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New Full Wave Theory for Plane Wave Scattering by a Rough Dielectric Surface – The Correction Current Method

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New Full Wave Theory for Plane Wave Scattering by a Rough Dielectric Surface – The Correction Current Method

Felix Schwering^α, Gerald Whitman^σ & Henry Tsai^ρ

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I. FORMULATION OF PROBLEM BY USE OF CC-METHOD

Subject of this paper is a new theory of scattering from rough dielectric surfaces of the type shown in Fig 1. The interface between an air halfspace and a dielectric halfspace is rough over a length $2L$ and planar beyond this region. The roughness profile is one-dimensional, i.e. the local height D varies with z but is constant with y .

A plane wave is incident under the angle ϕ_0 upon the interface; it may be incident from above or below. This incident wave and the resulting scattering field are TE- polarized, i.e. these fields consist of components E_y , H_x and H_z .

The technique used in this paper to formulate the problem is the Correction Current (CC) method, a new full wave method which has recently been established for plane wave scattering from metal surfaces [1] and subsequently extended to dielectric surfaces, with the lower halfspace at first assumed to be lossless but in the work presented here allowed to be lossy¹.

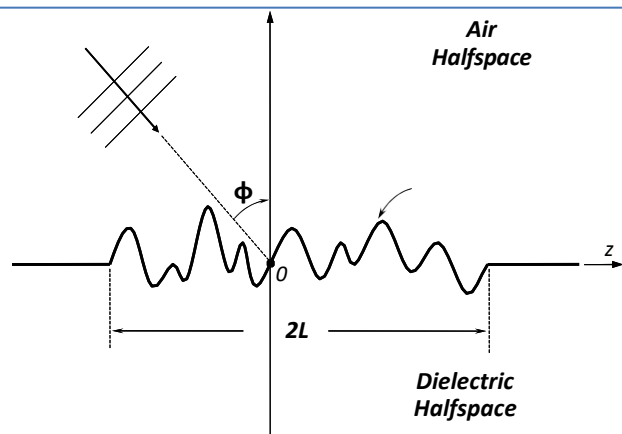


Fig. 1: Scatter Problem Geometry

The CC-method defines a primary field and a complete system of scatter fields, called radiation modes. Each of these fields consists of an incident, reflected and transmitted plane wave. These component waves, however, are modified such that each of the radiation modes as well as the primary field satisfies the boundary conditions at the interface, individually and rigorously. Since these fields are simple in structure (i.e. consisting of suitably modified plane waves) and comply with the boundary conditions at the rough interface, they will no longer satisfy Maxwell's equations¹. This is remedied by introducing fictitious current distributions, called passive currents that are assumed to be associated with each of these fields and chosen such that they exactly cancel the field errors of these fields; the passive current distribution of course varies from mode to mode. Thus, while not being solutions of Maxwell's source free equations, the primary field and radiation modes are solutions of Maxwell's equations with sources². The passive currents exist only in the corrugation region $|z| \leq L$; outside this region, where the interface is planar, the primary field and radiation modes are exact solutions of Maxwell's source free equations and no correction is needed.

¹ For a review of established theoretical methods for the analysis of rough surface scattering see [2]-[4].

² Alternatively, the passive currents may be interpreted as quantifying the field errors.

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In addition, it is assumed that each radiation mode is generated by an “active” sheet current distribution residing in a plane $z =$ termed the reference plane of this mode. These reference planes exist in the corrugation region $|Z| \leq L$ only, and all active currents of the mode system are limited to the same region as the passive currents. An important feature of these active currents is that taken together they form a complete orthogonal system so that any (passive) current distribution in the region $|Z| \leq L$ can be expanded into, or nullified by the active currents.

The total field is then written as a superposition of the primary field and the radiation modes. This combined field, of course, is not allowed to contain any active or passive current distribution, and this “zero-current” condition is then used to determine the unknown amplitudes of the radiation modes by an iterative procedure which in, a step-by-step fashion, eliminates the passive currents of the primary field and the radiation modes by the active currents of the radiation modes. The completeness, mentioned above, of the active currents allows us to do this consistently. The resulting combined field will satisfy the boundary conditions at the interface and the radiation condition at infinity while all active and passive currents are eliminated by mutual compensation. Thus the combined field is the solution of the scatter problem. For numerical efficiency, the iterative process is typically cut-off after the first iteration.

