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Compressible Flow Analysis through Spreadsheet

By Bhupinder Singh Gill

Abstract- The compressible flow analysis is traditionally done by referring to tables where various ratios are listed for various values of Mach number M . Here a different approach is presented which does not require any reference to any tables and the problems can be solved much more comprehensibly and which gives more accurate values.

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Compressible Flow Analysis through Spreadsheet

Bhupinder Singh Gill

Abstract- The compressible flow analysis is traditionally done by referring to tables where various ratios are listed for various values of Mach number M. Here a different approach is presented which does not require any reference to any tables and the problems can be solved much more comprehensively and which gives more accurate values.

I. INTRODUCTION

In aerodynamics the air flows over (aircraft wing) or inside (pipes and nozzles) solid bodies. Low speed flows are usually treated as 'incompressible', that is, changes in density of air as it flows are ignored (density is assumed constant). However, that is only an assumption and not a reality; in fact Anderson¹ calls it a myth. The analysis of flows as incompressible will always be in error, howsoever small, but the results may be in error by a small amount which may be acceptable in practical life. It would be preferable if the analysis can be carried out as compressible flow provided the mathematics is not too involved. That is an aim of this communication.

II. ANALYSIS OF NORMAL SHOCK WAVE

Fig. 1 shows the sketch of a normal shock wave. V_1 , P_1 , T_1 and ρ_1 are the velocity, pressure, temperature and density of air just before the shock wave. V_2 , P_2 , T_2 and ρ_2 are the parameters just after the shock wave.

The parameters are related to each other by the following equations:

$$\text{Continuity: } \rho_1 \cdot V_1 = \rho_2 \cdot V_2 \quad (1)$$

$$\text{Momentum: } P_1 + \rho_1 \cdot V_1^2 = P_2 + \rho_2 \cdot V_2^2 \quad (2)$$

$$\text{Gas law: } P_1 = \rho_1 \cdot R \cdot T_1 \text{ and } P_2 = \rho_2 \cdot R \cdot T_2 \quad (3)$$

Where R is the gas constant. $R = 287 \text{ J/kg.K}$ for air.

Energy equation:

$$c_p \cdot T_1 + \frac{V_1^2}{2} = c_p \cdot T_2 + \frac{V_2^2}{2} \quad (4)$$

Dividing eqn 2 by eqn 1:

$$\frac{P_1}{\rho_1 V_1} + V_1 = \frac{P_2}{\rho_2 V_2} + V_2 \quad (5)$$

Using gas law (eqn.3), eqn 5 can be simplified to:

$$R \cdot \left(\frac{T_1}{V_1} - \frac{T_2}{V_2} \right) = V_2 - V_1 \quad (6)$$

$$\text{Or } \frac{T_2}{V_1^2} = T_1 \frac{V_2}{V_1^3} - \left(\frac{V_2}{V_1} \right)^2 \frac{1}{R} + \left(\frac{V_2}{V_1} \right) \frac{1}{R} \quad (7)$$

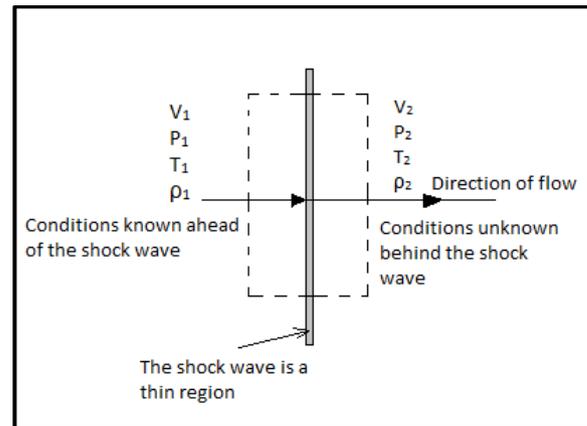


Fig. 1: Sketch of a normal shock wave

Knowing that $c_p = \frac{\gamma R}{\gamma - 1}$ energy equation (4) can be simplified to:

$$\left(\frac{V_2}{V_1} \right)^2 + \frac{2\gamma R}{\gamma - 1} \frac{T_2}{V_1^2} = 1 + \frac{2\gamma R}{\gamma - 1} \frac{T_1}{V_1^2} \quad (8)$$

Substituting T_2 from eqn 7 into eqn 8 and simplifying:

$$\left(\frac{V_2}{V_1} \right)^2 \left(1 - \frac{Z}{R} \right) + Z V_2 \left(\frac{T_1}{V_1^3} + \frac{1}{V_1 R} \right) = 1 + \frac{Z T_1}{V_1^2} \quad (9)$$

where $Z = \frac{2\gamma R}{\gamma - 1}$.

It may be noted that V_2 can be determined from eqn 9 since all other parameters pertain to upstream of the shock wave and are known.

With V_2 known, T_2 can be calculated from eqn 7. Eqn 5 can be simplified with the help of eqn 1:

$$P_2 = P_1 + \rho_1 V_1^2 - \rho_1 V_1 V_2 \quad (10)$$

Downstream pressure P_2 can be calculated from eqn 10.

Downstream density ρ_2 can be calculated from eqn 3 as all parameters are known.

III. SOLUTION OF EQUATIONS

V_2 can be determined from eqn 9. It is, however, not easy to solve the equation. The 'goal seek' facility of spreadsheet (like Excel of Microsoft Office) is used for this purpose. Once V_2 is known, temperature, pressure and density downstream of the shockwave are easily calculated.

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It can thus be seen that it is not necessary to read Mach number from any table for solving the normal shock wave problem. The sonic velocity and Mach number at both upstream and downstream regions of the shock wave can be easily calculated.

Table 1 gives one example of solution of a normal shock wave problem.

	A	B	C	D
1		Table 1: Solution of normal shock wave problem		
2			Case 1	Case 2
3		γ	1.4	1.4
4		R, J/kg.K	287	287
5		V1, m/s	680	680
6		T1, K	288	288
7		P1, Pa	101320	101320
8		P1, atm	1	1
9		Ro1, kg/m3	1.226	1.226
10		Z	2009.000	2009.000
11		RHS	2.251	2.251
12		a	-6.000	-6.000
13		b	0.012134236	0.012134236
14		V2	680.042	255.139
15		LHS	2.251	2.251
16		c	288.018	108.059
17		d	1611.351	226.816
18		e	1611.250	604.512
19		T2, K	287.917	485.755
20		P2, Pa	101284.592	455460.903
21		P2, atm	1.000	4.495
22		Ro2, kg/m3	1.226	3.267
23		c1	340.174	340.174
24		c2	340.125	441.788
25		M1	1.999	1.999
26		M2	1.999	0.578
27		P01, atm	3.797	3.797
28		P02, atm	3.797	5.545

Procedure for arriving at the solutions is described below.

In this example $V_1 = 680$ m/s which means the flow is supersonic before the shock wave. The flow will then be subsonic after the shock wave. This means there will be two solutions of the equations: one where there is no shock and the second where a shock wave occurs. To arrive at the first solution (case 1), assume an arbitrary value of V_2 greater than 680 m/s, say 800 m/s. In this case LHS is 1.403 (cell C15). Click on Data and then What-if Analysis in the spreadsheet. Then select Goal Seek. In Set Cell enter C15, the cell that contains value of LHS. In 'To value' enter 2.251, the value of RHS in cell C11. In 'By changing cell' enter C14, the cell that contains the value of V_2 . Click OK. The cell C14 will now

show value close to 680 which is the same as value of V_1 . This means that no shock has occurred in this case. This shows that the Excel has been properly filled. Click OK. The values of T_2 , P_2 , ρ_2 , c_2 , M_2 and P_{02} in cells C19, C20, C22, C24, C26 and C28 respectively will automatically update.

If a shock has occurred then, downstream of the shock wave, the velocity will be subsonic. Refer to values in column D (case 2). Enter a small value, say 50, in cell D1. Values in subsequent cells will change. Use Data-What if analysis-Goal Seek as before and enter D15 in Set Cell, 2.251 in 'To Value' and D14 in 'By changing cell' and click OK. The value of V_2 in cell D14 is now 255.139 m/s. The values of T_2 , P_2 , ρ_2 , c_2 , M_2 and P_{02} are automatically updated. It can be noted that $P_2 = 4.495$ atm and $M_2 = 0.578$. A complete solution of the problem is thus in the Spreadsheet.

It can be observed that there was no need to read Mach number value from charts; the problem could be easily solved in a normal mathematical fashion. However, if desired values of sonic velocity and mach number before and after the shock could be easily calculated as shown in Table 1. Even total pressure values upstream and downstream of the normal shock could be calculated.

IV. PITOT TUBE ANALYSIS

Pitot static tubes are used to measure flow velocities. There are three cases, viz. incompressible subsonic, compressible subsonic and compressible supersonic. These cases are analyzed below.

a) Incompressible subsonic flow

The instrument used to measure velocity of fluid at any location is called Pitot-static tube. The Pitot has two openings, one which faces opposite to the flow direction and another which is perpendicular to the flow direction. The former measures the total or stagnation pressure at the location of the Pitot tube and the latter measures the static pressure. No flow takes place through any of these two openings. The stagnation pressure represents a sum of the static pressure and the dynamic pressure. The difference between the stagnation pressure and the static pressure is measured by appropriate means. Bernoulli's equation is applied to the two readings. The stagnation pressure P_0 is related to the static pressure P_1 by the equation:

$$P_1 + \frac{1}{2} \rho V_1^2 = P_0 \tag{11}$$

from which velocity V_1 is determined as:

$$V_1 = \sqrt{\frac{2(P_0 - P_1)}{\rho}} \tag{12}$$

As an example, if $P_0 = 104326$ Pa, $P_1 = 101325$ Pa and $\rho = 1.223$ kg/m³,

$$V_1 = \sqrt{\frac{2(104326-101325)}{1.225}} = 70 \text{ m/s} \quad (13)$$

b) *Compressible subsonic flow*

The Pitot tube can also be used to determine flow velocity in high speed but subsonic flow. The formula for velocity V1 in a compressible subsonic flow is:

$$V_1 = \left[\frac{2\gamma P_1}{\rho(\gamma-1)} \left\{ \left(\frac{P_0}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \right]^{\frac{1}{2}} \quad (14)$$

For the case with P0 = 1104326 Pa, P1 = 101325 Pa, γ = 1.4 and ρ = 1.225 kg/m3,

V1 = 69.61 m/s. It may be noted that the difference between incompressible velocity and compressible velocity is hardly 0.39 m/s or 0.56%.

In case P0 = 140000 Pa, the incompressible velocity will be 251.28 m/s whereas compressible velocity will be 236.63 m/s. The difference is 14.65 m/s or 6.19% which may be difficult to ignore.

c) *Supersonic flow*

When a Pitot tube is used to measure flow velocity in supersonic flow, it acts as an obstruction to the flow and velocity is brought down to zero in front of it. But transition from supersonic flow to subsonic flow means a shock has occurred somewhere on the way. It is apparent that the Pitot tube will measure total pressure behind the shock; hence it is necessary to determine relationship between dynamic or total pressure behind the shock and flow velocity before the shock. Table 2 shows the calculation.

Column C gives data for a random case in supersonic flow. In the case shown, flow velocity of 350 m/s (cell C7) will give rise to Pitot tube reading of 198845 Pa (cell C26). Consider a Pitot tube reading of 275000 Pa for which we need to determine flow velocity. A two or three step iteration will be needed.

	A	B	C	D	E	F
1						
2		<i>Table 2: Supersonic Pitot Tube</i>				
3				Step 1	Step 2	Step 3
4		cp, J/kg.K	1004.5	1004.5	1004.5	1004.5
5		γ	1.4	1.4	1.4	1.4
6		R, J/kg.K	287	287	287	287
7		V1, m/s	350.0	446.8	441.3	441.5
8		T1, K	287	287	287	287
9		P1, Pa	101320	101320	101320	101320
10		P1, atm	1	1	1	1
11		Ro1, kg/m3	1.230	1.230	1.230	1.230
12		Z	2009	2009	2009	2009
13		RHS	5.707	3.889	3.961	3.958
14		a	-6.000	-6.000	-6.000	-6.000
15		b	0.033448	0.022135	0.022572	0.022552
16		V2, m/s	332.9	289.7	291.4	291.4
17		LHS	5.707	3.890	3.961	3.958
18		c	272.959	186.133	189.493	189.387
19		d	386.088	292.512	295.794	295.794
20		e	405.948	451.028	448.000	448.252
21		T2, K	292.8	344.6	341.7	341.8
22		P2, Pa	108692	187607	182703	182884
23		P2, atm	1.073	1.852	1.803	1.805
24		Ro2, kg/m3	1.293	1.897	1.863	1.864
25		T02, K	348.0	386.4	384.0	384.1
26		P02, Pa	198845	280029	274774	275000
27		P02, atm	1.962	2.764	2.712	2.714
28		c1, m/s	339.6	339.6	339.6	339.6
29		c2, m/s	343.0	372.1	370.5	370.6
30		M1	1.031	1.316	1.300	1.300
31		M2	0.970	0.779	0.786	0.786

Under Data-What if Analysis-Goal seek, 'Set Cell' to C26, 'To Value' to 275000, 'By changing cell' to C7, and enter. The values in the column C will change. Use Data-What if Analysis-Goal seek again and ensure that cell C17 reads 3.889 (same as in cell C13) and cell C16 reads 289.7 m/s. (Column D in Table 2 depicts values that one will see in column C). The value in column C26 will now read 280029 which is somewhat different from the desired value of 275000. Repeat the iteration and we get the value of 274774 in cell C26 (as depicted in cell E26 in Table 2). One more iteration gives value of 275000 (the desired figure) in cell C26 and the corresponding figure of velocity V_1 (441.5 m/s) in cell C7 (as depicted in cell F7 in Table 2). Thus a Pitot tube reading of 275000 Pa (equivalent to 2.714 atm)

corresponds to $V_1 = 441.5$ m/s. Sonic velocity upstream of shock is calculated as 339.6 m/s and downstream of shock as 370.6 m/s. Mach numbers upstream and downstream of shock are 1.300 and 0.786 respectively.

Another example is shown in Table 3. It can be seen that Pitot tube reading of 1221980 Pa (12.06 atm) represents a velocity upstream of shock as 1018.7 m/s. Sonic velocity upstream of shock is calculated as 339.6 m/s and downstream of shock as 555.8 m/s. Mach numbers upstream and downstream of shock are 3.0 and 0.475 respectively. Complete information about the example like downstream velocity, pressure, temperature and density is available in the spreadsheet. There never was a need to refer to any tables.

	A	B	C	D	E	F
1		<i>Table 3: Supersonic Pitot Tube</i>				
2				Step 1	Step 2	Step 3
3		cp, J/kg.K	1004.5	1004.5	1004.5	1004.5
4		γ	1.4	1.4	1.4	1.4
5		R, J/kg.K	287	287	287	287
6		V_1 , m/s	350.0	1034.3	1019.0	1018.7
7		T_1 , K	287	287	287	287
8		P_1 , Pa	101320	101320	101320	101320
9		P_1 , atm	1	1	1	1
10		ρ_1 , kg/m ³	1.230	1.230	1.230	1.230
11		Z	2009	2009	2009	2009
12		RHS	5.707	1.539	1.555	1.556
13		a	-6.000	-6.000	-6.000	-6.000
14		b	0.033448	0.007289	0.007414	0.007417
15		V_2 , m/s	332.9	265.4	264.1	264.1
16		LHS	5.707	1.540	1.555	1.555
17		c	272.959	73.649	74.377	74.400
18		d	386.088	245.443	243.002	243.002
19		e	405.948	956.451	937.674	937.393
20		T_2 , K	292.8	784.7	769.0	768.8
21		P_2 , Pa	108692	1079458	1047631	1046964
22		P_2 , atm	1.073	10.654	10.340	10.333
23		ρ_2 , kg/m ³	1.293	4.793	4.746	4.745
24		T_2 , K	348.0	819.7	803.8	803.5
25		P_02 , Pa	198845	1257929	1222696	1221980
26		P_02 , atm	1.962	12.415	12.067	12.060
27		c_1 , m/s	339.6	339.6	339.6	339.6
28		c_2 , m/s	343.0	561.5	555.9	555.8
29		M1	1.031	3.046	3.001	3.000
30		M2	0.970	0.473	0.475	0.475



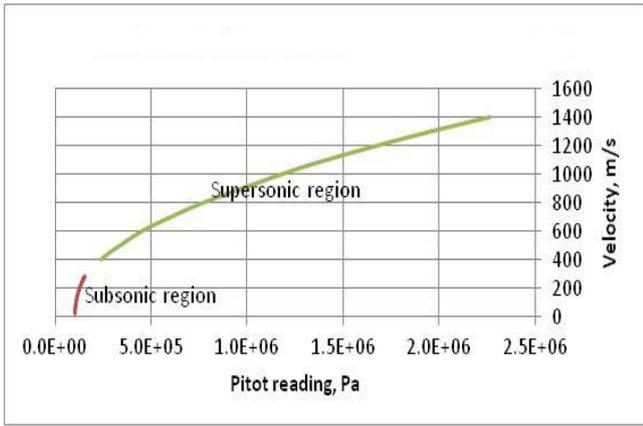


Fig. 2: Flow velocity as a function of Pitot tube reading

Fig. 2 shows the variation of flow velocity as a function of Pitot tube reading for both subsonic and supersonic regions.

V. FLOW THROUGH NOZZLES

The spreadsheet solution of compressible flow is covered in literature². What follows is an improved version of the same. Consider a convergent-divergent nozzle as shown in fig. 3.

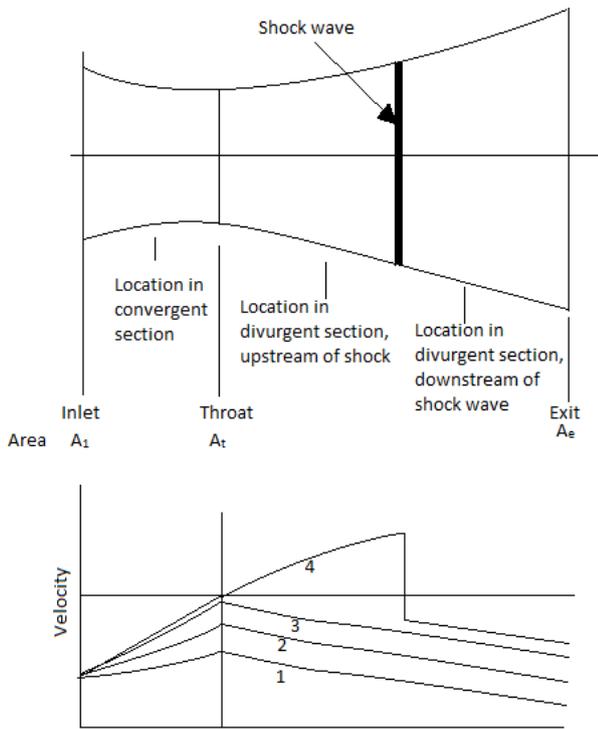


Fig. 3: A typical convergent-divergent nozzle. Curves 1, 2, 3 and 4 are for different pressure values at exit

Equation of state: $p = \rho RT$ where ρ is density of the fluid, p is the absolute pressure and T is the absolute temperature. R is the gas constant.

