kazuto Yukita & Yasuyuki Goto
Aichi Institute of Technology

Abstract- This paper describes an application of neural networks for forecasting the flow rate upper district of dams for hydropower plants. The forecasting of recession characteristics of the river flow after rainfalls is important with respect to system operation and dam management. We present a method for improving the precision of forecasting flow rate upper district of dams by utilizing steady-state estimation and recession time constant of the river flow. A case study was carried out on the upper district of the Yahagi River in Central Japan. It is found from our investigations that the forecasting accuracy is improved to 18.6% from 25.8% with a forecasted error of the total amount of river flow by using steady-state estimation.

Keywords: river flow rate, recession time constant, estimation, forecasting, steady state of river flow, neural network.

GJRE-F Classification: FOR Code: 090699
Improvement of Forecasting Method of Recession Characteristics of River Flow Rate into a Dam by using Estimation of Steady State

Tomonari Kawai, Katsuhiro Ichianagi, Takuo Koyasu, Kazuto Yukita & Yasuyuki Goto

Abstract: This paper describes an application of neural networks for forecasting the flow rate upper district of dams for hydropower plants. The forecasting of recession characteristics of the river flow after rainfalls is important with respect to system operation and dam management. We present a method for improving the precision of forecasting flow rate upper district of dams by utilizing steady-state estimation and recession time constant of the river flow. A case study was carried out on the upper district of the Yahagi River in Central Japan. It is found from our investigations that the forecasting accuracy is improved to 18.6% from 25.8% with a forecasted error of the total amount of river flow by using steady-state estimation.

Keywords: river flow rate, recession time constant, estimation, forecasting, steady state of river flow, neural network.

I. INTRODUCTION

Recently, natural energy is paid to attention because environmental problems such as global warming and acid rain become remarkable. In such situation, it is necessary that the hydro-energy stored in water reservoirs is converted into electric energy as effectively as possible in hydropower plants [1].

On the other hand, it is important to accurately grasp the inflow to the dam due to rainfall from the viewpoint of safety in the downstream area and efficient operation of the reservoir [2]. So far, “unit-hydrograph” [3], [4], “tank model method”[5],[6], “storage function method” [7], “Kalman filter method”[8], etc. have been used for river flow forecasting. These have been used for dam discharge control and power supply operation [2], [9]. However, in these forecasting methods, the flooding phenomenon due to rainfall and snowmelt is expressed using various mathematical models, but it is difficult to determine the parameters of the model [10].

Until now, we have developed a practical forecasting method of time series of river flow rate following rainfall upstream of a dam. The method is based on the artificial neural network theory [4] [5].

It is important that the water level on dam operation and management after peak rainfalls is forecasted. This paper describes an application of neural network for estimation of the recession time constant of river flow rate into a dam after the rainfall. We proposed the forecasting system of recession characteristics of river flow rate by using estimated recession time constant. An estimation system of recession time constant composed by neural network is developed through a case study on a dam for hydropower plant located the upper district of the Yahagi River in Central Japan. The estimation possibility of recession time constant and forecasting possibility of water level of dam is discussed.

II. STEADY STATE OF RIVER FLOW AND RECESSON TIME CONSTANT

a) Basin used as a Case Study

In order to confirm the forecasting of the river flow rate, we used the upper district of the Yahagi River in Central Japan as shown in Figure 1. The basin is 505 km2 area and gradually elevated from west to east. There are five rain gauges as shown by A to F in Figure 1.

![Figure 1: Basin used as a case study (Upper district of Yahagi Dam in Central Japan)](image)

b) Recession Characteristics of River Flow Rate

A method of expressing the diminishing part of the flow rate by a mathematical model has been introduced in many documents for a long time [11]-[16]. In this paper, regarding the flow rate prediction after the
rainfall stops, we propose a forecasting method of recession characteristics of river flow rate into a dam by using the information obtained at the peak flow time.

