Inventory Production Control Model With Back-Order When Shortages Are Allowed

K.A Adeleke[1],D.A Agunbiade[2]

GJSFR- F Clasification FOR 010201,010205,010206,150202

***Abstract*-It is prohibited to have shortage of inventory since inventory cost is induced from the amount of product stored. This paper presents inventory control theory in production inventory problem when shortages are allowed and backorder takes place. Three assumptions are considered here on shortage and backorders and this leads to three models. The first: when demand is fixed and known, production is infinite and shortages are allowed although the cost of shortage is finite. Second when time (t) interval is fixed, replenishment is allowed and production rate is infinite. Third when production rate is finite. It makes economic sense from the applications that for any production where shortages are allowed, backorder must follows to avoid lost in sales.**

Keywords-Inventory control ,Backorder, Production, Shortage, Demand

I. Introduction

Due to the quest for efficiency accelerated by the so called financial crisis, inventory control is a vital function in almost all kinds of productions. Inventory models majorly focused on minimizing the total inventory cost and to balance the economics of large orders or large production runs against the cost of holding inventory and the cost of going short. The method has been efficiently and successfully applied by some researchers in many areas of operation [2, 3, 5, 6, 7, 8 and 10]. Production and inventory planning and control procedures for a target firm depends on (i). Whether production is make-to-stock or make-to-order (which in turn depends on the relation between customer promise time and production lead time.) and (ii). Whether demand is for known production or anticipated production.

II. Literature Review

This paper introduce some typical papers involve in the topics with different subcategories, lost sales, backorders, shortages and deterioration as well as periodic review and continuous review. One critical factor playing major roles on the inventory theory is backorders. Much of the literatures on inventory models ignore backorders. Backorders means reordering and satisfying only part of the unsatisfied demand at a later stage when there is delay in meeting demands or inability to meet it at all. Most inventory models discuss two extreme situations when items are stock out. They are: (i). All demand within shortage period is backorder And (ii). All demand within shortage period is lost sales.

In real inventory systems,demands during the period of stock out can be partially captive. If demand is fully captive, the next replenishment will fulfill unsatisfied demands during the period of backorders. On the contrary, unsatisfied demands will be completely lost if demand cannot be fully captive, yet demand rate during the period of stock out is not a fix constant if to take backorders into consideration. The recent survey of [3, 4] and many other scholars have developed inventory models on related field, and initiated the concept of demands which will be changed through time cycle into model, also included backorder status, study on [9] optimal control of production inventory system with deteriorating items and dynamic cost, and a study on optimal control of production inventory system with deterioration items using Weibull distribution [1, 11]. Also there was an inventory model of replenishing the stock after a period of backorder [13], which is that deplete cycle always started from the period of backorder. A modification of the complete backorder assumptions and proposed the concept of partial backorders [12], which assumed the backorder ratio is a constant between 0 and 1. The assumption is that usually the time scale of backorder will become consumers' main pondering factor to accept backorder. This paper looked into assumptions and models for production inventory of a single item when shortages are allowed and there is an order to meet exogenous demand at a minimum cost.

III. Methodology

Inventory control models assumed that demand from customer are known for planning period both at present and past period. It is prohibited to have shortage of inventory since inventory cost is induced from the amount of product storage. Three assumptions were considered.

Notations and Assumptions

To develop the proposed models, the following notations and assumptions are used in this paper.

I (t) = inventory level at time *t*.

: Demand rate or the number of items required per unit time.

: Holding cost per unit time

Shortage cost per unit item per time

Production Set up cost per run

t : interval between runs.

q : Number of items produced per production run

if a production is made at time interval t, a quantity

q = Rt must be produced in each run. Since the stock in small time *dt is Rtdt, the stock in time period t is*

About[1] Department of Mathematical SciencesOlabisi Onabanjo University, Ago-Iwoye, Nigeria
Kay7bright2002@yahoo.com
About[2] Department of Mathematical SciencesOlabisi Onabanjo University, Ago-Iwoye, Nigeria bayoagunbiade@gmail.com

$$\int Rtdt = \frac{1}{2}Rt^2 = \frac{}{2}qt$$

Assumptions 1

In this model, we assume that demand is fixed and known, production is infinite and shortages are allowed although the cost of shortage is finite. i.e.

