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highlights

- Generalized Wave Transformation
- Ellipse‟s Perimeter Evaluation
- Integral Inequality
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From the Chief Author's Desk

We see a drastic momentum everywhere in all fields now a day. Which in turns, say a lot to everyone to excel with all possible way. The need of the hour is to pick the right key at the right time with all extras. Citing the computer versions, any automobile models, infrastructures, etc. It is not the result of any preplanning but the implementations of planning.

With these, we are constantly seeking to establish more formal links with researchers, scientists, engineers, specialists, technical experts, etc., associations, or other entities, particularly those who are active in the field of research, articles, research paper, etc. by inviting them to become affiliated with the Global Journals.

This Global Journal is like a banyan tree whose branches are many and each branch acts like a strong root itself.

Intentions are very clear to do best in all possible way with all care.

Dr. R. K. Dixit
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The Factors Affecting Training Effectiveness And Research Findings

Dr. Osman Yildirim

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***Abstract*-This study aims at indicating the factors having effect on the efficiency of the training received by the employees. The study was conducted on 57 employees of banking sector in which bank benefited extensively from training activities. In this study, survey method has been applied to all participants. The scales in this study are the measures that have been used in other studies before. In this study, discriminant analyzes have been used to determine the variables that separate the individuals with progress from the others. According to the results of analysis for the determination of variables discriminating the persons to whom the training transfer has been achieved and to whom the training transfer has not been achieved, amongst the employees received training. The scope of this study is limited to the technical trainings since the effects of the behavioral trainings cannot be monitored in short periods and they are hard to observe. Facts of the study showed that companies should consider factors affecting on the tasks for their staff while designing training programs in terms of effectiveness. Increasing effect of training and development programs and its direct relationship to employee performance lead organizations to enhance their current employees' training programs and to hire new ones with higher effectiveness. This paper contributes to HR and training specialists by providing them with knowledge about how to design training programs effectively in their applications.**

Keywords- Human resource management; organizational performance; personal traits; training transfer; training effectiveness

I. INTRODUCTION

In the organizations convened particularly for profit, it is essential to get in return for the investment made or to measure the return of the investment made. With the request of the human resource managers of the enterprises, educating and developing trainings are designed for the employees of the enterprise. Educating and developing programmes, for which the enterprises allocate large amounts of resources, are expected to be efficient. As for the efficiency of training, it depends on many internal and external factors. These factors may result from the employee as well as the factors beyond the control of employee. Furthermore, the factors resulting from the employee or organization/environment are also in interaction with each other. So then, even if the isolation of organizational performance from the training is put into practice; in such a case individual, organizational/environmental factors need to be introduced, which enable the training to be efficient namely enable the training to achieve its goal. The calculation of the return of the investments made in training is a quite difficult process due to many reasons. The discrimination of the effect of the investment made in training from that of the other investments leads to a major difficulty in itself. Another reason raising difficulty in the calculation of contribution made by the investment in training to the performance of enterprise, on the other hand, is that a very long period of time is needed in order to monitor the effect made by the investments in training. On the other hand, it may not be revealed easily whether the contribution of the training to the performance of the enterprise results from the characteristics of the employee, or organizational structure of the enterprise or environmental factors beyond the control of the employee. Various research performed until today have found out that there are different factors having effect on the efficiency of training. In some of the research taking part in the literature, the factors resulting from the employee (individual factors), the factors encompassing the employee (organizational and/or environmental factors) have been taken into account for the purpose of researching the efficiency of the training, while the effects of aforementioned factors on the training transfer have been studied in some research.In the research performed by Elangovan and Karakowsky (1999), the factors resulting from the employee were examined dividing as the ones relating to motivation (Perceived Relevance, Desire of Participating in the Training, Expected Results, Self-Efficacy, Bondage) and the ones relating to ability (Knowledge Acquisition, Identifying Situations). As for the Environmental Factors, they were divided into two parts as the ones relating to the job (Job specifications, Norm and Group Pressure, Contextual Similarity, Manager Support) and the ones relating to the organization (Reward System, Organizational Culture). For the purpose of introducing the training transfer, the factors having influence on the learning such as their attitudes to training, their self-efficacies, ability, auditing focus, bondage, career etc. were handled in relation to the ones participated in the training. (Machin and Fogarty, 2004). In another research, the factors having effect on learning motivation and transfer motivation were examined and afterwards the effect of learning motivation and transfer motivation on the training transfer were attempted to be ascertained. (Kontoghiorghes, 2004). In the aforementioned study; personal ability, personality features, personal motivation, learning styles, training concept, manager support, colleague support, job definition, career, actuality of training, possibility of applying new knowledge and abilities, reward system, organizational strategy etc.