Continuity equation: $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ or $\rho_1/\rho_2 = (A_2/A_1) (V_2/V_1)$ (A_1 and A_2 are cross sectional areas of two

locations in the nozzle and V_1 and V_2 are velocities at these locations respectively. Location 2 could be anywhere in the nozzle between inlet and exit).

From equation of state:

$$T_2 = (p_2/p_1) (A_2/A_1) (V_2/V_1) \cdot T_1 \tag{15}$$

Steady state energy equation (neglecting potential energy):

$$\frac{V_2^2}{2} + \frac{\gamma}{\gamma-1} RT_2 = \frac{V_1^2}{2} + \frac{\gamma}{\gamma-1} RT_1$$

Or

$$V_2^2 + Z \cdot T_2 = V_1^2 + Z \cdot T_1 \tag{16}$$

where $Z = 2\gamma R/(\gamma-1) = 2009$ for air.

For isentropic flow: $\frac{p_1}{\rho_1^\gamma} = \frac{p_2}{\rho_2^\gamma}$

$$\left(\frac{p_1}{p_2}\right) = \left(\frac{A_2}{A_1}\right)^\gamma \left(\frac{V_2}{V_1}\right)^\gamma \tag{17}$$

From equations 1, 2 and 3,

$$\left(\frac{V_2}{V_1}\right)^2 + Z \cdot \left(\frac{V_2}{V_1}\right)^{(1-\gamma)} \left(\frac{A_1}{A_2}\right)^{(\gamma-1)} \frac{T_1}{V_1^2} = 1 + \frac{Z \cdot T_1}{V_1^2} \tag{18}$$

Velocity V_2 at the downstream location can be calculated from eqn (18) for any value of A_2 since all other information pertains to upstream and is known.

	A	B	C	D
1		<i>Table 4: Nozzle flow analysis</i>		
2				
3		cp	1004.5	1004.5
4		γ	1.4	1.4
5		R	287	287
6		A1	0.007854	0.00785398
7		P1, atm	1	1
8		P1, Pa	1.01E+05	1.01E+05
9		T1, K	313	313
10		V1, m/s	100	100
11		Ro1, kg/m3	1.13E+00	1.13E+00
12		Z	2009	2009
13		RHS	63.8817	63.8817
14		V2, m/s	99.999787	258.772587
15		V2/V1	0.9999979	2.58772587
16		A2	0.007854	0.00384845
17		A1/A2	0.9999977	2.04081633
18		LHS	63.88169	63.8808525
19		P2, Pa	1.01E+05	7.27E+04
20		P2, atm	1.00E+00	7.17E-01
21		P1/P2	1.00E+00	1.39E+00
22		T2, K	312.99997	284.641749
23		T1/T2	1.0000001	1.09962787
24		Ro2, kg/m3	1.13E+00	8.90E-01
25		Ro1/Ro2	1.00E+00	1.27E+00
26		c1, m/s	354.63136	354.631358
27		c2, m/s	354.63134	338.184941
28		M1	0.281983	0.28198296
29		M2	0.2819824	0.76518069

Pressure P_2 at the downstream location can be calculated from eqn (17) and temperature T_2 can be calculated from eqn (15). All the information is available to determine the density at the downstream location. Table 4 is a sample of the nozzle flow analysis. It may be noted that column C is for parameters at the first section and column D is for second section where areas $A_1 = 0.00785$ sq m and $A_2 = 0.00385$ sq m. Also this table represents data for the subsonic solution of the problem. A second solution exists for the supersonic section of the problem. To arrive at the supersonic section of the solution, give a large value (say 1000) of V_2 (row 14) as the initial guess of V_2 and solve the problem through 'What-if-Analysis'.

VI. CONCLUSION

It is shown that compressible flow can be analyzed without going through the step of referring to pre-calculated tables. Use of spreadsheet makes it possible. In general, the reading of Mach number M may lie between two values and interpolation is required; the interpolated value could be in error. Also the complete problem is solved in one step, i.e. all parameters are determined in one step.

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About the Presence of Irregular Precession Motions in a Symmetric Euler Gyroscope

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Annotation- It is generally accepted that the only type of motion present in a symmetric Euler gyroscope (SEG) is regular precession. This paper proves that regular precession is not the only type of motion present, but corresponds only to the well-known initial coordinated Euler angles. At any other initial angles, motions that differ from regular precession occur. In the article, the problem is solved analytically in two stages: first, angular velocities of the gyroscope are determined using differential dynamic equations, at the second stage, as a result of integration of differential matrix kinematic and differential matrix Poisson equations (both with periodic coefficients), final relations about the SEG motion with arbitrary initial Euler angles are derived. Periodic coefficients are the SEG angular velocities that are found as a solution to the dynamic equations. From the obtained general formulas, special formulas of regular precession for particular coordinated initial Euler angles that coincide with the well-known ones are derived.

GJRE-D Classification: FOR Code: 090199



ABOUT THE PRESENCE OF IRREGULAR PRECESSION MOTIONS IN A SYMMETRIC EULER GYROSCOPE

Strictly as per the compliance and regulations of:



RESEARCH | DIVERSITY | ETHICS

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About the Presence of Irregular Precession Motions in a Symmetric Euler Gyroscope

P.K. Plotnikov

I. ANNOTATION

It is generally accepted that the only type of motion present in a symmetric Euler gyroscope (SEG) is regular precession. This paper proves that regular precession is not the only type of motion present, but corresponds only to the well-known initial coordinated Euler angles. At any other initial angles, motions that differ from regular precession occur. In the article, the problem is solved analytically in two stages: first, angular velocities of the gyroscope are determined using differential dynamic equations, at the second stage, as a result of integration of differential matrix kinematic and differential matrix Poisson equations (both with periodic coefficients), final relations about the SEG motion with arbitrary initial Euler angles are derived. Periodic coefficients are the SEG angular velocities that are found as a solution to the dynamic equations. From the obtained general formulas, special formulas of regular precession for particular coordinated initial Euler angles that coincide with the well-known ones are derived. For other initial angles, formulas for irregular precession are obtained. In addition to the solutions for the Euler angles, solutions for the Euler-Krylov angles were found, which in some cases provide a more explicit geometric interpretation of motion. The analytical results are supported by mathematical modeling. In particular, certain conditions were found – the “strong impact” condition when irregular SEG precession for the Euler-Krylov angles occurs in the direction of the rotational pulse, and the sign of the angular velocity of the gyroscope proper rotation changes to the opposite. At the Euler angles, the motions of irregular precession during the “strong” and “weak” impact conditions are qualitatively identical. In relation to the case of regular precession under the “strong” impact conditions, the changes are significant: the angles of precession and nutation become oscillatory, and the angular velocity and the angle of proper rotation change their sign to the opposite.

a) Relevance

Modern gyroscopic technology has achieved the highest accuracy in measuring angular motion parameters of moving objects (MO) in the field of classical symmetric Euler gyroscopes with electrostatic

suspension. In the US Gravity Probe experiment, the four axially symmetric Euler gyroscopes with electrostatic cryogenic suspension mounted on the astronomical Earth satellite had values of drift angular velocities of less than 10^{-11} angular deg/hr. This, together with the telescope readings, experimentally confirms the Einsteinian general theory of relativity (GTR) by detecting a gyro axis shift with the accuracy of 1% equal to 6.6 angular seconds per year, which is effectively predicted by the GTR [1, 2]. It is noted that classical symmetric Euler gyroscopes (SEG) with electrostatic suspensions have drift angular velocities values of 10^{-5} angular deg/hr in terrestrial conditions, which is a better accuracy level than that of fiber optic (FOG) and laser (LG) gyroscopes, i.e. gyros based on new physical measurement principles in which drift angular velocities values are in the range of 10^{-4} - 10^{-3} angular deg/hr, respectively [2]. Considering the fact that rotary classical Euler gyroscopes with magnetic active and magnetic resonance suspensions are still being developed and manufactured, it can be stated that studies concerning angular motions of the rotor's axis of proper rotation, which characterize its errors, are relevant. In this aspect, for a symmetric Euler gyroscope designed for GTR validation [1, 4], the parameters of its regular precession are evaluated, i.e. its errors, including the Poinot analysis. A fundamental presentation of the theory of symmetric Euler gyroscopes with the Poinot and McCullagh analyses of motion is given in [5–6].

It should be recalled that elementary particles - electrons, protons, etc. are essentially Euler gyroscopes [3] (one can say that the entire Universe consists of corpuscular Euler gyroscopes), which also emphasizes the relevance of this study.

b) Formulation of the problem

The solution to the problem of inertial motion of a symmetric Euler gyroscope is well known and described in many works, in particular, in [1-2]. This motion is regular precession, characterized by a constant angle of nutation between the kinetic moment axis, superimposed with the inertial basis axis, and the axis of SEG proper rotation. At the same time, the angular velocities of precession and nutation are constant.

The indicated properties have found application in [4] in the process of preparation of an experiment to validate the general theory of relativity using a SEG and

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a telescope on an artificial Earth satellite when solving the problem of selection of relations between the primary moments of inertia that provide very low angular precession velocities. In the experiment [1], drift angular velocities values were less than 10^{-11} angular deg/hr, which validated the Einsteinian general theory of relativity with an error of less than 1%.

It should be noted that the solution to the problem of regular precession was possible with the following restrictions on the initial Euler angles [6, formulas (2.39), (2.41)]:

$$\psi_0 = const; \quad \varphi_0 = 0; \quad \cos \theta_0 = const = \frac{Cr_0}{G},$$

where G is the kinetic moment; r_0 is the SEG proper rotation angular velocity component; C is the primary moment of SEG inertia around the same axis.

This paper sets the task of finding the solution to the problem of SEG motion for arbitrary initial angles not only along the precession angle ψ_0 , but also along the initial angles of nutation and proper rotation. The Poisson differential kinematic equations are used for this purpose. To clarify the problem formulation, let us cite a statement on this subject from the work [6, p. 79]. The first step in solving the problem is to determine the angular velocities of the body. This is solved analytically regardless of the Euler angles. The second step consists of determining the Euler angles by integrating the kinematic equations due to the angular velocities

found in the first step. This long and arduous process is eased by applying the kinetic moment theorem and the method of selection of a coordinate system, one of the axes of which coincides with the kinetic moment vector [5-6], etc. For this article, we chose the way of integration of the matrix differential equations in quaternions, as well as of Poisson equations by means of solving the Cauchy problem with arbitrary initial angles, which is not related to the special selection of a coordinate system, one of the axes of which is directed along the kinetic moment vector of the SEG.

II. ON THE INFLUENCE OF INITIAL CONDITIONS FOR KINEMATIC EQUATIONS ON THE NATURE OF MOTIONS IN A SYMMETRIC EULER GYROSCOPE

In this section, we set the task to clarify the range of values of the initial Euler angles for the kinematic equations of the symmetric Euler gyroscope, with which they are reduced to identities - after substituting their analytical solutions given in [7], as well as the solutions of dynamic equations given in [6]. Since these solutions describe regular precession, we are talking about the initial conditions under which it is observed, and under which it is not.

Dynamic equations for a symmetric Euler gyroscope have the form [7, p. 126]:

$$\begin{aligned} A \frac{dp}{dt} + (C - A)qr &= 0 \\ A \frac{dq}{dt} + (A - C)rp &= 0 \\ C \frac{dr}{dt} = 0; \quad \frac{dr}{dt} = 0; \quad r = r_0 = const \end{aligned} \tag{A.1}$$

The kinematic Euler equations [7, p. 115]:

$$\left. \begin{aligned} p &= \dot{\psi} \sin \theta \sin \varphi + \dot{\theta} \cos \varphi \\ q &= \dot{\psi} \sin \theta \cos \varphi - \dot{\theta} \sin \varphi \\ r &= \dot{\psi} \cos \theta + \dot{\varphi} \end{aligned} \right\} \tag{A.2}$$

The solutions of these equations obtained in [7, p. 37]:

$$\left. \begin{aligned} \psi &= nt + \psi_0; \quad n = \frac{G}{A} \\ \cos \theta &= \frac{Cr}{G} = \frac{Cr_0}{G}; \quad \theta = \theta_0 = const \\ \varphi &= n_1 t + \varphi_0; \quad n_1 = r_0 - n \cos \theta_0 \end{aligned} \right\} \tag{A.3}$$



Where r_0, G are the constants, $G^2 = (A^2 p^2 + A^2 q^2) + C^2 r_0^2$.

Equations (A.2), resolved in relation to $\dot{\psi}, \dot{\theta}, \dot{\varphi}$ [6, p. 46]:

$$\left. \begin{aligned} \frac{d\psi}{dt} &= (p \sin \varphi + q \cos \varphi) / \sin \theta \\ \frac{d\theta}{dt} &= (p \cos \varphi - q \sin \varphi) \\ \frac{d\varphi}{dt} &= r - ctg\theta \cdot (p \sin \varphi + q \cos \varphi) \end{aligned} \right\} \quad (A.4)$$

Considering the solutions of equations (A.1) and the designations from [6, p. 88] we have:

$$\begin{aligned} p &= \omega_1 = \omega_{10} \sin \nu t; & q &= \omega_2 = \omega_{10} \cos \nu t; & r &= r_0 = \omega_{30}; \\ G^2 &= A^2 \omega_{10}^2 + C^2 \omega_{30}^2; & G &= H; & \dot{\varphi} &= n_1 = \nu; \\ n &= \dot{\psi}; & \sin \theta_0 &= \frac{A \omega_{10}}{G}; & \tan \theta_0 &= \frac{A \omega_{10}}{C \omega_{30}}. \end{aligned} \quad (A.5)$$

Substitution of solutions into equations (A.4)

Substituting (A.5) into (A.4), we obtain, in consideration of (A.3) for the third equation in (A.4):

$$\begin{aligned} r_0 - \frac{G}{A} \cos \theta_0 &= r_0 - \omega_{10} \frac{\sin \dot{\varphi} t \cdot \sin(\dot{\varphi} t + \varphi_0) + \cos \dot{\varphi} t \cdot \cos(\dot{\varphi} t + \varphi_0)}{\tan \theta_0} \\ - \frac{G}{A} \sin \theta_0 &= -\omega_{10} \cos(\dot{\varphi} t + \varphi_0 - \dot{\varphi} t) = -\omega_{10} \cos \varphi_0 \\ - \frac{G}{A} \cdot \frac{A \omega_{10}}{G} &= -\omega_{10} \cos \varphi_0 \approx -\omega_{10} = -\omega_{10} \cos \varphi_0 \end{aligned} \quad (A.6)$$

The equality (A.6) is reduced to an identity when

$$\varphi_0 = 0; \pm 2\pi m \quad (m = 1, 2, 3, \dots) \quad (A.7)$$

That is, the angle φ_0 should be zero. For the equation (A.2) of the system (A.4) we have:

$$\begin{aligned} 0 &= \omega_{10} (\sin \dot{\varphi} t \cdot \cos(\dot{\varphi} t + \varphi_0) - \cos \dot{\varphi} t \cdot \sin(\dot{\varphi} t + \varphi_0)) \\ 0 &= \omega_{10} \sin(\dot{\varphi} t - \dot{\varphi} t - \varphi_0) = -\omega_{10} \sin(\varphi_0) \end{aligned} \quad (A.8)$$

The equality (A.8) is reduced to an identity at the angles $\varphi_0 = 0; \pm \pi m \quad (m = 1, 2, 3, \dots)$.

For the angle $\psi_0 + nt$ from the first equation in (A.4) we have:

$$n = (p \sin \varphi + q \cos \varphi) / \sin \theta = G / A \quad (A.9)$$

In consideration of (A.3) and (A.5) we obtain:

$$\begin{aligned} \frac{G}{A} \sin \theta_0 &= \omega_{10} \cos(\dot{\varphi} t + \varphi_0 - \dot{\varphi} t) = \omega_{10} \cos \varphi_0 \\ \frac{G}{A} \cdot \frac{A \omega_{10}}{G} &= \omega_{10} \cos \varphi_0 \Rightarrow \omega_{10} = \omega_{10} \cos \varphi_0 \end{aligned}$$

That is, we obtain the relations (A.7) once again. This means that the equation (A.9) is reduced to an identity with $\theta = \theta_0$; $\varphi = n_1 t$ for any value of ψ_0 . From (A.9) it also follows that when $\Delta\theta(0)$ of the initial value of the angle θ_0 is varied, i.e. for $\theta = \theta_0 + \Delta\theta(0)$, the equality (A.9) is not reduced to an identity.

From these calculations, we conclude that regular precession in a symmetric Euler gyroscope is possible only with the following values of the initial angles:

$$\psi_0 = const; \quad \varphi_0 = 0; \quad \cos \theta_0 = const = Cr_0/G \tag{A.10}$$

With any other initial values of the Euler angles, the equations (A.4) are reduced to identities with other solutions that do not coincide with the functions (A.3).

The relevance of the article is further reinforced by publications [4-7].

III. PROBLEM SOLUTION

a) Quaternion problem solution

Instead of integrating the degenerate Euler equations

$$\left. \begin{aligned} \frac{d\psi}{dt} &= (p \sin \varphi + q \cos \varphi) / \sin \theta \\ \frac{d\theta}{dt} &= (p \cos \varphi - q \sin \varphi) \\ \frac{d\varphi}{dt} &= r - ctg \theta \cdot (p \sin \varphi + q \cos \varphi) \end{aligned} \right\} \tag{1}$$

In this article, we use the method of integration of quaternion and Poisson matrices that are non-degenerate for any angle value of the equation:

$$2 \frac{dN^1}{dt} = P(t)N^1; \quad N^1(0) = E; \quad \frac{dA^1}{dt} = P(t)A^1; \quad A^1(t) = E; \tag{2}$$

The choice of the two types of equations is related to their widespread use in science and technology, it also enables comparison of their solutions. The coefficients and variables included in the differential equations (2) are indicated below.