Using this approach the dielectric surface scatter problem has recently been solved for the case that the dielectric halfspace is loss free. Scattering patterns have been obtained in the form of single integrals over elementary functions which are easily evaluated numerically. The patterns have been computed for both deterministic and random rough surfaces. Comparison with the corresponding patterns obtained by a Method of Moments (MoM) procedure has shown that the first order iteration theory is of good accuracy over a wide parameter region. A paper reporting on this study, which was conducted at CERDEC and NJIT, is close to completion and will be submitted for publication in the near future.

The extension of the theory to lossy dielectrics is currently pursued under the 2007 In-house Laboratory Independent Research (ILIR) program. This extension is not straight forward, however. The problem geometry consists of two halfspaces; and two groups of radiation modes are required for full characterization of the problem. This holds for the lossy as well as for the lossless case. But when these two mode groups for the lossy case are defined in direct analogy to the two mode groups for the lossless case one of the groups diverges, i.e. increases exponentially with $|x|$, which is unacceptable. Hence the radiation modes need to be redefined, which has a significant effect on the theory. In

the development of this theory it is consistently assumed that $[\epsilon_r]$ is finite; it may be small, but is not allowed to be arbitrarily small. Comparison to the results of the lossless theory will be made only after all formulas for fields and radiation patterns have been fully established.

II. ANALYTICAL RESULTS AND DISCUSSION

In the following it is assumed that the incident plane wave is situated in the upper (air) halfspace. If it is located in the lower halfspace, the expressions obtained for fields and patterns will be different, of course. But the overall trends observed are similar.

It is interesting to note that the scatter fields associated with the two locations of the incident wave are related by a simple symmetry relation. Assume that a plane wave of amplitude E_0 and phase constant (in the z -direction) incident in the air halfspace – generates a scatter field $\beta_0 - E^s(x, z; k_0, k_\epsilon, D(z))$. Assume, furthermore, that a plane wave of amplitude E_ϵ and phase constant β_ϵ – incident in the dielectric halfspace – creates a scatter field $\bar{E}_y^s(x, z; k_0, k_\epsilon, D(z))$. Then these two scatter fields are related by the condition.

$$\frac{\bar{E}_y^s(x, z; k_0, k_\epsilon, D(z))}{E_\epsilon} = \frac{E_y^s(-x, z; k_\epsilon, k_0, -D(z))}{E_0} \quad (1)$$

for $\beta_\epsilon = \beta_0$
with $k_\epsilon = k_0 \sqrt{\epsilon_r}$

This relation holds for all x and z , i.e. in both halfspaces. Thus, if E_y^s has been determined \bar{E}_y^s , follows by a simple substitution of coordinates and parameters³; and vice versa.

Symmetry relation (1) must be satisfied for the exact scatter fields. But it is satisfied already for the first order iteration approximation used in this paper which may be taken as an indication that the first order method is of good accuracy.

In this first order approximation the CC-theory leads to the following expression for the scatter field in the upper (air) halfspace $x > D$:

³ Note that since $\epsilon_r = (\kappa_\epsilon / \kappa_0)^2$ the parameter substitution replaces ϵ_r by $1 / \epsilon_r$.

$$\begin{aligned}
 E_y^{(1)}(x, z) = & -\frac{E_0 u_0}{\pi(u_0 + v_0)} \left[\int_{u=0}^{\infty} \int_{\tilde{z}=-L}^{+L} \left\{ \frac{1 - e^{+j(u_0+u)\tilde{D}}}{u_0 + u} \right. \right. \\
 & \cdot \left[1 + 2j\delta(z - \tilde{z}) \frac{\beta_1}{u_0^2 - u^2} \right] - \left(\frac{1}{u_0 + u} - \frac{1}{v_0 + v_1} \right) \\
 & \cdot \tilde{D}'^2 e^{+j(u_0+u)\tilde{D}} \left\{ (u + v_1)e^{+ju(x-2D)} + (u - v_1)e^{-ju(x-2D)} \right\} \\
 & \cdot e^{-j\beta_0\tilde{z} - j\beta_1|z-\tilde{z}|} \cdot \frac{u}{\beta_1} du d\tilde{z} - \\
 & - 2 \int_{u=0}^{\infty} \int_{\tilde{z}=-L}^{+L} \left\{ \frac{1 - e^{+j(u_0-u)\tilde{D}}}{u_0 - u} \left[1 + 2j\delta(z - \tilde{z}) \frac{\beta_2}{u_0^2 - v_2^2} \right] - \right. \\
 & \left. - \left(\frac{1}{u_0 - v_2} - \frac{1}{v_0 + u} \right) \tilde{D}'^2 e^{+j(u_0-u)\tilde{D}} \right\} e^{+jv_2x + j(u-v_2)D} \\
 & \left. \cdot e^{-j\beta_0\tilde{z} - j\beta_2|z-\tilde{z}|} \frac{u^2}{\beta_2} du d\tilde{z} \right] \tag{2a}
 \end{aligned}$$