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were handled as the factors having effect on learning motivation and transfer motivation. The relationship of each factor with learning motivation was examined, afterwards it was stated in this study that it contributed to training transfer. In the structural efficiency model developed by Mathieu, Tannenbaum and Salas (1992), "career planning", "bondage", "job satisfaction", "situational conditions", "attitude to training", "response against training" were taken into account as the factors having effect on training motivation and learning phenomenon.In the research performed by Clarke (2002), training transfer was attempted to be defined, by being handled as the factors affecting the individual properties of the employees participating in the training, the structure of the training and working environment, efficiency of learning. With the structural model for training assessment set forth by Carey and Gregory (2003) concerning the assessment of the efficiency of training, the considered factors consist of employee properties, learning environment, training contents, motivation, learning guide, active participation and the integration of the contents; and in this study, the efficiency of training was attempted to be determined by means of the factors related to certain individual properties, environmental conditions and training material. In addition to the application models explained briefly above regarding whether the educating and developing trainings, which the enterprises provide to their employees, achieve the desired goal or not; thousands of pilot studies on environmental and individual factors that might be important in the trainings of trainees have taken part in the literature. In the pilot studies, the participators were addressed statements related to individual and environmental factors for the purpose of revealing the efficiency of training activities intended for educating and developing. In a field research, again the statements including the factors having effect on educating and developing training were asked to an educating and developing training group of 56 persons in order to reveal the efficiency and/or transfer level of training. (Fortheringhame, 1986). Individual and environmental factors are sometimes held limited in field studies performed with respect to the educating and developing training activities while they are held broader in some research. (Tracey, 1997). It may not be exact and reliable if to what extent the level of the efficiency of training or its transfer takes place is reported by the trainees. However; Gist, Bavetta, and Stevens (1990) developed a model introducing the ability and performance increase, which are the indicators of training transfer to a subject group of 56 persons and ability gaining levels in general. In a study that they performed, Elangovan and Karakowsky (1999) asserted without performing field research that some factors related with workplace and personality have a role in training transfer and/or efficiency and researched the effect of conditions resulting from the employee and environmental conditions on the trainings and the coherences amongst each other of these factors.When Rochford (2003) performed a research taking 56 students in his field study in order for the assessment of the efficiency of learning acitiviy, he took into account the individual preferences and the factors concerning training environment. Different approaches and models introducing the training transfer were also constituted. For example, the models adding financial dimensions into the four stage assessment models of Kirkpatrick were suggested for the purpose of calculating the return of training investment. While in some studies, the return of investment made in training was calculated viewing the increase of knowledge and ability increase or the increase of behavioural abilities of the trainees. Russell, Wexley and Hunter (1984) attempted to measure the knowledge and abilities gained in the end of training of 44 subjects and 22 control subjects addressing statements in a study similar to that and the behavioural properties gained were attempted to be modelled. In this study, the researcher attempted to measure the knowledge and abilities by means of a limited number of subjects, which the trainees gained due to the training and attempted to model the behavioural properties in the end of training. In this research; desire of learning, desire of participating in training, career expectation, learning styles, attitudes to the job, attitude to training, self-efficacy, auditing focus, emotional dedication to organization were handled respectively as the factors contributing to the efficiency of training and directly resulting from the individual. Desire of the person to learn, who receives training, is handled as an important factor as one of the factors having effect on the efficiency of training. Some researchers attempted to explain the desire of learning in the efficiency of training within the scope of expectation theory of Vroom. In the studies performed, desire of learning was regarded as associated with reliance on the organization and job satisfaction.(Bowers, Salas, Tannenbaum and Mathieu, 1995). Voluntary participation in training is regarded as a significant factor in the employees' finding the training beneficial. The employees who are usually asked to participate in the training compulsorily keeps negative attitudes against training and the desirable benefit may not be obtained from the training. In a research which they performed regarding the decisions of trainees to participate in training programmes, Hicks and Klimoski (1987) revealed that the ones willing to participate in the training were considerably satisfied with the training, more than the ones willing less, and they had more will to learn, reacted more positively and showed better performance in achievement test. Similarly, Ryman and Biersner (1975), with the research they performed, discovered that having a preference for participating in a training or not ensured a higher training achievement. In a laboratory experiment they performed; Baldwin, Magjuka and Loher (1991) set forth that when the trainees were given the chance of making selection among a few programmes and participating in the programme according to their selections ultimately, these individuals had more learning motivation prior to the training and learned more in comparison to the ones given no chance of making selection. The research have revealed that career expectation of employee has a positive effect on desire of learning. Accordingly, desire of learning may be regarded as a direct function of perception of the employee participating in training, regarding whether the training will avail for his career (Kontoghiorghes, 2001). Therefore, career expectation has an effect on the efficiency of training indirectly. If the employee has a career expectation, he/she believes the training will bring career opportunities for him/her, and that increases his/her motivation towards training. The research have shown that various personality features play a role in the efficiency of training. One of the aforementioned personality features, on the other hand, is self-efficacy. Some research have revealed that self-efficacy correlates with job performance. From this point of view, the transfer of aspects learned in training appears easier in the persons having self-efficacy in comparison to the ones not having self-efficacy (Bowers, Salas, Tannenbaum, and Mathieu, 1995).Locus of control is another personality trait that is considered to be related to the effectiveness of training. The research of Baumgartel, Reynolds and Pathan proves that. Noe and Schmitt suggest that the managers, who has higher achievement needs and has an inner locus of control, are more capable of carrying the new information, which they acquired in training, to the workplace environment and they also suggest that locus of control is less effective on motivation before training (Noe and Schmitt, 1986).It is a fact that each individual has a different way of learning. Learning styles may vary from person to person. However, the training sessions in workplaces do not consider these differences in employees' learning styles. Consequently, the employee, who received a training that is appropriate for his/her learning style will derive more benefit than the employee, who received a training that is not appropriate for his/her learning style. From this point of view, the level of reflecting what he/she had learnt to the workplace environment may differ in accordance with the learning style. The attitude against work is an effective factor on internalizing the training and reflecting this to their behaviors. To explain the attitude agaist work better, the subtitles of attitude agaist work, such as work satisfaction, organisational commitment and commitment to work are tried to be presented. Work satisfaction is defined as "gracious and positive feelings that are generated as a result of the evaluation of work and work experience of the individual" (Locke, 1983). If we are to literally express work satisfaction, we can say that it shows the "gratification or dissatisfaction that the individuals feel at work" (Davis and Newstrom, 1998). If an individual's work environment and his/her attitudes are inconsistent, this individual will eventually feel disappointed; and this situation negatively affects the desire for learning (Clarke, 2002). On the other hand, the research prove that the essential factors, such as workload-related stress, resoruce deficiency, not being able to make their own decisions about work, are negatively related with efficiency of training (Awoniyi, Orlando and Morgan, 2002).