1. The inventory system involves only one item.
2. Replenishment occurs instantaneously on ordering i.e. lead-time is zero.
3. Demand rate *R(t)* is deterministic and given by *R (t)* ; $0 < t < T$.
4. Shortages are allowed and completely backlogged.
5. The planning period is of infinite length. The planning horizon is divided into sub-intervals of length T units. Orders are placed at time points $t_1, and\ t_2$, the order quantity at each re-order point being just sufficient to bring the stock height to a certain maximum level *S*

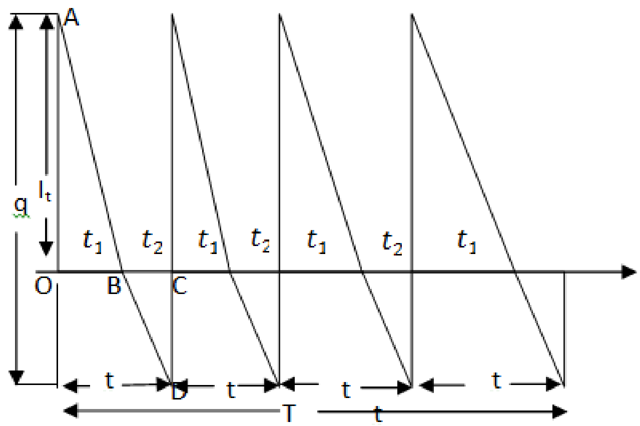

Figure 1.0 variation of inventory with time.

If $t = t_1 + t_2$ then

then $t_1 = \frac{I_t}{q}t$

Also,

then, $t_2 = \frac{q - I_t}{q}t$

Total inventory during time t = Area of triangle AOB

$$= \frac{1}{2}I_t\, t_1$$

Inventory holding ost during time t = $\frac{1}{2}C_1 I_t\, t_1$

Similarly, total shortage during time t = Area of $\Delta\ BCD$

$$= \frac{q - I_t}{2}t_2$$

Shortage cost during time t,

$$= C_2\frac{(q - I_t)}{2}t_2$$

Total cost during time

$$t = \frac{1}{2}C_1 I_t\, t_1 + C_2\frac{(q - I_t)}{2}t_2 + C_3$$

Average total cost during time t,

$$= \frac{1}{t}\left[\frac{1}{2}C_1 I_t\, t_1 + C_2\frac{(q - I_t)}{2}t_2 + C_3\right]$$

$$C(I_t, q) = \frac{C_1 I_t}{2q} + \frac{C_2(q - I_t)}{2q} + \frac{C_3 R}{q}$$

1.0

Differentiate equation 1.0 partially w.r.t I_t, q and equate to zero to obtain optimal inventory level (I_t) and optimum lot size (q)

$$\frac{(I_t, q)}{} = 0$$

$$I_{t0} = \frac{}{C_1 + C_2}q \qquad 1.1$$

Similarly, $\frac{\partial C(I_t, q)}{\partial q} = 0$

$$\sqrt{\frac{C_1 + C_2}{C\ C}}\cdot\sqrt{2C_3 R}$$

1.2 The optimal value of lot size q is

$$\sqrt{\frac{C_1 + C_2}{C\ C}}\cdot\sqrt{2C_3 R} = \sqrt{\frac{C_1 + C_2}{C}}\cdot\sqrt{\frac{2C_3 R}{C}}$$

1.3

Hence, equation 1.1 can be written as

$$\sqrt{\frac{}{C_1(C_1 + C_2)}}\cdot\sqrt{2C_3 R} \qquad 1.4$$

Substituting the values of I_{t0} , q_o in equation 1.0, we obtain the minimum average cost per unit time i.e

$$(I_{to}, q_o) = \sqrt{\frac{}{(C_1 + C_2)}}\cdot\sqrt{2C_1 C_3 R} \qquad 1.5$$

Optimum time interval between runs is given by

$$t_o = \frac{}{R} = \sqrt{\frac{C_1 + C_2}{C\ C}}\sqrt{\frac{2C_3}{R}} \qquad 1.6$$

Assumption II.