for $x > D$

Where E_0 is the amplitude of the incident plane wave \tilde{D} stands for $D(\tilde{z})$ and k_0 is the free-space wave number; furthermore

$$u_0 = k_0 \cos \varphi_0$$

$$\begin{aligned}
 v_0 &= k_0(\epsilon_r - \sin^2 \varphi_0)^{1/2}, \quad \beta_0 = k_0 \sin \varphi_0 \\
 v_1 &= [u^2 + k_0^2(\epsilon_r - 1)]^{1/2}, \quad \beta_1 = (k_0^2 - u^2)^{1/2} \\
 v_2 &= [u^2 - k_0^2(\epsilon_r - 1)]^{1/2}, \quad \beta_2 = (k_0^2 \epsilon_r - u^2)^{1/2}
 \end{aligned} \tag{2b}$$

The path of the u -integration runs along the positive real axis⁴. The propagation constants v_0 , v_1 , β_1 and β_2 are positive real and/or negative imaginary while β_0 is positive real and positive imaginary.

The first double integral in (2a) represents the contribution of the first mode group to the scatter field and the second double integral shows the contribution of the second mode group. A formally similar representation is obtained for the scatter field in the lower (lossy dielectric) halfspace but for brevity is not spelled out here.

In the far field region, where

$$k_0 \rho = k_0(x^2 + z^2)^{1/2} \gg k_0 L \text{ and } \gg 2\pi,$$

eq. (2) can be simplified by substituting

$$x = \rho \cos \varphi, \quad z = \rho \sin \varphi$$

where the scatter angle φ is in the region

$$-\frac{\pi}{2} \leq \varphi \leq +\frac{\pi}{2} \quad (\text{upper halfspace})$$

The formula may then be evaluated asymptotically for $k_0 \rho \rightarrow \infty$ using the method of stationary phase which eliminates the u -integration. Only the first double integral in (2a) contributes to the far field and one obtains

$$E_y^{(1)} = \left(\frac{2}{\pi}\right)^{1/2} E_0 \frac{e^{-j(k_0 \rho - \pi/4)}}{(k_0 \rho)^{1/2}} S_0(\varphi, \varphi_0) \tag{3a}$$

$$\text{for } -\frac{\pi}{2} \leq \varphi, \varphi_0 \leq +\frac{\pi}{2}, \quad k_0 \rho \rightarrow \infty$$

where the scatter pattern $S_0(\varphi, \varphi_0)$ takes the form

$$\begin{aligned}
 S_0(\varphi, \varphi_0) = & -k_0(\epsilon_r - 1) \frac{\cos \varphi \cos \varphi_0}{\cos \varphi + \cos \varphi_0} \\
 & \cdot \left[\frac{1}{\cos \varphi + (\epsilon_r - \sin^2 \varphi)^{1/2}} \right] \left[\frac{1}{\cos \varphi_0 + (\epsilon_r - \sin^2 \varphi_0)^{1/2}} \right]
 \end{aligned}$$

⁴ Except when a $(u-u)^{-1}$ singularity occurs on the real u -axis. In this case the u -integration bypasses the singularity by a local deformation of the integration path into the positive imaginary u -halfspace.

$$\cdot \int_{\tilde{z}=-L}^{+L} \left\{ 1 - \left[1 + \left(1 - \frac{\cos \varphi + \cos \varphi_0}{(\varepsilon_r - \sin^2 \varphi)^{1/2} + (\varepsilon_r - \sin^2 \varphi_0)^{1/2}} \right) \tilde{D}'^2 \right] \right. \\ \left. \cdot e^{+jk_0 \tilde{D}(\cos \varphi + \cos \varphi_0)} \right\} \cdot e^{+jk_0 \tilde{z}(\sin \varphi - \sin \varphi_0)} d\tilde{z} \quad (3b)$$

This expression for the scatter pattern in the air halfspace $x > D$ is identical to the one obtained by the previously developed theory for $\varepsilon_r = \text{real}$, but it is now confirmed to be valid for complex ε_r as well. Eq. (3b) also shows that S_0 vanishes, as required, in the no-contrast case that $\varepsilon_r \rightarrow 1$, and that it satisfies the reciprocity relation $S_0(\varphi, \varphi_0) = S_0(-\varphi_0, -\varphi)$.