Organisational commitment is the employee's empathy level with the organisation. On the other hand, organisational commitment implies the level of importance that the employer ascribes to the work (Blau, 1985). Comparing to others, it is considered that workaholics yearn to be more successful in trainings. Dixon ascertained that there is not any relationship between level of fancying the training sessions and after-training success tests (Dixon, 1990). Warr and Bunce, on the other hand, stated that there is not any relationship between the level of fancying the training sessions and test results that measure the learning levels (Warr and Bunce, 1995). Besides the individual factors mentioned above, sone of the factors that determine the efficiency of training may arise from environmental/organisational features beyond the employee. Environmental and/or organisational factors are not directly related to the employee. In this study, teaching style of the trainer, managerial support and peer support has been respectively discussed as the contributive and indirect environmental/organisational factors. The underlying thought behind this evaluation is the idea of the effects of environmental factors on learning motivation and on carrying these information into practice (Wendy, Leimbach, Holton and Bates, 2002). Therefore, learning is affected by both individual factors and environmental factors.Selecting appropriate teaching styles for learning styles is an important element for the efficiency of training. At this stage, it would be beneficial to help trainees to develop strategies that can adopt them to various situations (Vaughn ve Baker, 2001). Active learners can adapt to the teaching style. Relations of the employee with the social environment (colleagues, managers) in the work environment have an important role in putting the learnt knowledge into practice (Kontoghiorghes, 2004). It is known that the encouragement and guidance of the managers have an important role in employee's active learning process. One of the most important factors for effective learning is the support of managers and colleagues about carrying what the employee had learned into the work environment (Kontoghiorghes, 2004). Both the behaviors of the superiors and the colleagues have an important role in employee's success in carrying the knowledge and skills into work environment. It has a facilitating role when the superiors and colleagues give him/her a chance to carry the knowledge and skills into practice (Weiss, Huczynski and Lewis, 1980). Managerial support is an important factor that affects the efficiency of the training. When the employee thinks that he/she can apply the acquired knowledge and skills with the support of his/her manager, he/she will be more enthusiastic for training.Another factor that affects the efficient actualisation of the training is the support of colleagues. Support of the colleagues will help the employee to apply in the workplace what he/she had learned from training.

II. METHODOLOGY

This study is executed on employees of a medium scaled bank, who has participated in subject training, by taking one of the technical trainings as a basic. The purpose of this study is to reveal the individual and environmental factors related to the efficiency of the trainings.

III. DATA COLLECTION TOOL

To collect data, survey method has been used. The scales in this study are the measures that have been used in other studies before. The inclination to participate in this study, attitude towards training, managerial support and self-sufficiency scales have been developed by Guerro ve Sire (2001); will to learn scales have been developed by Tharenou (2001); locus of control scales have been developed by Burger (1986); work satisfaction scales have been developed by Currivan (1999); organisational commitment scales have been developed by Meyer, Allen and Smith (1993); commitment to work scales have been developed by Kanungo (1982); scales of support of colleagues and teaching style of the trainer have been developed by the researcher himself.All of the scales are on pentad Likert scale. On positive expressions, the evaluations have been made by giving 5 points to strongly and 1 point to strongly disagree. On negative expressions, exact opposite evaluation has been carried out. Cronbach α values that has been calculated to analyze the internal consistency of the scales and the scales are reliable within the scope of dimensions and all other expressions ($\alpha>0,60$). The sample of this study are constituted by the employees that the participant bank has determined (a total of 57 individuals). In this study, survey method has been applied to all participants. A placement test is applied by the institute to the participants before the training; in accordance with the test results, three different groups are constituted by the individuals who considered to be in need of training. Each group received the same training program by the same trainer. Before-training surveys are applied to the participants before training. The placement test that has been applied at the beginning is repeated and the survey that has been prepared for after-training is applied. To do the before and after-training matchings, the names of the participants have been asked in all tests and surveys applied. At the end of the second placement test, it has been proved that the knowledge levels of 57 employees were at the desired levels at the end of the second placement test. Therefore, it is agreed that effective training is schieved for 42% of the participants.

IV. Statistical Methods

When the structural matrix of the model, which is illustrated in Table 1, has been analyzed, it can be seen that the most meaningfully ($p<0,05$) related ($r>0,40$) variables are "considering the learning styles" ($r=0,82$), "will to participate in trainings" ($r=0,72$), "teaching style of the trainer" ($r=0,63$) and "attitude towards education" ($r=0,51$). Despite their coefficients are low ($r<0,40$), the meaningful variables ($p<0,05$) following the above-mentioned variables are respectively: "career expectations" ($r=0,30$), "managerial support" ($r=0,27$), "commitment to work" ($r=0,22$), "work satisfaction" ($r=0,21$), "organisational commitment" ($r=0,19$) and "locus of control" ($r=0,10$). "Will to learn", "self-sufficiency" and "support of the colleagues" veriables were not statistically meaningful.