When the rainfall stops and then the light or no rainfall continues, the flow rate gradually decreases from the peak value. In this paper, the characteristics during the recession period after the peak flow rate are expressed by the following equation using the recession time constant TRTC and the steady flow rate q_{fin} (see Figure 2).

$$ q(t) = (q_p - q_{\infty}) e^{-t/\text{RTC}} + q_{\infty} \quad (1) $$

Figure 2: Algorism of recession time constant

c) Steady States of River Flow Rate

The runoff of rainfall to the river is mainly composed of three components. The “Surface runoff” appears on the surface of the earth and the “Intermediate runoff” permeates shallowly and then flows out with a little delay. The “Groundwater runoff” gradually becomes groundwater and flows out[17]. In this paper, we study a method for predicting the time-dependent change in discharge for both surface and intermediate flow components that flow into a river in a relatively short period after rainfall.

In order to roughly understand the runoff component of the discharge in the basin used as a case study shown the Figure 1, the decreasing part after the peak of the discharge is plotted by semi-logarithm and shown in Figure 3. From the figure, the sudden change in slope can be confirmed around 12 hours (shown by term A: Surface runoff), around 48 hours (shown by term B: Intermediate runoff) and over 48 hours (shown by C: Groundwater runoff) in the recession characteristics.

In this paper, based on the analysis results of the runoff components in the recession term of the flow rate in the case study, it is assumed that rainfall in the basin flows directly to the river in about 48 hours, and the rest flows out as the groundwater for a long time. Therefore, it is also assumed that the steady flow rate is the flow rate after 48 hours of flow peak. Since the time until the steady flow rate is reached is a value that is peculiar to the basin, it should be determined by performing a river flow component analysis for each basin.

Figure 3: Analysis of outflow components in recession term of river flow rate

d) Calculated Results of Recession Time Constant and Steady States of River Flow Rate

As a pre-processing for forecasting the recession characteristics in flow rate, the recession time constant T_{RTC} and steady flow rate q_{fin} were simultaneously estimated from the past rainfall/flow data and used as the actual values of steady flow rate and recession time constant. In order to obtain the solutions of unknown parameters, T_{RTC} and q_{fin}, these may converge to values different from the actual values depending on the initial value given. Therefore, instead of q_{fin}, q_{BASE} (base flow rate) is given, the T_{RTC} is calculated using equations (2) and (3). Furthermore, to find the q_{fin}, the obtained the T_{RTC} value is given to equations (4) and (5). The mutual substitution and solution are repeated for the T_{RTC} and the q_{fin}, and finally a stable solution is obtained (the details are as shown by the flow chart in the Appendix).

$$ T_{RTC} = \frac{\sum_{i=1}^{n} t_i^2}{\sum_{i=1}^{n} \{ t_i \times z(t_i) \}} \quad (2) $$

$$ z(t_i) = \log \left( \frac{q(t_i) - q_{\infty}}{q_p - q_{\infty}} \right) \quad (3) $$

$$ q_{fin} = \frac{- \sum_{i=1}^{n} q(t_i) - q_p \cdot x_i}{\sum_{i=1}^{n} x_i - 1} \quad (4) $$

$$ x_i = e^{-\frac{t_i}{T_{RTC}}} \quad (5) $$
In equations (2) to (5), \( i = 1 \) is the peak time of the flow rate, and \( i = n \) is the time when the flow rate is reached at the steady states value after the flow peak. From the analysis result of outflow components in recession term of river flow rate as shown in Figure 3, \( n = 48 \) is used. Furthermore, it was assumed that there was no preceding rainfall within 48 hours before the start of rainfall, and the rainfall during the recession period of 48 hours after the peak of discharge was less than 30 mm. The base flow is the river flow rate at the start of a series of rainfall (beginning of rainfall), and in this paper, it was the average value of river flow for five hours before the start of rainfall.

In order to confirm the forecasting of the river flow rate, we used the upper district of the Yahagi River in Central Japan as shown in Figure 1. Therefore, we used 26 cases of rainfall from 2003 to 2008 with the peak flow rate of 100 \( \text{m}^3/\text{s} \) or more and a cumulative rainfall value of less than 30 mm after the river flow peak. Furthermore, using the time-series data at the recession period of flow rate after the peaks and the equations from (2) to (5), the steady flow rate \( q_{\text{fin}} \) and the recession time constant \( T_{\text{RTC}} \) are used as unknown parameters to calculate the equation (1). It was calculated by the method of least squares. In the following, the estimated values of the recession time constant \( T_{\text{RTC}} \) and steady state river flow rate \( q_{\text{fin}} \), obtained by using equations from (2) to (5) are used as the observed values, respectively.

