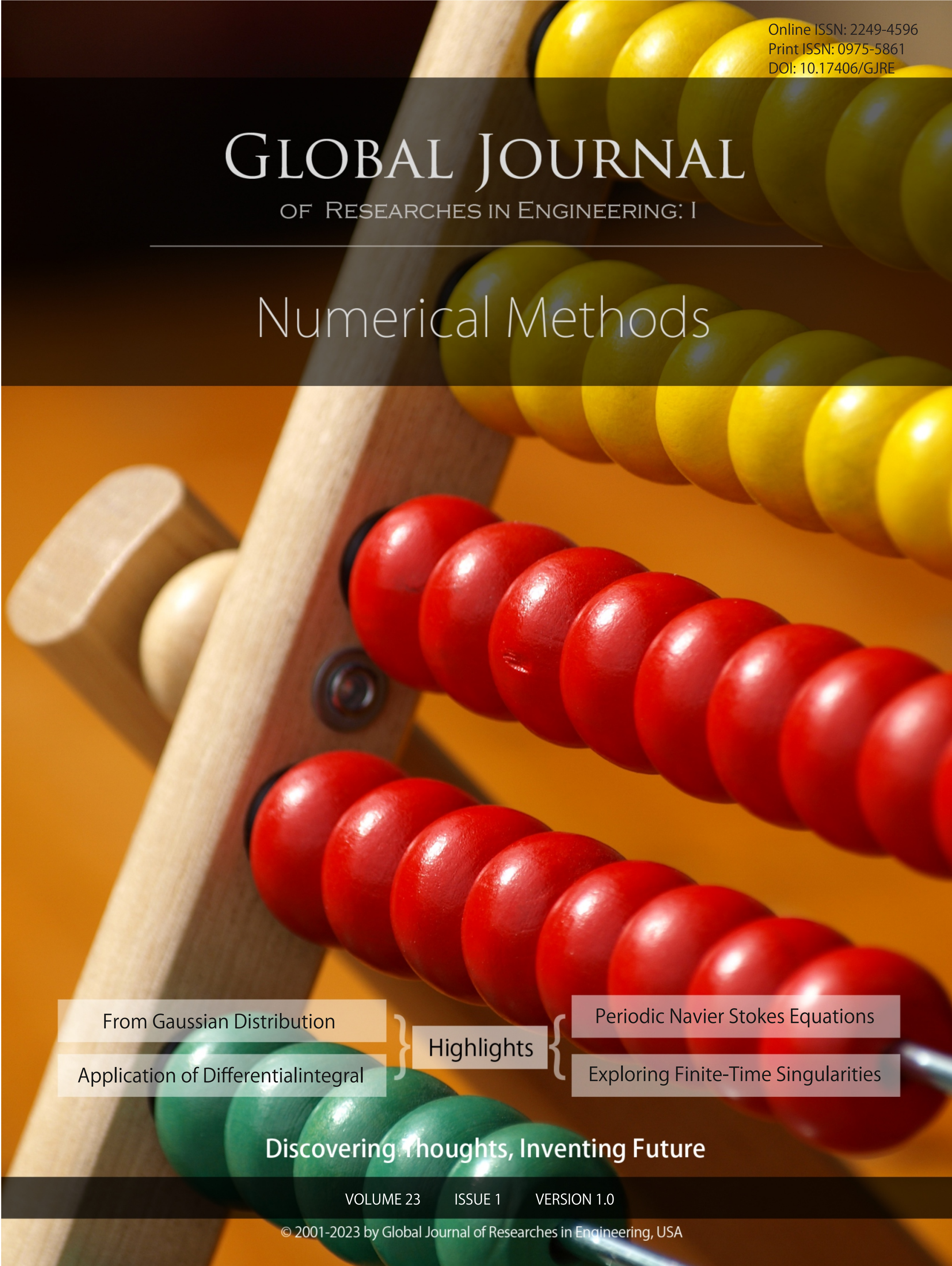

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From Gaussian Distribution

Application of Differentialintegral

Highlights

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Exploring Finite-Time Singularities

Discovering Thoughts, Inventing Future

VOLUME 23 ISSUE 1 VERSION 1.0

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From Gaussian Distribution to Weibull Distribution

By Xu Jiajin & Gao Zhentong
Beihang University

Abstract- The Gaussian distribution is one of the most widely used statistical distributions, but there are a lot of data that do not conform to Gaussian distribution. For example, structural fatigue life is mostly in accordance with the Weibull distribution rather than the Gaussian distribution, and the Weibull distribution is in a sense a more general full state distribution than the Gaussian distribution. However, the biggest obstacle affecting the application of the Weibull distribution is the complexity of the Weibull distribution, especially the estimation of its three parameters is relatively difficult. In order to avoid this difficulty, people used to solve this problem by taking the logarithm to make the data appear to be more consistent with the Gaussian distribution. But in fact, this approach is problematic, because from the physical point of view, the structure of the data has changed and the physical meaning has changed, so it is not appropriate to use logarithmic Gaussian distribution to fit the original data after logarithm. The author thinks that Z.T. Gao method can give the estimation of three parameters of Weibull distribution conveniently, which lays a solid mathematical foundation for Weibull distribution to directly fit the original data.

Keywords: gaussian distribution; three-parameter weibull distribution; full state distribution; safe life; Z.T. Gao (or GZT) method.

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From Gaussian Distribution to Weibull Distribution

Xu Jiajin [α] & Gao Zhentong [σ]

Abstract- The Gaussian distribution is one of the most widely used statistical distributions, but there are a lot of data that do not conform to Gaussian distribution. For example, structural fatigue life is mostly in accordance with the Weibull distribution rather than the Gaussian distribution, and the Weibull distribution is in a sense a more general full state distribution than the Gaussian distribution. However, the biggest obstacle affecting the application of the Weibull distribution is the complexity of the Weibull distribution, especially the estimation of its three parameters is relatively difficult. In order to avoid this difficulty, people used to solve this problem by taking the logarithm to make the data appear to be more consistent with the Gaussian distribution. But in fact, this approach is problematic, because from the physical point of view, the structure of the data has changed and the physical meaning has changed, so it is not appropriate to use logarithmic Gaussian distribution to fit the original data after logarithm. The author thinks that Z.T. Gao method can give the estimation of three parameters of Weibull distribution conveniently, which lays a solid mathematical foundation for Weibull distribution to directly fit the original data.

Keywords: gaussian distribution; three-parameter weibull distribution; full state distribution; safe life; Z.T. Gao (or GZT) method.

I. Introduction

The Gaussian distribution is also commonly known as the Gaussian distribution, and it is generally known that the height, weight, and even IQ of a group of people are relatively consistent with the Gaussian distribution. However, like fatigue life of structures is often far from the Gaussian distribution and more in line with the Weibull distribution. In [1] it was pointed out that the Weibull distribution is a full state distribution, i.e., it can depict not only left-skewed and right-skewed data, but to some extent also symmetric as well as data satisfying a power law. In this sense it is more versatile than the Gaussian distribution [2], [3] and plays a very important role especially in fitting the fatigue life of structures. However, because of the difficulties encountered in determining the three parameters of the Weibull distribution, the problem was solved by taking the logarithm to make the data appear to be more in line with the Gaussian distribution. In fact, this approach is problematic. This paper points out that logging the original data is only a spatial transformation from a mathematical point of view, but from a physical point of view, it changes the structure of the data, and the physical meaning is changed, so it is not appropriate to use logarithmic Gaussian distribution to fit the original data after logarithm. To determine the three parameters of the Weibull distribution, the graphical and analytical methods[4] were previously adopted, the former being inconvenient to use and with relatively large errors; the latter involves solving a system of three joint transcendental equations, which, despite the availability of computers to do so, still has the problem of being inconsistent. This problem can now be solved relatively well by using T.Z. Gao method proposed by [1].

II. The Characteristics of the Gaussian Distribution

It is well known[4] that the so-called Gaussian distribution is a distribution in which the random variable is a PDF of X with the form,

$$f(x)=[1/(2\pi)^{1/2}\sigma]\exp[-(x-\mu)^2/2\sigma^2] \quad (1)$$

where μ and σ^2 are the mean and variance of the Gaussian distribution, respectively. And when the mean $\mu = 0$ and the standard deviation $\sigma = 1$ is called the standard Gaussian distribution as follows,

$$[1/(2\pi)^{1/2}]\exp(-x^2/2) \quad (2)$$

From the definition of Gaussian distribution it is easy to see that Gaussian distribution has the following characteristics[5]:

1. Single-peaked, a distribution that is unimodal. And symmetry, with its Mode and median and mean are the same.
2. Universality, a significant proportion of random variables encountered in real life are or approximately conform to the Gaussian distribution. Even in an arbitrary distribution, in the case of a large sample, the distribution of the mean will approximate the Gaussian distribution.
3. Simplicity, i.e., only two parameters (μ, σ^2) are needed to determine the shape of the entire distribution.

Because the normal distribution has so many good characteristics, it has become the most studied and applied distribution. However, it is obvious that not all data conform to Gaussian distribution, and in most cases the data conform to Gaussian distribution is only

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a good approximation. In fact[4], the data of various fatigue lives are often not fit Gaussian distribution but better fit Weibull distribution, and sometimes the fatigue life is logarithmically distributed, but it is only an approximation. Because of this, Weibull distribution needs to be introduced and studied in more depth.

III. Brief Introduction of Weibull Distribution

There are various expressions for the Weibull distribution, and a more general form is taken here[1], with a probability density function:

$$f(x)=(b/\lambda)[(x-x_0)/\lambda)]^{b-1}\exp\{-[(x-x_0)/\lambda]^b\} \qquad (3)$$

where b is the shape parameter, λ is the scale or proportional parameter, and x_0 is called the position parameter. In the field of fatigue it is customary to use the fatigue life N instead of x, N_0 instead of x_0, and call it the safe life. In a non-strict sense[1], "when $0 < b < 1$ resembles a power-law function, while $1 < b < 3$ is a left-skewed distribution, $3 < b < 4$ approximates a Gaussian distribution, and $b > 4$ is a right-skewed distribution". This is the reason why the Weibull distribution is called the "full state distribution". As shown in the following fig.1[5]:

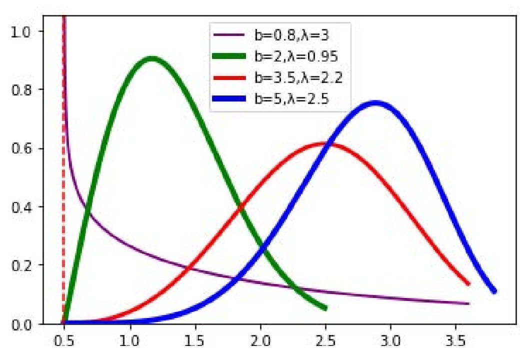

Fig. 1: PDF of various Three-Parameter Weibull distributions when $x_0=0.5$

It is easy to prove that the life is x_i and the corresponding reliability[1] is,

$$p_i=\exp\{-[(x_i-x_0)/\lambda]^b\} \qquad (4)$$

It can be seen that when $x=x_0$, $p_0=100\%$. This is the origin of 100% reliability safety life. If $p_{50}=50\%$, it means that the corresponding X is called the median value x_m of X, that is, there are,

$$50\%=\exp\{-[(x_m-x_0)/\lambda]^b\} \qquad (5)$$

It is not difficult to get the expectation and variance of Weibull distribution with three parameters according to the definition[4],

$$E(X)=x_0+\lambda\Gamma(1+1/b) \qquad (6)$$

$$\mathrm{Var}(X)=\lambda^2[\Gamma(1+2/b)-\Gamma^2(1+1/b)] \qquad (7)$$

In this way, the fatigue life data are given and the three parameters of Weibull distribution can be derived by (5), (6) and (7), which is the analytical metho[4]. In addition to the analytical method, the maximum likelihood method and some methods derived from it[6], [7] have been used more recently, but they have problems such as cumbersome derivation and inconvenient calculation, so we will not discuss them in depth here.

IV. Origin of Z.T. Gao Method and Fitting Standard

Theoretically if a set of fatigue life data *N* is given, then using the median (N_m), mean (N_{av}) and mean squared deviation (s) of this array, then using the three equations (5), (6) and (7) is possible to solve for the estimated values of the three parameters of the Weibull distribution. However, for convenience (5), (6) and (7) can be reduced to a transcendental equation[1] with respect to b:

$$(N_{av}-N_m)[\Gamma(1+2/b)-\Gamma^2(1+1/b)]+s[D^{1/b}-\Gamma(1+1/b)]^{1/2}=0 \qquad (8)$$

where D = ln2. This equation is solvable by Newton's method, and after obtaining b, then λ and N_0 can be found by (7),(6).

Example 1: The data in Table 8-2 in [4] are used to find the three parameters of the Weibull distribution by analytical method.

Table 1: A set of fatigue life data (10^3c)

124	134	135	138	140
147	154	160	166	181
N_{av}=148, N_m=144, s=17.3				

You can get it through Python code, Parameter estimation: b=1.221,N_0=127,λ=22.46

It is not difficult to find that N_0(=127) derived from the analytical method is greater than the minimum value of 124 for this group of fatigue lives. And this is in contradiction with the definition of safe life N_0. That is, the problem of inconsistent occurs. Another question is what happens if we fit this set of data with a Gaussian distribution? That is, which is the more appropriate distribution to fit?

The second problem can be judged by the magnitude of the determination coefficient[8] R^2 fitting the ideal reliability based on the so-called "average rank"[4]. The so-called ideal reliability means that the following formula is independent of the specific distribution,

$$p_i=1-i/(n+1) \qquad (9)$$

where *i* is the order of the data from smallest to largest, and n is the number of data.

And the first problem is solved by the Z. T. Gao method[1]. The basic idea of the method is briefly described below. Taking the logarithm of both sides of (4) twice yields that

In $$(\ln(1/p_i))=b\ln(N_i-N_0)-b\ln(\lambda) \quad (10)$$

if set, $$Y_i=\ln(\ln(1/p_i)),\ X_i=\ln(N_i-N_0) \quad (11)$$

$$d=-b\ln(\lambda),\ \lambda=\exp(-d/b) \quad (12)$$

So (10) could been become,

$$Y_i=bX_i+d \quad (13)$$

This is a system of linear regression equations that can be derived by the least squares method with coefficients *b* and *d*. However, it is important to note that here X_i is related not only to the data *N* already given, but also to the required safety lifetime N_0 of Weibull distribution. This problem can be solved by determining the extreme value of the absolute value of the relative coefficient r of the regression line to determine the corresponding N_0, but the mathematical derivation of this method is complex and error-prone [9]. It is better to use a different idea to use Python to find the series of r about N_0 directly in the interval $0\leq N_0 < N_{min}$ (here N_{min} is taken as the minimum value of the given data). Then Python intelligently finds the N_0 of r with the largest correlation coefficient, and at the same time determines *b* and λ. This is known as the Z.T. Gao algorithm. It is abbreviated as the Z.T. Gao method [1], [5] or GZT method.

Example 2: Now, using the data of Example 1, three parameters of Weibull distribution are determined by using GZT method, and the results are compared with Gaussian distribution. The results are as follows:

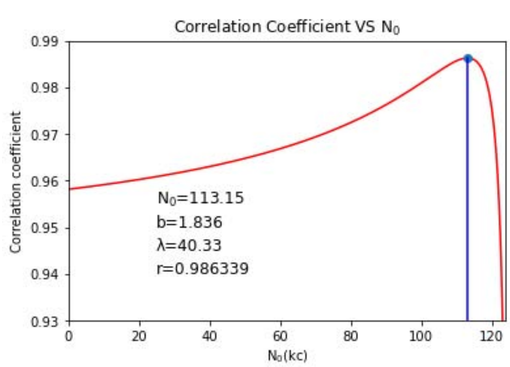

Fig. 2: Schematic graph of Z.T. Gao method

This figure graphically demonstrates how GZT method finds the corresponding safe lifetime that maximizes the correlation coefficient. Since it is clear at the beginning of the process that N_0 cannot be greater than the minimum lifetime of the data, it is not possible to have a situation where it is inconsistent. Again, if the data are fitted with a Gaussian distribution and the coefficient of determination of the Weibull distribution estimated by GZT method, respectively, fitted with the ideal reliability (9):

Coefficient of determination obtained by fitting the Weibull distribution = 0.97999

Coefficient of determination obtained by fitting the Gaussian distribution = 0.95044

It can be seen that the fitted coefficient of determination of the Weibull distribution obtained by GZT method is greater than that of the Gaussian distribution. That is, in this sense the data are more realistically depicted by the Weibull distribution.

The advantage of GZT method is that the physical meaning is very intuitive, and there is no problem of "inconsistent". This method is not only convenient for solving the problem of estimating the three parameters of the Weibull distribution, but also easy to determine whether the original data fits better with the Weibull distribution or with the Gaussian distribution. It is also easy to extend to solve similar problems, such as fitting fatigue performance curves with three parameters[1], and the confidence intervals of these three parameters will be discussed in separate papers[10], [11].

V. Problems of Logarithmization of Original Data

Due to the complexity of the Weibull distribution, when the original data is not so consistent with the Gaussian distribution, often take its logarithmic, from a mathematical point of view is equivalent to do a spatial transformation, at this time because the data "compressed", it may be closer to the Gaussian distribution[4]. This has the advantage of making the PDF of the original data taken logarithmically will be fitted quite well by the Gaussian distribution, which will be more convenient for people to study and apply. However, this will lose the physical meaning of the safety lifetime, while making the original data density distribution is "distorted". This is illustrated in the following two examples.

Example 3: Using the (large sample) 100 fatigue life data of a structure from Table 12-3 of [1] P253, the Python code gives:

Fatigue life (original data) N= [3.08, 3.26, 3.32, 3.48, 3.49, 3.56, 3.69, 3.7, 3.78, 3.79, 3.8, 3.87, 3.95, 4.07, 4.08, 4.1, 4.12, 4.2, 4.24, 4.25, 4.28, 4.31, 4.31, 4.36, 4.54, 4.58, 4.6, 4.62, 4.63, 4.65, 4.67, 4.67, 4.72, 4.73, 4.75, 4.77, 4.8, 4.82, 4.84, 4.9, 4.92, 4.93, 4.95, 4.96, 4.98, 4.99, 5.02, 5.03, 5.06, 5.08, 5.06, 5.1, 5.12, 5.15, 5.18, 5.2, 5.22, 5.38, 5.41, 5.46, 5.47, 5.53, 5.56, 5.6, 5.61, 5.63, 5.64, 5.65, 5.68, 5.69, 5.73, 5.82, 5.86, 5.91, 5.94, 5.95, 5.99, 6.04, 6.08, 6.13, 6.16, 6.19, 6.21, 6.26, 6.32, 6.33, 6.36, 6.41, 6.46, 6.81, 7.0, 7.35, 7.82, 7.88, 7.96, 8.31, 8.45, 8.47, 8.79, 9.87] (10^5cycle).

Also the following parameter table and histogram can be obtained.

Table 2: Gaussian distribution parameters (large sample)

	Mean	s	Median	r	R^2
Original	5.315	1.289	5.07	0.99051	0.97911
Take Log10	5.713	0.101	5.705	0.99702	0.99385
Recover 10^{log10}	5.164	1.262	5.07	0.99515	0.98805

(Where s is sample standard deviation)

Table 3: Weibull distribution parameters (large sample)

	b	N_0	λ	r	R^2
Original	2.147	2.78	2.8	0.99515	0.98903
Take Log10	3.346	5.408	0.34	0.9961	0.99121

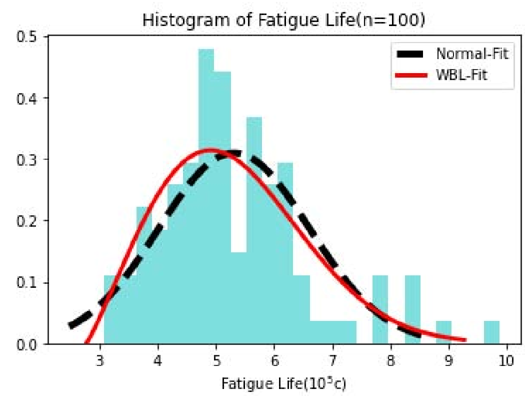

Fig. 3: Histogram of original data (large sample) and fitting diagram of Gaussian and Weibull distribution

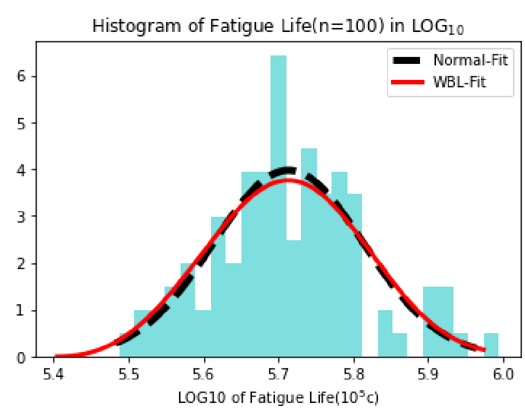

Fig. 4: Histogram after logarithm of the original data (large sample) and fitting diagram of Gaussian and Weibull distribution

As seen in Fig. 3, the histogram of the original data is asymmetric and left-skewed, and fitting it with a Gaussian distribution would be less appropriate, as in fact demonstrated with the chi-square test[4]. At this point it would be more appropriate to use the Weibull distribution. Looking at the logarithm of the data, we can see from Fig. 4 that the data do appear to be symmetric, and the Gaussian distribution is indeed a good fit. The problem is that the fatigue life PDF left-skewed features are lost, and the physical meaning of safe life is lost. Even if the results obtained in the logarithmic case "back" to the original state, only the median can "recover" (see Table 2, line 3), and the mean is left-skewed, the relative coefficient and the coefficient of determination is improved. Nevertheless, it is still not possible to obtain a 100% safe lifetime. In contrast, the fit with the Weibull distribution, as seen in Table 3, is a fairly good fit. Even after taking the logarithm, the fit is almost the same as that of the Gaussian distribution. From the data in row 2 of the Weibull distribution parameters in Table 3 and (6) and (7), we can calculate that $\hat{\mu}=5.7137$; $\hat{\sigma}=0.1005$

And this result is almost the same as the data in row 2 of Table 2. In this sense the Weibull distribution is indeed more general than the Gaussian distribution, which can be seen as a first-order approximation to the Weibull distribution. It can be seen that using the Weibull distribution to fit this set of fatigue life data does not require any logarithm of the data at all and the physical meaning of each parameter is very clear.

Example 4: Looking again at the case of a small sample, 20 data for the life of a structure using Table 8-4 of P136 in [2]. Again, this can be obtained by Python code as follows

Fatigue life (raw data) N= [3.5, 3.8, 4.0, 4.3, 4.5, 4.7, 4.8, 5.0, 5.2, 5.4, 5.5, 5.7, 6.0, 6.1, 6.3, 6.5, 6.7, 7.3, 7.7, 8.4] (10^5cycle)

Also the following parameter table and histogram can be obtained.

Table 4: Gaussian distribution parameters (small sample)

	Mean	s	Median	r	R^2
Original	5.57	1.322	5.45	0.99675	0.98948
Take Log10	5.734	0.103	5.736	0.99892	0.99257
Recover 10^{log10}	5.42	1.268	5.45	0.99648	0.99094

(Where s is sample standard deviation)

Table 5: Weibull distribution parameters (small sample)

	b	N_0	λ	r	R^2
Original	2.04	2.766	3.21	0.99914	0.99824
Take Log10	3.588	5.36	0.41	0.99921	0.9984

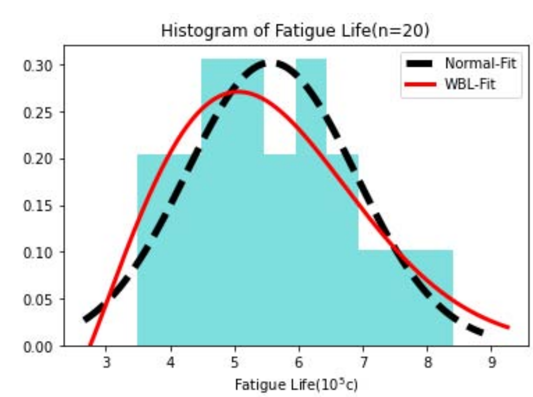

Fig. 5: Histogram of original data (small sample) and fitting diagram of Gaussian and Weibull distribution

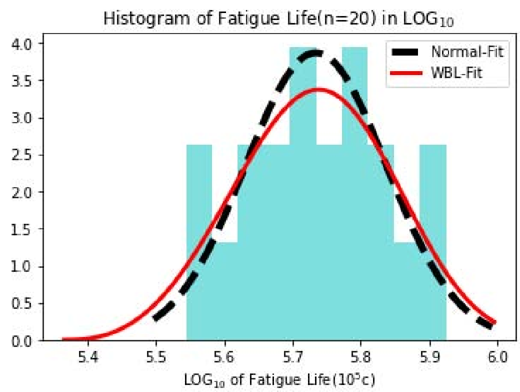

Fig. 6: Histogram after logarithm of the original data (small sample) and fitting diagram of Gaussian and Weibull distribution

As seen in Fig. 5 and 6, similar to the case of the large sample, the original data are also left-skewed and appear symmetric after taking the logarithm. However, if the Weibull distribution is fitted, there is no need to take the logarithm of the original data. Even if the logarithm is taken, the data looks more symmetric, but the Weibull distribution does not fit worse than the Gaussian distribution. So in this sense, even for symmetric data, fitting with the Weibull distribution is possible. However, the difficulty in fitting the Weibull distribution is that it is more difficult to estimate the three parameters, but now there is no problem with GZT method.

VI. Conclusion

1. The three-parameter Weibull distribution is a more general full state distribution than the Gaussian distribution. In the field of reliability, the physical meaning of its position parameter is particularly important, that is, the safe life under 100% reliability.
2. Based on the complexity of the three-parameter Weibull distribution, the previous methods to determine its three parameters by test data are complicated. The graphical method is more error-prone and inconvenient to use; while the analytical method may be inconsistent; and the GZT method makes full use of the advantages of Python, which solves this problem better.
3. In the past, the fatigue life data that were not so well fitted with Gaussian distribution were taken logarithmically so that they might be more consistent with Gaussian distribution, but the result of doing so made the 100% reliability of the safe life no longer exist. The fact is that the data itself is more consistent with Weibull distribution. Since Weibull distribution is a full state distribution, it is generally not necessary to take the fatigue life as logarithm in the future and directly fit the fatigue life data with the three-parameter Weibull distribution to get a better fit.
4. The two parameters of Gaussian distribution (mean and variance) are not very significant for asymmetric data, while for asymmetric data like structural fatigue life the three parameters of Weibull distribution (safety life, shape and scale parameters) will be much more significant, and in a sense these three parameters "contain" the two parameters of Gaussian distribution. This is probably the reason why the Weibull distribution can "contain" the Gaussian distribution.
5. Finally, it can be concluded that for asymmetric fatigue life, it is not necessary to take logarithms to fit with Gaussian distribution, but can be directly fitted with three-parameter Weibull distribution. Further even for the more symmetric data, it is better to fit directly with the three-parameter Weibull distribution.

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Application of Differentialintegral Functions

By Alexey S. Dorokhov, Solomashkin Alexey Alekseevich,
Vyacheslav A. Denisov & Kataev Yuri Vladimirovich

Summary- The article is devoted to the development and implementation of new mathematical functions, differentialintegral functions that provide differentiation and integration operations not only according to existing algorithms described in textbooks on higher mathematics, but also by substituting a certain parameter k into formulas developed in advance, forming the necessary derivatives and integrals from these formulas.

The Purpose of the Research: The expansion of the concept of number, in particular, in classical mechanics, physics, optics and other sciences, including biological and economic, which makes it possible to expand some understanding of the essence of space, time and their derivatives.

Materials and Methods: The idea of fractional space, time and its application is given. The usual elementary functions and the Laplace transform were chosen as the object of research. New functions, differentialintegral functions, have been developed for them. A graphical representation of these functions is given, based on the example of the calculation of the sine wave. Examples of calculating these functions for elementary functions are given.