Table 1. Discriminant Analyze Results

	Structural r	*F*	*p*
Considering the learning styles	0,82	30,08	0,00**
Will to participate in trainings	0,72	23,26	0,00**
Teaching style of the trainer §	0,63	9,66	0,00**
Attitude towards education §	0,51	6,86	0,01*
Career expectations §	0,30	11,76	0,00**
Managerial support §	0,27	5,41	0,02*
Will to learn §	0,24	2,07	0,16
Commitment to work §	0,22	5,88	0,02*
Work satisfaction §	0,21	9,58	0,00**
Organisational commitment §	0,19	13,35	0,00**
Self-sufficency §	0,11	2,96	0,09
Locus of control §	0,10	9,16	0,00**
Support of colleagues §	0,00	3,25	0,08

Box's M= 4,310 F=1,376 p=0,248>0,05
Canonical Correlation=0,680
Wilks Lamda=0,537 Chi-square=31,70 p=0,000<0,01§ It is not included to the model.**p<0,01 *p<0,05

Stepwise method revealed a model by using only two variables as base. These variables are "considering the learning style" (standardized coefficient = 0,71) and "will to participate in training" (standardized coefficient = 0,58). Other variables are not included in this model. Accordingly, consideration of learning styles is the most important factor for the technical trainings to be efficient, in other words for the employees to learn new information and skills and to put these two into practice at workplace. The second most important factor is the will to participate in the trainings. So, it is important for the employee to willingly participate in these trainings; it is also important that the employee is eager to request to participate in these trainings. According to the classification matrix results, which have been acquired by applying the model to the existing data, the model is able to predict the efficiency of the training with 80,7% of success, considering the attitudes of the employees.

V. Conclusion

Recently, human resources managers have been making important efforts to measure the feedback of the trainings that they carry out. Notwithstanding that the effort to prove whether the trainings positively contribute the results or not is an important development; the factors, which are related to the employees that cause these contribution to be proved, are not decently considered.According to the results that are related to individual and environmental factors, it has been determined that the employees are eager to participate in and benefit from trainings within their institutes; that they are partly participating with their own consent; however, it has also been determined that the employees consider these trainings as contributors to their careers. It can be clearly seen after the training that the participants generally have a positive attitude towards the training, that they find the trainer adequate in terms of providing active participation and that they think that the trainer considered the learning styles of the participants. Discriminant analysis revealed a model by using only two variables as base. These variables are "considering the learning styles" and "will to participate in trainings". Other variables were not included in this model. Accordingly, consideration of learning styles is the most important factor for the technical trainings to be efficient, in other words for the employees to learn new information and skills and to put these two into practice at workplace. The second most important factor is the will to participate in the trainings. So, it is important for the employee to willingly participate in these trainings; it is also important that the employee is eager to request to participate in these trainings.

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Exact Traveling Wave Solutions Of Variable Coefficients Nonlinear Pdes Using The Tanh-Function Method With A Generalized Wave Transformation

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***Abstract* - In this article, we have applied the tanh-function method using a generalized wave transformation to obtain the exact traveling wave solutions of nonlinear partial differential equations with variable coefficients via the KdV equation with forcing term and the generalized Gardner equation. It is shown that this method provides straightforward and powerful mathematical tools for solving nonlinear partial differential equations with variable coefficients.**

Keywords-The tanh-function method; Nonlinear partial differential equations with variable coefficients; Exact traveling wave solutions; Generalized wave transformation.

I. Introduction

In the nonlinear science, many important phenomena in various fields can be described by the nonlinear evolution equations (NLEES). Searching for exact soliton solutions of NLEEs plays an important and a significant role in the study on the dynamics of those phenomena. With the development of soliton theory, many powerful methods for obtaining the exact solutions of NLEES have been presented, such as the extended tanh-function method [1-4], the tanh-sech method [5-7], the sine-cosine method [8-10], the homogeneous balance method [11,12], the exp-function method [13-16], the Jacobi elliptic function method [17-20], the F-expansion method [21], the homotopy perturbation method [22,23], the variational iteration method [24], the inverse scattering transformation method [25], the Bäcklund transformation method [26], the Hirota bilinear method [27] and so on. To our knowledge, most of the aforementioned methods are related to constant coefficients models. Recently, much attention has been paid to the variable-coefficient nonlinear equations which can describe many nonlinear phenomena more realistically than their constant-coefficient ones.

The objective of this article is to apply the tanh-function method using a generalized wave transformation to find the exact solutions of the following KdV equation with variable coefficients and forcing term [28]:

$$u_t + f(t) u_{xxx} + g(t) u u_x = F(t), \qquad (1.1)$$

as well as the following generalized Gardner equation with variable coefficients [29]:

$$u_t + a(t) u u_x + b(t) u^2 u_x + c(t) u_{xxx} + d(t) u_x + f(t) u = 0, \qquad (1.2$$

where *f(t)*, *g (t)*, *a(t)*, *b(t)*, *c(t)* and *d(t)* are all functions of t while *F(t)* is an external forcing function varying with time t. Salas et al [28] have discussed Eq. (1.1) using the exp-function method and the projective Riccati equation method, while Li et al [29] have discussed Eq. (1.2) and obtained the exact solutions using the Hirota bilinear method. As we all know, Eq. (1.1) is the best typical representation of nonlinear dispersive wave equation. Eq. (1.2) includes KdV–equation, mKdV–equation and Gardner equation. These two equations are widely used in various branches of physics, such as plasma physics, fluid physics, quantum field theory, solid state and so on.The rest of this article is organized as follows: In section 2, we describe the tanh-function method using a generalized wave transformation. In section 3, we apply this method to Eqs. (1.1) and (1.2). In section 4, some conclusions are given.

