MONSDA: - A Novel Multi-Objective Non-Dominated Sorting Dragonfly Algorithm

By Pradeep Jangir

Abstract- This novel article presents the multi-objective version of the recently proposed Dragonfly Algorithm (DA) known as Non-Dominated Sorting Dragonfly Algorithm (NSDA). This proposed NSDA algorithm works in such a manner that it first collects all non-dominated Pareto optimal solutions in achieve till the evolution of last iteration limit. The best solutions are then chosen from the collection of all Pareto optimal solutions using a crowding distance mechanism based on the coverage of solutions and swarming strategy to guide dragonflies towards the dominated regions of multi-objective search spaces. For validate the efficiency and effectiveness of proposed NSDA algorithm is applied to a set of standard unconstrained, constrained and engineering design problems. The results are verified by comparing NSDA algorithm against Multi objective Colliding Bodies Optimizer (MOCBO), Multi objective Particle Swarm Optimizer (MOPSO), non-dominated sorting genetic algorithm II (NSGA-II) and Multi objective Symbiotic Organism Search (MOSOS). The results of proposed NSDA algorithm validates its efficiency in terms of Execution Time (ET) and effectiveness in terms of Generalized Distance (GD), Diversity Metric (DM) on standard unconstraint, constraint and engineering design problem in terms of high coverage and faster convergence.

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I. Introduction

Optimization is a work of achieving the best result under given limitation or constraints. Now a day, optimization is used in all the fields like construction, manufacturing, controlling, decision making, prediction etc. The final target is always to get feasible solution with minimum use of resources. In this field computers make a revolutionary impact on every field as it provides the facility of virtual testing of all parameters that are involved in a particular design with less involvement of human efforts, benefits in less time consuming, human efforts and wealth as well.

Today we use computer-aided design where a designer designs a virtual system on computer and gives only command to test all parameters involved in that design without even the need for a single prototype. A designer only to design and simulate a system and set all the parameter limitation for the computer.

Computer-aided design technique becomes more effective with the additional feature of automatic generation of solutions after it’s mathematically formulation of any system or design problem. Auto generation of solution, this feature is come into nature with the development of algorithms. In past years, real world designing problems are solved by gradient descent optimization algorithms. In gradient descent optimization algorithm, the solution of mathematically formulated problem is achieved by obtaining its derivative. This technique is suffered from local minima stagnation [1, 2] more time consuming and their solution is highly dependent on their initial solution.

The next stage of development of optimization algorithms is population based stochastic algorithms. These algorithms had number of solutions at a time so embedded with a unique feature of local minima avoidance. Later population based algorithms are developed to solve single objective at a time either it may be maximization or minimization on accordance the problems objective function. Some popular algorithms for single objective problems are Moth-Flame optimizer (MFO) [3], Bat algorithm (BA) [4], Particle swarm optimization (PSO) [5], Ant colony optimization (ACO) [6], Genetic algorithm (GA) [7], Cuckoo search (CS) [8], Mine blast algorithm (MBA) [9], Krill Herd (KH) [10], Interior search algorithm (ISA) [11] etc. These algorithms have capabilities to handle uncertainties [12], local minima [13], misleading global solutions [14], better constraints handling [15] etc. To overcome these difficulties different algorithms are enabled with different powerful operators. As mention above here is only objective then it is easy to measure the performance in terms of speed, accuracy, efficiency etc. with the simple operational operators.

In general, real world problems are nonlinear and multi-objective in nature. In multi-objective problem there may be some objectives are consisting of maximization function while some are minimization function. So now a day, multi-objective algorithms are in firm attention.

Let’s take an example of buying a car, so we have many objectives in mind like speed, cost, comfort level, space for number of people riding, average fuel consumption, pick up time required to gain particular speed, type of fuel requirement either it is diesel driven, petrol driven or both etc. To simply understand multi-objective problem, from Fig. 1, we consider two objectives, first cost and second comfort level. So we go for sole objective of minimum cost possible then we...
have to deny comfort level objective and vice-versa. It means real word problems are with conflicting objectives. So as, we are disabled to find an optimal solution like single objective problems. About multi-objective algorithm and its working is detailed described in next portion of the article.

Fig. 1: Car-buying decision-making problem (Hypothetical Optimal solutions)

The No free lunch [16] theorem that logically proves that none of the only algorithm exists equally efficient for all engineering problem. This is the main reason that it allows all researcher either to propose new algorithm or improve the existing ones. This paper proposed the multi-objective version of the well-known dragonfly algorithm (DA) [17]. In this paper non-sorted DA (NSDA) is tested on the standard un-constraint and constraint test function along with some well-known engineering design problem, their results are also compared with contemporary multi-objective algorithms Multi objective Colliding Bodies Optimizer (MOCBO)[18], Multi objective Particle Swarm Optimizer (MOPSO)[19-20], Non-dominated Sorting Genetic Algorithm (NSGA) [21-23], non-dominated sorting genetic algorithm II (NSGA-II)[24] and Multi objective Symbiotic Organism Search (MOSOS)[25]that are widely accepted due to their ability to solve real world problem.

The structure of the paper can be given as follows: - Section 2 consists of literature; Section 3 includes the proposed novel NSDA algorithm; Section 4 consists of competitive results analysis of standard test functions as well as engineering design problem and section 5 includes real world application, finally conclusion based on results and future scope of work is drawn.

II. Literature Review

As the name describes, multi-objective optimization handles simultaneously multiple objectives. Mathematically minimize/maximize optimization problem can be written as follows:

\[
\begin{align*}
\text{Minimize}/\text{maximize:} & \quad \quad F_n(\vec{x}) = \{f_n(\vec{x}), f_n(\vec{x}), \ldots, f_{no}(\vec{x})\} \\
\text{Subject to:} & \quad \quad p_i(\vec{x}) \geq 0, \quad i = 1,2,\ldots,q \\
& \quad \quad t_i(\vec{x}) = 0, \quad i = 1,2,\ldots,r \\
& \quad \quad L_i^{lb} \leq x_i \leq U_i^{ub}, \quad i = 1,2,\ldots,k
\end{align*}
\]

Where \( q \) is the number of inequality constraints, \( r \) is the number of equality constraints, \( k \) is the number of variables, \( p_i \) is the \( i^{th} \) inequality constraints, \( no \) is the number of objective functions, \( t_i \) indicates the \( i^{th} \) equality constraint, and \( [L_i^{lb}, U_i^{ub}] \) are the boundaries of \( i^{th} \) variable.

Obviously, relational operators are ineffective in comparing solutions with respect to multiple objectives.
The most common operator in the literate is Pareto optimal dominances, which is defined as follows for minimization problems:

\[
\forall n \in \{1, 2, \ldots, k\}: f_n(\vec{x}) \leq f_n(\vec{y}) \quad \land \quad \exists n \in \{1, 2, \ldots, k\}: f_n(\vec{x}) < f_n(\vec{y})
\]  

(2.5)

where \( \vec{x} = (x_1, x_2, \ldots, x_k) \) and \( \vec{y} = (y_1, y_2, \ldots, y_k) \).

For maximization problems, Pareto optimal dominance is defined as follows:

\[
\forall n \in \{1, 2, \ldots, k\}: f_n(\vec{x}) \geq f_n(\vec{y}) \quad \land \quad \exists n \in \{1, 2, \ldots, k\}: f_n(\vec{x}) > f_n(\vec{y})
\]

(2.6)

where \( \vec{x} = (x_1, x_2, \ldots, x_k) \) and \( \vec{y} = (y_1, y_2, \ldots, y_k) \).