e) Forecasting System of River Flow Rate at Recession period

The flow rate after the rainfall stops is expressed by equation (1) using the recession time constant \( T_{\text{RTC}} \) and steady state value of river flow rate \( q_{\text{fin}} \). Then, we propose a method of estimating \( T_{\text{RTC}} \) and \( q_{\text{fin}} \) by giving various quantities obtained at the peak time as input information of the neural network, and a method of forecasting the recession characteristics of the flow rate. As shown in Figure 4, a system is constructed to forecast the flow rate from both the \( T_{\text{RTC}} \) and \( q_{\text{fin}} \) estimated values and the peak flow rate. Both the \( T_{\text{RTC}} \) and \( q_{\text{fin}} \) estimation methods, and forecasting system of the recession characteristics of river flow rate using these estimated values are described in the following chapters as case study.

III. Estimation of Steady State of River Flow and Recession Time Constant

We have constructed a estimating system for the steady flow rate and recession time constant using a neural network with various features. Actually, the verification result of the proposed method is described below for the upper district of the Yahagi River in a case study.

![Figure 4: Forecasting system of recession characteristics of river flow rate](image)

![Figure 5: Correlation between various quantities at flow rate peak and steady flow rate](image)
a) Steady State Values of River Flow

The steady state value of river flow rate \( q_{\text{fin}} \) shown in equation (1) is a parameter necessary for forecasting the recession characteristics of the flow rate. Therefore, the value of \( q_{\text{fin}} \) is estimated at the peak of the flow rate.

i. Correlation between \( q_{\text{fin}} \) and various quantities

In order to select the optimum input information for the estimation of \( q_{\text{fin}} \), we investigated the correlation between \( q_{\text{fin}} \) and various quantities obtained at the peak flow rate. Among the results, the peak flow rate, the accumulated rainfall up to the peak, and the correlation between the base flow and the steady state value after the peak are shown in (a) to (c) of Figure 5, respectively. (The value of the correlation coefficient is shown in each figure).

According to these figures, it can be confirmed that although the correlation coefficient \( R = 0.66 \) to 0.39, there are many variations, but the steady flow rate tends to show a large value when there are many quantities. When the correlation with other quantities was investigated, it was confirmed that the correlation with the steady state flow rate was lower than that of the quantities shown in Figure 5.

ii. Estimating System of Steady State Value of River Flow

The top three information (base flow \( R = 0.66 \), peak flow \( R = 0.49 \), total amount of rainfall \( R = 0.39 \)) in descending order of the value of the correlation coefficient between the steady state of river flow rate and various quantities are taken up. An estimation system of steady state value of river flow \( q_{\text{eval}} \) is constructed using these as input information is shown in Figure 6. The system consists of a three-layer and simple hierarchical neural network, which gives "peak flow", "total amount of rainfall" and "base flow" as corresponding to the input layer, and "steady state value of river flow" to the output layer. As for the middle layer, as in Figure 6, we selected three units that give a small estimation error at the early learning stage of the neural network. The back propagation method \( [18] \) was used for learning the estimation system. For the hidden layer, we selected three units that gave a small estimation error at the early stage of learning when learning the neural network.

iii. Estimated Results of Steady State Value of River Flow

The estimation system was trained using 16 cases (No. 1 to 16) in Table 1, and the steady state of river flow rate was estimated by the rainfall of the remaining 10 cases (No. 17 to 26). As a result, the error of estimated value was calculated by equation (6) and summarized by the "Estimated-value-1" in Table-2.

\[
\text{Error of estimated value} = \left( \frac{\text{Estimated value} - \text{Observed value}}{\text{Observed value}} \right) \times 100 \ [\%] \quad (6)
\]

![Figure 6: Estimating system for steady states of river flow rate](image)

![Table 1: Observed data of rainfall and flow rate used for studying](table)

![Table 2: Estimated results of steady states of flow rate](table)
In addition, the results of using the base flow rate instead of the steady state of river flow rate are also shown by the “Estimated-value-2” in the same Table-2. From the table, the estimation error of the steady state of river flow rate decreases from 47.0% to 17.1%. From this result, it is expected that the forecasted accuracy of river flow rate will be improved significantly by using the estimated value of steady flow rate.