Following [6], we present the Euler rotation angles diagram depicting the inertialess frames of the cardan suspension according to Fig. 1. Let us associate the moving coordinate system Oxyz (corresponds to the coordinate system O1'2'3' in [6]) with the gyroscope body, and also introduce inertial coordinate systems: the expanded Oξηζ, system, which coincides with the coordinate system Oxyz at the initial moment, and the original system Oξ₀η₀ζ₀, relative to which the coordinate system Oξηζ is rotated at the initial angles Ψ₀, Θ₀, Φ₀. Figure 2 shows a similarly constructed diagram of the same gyroscope, but for the Euler - Krylov angles (ψ, Θ, φ).



equations. Note that the quaternion matrices are related to the matrices of directional cosines of the angles by the relation $A=M^T N$ [10, 11]. In the article [8], the formulas for the angular velocities p, q, r of the gyroscope are solutions of the dynamic equations of the SEG, which had the initial angular velocity $p(0)=q(0)=0$;

$r(0)=R$, and which was affected by the impact to the axis of the gyroscope figure in the form of a rotational pulse M_0 around the axis Ox (hereinafter, $M_0=H_x$ is the kinetic moment from the impact). The dynamic Euler equations for a gyroscope with a dynamic axis of symmetry have the following form [8]:

$$\left. \begin{aligned} \frac{dp}{dt} + \Omega q &= \frac{M_0}{A} \cdot \frac{d}{dt} [I(t)] \\ \frac{dq}{dt} - \Omega p &= 0 \\ \frac{dr}{dt} &= 0; \quad \Omega = r \cdot \frac{C - A}{A} \end{aligned} \right\} \tag{4}$$

p, q, r are the components of the vector of angular velocity of rotation of the gyroscope in the axes associated with it; $I(t)$ is the unit function.
For initial conditions

$$t=0; p(0)=0; q(0)=0; r(0)=R$$

The solution to the system of differential equations (2) has the following form:

$$p = a \cos \Omega t; \quad q = a \sin \Omega t; \quad r = R; \quad a = \frac{M_0}{A} = \frac{H_x}{A}. \tag{5}$$

The transformation of coordinate systems from the inertial $O\xi\eta\zeta$ to the moving $Oxyz$, in consideration of the initial inertial coordinate system $O\xi_H\eta_H\zeta_H$, according to (3) is determined by the relations:

$$[xyz]^T = A^1 A(0) [\xi_H \eta_H \zeta_H]^T = A [\xi_H \eta_H \zeta_H]^T$$

or, equivalently, through quaternion matrices [10], [11]:

$$[xyz]^T = N^1 M^{1T} M^T(0) N(0) [\xi_H \eta_H \zeta_H]^T; \tag{6}$$

$$N^1 = N^\Phi N^\Theta N^\Psi; \quad N = N^1 N(0); \quad A^1 = M^{1T} N^1; \quad A(0) = M^T(0) N(0),$$

where N, A are the quaternion matrix and the matrix of directional cosines of the resulting rotation; N^1, A^1 are the matrixants; N^Φ, N^Θ, N^Ψ are the quaternion matrices of the corresponding simplest rotations. At the same time, M and N are the corresponding types of quaternion matrices [10, 11].

The matrix of directional cosines of the Euler angles for Fig. 1 when combining the coordinate systems $\xi\eta\zeta$ and $\xi_H\eta_H\zeta_H$:

$$A^1 = \begin{bmatrix} \cos \Psi_1 \cos \Theta_1 \cos \Phi_1 - \sin \Psi_1 \sin \Phi_1 & \sin \Psi_1 \cos \Theta_1 \cos \Phi_1 + \cos \Psi_1 \sin \Phi_1 & -\sin \Theta_1 \cos \Phi_1 \\ -\cos \Psi_1 \cos \Theta_1 \sin \Phi_1 - \sin \Psi_1 \cos \Phi_1 & -\sin \Psi_1 \cos \Theta_1 \sin \Phi_1 + \cos \Psi_1 \cos \Phi_1 & \sin \Theta_1 \sin \Phi_1 \\ \cos \Psi_1 \sin \Theta_1 & \sin \Psi_1 \sin \Theta_1 & \cos \Theta_1 \end{bmatrix}. \tag{7}$$

The matrix of directional cosines of the Euler-Krylov angles (Fig. 2), which is equal to the matrix (7), has the form:

$$A^k = \begin{bmatrix} \cos \varphi \cos \theta & \sin \psi \sin \theta \cos \varphi + \cos \psi \sin \varphi & -\cos \psi \sin \theta \cos \varphi + \sin \psi \sin \varphi \\ -\sin \varphi \cos \theta & -\sin \psi \sin \theta \sin \varphi + \cos \psi \cos \varphi & \cos \psi \sin \theta \sin \varphi + \sin \psi \cos \varphi \\ \sin \theta & -\sin \psi \cos \theta & \cos \theta \cos \psi \end{bmatrix}. \tag{8}$$

The matrix N^1 corresponding to $N^1(0)=E$, i.e. to the angles $\Psi(0)=\Theta(0)=\Phi(0)=0$ (that is, the matrixant), can be determined by integrating the quaternion matrix equation [10, 11]:

$$2 \frac{dN^1}{dt} = P(t) N^1; \quad N^1(0) = E. \tag{9}$$



$$P(t) = \begin{bmatrix} 0 & -p & 0 & -r \\ p & 0 & r & 0 \\ 0 & -r & 0 & p \\ r & 0 & -p & 0 \end{bmatrix}; \quad N^1 = \begin{bmatrix} \lambda_0^1 & -\lambda_1^1 & -\lambda_2^1 & -\lambda_3^1 \\ \lambda_1^1 & \lambda_0^1 & \lambda_3^1 & -\lambda_2^1 \\ \lambda_2^1 & -\lambda_3^1 & \lambda_0^1 & \lambda_1^1 \\ \lambda_3^1 & \lambda_2^1 & -\lambda_1^1 & \lambda_0^1 \end{bmatrix}.$$

The angular velocity matrix in consideration of (5) has the form:

$$P(t) = \begin{bmatrix} 0 & -a \cos \Omega t & -a \sin \Omega t & -R \\ a \cos \Omega t & 0 & R & -a \sin \Omega t \\ a \sin \Omega t & -R & 0 & a \cos \Omega t \\ R & a \sin \Omega t & -a \cos \Omega t & 0 \end{bmatrix}, \tag{10}$$

Which means that it satisfies the condition $P(t) = P(t + \tau)$; $\tau = 2\pi/\Omega$.

Therefore, the system (9) is Lyapunov reducible [8]. By means of substitution

$$N_Z = N_\Phi N^1 \tag{11}$$

The system (9), (10) is reduced to an equivalent differential equation with constant coefficients

$$\frac{dN_Z}{dt} = P_B N_Z. \tag{12}$$

$$N_\Phi = \begin{bmatrix} v_0 & -v_1 & -v_2 & -v_3 \\ v_1 & v_0 & v_3 & -v_2 \\ v_2 & -v_3 & v_0 & v_1 \\ v_3 & v_2 & -v_1 & v_0 \end{bmatrix}; \quad P_B = \begin{bmatrix} 0 & -a & 0 & -R_1 \\ a & 0 & R_1 & 0 \\ 0 & -R_1 & 0 & a \\ R_1 & 0 & -a & 0 \end{bmatrix}. \tag{13}$$

$$R_1 = R \frac{C}{A}; \quad v_0 = \cos \Omega t / 2; \quad v_1 = v_2 = 0; \quad v_3 = \sin \Omega t / 2.$$

Given these formulas, we have:

$$N_\Phi = \begin{bmatrix} \cos \Omega t / 2 & 0 & 0 & -\sin \Omega t / 2 \\ 0 & \cos \Omega t / 2 & \sin \Omega t / 2 & 0 \\ 0 & -\sin \Omega t / 2 & \cos \Omega t / 2 & 0 \\ \sin \Omega t / 2 & 0 & 0 & \cos \Omega t / 2 \end{bmatrix}. \tag{14}$$

The equivalence of equations (9) and (12), (13) is confirmed by the fulfillment of the identity

$$N_\Phi (PN_\Phi^{-1} - \dot{N}_\Phi^{-1}) \equiv P_B \tag{15}$$

The solution to the equation (12) with constant coefficients is the Cauchy formula:

$$N_Z = L(t)L^{-1}(0)N_Z(0), \tag{16}$$

where $L(t)$ is the fundamental matrix of solutions; $N_Z(0)$ is the matrix of initial values of the angles, equal, by condition, to the identity matrix: $N_Z(0) = E$.

After finding the fundamental matrix of solutions and a number of transformations, let us write down the expression (16) in the form:

$$N_Z = \left(E \cos \frac{\chi}{2} + D \frac{\sin \chi/2}{\chi} \right) N_Z(0);$$

$$D = \int_0^t P_B(\tau) d\tau; \chi = (\chi_1^2 + \chi_3^2)^{1/2} = (a^2 + R_1^2)^{1/2} t = n; t$$

$$\chi_1 = \int_0^t a(\tau) d\tau; \chi_3 = \int_0^t R_1(\tau) d\tau; (a^2 + R_1^2)^{1/2} = n; \chi = nt.$$
(17)

After transformations, the matriciant takes the form:

$$N_Z = \begin{bmatrix} \cos \chi/2 & -\frac{a}{n} \sin \chi/2 & 0 & -\frac{R_1}{n} \sin \chi/2 \\ \frac{a}{n} \sin \chi/2 & \cos \chi/2 & \frac{R_1}{n} \sin \chi/2 & 0 \\ 0 & -\frac{R_1}{n} \sin \chi/2 & \cos \chi/2 & \frac{a}{n} \sin \chi/2 \\ \frac{R_1}{n} \sin \chi/2 & 0 & -\frac{a}{n} \sin \chi/2 & \cos \chi/2 \end{bmatrix}$$
(18)

From the expression (11) we have:

$$N^1 = N_\Phi^T N_Z; N = N_\Phi^T N_Z N(0) = N^1 N(0); N^1 = N^1(\lambda_{ak}) (k=0,1,2,3).$$
(19)

In consideration of (13), (14) and (18), the expanded expression for the quaternion matriciant N^1 is derived below.

Since $N = N^1 N(0) = N_\Phi^T N_Z N(0)$, we have the following expression for the quaternion matrix of the resulting rotation N for nonzero initial conditions:

$$N = \begin{bmatrix} n_{a0} & -n_{a1} & -n_{a2} & -n_{a3} \\ n_{a1} & n_{a0} & n_{a3} & -n_{a2} \\ n_{a2} & -n_{a3} & n_{a0} & n_{a1} \\ n_{a3} & n_{a2} & -n_{a1} & n_{a0} \end{bmatrix} \cdot \begin{bmatrix} n_{00} \\ n_{01} \\ n_{02} \\ n_{03} \end{bmatrix}.$$

Formulas for the components of the quaternion matrix N:

$$\left. \begin{aligned} n_0 &= n_{a0} \cdot n_{00} - n_{a1} \cdot n_{01} - n_{a2} \cdot n_{02} - n_{a3} \cdot n_{03} \\ n_1 &= n_{a1} \cdot n_{00} + n_{a0} \cdot n_{01} + n_{a3} \cdot n_{02} - n_{a2} \cdot n_{03} \\ n_2 &= n_{a2} \cdot n_{00} - n_{a3} \cdot n_{01} + n_{a0} \cdot n_{02} + n_{a1} \cdot n_{03} \\ n_3 &= n_{a3} \cdot n_{00} + n_{a2} \cdot n_{01} - n_{a1} \cdot n_{02} + n_{a0} \cdot n_{03} \end{aligned} \right\}$$
(20)

By marking

$$\lambda_{ai} = n_{ai} \quad (i = \overline{0,3}),$$
(21)

We have the explicit form of the formulas for the components of the quaternion matriciant N^1 :

$$\begin{aligned}
 n_{a0} &= \lambda_{a0} = \cos \frac{\Omega t}{2} \cos \frac{nt}{2} + \frac{R_1}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \\
 n_{a1} &= \lambda_{a1} = \frac{a}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \\
 n_{a2} &= \lambda_{a2} = \frac{a}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \\
 n_{a3} &= \lambda_{a3} = -\sin \frac{\Omega t}{2} \cos \frac{nt}{2} + \frac{R_1}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2}
 \end{aligned} \tag{22}$$

For regular precession, the angles of the initial orientation and the components of the initial quaternion are expressed by the formulas:

$$\begin{aligned}
 \Psi_0 &= 0; \quad \Phi_0 = 0; \quad \Theta(0) = \Theta_0; \quad \operatorname{tg} \Theta_0 = -\frac{H_x}{H}; \\
 \lambda_0 &= n_{00} = \cos \frac{\Theta_0}{2}; \quad \lambda_1 = n_{01} = 0; \quad \lambda_2 = n_{02} = \sin \frac{\Theta_0}{2}; \quad \lambda_3 = n_{03} = 0.
 \end{aligned} \tag{23}$$

In this regard, we have:

$$\begin{aligned}
 n_0 &= n_{a0} \cos \frac{\Theta_0}{2} - n_{a2} \sin \frac{\Theta_0}{2} \\
 n_1 &= n_{a1} \cos \frac{\Theta_0}{2} + n_{a3} \sin \frac{\Theta_0}{2} \\
 n_2 &= n_{a2} \cos \frac{\Theta_0}{2} + n_{a0} \sin \frac{\Theta_0}{2} \\
 n_3 &= n_{a3} \cos \frac{\Theta_0}{2} - n_{a1} \sin \frac{\Theta_0}{2}
 \end{aligned}$$

In consideration of (23) we obtain:

$$\begin{aligned}
 n_0 &= \cos \frac{\Omega t}{2} \cos \frac{nt}{2} \cos \frac{\Theta_0}{2} + \frac{R_1}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \cos \frac{\Theta_0}{2} - \frac{a}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \sin \frac{\Theta_0}{2} \\
 n_1 &= \frac{a}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \cos \frac{\Theta_0}{2} - \sin \frac{\Omega t}{2} \cos \frac{nt}{2} \sin \frac{\Theta_0}{2} + \frac{R_1}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \sin \frac{\Theta_0}{2} \\
 n_2 &= \frac{a}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \cos \frac{\Theta_0}{2} - \sin \frac{\Omega t}{2} \cos \frac{nt}{2} \sin \frac{\Theta_0}{2} + \frac{R_1}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \sin \frac{\Theta_0}{2} \\
 n_3 &= -\sin \frac{\Omega t}{2} \cos \frac{nt}{2} \cos \frac{\Theta_0}{2} + \frac{R_1}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \cos \frac{\Theta_0}{2} - \frac{a}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \sin \frac{\Theta_0}{2}
 \end{aligned} \tag{24}$$

After that, let us similarly determine the trigonometric functions for the Euler-Krylov angles ψ , Θ , φ on the basis of the matrix (8) and its quaternion counterpart [8, 9]. We have:

$$\tan \psi = -\frac{a_{32}}{a_{33}} = \frac{2(\lambda_0 \lambda_1 - \lambda_2 \lambda_3)}{\lambda_0^2 + \lambda_3^2 - \lambda_1^2 - \lambda_2^2}$$

$$\begin{aligned} \sin \theta &= a_{31} = 2(\lambda_0 \lambda_2 + \lambda_1 \lambda_3) \\ \tan \varphi &= -\frac{a_{21}}{a_{11}} = \frac{2(\lambda_0 \lambda_3 - \lambda_1 \lambda_2)}{\lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2} \end{aligned} \tag{25}$$

Substituting the quaternion components (22) into these formulas, we obtain

$$\begin{aligned} \tan \psi &= \frac{\frac{a}{n} \sin n t}{\frac{a^2}{n^2} \cos n t + \frac{R_1^2}{n^2}}; \\ \sin \theta &= \frac{a R_1}{n^2} (1 - \cos n t); \\ \tan \varphi &= \frac{-\sin \Omega t \left(\frac{R_1^2}{n^2} \cos n t + \frac{a^2}{n^2} \right) + \frac{R_1}{n} \sin n t \cos \Omega t}{\cos \Omega t \left(\frac{R_1^2}{n^2} \cos n t + \frac{a^2}{n^2} \right) + \frac{R_1}{n} \sin n t \sin \Omega t} \end{aligned} \tag{26}$$

These expressions coincide with formulas (18) [8], confirming the fidelity of the solutions to the problem for zero initial Euler-Krylov angles both in the quaternion form and in the form associated with the application of the Poisson differential kinematic equations.

For arbitrary initial Euler-Krylov angles, explicit solutions can be obtained from relations (24), (25) (in (25), the λ_i must be replaced by values n_i ($i = \overline{0,3}$)).

In turn, for the Euler angles we have the following solutions:

$$\begin{aligned} \tan \Psi &= \frac{a_{32}}{a_{31}} = \frac{\lambda_2 \lambda_3 - \lambda_0 \lambda_1}{\lambda_0 \lambda_2 + \lambda_1 \lambda_3} \\ \cos \Theta &= a_{33} = \lambda_0^2 + \lambda_3^2 - \lambda_1^2 - \lambda_2^2 \\ \tan \Phi &= -\frac{a_{23}}{a_{13}} = \frac{\lambda_0 \lambda_1 + \lambda_2 \lambda_3}{\lambda_1 \lambda_3 + \lambda_0 \lambda_2} \end{aligned} \tag{27}$$

In consideration of (22), we obtain the solutions in explicit form:

$$\begin{aligned} \tan \psi &= \frac{\frac{a}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \left(-\sin \frac{\Omega t}{2} \cos \frac{nt}{2} + \frac{R_1}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \right) - \frac{a}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \left(\cos \frac{\Omega t}{2} \cos \frac{nt}{2} + \frac{R_1}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \right)}{\frac{a}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \left(\cos \frac{\Omega t}{2} \cos \frac{nt}{2} + \frac{R_1}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \right) + \frac{a}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \left(-\sin \frac{\Omega t}{2} \cos \frac{nt}{2} + \frac{R_1}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \right)} \\ \cos \theta &= \left(\cos^2 \frac{\Omega t}{2} \cos^2 \frac{nt}{2} + 2 \frac{R_1}{n} \sin \frac{\Omega t}{2} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \cos \frac{nt}{2} + \frac{R_1^2}{n^2} \sin^2 \frac{\Omega t}{2} \sin^2 \frac{nt}{2} + \sin^2 \frac{\Omega t}{2} \cos^2 \frac{nt}{2} - \right. \\ &\quad \left. - 2 \frac{R_1}{n} \sin \frac{\Omega t}{2} \cos \frac{\Omega t}{2} + \frac{R_1^2}{n^2} \cos^2 \frac{\Omega t}{2} \sin^2 \frac{nt}{2} - \frac{a^2}{n^2} \cos^2 \frac{\Omega t}{2} \sin^2 \frac{nt}{2} - \frac{a^2}{n^2} \sin^2 \frac{\Omega t}{2} \sin^2 \frac{nt}{2} \right) \\ \tan \varphi &= \frac{\frac{a}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \left(\cos \frac{\Omega t}{2} \cos \frac{nt}{2} + \frac{R_1}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \right) + \frac{a}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \left(-\sin \frac{\Omega t}{2} \cos \frac{nt}{2} + \frac{R_1}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \right)}{\frac{a}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \left(-\sin \frac{\Omega t}{2} \cos \frac{nt}{2} + \frac{R_1}{n} \cos \frac{\Omega t}{2} \sin \frac{nt}{2} \right) - \frac{a}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \left(\cos \frac{\Omega t}{2} \cos \frac{nt}{2} + \frac{R_1}{n} \sin \frac{\Omega t}{2} \sin \frac{nt}{2} \right)} \end{aligned} \tag{28}$$

For regular precession in (24), (25), it is necessary to consider λ_i ($i = \overline{0,3}$) according to the expressions (23), and then, after transformations, we obtain:

$$\begin{aligned} \psi &= \psi_0 + nt; \\ \tan \theta &= \frac{aA}{\sqrt{(aA)^2 + CR_1^2}} = a/n; \end{aligned}$$

$$\Phi = (1 - C/A)Rt. \tag{29}$$

The result coincided with the classical one, which is expressed by the formulas (A.3, A.5).