These equations show that a solution is better than another in a multi-objective search space if it is equal in all objective and better in at least one of the objectives. Pareto optimal dominance is denoted with \( \prec \) and \( \succ \). With these two operator’s solutions can be easily compared and differentiated.

Population based multi-objective algorithm’s solution consists of multiple solution. But with multi-objective algorithm we cannot exactly determine the optimal solution because each solution is bounded by other objectives or we can say there is always conflict between other objectives. So the main function of stochastic/population based multi-objective algorithm is to find out best trade-offs between the objectives, so called Pareto optimally set [26-28].

The principle of working for an ideal multi-objective optimization algorithm is as shown in Fig. 2.

**Step No. 1** Find maximum number of non-dominated solution according to objective, it expresses the number of Pareto optimal set so as shows higher coverage.

**Step No. 2** Choose one of the Pareto optimal solution using crowding distance mechanism that fulfills the objectives.

---

**Fig. 2:** Multi-objective optimization (Ideal) procedure.
Now a day recently proposed sole objective algorithms are equipped with powerful operators to provide them a capability to solve multi-objective problems as well. In the same manner we proposed NSDA algorithm in a hope that it will perform efficiently for multi-objective problems. These are: Multi-objective GWO [29], Multi-objective Bat Algorithm [30], Multi-objective Bee Algorithm [31], Pareto Archived Evolution Strategy (PAES) [32], Pareto-frontier Differential Evolution (PDE) [33], Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) [34], Strength-Pareto Evolutionary Algorithm (SPEA) [35, 36] and Multi-objective water cycle algorithm with unconstraint and constraint standard test functions [37][38]. Performance measurement for approximate robustness to Pareto front of multi-objective optimization algorithms in terms of coverage, convergence and success metrics.

The computational complexity of NSDA algorithm is order of $O(mn^2)$ where $N$ is the number of individuals in the population and $M$ is the number of objectives. The complexity for other good algorithms in this field: NSGA-II, MOPSO, SPEA2 and PAES are $O(mn^2)$. However, the computational complexity is much better than some of the algorithms such as NSGA and SPEA which are of $O(mn^3)$.

### III. Non-Dominated Sorting Dragonfly Algorithm (NSDA)

Dragonfly Algorithm (DA) with sole objective was proposed by Mirjalili Seyedali in 2015 [17]. It is basically a stochastic population based, nature inspired algorithm. In this algorithm the basic strategy based on swarming nature of dragonflies for exploration and exploitation. DA algorithm originated from the static and dynamic swarming behaviors of dragonflies. These two swarming behaviors are similar to the basic stage of working of any optimization algorithm in all meta-heuristic algorithms as: exploration and exploitation. Dragonflies build small number of group and fly in different directions in search of food is known as static swarm, this function is very similar to exploration phase in meta-heuristic techniques. Whereas, dragonflies make a big group and fly in only direction for either attacking to prey or migration to other place is known as dynamic swarm, this function is very similar to exploitation phase.

Mathematical modelling of Dragonfly Algorithm:

Each portion of Dragonfly Algorithm is formulated by algebraic equations are:

1. For Separation part formulating equation:

   $$\text{SEP.}_j = \sum_{i=1}^{N} L_i - L_j$$  

2. For Alignment part formulating equation:

   $$\text{Alig.}_j = \sum_{i=1}^{N} \frac{L_i}{N}$$  

3. For cohesion part formulating equation:

   $$\text{Coh.}_j = \sum_{i=1}^{N} \frac{L_i}{N} - L$$
4. For Attraction towards a food source part formulating equation:
\[ F_j = L^* - L \]  
(3.4)

5. For Attraction towards a food source part formulating equation:
\[ E_j = L^- + L \]  
(3.5)

6. Step vector is formulating equation:
\[ \Delta L_{i+1} = \left( sSep. \_j + aAlign. \_j + cCoh. \_j + fF \_j + eE \_j \right) + w\Delta L_i \]  
(3.6)

7. Position vector is calculated using equation
\[ L^d_{i+1} = L_i + \Delta L_{i+1} \]  
(3.7)

8. Position of dragonfly updated using equation
\[ L_{i+1} = L_i + \text{Levy}(L)^*L_i \]  
(3.8)

Where:
L=Location of the current individuals, N= Neighboring individuals, L*=positions of food source, L-=positions of enemy, s=separation weight, a=alignment weight, c=cohesion weight, f=food weight, e=enemy weight, w=inertia weight, t=iteration counter and d=dimension of position vectors that levy flight step calculated.

Fig. 4: Dragonfly Algorithm principle

Basic working of NSDA algorithm is as follows:

- **Stage 1**
  - First of all, initialize the population of dragonflies
  - Randomly generated sets of dragonflies & position vectors are represented in matrix for convenience to understand
  - Then fitness of step vector & position vectoris calculated on an according as objective function

- **Stage 2**
  - Position of dragonflies are updated as a function of levy flight motion and so as value of position vector is decided

- **Stage 3**
  - The value of absolute distance is achieved which is basically a distance between the current best solution to the final optimal solution
  - Step vector is a function of both static and dynamic swarming behavior of dragonflies where some constant weight is assigned to the step vector function according to their swarming nature
  - Termination counter in integrated to limit/forcefully stop the search in uncertain search space (max. iteration counter to forcefully converge the search to optimal one)
The main drawback of non-dominated sorting is their rank. The better rank, the higher probability to be assigned rank 3, and so on. Afterwards, solutions are solutions that are dominated by only two solutions are assigned rank 2, the solutions is assigned with rank 1, the solutions that are not dominated by any on the domination level and give them a rank. This non-dominated sorting sort Pareto optimal solutions based objective optimization. As its name implies, non-dominated sorting has been of the most popular version of the DA algorithm called NSDA algorithm. The solutions in the archive across all objectives.

This subsection proposes multi-objective version of the DA algorithm called NSDA algorithm. The non-dominated sorting has been of the most popular and efficient techniques in the literature of multi-objective optimization. As its name implies, non-dominated sorting sort Pareto optimal solutions based on the domination level and give them a rank. This means that the solutions that are not dominated by any solutions is assigned with rank 1, the solutions that are dominated by only one solution are assigned rank 2, the solutions that are dominated by only two solutions are assigned rank 3, and so on. Afterwards, solutions are chosen to improve the quality of the population base on their rank. The better rank, the higher probability to be chosen. The main drawback of non-dominated sorting is its computational cost, which has been resolved in NSGA-II.

The success of the NSGA-II algorithm is an evidence of the merits of non-dominated sorting in the field of multi-objective optimization. This motivated our attempts to employ this outstanding operator to design another multi-objective version of the DA algorithm. In the NSDA algorithm, solutions are updated with the same equations presented in equation 3.9. In every iteration, however, the solutions to have optimal position of dragonflies are chosen using the following equation:

$$P_i = \frac{c}{Rank_i}$$  (3.9)

where $c$ is a constant and should be greater than 1 and $Rank_i$ is the rank number of solutions after doing the non-dominated sorting.

This mechanism allows better solutions to contribute in improving the solutions in the population. It should be noted that non-dominated sorting gives a probability to dominated solutions to be selected as well, which improves the exploration of the NSDA algorithm. Flow chart of NSDA algorithm is represented as Fig. 5.