b) Recession time constant

As in the previous section, the recession time constant $T_{RTC}$ shown in equation (1) is taken as a parameter necessary for forecasting the recession characteristics of the river flow rate, and the value of $T_{RTC}$ is estimated at the peak of the flow rate. In order to select the optimum input information for $T_{RTC}$ estimation, we investigate the correlation between various quantities obtained at the peak flow rate and $T_{RTC}$, and propose an estimation system of recession time constant.

i. Correlation between $T_{RTC}$ and various quantities

In estimating the recession time constant, we investigated the correlation with various quantities obtained at the peak flow rate. Of the results, the correlations with peak flow rate discharge, total rainfall, and rainfall intensity are investigated and shown in (a) to (c) in Figure 7 respectively (The value of the correlation coefficient is shown by the caption $R$ in each figure). According to these figures, although there are many variations in the correlation coefficient $R = -0.56$ to -0.38, it can be confirmed that the decreasing time constant tends to show a small value when there are many quantities. When the correlation with other quantities, such as the duration of rainfall to the peak, was also examined, it was confirmed that the correlation with the decreasing time constant was lower than the quantities shown in Figure 7.

ii. Estimating System of Recession Time Constant

The top three information (rainfall intensity $R = -0.56$ up to the peak flow rate, peak flow rate $R = -0.49$, cumulative rainfall up to the peak $R = -0.38$) in descending order of the absolute value of the correlation coefficient between the recession time constant and various quantities are taken up. An estimation system of recession time constant $T_{RTC}$ is constructed using these as input information is shown in Figure 8. The system consists of a three-layer and simple hierarchical neural network, which gives "peak flow", "total amount of rainfall" and "rainfall intensity" are corresponded to the input layer, and "recession time constant" to the output layer. As for the middle layer, as in Figure 6, we selected three units that give a small estimation error at the early learning stage of the neural network.

iii. Estimated Results of Recession Time Constant of River Flow

The estimation system was trained using 16 cases (No. 1 to 16) in Table 1, and the recession time constant of river flow rate was estimated by the rainfall of the remaining 10 cases (No. 17 to 26). As a result, the estimation error is summarized in Table-3. The estimation error of the recession time constant is represented by the subtraction (hours) from the actual value, which is reduced from 8.2h to 1.4h.
IV. Forecasting River Flow Rate for Recession Term

The river flow after the peak flow rate was forecasted based on the estimated results of the steady-state of flow rate and the recession time constant obtained up to the previous section. Based on the forecasting system shown in Figure 4, the time variation of flow rate was calculated using the steady-state ones $q_{\text{fin}}$ in Table 2, the recession time constant $T_{\text{RTC}}$ in Table 3, and the peak flow rate $q(t_1)$ as input information.

Table 3: Estimated results of recession time constant

<table>
<thead>
<tr>
<th>Rain No.</th>
<th>Observed $h$</th>
<th>Estimated-value-1</th>
<th>Estimated-value-2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(Neural network)</td>
<td>(Base flow)</td>
</tr>
<tr>
<td>17</td>
<td>6.3</td>
<td>6.2</td>
<td>6.7</td>
</tr>
<tr>
<td>18</td>
<td>9.3</td>
<td>6.7</td>
<td>17.3</td>
</tr>
<tr>
<td>19</td>
<td>4.3</td>
<td>8.0</td>
<td>13.4</td>
</tr>
<tr>
<td>20</td>
<td>5.1</td>
<td>6.8</td>
<td>16.8</td>
</tr>
<tr>
<td>21</td>
<td>4.1</td>
<td>5.0</td>
<td>13.0</td>
</tr>
<tr>
<td>22</td>
<td>3.5</td>
<td>4.0</td>
<td>11.6</td>
</tr>
<tr>
<td>23</td>
<td>6.7</td>
<td>4.2</td>
<td>13.1</td>
</tr>
<tr>
<td>24</td>
<td>3.3</td>
<td>3.4</td>
<td>15.3</td>
</tr>
<tr>
<td>25</td>
<td>3.8</td>
<td>3.9</td>
<td>13.8</td>
</tr>
<tr>
<td>26</td>
<td>3.6</td>
<td>5.9</td>
<td>11.2</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>8.2</td>
</tr>
</tbody>
</table>

| Estimated-value-1: Estimated values of steady states of river flow rate by using neural network |
| Estimated-value-2: Estimated values of steady states of river flow rate by using base flow instead of steady flow rate |