Let us now consider a variant of the solution to the problem for irregular precession. It corresponds to the initial angles $\Phi_0 = \Psi_0 = 0; \tan\theta_0 = \frac{Aa}{CR}$ that differ from the angles (23), which generate regular precession, only by the sign of the angle of nutation. After transformations, the formulas for determining the Euler angles for the SEG are:

$$\begin{aligned} \tan\Psi^* &= -\frac{\sin nt}{2 \cos^2 \theta_0 - \cos 2 \theta_0 \cos nt} \\ \cos \Theta^* &= \frac{\cos \theta_0 \cos 2 \theta_0}{\tan^2 \theta_0} + 2 \sin^2 \theta_0 \cos \theta_0 \cos nt \\ \tan\Phi^* &= \frac{\sin \theta_0 \cos 2\theta_0 \sin \Omega t + \sin 2\theta_0 \sin nt \cos \Omega t}{\sin \theta_0 \cos 2\theta_0 \cos \Omega t - 2 \sin 2\theta_0 \cos nt \cos \Omega t} \end{aligned} \tag{30}$$

The expressions (30) suggest that only the change of the sign of the initial angle of nutation – with the other two initial angles unchanged – caused the appearance of irregular precession motions in the Euler gyroscope.

b) Solution for the Poisson matrix differential equation

The transformation of the coordinate system Oxyz from the initial position Oξηζ is characterized by the formulas:

$$[xyz]^T = A^1[\xi\eta\zeta]; \quad A^1 = A^\Phi A^\Theta A^\Psi, \tag{31}$$

Where A^Φ, A^Θ, A^Ψ are the transformation matrices of the coordinates of the simplest rotations. On the other hand, this matrix can be determined by integrating the Poisson matrix kinematic equation:

$$\frac{dA^1}{dt} = P(t)A^1; \quad A^1(t) = E; \tag{32}$$

$$A^1 = \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 \end{bmatrix}; \quad P(t) = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \tag{33}$$

The matrix of directional cosines of the Euler angles for Fig. 1 when combining the coordinate systems $\xi\eta\zeta$ and $\xi_H\eta_H\zeta_H$ – form (32), and the matrix of directional cosines of the Euler-Krylov angles (Fig. 2) – form (33). The angular velocity tensor for gyroscopes with a dynamic axis of symmetry has the form:

$$P(t) = \begin{bmatrix} 0 & R & -a \sin \Omega t \\ -R & 0 & a \cos \Omega t \\ a \sin \Omega t & -a \cos \Omega t & 0 \end{bmatrix}, \tag{34}$$

That is, it satisfies the condition $P(t) = P(t + \tau); \quad \tau = \frac{2\pi}{\Omega}$.

As a result of this condition, the system (32) - (33) is Lyapunov reducible [13]. Indeed, by substitution

$$Z = \Phi(t)A^1 \tag{35}$$

it is reduced to a matrix equivalent differential equation with constant coefficients

$$\frac{dZ}{dt} = BZ, \tag{36}$$



$$\Phi(t) = \begin{vmatrix} \cos \Omega t & \sin \Omega t & 0 \\ -\sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 1 \end{vmatrix}; \quad B = \begin{vmatrix} 0 & R_1 & 0 \\ -R_1 & 0 & a \\ 0 & -a & 0 \end{vmatrix}; \quad Z = \|Z_{ij}\|; \quad R_1 = R + \Omega; \quad (i, j) = 1; 2; 3. \quad (37)$$

The equivalence of the equations (32) and (36) is confirmed by the validity of the identity $\Phi(t) \cdot (P\Phi^{-1}(t) - \Phi^{-1}(t)) \equiv B$. The differential linear homogeneous equation (36) is solved by the Cauchy formula

$$Z(t) = Q(t) \cdot Q^{-1}(0) \cdot Z(0), \quad (38)$$

Where Q(t) is the fundamental matrix of solutions; Z(0) is the matrix of initial values of directional cosines, and as provided by the condition, Z(0)=E. After finding the fundamental matrix and performing a number of transformations, the solution (38) takes the form:

$$Z = \begin{vmatrix} \frac{R_1^2}{n^2} \cos nt + \frac{a^2}{n^2} & \frac{R_1}{n} \sin nt & -\frac{aR_1}{n^2} (1 - \cos nt) \\ -\frac{R_1}{n} \sin nt & \cos nt & \frac{a}{n} \sin nt \\ \frac{aR_1}{n^2} (1 - \cos nt) & -\frac{a}{n} \sin nt & \frac{a^2}{n^2} \cos nt + \frac{R_1^2}{n^2} \end{vmatrix}; \quad (39)$$

$$n^2 = a^2 + R_1^2; \quad R_1 = R \frac{C}{A}.$$

From (37) it follows that $A^1 = \Phi^{-1}(t) \cdot Z$, as a result, the solution to the equation (32) for a gyroscope with a dynamic axis of symmetry is the matrix (matriciant):

$$A^1 = \begin{vmatrix} \cos \Omega t \left(\frac{R_1^2}{n^2} \cos nt + \frac{a^2}{n^2} \right) + \frac{R_1}{n} \cos \Omega t \cdot \sin nt - \frac{aR_1}{n^2} (1 - \cos nt) \cos \Omega t - \\ + \frac{R_1}{n} \sin nt \cdot \sin \Omega t & -\sin \Omega t \cdot \cos nt & -\frac{a}{n} \sin nt \sin \Omega t \\ \sin \Omega t \left(\frac{R_1^2}{n^2} \cos nt + \frac{a^2}{n^2} \right) - \frac{R_1}{n} \sin \Omega t \cdot \sin nt + \frac{aR_1}{n^2} (1 - \cos nt) \sin \Omega t + \\ -\frac{R_1}{n} \sin nt \cdot \cos \Omega t & +\cos \Omega t \cdot \cos nt & +\frac{a}{n} \sin nt \cos \Omega t \\ \frac{aR_1}{n^2} (1 - \cos nt) & -\frac{a}{n} \sin nt & \frac{a^2}{n^2} \cos nt + \frac{R_1^2}{n^2} \end{vmatrix}, \quad (40)$$

For the initial Euler angles, the matrix A₀ has the form:

$$A_0 = \begin{bmatrix} \cos \Psi_0 \cos \Theta_0 \cos \Phi_0 - \sin \Psi_0 \sin \Phi_0 & \sin \Psi_0 \cos \Theta_0 \cos \Phi_0 + \cos \Psi_0 \sin \Phi_0 & -\sin \Theta_0 \cos \Phi_0 \\ -\cos \Psi_0 \cos \Theta_0 \sin \Phi_0 - \sin \Psi_0 \cos \Phi_0 & -\sin \Psi_0 \cos \Theta_0 \sin \Phi_0 + \cos \Psi_0 \cos \Phi_0 & \sin \Theta_0 \sin \Phi_0 \\ \cos \Psi_0 \sin \Theta_0 & \sin \Psi_0 \sin \Theta_0 & \cos \Theta_0 \end{bmatrix}. \quad (41)$$

Formulas for determining the Euler angles:

$$\tan\Psi = \frac{a_{32}}{a_{31}} = \frac{\sum_{k=1}^3 a_{3k}^1 a_{k2}^0}{\sum_{k=1}^3 a_{3k}^1 a_{k1}^0}; \cos\Theta = a_{33} = \sum_{k=1}^3 a_{3k}^1 a_{k3}^0; \tan\Phi = -\frac{a_{23}}{a_{11}} = -\frac{\sum_{k=1}^3 a_{2k}^1 a_{k3}^0}{\sum_{k=1}^3 a_{1k}^1 a_{k1}^0}. \tag{42}$$

The following kinematic Euler equations correspond to the Poisson equations:

$$\left. \begin{aligned} \dot{\Psi} &= (q \sin\Phi - p \cos\Phi) / \sin\Theta; & p &= a \cos\Omega t; \\ \dot{\Theta} &= p \sin\Phi + q \cos\Phi; & q &= a \sin\Omega t; \\ \dot{\Phi} &= r - (q \sin\Phi - p \cos\Phi) \operatorname{ctg}\Theta; & r &= R \end{aligned} \right\}. \tag{43}$$

$$t = t_0; \quad \Psi(t_0) = \Psi_0; \quad \Theta(t_0) = \Theta_0; \quad \Phi(t_0) = \Phi_0$$

Let us now apply the obtained formulas to the case of regular precession.

We use the initial values $\Phi_0 = \Psi_0 = 0; \tan\Theta_0 = -\frac{Aa}{CR}$ in the matrix A_0 associated with this type of precession

$$A_0 = \begin{bmatrix} \cos\Theta_0 & 0 & -\sin\Theta_0 \\ 0 & 1 & 0 \\ \sin\Theta_0 & 0 & \cos\Theta_0 \end{bmatrix}. \tag{44}$$

In consideration of this we obtain:

$$A = \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 \end{bmatrix} \cdot \begin{bmatrix} \cos\Theta_0 & 0 & -\sin\Theta_0 \\ 0 & 1 & 0 \\ \sin\Theta_0 & 0 & \cos\Theta_0 \end{bmatrix} =$$

$$= \begin{bmatrix} a_{11}^1 \cos\Theta_0 + a_{13}^1 \sin\Theta_0 & a_{12}^1 & -a_{11}^1 \sin\Theta_0 + a_{13}^1 \cos\Theta_0 \\ a_{21}^1 \cos\Theta_0 + a_{23}^1 \sin\Theta_0 & a_{22}^1 & -a_{21}^1 \sin\Theta_0 + a_{23}^1 \cos\Theta_0 \\ a_{31}^1 \cos\Theta_0 + a_{33}^1 \sin\Theta_0 & a_{32}^1 & -a_{31}^1 \sin\Theta_0 + a_{33}^1 \cos\Theta_0 \end{bmatrix} \tag{45}$$

$$\tan\Psi = \frac{a_{32}}{a_{31}} = \frac{a_{32}^1}{a_{31}^1 \cos\theta_0 + a_{33}^1 \sin\theta_0} \tag{46}$$

After conversion we obtain:

$$\tan\Psi = \frac{-\frac{a}{n} \sin nt}{\cos\theta_0 \cdot \frac{aR}{n^2} (1 - \cos nt) + \sin\theta_0 \cdot \left(\frac{a^2}{n^2} \cos nt + \frac{R^2}{n^2} \right)} \tag{47}$$

$$\cos\Theta_0 = \frac{CR}{H}; \quad \sin\Theta_0 = -\frac{Aa}{H}. \tag{48}$$

$$\tan\Psi = \tan nt; \quad \Psi = nt = \frac{H}{A} t; \quad \dot{\Psi} = \frac{H}{A} = n \tag{49}$$

The solution (49) coincided with the classical one.

Let us now determine the value of the angle of nutation Θ :

$$\cos\theta = a_{33} = -a_{31}^1 \sin\theta_0 + a_{33}^1 \cos\theta_0$$

After calculations we obtain:

$$\cos\Theta = \cos\Theta_0 = \frac{RC}{H}. \tag{50}$$

The solution to Θ by the formula (50) also coincides with the classical solution for regular precession.

Let us now consider a solution in consideration of the angle of proper rotation Φ .

$$\tan\Phi = -\frac{a_{23}}{a_{11}} = \frac{-a_{21}^1 \sin\Theta_0 + a_{23}^1 \cos\Theta_0}{-a_{11}^1 \sin\Theta_0 + a_{13}^1 \cos\Theta_0} \tag{51}$$

After calculations we have:

$$\tan\Phi^* = -\tan\Omega t \quad \Phi^* = -\Omega t, \quad \dot{\Phi}^* = -\Omega. \tag{52}$$

The obtained formulas coincide with the formulas of the classical solution, but with zero initial angles of precession and proper rotation.

Let us now consider a variant of the solution to the problem for irregular precession.

For the initial angles $\Phi_0 = \Psi_0 = 0; \tan\theta_0 = \frac{Aa}{cR}$ that differ from the angles (45), which generate regular precession, only by the sign of the angle of nutation. After transformations, the formulas for determining the Euler angles for the SEG are:

$$\begin{aligned} \tan\Psi^* &= -\frac{\sin n t}{2 \cos^2 \Theta_0 - \cos 2 \Theta_0 \cos n t} \\ \cos \Theta^* &= \frac{\cos \Theta_0 \cos 2 \Theta_0}{\tan^2 \Theta_0} + 2 \sin^2 \Theta_0 \cos \Theta_0 \cos n t \\ \tan\Phi^* &= \frac{\sin \Theta_0 \cos 2\Theta_0 \sin \Omega t + \sin 2\Theta_0 \sin n t \cos \Omega t}{\sin \Theta_0 \cos 2\Theta_0 \cos \Omega t - 2 \sin 2\Theta_0 \cos n t \cos \Omega t} \end{aligned} \tag{53}$$

The expressions (53) suggest that only the change of the sign of the initial angle of nutation – with the other two initial angles unchanged – caused the appearance of irregular precession motions in the Euler gyroscope.

IV. MATHEMATICAL MODELING

Figures 3 – 8 show the results of mathematical modeling using the kinematic Euler equations, which confirm the obtained analytical results.

Figures 3 and 4 present graphs of the modeling process for the Euler Ψ, Θ, Φ and the Euler-Krylov angles change, respectively, for the initial angles

$$\Theta(0) = \Theta_0; \theta(0) = \theta_0 = \Theta_0; \Psi_0 = \Phi_0 = \psi_0 = \varphi_0 = 0, \tag{M.1}$$

That is, corresponding to the conditions (23) of regular precession in the Euler angles. The relationship between the Euler and the Euler-Krylov angles is established due to the equality of the respective elements of the matrices (7) and (8).

SEG parameters

$$A = 0.1, \quad sN \cdot cm \cdot s; \quad s = 0.2, \quad sN \cdot cm \cdot s; \quad a = 10^3, \text{rad/s}; \quad R = 1570, \text{rad/s};$$

$$\Omega = (c/A - 1)R = 1.57 \cdot 10^3, \text{rad/s} \tag{M.2}$$

$$\Theta_0 = -\arctan\left(\frac{aA}{Rc}\right) = -0.308, \text{ rad.}$$

The graphs in Fig. 3 depict the change of the Euler angles for regular precession. The same cannot be said about the graphs in Fig. 4 for the Euler-Krylov angles – where one can see harmonic oscillations for the angles Ψ and θ with a frequency slightly higher than 500 Hz, and for the angle Φ , its increscent property is evident.