**Constraint Handling Approach:**

With the extended literature survey we find that the population based algorithms are the common way to solve the multi-objective problems as they are more commonly provides the global solution and capable of handling both continuous and combinational optimization problem with a very high coverage and convergence. Multi-objective problems are subjected to various type of constraints like linear, non-linear, equality, inequality etc. So with these problems embedded it is very difficult to find simple and good strategy to achieve considerable solutions in the acceptable criterion. So in this paper NSDA algorithm uses a very simple approach to get feasible solutions. In this mechanism, after generating number of solutions at each generation, all the desirable constraint checked and then some solution that fulfills the criterion of acceptable solution are selected and collected them in archive. Afterward non dominated solutions added in archive as we find more suitable solution to get acceptable solution. So as if achieve is full then less dominated solutions are removed. Finally according to crowding distance mechanism all these solutions (more suitable position of dragonflies) from archive is selected to get desired solution.
IV. Results Analysis on Test Functions

For determine the performance of proposed NSDA algorithm is applied to:

- A set of unconstraint and constraint standard multi-objective test functions
- Tested on well-known engineering design problems
- Non-linear, highly complex practical application known as formulation of economic constrained emission dispatch (ECED) with stochastic integration of wind power (WP) in the next section

NSDA algorithm is tested on seventeen different multi-objective case studies, including eight unconstrained test functions, five constrained test functions, and four real world engineering design problem, later algorithm is applied to the main application economic constrained emission dispatch with wind power (ECEDWP). These can be classified into four groups given below:
• Standard multi-objective unconstrained test functions (KUR, FON, ZDT1, ZDT2, ZDT3, ZDT4, SCHN1, and SCHN2)
• Standard multi-objective constrained test functions (TKN, OSY, BNH, SRN, and CONST)
• Real world engineering multi-objective design problem (Four bar truss design, welded beam design, speed reducer and disk brake design problem)
• Modeling of ECEDWP problem

Mathematical representation of these standard test functions are given in Appendix 1. (Multi-objective unconstrained test functions), 2. (Multi-objective constrained test functions), 3. (Engineering multi-objective design problem) with distinct characteristics like non-linear, non-convex, discrete pareto fronts and convex etc. are selected to measure the performance of algorithms like MOWCA, NSGA-II, MOPSO, PAES and performance is compared with various well known algorithms like MOWCA, NSGA-II, MOPSO, PAES and μ-GA multi-objective algorithms. Each algorithm is separately runs fifteen times and numeric results are listed in tables below. To measure the quality of obtained results we match their coverage of obtained true pareto front with respect to their original or true pareto fronts.

For numeric as well as qualitative performance of proposed NSDA algorithm on various case studies we consider Generational Distance (GD) given by Veldhuizen in 1998 [39] for measuring the deviation of the distance between true pareto front and obtained pareto front, Diversity matrix (Δ) also known as matrix of spread to measure the uniformly distribution of non-dominated solution by Deb [24] and Metric of spacing (S) to represent the distribution of non-dominated distribution of obtained solutions by proposed algorithm given by Schott [40].

The mathematical representation of these performance indicating metric are as follows:

\[ GD = \sqrt{\frac{1}{n} \sum_{i=1}^{n_{PFs}} d_i^2} \] (4.1)

where \( d_i \) shows the Euclidean distance (calculated in the objective space) between the \( i^{th} \) Pareto optimal solution achieved and the nearest true Pareto optimal solution in the reference set. \( n_{PFs} \) is the total number of achieved Pareto optimal solutions.

\[ \Delta = \frac{d_i + d_m + \int_{u=1}^{n_{PFs}} |d_{i} - d|}{d_i + d_m + \int (n-1)d} \] (4.2)

where, \( d_i, d_m \) are Euclidean distances between extreme solutions in true pareto front and obtained pareto front. \( n_{PFs} \) and ‘d’ are the total number of achieved Pareto optimal solutions and averaged distance of all solutions.

\[ S = \frac{1}{n_{PFs}} \sum_{i=1}^{n_{PFs}} (d_i - d)^2 \] (4.3)

where “d” is the average of all \( d_i \), \( n_{PFs} \) is the total number of achieved Pareto optimal solutions, and \( d_i = \min_j \left( \| f_j(x) - f_j^*(x) \| + \| f_j^*(x) - f_j^*(\tilde{x}) \| \right) \) for all \( i,j=1,2,...,n \). Smallest value of “S” metric gives the global best non-dominated solutions are uniformly distributed, thus if numeric value of \( d_i \) and \( d \) are same then value of “S” metric is equal to zero.

a) Results on unconstrained test problems

Like as above mentioned, the first set of test problems consist of unconstrained standard test functions. All the standard unconstrained test functions mathematical formulation is shown in Appendix A. Later, the numeric results are represented in Table 1 and best optimal pareto front is shown in Fig. 6.

All the statistical results are shown Table 1 suggests that the NSDA algorithm effectively outperforms with most of the unconstraint test functions compare to the MOSOS, MOCBO, MOPSO and NSGA-II algorithm. The effectiveness of proposed non-dominated version of DA (NSDA algorithm) can be seen in the Table 1, represents a greater robustness and accuracy of NSDA algorithm in terms of mean and standard deviation with the help of GD, diversity matrix along with computational time. However, proposed NSDA algorithm shows very competitive results in comparison with the MOPSO, MOCBO and MOSOS algorithms and in some cases these algorithms performs better than proposed one. Pareto front obtained by proposed NSDA algorithm shows almost complete coverage with respect to true pareto front.

**Table 1:** Results of the multi-objective NSDA algorithms (using GD, Δ, CT) on the unconstrained test functions employed

<table>
<thead>
<tr>
<th>Algorithm → Function ↓</th>
<th>PFs</th>
<th>NSDA MEAN±SD</th>
<th>MOSOS MEAN±SD</th>
<th>MOCBO MEAN±SD</th>
<th>MOPSO MEAN±SD</th>
<th>NSGA-II MEAN±SD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KUR</td>
<td></td>
<td>0.00729±0.00241</td>
<td>0.0075±0.0042</td>
<td>0.0083±0.0062</td>
<td>0.015±0.0075</td>
<td>0.0301±0.0043</td>
</tr>
<tr>
<td>Δ</td>
<td></td>
<td>0.02704±0.01025</td>
<td>0.0295±0.0122</td>
<td>0.0357±0.0236</td>
<td>0.0991±0.031</td>
<td>0.0362±0.0240</td>
</tr>
<tr>
<td>CT</td>
<td></td>
<td>7.65853±0.44369</td>
<td>10.7413±0.822</td>
<td>7.9531±0.5823</td>
<td>8.0532±0.621</td>
<td>20.4368±3.102</td>
</tr>
<tr>
<td>GD</td>
<td></td>
<td>0.00173±0.00032</td>
<td>0.0019±0.0002</td>
<td>0.0022±0.0003</td>
<td>0.0042±0.000</td>
<td>0.0026±0.0003</td>
</tr>
</tbody>
</table>

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The next set of standard test functions consisting of constrained functions. For constrained test function it should be necessary that NSDA algorithm has a capability of handling constraints so algorithm is equipped with a death penalty function to search agents that violate any of the constraints at any level [41]. For comparing the results of different algorithms, we have utilized GD and $\Delta$ metrics.