Keywords: differentialintegral functions, derivative, fractional derivative, integral, fractional integral.

GJRE-I Classification: ACM: (G.1), (G.2), (G.4), (F.1)

APPLICATIONOFDIFFERENTIALINTEGRALFUNCTIONS

Strictly as per the compliance and regulations of:

Application of Differentialintegral Functions

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Summary- The article is devoted to the development and implementation of new mathematical functions, differentialintegral functions that provide differentiation and integration operations not only according to existing algorithms described in textbooks on higher mathematics, but also by substituting a certain parameter k into formulas developed in advance, forming the necessary derivatives and integrals from these formulas.

The Purpose of the Research: The expansion of the concept of number, in particular, in classical mechanics, physics, optics and other sciences, including biological and economic, which makes it possible to expand some understanding of the essence of space, time and their derivatives.

Materials and Methods: The idea of fractional space, time and its application is given. The usual elementary functions and the Laplace transform were chosen as the object of research. New functions, differentialintegral functions, have been developed for them. A graphical representation of these functions is given, based on the example of the calculation of the sine wave. Examples of calculating these functions for elementary functions are given. Of particular interest is the differentialintegral function, in which the parameter k is a complex number s, s = a + i · b, although in general, the parameter k can be any function of a real or complex argument, as well as the differentialintegral function itself.

Research Results: As a result of the research, it is shown how the Laplace transform and Borel's theorem are used to calculate differentialintegral functions. It is shown how to use these functions to carry out differentiation and integration. It is presented how fractional derivatives and fractional integrals should be obtained. Dependencies for their calculation are obtained. Examples of their application for such functions as cos(x), exp(x) and loudness curves in music, Fletcher-Manson or Robinson-Dadson curves are shown.

Conclusions: Studies show the possibility of a wide application of differentialintegration functions in modern scientific research. These functions can be used both in office and in specialized programs where calculations of fractional derivatives and fractional integrals are needed.

Keywords: *differentialintegral functions, derivative, fractional derivative, integral, fractional integral.*

I. Introduction

In modern sciences, such as mathematics, physics, astronomy, economics and other sciences, there is little use of differential functions in calculations, because with the help of fractional derivatives and integrals, very few physical, natural, social and other processes are described that use not only the first and second derivatives, single and double integrals, but fractional derivatives and fractional integrals. So in classical mechanics, the first derivative is used as velocity, the second as acceleration, and the third as a jerk. A one-time integral is used to calculate the area under the curve, the mass of an inhomogeneous body, a two–time integral is used to calculate the volume of a cylindrical beam, a three-time integral is used to calculate the volume of the body.

They can be found in the equations of mathematical physics, where, in particular, generalized functions and convolutional operations on them are used, and in spectral analysis, and in operational calculus based on integral Fourier and Laplace transformations, and in many other methods where differentiation and integration of functions are used.

The basis of all these concepts is the derivative and integral[1]. Two mathematical operations that are "opposite" to each other, like addition and subtraction, multiplication and division. Two "reciprocal" functions like *sin(x)* and *arcsin(x)*, x^2 and $\sqrt{x}$, e^x and *ln(x).* Two mathematical operations that logically complement each other, the derivative of the integral does not change the integrable function, as does the integral of the derivative, leaves it unchanged.

Let us recall the symbols on graphs and in computer programs. Like any mathematical operation, they have their symbols (designations) on a piece of paper, like ordinary symbols on a computer screen. So, differentiation is denoted as *y'* or *d/dx*, and integration is $\int y(x)dx$. In this case, a one – time integral is denoted as $\int y(x)dx$, and a two - time integral is $\iint y(x)z(t)dxdt$. With the derivative, the situation is more complicated, it has two designations:

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[1] And also, definitions of derivatives/integrals based on such concepts as the Riemann-Liouville, Grunwald-Letnikov and Weyl differentialintegrals.

y' and *d/dx*. Figure 1 shows (as one of the options) the currently existing designations of differentials and integrals, widely used in the literature.

---	---	y	y'	---	y"	(1)
$\int_a^b y\,dx$	---	y	$\frac{d}{dx}y$	---	$\frac{d^2}{dx^2}y$	(2)
$f_1 * y$	$f_{0,46} * y$	$f_0 * y$	$f_{-1} * y$	$f_{-1,35} * y$	$f_{-2} * y$	(3)
$y^{<-1>}$	$y^{<-0,46>}$	$y^{<0>}$	$y^{<+1>}$	$y^{<+1,35>}$	$y^{<+2>}$	(4)

```
---|------------------x------------------|-----------------------------|---------------x--------------------|-------->
K- 1          - 0,46              0                        + 1         + 1,35                    + 2
```

Figure 1: Notation of integrals and derivatives

As can be seen from Figure 1, all the variety of these notations has one property common to all: they try to reflect in various ways, either with the help of numbers or graphically, the order of derivatives or the multiplicity of the integral.

In order to unify the record of derivatives and integrals, consider them relative to a certain numerical axis "*K*" (Figure 2), where the value of the parameter *k* corresponds to the multiplicity of the integral or the order of the derivative. So, in this scenario of notation, $k = -1$ corresponds to the designation of a single integral $\int y(x)dx$ from the 2nd line and the designation of the same integral $f_1 * y$ from the 3rd row, and for $k = 1$- we have the designation of the first derivative y' from the 1st row and the designation of the same first derivative *d /dx* from the 2nd row.

The third line contains the notation of differentials and integrals based on convolutional operations of generalized functions: $y^{(k)} = f_{-k} * y$, where $k > 0$, a value unequal to an integer is called a fractional derivative of order *k*. An expression of the form: $y_{(k)} = f_k * y$ is called a primitive of order *k*, i.e. an integral of multiplicity *k* [1].

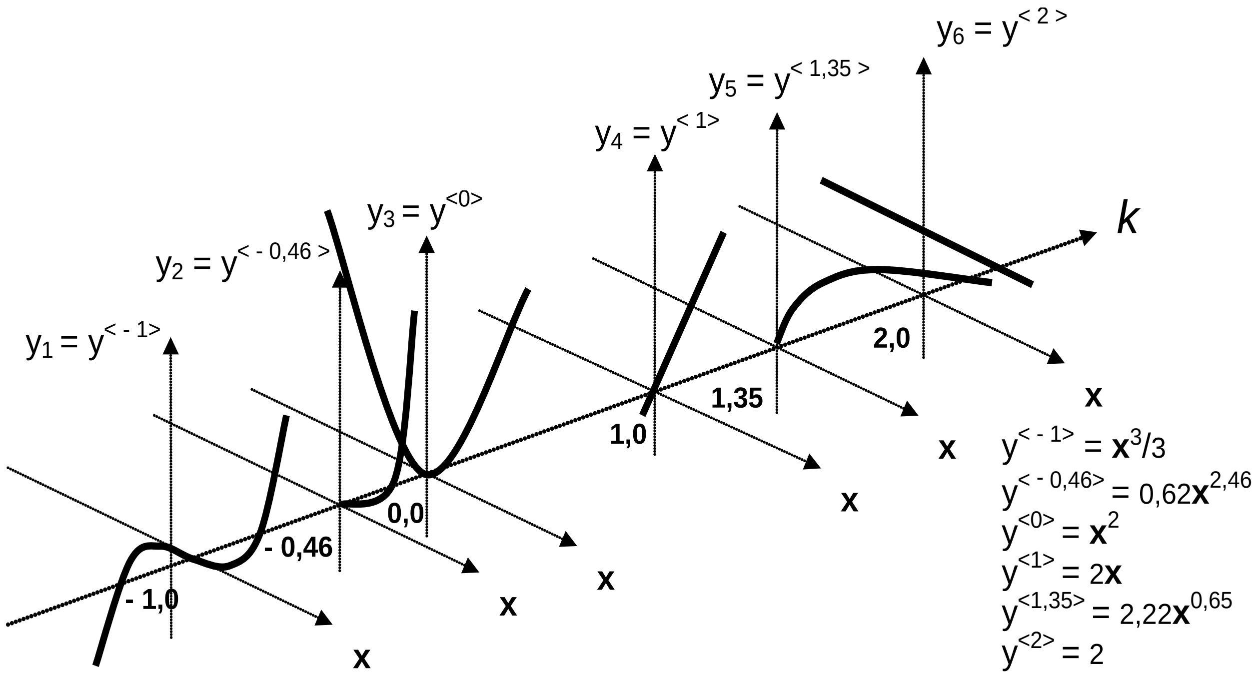

Figure 2: Notation of derivatives and integrals for a parabola$y(x) = x^2$

At the same time, all derivatives, including fractional ones, having a negative index, are located on the numerical axis on the right, and all integrals with a positive index - on the contrary, on the left. It was possible to arrange the designations differently, change the plus to minus, but the essence would not change at the same time. There are many types of symbols, binding to the numeric axis requires clarification.

To bring these notations in line with the numerical axis "*K*", the 4th line contains universal notations for derivatives of any order and integrals of any multiplicity, using angle brackets.

The angle brackets denote the order of the derivative or the multiplicity of the integral, for example, $y^{<0>} = y(x)$ is the function under study, and $y^{<-1>} = \int y(x)dx$ is its integral, multiplicity 1. So $y^{<2>} = d^2/dx^2 = y''$ is the second derivative, and $y^{<-0,46>}$ is the integral, multiplicity *0,46*. For example, a certain derivative of the order of 1,35 is denoted as $y^{<1,35>}$. In other words, if there is a positive number in the angle brackets, it means it is some kind of derivative, and if it is negative, it means it is an integral. And it is easy to read, and it is located correctly on the numeric axis: negative values of the *k* index are on the left, and positive values are on the right. This form of writing integrals and derivatives is very convenient, for example, for their designation on graphs or diagrams.

Figure 2 shows an example of the notation of derivatives and integrals for the parabola $y(x) = x^2$.

In addition to notation on graphs, this method can be used for programmers writing programs in various programming languages, for example,

```
...
int main () {
float y, u, z;
int n3;
...
z= y (4) <1.5>;
u=n3 <-0,25>;
…
```

where $y^{<1,5>}$ is the derivative of the function *y(4)* of order *1,5* and $n3^{<-0.25>}$ is the integral of multiplicity *0,25* of the function *n3*.

In Figure 2, the integral of multiplicity *–0,46* and the derivative of the order of *1,35* are shown for $x > 0$.

It should be borne in mind that when calculating a derivative of a "high" order, say, *123* orders – $y^{<123>}$, previously it was necessary to perform *122* differentiation operations beforehand. This is due to the fact that the definition of the derivative/integral implies an increase in the order of the derivative/integral by only 1. It is impossible, using the existing definition of the derivative, to immediately calculate a high-order derivative from it. Only with the help of sequential multiple calculations can the order of the derivative be increased to the desired value. The same applies to integration.

II. Materials and Methods

This method of calculating derivatives reduces the efficiency of using the differentiation operation, for example, in series expansions, because it requires calculating derivatives of a "high" order, and this is time-consuming and involves calculation errors. Therefore, in such calculations, only the first few terms of the decomposition are taken, and the rest are discarded, which increases the calculation error.

As for calculating integrals, especially multiplicities greater than *2*, this is an even more difficult task. Thus, the lack of a simple, reliable and accurate method of differentiation and/or integration significantly hinders computational progress in mathematics.

The same problem is observed in physics. Many laws of mathematical physics, most often appearing in simple, accessible calculations, are based on the use, mainly, of the 1st, maximum 2nd derivative (for example, current $i = dq / dt$, force $F = m \cdot d^2x / dt^2$) and a single integral, for example, voltage across the capacitor $u(t) = 1 / C \cdot \int i(t)dt$.

It is very rare in everyday physics or mathematics to find a 3rd derivative or a 3-fold integral. This does not happen often. One of the ways to use a 3-fold integral is the Ostrogradsky-Gauss integral to calculate the volume of a body if the surface bounding this body is known.

And if you look more broadly, then neither in physics nor in mathematics have the everyday laws of the universe using fractional derivatives and integrals been discovered so far, because their calculation is fraught with great difficulties [1]. At the same time, it is possible that all the diversity of the world exists exactly there, in a fractional dimension, which can be described and studied, precisely with the help of fractional (analog), and not integer (discrete) integrals and differentials.

Take, for example, the mechanism of describing multidimensional structures, for example, multidimensional space. Our 3-dimensional space and one-dimensional time are described by discrete (integer) coordinate values, in this case one and three. At the same time, the question of the existence of a space having, not 3, but, say, 2,345 coordinates is of great scientific and practical interest. In other words, the structure of a special "fractional" space, no longer two-dimensional, is a plane (because to describe the plane, you need 2 coordinates, and we have more – 2,345), but also not a three-dimensional volume (where 3 coordinates are needed), i.e. something average between the plane and the volume. It is very difficult to imagine such a structure. In nature, such a space does not seem to exist.

It is even more difficult to determine the velocity or acceleration in such a space, i.e. to describe the kinematics of the motion of bodies. If it is possible to define the force in such a space (or to use the already existing classical method of specifying forces), then we can count on success in creating the dynamics of such structures, i.e., in other words, to create the mechanics of multidimensional space. At the same time, our classical 3-dimensional mechanics will turn out to be a special case of a more general mechanics – the mechanics of multidimensional spaces. This can be said about other physical laws of the universe.

And whether our idea of the world will change with the emergence of a new, more general, idea of space. So far we don't know much about this, because our concepts are tied to a three-dimensional dimensional space, and all the diversity of the world "lies" in a multidimensional "fractional" world that has not been studied at all.

A number of legitimate questions arise:

- What kind of space is "located", say, between a plane (2-dimensional space) and a volume (3-dimensional), i.e. a substance with the dimension of space *R*, where $2 < R < 3$?
- What kind of physical quantity, which is between speed and acceleration between $y^{<1>}$ and $y^{<2>}$ from the move, i.e. a physical quantity, defined, for example, the fractional derivative of $y^{<1,23>}$, the order of *1,23* (not 1 or 2)?
- Whether Newton's laws are applicable to the so-called fractional space?
- How will the definition of force in fractional space change (if it changes)?
- Will it be possible to apply the classical laws of mechanics to fractional space, or will it be necessary to create a new, more general, mechanics of the macro and microcosm?
- Will the interaction between space and time change if we "replace" the classical concept of space with a fractional one?
- Will there be changes in Einstein's theory of relativity and will the concepts of "gravitational, electromagnetic and other interactions" and much, much more remain the same?

Answers to these and other questions can be obtained if you have a convenient apparatus for calculating derivatives/integrals of any order/multiplicity, including fractional ones. In other words, it is necessary to create such

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a calculation algorithm, simple and convenient, especially for novice researchers, where instead of calculating integrals/differentials, it would be possible to use the usual substitution of numbers, in which the desired order or multiplicity could be set without performing calculations, but simply substitute the desired parameter into the desired formula and get a ready derivative/integral without their calculations, i.e. immediately. Such a tool, which could be called, for example, functions - *SL(x, k)*, would greatly simplify the process of calculating derivatives and integrals and significantly expand the boundaries of our knowledge.

First, we introduce the concepts of a differential integral function based on the definition of a differential integral. The differential integral function *SL (x, k)* is an ordinary function of several arguments, where, separated by commas, its arguments (in this case one – *x*) and the parameter *k*, the order of future derivatives and/or the multiplicity of integrals are indicated[2].

For example, for a parabola $y(x) = x^2$, such a differentialintegral function *SL(x, k)* will have the form[3].

$$SL(x,k) := 2 \cdot \frac{x^{2-k}}{\Gamma(3-k)} \tag{1}$$

where, *x* is the argument of the function,
k is a parameter that specifies the order of the derivative or the multiplicity of the integral.

This is the differential integral function of a parabola, the usual function of 2 arguments, argument *x* and parameter *k*. It represents a whole set of integrals and derivatives of any order and multiplicity[4] (the main, mother function). How to use it? You need to set the parameter *k* and get the desired derivative or integral.

For example, for a parabola, we substitute $k = 0$ into it. Then, for $k = 0 y (x, k) = x^2$, $(\Gamma (3 - k) = 2)$[5] the function (parabola) does not change. When $k = 1 y (x, k) = 2x$ and the parabola is transformed into its 1st derivative - $y^{<1>}$. When $k = -1$ $y (x, k) = x^3/3$ and the function becomes its one-time integral $-y^{<-1>}$, and for $k = -2$ $y (x, k) = x^4/12$ - double - $y^{<-2>}$. No calculations, just substitution.

Fractional derivatives and integrals are of particular interest, because there is no simple and reliable way to calculate them, except for the method indicated above [2]. In this case, the method of obtaining is the same. To calculate them, it is enough to substitute the necessary value of the derivative instead of the parameter k, for example, k = 0.123 and the parabola becomes its derivative of the order 0.123 – y <0.123>:

$$SL(x,k) := 2 \cdot \frac{x^{2-0{,}123}}{\Gamma(3-0{,}123)}\ float, 3\ \rightarrow 1{,}12 \tag{2}$$

If it is necessary to obtain an integral of multiplicity *3,45* - $y^{<-3,45>}$, it is enough to substitute $k = -3{,}45$ into the differential function (1) and the parabola becomes its integral of multiplicity *3,45* - $y^{<-3,45>}$:

$$SL(x,k) := 2 \cdot \frac{x^{2+3{,}45}}{\Gamma(3+3{,}45)}\ float, 3\ \rightarrow 7{,}6060^{-3} x^{5{,}4} \tag{3}$$

This method of calculating fractional derivatives is no different from the method of obtaining integer (discrete) derivatives – the same substitution. There is no difference between an integer or fractional derivative/integral. Simple substitution to get a given result.

Consider another example: *y(x)=sin(x).* For a sine wave, the differentialintegral function *SL(x,k)* will have the following form:

$$SL(x,k) := \sin\left(x + k \cdot \frac{\pi}{2}\right) \tag{4}$$

This is a sine wave whose phase shift depends on the order of its derivative/multiplicity of its integral. At $k = 0$, the sine wave does not change, at $k = 1$, and becomes *cos(x)*, i.e. its the first derivative is $y^{<1>}$, and at $k = -1$ it becomes *-cos(x)*, i.e. its integral is $y^{<-1>}$. At $-1<k<1$, the function occupies an intermediate position between *-cos(x)* and *cos(x),* including *sin(x)* at $k = 0$.

The differential integral function for the sine wave (4) is a graphical representation of the differential integral function, namely, the parameter *k* represents a part of the right angle for unit orts. At $k = 1$, the function *SL(x,1)* becomes the 1st derivative, such a unit ort is perpendicular to the abscissa axis, and at $k = var$ it is a fractional derivative of *k* order and the angle *k* (in values from 0 to 1 or in % of 90 degrees) it is only a part of the right angle.

For the exponent $y(x) = e^x$, the differential integral function *SL(x, k)* does not depend on *k* and all its derivatives and integrals are equal to each other and equal to the exponent itself.

[2] Here *SL(x, k)* is another form of writing a power differential function, different from writing the formy<k>.
[3] Here and further calculations are performed in the MathCad program, so it uses a dot in its formulas instead of a comma.
[4] As the latter, there may be the differentialintegral functions themselves. In this case, the parameter k can also be a complex value.
[5] G(x) - gamma function.

These examples can be summarized in Table 1, where its derivatives and integrals are given for some elementary functions.

Table 1: Examples of calculation of derivatives and integrals

$y^{<-1>}$	$y^{<-0.5>}$	$y^{<0>}$	$y^{<0.5>}$	$y^{<1.5>}$	*SL (x, k)*
$\frac{\Gamma(n+1)\, x^{n+1}}{\Gamma(n+2)}$	$\frac{\Gamma(n+1)\, x^{n+0,5}}{\Gamma(n+1,5)}$	x^n	$\frac{\Gamma(n+1)\, x^{n-0,5}}{\Gamma(n+0,5)}$	$\frac{\Gamma(n+1)\, x^{n-1,5}}{\Gamma(n-0,5)}$	$\frac{\Gamma(n+1)\, x^{n-k}}{\Gamma(n+1-k)}$
$x^3/3$	$0{,}601x^{2.5}$	x^2	$1{,}504x^{1.5}$	$2{,}256x^{0.5}$	$\frac{2\, x^{2-k}}{\Gamma(3-k)}$
e^x	e^x	e^x	e^x	e^x	e^x
$\sin(x-\pi/2)$	$\sin(x-0{,}5\cdot\pi/2)$	$\sin(x)$	$\sin(x+0{,}5\cdot\pi/2)$	$\sin(x+1{,}5\cdot\pi/2)$	$\sin(x+k\cdot\pi/2)$

Differential functions can be a function of 2 or more arguments, for example, *SL (x, y, k),* where *(x)* and *(y)* are two arguments of the same function: $SL(x, y, k_x, k_y) = 2 \cdot k_y + (x - y) \cdot k_x$, and k_x and k_y- are still a parameter. In addition, any continuous elementary function can be used as a parameter, including the same differential integral function, for example:

$$(x, y, k1, k2) := x^{\sin\left(y \cdot k1 + \frac{\pi}{2} k2\right)} \tag{5}$$

Of particular interest is the differential integral function, in which the parameter *k* is a complex number *s*, $s = a + i \cdot b$, although in general, the parameter *k* can be any function of a real or complex argument.

III. Research Results

To obtain the differential integral function, we recall the Laplace integral transformation and Borel's theorem.

The integral Laplace transform has the form

$$L[f(t)] = F(s) = \int_0^{\infty} f(t) e^{-st}\, dt \equiv [f(t) \cdot e^{-st}\, dt]^{<-1>}{}_{0<t<\infty} \tag{6}$$

where $s = a + i * b$ is a complex quantity. Here *f (t)* is the original function, and *F(s)* is its Laplace image. This is a direct conversion of the original into an image. The inverse Laplace transform

$$f(t) := \frac{1}{2\pi i} \cdot \int_{\sigma - i\cdot\infty}^{\sigma + i\cdot\infty} e^{st} F(s) ds \;\equiv\; [e^{st} \cdot F(s) \cdot ds]^{<-1>}{}_{\sigma - i\infty < \sigma + i\infty} \tag{7}$$

it is necessary to find the original of the function by its image.

Let's consider one of the main properties of this transformation – the differentiation of the original function.

Let $L[f(t)] = F(s)$. Let's find $L[f(t)^{<1>}]$, where $f(t)^{<1>}$- is the 1st derivative, and $L[f(t)^{<1>}]$- is its image.

$$L[f(t)^{<1>}] = [f(t)^{<1>} \cdot e^{-st} dt]^{<-1>}{}_{0<t<\infty} = e^{-st} \cdot f(t)_{0<t<\infty} + s \cdot [f(t) \cdot e^{st} dt]^{<-1>}{}_{0<t<\infty} \tag{8}$$

If for $t \to \infty$ the function *f (t)* increases no faster than $M * e^{at}$, then

$e^{-st} * f(t) \to 0$ for $t \to \infty$ and is equal to *f (0)*, and

$$L[f(t)^{<1>}] = s * F(s) - f(0) \tag{9}$$

For $f(0) = 0$

$$L[f(t)^{<1>}] = s * F(s) \tag{10}$$

and the differentiation of the original function corresponds to the multiplication of the image of the function by *s*. Let's consider another important property – the integration of the original.

If $g(t) = [f(\tau)d\tau]^{<-1>}{}_{0<\tau<t}$, then under zero initial conditions $g(t)^{<1>} = f(t)$ and

$$L[g(t)^{<1>}] = L[f(t)] = s * L[g(t)] = s * L[[f(\tau)\, d\tau]^{<-1>}{}_{0<\tau<t}] \tag{11}$$

Since $L[f(t)] = F(s)$, then

$$L[[f(\tau) * d\tau]^{<-1>}{}_{0<\tau<t}] = F(s)/s \tag{12}$$

that is, the integration of the function corresponds to the division of the image *F (s)* by *s*.

Taking into account expressions (14) and (16), we can conclude that the operations of differentiation/integration of the original can be replaced by algebraic actions (multiplication/division by s) on their images [3]. Thanks to this replacement, this method has found the widest application in integral and differential calculus [4].

However, the case is of particular interest when the function is represented as

$$L\ [f\ (t)] = F(s)/(s^{-k}) \tag{13}$$

that is, the image is divided by *(s-k).* In this case, depending on *k*, we get fractional derivatives/integrals. For $k > 0$, fractional derivatives of the order *k* are formed, and for $k < 0$, fractional integrals of the same multiplicity are formed.

$$L[f(t)] = \frac{\mathrm{F(s)}}{s^{-k}} = 1/(\Gamma(-k)) \tag{14}$$

$$SL\ (x,\ k) = L\ [f\ (t)] \tag{15}$$

These expressions (18) and (19) define fractional derivatives/integrals of order *k*, and are the differential functions of the desired function *f(t).* Examples of these functions are shown in Table 1.

Let's consider some examples of the use of differential integral functions in solving approximation problems. Suppose must be approximated by a power series ***ряд***_*cos(x)* in a neighborhood of the point *x0,* the function *cos(x),* and choose the polynomial coefficients $a_0 ... a_5$ so as to minimize the mean square error of approximation of this polynomial are:

$$\_cos(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + a_3 \cdot x^3 + a_4 \cdot x^4 + a_5 \cdot x^5 \tag{16}$$

and at the selected point is known for its derivatives and differentials, as an integer and the fraction.

To do this, we fulfill the approximation conditions according to which the value of the polynomial _*cos(x)* and its fractional derivatives (for simplicity of calculation, only six (5) derivatives are used[6]. To increase the accuracy, you can use more, for example, several dozen derivatives, the computer allows it. Instead of derivatives, its integrals can also be used in the same way) in the vicinity of a given point x0, from the domain of the polynomial definition, should equal the corresponding values of the desired function *cos(x)* and its fractional derivatives (and integrals). 2 points are selected as points – $x = 3$ and $x = 15$.

The fractional derivatives/integrals for the elements of the polynomial are defined as

$$SL(x, n, k) := \frac{\Gamma(n+1) \cdot x^{n-k}}{\Gamma(n+1-k)} \tag{17}$$

where *x* -is the matrix of diagnostic information;
n - is the exponent of the polynomial;
k- is a parameter that sets the multiplicity of the integral or the order of derivatives.

Further, solving a linear algebraic equation of the form:

$$a = A1^{-1} \cdot B1 \tag{18}$$

we obtain the solution of this equation in the form of the desired coefficients $a_0 ... a_5$ (Application A Figure A.1).

The solution was made in the MathCad program, the calculation listing is given for the point $x = 3$ and additionally for $x = 15.$

Another example. In addition to the approximation at a point, using the differential integral functions, it is possible to approximate on a given segment. Examples of this approximation are given below.

Let it be necessary to approximate, for simplicity, the known functions cos (x) and the exponent *exp(x)*, as well as *cos(x)* on the plot $4 < x < 6$, as well as volume curves, according to the type of Fleicher-Manson or Robinson-Dadson curves. For ease of calculation, we approximate 6 points for 2 *cos (x)* functions, 4 (four) points for the exponent *exp(x)* and 23 for volume curves.

For a sine wave, the desired points will be of two types. In the first case, these are the points -5, -4, -2, 1, 3, 5. In the second case, this is -5, -3, -1, 1, 3, 5.

We will approximate the sinusoid with a polynomial (17).

Exponent – exponent.

[6] To approximate in this case, it is to decompose into a power series using differential integral functions in the vicinity of the point x_0, bearing in mind that these points are the values of the function *f (x) = cos (x).*

For the first case, for points -5, -4, -2, 1, 3, 5 the initial data obtained by formula (17) will have the following form.

$$A2 := \begin{pmatrix} 1 & -5 & 25 & -125 & 625 & -3125 \\ 1 & -4 & 16 & -64 & 256 & -1024 \\ 1 & -2 & 4 & -8 & 16 & -32 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 & 81 & 243 \\ 1 & 5 & 25 & 125 & 625 & 3125 \end{pmatrix} \qquad B2 := \begin{pmatrix} \cos(-5) \\ \cos(-4) \\ \cos(-2) \\ \cos(1) \\ \cos(3) \\ \cos(5) \end{pmatrix} \qquad b2 := A2^{-1} \cdot B2 \quad b2 = \begin{pmatrix} 0.615 \\ 0.207 \\ -0.257 \\ -0.036 \\ 9.731 \times 10^{-3} \\ 1.117 \times 10^{-3} \end{pmatrix}$$

As a result of calculating the series *rjad_cos (x),* we get the values of *cos (x).*

$$rjad\_1\_cos\,(x) := b2_0 + b2_1 + b2_2 \cdot x^2 + b2_3 \cdot x^3 + b2_4 \cdot x^4 + b2_5 \cdot x^5 \tag{19}$$

The graphs of these two functions *cos (x)* and *rjad_1_cos(x)* and some values of these graphs are shown in Figure 3.