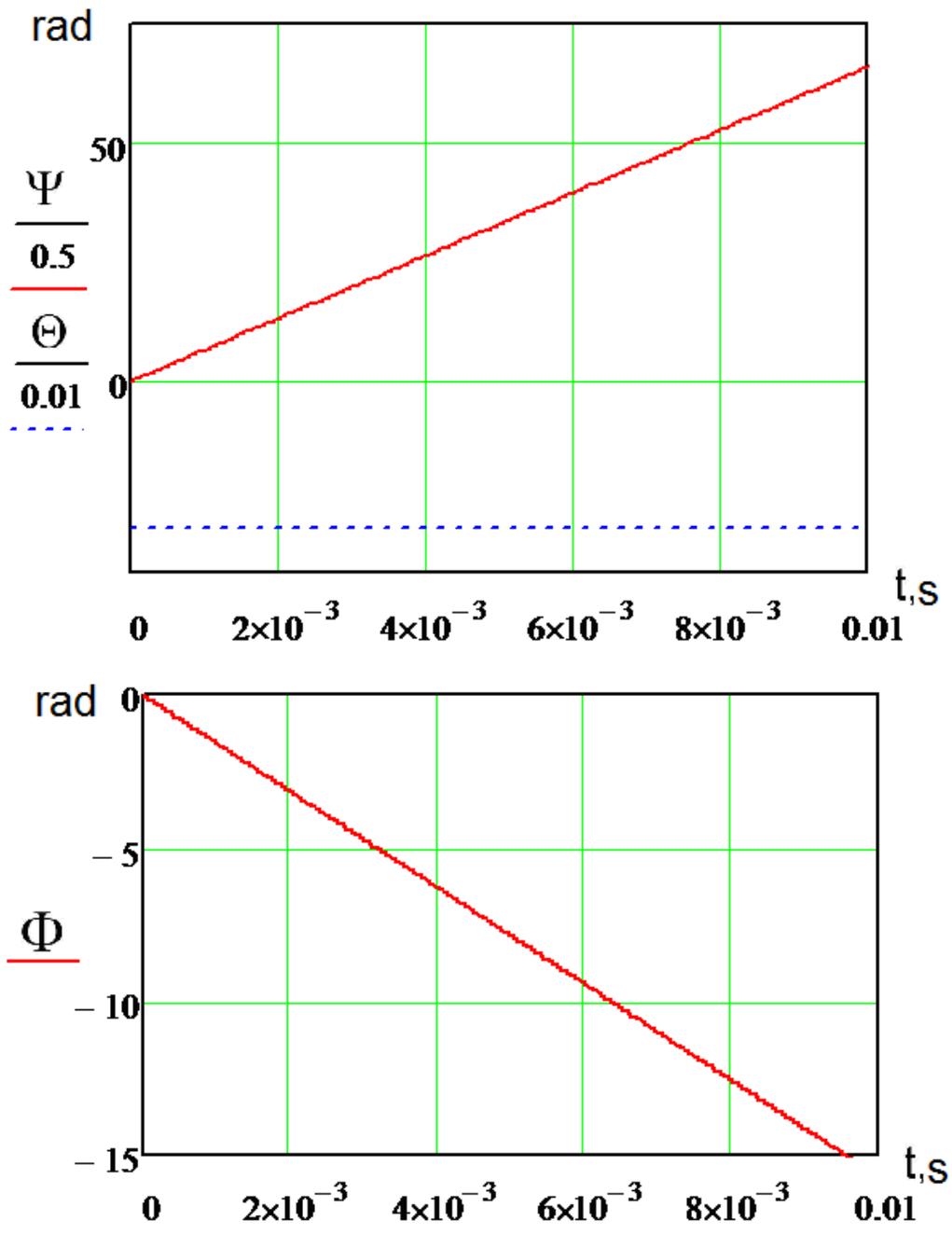


Fig. 3



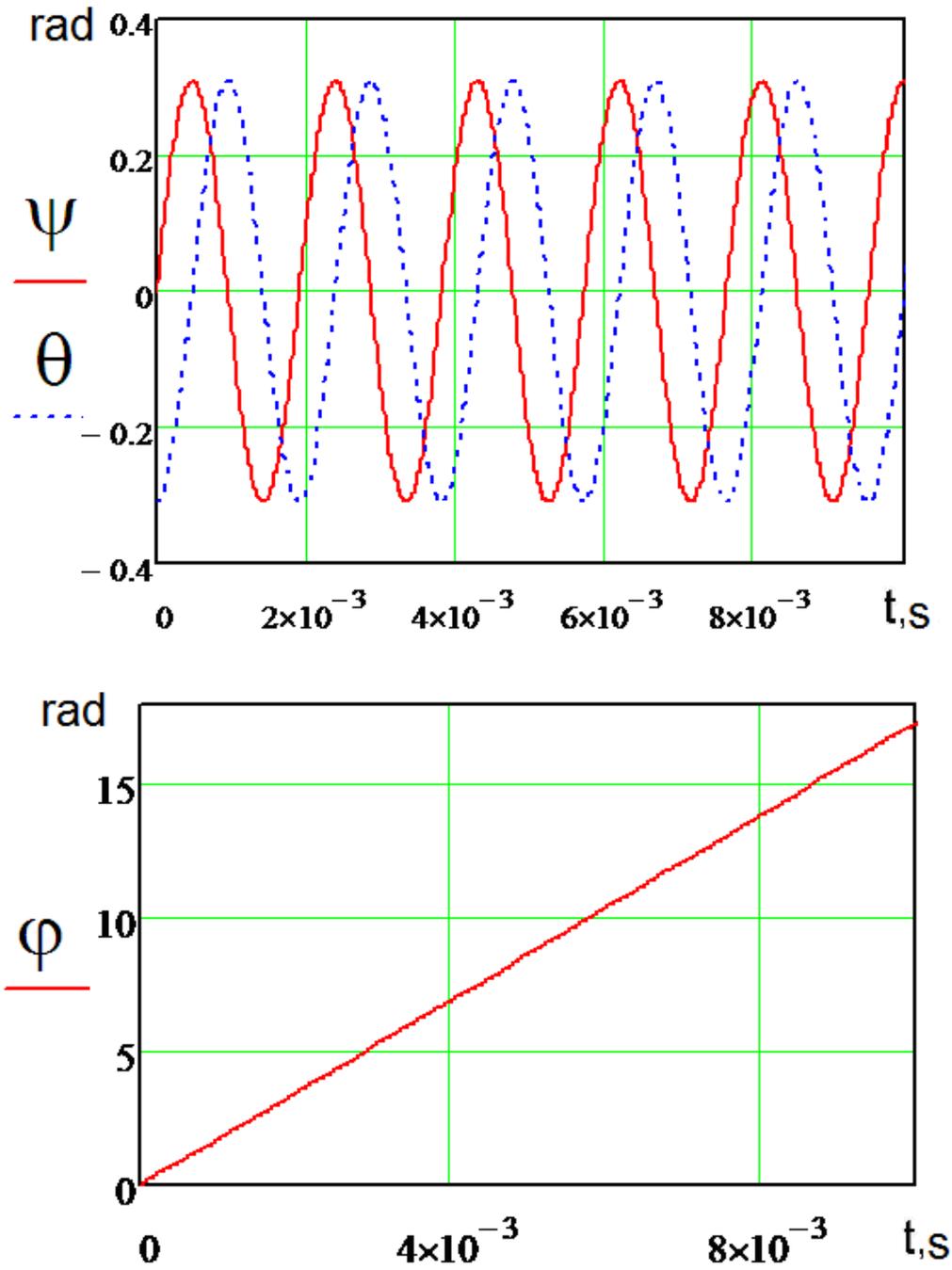


Fig. 4

When applying a stronger rotational pulse around the axis Ox for which $a = 4000, rad/s > R$, with unchanged other conditions for Fig. 3 and 4, the nature

of the motion does not change (therefore, the graphs are not shown), however, for the Euler angles we have:

$$\Psi_{\max}(0.01) \cong 50rad, \Theta = \Theta_0 = -0.905rad = const, \Phi_{\max}(0.01) = -15.7rad.$$

For the Euler-Krylov angles in Fig. 6, the oscillation amplitudes along ψ and θ are equal to 0.905 rad, the frequencies are approximately equal to 870 Hz. The angle φ is increscent with superimposed frequency fluctuations of 1740 Hz.

Additionally, with unchanged parameters of modeling of SEG motions according to (M.1), (M.2) (figures 3 and 4), but with the sign of the initial angles of nutation reversed and equal to $\theta_0 = \theta_0 = 0.308rad$, the motion patterns shown in figures 5 and 6 were obtained.

In Fig. 5, for the Euler angles, the motion has acquired the character of irregular precession, namely, along Ψ and Θ - a vibrational pattern with frequencies slightly above 500 Hz of different amplitudes with oscillation centers shifted by about 0.3 rad. For the angle Φ , the

velocity sign in Fig. 3 has changed to the opposite, and the angle become incresecent. The graphs confirm the derived formulas (30).

For the Euler - Krylov angles, the motion is of a qualitatively similar character.

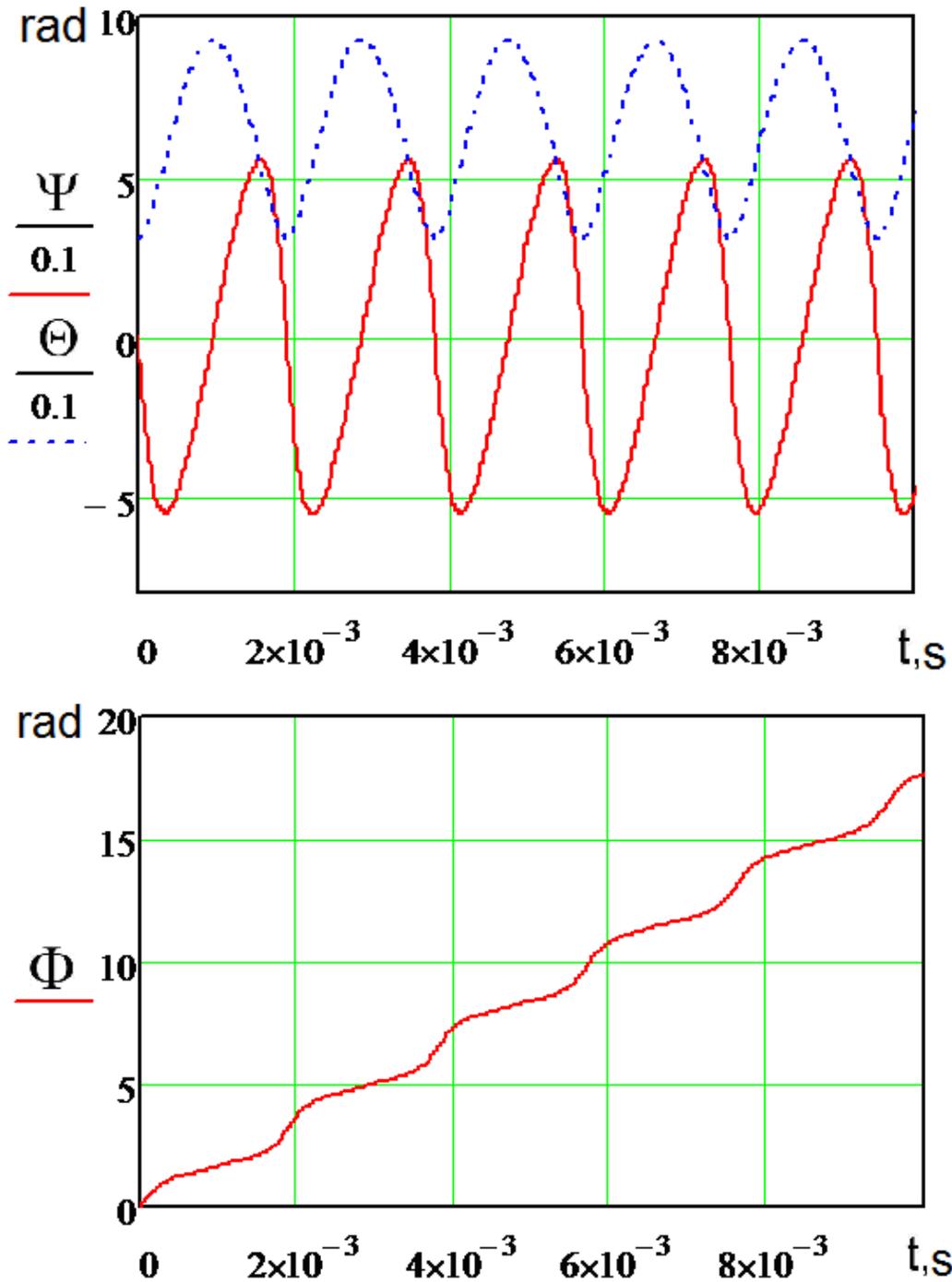


Fig. 5

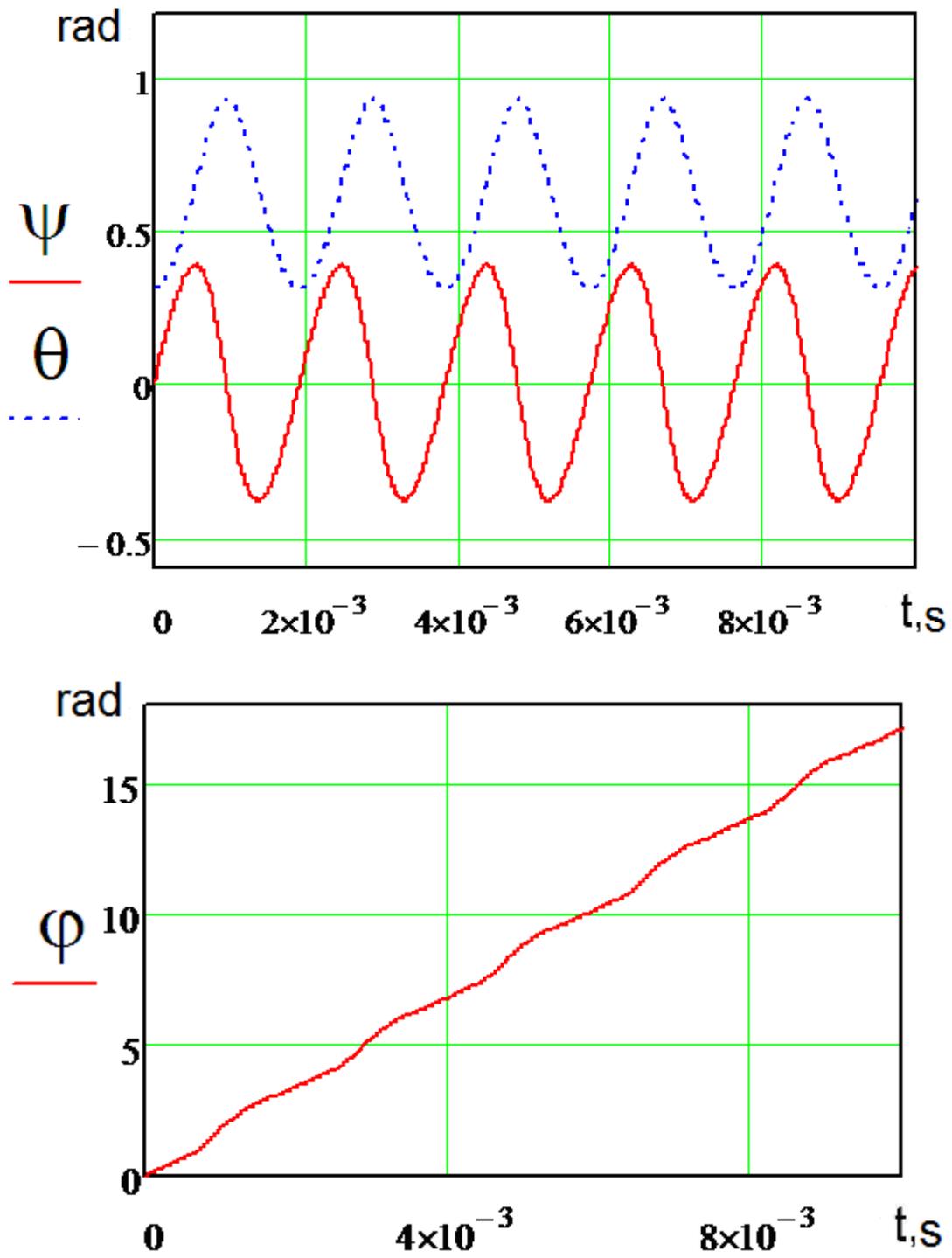


Fig. 6



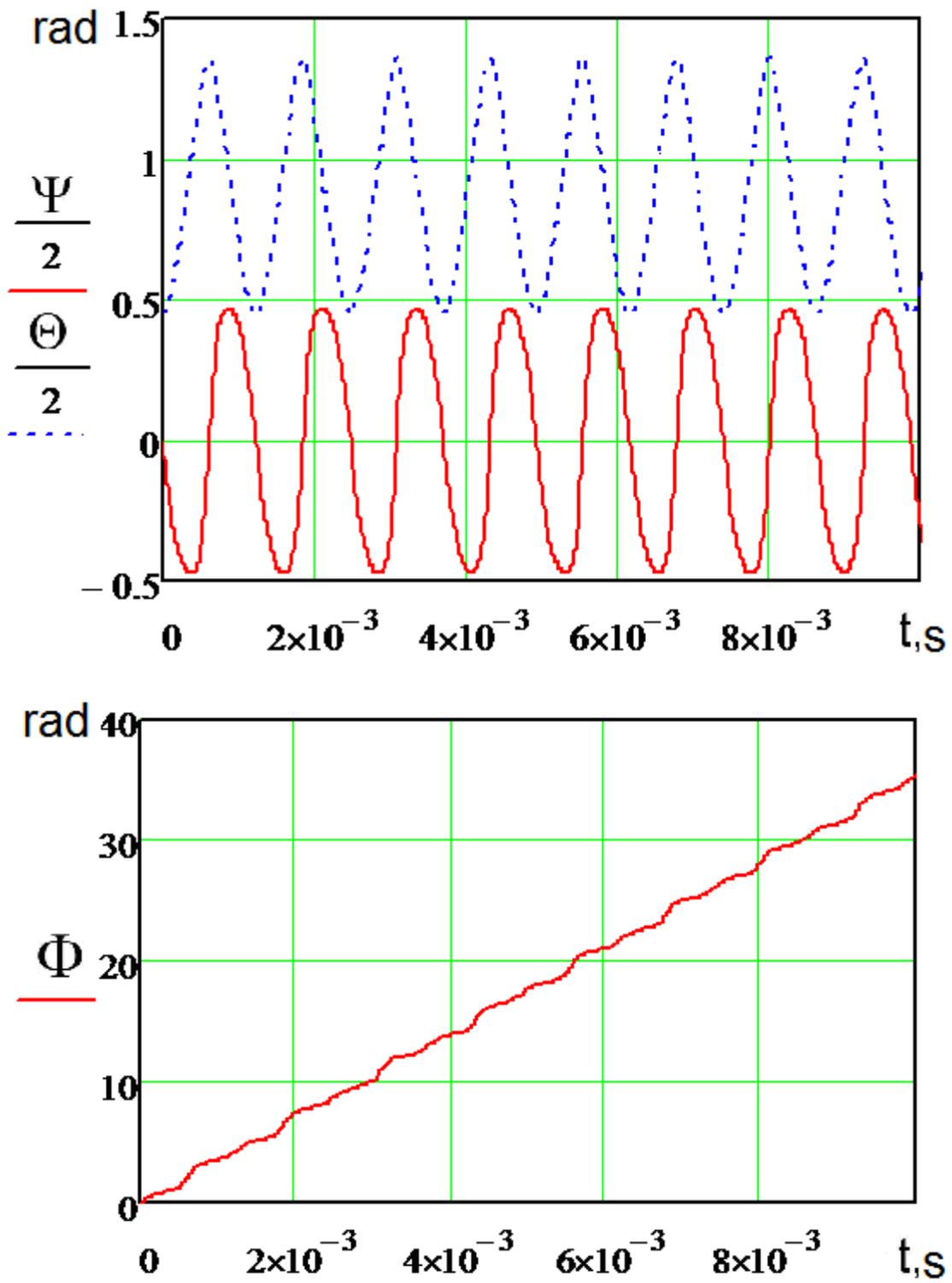


Fig. 7

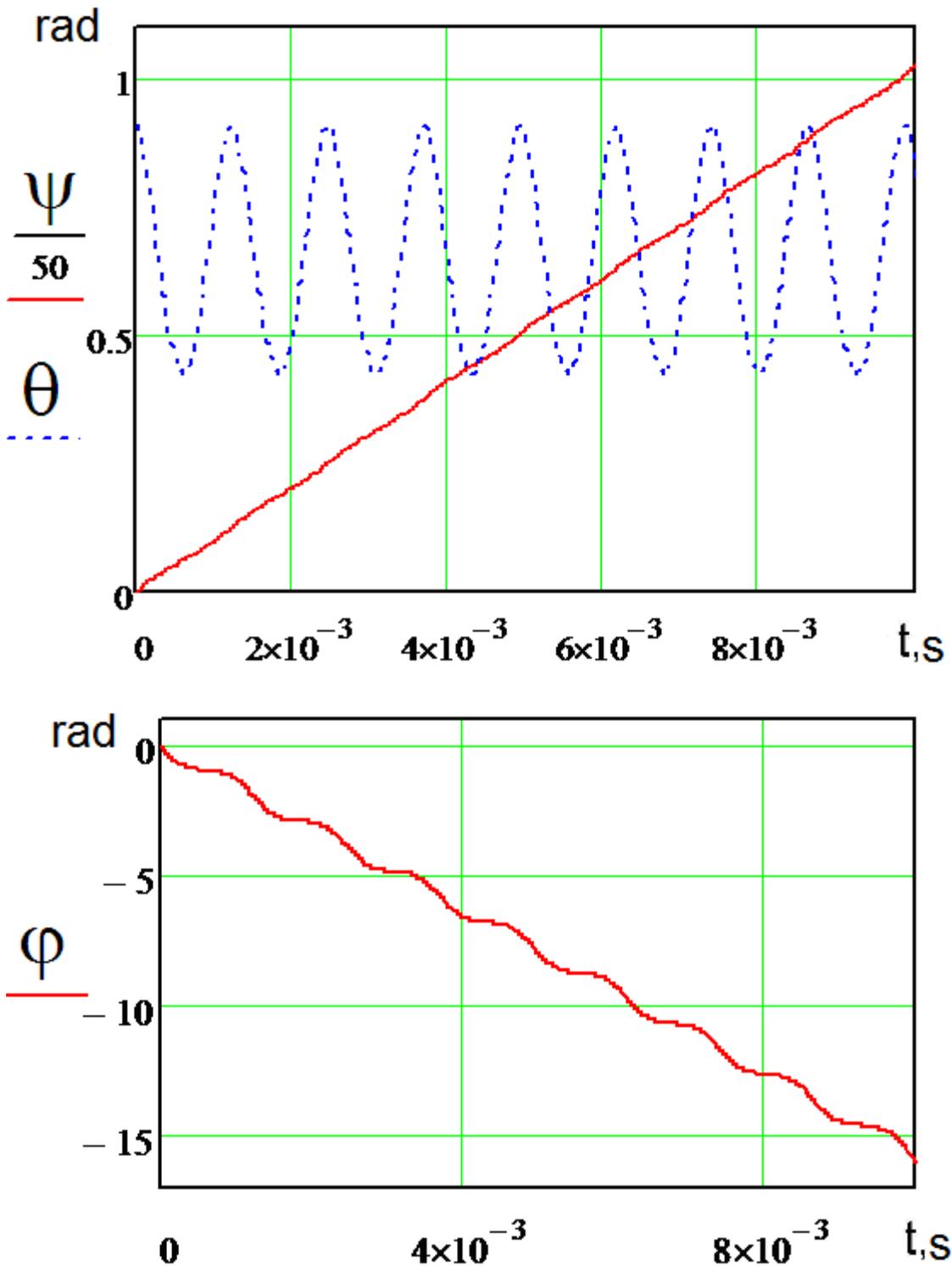


Fig. 8

Figures 7 and 8 show the results of modeling of the SEG parameters and motions that correspond to figures 5 and 6 with the only difference: angular velocity is provided equal to $a = 4000, rad/s$, $a > R$. As the result, the nature of motions along the Euler angles (Fig. 7) did not change qualitatively, while quantitatively, the vibration centers moved apart to the angles Ψ and to

Θ up to 0.45 rad, and the oscillation frequencies increased up to 820 Hz. The angle Φ remains to be in crescent with superimposed oscillations.