**Table 2:** Results of the multi-objective NSDA algorithms on constrained test problems

<table>
<thead>
<tr>
<th>Algorithm/Function</th>
<th>PFs</th>
<th>NSDA MEAN±SD</th>
<th>MOSOS MEAN±SD</th>
<th>MOCBO MEAN±SD</th>
<th>MOPSO MEAN±SD</th>
<th>NSGA-II MEAN±SD</th>
</tr>
</thead>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GD</td>
<td></td>
<td>0.14466±0.00210</td>
<td>0.1508±0.0040</td>
<td>0.1528±0.0051</td>
<td>0.1576±0.0062</td>
<td>0.1542±0.0072</td>
</tr>
<tr>
<td>TNK</td>
<td>$\Delta$</td>
<td>0.57896±0.05587</td>
<td>0.1206±0.0432</td>
<td>0.1242±0.0512</td>
<td>0.1286±0.0522</td>
<td>0.126±0.0624</td>
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<tr>
<td>CT</td>
<td></td>
<td>10.7895±0.04748</td>
<td>15.1286±0.063</td>
<td>11.0104±0.052</td>
<td>12.0212±0.054</td>
<td>17.4204±0.055</td>
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<tr>
<td>GD</td>
<td></td>
<td>0.10054±0.00200</td>
<td>0.1196±0.0031</td>
<td>0.121±0.0041</td>
<td>0.1282±0.0042</td>
<td>0.1242±0.0043</td>
</tr>
<tr>
<td>OSY</td>
<td>$\Delta$</td>
<td>0.54789±0.05679</td>
<td>0.5354±0.0616</td>
<td>0.5422±0.0712</td>
<td>0.5931±0.0721</td>
<td>0.5682±0.0751</td>
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<tr>
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<td>15.5578±0.02047</td>
<td>20.2124±0.032</td>
<td>12.2104±0.030</td>
<td>14.6420±0.042</td>
<td>24.2204±0.039</td>
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<tr>
<td>GD</td>
<td></td>
<td>0.14458±0.00375</td>
<td>0.1436±0.0062</td>
<td>0.1498±0.0076</td>
<td>0.1644±0.0078</td>
<td>0.1566±0.0042</td>
</tr>
<tr>
<td>BNH</td>
<td>$\Delta$</td>
<td>0.44587±0.03789</td>
<td>0.4288±0.0625</td>
<td>0.4798±0.0721</td>
<td>0.4975±0.0632</td>
<td>0.4892±0.0832</td>
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<tr>
<td>CT</td>
<td></td>
<td>07.5254±0.04587</td>
<td>16.2664±0.054</td>
<td>9.1544±0.0420</td>
<td>9.7452±0.0464</td>
<td>19.652±0.0511</td>
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<tr>
<td>GD</td>
<td></td>
<td>0.05001±0.01478</td>
<td>0.0988±0.0014</td>
<td>0.1018±0.0015</td>
<td>0.1125±0.0026</td>
<td>0.1024±0.0032</td>
</tr>
<tr>
<td>SRN</td>
<td>$\Delta$</td>
<td>0.20458±0.00090</td>
<td>0.2295±0.0017</td>
<td>0.2352±0.0019</td>
<td>0.2730±0.0023</td>
<td>0.2468±0.0018</td>
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<td>CT</td>
<td></td>
<td>7.24456±0.00102</td>
<td>12.3254±0.012</td>
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<td>9.2134±0.0083</td>
<td>17.0231±0.023</td>
</tr>
<tr>
<td>GD</td>
<td></td>
<td>0.32145±0.04002</td>
<td>0.5162±0.0021</td>
<td>0.5202±0.0034</td>
<td>0.5854±0.0036</td>
<td>0.5532±0.0041</td>
</tr>
<tr>
<td>CONST</td>
<td>$\Delta$</td>
<td>0.7056±0.000706</td>
<td>0.7122±0.0072</td>
<td>0.7235±0.0083</td>
<td>0.7344±0.0084</td>
<td>0.8126±0.0087</td>
</tr>
<tr>
<td>CT</td>
<td></td>
<td>16.8556±0.00054</td>
<td>10.0112±0.003</td>
<td>5.2252±0.0028</td>
<td>6.4766±0.0035</td>
<td>14.0892±0.003</td>
</tr>
</tbody>
</table>
Fig. 7: Best Pareto optimal front TNK, OSY, BNH, SRN and CONST obtained by NSDA algorithm

Table 2 suggests that the NSDA algorithm comparatively performs better than other four algorithms for most of the standard constrained test functions employed. The best Pareto optimal fronts in Fig. 7 also helps in proving since all the Pareto optimal solutions exactly follow the true pareto fronts obtained from the NSDA algorithm.

CONST function consists of concave front with linear front, OSY is similar to CONST but consists of many linear regions with different slopes while TNK almost similar to wave shaped. These also suggests that NSDA algorithm has a capability to solve various type of constraint problem. All the constraint test functions are mathematically given in Appendix B.

c) Results on constrained engineering design problems

The third set of test functions is the most complicated one and consists of four real engineering design problems. Mathematical model of all the four engineering design problem are given in Appendix C. Same as before both GD and diversity matrix is employed to measure the performance of NSDA algorithm with respect to other algorithms to solve them, numeric results are given in Tables and Figure respectively shows the best optimal front obtained by NSDA algorithm.

i. Four-bar truss design problem

The statistical results of four bar truss design problem [42] in given in Table 3 and best optimal front is given in Fig. 8. It consists of two minimization objectives displacement and volume with four design control variable mathematically given in Appendix C.

Table 3: Results of the multi-objective NSDA algorithm on four-bar truss design problem in terms mean and standard deviation

<table>
<thead>
<tr>
<th>Methods</th>
<th>GD MEAN±SD</th>
<th>S MEAN±SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSDA</td>
<td>0.1756±0.0235</td>
<td>1.8717±0.1205</td>
</tr>
<tr>
<td>MOWCA</td>
<td>0.2076±0.0055</td>
<td>2.5816±0.0298</td>
</tr>
<tr>
<td>NSGA-II</td>
<td>0.3601±0.0470</td>
<td>2.3635±0.2551</td>
</tr>
<tr>
<td>MOPSO</td>
<td>0.3741±0.0422</td>
<td>2.5303±0.2275</td>
</tr>
<tr>
<td>μ-GA</td>
<td>0.9102±1.7053</td>
<td>8.2742±16.831</td>
</tr>
<tr>
<td>PAES</td>
<td>0.9733±1.8211</td>
<td>3.2314±5.9555</td>
</tr>
</tbody>
</table>
ii. Speed-reducer design problem

The statistical results of speed reducer design problem[43] is given in Table 4 and best optimal front is given in Fig. 9. It is a well-known mechanical design problem consists of two minimization objectives stress and weight with seven design control variable mathematically given in Appendix C.

Table 4: Results of the multi-objective NSDA algorithm on speed-reducer design problem in terms mean and standard deviation

<table>
<thead>
<tr>
<th>Methods</th>
<th>GD MEAN±SD</th>
<th>S MEAN±SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSDA</td>
<td>0.95578±0.32458780</td>
<td>1.578354±0.947475</td>
</tr>
<tr>
<td>MOWCA</td>
<td>0.98831±0.17894217</td>
<td>16.68520±2.6969443</td>
</tr>
<tr>
<td>NSGA-II</td>
<td>9.843702±7.0810303</td>
<td>02.7654494±3.534978</td>
</tr>
<tr>
<td>µ-GA</td>
<td>3.117536±1.6781086</td>
<td>47.80096±3.8015157</td>
</tr>
<tr>
<td>PAES</td>
<td>77.99834±4.2102608</td>
<td>16.20129±4.26842769</td>
</tr>
</tbody>
</table>

**Fig. 8:** Pareto optimal front obtained by the NSDA Algorithm for “Four –bus truss design problem”

**Fig. 9:** Pareto optimal front obtained by the NSDA Algorithm for “Speed Reducer design problem”
iii. **Welded-beam design problem**

The statistical results of welded beam design problem [44] is given in Table 5 and best optimal front is given in Fig. 10. It is a well-known mechanical design problem consists of two minimization objectives fabrication cost and deflection of beam with four design control variable mathematically given in Appendix C.