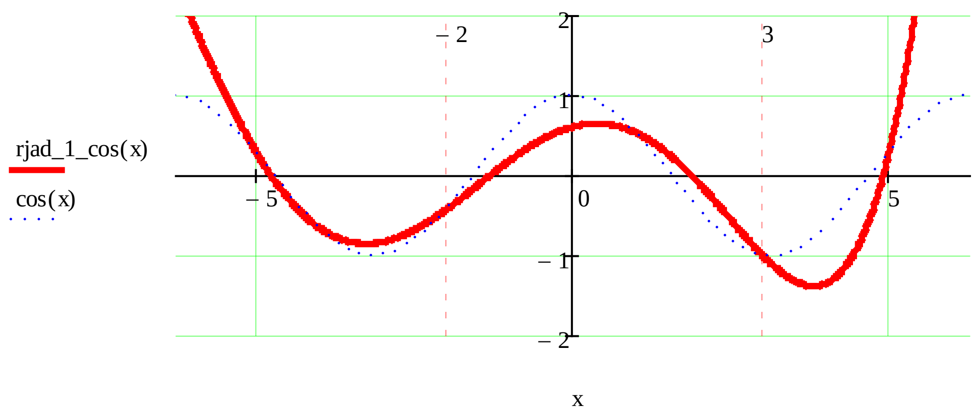

Figure 3: Values of the functions-*rjad_1_cos (x)* and *cos (x)*

$rjad\_1\_cos\,(-5) = 0.284$
$\cos(-5) = 0.284$

$rjad\_1\_cos\,(-4) = -0.654$
$\cos(-4) = -0.284$

$rjad\_1\_cos\,(-2) = -0.416$
$\cos(-2) = -0.416$

$rjad\_1\_cos\,(-1) = 0.54$
$\cos(-1) = 0.54$

$rjad\_1\_cos\,(3) = -0.99$
$\cos(3) = -0.99$

$rjad\_1\_cos\,(5) = 0.284$
$\cos(5) = 0.284$

For another cosine, for the values -5, -3, -1, 1, 3, 5 the initial data obtained by the formula (17) will have the following form:

$$A3 := \begin{pmatrix} SL(\_x_0,\_n_0,0) & SL(\_x_0,\_n_1,0) & SL(\_x_0,\_n_2,0) & SL(\_x_0,\_n_3,0) & SL(\_x_0,\_n_4,0) & SL(\_x_0,\_n_5,0) \\ SL(\_x_1,\_n_0,0) & SL(\_x_1,\_n_1,0) & SL(\_x_1,\_n_2,0) & SL(\_x_1,\_n_3,0) & SL(\_x_1,\_n_4,0) & SL(\_x_1,\_n_5,0) \\ SL(\_x_2,\_n_0,0) & SL(\_x_2,\_n_1,0) & SL(\_x_2,\_n_2,0) & SL(\_x_2,\_n_3,0) & SL(\_x_2,\_n_4,0) & SL(\_x_2,\_n_5,0) \\ SL(\_x_3,\_n_0,0) & SL(\_x_3,\_n_1,0) & SL(\_x_3,\_n_2,0) & SL(\_x_3,\_n_3,0) & SL(\_x_3,\_n_4,0) & SL(\_x_3,\_n_5,0) \\ SL(\_x_4,\_n_0,0) & SL(\_x_4,\_n_1,0) & SL(\_x_4,\_n_2,0) & SL(\_x_4,\_n_3,0) & SL(\_x_4,\_n_4,0) & SL(\_x_4,\_n_5,0) \\ SL(\_x_5,\_n_0,0) & SL(\_x_5,\_n_1,0) & SL(\_x_5,\_n_2,0) & SL(\_x_5,\_n_3,0) & SL(\_x_5,\_n_4,0) & SL(\_x_5,\_n_5,0) \end{pmatrix}$$

$$B3 := \begin{pmatrix} \cos(-5) \\ \cos(-3) \\ \cos(-1) \\ \cos(1) \\ \cos(3) \\ \cos(5) \end{pmatrix} \qquad \_x := \begin{pmatrix} -5 \\ -3 \\ -1 \\ 1 \\ 3 \\ 5 \end{pmatrix} \qquad d := A3^{-1} \cdot B3$$

$$rjad\_2\_\cos(x) := d_0 + d_1x + d_2x^2 + d_3x^3 + d_4x^4 + d_5x^5 \tag{20}$$

The graphs of these two functions *cos (x)* and *rjad_2_cos(x)* and some values of these graphs are shown in Figure 4.

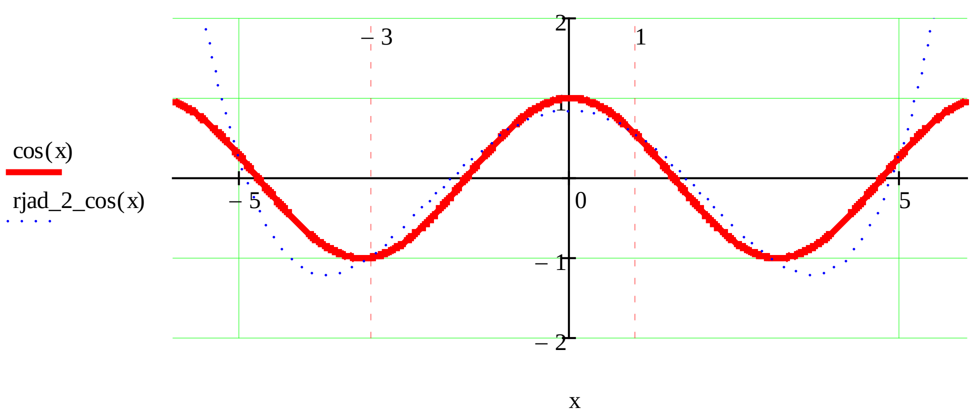

Figure 4: Values of the functions - *rjad_2_cos (x)* and *cos (x)*

$$rjad\_2\_\cos(-5) = 0.284 \qquad \cos(-5) = 0.284$$

$$rjad\_2\_\cos(1) = 0.54 \qquad \cos(1) = 0.54$$

$$rjad\_2\_\cos(-3) = -0.99 \qquad \cos(-3) = -0.99$$

$$rjad\_2\_\cos(-3) = -0.99 \qquad \cos(3) = -0.99$$

$$rjad\_2\_\cos(-1) = 0.54 \qquad \cos(-1) = 0.54$$

$$rjad\_2\_\cos(5) = 0.284 \qquad \cos(5) = 0.284$$

If we look at the same graphs in other coordinates, we can say that at these points the graphs coincide with their values, and at other points they do not, and they differ significantly.

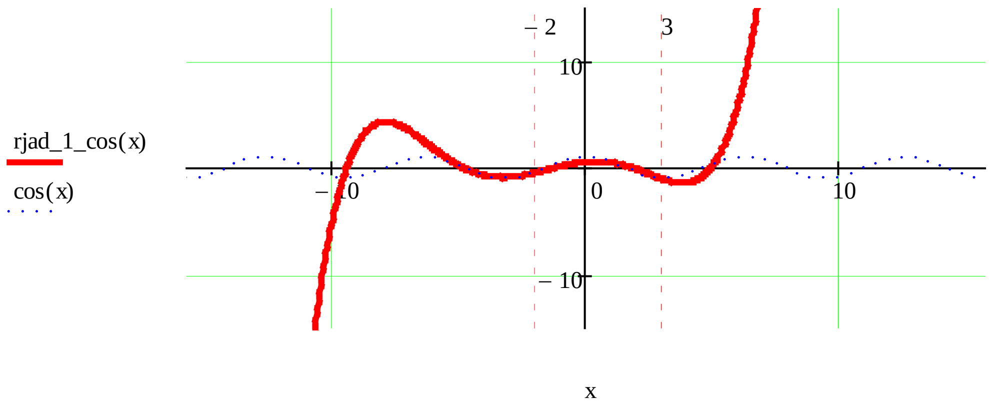

Figure 5: Values of the functions-*rjad_1_cos (x)* and *cos (x)*

The values of these two functions-*rjad_1_cos(x)* and *cos (x)* in other coordinate systems coincide only in this section in $\pm 2\pi$, and for other values of the argument they differ greatly.

Figure 6 shows the values of these two functions *rjad_2_cos(x)* and *cos (x).*

In the given figure shows that the values of these two functions *rjad_2_cos(x)* and *cos (x)* in different coordinate systems coincide only in this region of ± 6, and for other values of the argument vary greatly.

This suggests that approximation by differential integral functions is possible both at a point and at a certain area. The approximation error is minimal and can be reduced by increasing the number of terms of the polynomial.

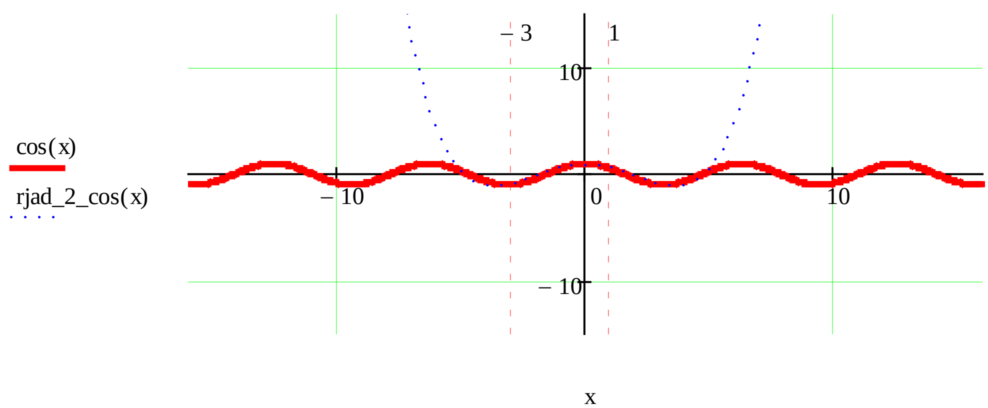

Figure 6: Values of the functions-rjad_2_cos (x) and cos (x)

The exponent can be approximated by the exponent itself. An example is shown below in Figure 7.

$$\begin{array}{cc} n & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ A1 := & \begin{pmatrix} 1^0 & 1^1 & 1^2 & 1^3 \\ 2^0 & 2^1 & 2^2 & 2^3 \\ 7^0 & 7^1 & 7^2 & 7^3 \\ 10^0 & 10^1 & 10^2 & 10^3 \end{pmatrix} \end{array} \qquad B1 := \begin{pmatrix} e^1 \\ e^2 \\ e^7 \\ e^{10} \end{pmatrix} \qquad a := A1^{-1} \cdot B1 \qquad a = \begin{pmatrix} -1.19 \times 10^3 \\ 1.966 \times 10^3 \\ -863.71 \\ 89.924 \end{pmatrix}$$

$$k := (1 \ \ 2 \ \ 7 \ \ 10)^T$$

$$e^k = \begin{pmatrix} 2.718 \\ 7.389 \\ 1.097 \times 10^3 \\ 2.203 \times 10^4 \end{pmatrix}$$

$$rjad\_\exp(x) := a_0 + a_1 x + a_2 x^2 + a_3 x^3 \tag{21}$$

$$rjad\_exp\,(1) = 2.718 \qquad e^1 = 2.718$$

$$rjad\_exp\,(2) = 7.389 \qquad e^2 = 7.389$$

$$rjad\_exp\,(7) = 1.097 \times 10^3 \qquad e^7 = 1.097 \times 10^3$$

$$rjad\_exp\,(10) = 2.203 \times 10^4 \qquad e^{10} = 2.203 \times 10^4$$

Figure 7 shows the values of these two functions - *rjad_exp(x)* and *exp (x).*

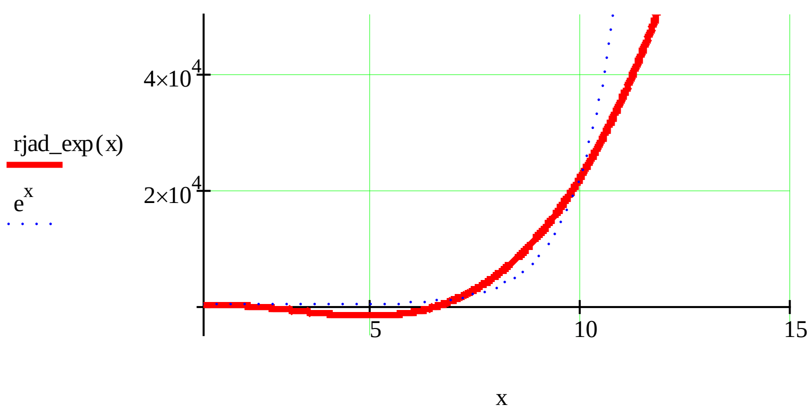

Figure 7: Values of the functions - *rjad_exp (x)* and *exp (x)*

The graph of the cos(x) function on the section from $x = 4$ to $x = 6$ and the initial data are shown below in Figure 8.

$$k\_1 := 1 + 1\cdot 10^{-6} \qquad SL(x,k,n) := \frac{x^{n-k}\cdot\Gamma(n+1)}{\Gamma(n-k+1)}$$

The set point - μ

$$\mu := 5$$

$$A1 := \begin{pmatrix} 1 & \mu & \mu^2 & \mu^3 & \mu^4 & \mu^5 \\ \frac{\mu^{-0.25}\cdot\Gamma(1)}{\Gamma(1-0.25)} & \frac{\mu^{1-0.25}}{\Gamma(2-0.25)} & \frac{2\cdot\mu^{2-0.25}}{\Gamma(3-0.25)} & \frac{6\cdot\mu^{3-0.25}}{\Gamma(4-0.25)} & \frac{24\cdot\mu^{4-0.25}}{\Gamma(5-0.25)} & \frac{120\cdot\mu^{5-0.25}}{\Gamma(6-0.25)} \\ \frac{\mu^{-0.5}\cdot\Gamma(1)}{\Gamma(1-0.5)} & \frac{\mu^{1-0.5}}{\Gamma(2-0.5)} & \frac{2\cdot\mu^{2-0.5}}{\Gamma(3-0.5)} & \frac{6\cdot\mu^{3-0.5}}{\Gamma(4-0.5)} & \frac{24\cdot\mu^{4-0.5}}{\Gamma(5-0.5)} & \frac{120\cdot\mu^{5-0.5}}{\Gamma(6-0.5)} \\ \frac{\mu^{-0.75}\cdot\Gamma(1)}{\Gamma(1-0.75)} & \frac{\mu^{1-0.75}}{\Gamma(2-0.75)} & \frac{2\cdot\mu^{2-0.75}}{\Gamma(3-0.75)} & \frac{6\cdot\mu^{3-0.75}}{\Gamma(4-0.75)} & \frac{24\cdot\mu^{4-0.75}}{\Gamma(5-0.75)} & \frac{120\cdot\mu^{5-0.75}}{\Gamma(6-0.75)} \\ \frac{\mu^{-k\_1}\cdot\Gamma(1)}{\Gamma(1-k\_1)} & \frac{\mu^{1-1}}{\Gamma(2-1)} & \frac{2\cdot\mu^{2-1}}{\Gamma(3-1)} & \frac{6\cdot\mu^{3-1}}{\Gamma(4-1)} & \frac{24\cdot\mu^{4-1}}{\Gamma(5-1)} & \frac{120\cdot\mu^{5-1}}{\Gamma(6-1)} \\ \frac{\mu^{-1.25}\cdot\Gamma(1)}{\Gamma(1-1.25)} & \frac{\mu^{1-1.25}}{\Gamma(2-1.25)} & \frac{2\cdot\mu^{2-1.25}}{\Gamma(3-1.25)} & \frac{6\cdot\mu^{3-1.25}}{\Gamma(4-1.25)} & \frac{24\cdot\mu^{4-1.25}}{\Gamma(5-1.25)} & \frac{120\cdot\mu^{5-1.25}}{\Gamma(6-1.25)} \end{pmatrix}$$

$$B1 := \begin{pmatrix} \cos\left(\mu + 0.00\cdot\frac{\pi}{2}\right) \\ \cos\left(\mu + 0.25\cdot\frac{\pi}{2}\right) \\ \cos\left(\mu + 0.50\cdot\frac{\pi}{2}\right) \\ \cos\left(\mu + 0.75\cdot\frac{\pi}{2}\right) \\ \cos\left(\mu + 1.00\cdot\frac{\pi}{2}\right) \\ \cos\left(\mu + 1.25\cdot\frac{\pi}{2}\right) \end{pmatrix} \qquad a := A1^{-1}\cdot B1$$

$$\_cos(x) := a_0 + a_1\cdot x + a_2\cdot x^2 + a_3\cdot x^3 + a_4\cdot x^4 + a_5\cdot x^5 \tag{22}$$

$\_cos\ (5) = 2{,}836622\cdot 10^{-1}$

$\cos\ (5) = 2{,}836622\cdot 10^{-1}$

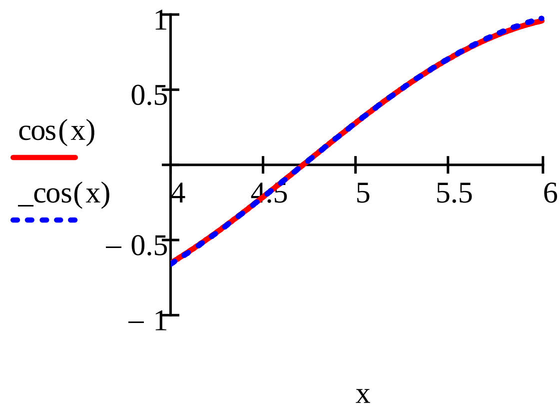

Figure 8: Values of the functions – *cos(x)* and *_cos (x)*

Additionally, the application of differential integration functions in music, curves of equal loudness, for example, Fletcher-Manson curves or Robinson-Dudson curves, Figure 9, is presented.

KRG=

	0	1	2	3	4	5	6	7	8	9	10	11
0	20	40	63	100	160	200	250	315	400	500	630	800
1	2.996	3.689	4.143	4.605	5.075	5.298	5.521	5.753	5.991	6.215	6.446	6.685
2	119	105.3	98.4	92.5	87.8	85.9	84.3	82.9	81.7	80.9	80.2	79.7

	12	13	14	15	16	17	18	19	20	21	22	23
0	$1\cdot10^3$	$1.25\cdot10^3$	$1.6\cdot10^3$	$2\cdot10^3$	$2.5\cdot10^3$	$3.15\cdot10^3$	$4\cdot10^3$	$5\cdot10^3$	$6.3\cdot10^3$	$8\cdot10^3$	$1\cdot10^4$	$1.25\cdot10$
1	6.908	7.131	7.378	7.601	7.824	8.055	8.294	8.517	8.748	8.987	9.21	9.43
2	80	82.5	83.7	80.6	77.9	77.1	78.3	81.6	86.8	91.4	91.7	85.

$$SL(x,n,k) := \frac{x^{n-k}\cdot\Gamma(n+1)}{\Gamma(n-k+1)}$$

$i := 0..23$

$j := 0..23$

$k := 0$

$0, 2, 7, 11, 12, 14, 16, 17, 19, 21, 22, 23$

$$n := \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \end{pmatrix} \qquad x := \begin{pmatrix} 2.996 \\ 4.143 \\ 5.753 \\ 6.685 \\ 6.908 \\ 7.378 \\ 7.824 \\ 8.055 \\ 8.517 \\ 8.987 \\ 9.21 \\ 9.433 \end{pmatrix} \qquad y := \begin{pmatrix} 119 \\ 98.4 \\ 82.9 \\ 79.7 \\ 80 \\ 83.7 \\ 77.9 \\ 77.1 \\ 81.6 \\ 91.4 \\ 91.7 \\ 85.4 \end{pmatrix}$$

$$\begin{aligned} \_A := & \text{ for } j \in 0..11 \\ & \quad \text{for } i \in 0..11 \\ & \quad\quad \_SL(i,j) \leftarrow SL\left(x_i, n_j, 0\right) \end{aligned}$$

$$a := \_\_A^{-1}\cdot B$$

$$A := \begin{pmatrix}
SL\ x_0,n_0,k & SL\ x_0,n_1,k & SL\ x_0,n_2,k & SL\ x_0,n_3,k & SL\ x_0,n_4,k & SL\ x_0,n_5,k & SL\ x_0,n_6,k & SL\ x_0,n_7,k & SL\ x_0,n_8,k & SL\ x_0,n_9,k & SL\ x_0,n_{10},k & SL\ x_0,n_{11},k \\
SL(x_1,n_0,k) & SL(x_1,n_1,k) & SL(x_1,n_2,k) & SL(x_1,n_3,k) & SL(x_1,n_4,k) & SL(x_1,n_5,k) & SL(x_1,n_6,k) & SL(x_1,n_7,k) & SL(x_1,n_8,k) & SL(x_1,n_9,k) & SL(x_1,n_{10},k) & SL(x_1,n_{11},k) \\
SL(x_2,n_0,k) & SL(x_2,n_1,k) & SL(x_2,n_2,k) & SL(x_2,n_3,k) & SL(x_2,n_4,k) & SL(x_2,n_5,k) & SL(x_2,n_6,k) & SL(x_2,n_7,k) & SL(x_2,n_8,k) & SL(x_2,n_9,k) & SL(x_2,n_{10},k) & SL(x_2,n_{11},k) \\
SL(x_3,n_0,k) & SL(x_3,n_1,k) & SL(x_3,n_2,k) & SL(x_3,n_3,k) & SL(x_3,n_4,k) & SL(x_3,n_5,k) & SL(x_3,n_6,k) & SL(x_3,n_7,k) & SL(x_3,n_8,k) & SL(x_3,n_9,k) & SL(x_3,n_{10},k) & SL(x_3,n_{11},k) \\
SL(x_4,n_0,k) & SL(x_4,n_1,k) & SL(x_4,n_2,k) & SL(x_4,n_3,k) & SL(x_4,n_4,k) & SL(x_4,n_5,k) & SL(x_4,n_6,k) & SL(x_4,n_7,k) & SL(x_4,n_8,k) & SL(x_4,n_9,k) & SL(x_4,n_{10},k) & SL(x_4,n_{11},k) \\
SL(x_5,n_0,k) & SL(x_5,n_1,k) & SL(x_5,n_2,k) & SL(x_5,n_3,k) & SL(x_5,n_4,k) & SL(x_5,n_5,k) & SL(x_5,n_6,k) & SL(x_5,n_7,k) & SL(x_5,n_8,k) & SL(x_5,n_9,k) & SL(x_5,n_{10},k) & SL(x_5,n_{11},k) \\
SL(x_6,n_0,k) & SL(x_6,n_1,k) & SL(x_6,n_2,k) & SL(x_6,n_3,k) & SL(x_6,n_4,k) & SL(x_6,n_5,k) & SL(x_6,n_6,k) & SL(x_6,n_7,k) & SL(x_6,n_8,k) & SL(x_6,n_9,k) & SL(x_6,n_{10},k) & SL(x_6,n_{11},k) \\
SL(x_7,n_0,k) & SL(x_7,n_1,k) & SL(x_7,n_2,k) & SL(x_7,n_3,k) & SL(x_7,n_4,k) & SL(x_7,n_5,k) & SL(x_7,n_6,k) & SL(x_7,n_7,k) & SL(x_7,n_8,k) & SL(x_7,n_9,k) & SL(x_7,n_{10},k) & SL(x_7,n_{11},k) \\
SL(x_8,n_0,k) & SL(x_8,n_1,k) & SL(x_8,n_2,k) & SL(x_8,n_3,k) & SL(x_8,n_4,k) & SL(x_8,n_5,k) & SL(x_8,n_6,k) & SL(x_8,n_7,k) & SL(x_8,n_8,k) & SL(x_8,n_9,k) & SL(x_8,n_{10},k) & SL(x_8,n_{11},k) \\
SL(x_9,n_0,k) & SL(x_9,n_1,k) & SL(x_9,n_2,k) & SL(x_9,n_3,k) & SL(x_9,n_4,k) & SL(x_9,n_5,k) & SL(x_9,n_6,k) & SL(x_9,n_7,k) & SL(x_9,n_8,k) & SL(x_9,n_9,k) & SL(x_9,n_{10},k) & SL(x_9,n_{11},k) \\
SL(x_{10},n_0,k) & SL(x_{10},n_1,k) & SL(x_{10},n_2,k) & SL(x_{10},n_3,k) & SL(x_{10},n_4,k) & SL(x_{10},n_5,k) & SL(x_{10},n_6,k) & SL(x_{10},n_7,k) & SL(x_{10},n_8,k) & SL(x_{10},n_9,k) & SL(x_{10},n_{10},k) & SL(x_{10},n_{11},k) \\
SL(x_{11},n_0,k) & SL(x_{11},n_1,k) & SL(x_{11},n_2,k) & SL(x_{11},n_3,k) & SL(x_{11},n_4,k) & SL(x_{11},n_5,k) & SL(x_{11},n_6,k) & SL(x_{11},n_7,k) & SL(x_{11},n_8,k) & SL(x_{11},n_9,k) & SL(x_{11},n_{10},k) & SL(x_{11},n_{11},k)
\end{pmatrix}$$

$$B := \begin{pmatrix} 119 \\ 98.4 \\ 82.9 \\ 79.7 \\ 80 \\ 83.7 \\ 77.9 \\ 77.1 \\ 81.6 \\ 91.4 \\ 91.7 \\ 85.4 \end{pmatrix}$$

$$rjad(x) := a_0 + a_1 \cdot x + a_2 \cdot x^2 + a_3 \cdot x^3 + a_4 \cdot x^4 + a_5 \cdot x^5 + a_6 \cdot x^6 + a_7 \cdot x^7 + a_8 \cdot x^8 + a_9 \cdot x^9 + a_{10} \cdot x^{10} + a_{11} \cdot x^{11} \tag{23}$$

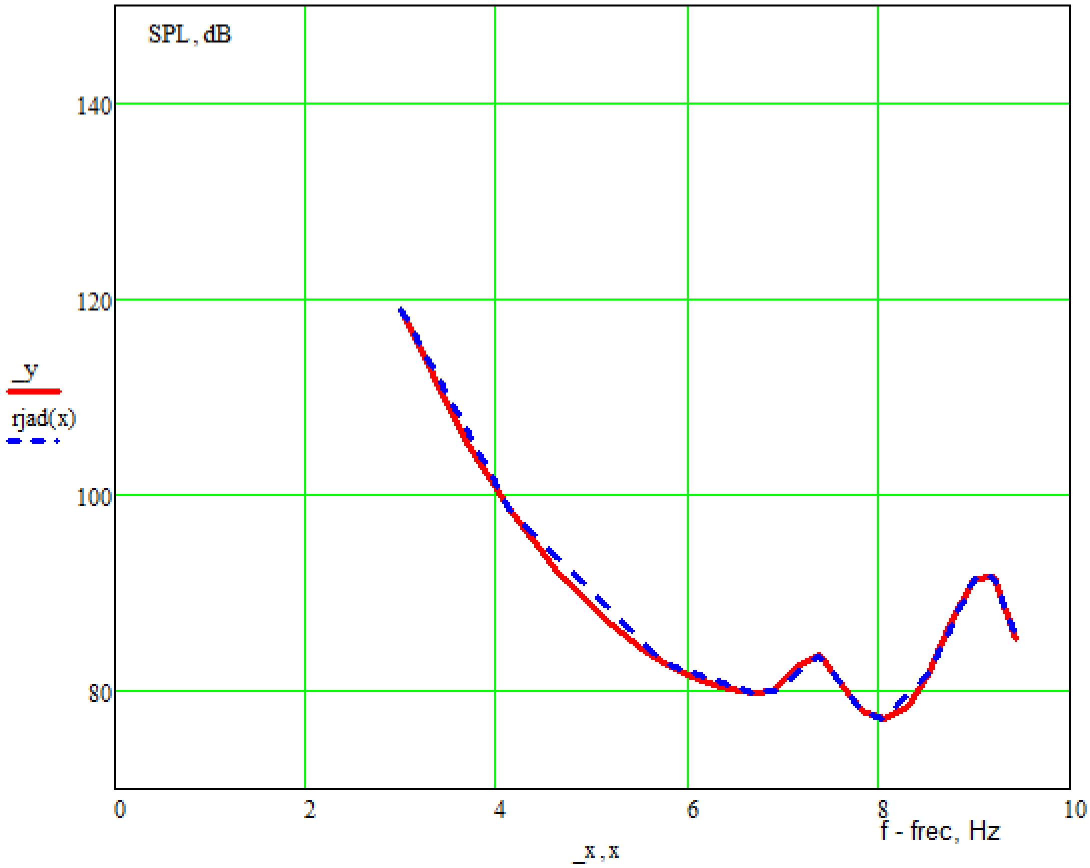

Figure 9: Curves of equal loudness *_y, dB* and a curve approximating them *rjad(x)*, Hertz

From the materials presented in the figures, it can be seen that for a given number of points, the approximation is satisfactory.

