CHIARA: Cost of Holding Interruptions of Availability via Reliability Analysis

By F. Galetto

Abstract- Attending seminars, Conferences, looking at "television lessons" the author saw many times many people (often Professors) that did not know the matter they were talking about [as Deming wrote "the 1st requisite for a good teacher is that he have something to teach… must possess knowledge of the subject"]; nevertheless many of them still write papers, suggest "wrong" books to students, provide "wrong" lessons, make consultancy. Visiting Companies the author saw many times many Companies lacking Quality of Management, a big problem against Quality achievement.

Many lecturers on "quality matters" and on "reliability matters" do not know, in a scientific way, reliability theory; therefore, they propose wrong methods to students. The basic reliability ideas are easily understandable, but when you need more sophisticated methods many people do more harm than good. In the paper we present a case related to contractual clauses on failures and related costs; we show that even in this simple case, the Reliability Integral Theory (devised by F. Galetto to overcome limitations on the usual methods in reliability) is needed.

Companies’ solutions and real applications are an important problem: wrong solutions depend on the lack of scientific knowledge.

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Strictly as per the compliance and regulations of:
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I. Introduction

As said by the author [2], “Higher Education is seen many times as a Production System, and students are considered as its “Customers”. Books and magazines are suggested to students attending “Quality Courses” at Universities. Some of them are good, some are not so good. Students use papers from magazines for their teaching; some have good Quality; some are not very good. Therefore it seems important to stand-back a bit and meditate, starting from a managerial point of view.”

In order not to be cheated, any person should use the SPQR (Semper Paratus ad Qualitatem et Rationem) Principle [1]: anybody must be attentive to use his Rationality to find if the quality is present or absent in any activity.

Generally, engineers do not learn Quality matters and specially they know very little about Reliability: System Reliability, Reliability Theory, Reliability Tests, Availability Theory, Cost related to Un-Reliability and UN-Availability, even though the lessons are provided by professors members of the "Politecnico Quality Engineering Group (QEG)" (all graduated CUM LAUDE) [Fausto Galetto, who always was striving for Quality scientific applications, is not a member of QEG]!

Many professors teaching "quality" do not have enough knowledge: to deal properly with those matters, Probability Theory and Statistics are essential [as stated by W.E. Deming].

“"To “measure” Quality (?), various bibliometric indices (e.g., h-index, s-index, …) have been devised, based on informetric models. Quality (?) of Research, in many universities, is based on these indexes: if you are cited many times, you are a better professor than if you are not!” [2].

Figure 1: Deming’s and Galetto’s ideas about Quality and Teaching
Galileo Galilei, Einstein, Jay, Deming, Berne, Feynman (to mention only very few) were champions before F. Galetto, in the SPQR Principle, without naming it.

“To grasp the importance of these ideas, let’s imagine that in one university there is a Quality Engineering Group (QEG, comprising four lecturers, all graduated CUM LAUDE, and teaching “Quality matters”; they are also in the Research Gate with high Impact Points!). Any rational person shall expect that those people teach good ideas and will write “Quality papers on Quality matters”. QEG experts do think firmly that only “Peer Reviewed papers” and “Citations” are important for Quality... Do they act correctly or wrongly?” [2] Consider the following case: Minitab software computes the T Charts Control Limits for exponential and Weibull distributed data for the so-called “rare events”; it happens that, using the SPQR approach [1]. The same happens for other software. If professors use that software for teaching either Quality or Statistics do they act correctly or wrongly?

“Is there any Quality in wrong teaching? Teaching must be scientific for future managers, as Deming and Galetto say (fig. 1).” [2] If the reader want he can find some cases in the references [from 10 to 21].

In the appendix, we provide some ideas about scientifically; we suggest reading it, before going on: it is useful but not compulsory.

“To show how teaching fails to attain his goal (i.e. to prepare students for the future), the paper will use a simple case:” [2] the analysis of a 2-state system, where requirements on the number of failures(Ni) on the length of downtime (x) and on the maximum number of Long Downtimes (Nn,0) are fixed in a supply contract of the system; if the supplier does not meet the stipulated goals he must pay the penalty. We use the SPQR approach [1].

In this introduction, we provide here some basic ideas of Reliability Theory [from 22 to 31], useful for Reliability Analysis and other methods (e.g., inventory, the Bass model analysis, ...). The following concepts are taken from [2].

Let T be the random variable “Time to failure” of an item, and 0 < t the mission interval, whose duration is t. The reliability R(t) is the probability that failure happens during the mission, with f(t) being the pdf,

\[ R(t) = P(T > t) = 1 - F(t) = \int_{t}^{\infty} f(x)dx \]  

The mean of the r.v. T is the Mean Time To Failure

\[ MTTF = E[T] = \int_{0}^{\infty} x f(x)dx = R(0) \]  

The failure rate h(t), as any good student knows, is neither a (conditional) probability density nor a (conditional) probability; it is the ratio

\[ h(t) = f(t) / R(t) \]  

Hence it is easily derived that

\[ R(t) = \exp\left[-\int_{0}^{t} h(x)dx\right] \]  

When the failure rate is constant, the failures are distributed “in the most random manner”: the conditional reliability does not depend on the item past life.

It is easily seen that the knowledge of the failure rate h(t) is enough to obtain any reliability characteristic [R(t), MTTF, MTTF(t), F(t), f(t)].

The Mean Time to Failure, related to the interval 0 – t, is

\[ MTTF(t) = \int_{0}^{t} R(x)dx \]  

The same ideas are also valid for maintenance. Let T, be the random variable “Time to repair” of an item, and 0 < t the interval considered for repair, whose duration is t. The reparability G(t) is the probability that a repair happens in the mission, g(t) being the pdf (the time 0 is the instant at which the item fails)

\[ G(t) = P[T < t] = \int_{0}^{t} g(x)dx \]  

The mean of the r.v. is the Mean Time To Repair

\[ MTTR = E[T_r] = \int_{0}^{\infty} x g(x)dx = \int_{0}^{\infty} [1 - G(t)]dt \]  

The repair rate, as any good student knows, is neither a (conditional) probability density nor a (conditional) probability; it is the ratio

\[ r(t) = g(t) / [1 - G(t)] \]  

Hence it is easily derived that

\[ G(t) = 1 - G(t) = \exp \left[-\int_{0}^{t} r(x)dx\right] \]  

When the repair rate is constant, the repairs are distributed “in the most random manner”: the conditional reparability does not depend on the past repairs.

The Mean Time To Repair, related to the interval 0 – t, is

\[ MTTR(t) = \int_{0}^{t} \bar{G}(x)dx \]  

Let’s now see the concept of Availability.

Let’s assume that we have a system that is repaired after any failure; let U, the time of survival to the i-th failure, measured from the previous repair [Up time]; let D, the time from the i-th failure to the next repair.
[Down time]; both are random variable: their means are the Mean Up Time MUT, and the Mean Down Time MDT; the sum D, i+U, is the Time Between Failures, from the (i-1)-th failure to the i-th failure [it is a random variable]: the mean of it is the MTBF i-1, i = Mean Time Between Failures, from the (i-1)-th failure to the i-th failure.