Table 4: Forecasted results for the recession term after the peak of the flow rate

<table>
<thead>
<tr>
<th>Rain No.</th>
<th>Observed $x 10^6$ m$^3$</th>
<th>Estimated-value-1 (Neural network) $x 10^6$ m$^3$</th>
<th>Error %</th>
<th>Estimated-value-2 (Base flow) $x 10^6$ m$^3$</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>7.6</td>
<td>8.3</td>
<td>9.2</td>
<td>5.9</td>
<td>-22.4</td>
</tr>
<tr>
<td>18</td>
<td>9.4</td>
<td>8.6</td>
<td>-8.5</td>
<td>10.4</td>
<td>10.6</td>
</tr>
<tr>
<td>19</td>
<td>7.0</td>
<td>8.9</td>
<td>27.1</td>
<td>8.8</td>
<td>25.7</td>
</tr>
<tr>
<td>20</td>
<td>8.6</td>
<td>8.6</td>
<td>0.0</td>
<td>10.9</td>
<td>26.7</td>
</tr>
</tbody>
</table>

Figure 9: Forecasted result for the recession term after the peak of the flow rate (rain fall No.22)

Figure 10: Forecasted result for the recession term after the peak of the flow rate (rain fall No.23)

Among the results, Figure 9 shows an example of rainfall concentrated in a relatively short time (33 mm in 5 hours), and Figure 10 shows an example of rainfall in a relatively long time (84 mm in 16 hours). In addition, the measured value of the flow rate is indicated by a circle, and the forecasted value of the decreasing part is indicated by a broken line. The predicted values in both figures as a whole are relatively close to the actual values. In detail, there is an increase or decrease in the measured flow rate during the time when rainfall is not observed. The reason seems to be the rainfall at points other than the rainfall observation point. Table 4 shows the forecasted error seen from the total amount of runoff. The table also shows the forecasted results when the basal flow rate is used instead of the steady flow rate value.

According to the table, the forecasted error is reduced from 25.0% to 18.6% on average of the absolute values when the steady flow rate is used compared to when the base flow rate is used. This 6% reduction in prediction error corresponds to the amount of electricity used by approximately 22,800 households per day, which can improve the economic operation of thermal power generation (Appendix-2). In addition, the forecasted error is smaller when the estimated steady-state value of flow rate is used for seven of the ten cases than when the basal flow rate is used. From the above results, the effectiveness of using the estimated steady-state value of flow rate can be confirmed.
Estimated-value-1: Estimated values of steady states of river flow rate by using neural network

Estimated-value-2: Estimated values of steady states of river flow rate by using base flow instead of steady flow rate

V. Conclusions

In this paper, we proposed a method for estimating the recession time constant of the river flow rate and the steady flow value for the outflow after rainfall and improved the accuracy of the flow forecasting. We confirmed the effectiveness of the forecasted method for the upper section of the Yahagi River in central Japan. The features of the proposed method are as follows.

1. Regarding the prediction of the recession characteristics of the river flow rate into a dam, the steady state flow rate \( q_{\text{inf}} \) is taken up as a necessary parameter. The correlation between the various quantities obtained at the peak flow rate and \( q_{\text{inf}} \) was investigated. As a result, although there were many variations in peak flow rate, total rainfall up to the peak, and base flow rate, it was confirmed that the larger the various quantities, the larger the steady-state flow rate.

2. We proposed a estimation system of the steady-state value of the flow rate by giving "total rainfall up to the flow rate peak", "peak flow rate" and "base flow rate" to the input layer and "steady-state value of flow rate" to the output layer. According to the simulation results of the steady-state estimation of river flow rate, the estimation error was significantly reduced from 47.0% to 17.1% compared with the case where the base flow rate was used as the estimated value instead of the steady-state flow rate.

3. The recession time constant \( T_{\text{RTC}} \) is taken up as a parameter necessary for predicting the recession characteristic of the flow rate. Although there were many variations in the peak flow rate obtained at the peak flow rate, the cumulative rainfall up to the peak, and the rainfall intensity, it was confirmed that the larger the various quantities, the smaller the recession time constant.