IV. Conclusions

Differential integral functions, this is the Riemann-Liouville differential integral, written in a convenient form, as a function of two variables[7]: the usual argument x and the parameter k, which sets the multiplicity of the integral or the order of the derivative. These functions allow you to calculate the desired integral or derivative by substituting the parameter k into the established formula. The formula does not change, only one parameter changes. Classical tables of integrals and differentials are not required. Only tables of pre-prepared formulas of differential functions are used, which can be represented in simple calculations in the form of icons, and in the form of SL (x, k) functions in computer programs written in programming languages such as VBasic, C++, Excel, MathCad, Python, etc.

These differential integral functions are of great practical importance, for example, they allow us to approximate a certain given function in the vicinity of the desired point (by the type of decomposition into a Taylor, Maclaurin, Fourier series or Z transformation) or on a segment. At the same time, the conditions of equality of not only the function itself, but also the selected derivatives and differentials, integer and fractional, are observed at the desired approximation points themselves.

Examples of approximation of some elementary functions are shown, for example, using a standard polynomial. It is also possible to approximate trigonometric, power functions and their combinations.

To simplify working with differential integral functions, they can be represented in two forms: for a graphic image-as a function with angle brackets, and for writing in the program text-as a function *SL (x, k)* of two or more arguments (Application B).

[7] There may be other parameters, for example, integration limits, constants, etc.

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Application A

$$k\_1 := 1 + 1\cdot 10^{-6} \qquad SL(x,k,n) := \frac{x^{n-k}\cdot\Gamma(n+1)}{\Gamma(n-k+1)}$$

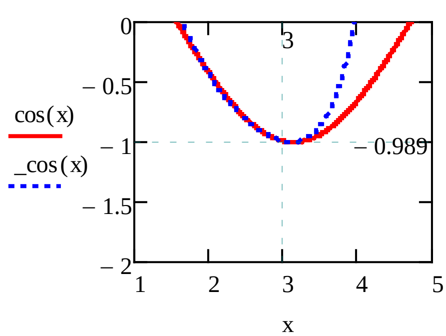
The set point - μ

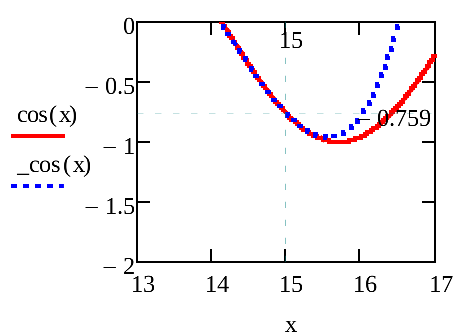
$$\mu := 3$$

$$
\begin{array}{ll}
k = 0 & \text{->} \\
k = 0{,}25 & \text{->} \\
k = 0{,}50 & \text{->} \\
k = 0{,}75 & \text{->} \\
k = 1{,}00 & \text{->} \\
k = 1{,}25 & \text{->}
\end{array}
\qquad
A1 := \begin{pmatrix}
1 & \mu & \mu^2 & \mu^3 & \mu^4 & \mu^5 \\
\frac{\mu^{-0.25}\cdot\Gamma(1)}{\Gamma(1-0.25)} & \frac{\mu^{1-0.25}}{\Gamma(2-0.25)} & \frac{2\cdot\mu^{2-0.25}}{\Gamma(3-0.25)} & \frac{6\cdot\mu^{3-0.25}}{\Gamma(4-0.25)} & \frac{24\cdot\mu^{4-0.25}}{\Gamma(5-0.25)} & \frac{120\cdot\mu^{5-0.25}}{\Gamma(6-0.25)} \\
\frac{\mu^{-0.5}\cdot\Gamma(1)}{\Gamma(1-0.5)} & \frac{\mu^{1-0.5}}{\Gamma(2-0.5)} & \frac{2\cdot\mu^{2-0.5}}{\Gamma(3-0.5)} & \frac{6\cdot\mu^{3-0.5}}{\Gamma(4-0.5)} & \frac{24\cdot\mu^{4-0.5}}{\Gamma(5-0.5)} & \frac{120\cdot\mu^{5-0.5}}{\Gamma(6-0.5)} \\
\frac{\mu^{-0.75}\cdot\Gamma(1)}{\Gamma(1-0.75)} & \frac{\mu^{1-0.75}}{\Gamma(2-0.75)} & \frac{2\cdot\mu^{2-0.75}}{\Gamma(3-0.75)} & \frac{6\cdot\mu^{3-0.75}}{\Gamma(4-0.75)} & \frac{24\cdot\mu^{4-0.75}}{\Gamma(5-0.75)} & \frac{120\cdot\mu^{5-0.75}}{\Gamma(6-0.75)} \\
\frac{\mu^{-k\_1}\cdot\Gamma(1)}{\Gamma(1-k\_1)} & \frac{\mu^{1-1}}{\Gamma(2-1)} & \frac{2\cdot\mu^{2-1}}{\Gamma(3-1)} & \frac{6\cdot\mu^{3-1}}{\Gamma(4-1)} & \frac{24\cdot\mu^{4-1}}{\Gamma(5-1)} & \frac{120\cdot\mu^{5-1}}{\Gamma(6-1)} \\
\frac{\mu^{-1.25}\cdot\Gamma(1)}{\Gamma(1-1.25)} & \frac{\mu^{1-1.25}}{\Gamma(2-1.25)} & \frac{2\cdot\mu^{2-1.25}}{\Gamma(3-1.25)} & \frac{6\cdot\mu^{3-1.25}}{\Gamma(4-1.25)} & \frac{24\cdot\mu^{4-1.25}}{\Gamma(5-1.25)} & \frac{120\cdot\mu^{5-1.25}}{\Gamma(6-1.25)}
\end{pmatrix}
$$

$$
B1 := \begin{pmatrix}
\cos\left(\mu + 0.00\cdot\frac{\pi}{2}\right) \\
\cos\left(\mu + 0.25\cdot\frac{\pi}{2}\right) \\
\cos\left(\mu + 0.50\cdot\frac{\pi}{2}\right) \\
\cos\left(\mu + 0.75\cdot\frac{\pi}{2}\right) \\
\cos\left(\mu + 1.00\cdot\frac{\pi}{2}\right) \\
\cos\left(\mu + 1.25\cdot\frac{\pi}{2}\right)
\end{pmatrix}
$$

$$a := A1^{-1}\cdot B1 \qquad \_cos(x) := a_0 + a_1\cdot x + a_2\cdot x^2 + a_3\cdot x^3 + a_4\cdot x^4 + a_5\cdot x^5$$





$$\_cos(3) = -9{,}899925 \times 10^{-1} \qquad \_cos(15) = -7.596879\times 10^{-1}$$

$$cos(3) = -9{,}899925 \times 10^{-1} \qquad cos(15) = -7.596879\times 10^{-1}$$

Figure A.1: Decomposition of the function *cos (x)* into a series *_cos(x)* in the vicinity of two different points $\mu = 3$ and $\mu = 15$

The system consists of the polynomial *cos (x)* and its six fractional derivatives *ki*, with a maximum multiplicity of 1.25. The order of the derivatives of *k* changes after 0.25.

Application B

Differential integral functions of SL().

The text of the VBasic program for calculating the differentialfunctions of SL() is given below.

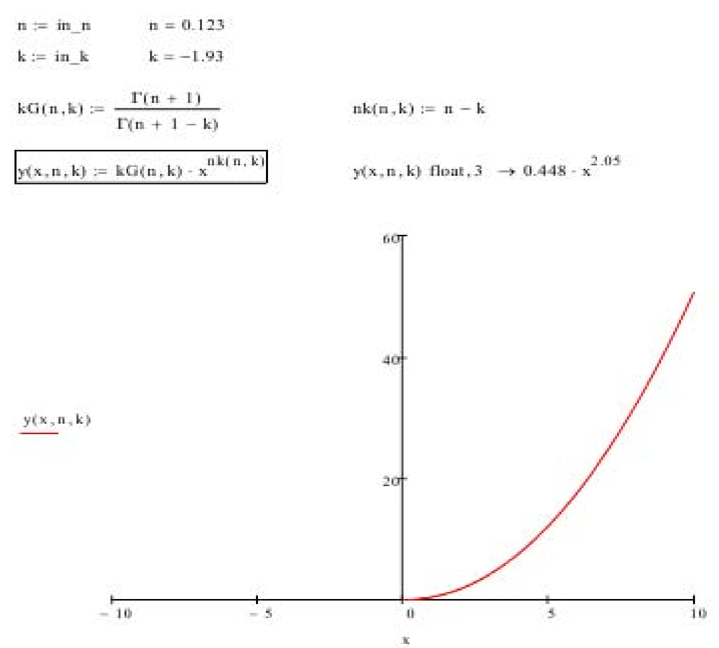
The text of the program in VBasic for calculating the differential functions of SL ().

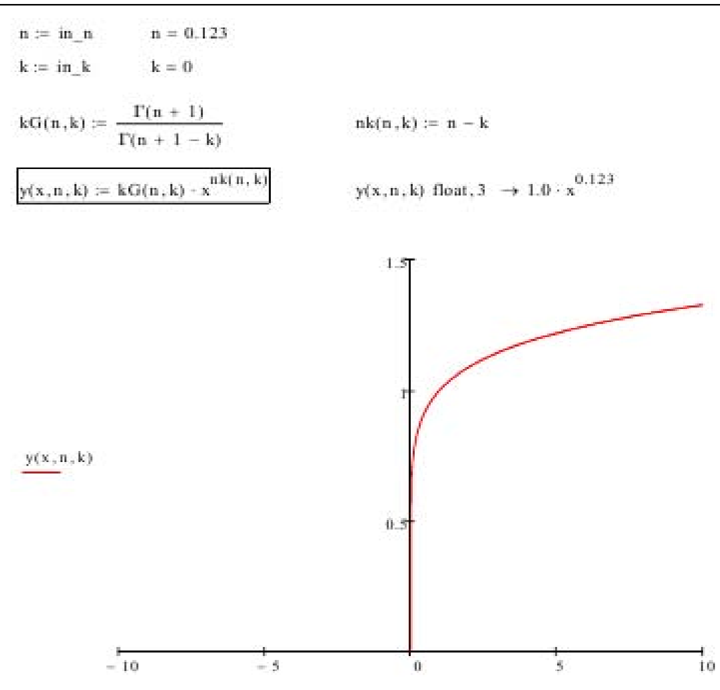
```
Option Explicit

Dim n, k As Double
Dim in_n, in_k As Double
Dim Message1, Title1, Default, MyValue
Dim Message2, Title2
Dim MathcadObj
Dim MCWSheet

Private Sub Form_Load ()
Form1.Enabled = True
Form1.Cls
Form1.Visible = False
Form1.Appearance = 0
Form1.WindowState = 2
Call nk
End Sub

Private Sub nk ()
Message1 = "Enter the degree <n> for the power function y = x^n"
Title1= "Default n =2"
Default= "2"
MyValue = InputBox (Message1, Title1, Default)
n = CDbl (MyValue)
'----------------------------------------------------
Message2 = ""Enter K. If K < 0, then it is an integral of multiplicity K, and if K > 0, then it is a derivative of order K"
k = InputBox (Message2, Title1, Default)
Call Gam
End Sub

Private Sub Gam()
'Setting a custom function
Set MathcadObj = OLE1.object
Set MCWSheet = MathcadObj.Worksheet
in_n = n
in_k = k
Call MathcadObj.setcomplex("in_n", n, 0)
Call MathcadObj.setcomplex("in_k", k, 0)
'Recalculating results in MathCad and getting a custom SLFunctions function
Call MathcadObj.Recalculate
'End of the program
Dim Msg, Style, Title, Response
Msg = "Continue? Yes"
Style = vbYesNo + vbCritical + vbDefaultButton2
Title = "The program has finished working. Viewing the result"
Response = MsgBox (Msg, Style, Title)
If Response = 6 Then Form1.Enabled = False
Set MathcadObj = Nothing
Set MCWSheet = Nothing
End
End Sub
```

Below, as an example, is a table (Table 1) with the results of calculating the differential functions on VBasic, where n is the exponent of the power function, and k is the parameter of the differential function. For k < 0 it is a fractional integral, k = 0 is the parent function, and for k > 0 it is a fractional derivative.

Table B.1: Values of functions $x^{0,123}$, x^2, $x^{12,3}$ and *sin(x)*

<table>
<tr><th>PDF File</th><th>n</th><th>k</th><th>Figure</th></tr>
<tr><td>(0,123) <-1,93>
Fractionalintegral 1.1

SLFunction n=0,123 k=-1,93.pdf</td><td>0,123</td><td>-1,93</td><td>
</td></tr>
<tr><td>0,123 <0>
Maternalfunction No. 1

SLFunction n=0,123 k=0.pdf</td><td>0,123</td><td>0</td><td>
</td></tr>
</table>

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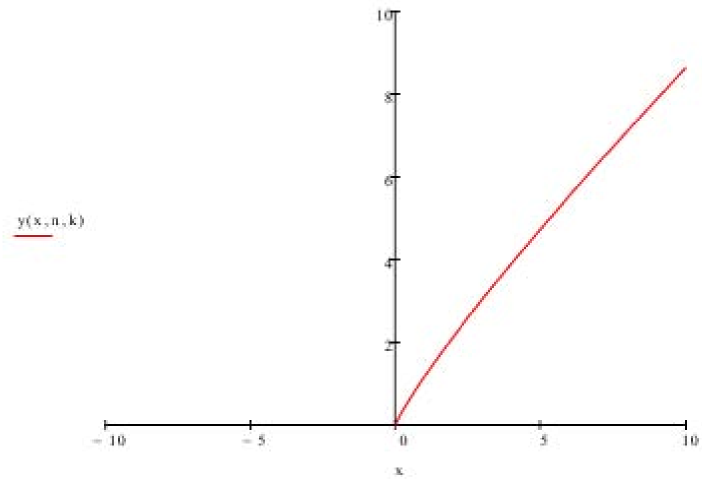
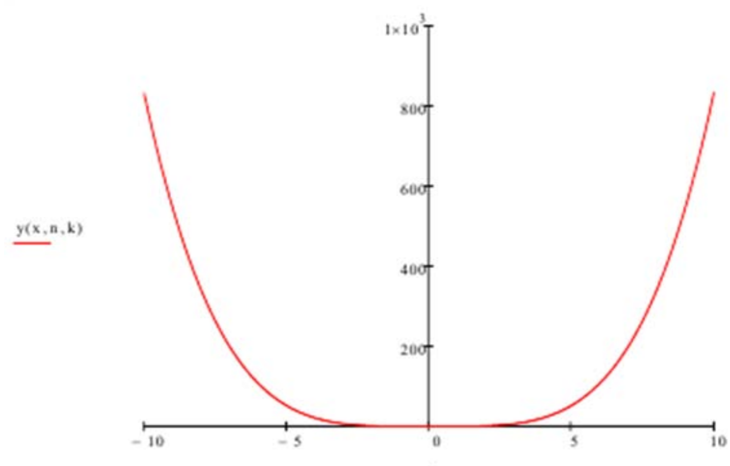
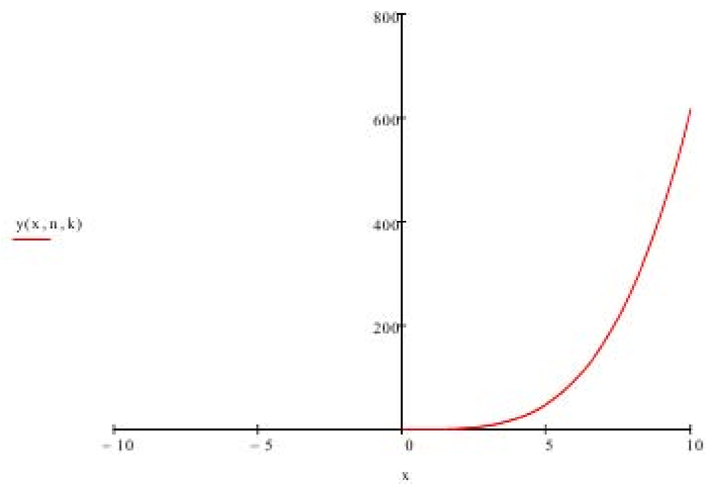
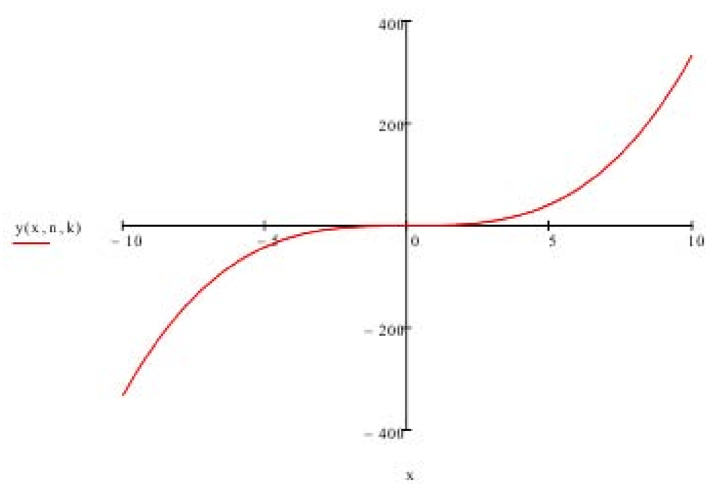
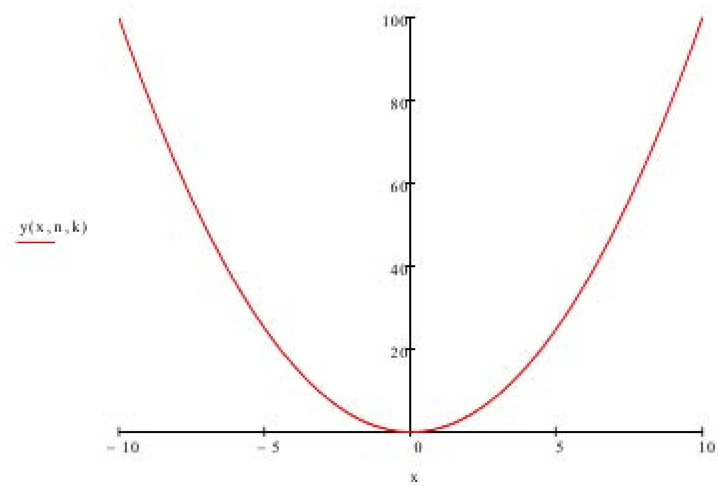
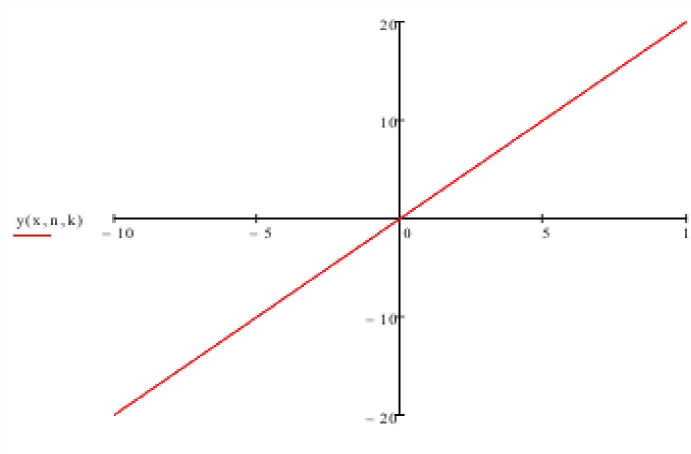
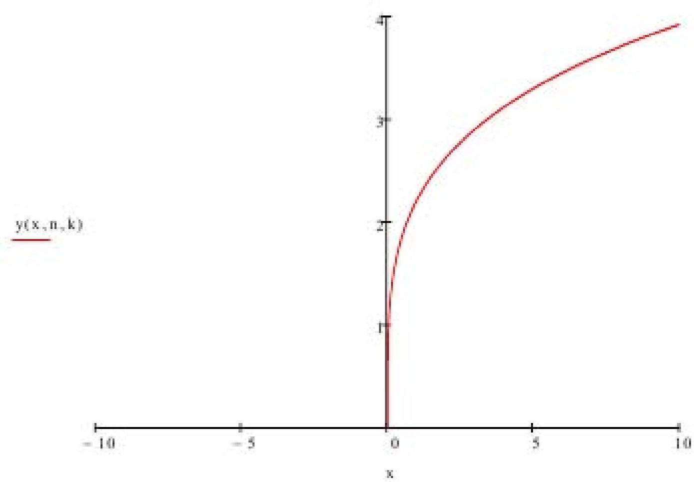
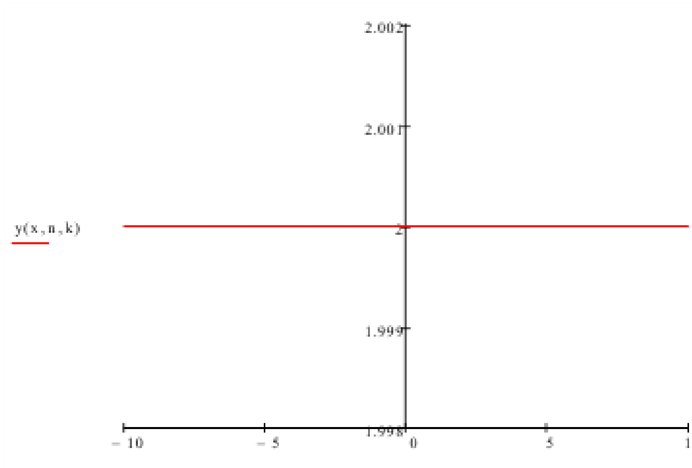
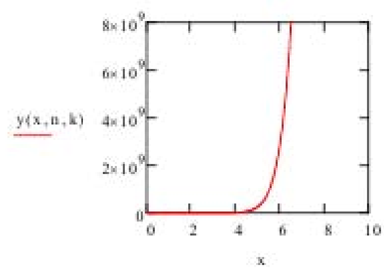
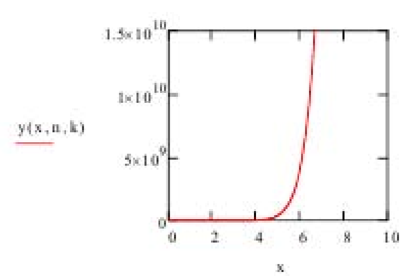
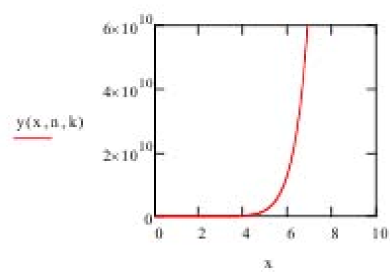
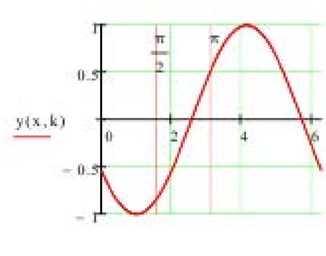
$(0{,}123)^{<0{,}37>}$ Fractionalderivative 1.1 SLFunction n=1,234 k=0,37.pdf	0,123	0,37	$n := in\_n \quad n = 1.234$ $k := in\_k \quad k = 0.37$ $kG(n,k) := \frac{\Gamma(n+1)}{\Gamma(n+1-k)} \qquad nk(n,k) := n - k$ $\boxed{y(x,n,k) := kG(n,k)\cdot x^{nk(n,k)}} \qquad y(x,n,k)\ \mathrm{float},3 \rightarrow 1.18\cdot x^{0.864}$ 
$2^{<-2>}$ The two-foldintegral 2.1 SLFunction n=2 k=-2.pdf	2	-2	$n := in\_n \quad n = 2$ $k := in\_k \quad k = -2$ $kG(n,k) := \frac{\Gamma(n+1)}{\Gamma(n+1-k)} \qquad nk(n,k) := n - k$ $\boxed{y(x,n,k) := kG(n,k)\cdot x^{nk(n,k)}} \qquad y(x,n,k)\ \mathrm{float},3 \rightarrow 0.0833\cdot x^{4}$ 

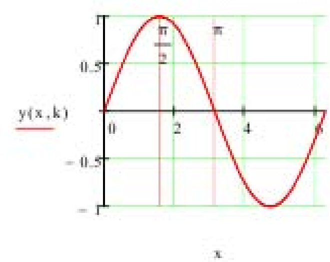
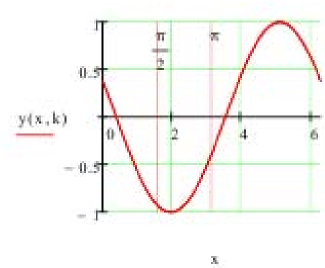
2 <-1,64> Fractionalintegral 2.2 SLFunction n=2 k=-1,64.pdf	2	-1,64	$n := in\_n \quad n = 2$ $k := in\_k \quad k = -1.64$ $kG(n,k) := \frac{\Gamma(n+1)}{\Gamma(n+1-k)} \quad nk(n,k) := n - k$ $\boxed{y(x,n,k) := kG(n,k)\cdot x^{nk(n,k)}} \quad y(x,n,k)\ \text{float},3 \ \rightarrow 0.141\cdot x^{3.64}$ 
2 <-1> Singleintegral 2.3 SLFunction n=2 k=-1.pdf	2	-1	$n := in\_n \quad n = 2$ $k := in\_k \quad k = -1$ $kG(n,k) := \frac{\Gamma(n+1)}{\Gamma(n+1-k)} \quad nk(n,k) := n - k$ $\boxed{y(x,n,k) := kG(n,k)\cdot x^{nk(n,k)}} \quad y(x,n,k)\ \text{float},3 \ \rightarrow 0.333\cdot x^{3}$ 

2 <0> Maternal functionNo.2 SLFunction n=2 k=0.pdf	2	0	$n := in\_n \quad n = 2$ $k := in\_k \quad k = 0$ $kG(n,k) := \frac{\Gamma(n+1)}{\Gamma(n+1-k)} \quad nk(n,k) := n - k$ $\boxed{y(x,n,k) := kG(n,k)\cdot x^{nk(n,k)}} \quad y(x,n,k)\ \mathrm{float},3\ \rightarrow x^{2}$ 
2 <1> The firstderivative 2.1 SLFunction n=2 k=1.pdf	2	1	$n := in\_n \quad n = 2$ $k := in\_k \quad k = 1$ $kG(n,k) := \frac{\Gamma(n+1)}{\Gamma(n+1-k)} \quad nk(n,k) := n - k$ $\boxed{y(x,n,k) := kG(n,k)\cdot x^{nk(n,k)}} \quad y(x,n,k)\ \mathrm{float},3\ \rightarrow 2.0\cdot x$ 