1. Fixed time interval t.

When time interval t is fixed, it means inventory is to be replenished after every fixed time t. All other assumptions in I above hold.

Total inventory holding cost during time $t = \frac{1}{2}C_1 I_t t_1$

Total shortage cost during time t = $\frac{1}{2}C_2(q - I_t)t_2$

Set up cost C_3 and time interval t are both constant therefore, average set up cost per unit time $\frac{C_3}{t}$ is also constant. It needs not to be considered.

Total average cost per unit

$$(I_t) = \frac{1}{t}\left[\frac{1}{2}C_1 I_t t_1 + \frac{1}{2}C_2(q - I_t)t_2\right]$$

Or

$$= \frac{C_1}{2q}.I_t^2 + \frac{C_2}{2q}(q - I_t)^2$$

2.0

$$(CI_t) = 0$$

2.1

Hence the minimum inventory level or order quantity given is

2.2

The minimum average cost per unit time from equation 2.0 is

$$(I_t) = \frac{}{2q}\left(\frac{}{C_1 + C_2}\right) q^2 + \frac{}{2q}\left(q - \frac{}{C_1 + C_2} . q\right)$$

$= \frac{1}{2} . \frac{}{C_1 + C_2} . q$ *or* $\frac{1}{2} . \frac{}{C_1 + C_2} Rt$ 2.3

Assumption III.

Finite production / planning rate.
The model here follows the assumptions in I except that production rate is finite. With this assumption, we found that inventory is zero at the beginning. It increases at a constant rate (K-R) for time t_1 until it reaches a level I_t. No replenishment during time t_2, inventory decreases at the rate R until it reaches zero. Shortage start piling up at constant rate R during t_3 until this backlog reaches a level s. Lastly, production start and backlog is filled at a constant rate K-R during t_4 till backlog become zero. This completes cycle.
The total time taken is $t = t_1 + t_2 + t_3 + t_4$.

Holding cost $= \frac{1}{2} C_1 I_t (t_1 + t_2)$

Shortage cost during time interval t = $\frac{1}{2} C_2 s(t_3 + t_4)$

Set up cost = C_3

Hence, total average cost per unit time t

$$\frac{- \quad (t_1+t_2) + \frac{}{2} C_2 s(t_3+t_4) + C_3}{} \quad 3.0$$

equation 3.0 *is a function of six* (6) *variables.*

Inventory level at time t_1 *is*

$I_t = (K - R)t_1$ 3.1

Also at time t_2 is

$I_t = Rt_2$ 3.2

$\therefore (K - R)t_1 = Rt_2$ 3.3

Also, S = t_3 , 3.4

and s = $(K - R)t_4$ 3.5

$(K - R)t_4 = Rt_3$ 3.6

Adding equation 3.3 & 3.6,

$(K - R)(t_1 + t_4) = R(t_2 + t_3)$.

Manufacturer's rate multiply by manufacturer's time gives manufactured quantity produced

q = $Kt_1 + Kt_4 = (t_1 + t_4)K$

$(t_1 + t_4) = \frac{q}{K}$, 3.7

Adding equations 3.2 and 3.4

$(t_2 + t_3)$

$(t_2 + t_3)$ – s

= (K-R) $(t_1 + t_4)$ - s

= $\frac{q}{K}$(K – R) – s

$\left(\frac{q}{K}\right)(K - R) - s$

$I_t = q\left(1 - \frac{R}{K}\right) - s$ 3.8

From equation 3.1 & 3.2

$t_1 + t_2 = \frac{}{K-R} + \frac{I_t}{R} -$ 3.9

And $(t_2 + t_3) = \frac{}{K-R} + \frac{s}{R}$

Hence, $t = t_1 + t_2 + t_3 + t_4$

$\left(\frac{}{K-R} + \frac{1}{R}\right)\left(q.\frac{K-R}{K}\right) = \frac{q}{R}$