If f(t) is the density of the Up time U, we have

$$MUT_i = E[U_i] = \int_0^\infty xf_i(x)dx$$  \hspace{1cm} (11)

while if g(t) is the density of the Down time D, we have

$$MDT_i = E[D_i] = \int_0^\infty xg_i(x)dx$$  \hspace{1cm} (12)

By defining z(t) is the density of the “Cycle time”, from an up-state of the system (when it works well) to the next up-state of the system (when it works well, again), we have that z(t) is the convolution f(t)*g(t) of the two densities f(t) and g(t); then the Mean Cycle Time MCT, is

$$MCT_i = \int_0^\infty z_i(x)dx = E[U_i + D_i]$$  \hspace{1cm} (13)

When all the r.v. U, are identically distributed, we indicate with f(t) the probability density of the r.v. U; when all the r.v. D, are identically distributed, we indicate with g(t) the probability density of the r.v. D; when the r.v. U and D are identically distributed, we indicate with z(t) the probability density of the r.v. U+D.

In that case, we have the means MUT, MDT, MCT=MTBF (Mean Time between Failures).

In the next sections, we shall start to deal with our the analysis of a 2-state system, where requirements on the number of failures and the length of downtime are stated goals in a supply contract of the system (if the supplier does not meet the stipulated goals he must pay the penalty), by working, step by step, from a simple model to a more complete model.

We will use the Reliability Integral Theory devised by Fausto Galetto to overcome the Markov process theory used for reliability calculations. [22-24, 28-31]

In the next section, we provide some concepts on reliability and availability.

II. Reliability and Availability

Let’s consider now our system, as depicted in the following flow graph.

![Flow graph diagram](image)

**Figure 2:** A 2-state system, with failure rate h(t) and repair rate r(t)

We consider only a very simple system to provide fundamental concepts.

State 0 of the system is the state where it works well and can fail with failure rate h(t), while state 1 is the state where the system is failed and under repair with repair rate r(t): in 0 the system is up, in 1 the system is down.

We assume that the process failure-repair is regenerative: any time the system enters a state, the process starts from scratch: the system is GAN, as Good As New. The failure-repair process is a Semi-Markov process. [22-24, 28-31]

Let A0(t) be the Availability of the system, i.e., the probability that the system is working well at time t, when it entered the state 0 at time t=0; let A1(t) be the Availability of the system, i.e., the probability that the system is working well at time t, when it entered the state 1 at time t=0.

Using the Availability Integral Theory [F. Galetto, 22-24, 28-31] we write the following system of “INTEGRAL” equations

$$A_0(t) = R(t) + \int_0^t f(s)A_1(t-s)ds$$  \hspace{1cm} (14)

$$A_1(t) = \int_0^t g(s)A_0(t-s)ds$$

We can reduce it to a single “INTEGRAL” equation [using the cycle density z(t)]

$$A_0(t) = R(t) + \int_0^t z(r)A_0(t-r)dr$$  \hspace{1cm} (15)

Using the method of Peano-Picard, we can derive the solution in the form

$$A_0(t) = R(t) + \int_0^t m(r)R(t-r)dr$$  \hspace{1cm} (16)

where the “intensity” m(t) is given by

$$m(t) = z(t) + \int_0^t m(r)z(t-r)dr$$  \hspace{1cm} (17)

Notice that the product m(t)dr= probability that a cycle is completed in the interval t→t+dr. The integral

$$M(t) = \int_0^t m(r)dr = 1 - R(t) + \int_0^t z(r)M(t-r)dr$$  \hspace{1cm} (18)

is the Mean Number of Cycles in the interval 0→t. Letting t→∞ one gets [22-24, 28-31] the Steady State Availability

$$A_0(\infty) = A_{ss} = \frac{MUT}{MUT + MDT} = \frac{MUT}{MTBF}$$  \hspace{1cm} (19)

and asymptotic rate
The failure and repair rate are both constant, \( h(t) = \lambda \) and \( r(t) = \mu \), it is easily found [where \( A(t) = A_0(t) \)]

\[
A(t) = \frac{\mu}{\mu + \lambda} + \frac{\lambda}{\mu + \lambda} \exp[-(\mu + \lambda)t] \tag{21}
\]

It is easily seen that \( A(t) = MTTF/(MTTF + MTTR) = MTTF/MTBF \), as it must be.

Notice: the relationship \( m(t) = \lambda A(t) \) is valid only when the failure and repair rate are both constant, \( h(t) = \lambda \) and, \( r(t) = \mu \). There are incompetent professors who teach the formula \( m(t) = \lambda A(t) \) for variable failure and repair rates. [30, 31]

Before leaving this section, let’s see what five “reliability experts” (are they experts?) of four different universities wrote in a booklet! Use the SPQR Principle.

The author gave it as Esercizio n. 5 (Exercise 5) to his students at the Quality Exam.

“A system is made by three units named GPS, TV e SC; the system performs properly when the items GPS and TV work well; if SC fails it is repaired; the items failures are considered independent; the professors draw the diagram on the left (“riparazione” = repair) and compute the system reliability. Then they (BMWists) have the “GREAT IDEA” that some failure could be dependent and draw the diagram on the right.

Figure 3: A 6-state system (3 units, “riparazione” = repair), in booklet written by 5 “reliability experts” of 4 different universities”

Notice the failure rates \( \lambda_{SC/TV} \) and \( \lambda_{SC/GPS} \) (dotted arrows), by the 4 professors were written to be

\[
\lambda_{SC/TV} = \frac{\lambda_{SC}^2 \lambda_{TV} + \lambda_{SC} \lambda_{TV}^2}{\lambda_{SC}^2 + \lambda_{TV}^2 + \lambda_{TV} \lambda_{SC}}
\]

\[
\lambda_{SC/GPS} = \frac{\lambda_{SC}^2 \lambda_{GPS} + \lambda_{SC} \lambda_{GPS}^2}{\lambda_{SC}^2 + \lambda_{GPS}^2 + \lambda_{GPS} \lambda_{SC}}
\]

The students had to be better than those professors, and FIND that the failure rates \( \lambda_{SC/TV} \) and \( \lambda_{SC/GPS} \) (dotted arrows) are ACTUALLY the formulae \( 1/MTTF \) of the PARALLEL of the units SC/TV and SC/GPS!WRONG!

Can anyone believe to such professors? Use the SPQR Principle [1].

III. THE 1ST STEP: A POISSON PROCESS

Let’s consider now our system, as depicted in the following flow graph.
We assume that the reliability goals are as follows:
1. During the mission time 0--t, a maximum number N ≥ N_{f} of failures is accepted; if the number of failures n is >N_{f}, the supplier will pay a penalty P_{n}. 
2. During the mission time 0--t, at any failure, a maximum length of the downtime < x is accepted; if the downtime is > x (a stated value, named “Long Downtime”), the supplier will pay a penalty P_{D}. 
3. During the mission time 0--t, a maximum number N_{D} of Long Downtimes is accepted; if the number of downtimes m is >N_{D}, the supplier will pay a penalty P_{NLD}.

The 1st step for building our model is to consider only the number of failures; so doing we assume that the downtimes are very short (we assume them as not important; the repair rate μ>>λ is a strong assumption!): in this case t, the duration of the mission, is “almost” the total up time.

The probability of the random variable N “number of failures”, in the mission time 0--t, is

\[ P[N = n] = \frac{(\lambda t)^n}{n!} \exp(-\lambda t) \]  (23)

Therefore the probability that the “number of failures” N, in the mission time 0--t, is >N_{f} is

\[ P[N \geq N_{f}] = 1 - \sum_{n=0}^{N_{f}} \frac{(\lambda t)^n}{n!} \exp(-\lambda t) \]  (24)

In this case, the supplier will pay a penalty P_{n}.