$2^{<1,75>}$ Fractionalderivative 2.2 SLFunction n=2 k=1,75.pdf	2	1,75	$n := in\_n$ $\quad n = 2$ $k := in\_k$ $\quad k = 1.75$ $kG(n,k) := \frac{\Gamma(n+1)}{\Gamma(n+1-k)}$ $\quad nk(n,k) := n - k$ $\boxed{y(x,n,k) := kG(n,k) \cdot x^{nk(n,k)}}$ $\quad y(x,n,k)$ float, 3 $\rightarrow 2.21 \cdot x^{0.25}$ 
$2^{<2>}$ The secondderivative 2.3 SLFunction n=2 k=2.pdf	2	2	$n := in\_n$ $\quad n = 2$ $k := in\_k$ $\quad k = 2$ $kG(n,k) := \frac{\Gamma(n+1)}{\Gamma(n+1-k)}$ $\quad nk(n,k) := n - k$ $\boxed{y(x,n,k) := kG(n,k) \cdot x^{nk(n,k)}}$ $\quad y(x,n,k)$ float, 3 $\rightarrow 2.0$ 

$(12{,}34)^{<-0{,}45>}$ Fractionalintegral 3.1 SL Fun 12,34 -0,45.xmcd	12,34	-0,45	Введите показатель степенной функции < n > $n := 12{,}34$ Введите порядок производной или интеграла < k > Для производной k > 0 для интеграла k < 0 $k := -0{,}45$ Дифферинтегральная Функция Y равна $y(x,n,k) := \frac{\Gamma(n+1)}{\Gamma(n+1-k)} \cdot x^{n-k}$ $y(x,n,k)\ \text{float},3 \rightarrow 0.315 \cdot x^{12.8}$ Вид этой функции представлен на графике 
$12{,}34^{<0>}$ Maternal functionNo.3 SL Fun 12,34 0.xmcd	12,34	0	Введите показатель степенной функции < n > $n := 12{,}34$ Введите порядок производной или интеграла < k > Для производной k > 0 для интеграла k < 0 $k := 0$ Дифферинтегральная Функция Y равна $y(x,n,k) := \frac{\Gamma(n+1)}{\Gamma(n+1-k)} \cdot x^{n-k}$ $y(x,n,k)\ \text{float},3 \rightarrow 1.0 \cdot x^{12.3}$ Вид этой функции представлен на графике 

$(12,34)^{<1,75>}$ Fractionalderivative 3.1 SL Fun 12,34 1,75.xmcd	12,34	1,75	Введите показатель степенной функции < n > $n := 12,34$ Введите порядок производной или интеграла < k > Для производной k > 0 для интеграла k < 0 $k := 1,75$ Дифферинтегральная Функция Y равна $y(x,n,k) := \frac{\Gamma(n+1)}{\Gamma(n+1-k)} \cdot x^{n-k}$ $y(x,n,k)$ float, 3 $\to 76.9 \cdot x^{10.6}$ Вид этой функции представлен на графике 
$\sin(x)^{<-1,64>}$ Fractionalintegral 4.1 sin x -1,64.pdf		-1,64	Введите порядок производной или интеграла < k > Для производной k > 0 для интеграла k < 0 $k := -1,64$ Дифферинтегральная Функция Y равна $y(x,k) := \sin\left(x + k \cdot \frac{\pi}{2}\right)$ $y(x,k)$ float, 3 $\to \sin(x - 2.58)$ Вид этой функции представлен на графике 

$\sin(x)^{<0>}$ Maternal function No.4 sin x 0.pdf		0	Введите порядок производной или интеграла < k > Для производной k > 0 для интеграла k < 0 $k := 0$ Дифферинтегральная Функция Y равна $y(x,k) := \sin\left(x + k\cdot\frac{\pi}{2}\right)$ $y(x,k)\ \text{float},3 \rightarrow \sin(x)$ Вид этой функции представлен на графике 
$\sin(x)^{<1,75>}$ Fractionalderivative 4.1 sin x 1,75.pdf		1,75	Введите порядок производной или интеграла < k > Для производной k > 0 для интеграла k < 0 $k := 1{,}75$ Дифферинтегральная Функция Y равна $y(x,k) := \sin\left(x + k\cdot\frac{\pi}{2}\right)$ $y(x,k)\ \text{float},3 \rightarrow \sin(x + 2.75)$ Вид этой функции представлен на графике 

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Investigating the Effects of Physical Parameters on First and Second Reflected Waves in Air-Saturated Porous Media under Low-Frequency Ultrasound Excitation

By Mustapha Sadouki & Abd El Madjid Mahiou
Khemis-Miliana University

Abstract- This simulation study investigates the impact of a 20% variation in physical parameters, including porosity, tortuosity, viscous and thermal characteristic lengths, and two newly introduced viscous and thermal shape factor parameters, on reflected waves at the first and second interfaces in air-saturated porous media under low-frequency ultrasound excitation. The acoustic behavior of air-saturated porous media is modeled using the equivalent fluid theory and the Johnson-Allard model, refined by Sadouki [Phys. Fluids 33, (2021)]. Our results demonstrate that a 20% variation in certain physical parameters significantly affects the reflected waves at the first and second interfaces in the low-frequency domain of ultrasound. This study enhances our understanding of the underlying mechanisms governing acoustic wave propagation in air-saturated porous media, which is valuable for optimizing ultrasound-based techniques in a range of applications, such as nondestructive testing, medical imaging, and noise pollution control in buildings, aircraft, automobile industry, and civil engineering sectors.

Keywords: *air-saturated porous media, ultrasound, physical parameters, reflected waves, simulation study, equivalent fluid theory, Johnson-Allard model.*

GJRE-I Classification: *LCC: TA705*

INVESTIGATINGTHEEFFECTSOFPHYSICALPARAMETERSONFIRSTANDSECONDREFLECTEDWAVESINAIRSATURATEDPOROUSMEDIAUNDERLOWFREQUENCYULTRASOUNDEXCITATION

Strictly as per the compliance and regulations of:

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Mustapha Sadouki [α] & Abd El Madjid Mahiou [σ]

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I. Introduction

Porous materials have a rich history that dates back to ancient times, and they continue to be of great importance in modern chemistry and materials science [1]. These materials exhibit unique properties that make them valuable across a wide range of applications, including biomedical, building and construction, aerospace, and environmental domains. Their diverse classifications, such as fibrous, granular agglomerates, polymeric, and construction materials, contribute to their widespread use in our daily lives.

In recent years, there has been a growing interest in the use of porous materials due to their versatility and unique properties. For example, in the biomedical field [2], porous materials have shown tremendous potential for drug delivery and tissue engineering. The porous structure of these materials allows for controlled drug release and promotes cell growth, making them ideal candidates for advanced medical applications. In the building and construction industry [3], porous materials are commonly used for insulation and soundproofing. Their ability to absorb sound waves through viscous friction and thermal exchanges makes them an excellent choice for reducing noise pollution. Similarly, in the aerospace industry[4], porous materials are used for thermal insulation and noise reduction.

The physical and mechanical parameters used to characterize the properties of porous materials include geometric tortuosity, viscous and thermal characteristic lengths [5-8], Young's modulus of elasticity, and Poisson's ratio. In the field of acoustics, porous materials are widely used to reduce noise pollution by absorbing a part of the sound waves through viscous friction and thermal exchanges [4]. Previous studies have been conducted to investigate the influence of physical parameters describing porous media on the transmitted signal in the low-frequency ultrasound regime [9-11]. However, there is a need for a more comprehensive numerical simulation study to determine the effect of physical parameters on the low-frequency ultrasonic signal reflected by the first and second interfaces of the medium.

In this study, we address this gap by investigating the impact of a 20% variation in physical parameters, including porosity, tortuosity, viscous and thermal characteristic lengths, and two newly introduced viscous and thermal shape factor parameters, on the reflected waves at the first and second interfaces in air-saturated porous media in the low-frequency domain of ultrasound. The acoustic behavior of air-saturated porous media is modeled using the equivalent fluid theory and the Johnson-Allard model refined by Sadouki [12]. This study enhances our understanding of the underlying mechanisms governing acoustic wave propagation in porous media, providing valuable insights for optimizing ultrasound-based techniques in a range of applications, such as nondestructive testing, medical imaging, and noise pollution control in

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Author σ: Acoustics and Civil Engineering Laboratory. Khemis-Miliana University. Ain Defla, Algeria. Theoretical Physics and Radiation Matter Interaction Laboratory, Soumaa, Blida, Algeria.

buildings, aircraft, automobile industries, and civil engineering.

II. Model

Acoustic propagation in porous materials is a complex phenomenon that involves the interaction of sound waves with fluid and solid components of the porous medium. When considering air-saturated porous materials with immobile solid skeletons, wave propagation is confined to the fluid, and this behavior is typically modeled using the equivalent fluid model [5,6], which is a particular case of Biot theory [13-15]. The two frequency response factors, the dynamic tortuosity of the medium $\alpha(\omega)$ and the dynamic compressibility of air in the porous material $\beta(\omega)$, are used to account for structure-fluid interactions. The dynamic tortuosity is provided by Johnson et al [5,6], while the dynamic compressibility is given by Allard [7]. In the frequency domain, these factors are multiplied by the density and compressibility of the fluid.

At extremely low and high frequencies, the equations governing the acoustic behavior of the fluid simplify and the parameters involved are different. In the high-frequency range [7], this simplification occurs when the viscous and thermal skin thicknesses $\delta(\omega) = \sqrt{\frac{2\eta}{\rho_0\omega}}$ and $\delta'(\omega) = \sqrt{\frac{2\eta}{P_r\rho_0\omega}}$ are smaller than the pore radius r. (Here, the density of the saturating fluid is represented by ρ_0, the viscosity by η, the pulse frequency by ω, and the Prandtl number by Pr). In the low-frequency range of the ultrasonic domain, the dynamic tortuosity and compressibility are given by [12]:

$$\alpha(\omega) = \alpha_\infty\left(1 + \frac{\delta(\omega)}{\Lambda}\left(\frac{2}{j}\right)^{\frac{1}{2}} + \xi\left(\frac{\delta(\omega)}{\Lambda}\right)^2\left(\frac{2}{j}\right) + \cdots\right) \tag{1}$$

$$\beta(\omega) = 1 + (\gamma - 1)\left(\frac{\delta'(\omega)}{\Lambda'}\left(\frac{2}{j}\right)^{1/2} + (\xi' - 1)\left(\frac{\delta'(\omega)}{\Lambda'}\right)^2\left(\frac{2}{j}\right) + \cdots\right) \tag{2}$$

where, $j = \sqrt{-1}$ and γ is the adiabatic constant.

The relevant physical parameters of the models are the high-frequency limit of the tortuosity α_∞, the viscous and thermal characteristic lengths Λ and Λ', respectively, and the dimensionless parameter ξ introduced by Sadouki [12], which is a shape factor related to the correction of the viscous skin depth of the air layer near the tube surface where the velocity distribution is significantly perturbed by the viscous forces generated by the stationary frame in the low-frequency ultrasonic regime. ξ' is the associated thermal counterpart.

Consider a homogeneous porous material that occupies the region $0 \le x \le L$. A sound pulse normally strikes the medium, generating an acoustic pressure field p(x,t) and an acoustic velocity field v(x,ω) within the material (Fig. 1). These fields satisfy the Euler equation and the constitutive equation along the x-axis:

$$\rho_0\alpha(\omega)j\omega\, v(x,\omega) = \frac{\partial p(x,\omega)}{\partial x}, \qquad \frac{\beta(\omega)}{K_a}j\omega\, p(x,\omega) = \frac{\partial v(x,\omega)}{\partial x}, \tag{3}$$

Here, K_a is the compressibility modulus of the fluid.

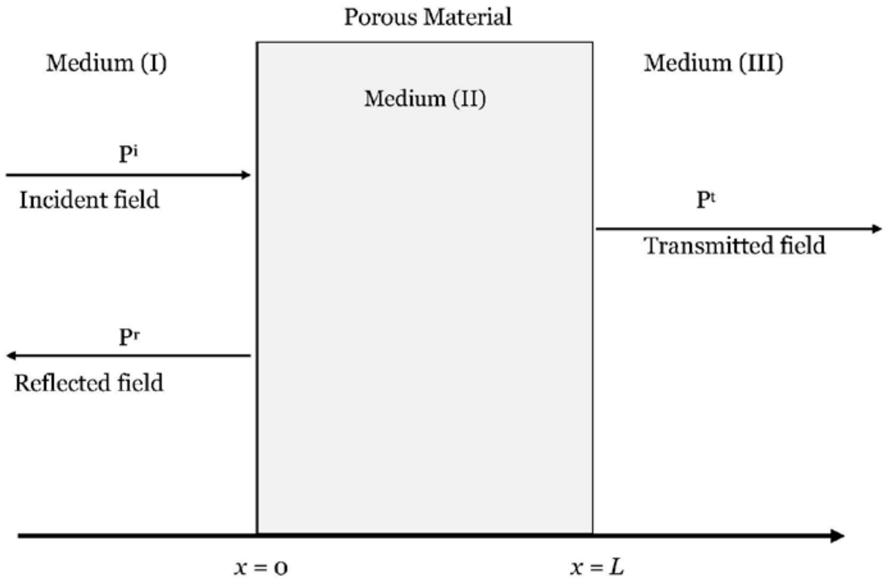

Figure 1: Problem geometry

The continuity of the pressure and velocity fields at the medium boundary gives the reflection coefficient of the porous material [17]:

$$R = \frac{(1-\tilde{Z}^2)\sinh(j\tilde{k}L)}{2\tilde{Z}\cosh(j\tilde{k}L)+(1+\tilde{Z}^2)\sinh(j\tilde{k}L)} \qquad (4)$$

Where $\tilde{Z} = \frac{1}{\phi}\sqrt{\frac{\alpha(\omega)}{\beta(\omega)}}$ is the normalized characteristic impedance of the material, ϕ is the porosity, and $\tilde{k} = \omega\sqrt{\frac{\rho_0\alpha(\omega)\beta(\omega)}{K_a}}$ is the wave number of the acoustic wave in the porous medium. The incident and reflected fields p^i and p^r_{sim} are related in the frequency domain by the reflection coefficient R:

$$p^r_{sim}(x,\omega) = R\, p^i(x,\omega) \qquad (5)$$

In the time domain, the reflected signal $p^r_{sim}(x,t)$ is obtained by taking the inverse Fourier transform of Eq. (5):

$$P^t(x,t) = \mathcal{F}^{-1}\left(R\, P^i(x,\omega)\right) \qquad (6)$$

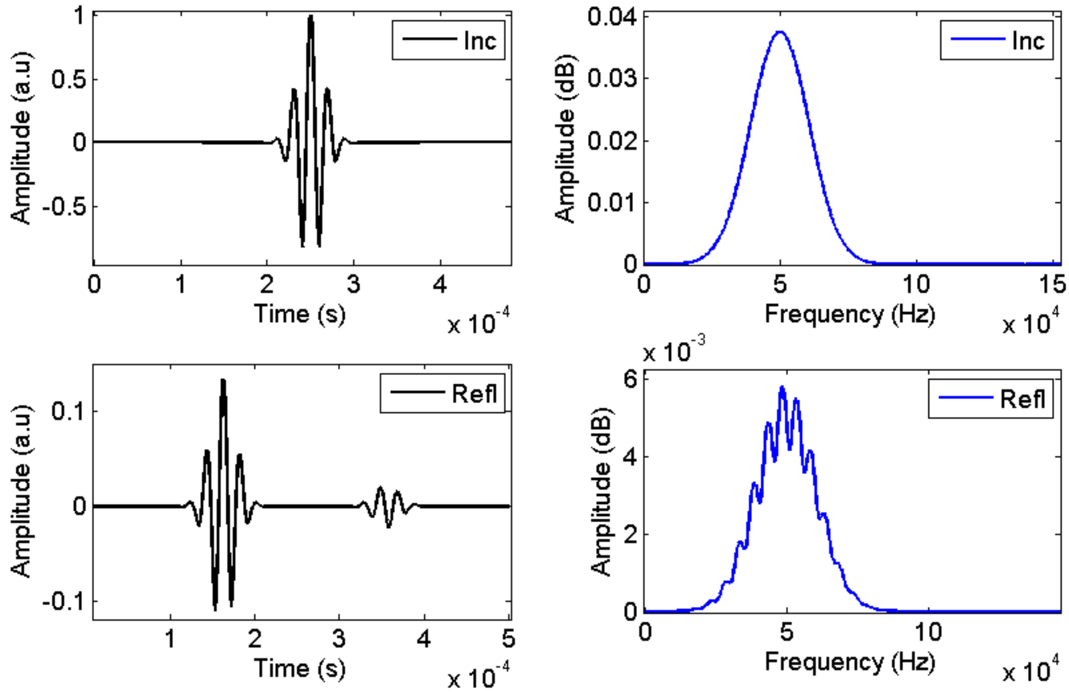

Figure 2: The incident and reflected signals of a monolayer porous medium constructed in frequency via expression (5) and in time via Eq. (6)

The simulated incident and reflected signals of a single-layer porous medium are shown in Fig. 2, and were obtained by expression (5) in the frequency domain and equation (6) in the time domain. The characteristic parameters of the porous medium are as follows: $L = 3.0$ cm, $\phi = 0.85$, $\alpha_\infty = 1.2$, $\Lambda = 300$ μm, $\Lambda'/\Lambda = 3$, $\xi = 10$, and $\xi/\xi' = 2$. These signals were generated using the Gauss function in Matlab with center frequencies of 50 kHz and 120 kHz. In the time domain, two successive reflections on the first and second interface can be clearly observed, as shown in black color below in Figure 2.

III. Simulation Study

To investigate the influence of physical parameters, such as porosity, tortuosity, viscous and thermal characteristic lengths, and newly introduced shape factors on the reflected waves, a parameter analysis was performed. Specifically, each parameter was varied while holding the others constant, and the impact on the first and second reflected waves in the time domain, as indicated by equation (6), was observed. By systematically varying each parameter and analyzing its effect on the reflected waves, we can better understand the individual contributions of these physical factors to the overall acoustic behavior of the porous material.

a) *Effect of Porosity* $\boldsymbol{\phi}$ *on the Reflected Signal*

Figure 3 shows the impact of varying the porosity (ϕ) on the amplitude of the first and second reflected waves through a rigid porous medium, while keeping the other parameters fixed at $\alpha_\infty = 1.2$, $\Lambda = 300$ μm, $\Lambda'/\Lambda = 3$, $\xi = 10$, and $\xi/\xi' = 2$. The porosity ϕ varies from +20% to -20% of its initial value ($\phi = 0.85$). Table 1 presents the variation ratio of the reflection coefficient compared to a ±20% variation of each parameter.

According to Table 1, a significant influence of porosity on the reflected signal is observed at frequencies of 50 kHz and 120 kHz. When the porosity increases by +20%, the modulus of the first and second reflected signals decrease by -66.84% and -65.06%, respectively. Conversely, when the porosity decreases by -20%, the amplitude of the 1st and 2nd reflected signals increases by +80.26% and +71.32% respectively. Moreover, the sensitivity of the porosity ϕ increases with frequency, as also shown in Table 1.

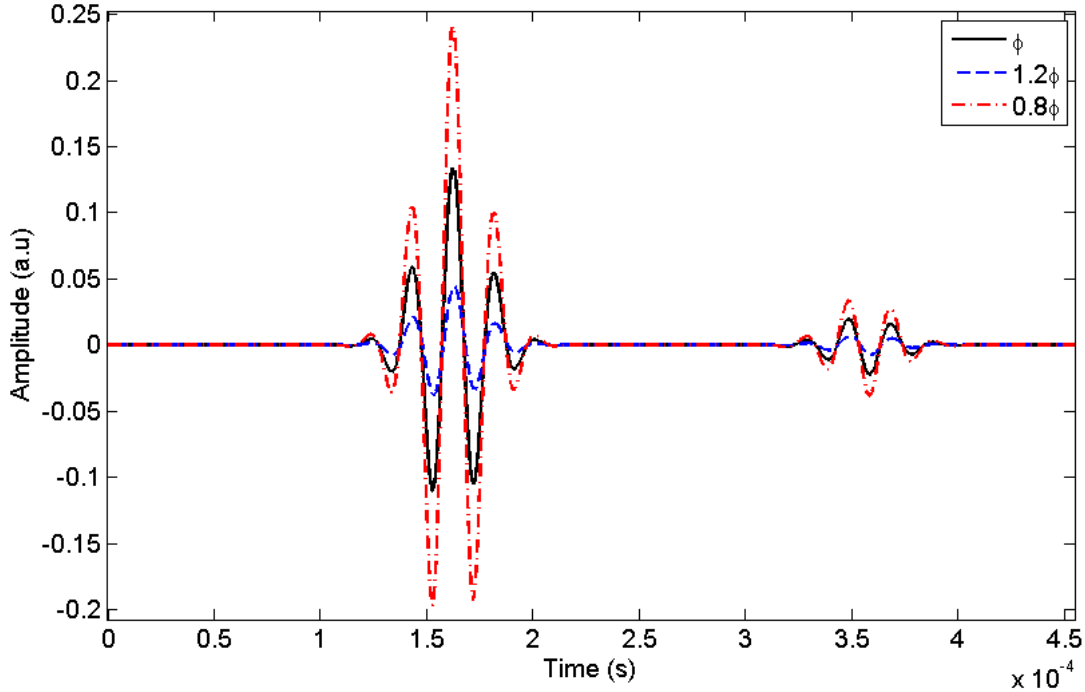

Figure 3: Sensitivity of porosity ϕ on the 1st and 2nd reflected signals at 50 kHz

b) *Effect of Tortuosity $\boldsymbol{\alpha}_{\infty}$ on the Reflected Signal*

Figure 4 illustrates the sensitivity of tortuosity $\boldsymbol{\alpha}_{\infty}$ on the 1st and 2nd reflected waves, for an excitation pulse of frequency 50 kHz. When the initial tortuosity value is increased by +20%, the amplitude of the 1st and 2nd reflected waves increases by +33.16% and 10.82%, respectively. Conversely, a decrease of -20% in tortuosity results in a decrease in the amplitude of the 1st and 2nd reflected waves by -40.98% and -27.47%, respectively. Notably, the impact of tortuosity is more pronounced on the 1st reflected signal than on the 2nd. Additionally, the sensitivity of tortuosity to the reflected signal increases with frequency, as detailed in Table 1.

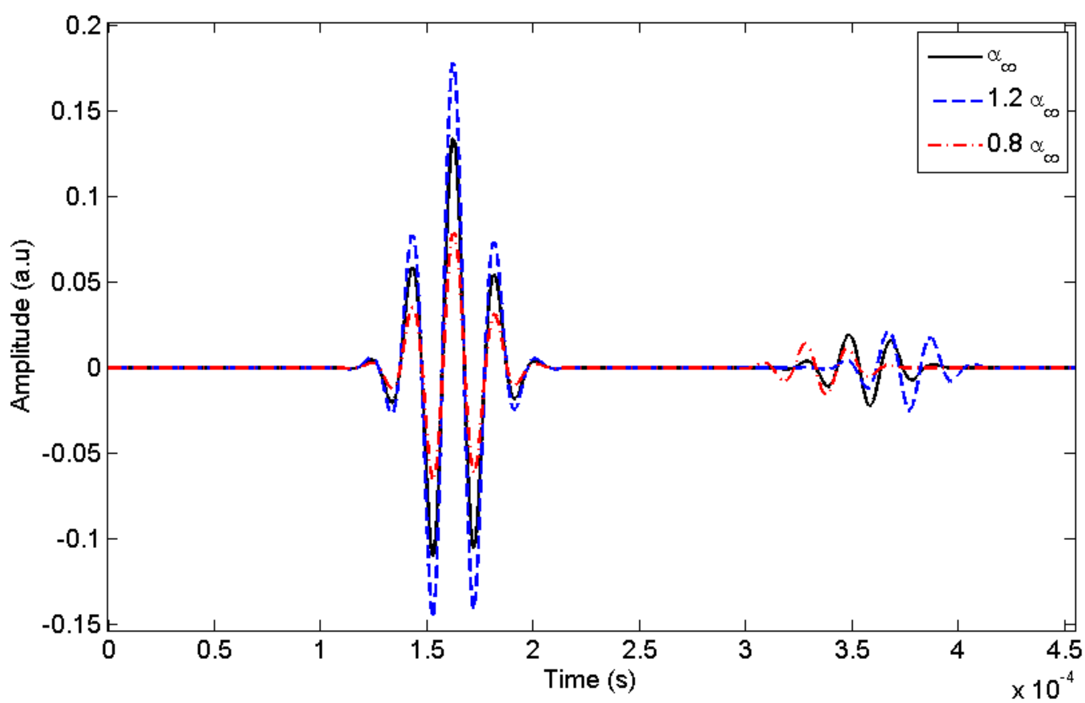

Figure 4: Sensitivity of Tortuosity $\boldsymbol{\alpha}_{\infty}$ on the 1st and 2nd Reflected Waves

c) Effect of Viscous Characteristic Length Λ *on the Reflected Waves*

The impact of varying the viscous characteristic length on the 1st and 2nd reflected waves at high frequency is shown in Figure 5. With an excitation frequency of 50 kHz, a +20% change in Λ results in a -1.07% decrease in the amplitude of the first reflected wave and a 42.76% increase in the amplitude of the second reflected signal. Furthermore, as the frequency increases, the sensitivity of the viscous characteristic length decreases for the 1st reflected wave and increases for the 2nd reflected wave. Therefore, we can conclude that the viscous characteristic length has a relatively small influence on the 1st reflected wave at high frequency but a high sensitivity on the 2nd reflected wave.

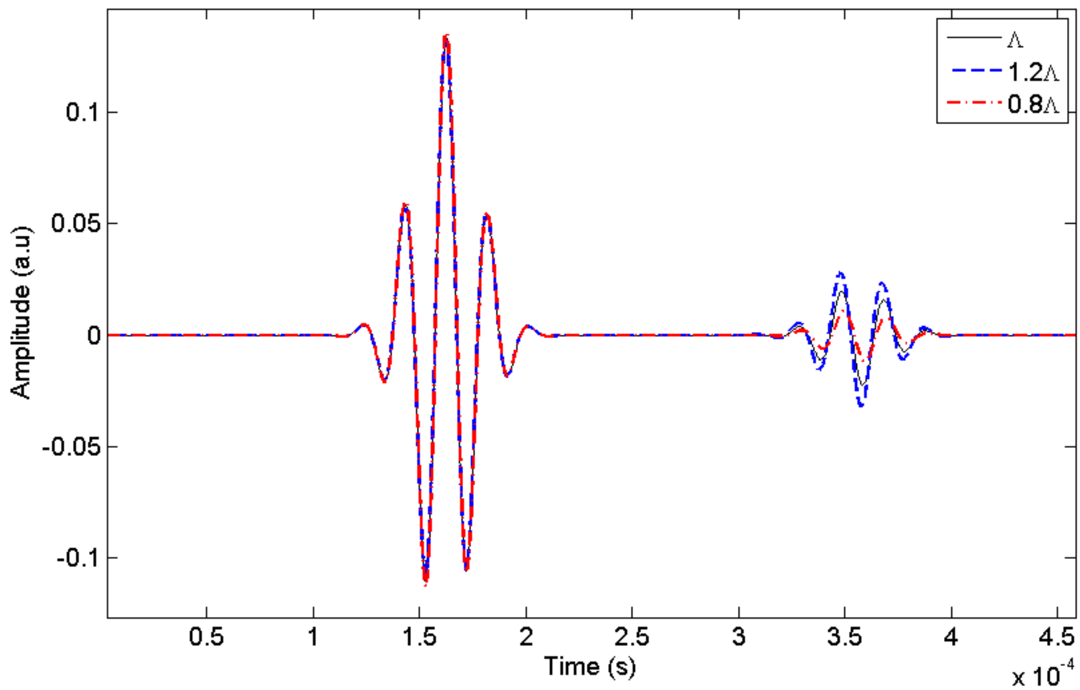

Figure 5: The sensitivity of the viscous characteristic length on the 1st and 2nd reflected waves at a frequency of 50 kHz

d) Effect of Thermal Characteristic Length Λ' *on the 1st and 2nd Reflected Waves*

The sensitivity of the thermal characteristic length Λ' on the two reflected waves at low ultrasonic frequency is shown in Figure 5 for a variation from +20% to -20% of its initial value. From Figure 6, we can see that for a frequency of 50 kHz, there is very little influence of the thermal characteristic length on the 1st reflected signal. An increase of +20% in Λ' results in a 0.17% increase in the modulus of the 1st reflected signal, while a variation of -20% results in a -0.26% decrease in the amplitude of the 1st reflected signal. However, the second reflected wave is more sensitive than the first. For a variation of +20% of Λ', the amplitude of the 2nd reflected wave increases by 3.29%. Moreover, according to Table 1, we observe that the sensitivity of the thermal characteristic length slightly decreases with frequency for the first reflection, while it increases for the second reflected wave.