3.10

Hence, equation 3.0 becomes,

$(q, s) = \frac{1}{2q} . \frac{}{K-R}\left[C_1\left\{q.\frac{K-R}{K} - s\right\} + C_2 s^2\right] + \frac{R}{q} C_3$ 3.11

$\frac{(q, s)}{\partial q} = 0$

Minimum lot size is

$\sqrt{\frac{2C_3(C_1+C_2)}{C\ C}} . \sqrt{\frac{KR}{K-R}}$

3.12

And

$\frac{\partial C(q,s)}{} = 0$, implies

$\frac{(C_1+C_2)}{}$,

$\sqrt{2C_3 . \frac{}{(C_1+C_2)C_2}} . \sqrt{\frac{R(K-R)}{K}}$.

3.13 Substituting q_o and s_o into equation 3.5 above, we have the optimum shortage cost cost

$$(q, s) = \sqrt{\frac{}{(C_1 + C_2)^3}} . \sqrt{\frac{(K - R)}{}}$$

$\sqrt{\frac{2C_1 C_2 C_3 R(K-R)}{K(C\ +C\ )}}$ 3.14

Optimum time interval t_o is

$- \quad \sqrt{\frac{(C_1 + C_2)}{(K-R)}}$ 3.15

Optimum inventory level

$\left(1 - \frac{R}{K}\right) - s_o$

$= \sqrt{\frac{}{C_1 + C_2}} . \sqrt{\frac{(K-R)}{K}} . \sqrt{\frac{2C_3 R}{C_1}}$

$\sqrt{\frac{2\ C_2\ C_3 R(K-R)}{KC\ (C\ +C\ )}}$ 3.16

IV. Numerical Applications

Example 1-
If a particular soap items has demand of 9000 units/year. The cost of one procurement is £100 and holding cost per unit is £2.40per year. The replacement is instantaneous and the cost of shortage is also £5per unit/year. We are required to determining the following:

i.) Economic lot size/Optimum lot size,
ii.) The number of orders per year ,
iii.) The time between the orders,
iv.) The total cost per year if the cost of one unit is £1.

Solution

Step I.

Demand rate R = £9000 units/year,

Holding cost, C_1= £2.40/unit/year,

Shortage cost C_2 = £5/unit/year,

Production set up cost per run = £100/procurement.

i.) From equation (1.3),

= 1,053units/run i.e. the optimum lot size / run is 1,053units.

ii.) The number of order per year = 8.55units/year

Recalled, equation (1.6)

Hence, the number of order per year is 8.55 or ≈ 9 number of times ordered per year.

iii. Time period between the order is as follows, From equation 1.6, = 0.117year

ere is approximately one month and thirteen days period between the order.

iv. From equation (1.5), the total cost per year if the cost of one unit is £1.

= £10710 per year

Hence, the total cost per year if the cost of one unit is £1 is £10,710

Example 2-

Consider an inventory system with the following data in usual Notations:

R = 20 engines/ day

= £12/ month or $\frac{12}{30}$ = £0.4/day.

$(fixed)$. We now want to check for the inventory level at the beginning of each month and the optimum cost per unit. Recall from equation (2.2)

Hence the optimum inventory level at the beginning of each month is 577 engines.

Also recall from equation 2.3

$$(I_t) = \; 2 \cdot C_1 + C_2 \quad Rt \quad 10$$

=

(I_t) =£115.38/unit of engines

i.e the minimum cost of producing an engine is £115.38

Example- 3.

A company has a demand of 12,000 units/year from an item and it can produce 2,000 such items per month. The cost of one set up is £400 and the holding cost/unit/month is £0.15. the shortage cost of one unit is £20 per year. Find the optimum lot size and the total cost per year, assuming the cost of one unit if £4. We can also find the maximum inventory manufacturing time and total time.

Given the following; R = 12,000

K = 2000 * 12 = 24,000/units/year,

=0.15 *12 = 1.8/unit/year,

= £20/year

=£400/set-up

Using equation 3.7

$$\sqrt{\frac{2 * 400 * (1.8 + 20)}{20}} \cdot \sqrt{}$$

= 3,410 units

The optimum lot size is 3,410units.