**Table 5:** Results of the multi-objective NSDA algorithms on welded-beam design problem in terms mean and standard deviation

<table>
<thead>
<tr>
<th>Methods</th>
<th>GD MEAN±SD</th>
<th>Δ MEAN±SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSDA</td>
<td>0.03325±0.01693</td>
<td>0.75844±0.03770</td>
</tr>
<tr>
<td>MOWCA</td>
<td>0.04909±0.02821</td>
<td>0.22476±0.09280</td>
</tr>
<tr>
<td>NSGA-II</td>
<td>0.16875±0.08030</td>
<td>0.88987±0.11976</td>
</tr>
<tr>
<td>pae-ODEMO</td>
<td>0.09169±0.00733</td>
<td>0.58607±0.04366</td>
</tr>
</tbody>
</table>

**Fig. 10:** Pareto optimal front obtained by the NSDA Algorithm for “Welded Beam Design problem”

iv. **Disk brake design problem**

The statistical results of welded beam design problem [44] is given in Table 6 and best optimal front is given in Fig. 11. It is a well-known mechanical design problem consists of two minimization objectives stopping time and mass of brake of a disk brake with four design control variable mathematically given in Appendix C.

**Table 6:** Results of the multi-objective NSDA algorithms on the Disk brake design problem in terms mean and standard deviation

<table>
<thead>
<tr>
<th>Methods</th>
<th>GD MEAN±SD</th>
<th>Δ MEAN±SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSDA</td>
<td>0.0587±0.27810</td>
<td>0.43551±0.08237</td>
</tr>
<tr>
<td>pae-ODEMO</td>
<td>2.6928±0.24051</td>
<td>2.6928±0.24051</td>
</tr>
<tr>
<td>NSGA-II</td>
<td>2.30771±0.10782</td>
<td>2.30771±0.10782</td>
</tr>
<tr>
<td>MOWCA</td>
<td>0.0244±0.12314</td>
<td>0.46041±0.10961</td>
</tr>
</tbody>
</table>
Due to high complexity of engineering design problem it is really hard to gain results alike true pareto front but we can clearly see that optimal pareto obtained by NSDA algorithm covers almost whole solutions that are the actual/true solutions of an engineering design problem. From all above tested function we can conclude that problem either it consists of constraints or unconstraint problem NSDA algorithm shows its capability to solve any kind of linear, non-linear and complex real world problem. So in the next section we attached a highly non-linear complex real problem to show its effectiveness regarding the real world complex application with many objectives.

\[ s(v) = (k/c) (v/c)^{k-1} \exp\left[-(v/c)^k\right], \quad v \geq 0 \]  

\[ S(v) = 1 - \exp\left[-(v/c)^k\right], \quad v \geq 0 \]  

Where, \( S(v) \) and \( s(v) \) are CDF and PDF respectively. Shape factor and scale factor are \( k \) and \( c \) respectively.

The wind speed and output wind power are related as:

\[ P_{\text{wind}} = \begin{cases} 0, & v < v_{\text{in}} \text{ or } v \geq v_{\text{out}} \\ \frac{v - v_{\text{in}}}{v_{\text{rated}} - v_{\text{in}}}, & v_{\text{in}} \leq v < v_{\text{rated}} \\ P_{\text{rated}}, & v_{\text{rated}} \leq v < v_{\text{out}} \end{cases} \]  

Where, \( v_{\text{rated}} \) and \( P_{\text{rated}} \) are the rated speed of wind and rated power output. \( v_{\text{out}} \) and \( v_{\text{in}} \) are cut-out and cut-in speed of wind respectively. The CDF of \( P_{\text{wind}} \) in the boundary of \([0, P_{\text{rated}}]\) on an accordance with the speed range of wind can be formulated as:

\[ S(P_{\text{wind}}) = 1 - \exp\left\{ -\left[\left(1 + \frac{v_{\text{in}} P_{\text{rated}}}{v_{\text{in}} P_{\text{rated}} P_{\text{wind}}}\right) \frac{v_{\text{in}}}{c}\right]^k \right\} + \exp\left[-(v_{\text{out}}/c)^k\right], \quad 0 \leq P_{\text{wind}} < P_{\text{rated}} \]  

Above equation is very meaningful to calculate the ECED problems with speculative wind power with variable speed.
ii. **Modeling of ECEDWP problem**

As wind power is formulated as system constraint, so the objective function of economic emission dispatch problem (EEDP) stays on unchanged as classical EEDP:

Fuel cost objective is given by:

$$ \text{Minimization } S(P_i) = \sum_{i}^{N}(a_i + b_i P_i + c_i P_i^2) $$

(4.5)

where, the thermal power generators cost coefficients are $a_i, b_i, c_i$ for i-th generator, Sum of the total fuel cost of the system and N is the total number of generators.

Total Emission is calculated by:

$$ \text{Minimization } E(P_i) = \sum_{i}^{N}[\{(a_i + \beta_i P_i + \gamma_i P_i^2) \times 10^{-2}\} + \delta_i \times \exp (\varphi_i \times P_i) ] $$

(4.6)

where, $\alpha_i, \beta_i, \gamma_i, \delta_i$ and $\varphi_i$ are emission coefficients with valve point effect taking into consideration for i-th thermal generator.

iii. **System Constraints**

As wind power generation is considered as system constraint with the summation of stochastic variables the classical power balance constraint changes to fulfill the predefined confidence level.

$$ \sum_{i}^{N}(P_i + P_{wind}) \geq P_D + P_{loss} \geq \eta_{pbc} $$

(4.7)

where, $\eta_{pbc}$ is confidence level that a power system must follow the load demand and so as it is selected nearer to unity as values lesser than unity represents high operational risk. $P_{loss}$ represents system losses can be calculated by B-coefficient method given below:

$$ P_{loss} = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} P_i B_{i0} + B_{00} $$

(4.8)

So as to change above described power balance constrained equation into deterministic form can be solved as:

$$ P_i \{P_{wind} < P_D + P_{loss} - \sum_{i=1}^{N} P_i \} = F(P_D + P_{loss} - \sum_{i=1}^{N} P_i) \leq 1 - \eta_{pbc} $$

(4.9)

Assume that the wind turbine have same speed and same direction and combination of Eqs. (4) and (9), the power balance constraint is represented as:

$$ P_D + P_{loss} - \sum_{i=1}^{N} P_i \leq \frac{cP_{rated}}{v_{rated} - v_{in}} \ln \left[ \eta_{pbc} + \exp \left( \frac{v_{in}^2}{cP_{rated}} \right) \right] \frac{1}{k} - \frac{v_{in}^2P_{rated}}{v_{rated} - v_{in}} $$

(4.10)

iv. **Reserve capacity system constraint**

So as to reduce the impact of stochastic wind power on system, up and down spinning reserve needs to be maintained [22]. Such reserve constraints formulated as [15] and [16] respectively:

$$ P_i \{\sum_{i=1}^{N} (P_i^{max} - P_i) \geq P_s + t_u \times P_{wind} \} \geq \eta_{urc} $$

(4.11)

$$ P_i \{\sum_{i=1}^{N} (P_i - P_i^{min}) \geq t_d \times (P_{rated} - P_{wind}) \} \geq \eta_{drc} $$