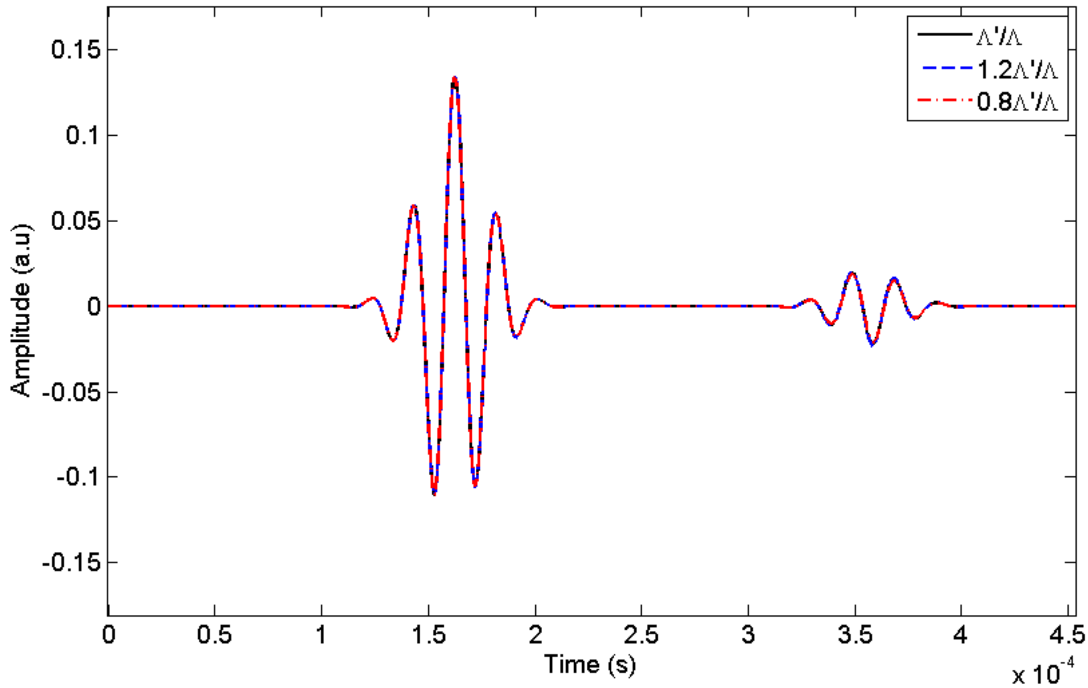

Figure 6: The sensitivity of the thermal characteristic length on the 1st and 2nd reflected waves for a frequency of 50 kHz

e) Effect of Viscous Shape Factor $\boldsymbol{\xi}$ *on the Reflected Signal*

The sensitivity of the shape factor ξ on the 1st and 2nd reflected waves in the Low-Frequency Ultrasound regime is presented in Figure 7. According to this figure, a -20% variation of ξ results in a regression of -0.26% and -4.89% in the amplitude of the 1st and 2nd reflected waves, respectively. For a variation of +20% of ξ, we observe a growth of +0.17% and 3.29% in the amplitude of the 1st and 2nd reflected waves. It can be concluded that the shape factor ξ has a weak influence on the first reflected wave but a strong sensitivity on the second wave in the low-frequency range of ultrasound. Moreover, the variation decreases for the first reflected wave and increases for the second wave as the frequency increases.

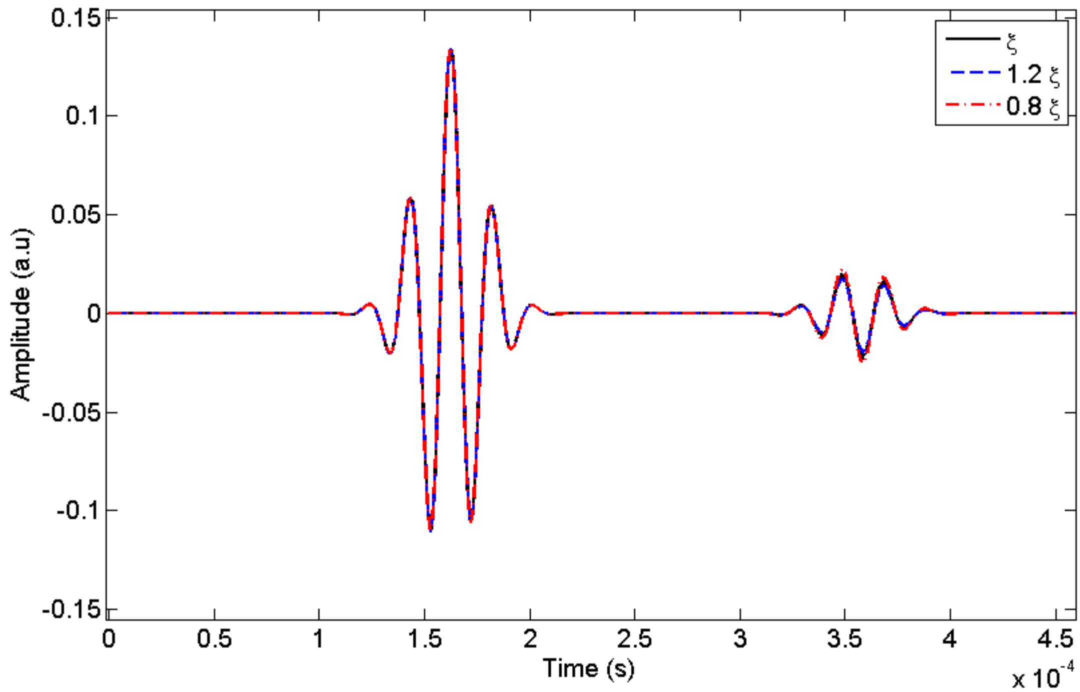

Figure 7: Sensitivity of the shape factor ξ on the 1st and 2nd reflected waves in the low-frequency ultrasound regime

f) Effect of Thermal Shape Factor ξ' on the Reflected Signal

To investigate the impact of varying the thermal shape factor ξ' on the 1st and 2nd high-frequency reflected waves, Figure 8 is presented. At an excitation frequency of 50 kHz, a +20% variation in ξ/ξ' results in a +0.10% increase and a -1.91% attenuation in the amplitude of the 1st and 2nd reflected waves, respectively. This parameter exhibits a weak influence on the 1st reflected wave but a more significant effect on the 2nd reflected wave in the low-frequency ultrasound regime. Table 1 summarizes the effects of porosity, tortuosity, viscous and thermal characteristic lengths, as well as the two shape factors on the 1st and 2nd reflected waves in the low-frequency regime of ultrasound (50-120 kHz).

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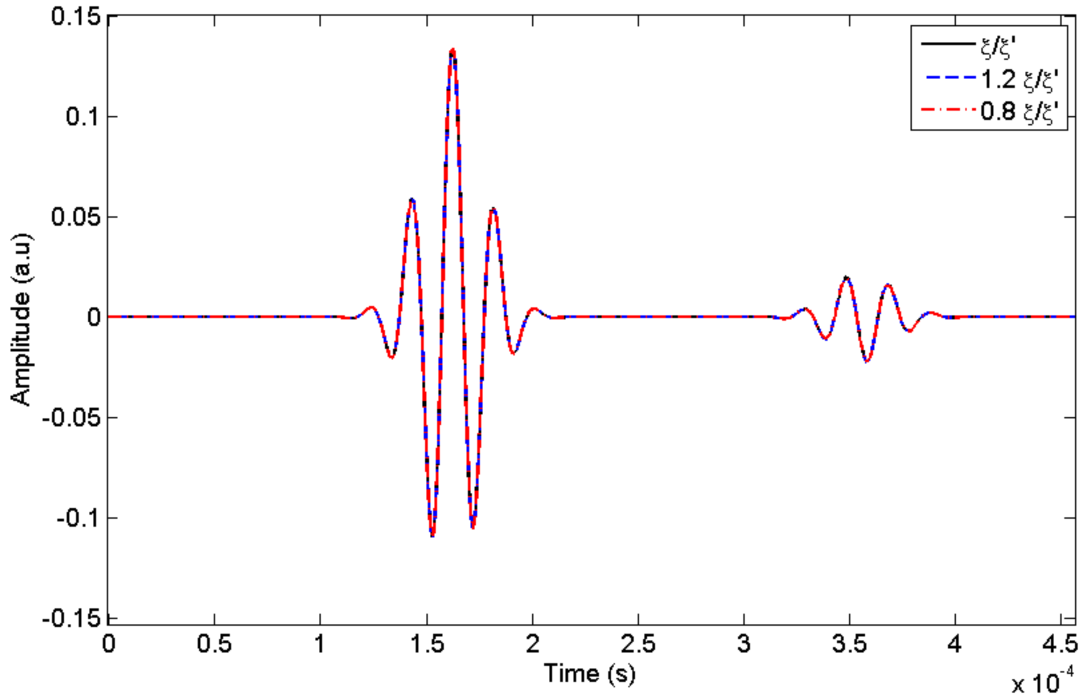

Figure 8: Sensitivity of thermal shape factor ξ' on the 1st and 2nd high-frequency reflected waves at an excitation frequency of 50 kHz

Based on the results presented in Table 1 and Figures 3-8, we can classify the sensitivity of each parameter on the reflected signal in the order of decreasing influence as presented in Table 2.

Table 1: Relative variation of the reflection coefficient $\frac{\Delta R}{R}$% corresponding to a variation of ± 20% of each physical parameter

		$\frac{\Delta R}{R}$ %			
		1st reflected wave		2nd reflected wave	
Parameters	Variations	50 kHz	120 kHz	50 kHz	120 kHz
Porosity φ	+20%	-66.84	-68.88	-65.06	-67.66
	-20%	80.26	82.30	71.32	74.12
Tortuosity α_∞	+20%	33.16	34.02	10.82	5.82
	-20%	-40.98	-42.10	-27.47	-24.77
Viscous characteristic length Λ (µm)	+20%	-1.07	-0.61	42.76	58.54
	-20%	1.77	0.93	-44.86	-52.95
Ratio thermal-viscous characteristic lengths (Λ'/Λ)	+20%	0.17	0.11	3.29	4.63
	-20%	-0.26	-0.16	-4.89	-6.57
Viscous shape factor ξ	+20%	0.10	0.19	-11.91	-12.16
	-20%	-0.97	-0.18	13.56	13.88
Ratio viscous-thermal shape factors ξ/ξ'	+20%	0.16	0.25	0.35	0.34
	-20%	-0.25	-0.38	-0.52	-0.51

Table 2: Classification of the sensitivity of each parameter on the 1st and 2nd reflected signal

Parameters	ϕ	α_∞	Λ	ξ	Λ'	ξ'
Influence on the 1st reflected signal	+++	++	+	~	~~	~~~
Influence on the 2nd reflected signal	+++	+	++	+	~	~~

+: Considerable
~: Weak

IV. Conclusion

In conclusion, this study investigated the influence of physical parameters on the reflected wave at the 1st and 2nd interface of rigid porous media in the low-frequency ultrasound regime. The results show that porosity and tortuosity are the most influential parameters affecting the two reflected signals. This influence varies proportionally with the frequency and inversely with the porosity for both the 1st and 2nd reflected waves. For the 1st reflected wave, the influence varies proportionally with the tortuosity and frequency, while for the 2nd reflected wave; it varies proportionally with the tortuosity and inversely with the frequency. However, the impact of porosity and tortuosity on the 1st reflected wave is greater than on the 2nd reflected wave. Moreover, the viscous characteristic length has a small effect on the 1st reflected wave but a substantial influence on the 2nd reflected wave, exceeding the impact of tortuosity. On the other hand, the shape factor has a minor impact on the 1st reflected wave and a significant sensitivity on the 2nd reflection. Concerning the thermal parameters, the thermal characteristic length and the thermal shape factor have a negligible impact on the 1st reflected wave, while the sensitivity of the thermal characteristic length on the 2nd reflected wave is considerable.

The study's strength is that it analyzed the two reflected waves separately and independently, which allows us to treat each wave individually. These results could have important implications for the design and optimization of ultrasound-based techniques in various applications such as medical imaging, non-destructive testing, and materials characterization. However, further research may be necessary to investigate the effect of these parameters in other frequency ranges and porous medium structures.

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Exploring Finite-Time Singularities and Onsager's Conjecture with Endpoint Regularity in the Periodic Navier Stokes Equations

By T. E. Moschandreou

Abstract- It has recently been proposed by the author of the present work that the periodic NS equations (PNS) with high energy assumption can breakdown in finite time but with sufficient low energy scaling the equations may not exhibit finite time blowup. This article gives a general model using specific periodic special functions, that is degenerate elliptic Weierstrass P functions whose presence in the governing equations through the forcing terms simplify the PNS equations at the centers of cells of the 3-Torus. Satisfying a divergence free vector field and periodic boundary conditions respectively with a general spatio-temporal forcing term $\boldsymbol{f}$ which is smooth and spatially periodic, the existence of solutions which blowup in finite time for PNS can occur starting with the first derivative and higher with respect to time. P. Isett (2016) has shown that the conservation of energy fails for the 3D incompressible Euler flows with Hölder regularity below 1/3. (Onsager's second conjecture) The endpoint regularity in Onsager's conjecture is addressed, and it is found that conservation of energy occurs when the Hölder regularity is exactly 1/3. The endpoint regularity problem has important connections with turbulence theory. Finally very recent developed new governing equations of fluid mechanics are proposed to have no finite time singularities.

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EXPLORINGFINITETIMESINGULARITIESANDONSAGERSCONJECTUREWITHENDPOINTREGULARITYINTHEPERIODICNAVIERSTOKESEQUATIONS

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I. Introduction to the Periodic Navier Stokes Equations

The Navier–Stokes equations are useful because they describe the physics of many phenomena of scientific and engineering interest. They may be used to model the weather, ocean currents, pipe flows and heat exchangers and air flow around a wing. The Navier–Stokes equations, in their full and simplified forms, help with the design of aircraft and automobiles, hemodynamics, the design of power stations, the analysis of pollution and fuel emissions and many other things.

In 1845, Stokes had derived the equation of motion of a viscous flow by adding Newtonian viscous terms and finalized the Navier–Stokes equations, which have now been used for almost two centuries. There are only a few studies to find how to understand the physical meaning of the viscous terms in NS equations. As is well known, Stokes had three assumptions: 1. The force on fluids is the stationary pressure when the flow is stationary. 2. Fluid viscosity is isotropic. 3. Fluid flow follows Newton's law that fluid stress and strain have linear relations. These assumptions lead to the NSE. In [1], since the regular NS equations are quite demanding in computational time and resources the vorticity part is considered as the only source of fluid stress for the purpose of computation cost reduction. In fact, fluid shear stress is contributed by both strain and vorticity. In mathematics, the computation of stress can be performed by strain only, vorticity only, or both. The computational results are exactly the same. The NSE equation adopts strain, which is symmetric and stress based on Stokes's assumption. In [1], a new governing equation which is based on a new assumption that accepts that fluid stress has a linear relation with vorticity, which is anti-symmetric. According to the mathematical analysis, the new governing equation is identical to NS equations in numerical analysis, but in a physical sense, the new governing equation is just the opposite to NSEs as it assumes that fluid stress is proportional to vorticity, where both are anti-symmetric, but not strain, contrary to Stokes's assumption and the current NSE.

Although both NSEs and the new governing equation in [1] lead to the same computational results for laminar flow, the new governing equation has several advantages: 1. The vorticity tensor is anti-symmetric, which has three elements, but NSEs use the strain tensor, which has six elements. It is shown that the computational cost is reduced to half for the viscous term. 2. The anti-symmetric matrix is independent of the coordinate system change or Galilean invariant, but the symmetric matrix that NSE uses is not. 3. The physical meaning is clear that the viscous term is generated by vorticity, not by strain only. 4. The viscosity is obtained by experiments, which are based on vorticity but not strain, since both strain and stress are hard to measure experimentally. 5. Vorticity can be further decomposed to rigid rotation and pure anti-symmetric shear, which is very useful for further study turbulent flow. However, the NS equation has no vorticity term, which is an impediment for further turbulence research. [ref [27] in [1]] studied the mechanism of turbulence generation and concluded that shear instability and transformation from shear to rotation are the paths of flow transition from laminar flow to turbulent flow. Using Liutex and the third generation of vortex identification methods, a lot of new physics has been found (see Dong et al., Liu et al., and Xu

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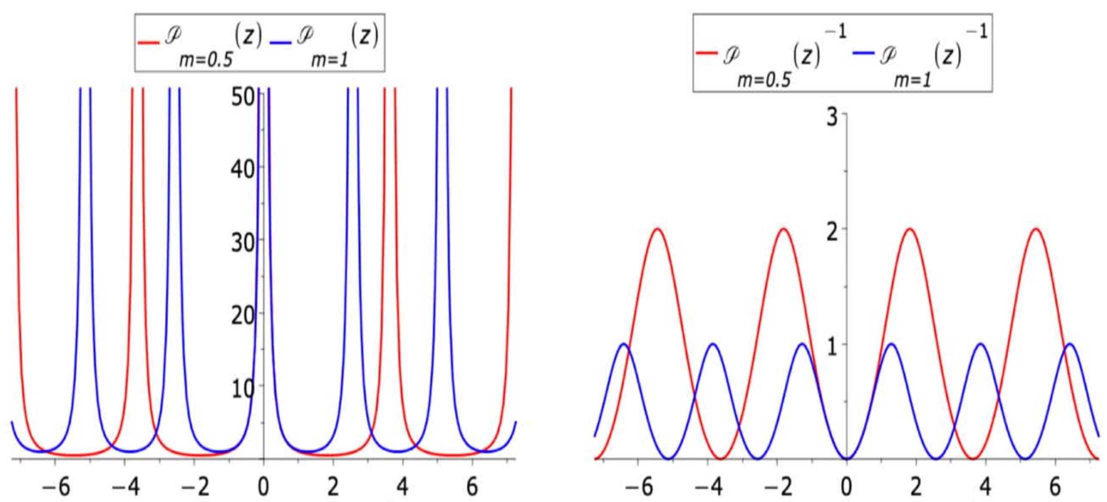
et al. references 24-26 in [1]) In Ref.28 in [1], Zhou et al. elaborated the hydrodynamic instability induced turbulent mixing in wide areas, including inertial confinement fusion, supernovae, and their transition criteria. Since the new governing equation has a vorticity term, which can be further decomposed to shear and rigid rotation, the new governing equation would be helpful in studying flow instability and transition to turbulence. Turbulence is rotational and characterized by large fluctuations in vorticity and thus it is important to accurately define vorticity. In the vorticity equation the vortex stretching term can be argued to be one of the most important mechanisms in the turbulence dynamics. It represents the enhancement of vorticity by stretching and is the mechanism by which the turbulent energy is transferred to smaller scales.

The purpose of this article is to refer to the periodic NS equations with high energy assumption as in the case of the continuum hypothesis being valid and can breakdown in finite time but with sufficient low energy scaling as in a fractal setting like for example on a Cantor set, the equations may not exhibit finite time blowup. It is known recently in the literature that the Cantor set with layers N (N can have up to two orders of magnitude) can be presented as a potential contender (analytical framework) for connecting the energy in a molecular level say C_1 at some cutoff length scale l_{cut} to the energy at a continuum level C_N with length scale L. The equipartition theorem of statistical mechanics has been used (Terrence Tao 2015) to relate the energy of a discrete block in say C_1 (molecular scale) to the energy in C_N (continuum scale). Additionally it has been shown that the ratio of the energy of the continuum scale to the molecular scale is a factor of 2^N. It then makes intuitive sense that the high energy PNS problem may breakdown in finite time. This article gives a general model using specific periodic special functions, that is degenerate elliptic Weierstrass P functions. See Figure 1.



(a) Weierstrass degenerate $\mathcal{P}$ function at $m = 0.5$ (red) and $m = 1.0$ (blue).

(b) Reciprocal Weierstrass degenerate $\mathcal{P}$ function at $m = 0.5$ (red) and $m = 1.0$ (blue).

Figure 1: Plots of the Degenerate Weierstrass P functions, given relative to the canonical Weierstrass P functions $\wp(z, g2, g3)$ as $\mathcal{P}_{m=n}(z) = \wp(z, 3n^2, n^3)$. Here, displayed as the standard function (Fig. 1a), and its reciprocal (Fig. 1b).

The definition of vorticity should be as defined in [1], which is that vorticity is a rotational part added to the sum of antisymmetric shear and compression and stretching. A vortex is recognized as the rotational motion of fluids. Within the last several decades, a lot of vortex identification methods have been developed to track the vortical structure in a fluid flow; however, we still lack unambiguous and universally accepted vortex identification criteria. It has been uncovered that the regions of strong vorticity and actual vortices are weakly related. It recently [1] has been concluded that a vorticity vector does not only represent rotation but also claims shearing and stretching components to be a part of the vortical structure, which is contaminated by shears in fluid. Satisfying a divergence free vector field and periodic boundary conditions respectively with a general spatio-temporal forcing term $f(x,t)$) which is smooth and spatially periodic, the existence of solutions of PNS which blowup in finite time can occur starting with the first derivative and higher with respect to time. On the other hand if u_0 is not smooth, then there exist globally in time solutions on $t \in [0, \infty)$ with a possible blowup at $t = \infty$. The control of turbulence is possible to maintain when the initial conditions and boundary conditions are posed properly for (PNS) ([5]). The endpoint regularity in Onsager's conjecture is addressed, and it is found that conservation of energy occurs when the Hölder regularity is exactly 1/3. Finally it is proposed that the periodic Liutex new equations[1] (The new equations referred to previously) do not exhibit finite time blow up. This is the focus of the ongoing work of the author to be presented in the near future.

II. Materials and Methods

Consider the incompressible 3D Navier Stokes equations defined on the three-Torus $\mathbb{T}^3 = \mathbb{R}^3/_{\mathbb{Z}^3}$. The periodic Navier Stokes system is,

$$(PNS)\begin{cases} \partial_t u - \Delta u + u\cdot\nabla u = -\nabla p + f \\ \operatorname{div}\, u = 0 \\ u_{t=0} = u_0. \end{cases}$$

where $u = u(x,y,z,t)$ is velocity, $p = p(x,y,z,t)$ is pressure and $f = f(x,y,z,t)$ is forcing vector. Here $u = (u_x, u_y, u_z)$, where u_x, u_y, and u_z denote respectively the x, y and z components of velocity.

Introducing Poisson's Equation (see [2], [3] and [5]), the second derivative P_{zz} is set equal to the second derivative obtained in the $\mathcal{G}_{\delta 1}$ expression further below, as part of $\mathcal{G}$, and

$$P_{zz} = -2u_z\nabla^2 u_z - \left(\frac{\partial u_z}{\partial z}\right)^2 + \frac{1}{\eta}\frac{\partial}{\partial z}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_z}{\partial y}\right) - \delta u_x\frac{\partial^2 u_z}{\partial z\,\partial x} - \delta u_y\frac{\partial^2 u_z}{\partial z\,\partial y} + \left(\frac{\partial u_x}{\partial x}\right)^2 + 2\frac{\partial u_x}{\partial y}\frac{\partial u_y}{\partial x} + \left(\frac{\partial u_y}{\partial y}\right)^2$$

where the last three terms on rhs can be shown to be equal to $-(P_{xx} + P_{yy})$. [4] Along with Equations below the continuity equation in Cartesian coordinates, is $\nabla^i u_i = 0$. The one parameter group of transformations on a critical space of PNS is given as,

Let
$$u_x = \frac{u_x^*}{\delta};\ u_y = \frac{u_y^*}{\delta};\ u_z = \frac{u_z^*}{\delta};\ P = \frac{P^*}{\delta^2}$$

$$x = x^*\delta;\ y = y^*\delta;\ z = z^*\delta;\ t = t^*\delta^2,$$

$$\frac{\partial}{\partial x} = \delta^{-1}\frac{\partial}{\partial x^*};\ \frac{\partial}{\partial y} = \delta^{-1}\frac{\partial}{\partial y^*};\ \frac{\partial}{\partial z} = \delta^{-1}\frac{\partial}{\partial z^*},\ \frac{\partial}{\partial t} = \delta^{-2}\frac{\partial}{\partial t^*}.$$

Furthermore the right hand side of the one parameter group of transformations are next mapped to η variable terms, (note that η and δ are not assumed to be arbitrarily small, they can be at most order one),

$$u_i^* = \frac{1}{\eta}v_i, P^* = \frac{1}{\eta^2}Q, x_i^* = \eta y_i, t^* = \eta^2 s, i = 1,2,3.$$

The double transformation is used for notational clarity. Note that the original Navier Stokes equations are preserved and simply rearranged in the following forms and Navier Stokes Equations become,

$$\mathcal{G}(\eta) = \mathcal{G}(\eta)_{\delta 1} + \mathcal{G}(\eta)_{\delta 2} + \mathcal{G}(\eta)_{\delta 3} + \mathcal{G}(\eta)_{\delta 4} = 0$$

where

$$\mathcal{G}(\eta)_{\delta 1} = \frac{1}{\eta^6}\left[(\delta^{-1}-1)\left(\frac{\partial v_3}{\partial s}\right)^2 + \frac{\mu\left(\frac{\partial v_3}{\partial s}\right)\left(\frac{\partial^2 v_3}{\partial y_1^2} + \frac{\partial^2 v_3}{\partial y_2^2} + \frac{\partial^2 v_3}{\partial y_3^2}\right)}{\rho}(1-\delta^{-1}) + \frac{(\delta^{-1}-1)\left(\frac{\partial v_3}{\partial s}\right)\frac{\partial Q}{\partial y_3}}{\rho}\right]$$

$$\mathcal{G}(\eta)_{\delta 2} = \frac{v_3}{\eta^6}\left(\frac{\partial v_3}{\partial y_3}\right)\frac{\partial v_3}{\partial s} + \frac{(v_3)^2}{\eta^6}\frac{\partial^2 v_3}{\partial y_3\,\partial s} + \frac{2\left(\frac{\partial v_1}{\partial s}\right)v_3\frac{\partial v_3}{\partial y_1} + 2\left(\frac{\partial v_2}{\partial s}\right)v_3\frac{\partial v_3}{\partial y_2} + 2\left(\frac{\partial v_3}{\partial s}\right)v_3\frac{\partial v_3}{\partial y_3}}{\delta\eta^6}$$

$$\mathcal{G}(\eta)_{\delta 3} = \frac{1}{\eta^3}\times\left[\iint_S\left(\frac{1}{\delta\rho}v_3^2\nabla_{y_1 y_2}Q + \frac{1}{\delta}\vec{v}\frac{1}{\rho}v_3\frac{\partial Q}{\partial y_3}\right)\cdot\vec{n}\,dS - \int_\Omega\frac{\left\|\frac{\partial v_3}{\partial s}\vec{b}\cdot(\vec{b}\otimes\nabla v_3)\right\|}{\|\vec{b}\|}dV\right]$$

$$\mathcal{G}(\eta)_{\delta 4}=\frac{1}{\eta^{3}}\left[\delta^{2}\overrightarrow{F_{T}}\cdot\nabla_{y_{1}y_{2}}v_{3}^{2}-\delta^{3}v_{3}\frac{\partial v_{3}}{\partial y_{3}}F_{z}+\delta^{2}\vec{v}\cdot\nabla(v_{3}F_{z})\right]$$

It has been shown in Moschandreou et al [5] that this decomposition holds and that,

$$\mathcal{G}(\eta)_{\delta 1}+\mathcal{G}(\eta)_{\delta 2}+\mathcal{G}(\eta)_{\delta 4}=3\,\Phi(s)$$

The function $\Phi(s)$ is the surface integral of pressure terms minus the volume integral of tensor product term.