The total cost per year is considered by using equation 3.9

$$(q,s) = 12000 * 4 + \sqrt{\frac{{}_3\ (K-R)}{(C_1 + C_2)}}$$

$$(q,s) = 12000 * 4 \sqrt{\frac{2 * 1.8 * 20 * 400 * 12000(24000 - 12000)}{(20 + 1.8)}}$$

$C_o(q,s)$ = £50,185per year

The total cost per year is £50,185 when the cost of one item is £4

Using the equation 3.11, optimum inventory level at time t is

$$\sqrt{\frac{2 * 20 * 400 * 12000 * 10}{}}$$

= 1,564unit/production run

iii. Manufacturing time interval $t_1 + t_4$

Recall from equation 3.3,

$$(t_1 + t_4) = \frac{q}{K}, = \frac{3410}{24000}$$

Hence, the optimum inventory level at time t is 0.1421 year

iv. Optimum time interval t_0 is given by

$t_o \frac{q}{R} = \frac{3410}{1200}$, 0.2842 years

This means that, the minimum time interval required is 103 days i.e 3 months and 8 days.

V. Conclusion

It can be deduced that when replenishment cost and demand rate per unit time R increase, order quantity q, and relevant total cost C will increase. An increment of inventory holding cost per unit (h), backorder cost and penalty cost will lead to the phenomenon of increasing before diminishing. This idea can induce cost items in inventory depletion period having a trade – off relationship with cost items in backorders status. Also the decision about when an order should be placed will also be based on how low the inventory should be allowed to be depleted before the order arrives. The idea is to place an order early enough so that the expected number of units demanded during the replenishment lead time will not result in stock out every often

VI. CONTRIBUTION TO KNOWLEDGE

This research work contribute to knowledge in many areas of production or daily life activities where failure to meet up with demand/supply (activities) induced a nebulous cost and pay-price or (replenishment has to be done). Many industries can benefit from this through proper implementations/applications.

VII. REFERENCES

1) Al-khedhairi and Tadj L., Optimal control of a production inventory system with Weibull distributed deterioration. Vol. 1 , no 35, 1703 – 1714 (2007)
2) E. Davis and D. J. Elzinga, The solution of an optimal control problem in financial modelling, Operations Research, 19(1972), 1419–1433.
3) Kaplan, A. (1969). Stock Rationing. Management Science, 15(5): 260-267.
4) Katircioglu, K. and Atkins, D. (2003). Management inventory with multiple customer classes requiring different levels of service. Working Paper.
5) M. Bounkhel and L. Tadj, Minimizing energy use for a road expansion in a transportation system using optimal control theory, Applied Mathematics E-Notes, 6(2006), 159–166.
6) N. A. Derzko and S. P. Sethi, Optimal exploration and consumption of a natural resource: deterministic case, Optimal Control Applications & Methods, 2(1)(1981),1–21.
7) P. K. GUPTA and D.S. HIRA “Text book on Operations Research” (2005), ISBN, 81-219-0218-9
8) Rapp, Models for Optimal Investment and Maintenance Decisions, Almqvist & Wiksell, Stockholm; Wiley, New York 1974.
9) S. P. Sethi, Optimal equity financing model of Krouse and Lee: corrections and extensions, Journal of Financial and Quantitative Analysis, 13(3)(1978), 487–505
10) S.K. Goyal and B.C. Giri, Recent trends in modeling of deteriorating inventory, *European Journal of Operational Research* 134 (2001) 1-16.
11) Yacine B., Tadj L. and Messaoud B. Optimal control of production inventory systems with deteriorating items and dynamic cost (2008), 194 – 202.
12) WEE (1995) studied an EOQ model with shortages, assuming that the demand declines. Journal of the OPERATIONAL RESEARCH Society 46 (1995), pp. 658–662.
13) GOYAL, S.K., 1992. An EOQ model for deteriorating items with time shortages and a linear trend in demand - a comment, Journal of the OPERATIONAL RESEARCH