(4.12)

where, $P_s$ represents the reserve demand of conventional thermal power plant system and it generally keeps the maximum value of thermal unit, $P_i^{max}$ and $P_i^{min}$ are maximum and minimum output level of operational generators of i-th unit, $\eta_{drc}$ and $\eta_{urc}$ are predefined down and upper confidence level parameter respectively, $t_u$ and $t_d$ are the demand coefficients of up and down spinning reserves.

v. **Generational capacity constraint**

The real output power is bounded by each generators upper and lower bounds given as:

$$ P_i^{Minimum} \leq P_i \leq P_i^{Maximum} $$

(4.13)

V. **40-Operational Thermal Generating Unit**

a) **Case study I- 40 thermal-generator lossless system without wind power**

In this case forty operational generating unit is consider without integration of wind power means all the generating units are coal fired. Input parameters like generators operating limit, fuel cost coefficients and emission coefficients are given in Appendix D extracted from [45]. System is considered lossless and its solution is compared with three well known multi-objective algorithms like SMODE [45], NSGA-II [45]and MBFA [46] in terms of various objectives such as best cost, best emission and best compromise between both objectives. Best compromise solution is then obtained...
by the fuzzy based method [47]. Total power demand for this system is 10500 MW. Results obtained by NSDA algorithm is added to table 7 and best pareto front obtained by NSDA algorithm is represented in Fig. 12.

**Table 7:** Results of the multi-objective NSDA algorithms for case study I- 40 thermal-generator lossless system without wind power

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Best emission</td>
<td>Best cost</td>
<td>Best compromise</td>
<td>Best emission</td>
<td>Best cost</td>
</tr>
<tr>
<td>Cost ($/h)</td>
<td>156,700</td>
<td>124,230</td>
<td>126,180</td>
<td>124,380</td>
</tr>
<tr>
<td>Emission (tons/h)</td>
<td>66,799</td>
<td>96,578</td>
<td>99,671</td>
<td>153,560</td>
</tr>
</tbody>
</table>

**Fig. 12:** Pareto optimal front obtained by the NSDA Algorithm for “40 thermal-generator lossless system without wind power”

b) **Case study II- 40 thermal-generator lossless system with wind power**

All the conditions are remaining same as case study I like input parameters and power demand. While integrating with wind power plant, the total rated output power of wind farm is set to 1000 MW [45, 47]. Statistical results obtained by NSDA algorithm is reported in Table 8 and best optimal front is represented in Fig. 13.

**Table 8:** Results of the multi-objective NSDA algorithms for case study II- 40 thermal-generator lossless system with wind power

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Best emission</td>
<td>Best cost</td>
<td>Best Compromise point</td>
<td>Best emission</td>
<td>Best cost</td>
</tr>
<tr>
<td>$\sum P_g$</td>
<td>$P_w$</td>
<td>Cost</td>
<td>Emission</td>
<td></td>
</tr>
<tr>
<td>10,244.76</td>
<td>254.24</td>
<td>153,830</td>
<td>54,925</td>
<td></td>
</tr>
<tr>
<td>10,225.72</td>
<td>241.72</td>
<td>123,560</td>
<td>71,894</td>
<td></td>
</tr>
<tr>
<td>10,241.76</td>
<td>257.91</td>
<td>122,810</td>
<td>76,880</td>
<td></td>
</tr>
<tr>
<td>10,244.43</td>
<td>258.37</td>
<td>123,820</td>
<td>78,860</td>
<td></td>
</tr>
<tr>
<td>10,244.3</td>
<td>257.32</td>
<td>122,610</td>
<td>75,880</td>
<td></td>
</tr>
<tr>
<td>10,242.71</td>
<td>257.29</td>
<td>121,850</td>
<td>74,880</td>
<td></td>
</tr>
<tr>
<td>10,224.18</td>
<td>274.82</td>
<td>118,680</td>
<td>78,860</td>
<td></td>
</tr>
<tr>
<td>10,236.58</td>
<td>263.42</td>
<td>123,459</td>
<td>80,880</td>
<td></td>
</tr>
</tbody>
</table>

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Fig. 13: Pareto optimal front obtained by the NSDA Algorithm for “40 thermal-generator lossless system with wind power”

### Table 9: Results of Wilcoxon test and simulation/computational time or speed

<table>
<thead>
<tr>
<th>Case Study</th>
<th>Cost Best</th>
<th>Cost Worst</th>
<th>Cost Mean</th>
<th>Emission Best</th>
<th>Emission Worst</th>
<th>Emission Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Study I</td>
<td>119,310</td>
<td>127,568</td>
<td>124,830</td>
<td>124,380</td>
<td>147,760</td>
<td>131,710</td>
</tr>
<tr>
<td></td>
<td>1/5.40e−10</td>
<td>11.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Study II</td>
<td>118,699</td>
<td>146,685</td>
<td>123,010</td>
<td>56,509</td>
<td>179,099</td>
<td>104,258</td>
</tr>
<tr>
<td></td>
<td>1/5.77e−10</td>
<td>09.785</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### VI. Result Discussion

In almost all the cases that we consider in this article where NSDA algorithm proves its effectiveness in both prospective quantitative and qualitative. From plots also evident that NSDA algorithm follows the exact pareto front similar to the true pareto front for all constrained, unconstrained and complex engineering design problem. So as for real world application of economic emission dispatch problem and its integration with stochastic wind power generation. So for this application Wilcoxon test (statistical test) is performed. In Table 9 the signed rank test is presented in third row of each results whereas the calculation time is represented in forth row. For this test null hypothesis cannot be rejected at 5% level for numeric value ‘0’ while null hypothesis is rejected at 5% level with the value of ‘1’. Where NSDA algorithm performs superior to other algorithms that are considered for comparative purpose. NSDA algorithm shows good performance in both coverage and convergence as main mechanism that guarantee convergence in DA and NSDA algorithms are continuously shrink its virtual limitation using Levy strategy in the movement of dragonflies for their random walk. Both mechanism emphasizes convergence and exploitation proportional to maximum number of
generation (iteration). Since this complex task might degrade its performance compare to without limitation or free movement should be a concern. However the numerical results expresses that NSDA algorithm has a little effect of slow convergence at all.

NSDA algorithm has an advantage of high coverage, which is the result of the selection of position of dragonflies and archive selection procedure. All the position are updated according to their fitness value that enable the algorithm to direct the search space in right direction to find the best solution without trapped in local solution. Archive selection criteria follow all the rules of the entrance and exhaust of any value in it for each iteration and updated when its size full. Solutions of higher fitness in archive have higher probability to thrown away first to improve the coverage of the pareto optimal front obtained during the optimization process.

VII. Conclusion

In this paper the non-dominated sorting dragonfly algorithm-multi-objective version of recently proposed dragonfly algorithm (DA) is proposed known as NSDA algorithm. This paper also utilizes the static and dynamic swarming strategy for exploration purpose used in its parent DA version. NSDA algorithm is developed with equipping dragonfly algorithm with crowding distance criterion, an archive and dragonflies position (accordance to ranking) selection method based on Pareto optimal dominance nature. The NSDA algorithm is first applied on 17 standard test functions (including eight unconstraint, five constraint and four engineering design problem) to prove its capability in terms of qualities and quantities showing numerical as well as convergence and coverage of pareto optimal front with respect to true pareto front. Then after NSDA algorithm is applied to real world complex ECEDWP problem where algorithm proves its dominance over other well recognized contemporary algorithms. The numeric results are stored and represented in performance indices: GD, metric of diversity, metric of spacing and computational time. The qualitative results are reported as convergence and coverage in best pareto optimal front found in 15 independent runs. To check effectiveness of proposed version of algorithm the results are verified with SMODE, MOSOS, MOCBO, MOPSO, NSGA-II and other well recognize algorithms in the field of multi-objective algorithms. We can also conclude from the standard test functions results that NSDA algorithm is able to find pareto optimal front of any kind of shape. Finally, the result of complex real world ECEDWP problem validates that NSDA algorithm is capable of solving any kind of non-linear and complex problem with many constraint and unknown search space. Therefore, we conclude that proposed non-dominated version of DA algorithm has various merits among the contemporary multi-objective algorithms as well as provides an alternative for solving multi or many objective problems.