II. Description of the Tanh-Function Method Using a Generalized Wave Transformation

Suppose that a nonlinear evolution equation is given by

$$F(u, u_t, u_x, u_{xx}, u_{tt}, \ldots) = 0, \qquad (2.1)$$

where $u = u(x,t)$ is an unknown function, F is a polynomial in u and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method:

Step1. Using the generalized wave transformation

$$u(x,t) = u(\xi), \qquad \xi = kx + \int \tau(t) dt, \qquad (2.2)$$

where k is a constant, while τ (t) is an integrable function of t to be determined. Then, Eq. (2.1) is reduced to the following ODE:

$$P(u, \tau(t) u', k u', k^2 u'', \ldots\ldots\ldots) = 0, \qquad (2.3)$$

where $' = \frac{d}{d\xi}$ and P is a polynomial in u and its total derivatives.

Step2. We suppose that Eq. (2.3) has the following formal solution:

$$u(\xi) = a_0 + \sum_{i=1}^{N} \left[a_i Y^{i}(\xi) + a_{-i} Y^{-i}(\xi) \right], \qquad (2.4)$$

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where N is a positive integer, and a_0, a_i, a_{-i} are constants, while Y(ξ) is given by

$$Y(\xi)=\tanh\xi. \qquad (2.5)$$

The independent variable (2.5) leads to the following derivatives:

$$\frac{d}{d\xi}=(1-Y^2)\frac{d}{dY},$$

$$\frac{d^2}{d\xi^2}=(1-Y^2)\left[-2Y\frac{d}{dY}+(1-Y^2)\frac{d^2}{dY^2}\right],$$

$$\frac{d^3}{d\xi^3}=(1-Y^2)\left[\left(6Y^2-2\right)\frac{d}{dY}-6Y(1-Y^2)\frac{d^2}{dY^2}+(1-Y^2)^2\frac{d^3}{dY^3}\right], \qquad (2.6)$$

and so on.

Step3. Determine the positive integer N in (2.4) by balancing the highest order derivatives and nonlinear terms in Eq. (2.3).

Step4. Substituting (2.4) along with (2.6) into (2.3) and equating the coefficients of powers of Y(ξ) to zero, we get a system of algebraic equations:

Step5. Solving these algebraic equations by *Maple* or *Mathematica*, we get the values of a_0, a_i, a_{-i}, k and τ (t).

Step6. Substituting these values into (2.4) and (2.2) we can obtain the exact traveling wave solutions of Eq. (2.1).

III. Applications

In this section, we use the method of Sec.2 to construct the exact solutions of two nonlinear evolution equations with variable coefficients via the KdV equation with forcing term and the generalized Gardner equation.

1) *On Solving The Kdv Equation With Variable Coefficients And Forcing Term (1.1)*

In order to obtain the exact solutions of Eq. (1.1), we assume that the solution of Eq. (1.1) can be written in the form

$$u(x,t)=\upsilon(x,t)+\int F(t)dt, \qquad (3.1)$$

where $\upsilon(x,t)$ is a new function to be determined. Substituting (3.1) into Eq. (1.1), we have

$$\upsilon_t+f(t)\upsilon_{xxx}+g(t)\beta(t)\upsilon_x+g(t)\upsilon\upsilon_x=0, \qquad (3.2)$$

where

$$\beta(t)=\int F(t)dt. \qquad (3.3)$$

Therefore, we use the generalized wave transformation (2.2) to reduce Eq. (3.2) to the following ODE:

$$\left[\tau(t)+kg(t)\beta(t)\right]\upsilon'(\xi)+k^3f(t)\upsilon'''(\xi)+kg(t)\upsilon(\xi)\upsilon'(\xi)=0. \qquad (3.4)$$

Balancing υ''' with $\upsilon\upsilon'$, we have N=2. Consequently, we have the solution of Eq. (3.4) in the form

$$\upsilon(\xi)=a_0+a_1Y(\xi)+a_{-1}Y^{-1}(\xi)+a_2Y^2(\xi)+a_{-2}Y^{-2}(\xi). \qquad (3.5)$$

From (2.6) and (3.5) we have the derivatives

$$\upsilon'=-2a_2Y^3-2a_{-2}Y^{-3}-a_1Y^2-a_{-1}Y^{-2}+2a_2Y+2a_{-2}Y^{-1}+a_1+a_{-1}, \qquad (3.6)$$

$$\upsilon''=6a_2Y^4+6a_{-2}Y^{-4}+2a_1Y^3+2a_{-1}Y^{-3}-8a_2Y^2-8a_{-2}Y^{-2}-2a_1Y-2a_{-1}Y^{-1}+2a_2+2a_{-2}, \qquad (3.7)$$

$$\upsilon'''=-24a_2Y^5-24a_{-2}Y^{-5}-6a_1Y^4-6a_{-1}Y^{-4}+40a_2Y^3+40a_{-2}Y^{-3}+8a_1Y^2+8a_{-1}Y^{-2}$$
$$-16a_2Y-16a_{-2}Y^{-1}-2a_1-2a_{-1}, \qquad (3.8)$$

and so on.