It is easily seen that in the chosen hypothesis we have a Homogenous Poisson Process. The probability of a “Long Downtime” x is

\[ p = \exp(-\mu x) \]  (25)

Formula (25) is found by the following argument: let our system be in state 1 and let \xi(t) = T_{u}--t the duration from the present time t, when the system enters state 1, to the time that the repair is completed and the system enters state 0; we want to compute the probability p(t+x|t) = P[\xi(t)>x] that the “time to repair” is longer than the stated “Long Downtime” x. From pages 169-173 of the book [24] one can write an integral equation whose solution is p(t+x|t); from there it is found that p(t+x|t) = p = \exp(-\mu x) when the repair rate is constant. The same result can be found in [23].

IF the repair rate μ>>λ then the Homogenous Poisson Process of intensity λ, where we pick its points \{\xi(t)>x\} with probability p, given by (25), becomes the Homogenous Poisson Process of intensity pλ; therefore (26)

\[ P[M \geq N_{LD}] = 1 - \sum_{n=0}^{N_{s}} \frac{(p\lambda t)^n}{n!} \exp(-p\lambda t) \]  (26)

Is the probability that the “number of Long Downtimes” M, in the mission time 0--t, is >N_{LD}; the supplier will pay a penalty P_{NLD}.

The formula (26) derives from the theory of Poisson Processes. We can get it using the Reliability theory, with the following arguments. Consider a process, named “auxiliary system”, as in the following figure 5:

The “auxiliary system” works as follows:
1. It starts in state 1 (Up-state)
2. It re-enters state 1 with probability p
3. It goes to state 2 (Down-state) with probability 1-p
4. The time to re-enter or to go out is provided by the exponential probability density with rate λ.

Let \Phi_{11}(n,t) be the probability of the joint event that the system will have made n transitions (re-entering) and will be in state 1, given that it started in state 1 at time t=0, and that time t has elapsed. We have

\[ \Phi_{11}(n,t) = p^{n} \frac{(\lambda t)^n}{n!} \exp(-\lambda t) \]  (27)

Let \Phi_{11}(t) be the probability of the system being in state 1 at time t, given that it started in state 1 at time t=0. By summing \Phi_{11}(n,t) for all the values of n (from 1 to \infty), we have

\[ \Phi_{11}(t) = \exp[-(1-p)\lambda t] \]  (28)

Figure 5: A 2-state, “auxiliary system”, with constant failure rate λ and repair rate μ>>λ.

The probability that the system experience n “long downtimes”, given that time t has elapsed, and it occupies state 1

\[ \frac{\Phi_{11}(n,t)}{\Phi_{11}(t)} = \frac{(p\lambda t)^n}{n!} \exp[-(1-p)\lambda t] \]  (29)

by doing the necessary operations, provides the formula (26).

We see then that, with the strong assumption μ>>λ (the downtimes are very short and t is the total up time), it is very easy to compute the costs involved.

The probability that the “number of failures” N(t), in the mission time 0--t, is >N_{f} (and the supplier will pay a penalty P_{n}) is
where $F_n(t)$ is the convolution $F(t) * F_{n-1}(t)$ with $F_1(t) = F(t)$.

From (30), one can derive the Mean Number of Failures in the interval $0 \leq t$:

$$M(t) = F(t) + \int_0^t f(r) M(t-r)dr$$ \hspace{1cm} (18b)

In the case that $h(t) = \lambda$, we have $M(t) = \lambda t$.

In all the sections, we consider a system with $\text{MTTF}=1000$ (units of time) and $\text{MTTR}=100$. These values do not conform “completely” with our strong assumption $\mu \gg \lambda$; despite that, they are chosen so that the graphs can show the different curves for the different cases. We state, $N=4$ and $x=100$. When the reliability is Weibull with $\text{MTTF}=1000$, the $M(t)$ depends on the shape parameter $\beta$. For our case, we chose $\beta=3$. See figure 6.

IV. THE 2ND STEP: A “MODIFIED” POISSON PROCESS

Let’s consider our system again, as depicted in the following flow graph.

Again we consider that the failure and the repair rates are both constant, $h(t) = \lambda$ and $r(t) = \mu$.

We assume that the reliability goals are the same as in section 3.

As in the 1st step, we consider the number of failures, but now we do not assume that the downtimes are very short; they depend on the repair rate $\mu$.

The probability of the random variable $N(t)$ “number of failures”, in the mission time $0 \leq t$, is no longer as the probability of the the number of points of a Homogenous Poisson Process.

We have a process with intensity $m(t)$ given by the formula (22), here repeated,

$$m(t) = \frac{\lambda \mu}{\lambda + \mu} + \frac{\lambda^2}{\lambda + \mu} \exp[-(\lambda + \mu)t] = \lambda A(t)$$ \hspace{1cm} (22)

It is easily seen that after two cycles, the Availability and the cycling intensity are almost constant; therefore, after few cycles, the stochastic process becomes a Homogenous Poisson Process, with intensity $m(\infty) = 1/(\text{MTTF} + \text{MTTR}) = 1/\text{MTBF} = \lambda A_{SS}$.

The same happens when the failure and repair rates are variable [provided the system is renewable]: $m(\infty) = 1/(\text{MTTF} + \text{MTTR}) = 1/\text{MTBF}$. See figure 7.
Therefore, when the failure and repair rate are both constant, $h(t) = \lambda$ and $r(t) = \mu$, the probability that the “number of failures” $N$, in the mission time $0 \rightarrow t$, is $> N_i$ is

$$P[N \geq N_i] = 1 - \sum_{n=0}^{N_i} \left( \frac{\lambda A_{sst} t^n}{n!} \right) \exp(-\lambda A_{sst} t) \quad (31)$$

The supplier will pay a penalty $P_r$.

The probability of a “Long Downtime” $x$ is, as in section 3, $p = \exp(-\mu x)$ and we pick the points (of the process) with probability $p$, given by (25); therefore (32)

$$P[M \geq N_{LD}] = 1 - \sum_{n=0}^{N_i} \left( \frac{p \lambda A_{sst} t^n}{n!} \right) \exp(-p \lambda A_{sst} t) \quad (32)$$

is the probability that the “number of Long Downtimes” $M$, in the mission time $0 \rightarrow t$, is $> N_{LD}$; the supplier will pay a penalty $P_{NLD}$.

We see again that it is very easy to compute the costs involved. See the related probabilities for $C_1$ and $C_2$.

![Figure 8: Example of the intensity $m(t)$ when the failure rate and the repair rate are variable (e.g., Weibull)](image)

![Figure 9: The probability that “number of Long Downtimes” $M$, in the mission time $0 \rightarrow t$, is $> N_{LD}$; case 1 and case 2](image)
V. The 3rd Step: A semi-Markov Process with Constant Failure Rate and General Repair Rate

Let's consider our system again, as depicted in the following flow graph.

![Flow graph](image)

Figure 10: A 2-state system, with constant failure rate \( \lambda \) and variable repair rate \( r(t) \).

Now we consider that the failure is constant, \( h(t) = \lambda \), while the repair rate is any positive function \( r(t) \).

We assume that the reliability goals are the same as in section 3.

We remind here that we found the integral equations for availability and using the method of Peano-Picard, we could derive the solution [here repeated for convenience]

\[
A_t = R(t) + \int_0^t m(r)R(t-r)dr
\]

where the “intensity” \( m(t) \) is given by

\[
m(t) = z(t) + \int_0^t m(r)z(t-r)dr
\]

and where the product \( m(t)dt \) is probability that a cycle is completed in the interval \( t------t+dt \).