At the end of this paper, a proof that on a volume of an arbitrarily small sphere embedded in each cell of the lattice centered at (a_i,b_i,c_i) (centers of cells) we have,

$$\mathcal{G}(\eta)_{\delta 1}+\mathcal{G}(\eta)_{\delta 2}+\mathcal{G}(\eta)_{\delta 4}=0$$

From this equation we then can solve for $\frac{\partial Q}{\partial y_3}$ algebraically and differentiating with respect to y_3 and using Poisson's equation (setting the representation of each of the two partial derivatives with respect to y_3 equal to each other we obtain, $L=0,$ which is precisely the following PDE,

$$\begin{aligned}
L=&\left(\frac{\partial v_3}{\partial s}\right)^2\mu(\delta-1)\frac{\partial^3 v_3}{\partial y_3\,\partial y_1^2}+\left(\frac{\partial v_3}{\partial s}\right)^2\mu(\delta-1)\frac{\partial^3 v_3}{\partial y_3\,\partial {y_2}^2}+\left(\frac{\partial v_3}{\partial s}\right)^2\mu(\delta-1)\frac{\partial^3 v_3}{\partial {y_3}^3}+\\
&\left(\frac{\partial v_3}{\partial s}\right)(v_3)^2\left(\frac{\partial^3 v_3}{\partial {y_3}^2\,\partial s}\right)\delta\rho-(v_3)^2\left(\frac{\partial^2 v_3}{\partial y_3\,\partial s}\right)^2\delta\rho-\\
&2\rho\Bigg(\left(\frac{\delta}{2}-\frac{1}{2}\right)\left(\frac{\partial v_3}{\partial s}\right)^2-v_3\left(\frac{\partial v_3}{\partial s}\right)\left(\frac{\partial v_3}{\partial y_3}\right)\delta+\Bigg(v_3\left(F_{T_1}(y_1,y_2,y_3,s)+\frac{\partial v_1}{\partial s}\right)\frac{\partial v_3}{\partial y_1}+\\
&v_3\left(F_{T_2}(y_1,y_2,y_3,s)+\frac{\partial v_2}{\partial s}\right)\frac{\partial v_3}{\partial y_2}+\frac{\Lambda(y_1,y_2,y_3,s)}{2}+\frac{\Phi(s)}{2}\Bigg)\delta\Bigg)\frac{\partial^2 v_3}{\partial y_3\,\partial s}+\\
&\Bigg(\Bigg((\delta-1)(\delta v_1(y_1,y_2,y_3,s)-1)\frac{\partial v_3}{\partial s}+2v_3\rho\delta\left(F_{T_1}(y_1,y_2,y_3,s)+\frac{\partial v_1}{\partial s}\right)\Bigg)\frac{\partial^2 v_3}{\partial y_3\,\partial y_1}+\\
&\left((\delta-1)(v_2(y_1,y_2,y_3,s)\delta-1)\frac{\partial v_3}{\partial s}+2v_3\rho\delta\left(F_{T_2}(y_1,y_2,y_3,s)+\frac{\partial v_2}{\partial s}\right)\right)\frac{\partial^2 v_3}{\partial y_3\,\partial y_2}+\\
&3v_3\left(-\frac{2}{3}+\left(\rho+\frac{2}{3}\right)\delta\right)\left(\frac{\partial v_3}{\partial s}\right)\frac{\partial^2 v_3}{\partial {y_3}^2}+2v_3\left(\frac{\partial v_3}{\partial s}\right)(\delta-1)\frac{\partial^2 v_3}{\partial {y_1}^{\,2}}+2v_3\left(\frac{\partial v_3}{\partial s}\right)(\delta-1)\frac{\partial^2 v_3}{\partial {y_2}^2}+\\
&2\left(\frac{\partial^2 v_1}{\partial y_3\,\partial s}\right)v_3\left(\frac{\partial v_3}{\partial y_1}\right)\rho\delta+2\left(\frac{\partial^2 v_2}{\partial y_3\,\partial s}\right)v_3\left(\frac{\partial v_3}{\partial y_2}\right)\rho\delta+\\
&\left((-1+(3\rho+1)\delta)\left(\frac{\partial v_3}{\partial y_3}\right)^2+(\delta-1)\left(\left(\frac{\partial v_1}{\partial y_1}\right)^2+2\left(\frac{\partial v_1}{\partial y_2}\right)\frac{\partial v_2}{\partial y_1}+\left(\frac{\partial v_2}{\partial y_2}\right)^2\right)\right)\frac{\partial v_3}{\partial s}+\\
&2\rho\Bigg(\Bigg(\left(F_{T_1}(y_1,y_2,y_3,s)+\frac{\partial v_1}{\partial s}\right)\frac{\partial v_3}{\partial y_1}+\left(\frac{\partial v_3}{\partial y_2}\right)\left(F_{T_2}(y_1,y_2,y_3,s)+\frac{\partial v_2}{\partial s}\right)\Bigg)\frac{\partial v_3}{\partial y_3}+\\
&v_3\left(\frac{\partial v_3}{\partial y_1}\right)\frac{\partial F_{T_1}}{\partial y_3}+v_3\left(\frac{\partial v_3}{\partial y_2}\right)\frac{\partial F_{T_2}}{\partial y_3}+\frac{1}{2}\frac{\partial\Lambda(y_1,y_2,y_3,s)}{\partial y_3}\Bigg)\delta\Bigg)\frac{\partial v_3}{\partial s}=0
\end{aligned}\tag{1}$$

and $\Lambda(y_1, y_2, y_3, s)$ is given as,

$$\Lambda(y_1, y_2, y_3, s) = 2\frac{f_0(s)F(y_1, y_2, y_3)v_3(y_1, y_2, y_3, s)\frac{\partial v_3}{\partial y_1}}{\delta} + 2\frac{f_0(s)G(y_1, y_2, y_3)v_3(y_1, y_2, y_3, s)\frac{\partial v_3}{\partial y_2}}{\delta} -$$

$$\delta^3 v_3\left(\frac{\partial v_3}{\partial y_3}\right)F_{sz}(y_1, y_2, y_3, s) + \delta^2\left(\left(\frac{\partial v_3}{\partial y_3}\right)F_{sz}(y_1, y_2, y_3, s) + v_3\frac{\partial F_{sz}}{\partial y_3}\right)$$

where $\vec{f} = \left(F_{T_1}, F_{T_2}, F_{sz}\right)$ is the forcing vector and $\vec{v} = (v_1, v_2, v_3)$ is the velocity in each cell of the 3-Torus.

For the three forcing terms, set them equal to products of reciprocals of degenerate Weierstrass P functions shifted in spatial coordinates from the center $(a_i, b_i, c_i), i = 1..N$.

Here the (a_i, b_i, c_i) is the center of each cell of the lattice belonging to the flat torus. Upon substituting the Weierstrass P functions and their reciprocals (unity divided by P-function) into Eq.(1) together with the forcing terms given by Λ, it can be observed that in the equation that terms in it are multiplied by reciprocal Weierstrass P functions which touch the centers of the cells of the lattice, thus simplifying Eq.(1). The initial condition in v_3 at $t = 0$ is instead of a product of reciprocal degenerate Weierstrass P functions for forcing, is a sum of these functions. The parameter m in the degenerate Weierstrass P function, if chosen to be small gives a ball,

$$B_r = \{y \in \mathbb{R}^3 : \|y\|_2 = (|y_1|^2 + |y_2|^2 + |y_3|^2)^{\frac{1}{2}} \leq r\}$$

Here we are in Cartesian space $\mathbb{R}^3$ with 2-norm L_2. Since the terms are squared in length in the initial condition for v_3 we require to multiply by dynamic viscosity μ to obtain units of velocity. In the above, the forcing is taken to be different than the gradient of pressure.

Introducing the space $\mathfrak{I}(y_3, s) = \left\{s \in \mathbb{R}^+, y_3 \in B\left(y_{3_{c_i}}; \varepsilon\right) : 2y_1v_1 + v_2 = 0 \,\&Ay_1 + By_2 + C = 0, \forall y_1, y_2 \in I \times I\ (I \subset \mathbb{R})\&y_2 = y_1^2 \,\&\, v_3(y_1, y_2, y_3, s) \in C^0(\mathbb{T}^3)\right\}$,

where $B\left(y_{3_{c_i}}; \varepsilon\right)$ is the 1-dimensional ε-ball centered at $y_{3_{c_i}}$, i=1,2,…N, ranging through the expanding lattice generated by the flat torus.. The point $y_{3_{c_i}}$ coincides with the center point (a_i, b_i, c_i), where $\vec{r} = (y_1 - a_i, y_2 - b_i, y_3 - c_i)$, $\quad i = 1,2, \ldots N$.

The y_3 points are along segments parallel to the y_3-axis, throughout the lattice. For points belonging to the space $\mathfrak{I}(y_3, s)$, the following part of Eq.(1) is exactly zero:

$$X = \left((\delta - 1)v_1\frac{\partial v_3}{\partial s} + 2\rho v_3\frac{\partial v_1}{\partial s}\right)\frac{\partial^2 v_3}{\partial y_3 \partial y_1} + \left((\delta - 1)v_2\frac{\partial v_3}{\partial s} + 2\rho v_3\frac{\partial v_2}{\partial s}\right)\frac{\partial^2 v_3}{\partial y_3 \partial y_2}$$

$$-\frac{\partial v_3}{\partial s}\left[v_3\frac{\partial v_3}{\partial y_1}\frac{\partial^2 v_1}{\partial y_3 \partial s} + v_3\frac{\partial v_3}{\underset{2}{\partial y_2}}\frac{\partial^2 v_2}{\partial y_3 \partial s} - \frac{\partial v_3}{\partial y_1}\frac{\partial v_3}{\partial y_3}\frac{\partial v_1}{\partial s} - \frac{\partial v_3}{\partial y_2}\frac{\partial v_3}{\partial y_3}\frac{\partial v_2}{\partial s}\right] + v_3\frac{\partial v_3}{\partial y_1}\frac{\partial v_1}{\partial s}\frac{\partial^2 v_3}{\partial s \partial y_3} + v_3\frac{\partial v_3}{\partial y_2}\frac{\partial v_2}{\partial s}\frac{\partial^2 v_3}{\partial s \partial y_3}$$

$$X = \left((\delta - 1)\ v_1\frac{\partial v_3}{\partial s} + 2\rho v_3\frac{\partial v_1}{\partial s}\right)\frac{\partial^2 v_3}{\partial y_3 \partial y_1} + \left((\delta - 1)\ v_2\frac{\partial v_3}{\partial s} + 2\rho v_3\frac{\partial v_2}{\partial s}\right)\frac{\partial^2 v_3}{\partial y_3 \partial y_2} + \frac{\partial v_3}{\partial s}\left(\frac{\partial v_3}{\partial y_1}\frac{\partial v_3}{\partial y_3}\frac{\partial v_1}{\partial s} + \frac{\partial v_3}{\partial y_2}\frac{\partial v_3}{\partial y_3}\frac{\partial v_2}{\partial s}\right) +$$

$$\left(\frac{\partial v_2}{\partial s}\right)^2 v_3\frac{\partial v_3}{\partial y_2}\left(\frac{\frac{\partial v_3}{\partial s}}{\frac{\partial v_2}{\partial s}}\right)_{y_3} + \left(\frac{\partial v_1}{\partial s}\right)^2 v_3\frac{\partial v_3}{\partial y_1}\left(\frac{\frac{\partial v_3}{\partial s}}{\frac{\partial v_1}{\partial s}}\right)_{y_3}$$

That is $X = 0$ on the subspace $\mathfrak{I}(y_3, s)$. v_1, v_2 are linearly dependent in this space. In the second equivalent expression for X, in the space $\mathfrak{I}(y_3, s)$, $X = 0$.

Next the sum of the two first vorticities is used together with the vorticity sum set to the sum of the first two components of the equivalent expression which is twice the angular velocity,

$$\omega_1 + \omega_2 = \frac{2y_2v_3 - 2y_1v_3 - 2y_3(v_2 - v_1)}{y_1^2 + y_2^2 + y_3^2}$$

Thus using the definition of vorticity we have the following equation in the space $\mathfrak{I}(y_3, s)$,

$$\frac{\partial v_3}{\partial y_1} - \frac{\partial v_3}{\partial y_2} = \frac{\partial v_1}{\partial y_3} - \frac{\partial v_2}{\partial y_3} - (\omega_1 + \omega_2)$$

Multiplying both sides of this equation by $y_1^2 + y_2^2 + y_3^2 = \varepsilon^2$ and letting ε approach zero gives,

$$2y_2 v_3 - 2y_1 v_3 - 2y_3(v_2 - v_1) = 0$$

so

$$v_3 = -\frac{y_3(v_2 - v_1)}{y_1 - y_2}$$

Introduce the following shifts, $(y_1 - a_1, y_2 - a_2, y_3 - a_3)$ ranging over all the centers of cells in the expanding lattice, and we set:

$$y_3 - a_3 = (y_1 - a_1) - (y_2 - a_2)$$

Cancellation occurs between y_3 and $y_1 - y_2$ terms leaving us with,

$$v_3 = -(v_2 - v_1)$$

Here we see clearly that we have an isotropic condition on the finite time blowup of the velocities. If the first derivatives and higher of the third component of velocity blows up then so do the corresponding derivatives of v_1 and v_2 respectively.

The third component of vorticity is calculated as twice the third component of angular velocity,

$$\left[2\frac{(\vec{r} \times \vec{v})_{y_3}}{y_1^2 + y_2^2 + y_3^2}\right] = 2\frac{-v_1 y_2 + v_2 y_1}{y_1^2 + y_2^2 + y_3^2}$$

$$\omega_3 = \frac{\partial v_1}{\partial y_2} - \frac{\partial v_2}{\partial y_1} = 2\frac{-v_1 y_2 + v_2 y_1}{y_1^2 + y_2^2 + y_3^2}$$

Substitute $v_2 = -2y_1 v_1$ into previous PDE,

$$\frac{\partial v_1}{\partial y_2} + 2v_1 + 2y_1\frac{\partial v_1}{\partial y_1} = 2\frac{(-2y_1^2 - y_2)}{\varepsilon^2} v_1$$

where the sphere of radius ε is introduced, at the center of each cell of the lattice.

Solving PDE, gives, for arbitrary function F_1,

$$v_1 = y_1^{-1-\frac{-\frac{ln(y_1)}{2} + y_2}{\varepsilon^2}} F_1\left(\frac{-\ln(y_1)}{2} + y_2, y_3, s\right) e^{-\frac{y_1^2}{\varepsilon^2}} e^{-\frac{\ln(y_1)^2}{4\varepsilon^2}}$$

A particular maximal class of solutions is obtained by setting,

$$F_1 = e^{\ln(y_1) - 2y_2} e^{\left({}^{\ln(y_1)}/_{2\varepsilon} - {}^{y_2}/_{\varepsilon}\right)^2} f(y_3, s)$$

which is in the required form of the general function and where f is an arbitrary function to be determined.
Back substituting F_1 into the solution for v_1, gives,

$$v_1 = e^{\frac{-2y_2\varepsilon^2 - y_1^2 - y_2^2}{\varepsilon^2}} f(y_3, s)$$

Here v_1 is Gaussian.

Substituting v_1 into $v_2 = -2y_1 v_1$, gives,

$$v_2 = -2y_1 e^{\frac{-2y_2\varepsilon^2 - y_1^2 - y_2^2}{\varepsilon^2}} f(y_3, s)$$

which is double sided Gaussian.

Near the center of each cell of the lattice, the solutions are non singular in spatial variables.

However $f(y_3, s)$, is yet to be determined and related to v_3 solution since $v_3 = -(v_2 - v_1)$.

Now the general form was reduced to a particular maximal class of solutions since as $y_1 \to 0$, $v_1 \to 0$, which is inadmissible according to a theorem of J.Y Chemin [6] ("Some remarks about the possible blowup for the Navier Stokes equations") If there is finite time blowup then it is impossible for one component of velocity to approach zero too fast. So we will show further that v_3 is not smooth. Thus v_1, v_2 blow up at the center of cells of lattice if we can conclude that $F(s) = \lim_{y_3\to 0} f(y_3,s)$ has finite time blowup. Again recall that $v_3 = -(v_2 - v_1)$, where in $\mathfrak{I}(y_3,s)v_3 = -(-2y_1v_1 - v_1) = (2y_1+1)v_1 \neq 0$ at the centers of cells of $\mathbb{R}^3/_{\mathbb{Z}^3}$ since $2y_1 + 1 \neq 0$ there and v_1 is also not zero there.

Define $F(s) = f(0,s) = \int H(s)\, ds$,

where $f(0,s) = \lim_{y_3\to 0} f(y_3,s)$ and $H(s)$ is the solution associated with v_3 in the ε −ball as $\varepsilon \to 0.$

Finally the solutions for v_1, v_2 satisfy the y_1, y_2 momentum equations for PNS when $-\frac{\partial P}{\partial y_1} + f_1 = \left(\frac{1}{P_{y_1}}\frac{1}{P_{y_2}}\frac{1}{P_{y_3}} + 1\right)H(s)$,

$$-\frac{\partial P}{\partial y_2} + f_2 = \left(-2\frac{1}{P_{y_1}}\frac{1}{P_{y_2}}\frac{1}{P_{y_3}} - 2\right)H(s)$$

for $\varepsilon > 0$ arbitrarily small and where f_1, f_2 are the forcing terms associated with the y_1, y_2 momentum equations. It remains to prove that the derivatives of $H(s)$ blowup in finite time.

The pressure gradient is oscillatory, that is it is written as a product of reciprocals of degenerate Weierstrass P functions added to a constant as is the forcing.

Finally the surface S given by $y_3 = \pm(Ay_1{}^2 + By_1 + C)$, plotted in $\mathbb{R}^3$ is such that by shifting and sweeping through y_1 values and heights along y_3 axis we can find intersection points between surface S and points or centers of cells (a_i, b_i, c_i).

Equation (1) together with $X = 0$ gives the following PDE which has viscosity in it and where in Eq.(8.21) we have condensed the PDE by collecting the terms that contribute to the Laplacian. Also the divergence theorem is applied to the volume integral of Eq(I) for the term with Laplacian multiplied by v_3. The calculations are taking into account that density is large, (fluids like water and higher densities.)

$$\left(\frac{\partial^3 v_3}{\partial y_3{}^3} + \frac{\partial^3 v_3}{\partial y_3\,\partial y_2{}^2} + \frac{\partial^3 v_3}{\partial y_3\,\partial y_1{}^2}\right)\mu + 2/3(v_3\left(\frac{\partial^2 v_3}{\partial y_3{}^2} + \frac{\partial^2 v_3}{\partial y_2{}^2} + \frac{\partial^2 v_3}{\partial y_1{}^2}\right) +$$

$$1/6\left(3\rho v_3\frac{\partial^2 v_3}{\partial y_3{}^2} + 3\left(\frac{\partial v_3}{\partial y_3}\right)^2\rho - \left(\frac{\partial v_3}{\partial y_3}\right)^2 + \frac{\partial^2 v_3}{\partial y_3\,\partial y_1} + \frac{\partial^2 v_3}{\partial y_3\,\partial y_2}\right)\frac{\partial v_3}{\partial s} = 0 \qquad \text{(I)}$$

In Equation (I) it is understood that in the top line with two expressions appearing there, that these both include a product of $(\delta - 1)\left(\frac{\partial v_3}{\partial s}\right)^2$ which has been set to a constant. Solving this implies that v_3 is a linear function in s. As $\delta \to 1$, v_3 approaches infinity from the right of a potential blowup point $s = s_0$. See Figure (1c) below,

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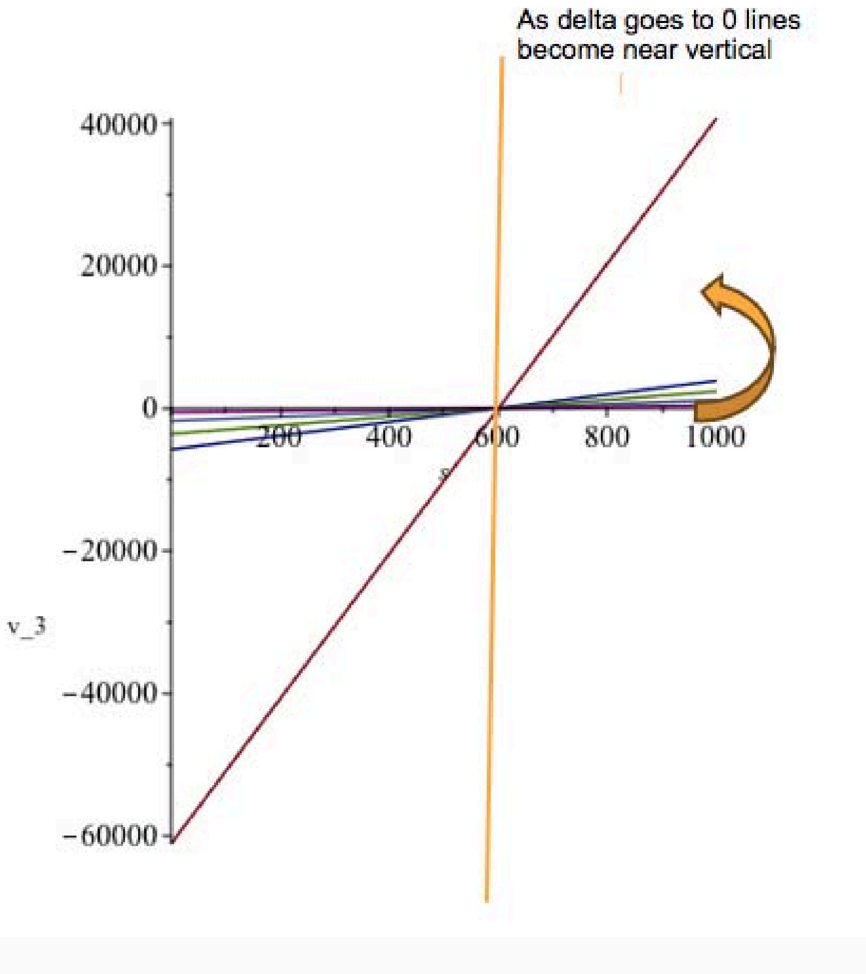

Figure 1c: Linear functions in the form $v_3 = (-abs(b) + (m * s - 600),\ m > 0, b$, y-intercept. It is shown that the right side limit approaches infinity as $\delta \rightarrow 1$

Equation (I) is confirmed to provide the left hand limit at $s = s_0$. We have two problems here. One is the solution for the Euler equation when $\mu = 0$. The solution is obtained by solving for one of the constants C_6. There are six unknown constants in the solution of the above PDE when $\mu = 0$. $(C_i, i = 1,2, \ldots 6)$ We use the fact that in the space $\Im(y_3, s)$, the set $\{1, y_1, y_1{}^2\}$ is linearly independent, implying that all the constants are zero in the solution except C_3 and C_4 associated with variables y_3, s respectively. The solution is expressed as linear sums of the spatial and time variables. Now y_3 is within an epsilon ball. The variable ζ appears in the initial condition when solving for the unknown constant C_6, and the initial condition for v_3 is given as the sum of arbitrarily large data ζ and sums of reciprocal degenerate Weierstrass P functions in the three directions for small m. We obtain the following solution,

$$D_1 = \ln\left(-6C_1{}^2C_3\zeta - 6C_2{}^2C_3\zeta - 6C_3^3\zeta - 2C_1^3 - 2C_1{}^2C_2 - 2C_1C_2{}^2 - 2C_1C_3{}^2 - 2C_2{}^3 - 2C_2C_3{}^2 + \mathrm{e}^{C_3{}^2C_4C_5}\right)C_3$$

$$D_2 = \left(12(s + \zeta/2)C_3{}^2 + 12(C_1/6 + C_2/6)C_3 + 12s(C_1{}^2 + C_2{}^2)\right)(C_1{}^2 + C_2{}^2 + C_3{}^2)\mathrm{e}^{-C_3{}^2C_4C_5}$$

$$\mathrm{C}_3{}^3\mathrm{C}_4\mathrm{C}_5 - \mathrm{C}_3$$

$$v_3(\zeta, s) = \frac{1}{6C_3({C_1}^2 + {C_2}^2 + {C_3}^2)}(-2C_1^3 - 2{C_1}^2C_2 - 2C_1{C_2}^2 - 2C_1{C_3}^2 - 2{C_2}^3 - 2C_2{C_3}^2 +$$

$$e^{C_3^2C_4C_5}W\left(-\exp\left(\frac{D_1 + D_2}{C_3}\right)\right) + e^{{C_3}^2C_4C_5}\Big)$$

where W is the Lambert W function. We replaced ζ by $-\zeta$+large shifts and found that the solution for v_3 for large s (example $s = 600$), the solution is locally Hölder continuous with Hölder constant 1/3 at arbitrary large values of ζ. (specifically in plot shown, $\zeta = 10000$).

In this analysis there is no restriction on the largeness of the data, thereby proving that the solution is admissible for arbitrary large data. The solution as seen in Figure 2 is not smooth from the first and higher derivatives in of ζ. This is discussed further in the chapter as it pertains to the Onsager regularity problem particularly the endpoint regularity problem.