Substituting (3.5) - (3.8) into (3.4), collecting the coefficients of powers of Y (ξ) and setting each coefficient to zero, we obtain the following system of algebraic equations:

$$Y^5:-24a_2k^3f(t)-2a_2^2kg(t)=0,$$

$$Y^{-5}:-24a_{-2}k^3f(t)-2a_{-2}^2kg(t)=0,$$

$$Y^4:-6a_1k^3f(t)-3a_1a_2kg(t)=0,$$

$$Y^{-4}:-6a_{-1}k^3f(t)-3a_{-1}a_{-2}kg(t)=0,$$

$$Y^3:-2a_2\left[\tau(t)+kg(t)\beta(t)\right]+40a_2k^3f(t)+(2a_2^2-a_1^2-2a_0a_2)kg(t)=0,$$

$$Y^{-3}:-2a_{-2}\left[\tau(t)+kg(t)\beta(t)\right]+40a_{-2}k^3f(t)+(2a_{-2}^2-a_{-1}^2-2a_0a_{-2})kg(t)=0,$$

$$Y^{2}:-a_1\left[\tau(t)+kg(t)\beta(t)\right]+8a_1k^{3}f(t)+(3a_1a_2-a_{-1}a_2-a_0a_1)kg(t)=0,$$
$$Y^{-2}:-a_{-1}\left[\tau(t)+kg(t)\beta(t)\right]+8a_{-1}k^{3}f(t)+(3a_{-1}a_{-2}-a_1a_{-2}-a_0a_{-1})kg(t)=0,$$
$$Y^{1}:2a_2\left[\tau(t)+kg(t)\beta(t)\right]-16a_2k^{3}f(t)+(a_1^2+2a_0a_2)kg(t)=0,$$
$$Y^{-1}:2a_{-2}\left[\tau(t)+kg(t)\beta(t)\right]-16a_{-2}k^{3}f(t)+(a_{-1}^2+2a_0a_{-2})kg(t)=0,$$
$$Y^{0}:(a_1+a_{-1})\left[\tau(t)+kg(t)\beta(t)\right]-2(a_1+a_{-1})k^{3}f(t)+(a_0a_1+a_0a_{-1}+a_1a_{-2}$$
$$+a_{-1}a_2)kg(t)=0. \qquad (3.9)$$

Solving the algebraic equations (3.9) by *Maple* or *Mathematica*, we have the following cases of solutions:

Case1.

$$a_0=a_0,\quad a_1=0,\quad a_{-1}=0,\quad a_2=0,\quad a_{-2}=a_{-2},\quad g(t)=g(t),\quad \beta(t)=\beta(t),$$
$$f(t)=\frac{-a_{-2}g(t)}{12k^{2}},\qquad \tau(t)=-kg(t)\left[\beta(t)+\frac{2a_{-2}}{3}+a_0\right]. \qquad (3.10)$$

In this case, the exact solution of Eq. (1.1) has the from:

$$u(\xi)=a_0+a_{-2}\coth^2\xi+\beta(t), \qquad (3.11)$$

where

$$\xi=kx-k\int g(t)\left(\beta(t)+\frac{2a_{-2}}{3}+a_0\right)dt. \qquad (3.12)$$

Case2.

$$a_0=a_0,\quad a_1=0,\quad a_{-1}=0,\quad a_2=a_2,\quad a_{-2}=0,\quad g(t)=g(t),\quad \beta(t)=\beta(t),$$
$$f(t)=\frac{-a_2g(t)}{12k^{2}},\qquad \tau(t)=-kg(t)\left[\beta(t)+\frac{2a_2}{3}+a_0\right]. \qquad (3.13)$$

In this case, the exact solution of Eq. (1.1) has the from:

$$u(\xi)=a_0+a_2\tanh^2\xi+\beta(t), \qquad (3.14)$$

where

$$\xi=kx-k\int g(t)\left(\beta(t)+\frac{2a_2}{3}+a_0\right)dt. \qquad (3.15)$$

Case3.

$$a_0=a_0,\quad a_1=0,\quad a_{-1}=0,\quad a_2=a_{-2},\quad a_{-2}=a_{-2},\quad g(t)=g(t),\quad \beta(t)=\beta(t),$$
$$f(t)=\frac{-a_2g(t)}{12k^{2}},\qquad \tau(t)=-kg(t)\left[\beta(t)+\frac{2a_2}{3}+a_0\right]. \qquad (3.16)$$

In this case, the exact solution of Eq. (1.1) has the from:

$$u(\xi)=a_0+a_2\left(\tanh^2\xi+\coth^2\xi\right)+\beta(t), \qquad (3.17)$$

where

$$\xi=kx-k\int g(t)\left(\beta(t)+\frac{2a_2}{3}+a_0\right)dt. \qquad (3.18)$$

2) On Solving The Generalized Gardner Equation With Variable Coefficients (1.2)