For future works, it is suggested to test NSDA algorithm on other real world complex problems. Also, it is worth to investigate and find the best constrained handling technique for this algorithm.

References Références Referencias


34. Qingfu Zhang,Hui Li ,MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition, IEEE transactions on evolutionary computation, vol. 11, no. 6, december 2007.


**Appendix A:** Unconstrained multi-objective test problems utilized in this work

**KUR:**

Minimize:

\[ f_1(x) = \sum_{i=1}^{2}\left[-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right] \]
\[ f_2(x) = \sum_{i=1}^{2}\left[|x_i|^{0.8} + 5\sin(x_i^3)\right] \]

\[-5 \leq x_i \leq 5\]
\[1 \leq i \leq 3\]

**FON:**

\[
\minimize = \begin{cases} 
 f_1(x) = 1 - \exp\left(-\sum_{i=1}^{n} (x_i - \frac{1}{\sqrt{n}})^2\right) \\
 f_2(x) = 1 - \exp\left(-\sum_{i=1}^{n} (x_i + \frac{1}{\sqrt{n}})^2\right) 
\end{cases} 
\]

\[-4 \leq x_i \leq 4\]
\[1 \leq i \leq n\]

**ZDT1:**

Minimise: \[ f_1(x) = x_1 \]
Minimise: \[ f_2(x) = g(x) \times h\left(f_1(x), g(x)\right) \]

Where:

\[
G(x) = 1 + \frac{9}{N-1} \sum_{i=2}^{N} i \times h(f_1(x), g(x)) = 1 - \frac{f_1(x)}{g(x)}
\]

\[0 \leq x_i \leq 1, 1 \leq i \leq 30\]
ZDT2:
Minimise: \( f_1(x) = x_1 \)
Minimise: \( f_2(x) = g(x) \times h(f_1(x), g(x)) \)
Where: \( G(x) = 1 + \frac{9}{N-1} \sum_{i=2}^{N} x_i h(f_1(x), g(x)) = 1 - \left( \frac{f_1(x)}{g(x)} \right)^2 \)
\( 0 \leq x_i \leq 1, 1 \leq i \leq 30 \)

ZDT3:
Minimise: \( f_1(x) = x_1 \)
Minimise: \( f_2(x) = g(x) \times h(f_1(x), g(x)) \)
Where: \( G(x) = 1 + \frac{9}{29} \sum_{i=2}^{N} x_i h(f_1(x), g(x)) = 1 - \sqrt[29]{f_1(x) g(x)} - \left( \frac{f_1(x)}{g(x)} \right) \sin(10\pi f_1(x)) \)
\( 0 \leq x_i \leq 1, 1 \leq i \leq 30 \)

ZDT4:
Minimise: \( f_1(x) = x_1 \)
Minimise: \( f_2(x) = g(x) \times h(f_1(x), g(x)) \)
\( h(f_1(x), g(x)) = 1 - \sqrt[29]{f_1(x) g(x)} g(x) = 91 + \sum_{i=2}^{10} (x_i^2 - 10 \cos(4\pi x_i)) \)

SCHN-1:
Minimize: \( f_1(x) = x_1^2 \)
\( f_2(x) = (x - 2)^2 \) Where: value of can be from 10 to 10^5.
\( -A \leq x \leq A \)

SCHN-2:
Minimize:
\[
\begin{cases}
-x, & \text{if} \quad x \leq 1 \\
 x - 2, & \text{if} \quad 1 < x \leq 3 \\
 4 - x, & \text{if} \quad 3 < x \leq 4 \\
 x - 4, & \text{if} \quad x > 4 \\
\end{cases}
\]
\( f_1(x) = (x - 5)^2 \)
\( -5 \leq x \leq 10 \)

Appendix B: Constrained multi-objective test problems utilised in this work

TNK:
Minimise: \( f_1(x) = x_1 \)
Minimise: \( f_2(x) = x_2 \)
Where:
\( g_1(x) = -x_1^2 - x_2^2 + 1 + 0.1 \cos(16\arctan(x_1/x_2)) \)
\( g_2(x) = 0.5 - (x_1 - 0.5)^2 - (x_2 - 0.5)^2 \quad 0.1 \leq x_1 \leq \pi, 0 \leq x_2 \leq \pi \)
BNH:
This problem was first proposed by Binh and Korn [48]:
Minimise:
\[ f_1(x) = 4x_1^2 + 4x_2^2 \]
Minimise:
\[ f_2(x) = (x_1 - 5)^2 + (x_2 - 5)^2 \]
Where:
\[ g_1(x) = (x_1 - 5)^2 + x_2^2 - 25 \]
\[ g_2(x) = 7.7 - (x_1 - 8)^2 - (x_2 + 3)^2 \]
\[ 0 \leq x_1 \leq 5, 0 \leq x_2 \leq 3 \]

OSY:
The OSY test problem has five separated regions proposed by Osyczka and Kundu [49]. Also, there are six constraints and six design variables.
Minimise:
\[ f_1(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 \]
Minimise:
\[ f_2(x) = -(25(x_1 - 2)^2 + (x_2 - 1)^2 + (x_3 - 1)^2 + (x_4 - 4)^2 + (x_5 - 1)^2) \]
Where:
\[ g_1(x) = 2 - x_1 - x_2 \]
\[ g_2(x) = -6 + x_1 + x_2 \]
\[ g_3(x) = -2 - x_1 + x_2 \]
\[ g_4(x) = -2 + x_1 - 3x_2 \]
\[ g_5(x) = -4 + x_4 + (x_3 - 3)^2 \]
\[ g_6(x) = 4 - x_6 - (x_5 - 3)^2 \]
\[ 0 \leq x_1 \leq 10, 0 \leq x_2 \leq 10, 1 \leq x_3 \leq 5, 0 \leq x_4 \leq 6, 1 \leq x_5 \leq 5, 0 \leq x_6 \leq 10 \]

SRN:
The third problem has a continuous Pareto optimal front proposed by Srinivas and Deb [50].
Minimise:
\[ f_1(x) = 2 + (x_1 - 2)^2 + (x_2 - 1)^2 \]
Minimise:
\[ f_2(x) = 9x_1 - (x_2 - 1)^2 \]
Where:
\[ g_1(x) = x_1^2 + x_2^2 - 255 \]
\[ g_2(x) = x_1 - 3x_2 + 10 \]
\[ -20 \leq x_1 \leq 20, -20 \leq x_2 \leq 20 \]

CONSTR:
This problem has a convex Pareto front, and there are two constraints and two design variables.
Minimise:
\[ f_1(x) = x_1 \]
Minimise:
\[ f_2(x) = (1 + x_2)/(x_1) \]
Where:
\[ g_1(x) = 6 - (x_2 + 9x_1), g_2(x) = 1 + x_2 - 9x_1 \]
\[ 0.1 \leq x_1 \leq 1.0, 0 \leq x_2 \leq 5 \]
Appendix C: Constrained multi-objective engineering problems used in this work

Four-bar truss design problem:

The 4-bar truss design problem is a well-known problem in the structural optimisation field [42], in which structural volume ($f_1$) and displacement ($f_2$) of a 4-bar truss should be minimized. As can be seen in the following equations, there are four design variables ($x_1$-$x_4$) related to cross sectional area of members 1, 2, 3, and 4.