The probability that the “number of Long Downtimes” \( M \), in the mission time \( 0------t \), is \( >N_{LD} \) (and the supplier will pay a penalty \( P_{N_{LD}} \)) is again [formula 26 repeated here]

\[
P[M \geq N_{LD}] = 1 - \sum_{n=0}^{N_{LD}} \left( \frac{p\lambda t}{n!} \right)^n \exp(-p\lambda t)
\]

where the probability \( p \) is obtained by the repair rate \( r(t) \).

For this case 3, the repair rate \( r(t) \) is of a Weibull distribution with \( \beta_{repair} = 2 \).

We want here to find the probability \( LD(x | t) \), that the system is still in the state 1 [downstate] for a time \( x \), given that the system entered state 1 at time \( t \); we name \( LD(x | t) \) “Long Downtime Complementary Distribution”.

As per F. Galetto, vol. 1, page 170, we can write the following equation (similar to 16)

\[
LD(x | t) = \int_x^{x+x} m(t+x-y)\overline{G(y)}dy
\]

(33)

When downtime \( D \) is \( >x \) (a stated value, named “Long Downtime”) the supplier will pay a penalty \( P_D \).

To prove (32), now we argue as in section 4: it is easily seen that after two cycles the Availability and the cycling intensity are almost constant; therefore, if \( t > 2 \) MTBF, we have

\[
LD(x | t) \approx \frac{MTTR(t+x)-MTTR(x)}{MTBF}
\]

(35)

where MTTR(t) is the Mean Time To Repair, related to the interval \( 0------t \).

For \( x \rightarrow \infty \), one gets \( LD(x | t) \rightarrow 0 \), as it must be. For \( t \rightarrow \infty \), one gets \( LD(x | t) \rightarrow MDT/MTBF \), as it must be.

To consider both the number of failures and the long downtime we need the probability \( LD(x | t, n) \): the probability that, in the mission interval \( 0------t \), the downtime is long \( x \), given that the number of failures is \( n \), is the formula (36)

\[
LD(x | t, n) = \int_x^{x+x} f_n(t+x-y)\overline{G(y)}dy
\]

(36)

where \( f_n(t)dt = P[t<T_n<t+dt] \) is the probability that the \( n \)-th failure happens in the interval \( t------t+dt \) (\( T_n \) is the “time to the \( n \)-th failure). The relationship between \( f_{n+1}(t) \) and \( f_n(t) \) [where \( f_1(t) = f(t) \)]

\[
f_{n+1}(t) = f(t) + \int_0^t f_n(r)z(t-r)dr
\]

(37)

Summing over all the number of failures from (37) one gets (33).

\( LD(x | t, N_f+1) \) provides the probability that, in the mission interval \( 0------t \), the downtime is long \( x \) and the number of failures is \( >\) maximum allowed number \( N_f \); the supplier will pay a penalty \( P_f + P_D \).
Using the very strong assumption $\mu > \lambda$ (the downtimes are very short and $t$ is the total up time) it is very easy to compute

$$LD(x|t, 1) = \int_x^{x+2t} f(t + x - y) \bar{G}(y) dy \quad (38)$$

Similarly, $LD(x|t, 2), \ldots \text{ Any } LD(x|t, n)$ is related to (27) via the probability $p$, given by the Weibull.

The cases 3 and 4 are similar to the ones 1 and 2, with the difference of the uses of a Weibull repair distribution.

VI. THE 4TH STEP: A GENERAL SEMI-MARKOV PROCESS

Let’s consider our system again, as depicted in the following flow graph.

```
0  h(t)  1
     r(t)
```

Figure 12: A 2-state system, renewable, with variable both failure rate $h(t)$ and repair rate $r(t)$

Now we consider that the failure is any positive function $h(t)$ and the repair rate is any positive function $r(t)$, both related to their Weibull distribution; we also assume that the system is renewed at any entrance into the state 0.

We assume that the reliability goals are the same as in section 3.

In the hypothesis of a general Semi-Markov process, the formulae are the same as those of section 5 (we do not repeat them here).
We see the probability in figure 13.

**Figure 13:** The probability that “number of Long Downtimes” $M$, in the mission time $0\rightarrow t$, is $>N_{LD}$; case 5

We see that the probability $P[M>N_{LD}]$ increases with the length $t$ of the “mission interval”: as $t$ increases, the “Long Downtimes” becomes more and more probable (as anybody should expect).

To appreciate the differences between the various cases, see figure 14.

**Figure 14:** Probability that “number of Long Downtimes” $M$, in the mission time $0\rightarrow t$, is $>N_{LD}$; cases 1-5

It is quite interesting to notice that the most general case 5 has time behavior “very similar” to the case 1: this is because the Steady State Availability $A_{SS}$ is the same value.
VII. The Cost Inserted in the General Semi-Markov Process

Let's consider our system again, as depicted in FIGURE 12.

Now we generate a model where the costs are inserted in the general equation of the model.

Let's indicate with the symbols \( b_{ik}(t)dt \) the transition probability from state \( i \) to state \( k \) (either 0 or 1, or vice versa) in the interval \( t \rightarrow t + dt \), \( p_{ik} \) the steady transition probability from state \( i \) to state \( k \), \( m \) the mean that the system stays in state \( i \) before making a transition, \( e_{ik}(0, s) \) the earning [or cost] of the system due the transition from state \( i \) to state \( k \) for the interval 0 to \( s \), \( d_{ik}(s) \) the earning [or a cost] of the system due the transition from state \( i \) to state \( k \) at the instant \( s \), \( v(t) \) the total expected profit [or cost] of the system for the interval 0 to \( t \) (mission time), if the system starts in state \( i \) at time 0. We define \( r(t) \) the expected reward (or cost) of the system related to state \( i \), due to its transitions in the interval 0 to \( t \)

\[
r_i(t) = \sum_{k=0}^{1} \int_0^t b_{ik}(s)[e_{ik}(0, s) + d_{ik}(s)]ds
\]

If the system makes its 1st transition out of the state \( i \), before the instant \( t \), it earns a profit

\[
\sum_{k=0}^{1} r_i(t) + \int_0^t b_{ik}(s)v_k(t - s)ds
\]

If the system makes its 1st transition out of the state \( i \), after the instant \( t \), it earns a profit

\[
\sum_{k=0}^{1} W_k(t)p_{ik}e_{ik}(0, t)
\]

Putting all together, we have the system of integral equations [notice the similarity with what done before for reliability] of the expected reward (or cost) of the system in the mission interval

\[
v_i(t) = \sum_{k=0}^{1} \int_0^t [e_{ik}(0, s) + d_{ik}(s) + v_k(t - s)]b_{ik}(s)ds + k - 01WK(t)pikeik01t
\] (39)

The general model (with \( n+1 \) states) was devised by the author and presented at an EOQC Conference [XXI EOQC (European Organisation for Quality Control)] held in Varna (Bulgaria), 1977, with a paper titled CLAUDIA Cost and Life Analysis via Uptime and Downtime Integral Approach.

In our case \( d_{01}(s)=0 \) (for the failures) and \( e_{01}(0, t)=0 \), while \( d_{00}(s)=0 \) and \( e_{10}(0, s) = H(s-x) \) (for the long downtimes > \( x \)), where \( H(s-x) \) is the Heaviside function.

\[
v_0(t) = \int_0^t b_{01}(s)[1 + v_1(t - s)]ds
\]

\[
= W_0(t) + \int_0^t b_{01}(s)v_1(t - s)ds
\]

\[
v_1(t) = W_1(t)H(t - x)
\]

\[
+ \int_0^t b_{10}(s)[H(s - x) + v_0(t - s)]ds
\]

\[
= \frac{W_1(t)}{H(t - x)} + W_1(t) - W_1(x)
\]

\[
+ \int_0^t b_{10}(s)v_0(t - s)ds
\]

The initial conditions:

\[
v_0(t) = 0
\]

\[
v_1(t) = 0
\]

These equations can be easily solved, in the case of exponential distributions.