In order to obtain the exact solutions of Eq. (1.2), we assume that the solution of Eq. (1.2) can be written in the form

$$u(x,t)=\upsilon(x,t)\exp\left\{-\int f(t)dt\right\}, \qquad (3.19)$$

where $\upsilon(x,t)$ is a new function to be determined later. The coefficients *a(t)* and *b(t)* in Eq. (1.2) can be also assumed to be in the forms

$$a(t)=k_1c(t)\exp\left\{\int f(t)dt\right\}, \qquad (3.20)$$

and

$$b(t)=k_2c(t)\exp\left\{2\int f(t)dt\right\}, \tag{3.21}$$

where k_1 and k_2 are nonzero constants.
Substituting (3.19) – (3.21) along with (2.2) into Eq. (1.2) we have the following ODE:

$$\left[\tau(t)+kd(t)\right]\upsilon'(\xi)+kk_1c(t)\upsilon(\xi)\upsilon'(\xi)+kk_2c(t)\upsilon^2(\xi)\upsilon'(\xi)+k^3c(t)\upsilon'''(\xi)=0. \tag{3.22}$$

Balancing υ''' with $\upsilon^2\upsilon'$ gives N=1. Consequently, we have the solution of Eq. (3.22) in the form

$$\upsilon(\xi)=a_0+a_1Y(\xi)+a_{-1}Y^{-1}(\xi). \tag{3.23}$$

From (2.6) and (3.23) we have the derivatives

$$\upsilon'=a_1-a_1Y^2(\xi)-a_{-1}Y^{-2}(\xi)+a_{-1}, \tag{3.24}$$

$$\upsilon''=-2a_1Y(\xi)+2a_1Y^3(\xi)+2a_{-1}Y^{-3}(\xi)-2a_{-1}Y^{-1}(\xi), \tag{3.25}$$

$$\upsilon'''=-2a_1+8a_1Y^2(\xi)-6a_1Y^4(\xi)-6a_{-1}Y^{-4}(\xi)+8a_{-1}Y^{-2}(\xi)-2a_{-1}. \tag{3.26}$$

Substituting (3.23) - (3.26) into (3.22), collecting the coefficients of powers of Y(ξ) and setting each coefficient to zero, we obtain the following system of algebraic equations:

$$Y^1: a_1^2kk_1c(t)+2a_0a_1^2kk_2c(t)=0,$$

$$Y^{-1}: a_{-1}^2kk_1c(t)+2a_0a_{-1}^2kk_2c(t)=0,$$

$$Y^2: -a_1\left[\tau(t)+kd(t)\right]-a_0a_1kk_1c(t)-\left[a_0^2a_1+a_1^2a_{-1}-a_1^3\right]kk_2c(t)+8a_1k^3c(t)=0,$$

$$Y^{-2}: -a_{-1}\left[\tau(t)+kd(t)\right]-a_0a_{-1}kk_1c(t)-\left[a_0^2a_{-1}+a_{-1}^2a_1-a_{-1}^3\right]kk_2c(t)+8a_{-1}k^3c(t)=0,$$

$$Y^3: -a_1^2kk_1c(t)-2a_0a_1^2kk_2c(t)=0,$$

$$Y^{-3}: -a_{-1}^2kk_1c(t)-2a_0a_{-1}^2kk_2c(t)=0,$$

$$Y^4: -a_1^3kk_2c(t)-6a_1k^3c(t)=0,$$

$$Y^{-4}: -a_{-1}^3kk_2c(t)-6a_{-1}k^3c(t)=0,$$

$$Y^0: (a_1+a_{-1})\left[\tau(t)+kd(t)\right]+\left[a_0a_1+a_0a_{-1}\right]kk_1c(t)+\left[a_{-1}^2a_1+a_1^2a_{-1}+a_0^2a_1+a_0^2a_{-1}\right]kk_2c(t)$$
$$-2(a_1+a_{-1})k^3c(t)=0. \tag{3.27}$$

On solving the algebraic equations (3.27) by *Maple* or *Mathematica*, we have the following sets of solutions:
Case1.

$$a_0=a_0,\quad a_1=a_1,\quad a_{-1}=0,\quad k=k,\quad c(t)=c(t),\quad d(t)=d(t),\quad k_1=\frac{12a_0k^2}{a_1^2},$$

$$k_2=\frac{-6k^2}{a_1^2},\qquad \tau(t)=\frac{-k}{a_1^2}\left[a_1^2d(t)+2k^2c(t)\left(3a_0^2-a_1^2\right)\right]. \tag{3.28}$$

In this case, the exact solution of Eq. (1.2) has the form:

$$u(\xi)=\left[a_0+a_1\tanh\xi\right]\exp\left\{-\int f(t)dt\right\}, \tag{3.29}$$

where

$$\xi=kx-\frac{k}{a_1^2}\int\left[a_1^2d(t)+2k^2c(t)\left(3a_0^2-a_1^2\right)\right]dt. \tag{3.30}$$

Case2.

$$a_0=a_0,\quad a_1=0,\quad a_{-1}=a_{-1},\quad k=k,\quad c(t)=c(t),\quad d(t)=d(t),\quad k_1=\frac{12a_0k^2}{a_{-1}^2},$$

$$k_2 = \frac{-6k^2}{a_{-1}^2}, \qquad \tau(t) = \frac{-k}{a_{-1}^2}\left[a_{-1}^2 d(t) + 2k^2 c(t)\left(3a_0^2 - a_{-1}^2\right)\right]. \tag{3.31}$$