4. We proposed a estimation system of the recession time constant of the flow rate by giving "peak flow rate", "total rainfall up to the flow rate peak" and "rainfall intensity up to the flow rate peak" to the input layer and "recession time constant" to the output layer. The simulation error of the recession time constant of river flow rate was significantly reduced from 47.0% to 17.1%.

5. Based on the estimation results of the steady flow rate and the recession time constant, we proposed a forecasting system of the flow rate after the peak flow. The error of the forecasting result using both estimated values of the steady states and the recession time constant reduced from 25.0% to 18.6%. A reduction of about 6% in the forecasting error corresponds to the daily power consumption by about 22,800 households, which can improve the economic operation of thermal power generation.

The proposed forecasting method of the recession characteristic can be applied when the rainfall after the forecasting point (peak flow rate point) is about 30 mm or less and can be ignored. If there is rainfall again after the flow rate peak, it is considered that the same prediction can be applied with the re-occurred flow rate peak point as the new flow rate peak point.

In the future, we would like to investigate the effect of rainfall after the forecasting point on the flow rate forecasted result, verify the forecasting method when the rainfall is 30 mm or more after the peak flow rate occurs. And using the forecasted rainfall after the peak we would like to study the forecasting method of the flow rate in the recession time. Furthermore, we will confirm the versatility by applying the proposed method to other rivers.

Acknowledgments

We would like to thank our gratitude for using the Hydrological and Water Quality Database of the Ministry of Land, Infrastructure, Transport and Tourism for the rainfall and river flow data used in the analysis.

Appendix

A-1 Estimating formula for steady flow rate and recession time constant

In this paper, the runoff characteristics to the river during the recession period after the peak flow rate are expressed by the following equation.

\[
q(t_i) = (q_p - q_{\text{fin}}) e^{-\frac{t_i}{T_{\text{RTC}}}} + q_{\text{fin}}
\]

where, \( T_{\text{RTC}} \) is the recession time constant, \( q_{\text{fin}} \) is the steady flow rate value, \( q(t_i) \) is the flow rate at time \( t_i \), \( q_p \) is the flow rate at the peak flow rate.

For the estimation of \( T_{\text{RTC}} \) and \( q_{\text{inf}} \) given \( q_{\text{inf}} = q_{\text{BASE}} \) (base flow rate), \( T_{\text{RTC}} \) is estimated by equation (A-1). Nextly, the obtained \( T_{\text{RTC}} \) is given to Eq (A-1) and \( q_{\text{inf}} \) is estimated. Here after, \( T_{\text{RTC}} \) and \( q_{\text{inf}} \) are calculated mutually to find a stable solution. The derivation of each estimation formula for \( T_{\text{RTC}} \) and \( q_{\text{inf}} \) is shown below.
i. Estimating the recession time constant $T_{RTC}$

Equation (A-1) is rewritten and expressed by the following equation.

\[
\frac{e^{-t_i}}{T_{RTC}} = \frac{q(t_i) - q_{fin}}{q_p - q_{fin}} - \frac{t_i}{T_{RTC}} = \log \left( \frac{q(t_i) - q_{fin}}{q_p - q_{fin}} \right)
\]

where

\[
a(t_i) = z(t_i) \quad (A-2)
\]

The objective function $J_1$ for deriving the estimation formula of the recession time constant $T_{RTC}$ is obtained by using the least squares method from the actual data of $q(t_i)$ in the recession period of the flow rate and equation (A-2) is as follows.

\[
J_1 = \sum_{i=1}^{n} (z(t_i) - a t_i)^2 \quad (A-5)
\]

The value of the coefficient $a$ that minimizes equation (A-5) is obtained by $\frac{\partial J_1}{\partial a} = 0$.

\[
\frac{\partial J_1}{\partial a} = 2 \sum_{i=1}^{n} (z(t_i) - a t_i)(-t_i)
\]

Therefore,

\[
a = -\frac{\sum_{i=1}^{n} z(t_i)t_i}{\sum_{i=1}^{n} t_i^2}
\]

\[
a = -\frac{1}{T_{RTC}}
\]

Therefore, the estimated value of the recession time constant $T_{RTC}$ is calculated by the following equation.