As a matter of fact, in such a case,

\[
v_0(t) = 1 - e^{-\lambda t} + \int_0^t \lambda e^{-\lambda s}v_1(t - s)ds
\]

\[
= 1 - e^{-\lambda t} + \lambda e^{-\lambda t} \int_0^t e^{\lambda s}v_1(s)ds
\]

\[
v_1(t) = e^{-\mu t}H(t - x) + (1 - e^{-\mu t}) - (1 - e^{-\mu x})
\]

\[
+ \int_0^t \mu e^{-\mu s}v_0(t - s)ds
\]

\[
= e^{-\mu t}H(t - x) - e^{-\mu t} + e^{-\mu x}
\]

\[
+ \int_0^t \mu e^{-\mu s}v_0(t - s)ds
\]

\[
= e^{-\mu t}H(t - x) - e^{-\mu t} + e^{-\mu x}
\]

\[
+ \mu e^{-\mu t} \int_0^t e^{\mu s}v_0(t - s)ds
\]

From these we can find two differential equations

\[
v'_0(t) + \lambda v_0(t) = \lambda v_1(t) + \lambda
\]

\[
v'_1(t) + \mu v_1(t) = \mu v_0(t) + e^{-\mu x} + \mu
\]

that are written in matrix form

\[
v'(t) = Av(t) + \begin{bmatrix} \lambda \\ \mu e^{-\mu x} \end{bmatrix}
\] (40)

with

\[
A = \begin{bmatrix} -\frac{\lambda}{\mu} \\ \frac{\lambda}{\mu} \end{bmatrix}
\] (41)

The solution is

\[
v(t) = e^{At} \int_0^t e^{-As} \begin{bmatrix} \lambda \\ \mu + e^{-\mu x} \end{bmatrix} ds
\] (42)
We can easily find the difference between the two components of the vector \( v(t) \). The difference becomes constant for \( t \to \infty \). The solution of (42) is increasing, linearly for \( t \to \infty \) (see figure 15): \[
v_0(t) = (2 \mu + e^{-\mu x}) \mu t - (2 \mu + e^{-\mu x} - \lambda) \mu t \left[ 1 - e^{-(\lambda + \mu)t} \right]
\]
\[
v_1(t) = (2 \mu + e^{-\mu x}) \mu t + (2 \mu + e^{-\mu x} - \lambda) \mu t \left[ 1 - e^{-(\lambda + \mu)t} \right]
\]

We see that the difference between the two curves becomes constant.

The type of behavior of the two curves, devised for constant failure and repair rates, is similar for variable rates; the proof can be found in the paper CLAUDIA Cost and Life Analysis via Uptime and Downtime Integral Approach.

### VIII. Conclusion

Any action speaks louder than words: professors teaching wrong ideas do a lot of harm to their students and the whole Society, although they are all graduated CUM LAUDE, with Ph.D. (CUM LAUDE), very appreciated by their followers (with their "likes") and have high scores with the informetric indexes (h-index, RG-index, s-index, and so on).[2, 10-21]

We showed that Theory is needed to solve correctly the problem of evaluating the cost of failures and downtimes in a very simple 2-state system, where requirements on the number of failures \( N_f \), on the length of downtime \( x \), and the maximum number of Long Downtimes \( N_{LD} \) are fixed in a supply contract of the system; if the supplier does not meet the stipulated goals, he must pay the penalty. We used the SPQR Principle and approach [1].

The method can be extended to more complex systems.

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**APPENDIX "Scientificity"

This appendix about Scientificity is derived from many sources of Fausto Galetto’s thinking. It is given here as a summary. We also show four cases of lack of Scientificity.

Here we want to provide the reader with some ideas about the need of the Scientific Attitude that all the teachers and managers must have: it starts with two Premises and one Entailment.

1st Premise: Ever since he was a young student, at the secondary school, Fausto Galetto was fond of understanding the matters he was studying; understanding for learning was his credo (φιλομαθηςσυνιηµι); for all his life he was keeping this attitude, studying more than one ton of pages: as manager and as consultant he studied several methods invented by professors, but never he used the (many) wrong ones; on the contrary, he has been devising many original methods needed for solving the problems of the Companies he worked for, and presenting them at international conferences [where he met many bad diverters, also professors "ASQC certified quality auditors" or "Master Black Belt (Six Sigma) Experts"]; after 25 years of applications and experience, he became professor, with a dream “improve the future managers (students) quality”: the incompetents he met since then grew dramatically (also with documents. F. Galetto got from ERASMUS students (Fijiu Antony et al., 2001, Sarin S. 1997).

2nd Premise: “The wealth of nations depends increasingly on the quality of managers.” (A. Jay [3]) and the fact that “Universities grow the future managers.” (F. Galetto)

Entailment: due to that, the author with this paper will try, again, to provide the important consequent message: let us, all of us, be scientific in all Universities, that is, let us all use our rationality. "What I want to teach is: to pass from a hidden non-sense to a non-sense clear." (L. Wittgenstein).

We have been seeing and we are still seeing the consequences of the lack of Scientificity during the Covid-19 pandemic… Remember Deming’s ideas.

"In my university studies …, in most of the cases, it seemed that students were asked simply to regurgitate at the exams what they had swallowed during the courses." M. Gell-Mann "The Quark and the Jaguar..." [1994]). Some of those students later could have become researchers and then professors, writing “scientific” papers and books … For these last, another statement of the Nobel Prize M. Gell-Mann is relevant: "Once that such a misunderstanding has taken place in the publication, it tends to become perpetual, because the various authors simply copy one each other."...
similar to “Imitatores, servumpecucus” [Horatius, 18 B.C.] and “Gravior et valido rest dececmviorum bonorum sententia quam to tiusmultitudinis imperitiae” [Cicero]. When they teach, "The result is that hundreds of people are learning what is wrong. I make this statement on the basis of experience, seeing every day the devastating effects of incompetent teaching and faulty applications." [Deming (1986)], because those professors are unable to practice maieutic [μαίευτικήτεχνη], the way used by Socrates for teaching [the same was for Galileo Galilei in his "Dialogue on the Two Chief World Systems"]. Paraphrasing P. B. Crosby, in his book "Quality is free", we could say: "Professors may or may not realize what has to be done to achieve quality. Or worse, they may feel, mistakenly, that they do understand what has to be done. Those types can cause the most harm." What do have in common Crosby, Deming and Gell-Mann statements? The fact that professors and students betray an important characteristic of human beings: rationality [the "Adult state" of E. Berne]. Human beings are driven by curiosity that demands that we ask questions ("why?", ..., why?) and we try to put things in order ("this is connected with that"): curiosity is one of the best ways to learn, but "learning does not mean understanding"; only twenty-six centuries ago, in Greece, people began to have the idea that the "world" could be "understood rationally", overcoming the religious myths: they were sceptic [σκέπτοµαι] and critic [κρίνω] and we assumed them implicitly. These ideas gave rise to the SPQR Principle and approach [1]

Till today, after so long time, we still do not use appropriately our brain! A peculiar, stupid and terrific non-sense! During his deep and long experience of Managing and Teaching (more than 40 years), F. Galetto always had the opportunity of verifying the truth of Crosby, Deming and Gell-Mann statements.