At the same time, the motion for the Euler-Krylov angles has changed dramatically (Fig. 8). The angle Ψ began to increase monotonically in the

direction of the rotational pulse action, which is novel. The angle θ is still oscillatory in nature with a frequency of 820 Hz around the shifted center of oscillations, and the angle φ has changed the sign to the opposite in relation to Fig. 6.

V. CONCLUSION

According to the results of mathematical modeling, it is shown that the motions that correspond to regular precession in the Euler angles are independent of the magnitude of the angular velocity a , which is caused by the action of the rotational pulse. However, a change of the sign of the initial angle of nutation leads to a sharp change in the nature of motion — it becomes irregular, which is reflected in the explanation for Fig. 5. The motion along the Euler-Krylov angles radically depends on a : with $a > R$, the angle Ψ becomes monotonically increasing in the direction of the pulse action, and the angle of proper rotation changes the sign of its monotonic rotation to the opposite. Additionally, in the article:

- It was proven that regular precession in SEG is possible only for the initial Euler angles determined by the known formulas:

$$\psi_0 = \text{const}, \quad \varphi_0 = 0; \quad \cos\theta_0 = \text{const} = C_k/G.$$

For any other initial angles regular precession is not possible.

- An analytical solution to the problem of the SEG motion was found by integrating the matrix differential quaternion kinematic equations, as well as the Poisson equations. Formulas for determining the Euler and the Euler-Krylov angles were derived.
- The obtained formulas and mathematical modeling confirmed that for the angles, that are different from the initial Euler angles (1), precession that is different from the regular one is present in SEG.

As for corpuscular gyroscopes, based on this study, it can be assumed that depending on the application of an external magnetic field over time, not only Larmor precession [14], but also “pseudo-Larmor” precession is possible in them.

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Evasive Maneuvers and Variables Technological Parameters in Orbital Regions Operational

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Abstract- In this paper we present results of an analysis on the dynamics of collision between operational vehicles and space debris on a mission in the regions LEO, MEO and GEO. The maneuvers are ideal because in the first instance, we do not consider the existing dissipative forces in these regions. The analysis established technological parameters of the propulsion system of the vehicle that enables the implementation of evasive maneuvers to debris of different sizes (from millimeters to kilometer), speed (0.5 to 20.0 km/s) and positions (3 to 300 km) initials. Furthermore, we assume that these collisional objects are separated by a small distance relative to the distance from the vehicle to earth. The results showed the possibility of collision from a distribution of the initial conditions, including the angles in-plain and out-plain. A policy of compromise between technological parameters and evasive maneuvers of the collisional debris varying size was established, verifying the existence of technological parameters minimum and characteristic for the orbital regions, in favor of schemes to avoid debris from millimeter sizes.

Keywords: *space debris; evasive maneuvers, parameters technological.*

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Evasive Maneuvers and Variables Technological Parameters in Orbital Regions Operational

Antonio Delson Conceicao de Jesus^α, Rafael Ribeiro de Sousa^σ & Ernesto Vieira Neto^ρ

Abstract- In this paper we present results of an analysis on the dynamics of collision between operational vehicles and space debris on a mission in the regions LEO, MEO and GEO. The maneuvers are ideal because in the first instance, we do not consider the existing dissipative forces in these regions. The analysis established technological parameters of the propulsion system of the vehicle that enables the implementation of evasive maneuvers to debris of different sizes (from millimeters to kilometer), speed (0.5 to 20.0 km/s) and positions (3 to 300 km) initials. Furthermore, we assume that these collisional objects are separated by a small distance relative to the distance from the vehicle to earth. The results showed the possibility of collision from a distribution of the initial conditions, including the angles in-plain and out-plain. A policy of compromise between technological parameters and evasive maneuvers of the collisional debris varying size was established, verifying the existence of technological parameters minimum and characteristic for the orbital regions, in favor of schemes to avoid debris from millimeter sizes. A characteristic curve which shows a relationship between the size of debris collisional and collision time was found and validated for any height and range of initial velocity.

Keywords: space debris; evasive maneuvers, parameters technological.

I. INTRODUCTION

Since the beginning of the space era, the space environment around Earth became a junk yard full of debris related with space missions. Debris were generated due to explosions (deliberated or not), launch vehicle upper stages, inoperative satellites and even tools and small objects. The millimetre and sub-millimetre source of debris are propellant residuals, fragmentation processes and ink fillets detached from spacecrafts surface. In 2004 about 40% of debris were generated by explosions and collision involving launch vehicle upper stage or spacecraft in orbit (Bendisich et al., 2004).

The satellite Sputnik 1 launched in 1957 became the first space debris produced by men. 4 years later this launch, the space around the earth had

become populated with 300 debris resulted by the explosion of the American Transist-4-A rocket, an explosion which happened 2 hours after reaching orbit. Today, after over 4,900 launches, the space activities produced about 240 further explosions which form the main source of production of space debris (ESA, 2013). With the space race, even with mitigation measures and the natural atmospheric drag in the objects in LEO the distribution of objects in the operating regions increased.

The vehicle propulsion system is responsible for 45.7% of the fragmentation that occur mainly due to catastrophic damage during orbit insertion or others manoeuvres, also there is factors related with failure of the active control system. The growth rate of fragmentation increased since 1970 achieving 5 fragmentation per year (Johnson et al., 2004). Collisions with debris larger than 10 cm are still considered catastrophic and consequently come up a cascade process of other collisions. This type of collision cascade process produces a critical density in long time, and without reduction perspectives, unless the amount of large objects is reduced.

In view of the great need of space missions some control techniques must be adopted to avoid the growth of debris in the operating regions (Kessler and CourPalais, 1978; Kessler, 1991). Simulations indicate that in a few decades debris from collision fragments will dominate the Earth space in attitudes between 800-1400 km at least (ESA, 2013). It is clear that the damage to space missions are large, even for smaller debris, since they may disable or burst operational vehicles and can make infeasible a space mission.

The distribution of space debris in LEO, MEO and GEO altitudes causes concerns about the safety of space activities operations in these regions. The size and altitude of debris are crucial in observing and tracking by radars. Depending on the accuracy and ability of the radar, smaller debris can be catalogued in higher altitudes. Optical instruments and radar are able to track and catalogue objects of sizes between 5 and 10 cm in LEO and sizes between 0.3 and 1 m in GEO (ESA, 2013). This indicates that the latter region can be populated by smaller and potentially destructive objects not catalogued. In fact, more than 99% of the mass and area of the population in orbit actually are debris which potentially capable of producing catastrophic rupture (Kessler, 1991).

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In MEO region orbits the navigation constellations (GPS, GLONASS, etc.) and about 16,000 debris with diameters larger than 1 cm are predicted to cross orbits in this region. Most of them have non-zero eccentricities, causing them to reach altitudes of GPS in its apogee. The energy produced in a collision is about 104 J which could cause severe damage to the spacecraft (Klinkrad, 2006; Rossi, 2005; Rossi and Valsecchi, 2006; Smirnov, 2001). The orbital perturbations in GEO region, different from the atmospheric drag in LEO, does not reduce the amount of debris (Milani et al., 1987). Since the launch of Syncom-3 in 1964, more than 800 satellites and rocket stages were placed in this region. The growth rate GEO debris without removal, increases 30 debris per year (Valk et al., 2009).

It is clear that security measures must be taken and evasive manoeuvres must be planned to prevent accidents among spatial objects. The collisions between operational vehicles with space debris became a reality, although in the point of statistical view they are rare. In most cases, on average ten risk alarms are generated per year, and fewer avoidance maneuver are implemented by year (ESA, 2009). But these estimates depend on a combination of many factors and specialized computer codes.

Beyond that there is a lot of uncertainties such as debris related with non-commercial missions, that is, upper stages and debris linked to certain classes of American secret missions and, more recently, a couple of Japanese reconnaissance spacecraft, which are not included in the catalogue version available (maintained by the Space Control Center, operated by the Air Force Space American Command) to commercial and foreign entities (Godwin, 2003). This account for approximately 4% of the catalogued objects. And there are other uncertainties that affect the missions in LEO, for example, modelling the uncontrolled satellites trajectory by re-entry, distortions in the observation of debris that occur in long time observation, the difficulty in predicting solar and geomagnetic activities which depends on the atmospheric density (Rossi et al., 1998; Anselmo and Trumpy, 1986).

This paper studies evasive manoeuvres in operating regions to establish technological parameters that are efficient to implement. This is a first model without dissipative forces in all regions.

Our study does not include dissipative forces, as it aims to establish optimal conditions for the evasive maneuvers of a vehicle in front of the possibility of collision with space debris. Moreover, the study is restricted to collisional bodies whose relative distance between them is smaller than the spacecraft to the Earth.

II. THE MATHEMATICAL MODEL

Our approach is based on the study of the relative movement between a vehicle and a space debris. The reference system in Figure 1 is fixed on the vehicle. We measure the positions and relative speeds w.r.t. this system, such that all technical control evasive maneuver is performed from it. The dynamics between these objects not consider active dissipative forces, only the Earth's gravitational force on the vehicle and the debris and the propulsive force of the vehicle.

We consider that the relative distance between objects is very small compared to the distance from the vehicle to the center of the Earth, that is, $(\frac{r}{R} \ll 1)$. In this condition, the resultant force of gravitational term can be expanded and analytic solution may be found with significant accuracy (Clohessy-Witshire, 1960). The thrust adopted in our model is proportional to the rate of exhaustion of the propellant vehicle system and, through it, the movement can be controlled. The equations for the Cartesian components of the relative acceleration are:

$$\ddot{x} - 2w\dot{y} - 3w^2x = -v_{ex} \frac{d}{dt} \ln(M(t)) \quad (1)$$

$$\ddot{y} + 2w\dot{x} = -v_{ey} \frac{d}{dt} \ln(M(t)) \quad (2)$$

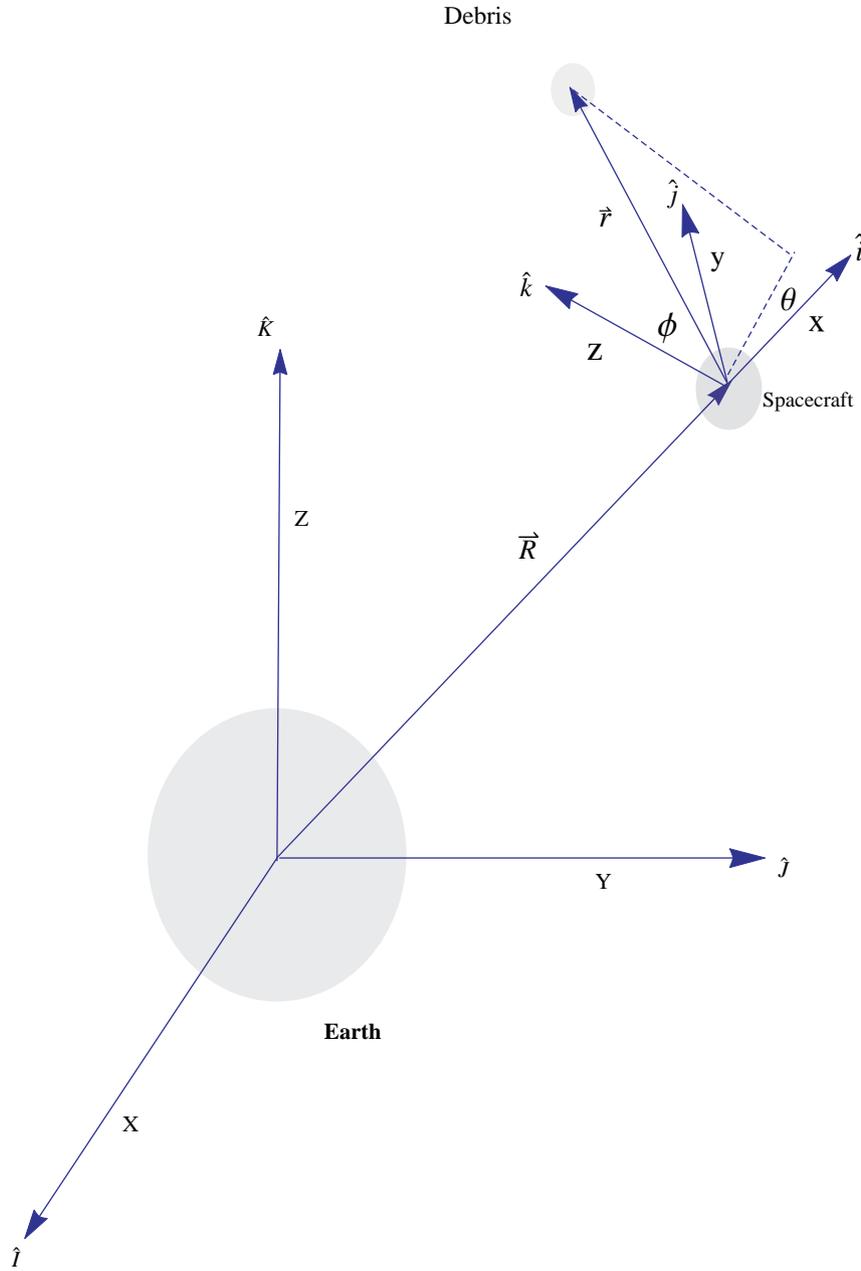


Figure 1: Reference frame for a vehicle around Earth and for a space debris. The frame with capital letters is positioned in the Earth center.

$$\ddot{z} + w^2 z = -v_{ez} \frac{d}{dt} \ln(M(t)) \quad (3)$$

These Equations model the relative dynamics between operating vehicle and space debris. For each orbital region (LEO, MEO, GEO) the vehicle orbits around the Earth with circular velocity, $\vec{w} = w\hat{k}$. The side of these Equations are the components of the non-gravitational acceleration, in which case the components of the acceleration of propulsion. For this

dynamics, we adopt exponential change in mass of the vehicle in time, such that,

$$m(t) = m_o e^{-\gamma t} \quad (4)$$

With m_o as the initial propellant mass and γ is the power factor of the engine. The total mass of the satellite is dependent of the mass of the propellant:

$$M(t) = M_o + m(t) \quad (5)$$

With M_o is the net mass of the satellite disregarding the propellant.

We write the mass factor as

$$\chi = \frac{M_o}{m_o} \tag{6}$$

And then we have

$$M(t) = m_o(\chi + e^{-\gamma t}) \tag{7}$$

The technological parameters identified for this dynamic are:

1. Components of the gas exhaust velocity (v_{ex}, v_{ey}, v_{ez}) ;
2. The power factor of the engine $\gamma > 0$ and;
3. And the mass factor (χ) , that is, the mass ratio between the spacecraft mass (M_o) and the initial propellant mass (m_o) ;

The components of the propulsion force are,

$$F_x = \gamma v_{ex} m_o e^{-\gamma t} \tag{8}$$

$$F_y = \gamma v_{ey} m_o e^{-\gamma t} \tag{9}$$

$$F_z = \gamma v_{ez} m_o e^{-\gamma t} \tag{10}$$

These Equations show that the evasive maneuver can be controlled by the power factor, maintaining the circular or quasi-circular orbit. We found that for circular orbits in LEO, the value of optimal power factor is 10-6 (Jesus et al, 2012). This factor should be reduced to that circular orbits nearly circular or are kept in MEO and GEO, as we shall see. In the Appendix, we show the evolution of the eccentricities of the orbits of these operating regions for our model.

$$x(t) = 2A \sin(nt) - 2B \cos(nt) + Et + \sum_{n=1}^{\infty} F_n e^{-n\gamma t} + G \tag{13}$$

$$y(t) = A \cos(nt) + B \sin(nt) - \sum_{n=1}^{\infty} C_n e^{-n\gamma t} + D \tag{14}$$

$$z(t) = H \cos(nt) + I \sin(nt) - \sum_{n=1}^{\infty} J_n e^{-n\gamma t} \tag{15}$$

These are the Cartesian components of the final relative position of the objects subject to gravitational and propulsion forces. This vector determines the separation of the objects at each instant. In t_c , determines whether a collision has occurred. All coefficients of these equations depend on the initial conditions and the technical parameters that can generally be written as $L = L(\vec{r}_o, \vec{v}_o, \vec{v}_e, \gamma, \chi, n)$. Therefore, the condition (5) can be controlled from the set of these coefficients through technological parameters suitable for the implementation of evasive maneuvers. These maneuvers are possible every time the final separation between the objects is comparable

III. EVASIVE MANEUVERS COLLISION CONDITIONS

The necessary condition for the collision between two space objects (operational vehicle and space debris) is its final position relative to cancel an instant t_c , ie,

$$x(t) = 0, y(t) = 0, z(t) = 0 \Rightarrow r(t) = 0 \tag{11}$$

This condition must be satisfied concerning the dynamics through a set of initial conditions that includes the components of velocity and position on the objects. The initial relative position is calculated by scanning the spherical angles (in-plane, θ , and out-plane, ϕ , distributed in the region of space where the collisional objects. The relative initial velocity can be found as a function of time through the homogeneous solution of Equations (1) to (3) to $F_x = F_y = F_z = 0$. We call this set, including the t_c , set of initial conditions collision course (CICC). The CICC elements constitute the collision possibilities in the dynamics on these objects, subject only to gravitational force. It does not determine the collision probability, because the equations of speeds are deterministic and not probabilistic. It admits the collision possibilities in the relative dynamic. Thus, the set (CICC) is obtained by the distribution possibilities in the collision time, mapped by a function of the type,

$$(r_o, \dot{r}_o) = [r_o(\theta_o, \phi_o), \dot{r}_o(\theta_o, \phi_o)] \tag{12}$$

Each pair (r_o, \dot{r}_o) provides a possibility of collision between objects. We found the inhomogeneous solution of the Equations (1) (3), whose Cartesian coordinates are:

to the dimensions of them. In this paper, we consider the collisional objects as known-radius spheres.

a) Implementation and Simulation

An operating vehicle in collision with space debris must implement an evasive maneuver to avoid it. The radar installed at the base on Earth or on the vehicle will provide information about the initial relatives position and velocity, and the collision time, t_c . The satellite onboard computer will perform calculations on the possibility of collision and also indicate the coordinates that allow the escape of the collision. In this work, we simulate these conditions, setting the time of collision

and getting the initial conditions in favorable velocities and positions to the collision. Hence, we simulate the dynamics described in Equations (7) to (9) to implement the evasive maneuver.