\[
T_{RTC} = \frac{\sum_{i=1}^{n} t_i^2}{\sum_{i=1}^{n} z(t_i)t_i} \quad (A-6)
\]

ii. Estimating the steady state value $q_{ss}$ of flow rate

Equation (A-1) is rewritten and expressed by $x(t_i) = e^{t_i}$

\[
\frac{\partial J_1}{\partial a} = 2 \sum_{i=1}^{n} (z(t_i) - a t_i)(-t_i)
\]

\[
q(t_i) = (q_p - q_{fin})x(t_i) + q_{fin}
\]

\[
= q_p x(t_i) - q_{fin}x(t_i) - 1 \quad (A-7)
\]

The objective function $J_2$ for deriving the estimation formula of the steady flow rate $q_{ss}$ from the actual data of $q(t_i)$ in the recession period of the flow rate and equation (A-7) using the least squares method is as follows.

\[
J_2 = \sum_{i=1}^{n} (q(t_i) - (q_p x(t_i) - q_{fin}x(t_i) - 1))^2 \quad (A-8)
\]

The value of $q_{ss}$ that minimizes equation (A-8) is obtained by $\frac{\partial J_2}{\partial q_{ss}} = 0$.

\[
\frac{\partial J_2}{\partial q_{ss}} = 2 \sum_{i=1}^{n} \{q(t_i) - (q_p x(t_i) - q_{fin}x(t_i) - 1)\} x(t_i) - 1
\]

Therefore,

\[
\sum_{i=1}^{n} \{q(t_i) - (q_p x(t_i) - q_{fin}x(t_i) - 1)\} = 0
\]

\[
\sum_{i=1}^{n} \{q(t_i) - q_p x(t_i) + q_{fin}x(t_i) - 1\} = 0
\]

Accordingly, the estimated value $q_{ss}$ is calculated by the following equation.

\[
q_{ss} = \frac{-\sum_{i=1}^{n} q(t_i) - q_p x(t_i)}{\sum_{i=1}^{n} x(t_i) - 1} \quad (A-9)
\]

To obtain the estimated values $T_{RTC}$ and $q_{ss}$, for the unknown parameters $T_{RTC}$ and $q_{ss}$, give $q_{BASE}$ (base flow rate) instead of $q_{ss}$, and obtain $T_{RTC}$ from equation (A-6).

Furthermore, the obtained $T_{RTC}$ value is given to (A-9) to obtain $q_{ss}$, $T_{RTC}$ and $q_{ss}$ are calculated mutually to find a stable solution.

Figure 1 shows the calculation flow for obtaining the estimated values $T_{RTC}$ and $q_{ss}$ for $T_{RTC}$ and $q_{ss}$.

A-2 Power generation

The amount of power generated for the flow rate equivalent to a forecasted error of 1% in the upper reaches of the Yahagi River (from Table 4 in the main text, the average total flow rate per rainfall is $0.1 \times 10^6 \text{m}^3$) is estimated as follows[32].

Power generation output; $P = 9.8\eta QH$ [kW]

Power generation; $S = P \times T$ [kWh]

However, $\eta$ is the total efficiency of the turbine and the generator, $Q$ is the flow rate [$\text{m}^3\text{s}^{-1}$], $H$ is the effective head [m], and $T$ is the generator operating time [h].

In addition, referring to the various quantities of the Yahagi Dam power plant [33], flow rate $Q=234 \text{m}^3\text{s}^{-1}$, Effective head $H=163\text{m}$, Efficiency $\eta=0.86$ are used.
When the flow rate equivalent to 1% error per rainfall (0.1 × 106m³) is converted into the amount of power generated at the target power plant, it can be calculated as follows.

Power generation time

\[ T = 0.1 \times 106/234/3600 = 0.12 \text{h} \]

Power generation output

\[ P = 9.8QH\eta \]

\[ = 9.8 \times 234 \times 163 \times 0.86 = 321,461 \text{ kW} \]

Power generation

\[ S = P \times T = 321,461 \times 0.12 = 38,575 \text{ kWh} \]

If the monthly power consumption of one household is 300kWh and the daily power consumption of one household is 10kWh, 38,575kWh is equivalent to the daily power consumption of about 3800 households. Therefore, an improvement in prediction accuracy of 6% is equivalent to the daily power consumption of about 3800 households, and the economic operation of thermal power generation can be improved.

**References**