To understand each other we need to define the word "scientific". A document (paper or book) is "scientific" if it "scientifically (i.e. with "scientific method") deals with matters concerning science (or science principles, or science rules)". Therefore to be "scientific" a paper must both concern "science matters" and be in accordance with the "scientific method".

The word "science" is derived from the Latin word "scientia" (to know for certain) {derived from the Greek words μαθησις, επιστηµη, meaning learning and knowledge, which, at that time, were very superior to "opinion" [δοξα], while today opinion of many is considered better than the knowledge of very few!}: think to the recent behaviour of people, they look for getting many "likes" in the web!!! Knowledge is strongly related to "logic reasoning" [λογικά], as it was, for ages, for Euclid, whose Geometry was considered the best model of "scientificity". Common (good) sense is not science! A lot of "likes" in the web is not science! Common sense does not look for "understanding", while science looks for "understanding"! "Understanding" is related to "intelligence" (from the Latin verb "intelligere" [intus+legere. to read into]: "intelligetcredas" i.e. understand to believe. Unfortunately "none so deaf as those that won't hear".

Let us give an example, the Pythagoras Theorem (figure 16):

"In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides." Is this statement scientific? It could be scientific because it concerns the science of Geometry and it can be proven true by mathematical arguments. It is not-scientific because we did not specify that we were dealing with the "Euclidean Geometry" (based, among others, on the "parallel axiom": from this only, one can derive that the sum of the interior angles of a triangle is always π): we did not deal "scientifically" with the axioms; we assumed them implicitly.

So we see that "scientificity" is present only if the set of statements (concerning a given "system") are non-contradictory and deductible from stated principles (as the rules of Logic and the Axioms).

Let us give another example, the 2nd law of Mechanics (figure 16):

"The force and the acceleration of a body are proportional vectors: F=ma, (m is the mass of the body)". Is this statement scientific? It could be scientific because it concerns the science of Mechanics and it can be proven "true" by well-designed experiments. It is not-scientific because we did not specify that we were dealing with "frames of reference moving relatively one to another with constant velocity" [inertial frames (with the so called "Galilean Relativity": the laws of Physics look the same for inertial systems)] and that the speed involved was not comparable with the "speed of light in the vacuum [that is the same for all observers]" (as proved by the Michelson-Morley experiment: in the Special Relativity Theory, F=dlv/dt is true, not F=ma) and not involving atomic or subatomic particles. We did not deal "scientifically" with the hypotheses: we assumed them implicitly. From the laws of Special Relativity we can derive logically the conservation laws of momentum and of energy, as could Newton for the "Galilean Relativity". For atomic or subatomic particles "quantum Mechanics" is needed (with Schrödinger equation as fundamental law).

So we see that "scientificity" is present only if the set of statements (concerning a given "system") do not contradict the observed data, collected through well designed experiments ["scientific" experiments]: only in the XVII century, due to Galilei, Descartes, Newton, … we learned that. Since that time only, science could really grow.
When we start trying to learn something, generally, we are in the “clouds”; reality (and truth) is hidden by the clouds of our ignorance, the clouds of the data, the clouds of our misconceptions, the clouds of our prejudices; to understand the phenomena we need to find out the reality from the clouds: we make hypotheses, then we deduct logically some consequences, predicting the results of experiments: if predictions and experimental data do match then we “confirm” our idea and if many other are able to check our findings we get a theory. To generate a theory we need Methods. Eric Berne, the psychologist father of “Transactional Analysis”, stated that everybody interacts with other people through three states P, A, C [Parent, Adult, Child, (not connected with our age, fig. 16)]: the Adult state is the one that looks for reality, makes questions, considers the data, analyses objectively the data, draws conclusions and takes logic decisions, coherent with the data, methodically. Theory [θεωρία] comes from the Adult state! Methods [μέθοδος] from μετα+οδός = the way through (which one finds out...)] used to generate a Theory come from the Adult state!

People who take for granted that the truth depends on “ipse dixit” [ειπε δειξα, “he said that” (F. Bass “said that”, and published his ideas on a very important Management Magazine, “Management Science”)], behave with the Parent state. People who get upset if one finds their errors and they do not consider them [“we are many and so we are right”, they say!] behave with the Child state. [see the books of the Palo Alto group]

To find scientifically the truth (out of the clouds) you must Focus on the problem, Assess where you are (with previous data and knowledge), Understand Scientifically the message in the data and find consequences that confirm (or disprove) your predictions, Scientifically design Test for confirmation (or disproval) and then Activate to make the Tests. If you and others Verify you prediction, anybody can Implement actions and Assure that the results are scientific (FAUSTA VIA): all of us then have a theory and scientificity is there (F. Galetto)

From the above two examples it is important to realise that when two people want to verbally communicate, they must have some common concepts, they agree upon, in order to transfer information and ideas between each other; this is a prerequisite, if they want to understand each other; what is true for them, what is their “conventional” meaning of the words they use, which are the rules to deduce statements (Theses) from other statements (Hypotheses and “previous” Theses): rigour is needed for science, not opinions.

Many people must apply Metanoia [μετάνοια = change their mind (to understand)] to find the truth.
Here we accept the rules of Logic, the deductive Logic, where the premises of a valid argument contain the conclusion, and the truth of the conclusion follows from the truth of the premises with certainty: any well-formed sentence is either true or false. We define as Theorem “a statement that is proven true by reasoning, according to the rules of Logic”; we must therefore define the term True: “something” (statement, concept, idea, sentence, proposition) is true when there is correspondence between the “something” and the facts, situations or state of affairs that verify it; the truth is a relation of coherence between a thesis and the hypotheses. Logical validity is a relationship between the premises and the conclusion such that if the premises are true then the conclusion is true. The validity of an argument should be distinguished from the truth of the conclusion (based on the premises).

This kind of truth is found in mathematics.

Human beings evolved because they were able to develop their knowledge from inside (the deductive logic, with analytic statements) and from outside, the external world, (the inductive logic, with synthetic statements), in any case using their intelligence; the inductive logic is such that the premises are evidence for the conclusion, but the truth of the conclusion follows from the truth of the evidence only with a certain probability, provided the way of reasoning is correct.

The scientific knowledge is such that any valid knowledge claim must be verifiable in experience and built up both through the inductive logic (with its synthetic statements) and the deductive logic (with its analytic statements); in any case, a clear distinction must be maintained between analytic and synthetic statements.

This was the attitude of Galileo Galilei in his studies of falling bodies. At first time, he formulated the tentative hypothesis that “the speed attained by a falling body is directly proportional to the distance traversed”; then he deduced from his hypothesis the conclusion that objects falling equal distances require the same amount of elapsed time. After “Gedanken Experimenten”, Designed Experiments made clear that this was a false conclusion: hence, logically, the first hypothesis had to be false. Therefore, Galileo framed a new hypothesis: “the speed attained is directly proportional to the time elapsed”. From this, he was able to deduce that the distance traversed by a falling object was proportional to the square of the time elapsed; through Designed Experiments, by rolling balls down an inclined plane, he was able to verify experimentally his thesis (it was the first formulation of the 2nd law of Mechanics).

Such agreement of a conclusion with an actual observation does not itself prove the correctness of the hypothesis from which the conclusion is derived. It simply renders that premise much more plausible.