Similar to expression (2) for the scatter field in the upper halfspace, the formula obtained for the lower halfspace (not shown here) consists of two double integrals representing the contributions of the two mode groups. However, due to the lossy nature of the dielectric halfspace, the scatter field will decrease here

exponentially away from the corrugated interface, and the two double integrals in general cannot be simplified. Simplification is possible only if $\text{Im}[\varepsilon_r]$ is either very small, so that a far field of reasonable magnitude exists, or if $\text{Im}[\varepsilon_r]$ is rather large, so that the scatter field in the lower halfspace is in effect confined to a neighborhood of the scatter surface.

In the case that $|\text{Im}[\varepsilon_r]|$ is very small, the scatter field in the region far away from the corrugated section of the interface can be obtained by the method of steepest descent, which eliminates the integration over \tilde{z} . One finds that $E_y^{(1)}$ in the lower halfspace $x < D$ is of the form

$$E_y^{(1)} = \sqrt{\frac{2}{\pi}} \cdot E_0 \left[\frac{e^{-j(k_\varepsilon \rho - \pi/4)}}{(k_\varepsilon \rho)^{1/2}} S_\varepsilon(\varphi, \varphi_0) + F_\varepsilon(\rho, \varphi) T_\varepsilon(\varphi_0) \right] \quad (4a)$$

where $k_\varepsilon = k_0 \sqrt{\varepsilon_r}$ and

$$F_\varepsilon(\rho, \varphi) \begin{cases} = \frac{e^{-jk_0 \rho [\sin \varphi - (\varepsilon_r - 1)^{1/2} \cos \varphi]}}{\left[k_0 \rho \left(\sin \varphi + \frac{\cos \varphi}{(\varepsilon_r - 1)^{1/2}} \right) \right]^{3/2}} & \text{for } \text{Im}[\varepsilon_r] < 0 \\ = 0 & \text{for } \text{Im}[\varepsilon_r] = 0 \end{cases} \quad (4b)$$

for $\frac{\pi}{2} \leq |\varphi| \leq \pi$, $k_0 \rho \rightarrow \infty$

The scatter field in this case consists of two parts which may be interpreted in the following way. In contrast to the scatter field in the upper halfspace, the scatter pattern in the lower halfspace is generated by two different mechanisms which are illustrated by Fig. 2. The first mechanism, as in the case of the lossless upper halfspace, is simply the scattering of the incident plane wave at the corrugated part of the interface. The second mechanism comes about because the scatter field in the lower, lossy halfspace decreases with ρ exponentially, i.e. much faster than the scatter field in the upper halfspace. But the scatter field must be continuous at the interface. As a consequence there is a continuous leakage of energy from the upper halfspace into the lower halfspace, which constitutes the second mechanism generating the scatter field in the lower halfspace⁵. The first mechanism causes a conventional scatter field where in the far zone the ρ - and φ -

dependence are separated, but with the ρ -dependence in this case decreasing exponentially; see eq.(4a). This field will dominate at scatter angle φ close to $\pm 180^\circ$. In the asymptotic field caused by the second mechanism, on the other hand, the ρ - and φ -dependence remain coupled also in the asymptotic region. This part of the field will be dominant near the interface, i.e. for φ near $\pm 90^\circ$.

⁵ No such energy transfer occurs when ε_r is real, since the energy density in both half spaces decreases with ρ^{-1} .

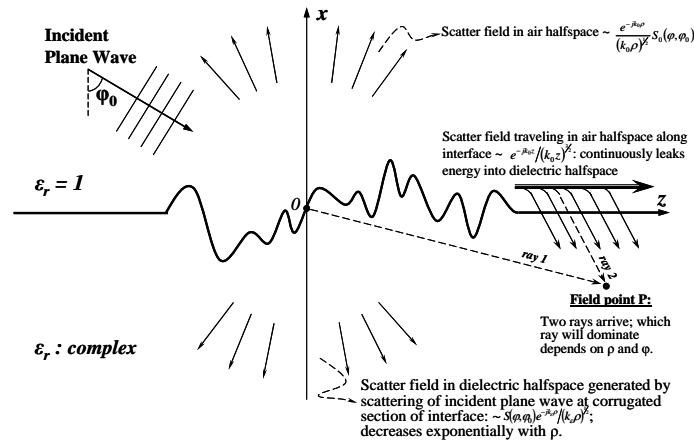


Fig. 2: Scatter field in lower (lossy) halfspaces generated by two mechanisms

- (1) Scattering of primary field at corrugated section of interface
 - (2) Energy transfer from scatter field in upper halfspace to asymptotic scatter field in lower half space
- ($x = \rho \cos \varphi, z = \rho \sin \varphi, k_x = k_0 \sqrt{\epsilon_r}$)

A possible problem is the following. If $|\text{Im}[\epsilon_r]|$ is very small then the denominator of F_ϵ in (4b) will have a near-zero at $\sin \varphi = |\epsilon_r|^{-1/2}$ resulting in a sharp peak of F_ϵ . The authors are not sure that this is correct and eq. (4b) will require further study.