Our numerical simulations of evasive maneuvers followed the following general steps:

1. Chose the orbital region (LEO, MEO, GEO) where collisional objects are and calculate the angular velocity that characterizes;
2. Use the solutions of the homogeneous equations of relative dynamics to calculate the initial relative velocity that allow collisions;
3. Set the collision time and the relative initial distance, r_o , between collisional objects and found angles (θ, ϕ) , consistent with collisions between objects;
4. Calculate the components of the initial relative position from their values;
5. Select a pair of initial conditions of the CICC set, Equation (6), and t_c ;
6. Chose specific technological parameters for the orbital region;
7. Implement the evasive maneuver the spacecraft, operating the propulsion system, numerically simulating the Equations (7) - (9);
8. Obtain the final value of the relative position between objects, testing the collision condition (5).

With these steps we intend to model approximately the realistic conditions of an evasive maneuver on the possibility of a collision. To scan the entire sphere of radius r_0 with pairs of angles (θ, ϕ) , is performed to select those that enable collision. Technological parameters are extracted from the catalog- curves produced in this paper, consistent with the limits for circular or nearcircular orbits. They characterize the propulsion system of the vehicle and are used to control their escape from the collision.

IV. NUMERICAL SIMULATION - RESULTS

The numerical simulation of evasive maneuvers should take into account information on the initial conditions, the time of collision and the characteristics of the propulsion system, represented by the technological parameters. The collision time should be small compared to the orbital period and sufficient for the evasive maneuver is implemented. Each element of the set CICC is a possibility of collision. The choice of this element is not arbitrary, why should characterize a real collision in orbital regions. For the numerical simulations, we chose the initial relative velocities equal to (7,76 km/s, 4,25 km/s, 1,01 km/s) in LEO, MEO and GEO, respectively, non-planar maneuvers which they are typical in these regions. We investigated the possibility of collision in a higher range of these velocities, that is, [1,0 - 20,0 km/s].

The evasive maneuver is characterized by technological parameters (ν_e, γ, χ) , with which you can

control these maneuvers front of an imminent collision. Equation (6) is valid for any time. The typical technological information space missions can restrict the solutions to a finite set. The observations of radar confirm average values of initial relative velocities specific to each operating region and also the minimum time required to perform the operations for evasive maneuvers. Thus, the simulations will be carried out in finite time and collision possibilities will also be finite, although in large numbers.

In this section we show the results of numerical simulations of the relative dynamics between two collisional objects (vehicle and space debris).

The results show: 1) the distribution of collisions in relative initial velocities ranges; 2) a parametric analysis of evasive maneuvers, characterizing the efficiency of the propulsion system that implements them. In the distribution of collisions we call collision possibilities. In implementing the evasive maneuver only use one of them. Figure 1 shows a reference system centered on the space vehicle. In its origin it focuses a sphere of radius equal to the relative distance between the vehicle and the space debris. Thus, our simulations vary these distances between 3 and 500 km, with initial relative velocities between 0 and 20 km/s. Knowing the collision possibilities in their collision time, we conducted a systematic study of technological parameters that are appropriate for the implementation of various evasive maneuvers. From there, curves-catalogue are generated, considering the variation of these parameters over time and size of collisional debris. For each operating region (LEO, MEO and GEO) choose an element of CICC to implement the evasive maneuvers in the region. In this work, we simulated various evasive maneuvers, taking into account debris and space vehicles of different dimensions. The evasive maneuvers will be confirmed based on the final values of the relative positions, comparing them to the sizes of the space objects.

a) Possibility of Collisions

The possibilities of collision between the space objects (vehicle and satellite) are high, considering a time interval of 10^6 s. Figure 2 below shows the distribution of collisions depending on the initial relative velocities interval between objects for the operating regions. For discussion purposes, we divide the range of relative initial velocities, as follows: 1) low velocities - between 0.0 and 4.0 km/s; 2) medium velocities - between 4.0 and 7.5 km/s; 3) high velocities - between 7.5 and 10.0 km/s, and; 4) high velocities - between 11.0 and 20.0 km/s.

In Figure 2, we note that for any operating region there is little chance of collision for high and very high initial velocities, if the objects move away initially 3 km. This result is consistent with the expected, since the velocities decrease with increasing altitude. Moreover, in

LEO there is greater chance of collision with small and average velocities in relation to other operating regions. The same is true for MEO regarding GEO. Obviously, the effect of the Earth gravitational field, favors the approach of objects in LEO by increasing the collision possibilities. For the range of initial velocities nulls or nearly nulls, the collision possibilities are virtually the

same for any operating region. Figures 3, 4 and 5 show the results of simulations for other initial relative positions between the space objects to collision time equal to 10^5 s. This collision time is compatible with orbital periods in MEO and GEO. For LEO it would be 10^3 s.

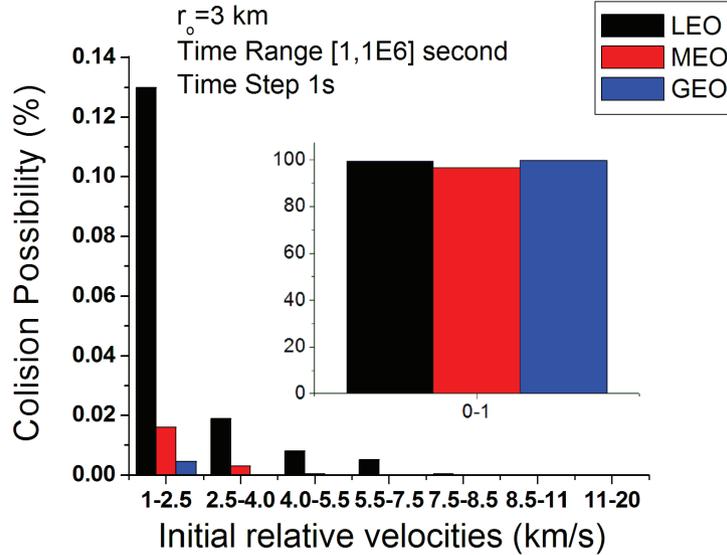


Figure 2: Collision Possibility in orbital regions vs. Initial relative velocities: 0.0 - 20.0 km/s, $r_0 = 3$ km.

These results generalize those for $r_0 = 3$ km. That is, the collision possibilities for high and very high velocities are small to any initial relative position. Proportionally, these collisions occur, possibly in LEO more than MEO, and MEO more than GEO. For medium velocities, there is greater possibility of collision if the objects are initially farther away and at higher altitudes. Thus, we say that a greater risk scenario is established for imminent collision in any relative initial velocity,

especially in LEO, MEO and GEO followed. Moreover, our model does not include dissipative forces occurring in these regions. These forces interfere with the distribution of physical initial conditions that enable collisions, sometimes favoring them, sometimes reducing them. Our simulations did not include these forces, and thus our results are an ideal initial test, establishing physical and technological conditions for complete dynamics with dissipative forces.

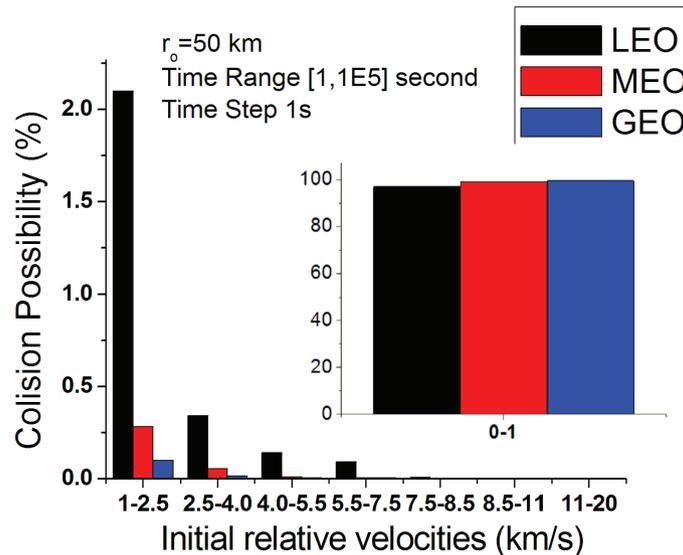


Figure 3: Collision Possibility in orbital regions vs. Initial relative velocities: 0.0 - 20.0 km/s, $r_0 = 50$ km.

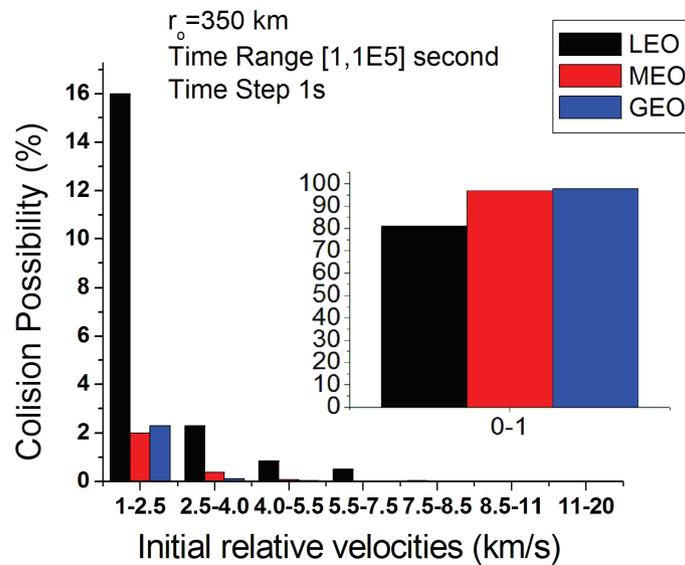


Figure 4: Collision Possibility in orbital regions vs. Initial relative velocities: 0.0 - 20.0 km/s, $r_o = 350 \text{ km}$.

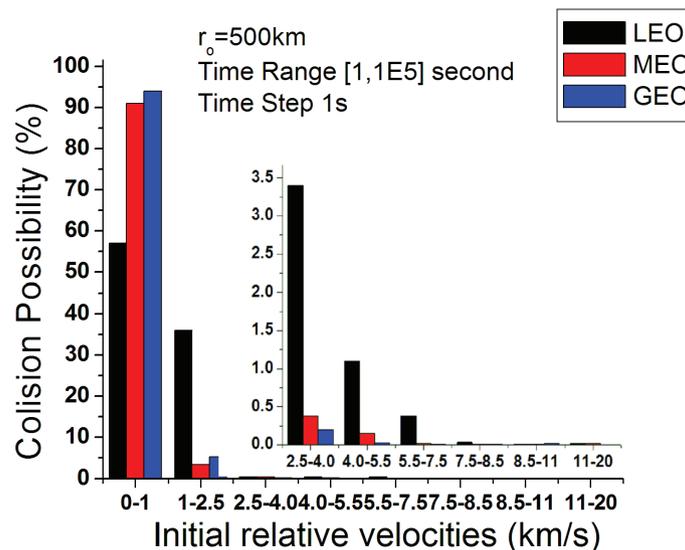


Figure 5: Collision Possibility in orbital regions vs. Initial relative velocities: 0.0 - 20.0 km/s, $r_o = 500 \text{ km}$.

b) Evasive Maneuvers In Meo And Geo

Knowing the dimensions of the collisional objects, our model determines if the evasive maneuver is sufficient to avoid collision between them. The technological parameters are used to evaluate the feasibility of evasive maneuvers for the set of initial conditions favorable to the collision, the collision time interval. For this study the technological parameters, simulated evasive maneuvers between objects initially separate to 3km, with angles $\theta = 68$ and $\varphi = 23$ degrees in MEO and GEO regions. The equivalent study for LEO was held earlier paper (Jesus et al., 2011). The red and pink curves, Figure 6, represent the course of a collision (homogeneous solution of Equations 1, 2 and 3) between the objects in the initial conditions for the

MEO and GEO regions, respectively. Curves in blue and green represent their evasive maneuvers, whose initial relative velocities are typical of these regions (4.15 km/s - MEO and 1.01 km/s - GEO). These curves characterize the dynamics with propulsion to escape collision with space debris. They inform the final relative position between collisional objects at each time, obtained by the performance of operational vehicle propulsion thrusters.



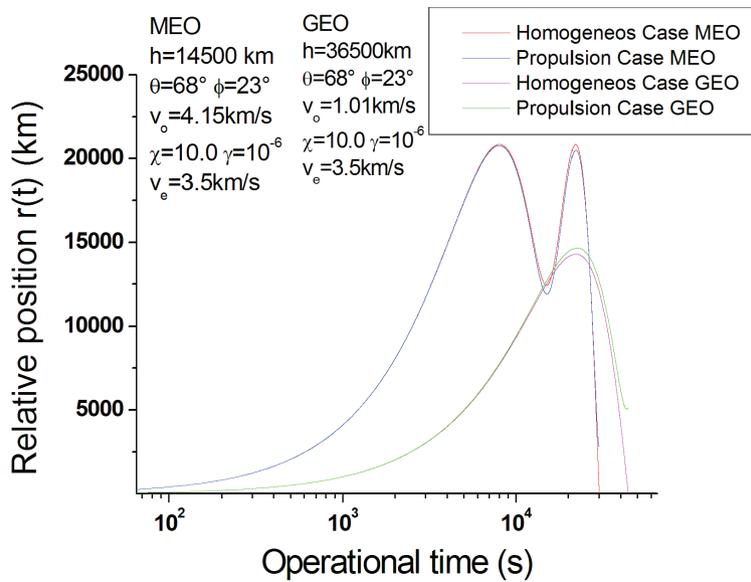


Figure 6: Relative position of the two objects as a function of time for different operational regions - MEO and GEO.

We note that collisional objects initially separated to 3km escape of the collision at different times and in different operating regions. Moreover, their final relative distances are different, which can characterize different sizes of debris. We note that the curves with propulsion characterize success of evasive maneuvers, as the final relative positions are not zero. This success is in large collision time, such that the continuous propulsion can overcome the gravitational effects on the objects, setting their final relative position and avoiding the collision between them. In all cases with propulsion, the collision is easily avoided in hundreds of kilometers of separation between the objects to these maneuvers under these initial conditions. The maneuver MEO escapes closer to the

collision in relation to maneuver in GEO. This fact is associated with the typical initial relative velocities and also the period orbits in these regions. The qualitative behavior shown is general, although the results are specific to these regions with the given initial conditions. Thus, we say that given the initial conditions favorable to the collision, the objects develop a relative dynamics to propulsion, such that as time passes, they leave the collision course, swaying their positions around it to fully escape, featuring the evasive maneuver. These initial conditions are overcome by the performance of the propulsion system. Therefore, the technological parameters of it were efficient in the implementation of the evasive maneuver.

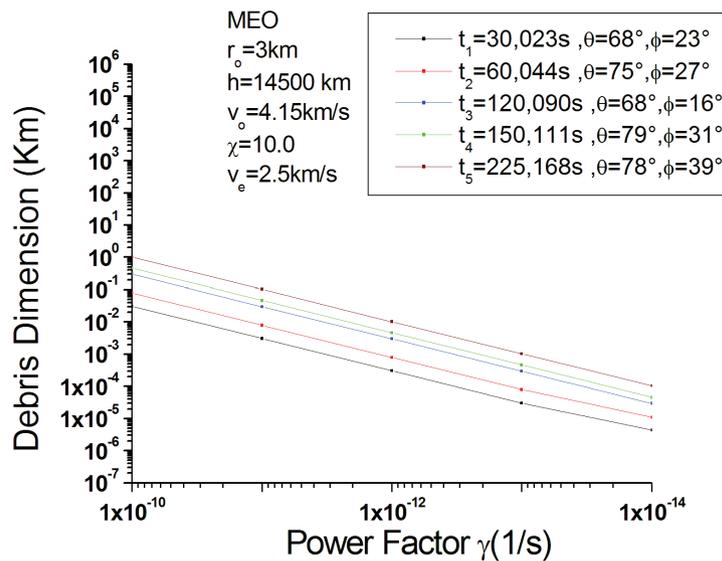


Figure 7: Behavior of the parameter vs. debris size vs. operation time, MEO.



c) *Catalog-Curves Technological Parameters*

Technological parameters are effectively crucial in the implementation of evasive maneuvers. They are related to the propulsion system of the space-craft and the correct choice of these parameters will allow efficient evasive maneuvers. The parameter characterizes the

frequency with which the fuel is consumed in time, such that the higher it is, more fuel will be spent on the evasive maneuver. Figures 7 and 8, below, are catalog-curves which show the behavior of this factor as a function of collision time and the size of the debris in MEO and GEO, respectively.