For rational people (like were the ancient Greeks) the criticism [κρίνω = to judge] is hoped for, because it permits improvement: asking questions, debating and looking for answers improves our understanding; we do not know the truth, but we can look for it and be able to find it, with our brain; to judge we need criteria [κριτέριον]. In this search, Mathematics [note μαθησις] and Logic can help us a lot: Mathematics and Logic are the languages that Rational Managers must know! Proposing the criterion of testability, or falsifiability, for scientific validity, Popper emphasized the hypothetic-deductive character of science. Scientific theories are hypotheses from which can be deduced statements testable by observation; if the appropriate experimental observations falsify these statements, the hypothesis is refused. If a hypothesis survives efforts to falsify it, it may be tentatively accepted. No scientific theory, however, can be conclusively established. A “theory” that is falsified, is no longer scientific.

“Good theories” are such that they complete previous “good” theories, in accordance with the collected new data.

A good example of that is Bell’s Inequality. In physics, this inequality was used to show that a class of theories that were intended to “complete” quantum mechanics, namely local hidden variable theories, are in fact inconsistent with quantum mechanics; quantum mechanics typically predicts probabilities, not certainties, for the outcomes of measurements. Albert Einstein [one of the greatest scientists] stated that quantum mechanics was incomplete, and that there must exist “hidden” variables that would make possible definite predictions. In 1964, J. S. Bell proved that all local hidden variable theories are inconsistent with quantum mechanics, first through a “Gedanken Experiment” and Logic, and later through Designed Experiments. Also, the great scientist, A. Einstein, was wrong in this case: his idea was falsified. We see then that the ultimate test of the validity of a scientific hypothesis is its consistency with the totality of other aspects of the scientific framework. This inner consistency constitutes the basis for the concept of causality in science, according to which every effect is assumed to be linked with a cause.

The scientific community as a whole must judge [κρίνω] the work of its members by the objectivity and the rigour with which that work has been conducted; in this way the scientific method should prevail.

In any case, the scientific community must remember: Any statement (or method) that is falsified, is no longer scientific.

Here we assume that the subject of a paper is concerning a science (like Mathematics, Statistics, Probability, Quality Methods, Management, …); therefore to judge [κρίνω] if a paper is scientific we have
to look at the “scientific method”: if the “scientific method” is present, i.e., the conclusions (statements) in the paper follow logically from the hypotheses, we shall consider the paper scientific; on the contrary, if there are conclusions (statements) in the paper that do not follow logically from the hypotheses, we shall not consider the paper scientific: a wrong conclusion (statement) is not scientific.

"To understand that an answer is wrong you don't need exceptional intelligence, but to understand that is wrong a question one needs a creative mind." (A. Jay). "Intelligeutecredas".

That was the way the author dealt with his students (in Universities, in Companies Courses, in Mater’s Lessons,…)

Right questions, with right methods, have to be asked to “nature”. "Intelligeutecredas”.

It is easy to show that a paper, a book, a method, is not scientific: it is sufficient to find an example that proves the wrongness of the conclusion. When there are formulas in a paper, it is not necessary to find the right formula to prove that a formula is wrong: an example is enough; to prove that a formula is wrong, one needs only intelligence; on the contrary, to find the right formula, that substitutes the wrong one, you need both intelligence and ingenuity. I will use only intelligence and I will not give any proof of my ingenuity: this paper is for intelligence …

For example, it’s well known (from Algebra, Newton identities) that the coefficients and the roots of any algebraic equation are related: it’s easy to prove that $\pm \sqrt{-c/a}$ is not the solution (even if you do not know the right solution) of the parabolic equation $ax^2 + bx + c = 0$, because the system $x_1 + x_2 = -b/a$, $x_1x_2 = c/a$ is not satisfied ($x_1$ and $x_2$ are the roots).

The literature on “Quality” matters is rapidly expanding. Unfortunately, nobody, but the author, as far as he knows, [he thanks any person that will send him the names of people who take care …], takes care of the "Quality of Quality Methods used for making Quality" (of product, processes and services). “Intelligeutecredas”.

Let’s give two others cases of lack of Scientificity.

See the following excerpts (figures 17-18, excerpts 1 and 2) taken from a book on reliability; they refer to the system we have analysed previously [w(t) is our m(t)].

\[
\begin{align*}
  w(t) &= f(t) + \int_0^t f(t-u)v(u)\,du \\
  v(t) &= \int_0^t g(t-u)w(u)\,du
\end{align*}
\]

(4.64)

The unconditional failure intensity $w(t)$ and the repair intensity $v(t)$ are calculated by an iterative numerical integration of (4.64) when densities $f(t)$ and $g(t)$ are given. If a rigorous, analytical solution is required, Laplace transforms can be used.

Figure 17: Excerpt 1 from a book…
We now differentiate the fundamental identity (4.64):

\[
\begin{align*}
\frac{w(t)}{dt} &= f'(t) + f(0)v(t) + \int_0^t f'(t-u)v(u)\,du \\
v(t) &= g(0)w(t) + \int_0^t g'(t-u)w(u)\,du 
\end{align*}
\]  

(4.124)

where \( f'(t) \) and \( g'(t) \) are defined by

\[
\begin{align*}
f'(t) &= \frac{f(t)}{dt}, & g'(t) &= \frac{g(t)}{dt}
\end{align*}
\]  

(4.125)

The differential equation (4.124) is now integrated, yielding the results shown in Fig. 4.23.

![Figure 4.23: Result of integration of (4.124).](image_url)
NOTICE the differentiation, FIRST, and the integration, SECOND!
Notice the symbols in the formula 4.124 and 4.125 (fig. 18)! Had the authors studied Mathematics?
WHAT are they for? Integration is the “opposite” operation of differentiation!
One sees very clearly that W(t) of excerpt 2 (the curve W(t) is M(t) in our formulae) does not have the behaviour that it MUST have according the reliability theory; the following graph shows the curve M(t) obtained via the Reliability Integral Theory.
The curves W(t) and M(t) are very different: only M(t) is according to the THEORY.
It is obvious that to compute correctly the cost of failures, of downtimes, of maintenance and of spare parts management one MUST compute correctly the function M(t). [23, 30, 31]
Will professors understand?
None so deaf as he does not want to hear…….
Wrong teaching: help or hoax for Quality?
HOAX, if people (professors, managers, consultants, …) do not use their own brain !!!!!
Let’s now consider another case related to the T Charts.
It is taken from the paper “Minitab T Charts and Quality Decisions”, submitted to a Journal, in 2020.
The T Charts are used for “rare events”: they are Individual Control Charts with Exponentially or Weibull distributed data.
Here we deal with the problem by considering Example 7.6 found in the Montgomery book [37]; he writes “A chemical engineer wants to set up a control chart for monitoring the occurrence of failures of an important valve. She has decided to use the number of hours between failures as the variable to monitor”. Here are the data (exponentially distributed), named lifetime; (we used Minitab 19 to see the arising problems):

<table>
<thead>
<tr>
<th>Table 1: Lifetime data (from Montgomery)</th>
</tr>
</thead>
<tbody>
<tr>
<td>286</td>
</tr>
<tr>
<td>2837</td>
</tr>
</tbody>
</table>

Since the data are few (20) and exponentially distributed one cannot use the usual formulae used for Normally distributed data. If one would [wrongly] do use those formulae he would find the figure 20 (Minitab used). According to it, the “process is Out Of Control” (OOC): two points are “above” UCL: If we had considered the Moving Ranges, we should have that two other points would be OOC.
Using SixPack or JMP; we would have the same picture of the process.