Minimise: $f_1(x) = 200 \times (2 \times x(1) + \sqrt{2 \times x(2)} + \sqrt{2 \times x(3)}) + x(4)$

Minimise: $f_2(x) = 0.01 \times \left( \frac{2}{x(1)} + \frac{2 \times \sqrt{2 \times x(2)}}{x(2)} \right) - ((2 \times sqrt(2)) / x(3)) + (2 / x(1))$

$1 \leq x_1 \leq 3.14142 \leq x_2 \leq 3.14142 \leq x_3 \leq 3.1 \leq x_4 \leq 3$

Speed reducer design problem:

The speed reducer design problem is a well-known problem in the area of mechanical engineering [43], in which the weight ($f_1$) and stress ($f_2$) of a speed reducer should be minimized. There are seven design variables: gear face width ($x_1$), teeth module ($x_2$), number of teeth of pinion ($x_3$ integer variable), distance between bearings 1 ($x_4$), distance between bearings 2 ($x_5$), diameter of shaft 1 ($x_6$), and diameter of shaft 2 ($x_7$) as well as eleven constraints.

Minimise: $f_1(x) = 0.7854 \times (1) \times (2) \times (3.3333 \times x(3)^2 + 14.9334 \times x(3) - 43.0934) - 1.508 \times x(1) \times (x(6)^2 + x(7)^2) + 7.4777 \times (x(6)^3 + x(7)^3) + 0.7854 \times (x(4) \times x(6)^2 + x(5) \times x(7)^2)$

Minimise: $f_2(x) = ((\sqrt{(((745 \times x(4)) / (x(2) \times x(3)))^2 + 16.96)} / (0.1 \times \ldots x(6)^3)) - 1$

Where:

$g_1(x) = 27 / (x(1) \times x(2)^2 \times x(3)) - 1$

$g_2(x) = 397.5 / (x(1) \times x(2) \times x(3)^2) - 1$

$g_3(x) = (1.93 \times x(4)^3) / (x(2) \times x(3) \times x(6)^4) - 1$

$g_4(x) = (1.93 \times x(5)^3) / (x(2) \times x(3) \times x(7)^4) - 1$

$g_5(x) = (\sqrt{(((745 \times x(4)) / (x(2) \times x(3)))^2 + 16.96)} / (110 \times x(6)^3)) - 1$

$g_6(x) = ((\sqrt{(((745 \times x(5)) / (x(2) \times x(3)))^2 + 157.5^6)) / (85 \times x(7)^3)) - 1$

$g_7(x) = ((x(2) \times x(3)) / 40) - 1$

$g_8(x) = (5 \times x(2) / x(1)) - 1$

$g_9(x) = (x(1) / 12 \times x(2)) - 1$

$g_{10}(x) = ((1.5 \times x(6) + 1.9) / x(4)) - 1$

$g_{11}(x) = ((1.1 \times x(7) + 1.9) / x(5)) - 1$

$2.6 \leq x_1 \leq 3.6, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 7.3 \leq x_4 \leq 8.3, 7.3 \leq x_5 \leq 8.3, 2.9 \leq x_6 \leq 3.9$

$5 \leq x_7 \leq 5.5$

Welded beam design problem:

The welded beam design problem has four constraints first proposed by Ray and Liew [44]. The fabrication cost ($f_1$) and deflection of the beam ($f_2$) of a welded beam should be minimized in this problem. There are four design variables: the thickness of the weld ($x_1$), the length of the clamped bar ($x_2$), the height of the bar ($x_3$) and the thickness of the bar ($x_4$).

Minimise: $f_1(x) = 1.10471 \times x(1)^2 \times x(2) + 0.04811 \times x(3) \times x(4) \times (14.0 + x(2))$

Minimise: $f_2(x) = 65856000 / (30 \times 10^6 \times x(4) \times x(3)^3)$

Where:

$g_1(x) = \tau = 13600$

$g_2(x) = \sigma = 30000$

$g_3(x) = x(1) - x(4)$
Disk Brake Design Problem:
The disk brake design problem has mixed constraints and was proposed by Ray and Liew [44]. The objectives to be minimized are: stopping time \( f_1 \) and mass of a brake \( f_2 \) of a disk brake. As can be seen in following equations, there are four design variables: the inner radius of the disk \( x_1 \), the outer radius of the disk \( x_2 \), the engaging force \( x_3 \), and the number of friction surfaces \( x_4 \) as well as five constraints.

Minimise:
\[
 g_4(x) = 6000 - P
\]
\[
 0.125 \leq x_1 \leq 5 \quad 0.1 \leq x_2 \leq 10 \quad 0.1 \leq x_3 \leq 10 \quad 0.125 \leq x_4 \leq 5
\]

\[
 Q = 6000 \times \left( 14 + \frac{x(2)}{2} \right) ; D = sqrt \left( \frac{(x(2))^2}{4} + \frac{(x(1) + x(3))^2}{4} \right)
\]
\[
 J = 2 \times \left( x(1) \times x(2) \times sqrt(2) \times \left( \frac{x(2)^2}{12} + \frac{(x(1) + x(3))^2}{4} \right) \right)
\]
\[
 alpha = \frac{6000}{sqrt(2) \times x(1) \times x(2)}
\]
\[
 beta = Q \times \frac{D}{J}
\]
\[
 tau = sqrt \left( alpha^2 + 2 \times alpha \times beta \times \frac{x(2)}{2 \times D} + beta^2 \right)
\]
\[
 sigma = \frac{504000}{x(4) \times x(3)^2}
\]
\[
 tmpf = 4.013 \times \frac{30 \times 10^6}{196}
\]
\[
 P = tmpf \times sqrt \left( \frac{x(3)^2 \times x(4)^6}{36} \right) \times \left( 1 - x(3) \times \frac{sqrt(30)}{28} \right)
\]

Minimise:
\[
 f_1(x) = 4.9 \times (10^{-5}) \times (x(2)^2 - x(1)^2) \times (x(4) - 1)
\]
Minimise:
\[
 f_2(x) = (9.82 \times (10^6)) \times (x(2)^2 - x(1)^2) / ((x(2)^3 - x(1)^3) \times x(4) \times x(3))
\]
Where:
\[
 g_1(x) = 20 + x(1) - x(2)
\]
\[
 g_2(x) = 2.5 \times (x(4) + 1) - 30
\]
\[
 g_3(x) = (x(3))/(3.14 \times (x(2)^2 - x(1)^2) \times 2) - 0.4
\]
\[
 g_4(x) = (2.22 \times 10^{-3}) \times x(3) \times (x(2)^3 - x(1)^3) / ((x(2)^2 - x(1)^2) \times 2) - 1
\]
\[
 g_5(x) = 900 - (2.66 \times 10^2) \times x(3) \times x(4) \times (x(2)^3 - x(1)^3) / ((x(2)^2 - x(1)^2))
\]
\[
 55 \leq x_1 \leq 80, 75 \leq x_2 \leq 110, 1000 \leq x_3 \leq 3000, 2 \leq x_4 \leq 20
\]