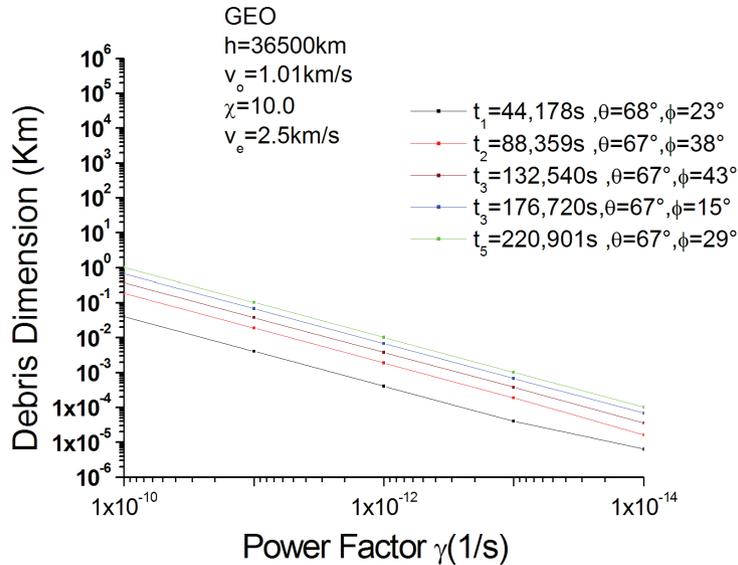


Figure 8: Behavior of the parameter vs. debris size vs. operation time, GEO.

The results show that small power factors favor evasive maneuvers on a collision course with small debris. Conversely, evasive maneuvers to larger debris will require major power factors. This phenomenon occurs in both operating regions, MEO and GEO. It should be noted, however, that the increase of this factor in continuously burning regime can remove the operating object of their nominal circular orbit. In this model we chose power factor operation MEO and GEO equal to 10^{-10} (see Appendix), which ensures the permanency of the operating object in a circular or nearly circular orbits. Moreover, very small power factors prevent the implementation of the evasive maneuver, since the propulsion force is virtually zero. But observing such limits, evasive maneuvers can be performed efficiently to escape collisions with debris of various sizes, depending on the time of collision and the propulsion system characterized by the parameter. We say that the high power factor would favor evasive maneuvers with less risks of collision with cloud debris of any sizes. Another technological parameter that we consider is the mass factor. This quantity measures how much the vehicle mass is greater than the initial mass of the fuel it carries. Figure 9 shows the relationship between this factor to the size of the debris which evasive maneuvers escape into GEO, and the exhaust velocity. The power factor used here is equal to 10^{-14} .

In this Figure we observe that evasive maneuvers with small mass factors are more efficient to

escape debris increasing relatively. Higher exhaust velocities are favorable to these maneuvers. To escape of smaller debris, lower exhaust velocities are preferable. However, large mass factors are favorable to the evasive maneuvers against collisions with smaller debris and this is independent, practically, the increasing values or not the exhaust velocities.

The results obtained here for MEO and GEO are equivalent to those found by Jesus et al. (2012) to LEO. Figure 10 below shows the risk curve in LEO, MEO and GEO, under the action of gravity forces and propulsion. They are ideal curves because it does not consider the dissipative forces occurring in these regions. In this curve the dimensions of the debris are distributed as a function of collision time, of altitude and of the vehicle exhaust velocities. We note that the qualitative behavior of the curves remains for any altitude of collisional objects and the quantitative results are a function of exhaust velocity.

We note that with higher exhaust velocities will be possible to escape from collisions with large debris. This means that more powerful propulsion systems are less efficient and less economic for evasive maneuvers in small sizes debris environments. In addition, we observed that the risk of colliding with large sized debris increases with low velocities. This risk is reduced by increasing the exhaust velocities. Therefore, it is preferable and more economical to use small exhaust velocity, since the power of the propulsion system must

be preserved for escape maneuvers against collisions with very large sizes of debris. Hence, the need for a propulsion system capable of controlling the

magnitude of velocity in non-uniform size distribution of debris environment.

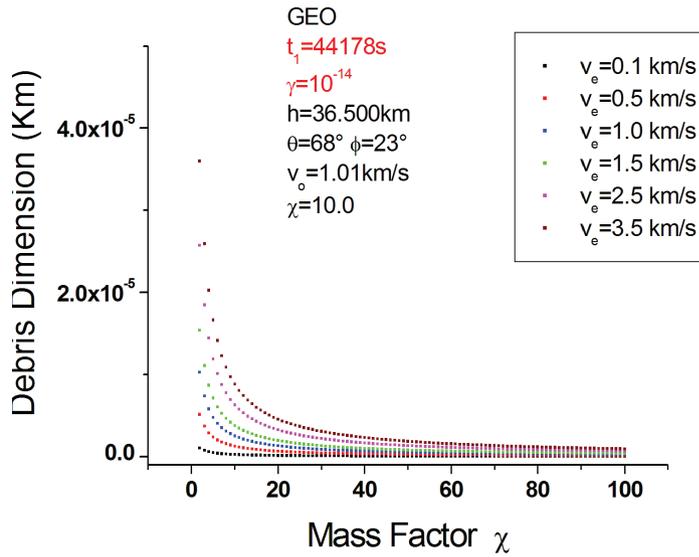


Figure 9: Debris Dimension vs. mass parameter, χ , $\gamma = 10-14$ 1/s, in GEO.

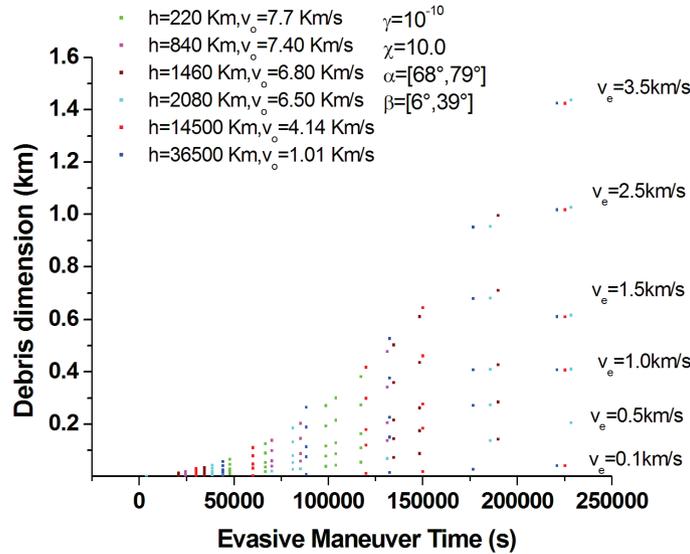


Figure 10: - Risk Curve - Debris dimension vs. Evasive Maneuver time.

d) The Risk Curve how to use it

The risk curve, shown in Figure 11, provides information that can be used in decision-making on a space mission to avoid collisions with space debris. To use this curve should be chosen initially, one octant. In this curve, we choose the angles in-plane and out-plane within the range 68 to 79 degrees and 6 and 39 degrees, respectively. Our results showed that the qualitative behavior displayed in this curve is general for all octants, where are located the collisional objects. There is a relationship between the dimensions of collisional debris with the collision time for every

operating region. The exhaust velocity is the coefficient between them. It is essential in the implementation of evasive maneuvers, since it is related to engine power. The use of the risk curve in space missions must follow an algorithm of operations that allow to perform an evasive maneuver. If the propulsion system for variable velocity, so the operating system of the vehicle will have the freedom to make decisions to escape from any debris, based on the information on the collision time. The radar informs the coordinates of the debris and the collision time. The vehicle would be in a specific altitude, for which there is a specific curve as a function of



exhaust velocity. Thus, based on the collision time and the vehicle altitude the operating system know whether or not escape the debris, depending on its size. The onboard computer will calculate the escape trajectory and the curve will inform if one can escape the specific size debris and this will depend on the velocity that the propellant system is able to implement. If the thruster system is not able to vary the exhaust speed, there will only be a range of sizes of debris it can escape. This range is as large as is the variation of the exhaust velocity values.

V. CONCLUSION

Our results showed that the relative dynamics between a vehicle and space debris is rich in collision possibilities in LEO, MEO and GEO. Thus, we say that a greater risk scenario is established for imminent collision

in any relative initial velocity, especially in LEO, MEO and GEO followed. Our simulations did not include dissipative forces, and thus are an ideal initial test. The continuous propulsion can be used to avoid collisions with space debris, provided that suitable technological parameters. These parameters can control the evasive maneuvers depending on the size of the debris and collision time. If the vehicle has short time to escape, the evasive maneuver occurs if in imminent collisions with small debris. In this case, small exhaust velocities are preferable. We found a risk curve for orbital maneuvers. This curve informs that a system propulsion with variable exhaust velocities is preferred for the evasive maneuvers. These maneuvers would be more economical and efficient to avoid the collisions with any debris.

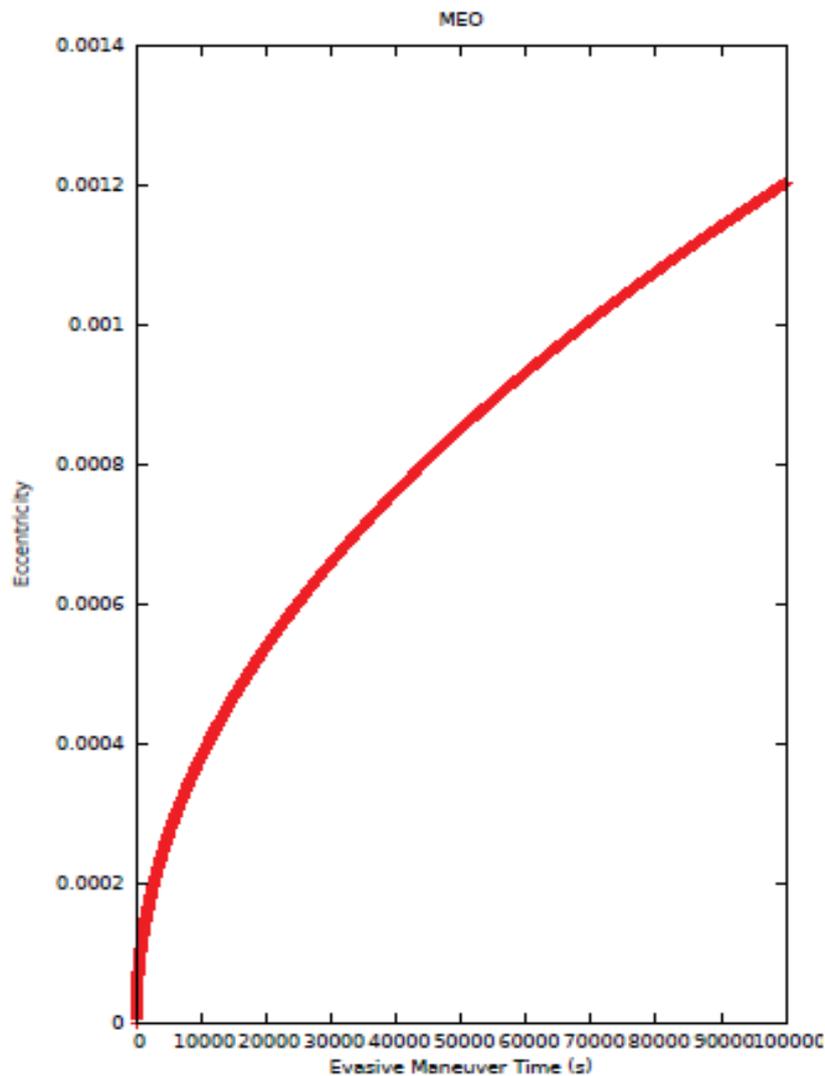


Figure 11A: Variation in the satellite eccentricity by the time. The satellite is in MEO, the gas exhaust velocity is 2.5 km/s, the mass factor of 10, the power factor of the engine is 10–10 1/s.

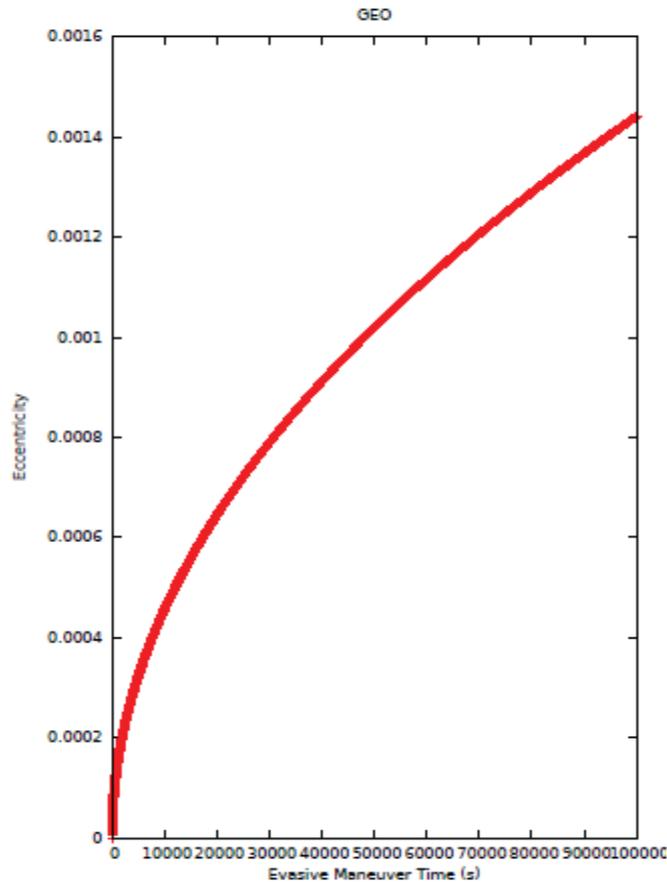


Figure 12A: Variation in the satellite eccentricity by the time. The satellite is in GEO, the gas exhaust velocity is 2.5 km/s, the mass factor of 10, the power factor of the engine is 10–10 1/s.

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APPENDIX A. SATELLITE ECCENTRICITY VARIATION

The satellite eccentricity (e) can be written as a function of specific energy (ϵ) of its circular orbit by the equation (Burns, 1976):

$$e = (1 + 2h^2\epsilon\mu^{-2})^{-1/2} \quad (\text{A.1})$$

Where h is the angular momentum of the satellite orbit by mass unit, and μ is the product of the Earth's mass by the gravitational universal constant. The satellite equation of motion with the gravitational force of the Earth and the propulsion system is:

$$\ddot{\vec{R}} = -\mu \frac{\vec{R}}{R^3} + f_x \hat{i} + f_y \hat{j} + f_z \hat{k} \quad (\text{A.2})$$

The propulsion accelerations are:

$$f_x = -v_{ex} \frac{d}{dt} \ln(M(t)) \quad f_y = -v_{ey} \frac{d}{dt} \ln(M(t)) \quad f_z = -v_{ez} \frac{d}{dt} \ln(M(t)) \quad (\text{A.3})$$

The power due to thrust produces variation in specific energy-orbit:

$$\dot{\epsilon} = \vec{V} \cdot (f_x \hat{i} + f_y \hat{j} + f_z \hat{k}) \quad (\text{A.4})$$

Where $\vec{V} = R\omega \hat{j}$ is the velocity of satellite in the its circular orbit.

The variation in the angular momentum is originated due to the applied torque is given by:

$$\dot{\vec{h}} = \vec{R} \times (f_x \hat{i} + f_y \hat{j} + f_z \hat{k}) \quad (\text{A.5})$$

The integration of the equations above show us how the energy and the angular momentum of the satellite are modified by the force of propulsion, these functions are respectively:

$$\epsilon = -\frac{\mu}{R} - R\omega v_{ey} \ln\left(\frac{\chi + e^{-\gamma t}}{\chi + 1}\right) \quad (\text{A.6})$$

$$h = (\mu R)^{1/2} + R(v_{ey}^2 + v_{ez}^2)^{1/2} \ln\left(\frac{\chi + e^{-\gamma t}}{\chi + 1}\right) \quad (\text{A.7})$$

Returned to equation (A.1) the time rate of change of eccentricity the satellite orbit is due to variation in the angular momentum and energy of its orbit. Therefore, we can calculate a function for the change in eccentricity in time:

$$e = \sqrt{1 - \left[1 - \frac{R\omega v_{ey}}{\epsilon_o} \ln\left(\frac{\chi + e^{-\gamma t}}{\chi + 1}\right)\right] \left[1 + \frac{R(v_{ey}^2 + v_{ez}^2)^{1/2}}{h_o} \ln\left(\frac{\chi + e^{-\gamma t}}{\chi + 1}\right)\right]^2} \quad (\text{A.8})$$

$\epsilon_o = -\mu/R$ and $h_o = (\mu R)^{1/2}$ are specific energy and the angular moment of satellite. The following shows the variation in the eccentricity of the satellite in function of time avoidance maneuver for the two orbital regions MEO and GEO.

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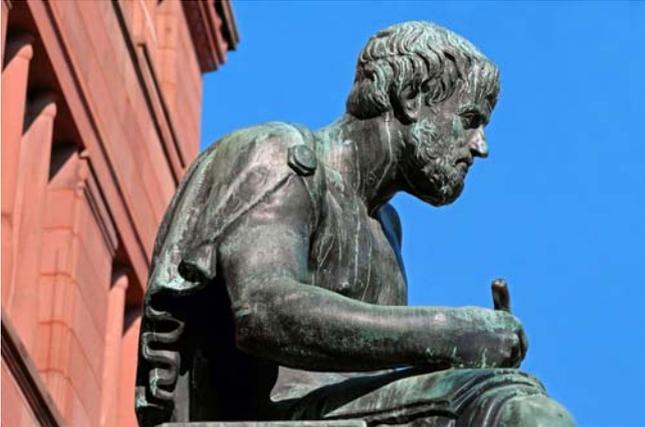
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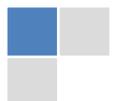
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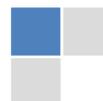
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Acknowledgments

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The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.



Manuscript Style Instruction (Optional)

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
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- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
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Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

- a) A title which should be relevant to the theme of the paper.
- b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
- c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
- d) An introduction, giving fundamental background objectives.
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- g) Suitable statistical data should also be given.
- h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

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The full postal address of any related author(s) must be specified.

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The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

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A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

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Numerical methods used should be transparent and, where appropriate, supported by references.

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Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

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Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

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Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.



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Techniques for writing a good quality engineering research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

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11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

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15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

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23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium through which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

INFORMAL GUIDELINES OF RESEARCH PAPER WRITING

Key points to remember:

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- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

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The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

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- Submitting a manuscript with pages out of sequence.
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- Align the primary line of each section.
- Present your points in sound order.
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- Use past tense to describe specific results.
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Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

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Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

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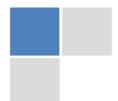
- Single section and succinct.
- An outline of the job done is always written in past tense.
- Concentrate on shortening results—limit background information to a verdict or two.
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Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

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This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

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Materials may be reported in part of a section or else they may be recognized along with your measures.

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- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.



Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
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What to stay away from:

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Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

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If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

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Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.



Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

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<i>Discussion</i>	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
<i>References</i>	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring



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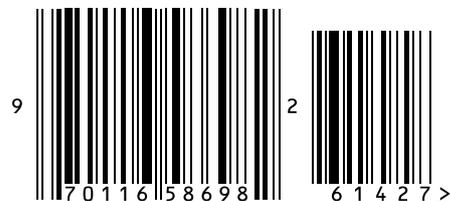


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