Is this a true picture of the process? Perhaps these OOC depends on the formulae used!
If we act as Montgomery did and we transform the exponential data into Weibull data with form parameter $\beta = 1/3.6$ (this ideas was copied by Montgomery from Nelson! This attitude of copying without knowledge is very general, as said by Deming, [4] “Management need to grow-up their knowledge because experience alone, without theory, teaches nothing what to do to make Quality” and “The result is that hundreds of people are learning what is wrong. I make this statement on the basis of experience, seeing every day the devastating effects of incompetent teaching and faulty applications.”)

Let be $y_i$ the original (exponential) data and $x_i = y_i^{1.36}$ the transformed (Weibull) data; Montgomery uses the I-MR Chart where in the upper graph the individual $x_i$ are plotted with their mean $\bar{x}$ and control limits and in the lower graph the individual $MR = |x_i - x_{i+1}|$ are plotted with their mean $\bar{MR}$ and control limits. [it is the same graph of Montgomery book]
Test Results for I Chart of lifetime

TEST 1. One point more than 3,00 standard deviations from center line.
Test Failed at points: 11; 19

Figure 20: Individual chart of Montgomery data. Minitab 19 used (assuming...)

Figure 21: Individual and Moving Range chart of “transformed” Montgomery data (as suggested by Nelson). Minitab 19 used (F. Galetto)

According to figure 21, Montgomery says “Note that the control charts indicate a state of control, implying that the failure mechanism for this valve is constant. If a process change is made that improves the failure rate (such as a different type of maintenance action), then we would expect to see the mean time between failures get longer. This would result in points plotting above the upper control limit on the individuals control chart”.

Using SixPack or JMP we would have the same picture of the process (of the transformed data).

Then we have two contradictory conclusions! See both the figure 20 and the figure 21...

The Control Limits of figures 20 and 21 are computed as shown in a Quality Management Course by the professors members of the “Politecnico Quality Engineering Group (QEG)” (all graduated CUM LAUDE), in figure 22
Now we calculate the control limits of the $\bar{X}$-chart:

$$UCL_X = \bar{x} + 3 \cdot \sigma_{\bar{x}}$$

$$CL_X = \bar{x} = \frac{\sum \bar{x}_i}{m} \quad \text{(grand average, estimator of } \mu \text{)}$$

$$LCL_X = \bar{x} - 3 \cdot \sigma_{\bar{x}}$$

Since $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad \text{(from statistics: central limit theorem)}$

Notice. Formulae for UCL, CL and LCL, in figure 22, hold only for Normally distributed data…. Hundred thousands of Master Black Belts, in the Six Sigma context, would use figure 22. Formula for $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ is always valid for samples of size $n$; it has nothing to do with the "Central Limit Theorem".

Thousands Master Black Belts, in the Six Sigma context, would suggest using the Minitab Software and the "T Charts", assuming that T Charts are the good method to deal with "rare events". See the Minitab T Chart (figure 23).

Is this a true picture of the process? Perhaps this In Control depends on the formulae used!

Actually, the process is Out Of Control! The Minitab "T Charts" are wrong.

To understand completely the lack of Sientificity the reader can usefully read the paper *Hope for the Future: Overcoming the DEEP Ignorance on the CI (Confidence Intervals) and on the DOE (Design of Experiments, Science J. Applied Mathematics and Statistics)*. Vol. 3, No. 3, pp. 70-95; the basic ideas, he can find there, are very useful.

*Figure 22*: Ideas given to students by professos of Quality Engineering Group (QEG)

*Figure 23*: T Chart of Montgomery data. Minitab 19 used (F. Galetto)
Comparing the figure 23 with 24 it is very clear that, for the Montgomery data, the T Charts are quite different from the one of Figure 24 and the process is OOC (Out Of Control). Notice the plural "T Charts" because also the differences \(|t_{i-1}|\) are exponentially distributed! Figure 24 is found by using the Reliability Integral Theory (RIT) [30, 31].

This proves the truth of Deming’s statements "The result is that hundreds of people are learning what is wrong," ,"It is a hazard to copy", "It is necessary to understand the theory...."

![Y Chart (F. Galetto)](image1)

![Range Chart (F. Galetto)](image2)

Figure 24: (F. Galetto) Scientific Control charts for valves [wrong control charts in Montgomery books]

We said before "The literature on "Quality" matters is rapidly expanding. Unfortunately, nobody, but the author, as far as he knows, [he thanks any person that will send him the names of people who take care ...], takes care of the "Quality of Quality Methods used for making Quality" (of product, processes and services). "Intelligentcredas".

The author is eager to meet one of them, fond of Quality as he is. If this kind of person existed, he would have agreed that "facts and figures are useless, if not dangerous, without a sound theory" (F. Galetto), "Management need to grow-up their knowledge because experience alone, without theory, teaches nothing what to do to make Quality"(Deming) because he had seen, like Deming, Gell-Mann and F. Galetto "The result is that hundreds of people are learning what is wrong. I make this statement on the basis of experience, seeing every day the devastating effects of incompetent teaching and faulty applications." [Deming (1986)]

During 2006 and 2020, F. Galetto experienced the incompetence of several people who were thinking that only the “Peer Review Process” is able to assure the scientificity of papers, and that only papers published in some magazines "good" are scientific: one is a scientist and gets funds if he publishes on those magazines! Using the scientific method one can prove that the referee analysis does not assure quality of publications in the magazines. You can see the incompetence level in Research Gate, in Academia.edu, iSixSigma and Minitab19 (wrong formulae for T_Charts)...

The symbol \(\varepsilon\) [which stands for the “epsilon Quality”] was devised by the author to show that Quality depends, at any instant, in any place, at any rate of improvement, on the Intellectual honesty of people who always use experiments and think well on the experiments before actually making them (Gedankenexperimenten) to find the truth [Gedankenexperimenten was a statement used by Einstein; but, if you look at Galileo life, you can see that also the Italian scientist was used to "mental experiments", the most important tool for Science; Epsilon (\(\varepsilon\)) is a Greek letter used in Mathematics and Engineering to indicate a very small quantity (actually going to zero); “epsilon Quality” conveys the idea that Quality is made of many and many prevention and improvement actions].

Many times the author spoiled his time and enthusiasm at conferences, in University and in Company courses, trying to provide good ideas on Quality and showing many cases of wrong applications of stupid methods [see references]. He will try to do it again ... by showing, step by step, one case (out of the hundreds he could document). ... in order people understand that Quality is a serious matter. The Nobel price R. Feynman (1965) said that "for the progress of Science are necessary experimental capability, honesty in providing the results and the intelligence of interpreting them... We need to take into account of the experiments even though the results are different from our expectations." It is apparent that Deming, Feynman, and Gell-Mann are in agreement with \(\varepsilon\) ideas of the author. Once upon a time, A. Einstein said “Surely there are two things infinite in the world: the Universe and the Stupidity of people. But I have some doubt that Universe is infinite". Let us hope that Einstein was wrong, this...
time. Anyway, before him, Galileo Galilei had said [in the Saggiatore] something similar “Infinite is the mob of fools”.

All the methods, devised by the author, were invented and have been used for preventing and solving real problems in the Companies he was working for, as Quality Manager and as Quality Consultant: several million € have been saved.

Companies will not be able to survive the global market if they cannot provide integrally their customer the Quality they have paid for. So it is of paramount importance to know correctly what Quality means. Quality is a serious and difficult business; it has to become an integral part of management.