As mentioned above, if $\text{Im}[\epsilon_r]$ is sizeable then eq. (4), even if mathematically correct, will lose its physical meaning. The equation holds in the far zone, i.e. at large distances from the corrugated part of the interface, and if $\text{Im}[\epsilon_r]$ is significant then the scatter field at such distances will be exceedingly small and undetectable for all practical purpose.

Conceptually it is obvious that, in the case of large $\text{Im}[\epsilon_r]$, the scatter field in the lower halfspace will be of significant magnitude only in a narrow region adjacent to the interface, while this field, whether generated by the first or the second mechanism mentioned above, will decrease rapidly with increasing distance from the interface. The general representation of the scatter fields in terms of two double integrals (i.e. a representation akin to eq. (2a)) allows to quantify this, but the analytical procedure is rather lengthy and tedious, and not included in this paper.

One last remark: The scatter pattern (3b) in the upper halfspace includes a factor $\cos \varphi$ and will be zero for $\pm 90^\circ$, i.e. at the interface, indicating that E_y along the

interface declines faster than $\frac{e^{-jk_0 z}}{|k_0 z|^{3/2}}$. The scatter field

in the direct vicinity of the interface can be determined by an asymptotic evaluation of (2) for

$$|k_0 z| \gg k_0 L \text{ and } \gg 2\pi, \quad |k_0 x| < |k_0 z|^{1/2}$$

i.e. in a region where z is large but x remains small compared to z (Fresnel Region). One obtains

$$E_y^{(1)} = -j\left(\frac{2}{\pi}\right)^{1/2} E_0 k_0^2 \frac{u_0}{u_0 + v_0} \int_{z=-L}^{+L} \frac{1 - e^{+ju_0 \tilde{D}}}{u_0} dz$$

$$-\left[\frac{1}{u_0} - \frac{1}{v_0 + k_0(\epsilon_r - 1)^{1/2}} \right] \tilde{D}'^2 e^{+ju_0 \tilde{D}} \int e^{-j(\beta_0 \mp k_0)z} dz \cdot \frac{e^{-j(k_0|z| - \frac{\pi}{4})}}{(k_0|z|)^{3/2}} \begin{cases} [1 + jk_0(\epsilon_r - 1)^{1/2}(x - D)], & x > D(z) \\ e^{+jk_0(\epsilon_r - 1)^{1/2}(x - D)}, & x < D(z) \end{cases}$$

The formula shows that E_y near the interface decrease with $\frac{e^{-jk_0|z|}}{|k_0 z|^{3/2}}$, i.e. the energy density will decrease here with ρ^{-3} rather than with ρ^{-1} , the rate of decrease for $|\varphi| < 90^\circ$. With x , the field below the surface decreases exponentially, indicating a transmitted wave while above the surface it varies linearly with x indicating the interaction between an incident and a reflected wave. All this is consistent with a continuous leakage of energy from the air halfspace into the lossy dielectric halfspace, i.e. with the second mechanism mentioned above for generating the scatter field in the lower halfspace.

As mentioned earlier, the discussion and formulas presented in this paper apply to the case that the primary wave is incident in the upper (air) halfspace. If the antenna generating the incident wave is situated in the lower (dielectric) halfspace, the symmetry relation (1) applies; the formulas are analogous; and similar overall trends are observed.

The theory for the lossy dielectric case summarized in this paper has been developed under the 2007 CERDEC ILIR program in cooperation with NJIT and is near completion, though some points need further investigation. Final results will be tested by comparison to data obtained by a MoM technique. This work is schedule for 2008.

Numerical Techniques such as the MoM and FDTD methods can be relied on to provide very

accurate results. Analytical methods as the one presented in this paper, even though approximate, have the advantage of showing parameter dependencies explicitly, thus providing physical insight. In addition, the field and pattern formulas – often obtained in the form of single integrals over elementary functions – are amenable to efficient computer evaluation and may be useful for real-time modeling.

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