See the following Figure 2, where the dashed line is the solution for v_3 and the non-dashed line is the Hölder solution, given for example as

$$(-0.52+\left(10000-\zeta\right)^{\frac{1}{3}})$$

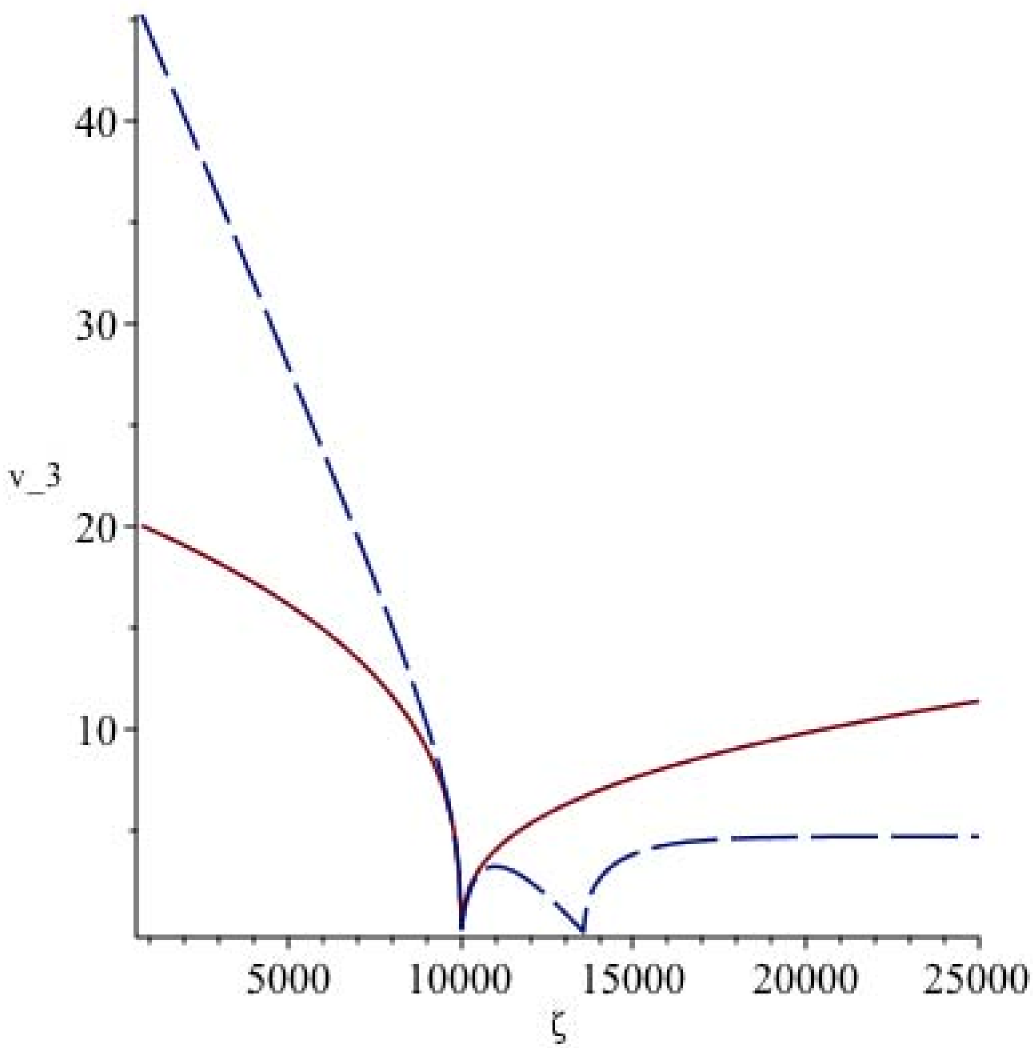

Figure 2: Locally Hölder continuous functions. $C_3 = -0.052C_4 = 0.05$

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III. On the Endpoint Regularity in Onsager's Conjecture

In order to obtain the solution previously shown as $v_3(\zeta, s)$ we let epsilon approach zero for solutions $v_3(\zeta, y_3, s)$ in the space $\Im(y_3, s)$. In this space a ball $B\left(y_{3_{c_i}}; \varepsilon\right)$ exists with $\varepsilon > 0$. Here ε is defined as a measure of how close one is to the center of a given cell in the lattice of the 3-Torus. Due to the definition of the space $\Im(y_3, s)$, the set $\{1, y_1, {y_1}^2\}$ is linearly independent, implying that all the constants are zero in the solution except C_3 and C_4 associated with variables y_3, s respectively. The constants C_i ranging from $i = 1..6$ in the solution of the Euler Equation (I) appear in the solution and in particular as an argument of the Lambert W function and is expressed as the following linear sum in spatial and time variables,

$$Y = C_1 y_1 + C_2 y_2 + C_3 y_3 + C_4 s + C_5$$

Note that the solution can be obtained by solving Eq.(I) when $3\left(\frac{\partial v_3}{\partial y_3}\right)^2 \rho - \left(\frac{\partial v_3}{\partial y_3}\right)^2 \approx 3\left(\frac{\partial v_3}{\partial y_3}\right)^2 \rho$, that is for $\rho \gg 100\frac{kg}{m^3}$. It is found that an exact solution is given by Maple 2023 software when this approximation is made for large enough density. It is also worthy to note that for lower densities when we retain both terms in the previous approximation, that for the locally Hölder continuous functions in time s, with Hölder constant equal to exactly 1/3, the product term $\left(\frac{\partial v_3}{\partial y_3}\right)^2 \frac{\partial v_3}{\partial s}$ in Eq.(I) becomes independent of time s and is only dependent on the spatial variables.

The Onsager conjecture suggested the value $\alpha = 1/3$ for the case of the Euler equations but the conjecture was mainly considering only the Hölder regularity with respect to the space variables. Here we consider a combination of velocity-time conditions (ζ, s), which depend precisely on the Hölder exponent. As outlined in the introduction, P. Isett's proof shows that if $\alpha < 1/3$ (strictly less than) then conservation of energy fails. The works of Eyink[7,8] and Constantin, E, Titi [9] on the Onsager conjecture describe results in a Fourier setting and in a space called a Besov space (slightly larger than Hölder spaces), respectively. A well known result is that if the velocity is a weak solution to the Euler equations such that,

$$u \in L^3(0, T; B_3^{\alpha,\infty}(\mathbb{T}^3)) \cap C(0, T; L^2(\mathbb{T}^3))$$

with $\alpha > 1/3$, (strictly greater than) then, $\|u(t)\| = \|u_0\|$, for all $t \in [0, T]$. This result is also true in Hölder spaces which was the setting that L. Onsager stated his conjecture rather than Besov spaces.

Hölder continuous functions, as defined in Berselli [10] with a focus on space-time properties of functions with "homogeneous behavior", that is the one of the Hölder semi-norm $[.]_\alpha$ (to be defined) and denote by $\dot{C}^\alpha$ the space of measurable functions such that this quantity is bounded. We say that,

$$u \in L^\beta(0, T;\ \dot{C}^\alpha(\mathbb{T}^3)),$$

if there exists $f_\alpha : [0, T] \to \mathbb{R}^+$ such that

1) $|u(x,t) - u(y,t)| \leq f_\alpha(t)|x - y|^\alpha, \forall\, x, y \in \mathbb{T}^3$, for a.e. $t \in [0, T]$,

2) $\int_0^T f_\alpha^\beta(t)dt < \infty$

and $f_\alpha(t) = [u(t)]_\alpha$ for almost all $t \in [0, T]$.

The space is endowed with the semi-norm

$$\|u\|_{L^\beta(0,T;\dot{C}^\alpha(\mathbb{T}^3))} := \left[\int_0^T f_\alpha^\beta(t)dt\right]^{1/\beta}$$

Finally

$$[u]_\alpha := \sup_{x \neq y} \frac{|u(x) - u(y)|}{|x - y|^\alpha}$$

In Berselli [10], (see Theorem 4.2 there) it is proven that if u is a weak solution to the Euler equation (in usual form), such that $u \in L^{1/\alpha}(0, T;\ \dot{C}_\omega^\alpha(\mathbb{T}^3))$ with $\alpha \in \left[\frac{1}{3}, 1\right]$ (where $\dot{C}_\omega^\alpha(\mathbb{T}^3) \subset C^\alpha(\mathbb{T}^3)$ is the slightly smaller space defined through the norm

$$\|u\|_{C_\omega^\alpha} = \max_{x \in \mathbb{T}^3} |u(x)| + [u]_{\omega,\alpha}$$

$$[u]_{\omega,\alpha} := \begin{matrix} sup \\ x \neq y \end{matrix} \frac{|u(x)-u(y)|}{\omega(|x-y|)|x-y|^{\alpha}},$$

with $\omega: \mathbb{R}^{+} \to \mathbb{R}^{+}$ a non-decreasing function such that $\lim_{s\to 0^{+}} \omega(s) = 0$.) then u conserves the energy.

In our proof of the endpoint regularity of Onsager's conjecture we are considering the Hölder continuous functions in the space $C^{\alpha}(\mathbb{T}^3)$.

There are two steps here. First we set $v_3(y_3, s)$ equal to the variable ζ appearing in the initial condition when solving for the unknown constant C_6 where $\zeta > 0$, and recall that the initial condition for v_3 is given as the sum of arbitrarily large data ζ and sums of reciprocal degenerate Weierstrass P functions in the three directions for small m. (By reciprocal we mean that unity is divided by the Weierstrass P functions with a bounded periodic result.) In the second step we solve for $v_3(\zeta, y_3, s)$ for arbitrarily large negative data $\zeta < 0$. In both steps separately we keep $y_3 \in B\left(y_{3_{c_i}}; \varepsilon\right)$ and integrate the square associated with energy of solution $v_3(\zeta, y_3, s)$, that is we will show that our solution satisfies conservation of energy, (for all times $s \in [0, T)$).

$$\iint_{C_2} \int_{y_3=-\varepsilon}^{\varepsilon} v_3^2\,(\zeta, y_3, s) \mathrm{d}y_3 dy_1 dy_2 = \iiint_{C(\vec{a};\varepsilon)} v_3^2(0)\, dy = \iiint_{C(\vec{a};\varepsilon)} (\zeta + (|y_1|^2 + |y_2|^2 + |y_3|^2))^2\, dy$$

The integrals are carried out over a cube $C(\vec{a}; \varepsilon) = [-\varepsilon, \varepsilon]^3$, centered about $\vec{a}$. For $\varepsilon = 1/2$ the scaled solutions and hence graphs are shown in Figure 3 and 4. It is seen that in either step in both figures that energy is conserved thereby proving the endpoint regularity in Onsager's Conjecture. In Figure 3 and 4, the thicker part of curves hides the energy (E) at $s = 0$, behind the solution curve. For $\zeta > 0$ there are two curves coinciding and the same is true for $\zeta < 0$.

The key empirical fact underlying the Onsager theory is the non-vanishing of turbulent energy dissipation in the zero-viscosity limit. The requirement for a non-vanishing limit of dissipation is that space-gradients of velocity must diverge. It is observed in experiment that when integrated over small balls or cubes in space the high-Reynolds limit of the the kinetic energy dissipation rate defines a positive measure with multifractal scaling. The solution for Euler's equation given in this paper agrees with this fact that gradient of v_3 with respect to spatial position y_3 does in fact diverge. This is a short-distance/ultraviolet (UV) divergence in the language of quantum field-theory, or what Onsager himself termed a "violet catastrophe" [12]. Since the fluid equations of motion (I.1) contain diverging gradients, they become ill-defined in the limit. In order to develop a dynamical description which can be valid even as ν → 0, some regularization of this divergence must be introduced.

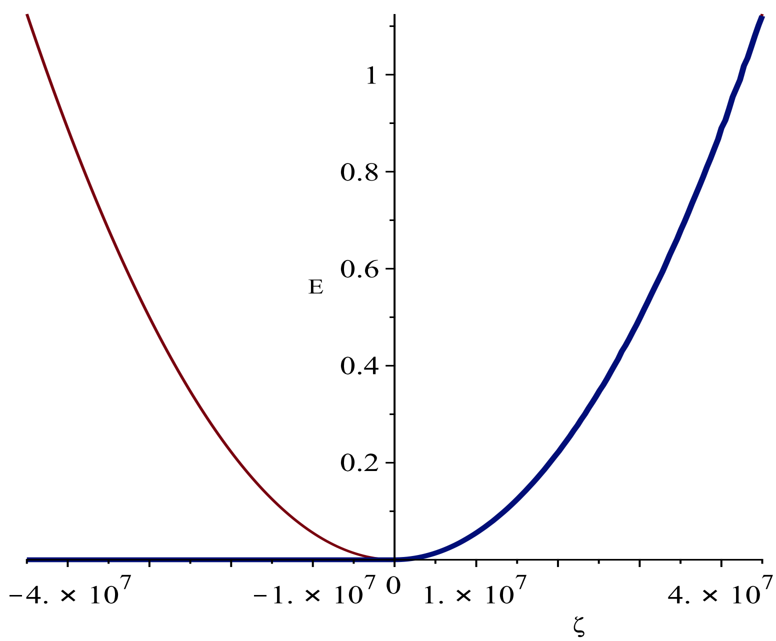

Figure 3: Energy of PNS system for arbitrarily large and positive data ζ

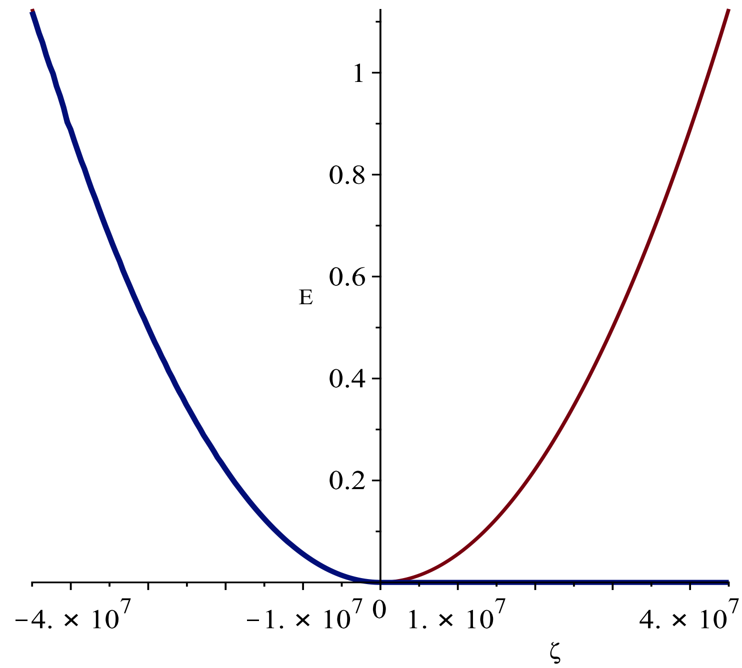

Figure 4: Energy of PNS system for arbitrarily large and negative data ζ

In the book "Theory of unitary symmetry" by Rumer and Fet[12], the Laplacian is defined an integration over a 3D-ball, in particular an epsilon ball.

The viscous solution when μ is non-zero is subject to a rewriting of Eq (I) and to use this result first we integrate Eq.(I) over an ε −ball, centered at each center of cells of the lattice of 3-Torus. Next using the divergence theorem for the term of Eq(I), that is specifically the expression $v_3\left(\frac{\partial^2 v_3}{\partial y_3{}^2}+\frac{\partial^2 v_3}{\partial y_2{}^2}+\frac{\partial^2 v_3}{\partial y_1{}^2}\right)$, gives $\int_{y\in B_\varepsilon(y)} |\nabla v_3|^2\, dy = 0$ where the surface integral is zero and since we are integrating a positive expression on an epsilon ball, at epsilon $=0$ the integral is zero.

Therefore Eq(I) becomes:

$$\left(\frac{\partial^3 v_3}{\partial y_3{}^3}+\frac{\partial^3 v_3}{\partial y_3\,\partial y_2{}^2}+\frac{\partial^3 v_3}{\partial y_3\,\partial y_1{}^2}\right)\mu(\delta-1)+$$

$$1/6\left(3\rho v_3\frac{\partial^2 v_3}{\partial y_3{}^2}+3\left(\frac{\partial v_3}{\partial y_3}\right)^2\rho-\left(\frac{\partial v_3}{\partial y_3}\right)^2+\frac{\partial^2 v_3}{\partial y_3\,\partial y_1}+\frac{\partial^2 v_3}{\partial y_3\,\partial y_2}\right)\frac{\partial v_3}{\partial s}=0 \qquad \text{(II)}$$

Equation (II) is integrated over an epsilon ball so we solve Eq.(II) in a neighborhood of epsilon $=0$ that is near the center of each cell of the lattice in the space $\Im(y_3,s)$. So we integrate Eq. (II) over an epsilon ball first and then take limit. We use the Fet theory on writing the Laplacian as an integral over an epsilon ball.

Here we know that there is an operator $\Delta_\varepsilon v_3=\frac{3}{4\pi\varepsilon^3}\int v_3(y)-v_3(0)dy$ such that in the limit as epsilon approaches zero, $\frac{10}{\varepsilon^2}\Delta_\varepsilon v_3{=}\Delta v_3$. Integral is over epsilon ball centered at $\vec{a}=(a,b,c)$.

Proof:

We take the Taylor expansion around 0 (or center $\vec{a}$ to second order, which gives terms proportional to y_1, y_1y_2 and y_1^2, however due to the symmetry of the y_1, y_1y_2 related terms these integrate to zero over the ball and thus we have that,

$$\Delta_\varepsilon v_3=\frac{3}{4\pi\varepsilon^3}\left[\frac{1}{2}\frac{\partial^2 v_3}{\partial y_1^2}\int y_1^2\,dy+\frac{1}{2}\frac{\partial^2 v_3}{\partial y_2^2}\int y_2^2\,dy+\frac{1}{2}\frac{\partial^2 v_3}{\partial y_3^2}\int y_3^2\,dy\right]+\mathcal{O}(\varepsilon^3)$$

where all derivatives are evaluated at the center $\vec{a}$. The integrals all give the same value,

$$\int y_1^2\,dy=\frac{1}{3}\int y_1^2+y_2^2+y_3^2\,dy=\frac{4\pi}{3}\int_0^\varepsilon r^4 dr=\frac{4\pi\varepsilon^5}{15}$$

where the differential has been transformed to spherical coordinates in 3D. Substituting this into the main statement of the theorem, we obtain,

$$\Delta_\varepsilon v_3 = \frac{3}{4\pi\varepsilon^3}\frac{4\pi\varepsilon^5}{15}\frac{1}{2}\left[\frac{\partial^2 v_3}{\partial y_1^2}+\frac{\partial^2 v_3}{\partial y_2^2}+\frac{\partial^2 v_3}{\partial y_3^2}\right]+\mathcal{O}(\varepsilon^3)=\frac{\varepsilon^2}{10}\Delta v_3 + +\mathcal{O}(\varepsilon^3)$$

Finally we take the limit,

$$\lim_{\varepsilon\to 0}\frac{10}{\varepsilon^2}\Delta_\varepsilon v_3 = \lim_{\varepsilon\to 0}[\Delta v_3 + \mathcal{O}(\varepsilon)] = \Delta v_3$$

In Eq.(II) the Laplacian is differentiated wrt to y_3. Using Fet theory, where we integrate $\Delta_\varepsilon v_3$ on an epsilon ball centered at zero and generalized to the center of any cell center of the lattice of the 3-Torus, we obtain the following PDE for large density:

$$1/6\left(3\rho v_3\frac{\partial^2 v_3}{\partial y_3{}^2}+3\left(\frac{\partial v_3}{\partial y_3}\right)^2\rho+\frac{\partial^2 v_3}{\partial y_3\,\partial y_1}+\frac{\partial^2 v_3}{\partial y_3\,\partial y_2}\right)\frac{\partial v_3}{\partial s}+\mu(\delta-1)\frac{\partial v_3}{\partial y_3}=0 \qquad \text{(III)}$$

with solution:

$$v_3 = \quad (1/3 - C_4C_1 - C_4C_2 + (-(6C_1y_1 + 6C_2y_2 + 6C_3y_3 + 6C_4s + 6C_5)C_3C_4^2C_5\rho + 6C_3C_4^2C_6\rho -$$
$$18(C_1y_1 + C_2y + C_3z + C_4s + C_5)^2C_3C_4\rho + C_1^2C_4^2 + 2C_1C_2C_4^2 + C_2^2C_4^2)^{1/2})/(C_3C_4\rho)$$

where C_i , i=1…6 are constants. Using the same initial condition in terms of ζ as in the first part in Eq(I), we can determine C_6 and on the space $\mathfrak{I}(y_3, s)$, all constants are zero and the only constants that survive are C_3, C_4 and arbitrarily large ζ. When the two constants are as follows,

$$C_3 = 0.052, C_4 = 0.05$$

for $\rho = 1000, \zeta = 10000$, the following result follows in Figure 5.

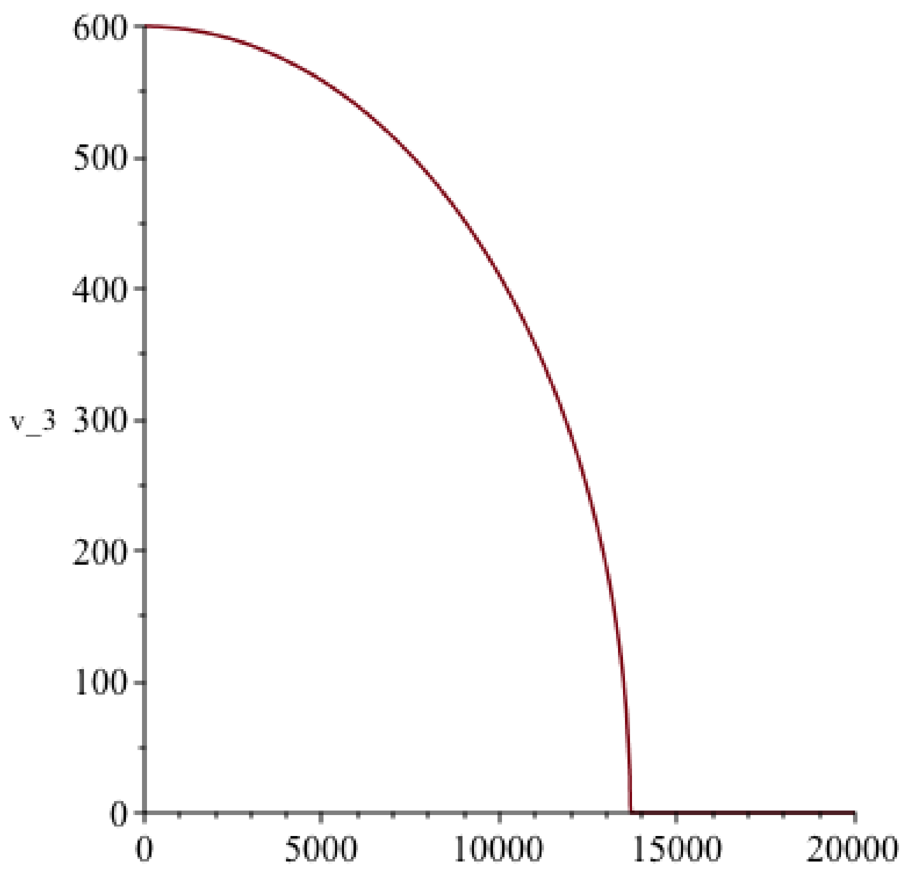

Figure 5: Locally Hölder continuous functions in s

Here it is clear that there exists a solution of PNS that is not smooth in time s for the first and higher derivatives.

Also if we specify the time, the solution is a Hölder continuous function in the data ζ with a Hölder constant equal to one half.

The Full Equation Proof for the Periodic Navier Stokes Equations

Integrating the Navier Stokes equations over an epsilon ball we obtain,

$$\int_{B_{y;\varepsilon}} \mathcal{G}_{\delta 1} + \mathcal{G}_{\delta 2} + \mathcal{G}_{\delta 4}\, dy = -\int_{B_{y;\varepsilon}} \mathcal{G}_{\delta 3}\, dy \tag{IV}$$

The first part of $\mathcal{G}_3$ becomes,

$$\mathcal{G}_3 = \nabla \cdot \nabla(\Xi),$$

where $\nabla(\Xi) = \frac{1}{\rho}\left(v_3^2 \nabla_{y_1 y_2} P + \vec{b} v_3 \frac{\partial P}{\partial y_3}\right)$

where P is the pressure and ρ is the density of the fluid.

Dividing Eq.(IV) by the measure or volume of the ball of radius epsilon centered at point $\boldsymbol{a}$.

$$B_{a;\varepsilon}$$

we know since Ξ is continuous everwhere on the 3-Torus (since integrals are continuous in inverting gradient), and in particular at the the center of the epsilon ball (note higher order derivatives of v_3 blowup, not v_3 and pressure), then,

$$\lim_{\varepsilon \to 0} \frac{1}{|B_{y;\varepsilon}|} \int_{B_{a;\varepsilon}} \Xi(y)\, dy = \Xi(\mathrm{a}) \tag{V}$$

However using the Fet theory, we can see that the integral on the RHS of Eq.(IV) divided by the volume of the ball is related to the integral over the ball centered at a of $\frac{1}{4\pi\varepsilon^2}(\Xi(y) - \Xi(\mathrm{a}))$. Using Eq.(V), we obtain a difference of exactly zero so that we are left with,

$$\lim_{\varepsilon \to 0} \frac{1}{|B_{y;\varepsilon}|} \int_{B_{a;\varepsilon}} \mathcal{G}_{\delta 1} + \mathcal{G}_{\delta 2} + \mathcal{G}_{\delta 4}\, dy = 0$$

$$\int_{B_{a;\varepsilon}} \mathcal{G}_{\delta 1} + \mathcal{G}_{\delta 2} + \mathcal{G}_{\delta 4}\, dy = 0 \tag{VI}$$

Eq.(VI) is the PDE we obtained previously and occurs at an arbitrarily small epsilon ball centered at each cell of the lattice of the 3-Torus.

In reference [5], we showed that,

$$\mathcal{G}_{\delta 1} + \mathcal{G}_{\delta 2} + \mathcal{G}_{\delta 4} = 3\Phi(s)$$

Since the negative pressure gradients are greater than or equal to zero being reciprocal Weierstrass P functions and $v_3^2 \geq 0$ and v_1 and v_2 cancel in the space $\mathfrak{I}(y_3, s)$ when integrating on the six faces of surface of a cell of $\mathbb{T}^3$, we have that,

$$\Phi(s) \geq 0$$

and

$$\mathcal{G}_{\delta 1} + \mathcal{G}_{\delta 2} + \mathcal{G}_{\delta 4} \geq 0$$

Also

$$\Xi_2 = \frac{\|Q\|}{\|\vec{b}\|} = \frac{\left\|\frac{\partial v_3}{\partial s}\vec{b} \cdot (\vec{b} \otimes \Delta v_3)\right\|}{\|\vec{b}\|}$$

where

$$\Xi_1 = \Xi + \Xi_2$$

Recall that the three velocities are isotropic and they are continuous on $B_{a;\varepsilon}$ and Ξ_2 is continuous on the epsilon ball. Also Ξ_2 is independent of s for Hölder continuous functions at $\alpha = 1/3$.

Theorem

$\mathcal{G}_{\delta 1}+\mathcal{G}_{\delta 2}+\mathcal{G}_{\delta 4}=0$ if and only if Ξ_1 is continuous on the epsilon ball

$$B_{a;\varepsilon}.$$

Proof:

Apply (V) to Ξ_1

IV. Conclusion

Satisfying a divergence free vector field and periodic boundary conditions respectively with a general spatio-temporal forcing term f which is smooth and spatially periodic, the existence of solutions which blowup in finite time for PNS can occur starting with the first derivative and higher with respect to time. P. Isett (2016) (see [13]) has shown that the conservation of energy fails for the 3D incompressible Euler flows with Hölder regularity below 1/3. (Onsager's second conjecture) The endpoint regularity in Onsager's conjecture has been addressed, and it is found that conservation of energy occurs when the Hölder regularity is exactly 1/3. The solution for Euler's equation given in this paper agrees with this fact that gradient of v_3 with respect to spatial position y_3 does in fact diverge. This is a short-distance/ultraviolet (UV) divergence in the language of quantum field-theory as L. Onsager proposed. Finally very recent developed new governing equations of fluid mechanics are proposed to have no finite time singularities. This is the focus of the ongoing work of the author to be presented in the near future. Finally future work to conclude the nature of flows in a non-epsilon or arbitrary small ball for the 3-Torus will be carried out.

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Acknowledgments

Contributors to the research other than authors credited should be mentioned in Acknowledgments. The source of funding for the research can be included. Suppliers of resources may be mentioned along with their addresses.

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Preparing your Manuscript

Authors can submit papers and articles in an acceptable file format: MS Word (doc, docx), LaTeX (.tex, .zip or .rar including all of your files), Adobe PDF (.pdf), rich text format (.rtf), simple text document (.txt), Open Document Text (.odt), and Apple Pages (.pages). Our professional layout editors will format the entire paper according to our official guidelines. This is one of the highlights of publishing with Global Journals—authors should not be concerned about the formatting of their paper. Global Journals accepts articles and manuscripts in every major language, be it Spanish, Chinese, Japanese, Portuguese, Russian, French, German, Dutch, Italian, Greek, or any other national language, but the title, subtitle, and abstract should be in English. This will facilitate indexing and the pre-peer review process.

The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.

Manuscript Style Instruction (Optional)

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11'", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

a) A title which should be relevant to the theme of the paper.
b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
d) An introduction, giving fundamental background objectives.
e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
j) There should be brief acknowledgments.
k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.

Format Structure

It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

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The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

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Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution at final image size ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs): >350 dpi; figures containing both halftone and line images: >650 dpi.

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Tips for Writing a Good Quality Engineering Research Paper

Techniques for writing a good quality engineering research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of research engineering then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

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6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. Use proper verb tense: Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.
22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.
23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.

- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- o Single section and succinct.
- o An outline of the job done is always written in past tense.
- o Concentrate on shortening results—limit background information to a verdict or two.
- o Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- o Explain the value (significance) of the study.
- o Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- o Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- o Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

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	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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