In this case, the exact solution of Eq. (1.2) has the form:

$$u(\xi) = \left[a_0 + a_{-1}\coth\xi\right]\exp\left\{-\int f(t)dt\right\}, \tag{3.32}$$

where

$$\xi = kx - \frac{k}{a_{-1}^2}\int \left[a_{-1}^2 d(t) + 2k^2 c(t)\left(3a_0^2 - a_{-1}^2\right)\right] dt. \tag{3.33}$$

Case3. $a_0 = a_0, \quad a_1 = a_{-1}, \quad a_{-1} = a_{-1}, \quad k = k, \quad c(t) = c(t), \quad d(t) = d(t), \quad k_1 = \frac{12a_0k^2}{a_{-1}^2},$

$$k_2 = \frac{-6k^2}{a_{-1}^2}, \qquad \tau(t) = \frac{-k}{a_{-1}^2}\left[a_{-1}^2 d(t) + 2k^2 c(t)\left(3a_0^2 - 4a_{-1}^2\right)\right]. \tag{3.34}$$

In this case, the exact solution of Eq. (1.2) has the form:

$$u(\xi) = \left[a_0 + a_1\left(\coth\xi + \tanh\xi\right)\right]\exp\left\{-\int f(t)dt\right\}, \tag{3.35}$$

where

$$\xi = kx - \frac{k}{a_1^2}\int \left[a_1^2 d(t) + 2k^2 c(t)\left(3a_0^2 - 4a_1^2\right)\right] dt. \tag{3.36}$$

Case4.

$$a_0 = a_0, \quad a_1 = -a_{-1}, \quad a_{-1} = a_{-1}, \quad k = k, \quad c(t) = c(t), \quad d(t) = d(t), \quad k_1 = \frac{12a_0k^2}{a_{-1}^2},$$

$$k_2 = \frac{-6k^2}{a_{-1}^2}, \qquad \tau(t) = \frac{-k}{a_{-1}^2}\left[a_{-1}^2 d(t) + 2k^2 c(t)\left(3a_0^2 + 2a_{-1}^2\right)\right]. \tag{3.37}$$

In this case, the exact solution of Eq. (1.2) has the form:

$$u(\xi) = \left[a_0 - a_1\left(\coth\xi - \tanh\xi\right)\right]\exp\left\{-\int f(t)dt\right\}, \tag{3.38}$$

where

$$\xi = kx - \frac{k}{a_1^2}\int \left[a_1^2 d(t) + 2k^2 c(t)\left(3a_0^2 + 2a_1^2\right)\right] dt. \tag{3.39}$$

IV. Conclusion

In this article, the tanh-function method using the generalized wave transformation (2.2) is applied to obtain the exact solutions for NLEEs with variable coefficients. By using this method, we have successfully obtained many exact solutions of two nonlinear PDEs with variable coefficients via the KdV equation with forcing term and the generalized Gardner equation. The two proposed nonlinear PDEs play an important role in various branches in physics.

V. References

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Study of Ultrasonic Parameters of Polyvinyl Butyral

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Abstract- **Ultrasonic studied have been widely used to know more about the properties of polymer solution. Ultrasonic method find application for characterizing aspects of physico-chemical behavior such as the nature of molecular interactions in liquids as well as in liquid mixtures. In the present study, the polymer polyvinyl butyral is taken because of its vast application. The ultrasonic velocity, density and viscosity of the polymer solution were measured at different concentration and at different temperatures. From these data, ultrasonic parameters such as adaibatic compressibility(β), acoustic impedance(Z), relaxation time(τ),ultrasonic attenuation(α/f2) and stiffness constant(c) have been estimated using standard relations. These results are interpreted in terms of molecular interaction between the components of the mixture. The variation of β, (Z), τ, α/f2 and c with concentration at several temperatures have been studied. The results have been compared with those obtained from various theories and the results are analysed and interpreted in terms of molecular interactions.**

Keywords-Ultrasonic velocity, Adiabatic compressibility, Acoustic impedance, Relaxation time, Ultrasonic attenuation, Stiffness constant

I. INTRODUCTION

Ultrasonic study is a subject of extensive research and finds its usefulness in the fields of biology, biochemistry, dentistry, engineering, geography, geology, medicine, polymers, industry, etc. The study of propagation of ultrasonic waves in polymer solutions has been carried out by several investigators(1-4). Ultrasonic waves propagating in amaterial, carry valuable information about its microstructure and mechanical properties. Various ultrasonic parameters such as velocity, attenuationand spectral analysis of the ultrasonic signals can be used to extract this information on the material(5). A review of literature (6-10) on the acoustical studies in polymer solutions reveals that ultrasonic velocity measurements are very useful to understand the nature of molecular interaction in these system and the ionic interactions in aqueous electrolytic solutions(11-12). The measurement of ultrasonic velocity enables the accurate determination of some useful acoustic and thermodynamic parameters, their excess functions, which are highly sensitive to molecular interaction, thus provide qualitative information about the physical nature and strength in a solutions(13). Ultrasonic velocity is the most important physical quantity that plays an important role to understand the functions of concentration, temperature, density and viscosity in the solutions. It is also useful in gaining insight into the structure and bonding of associated molecular composition. When acoustic waves are propagated through a liquid, dissipation of energy takes place. The dissipation of acoustic energy that is associated with change in the molecular structure of the medium results from the finite that is required for these changes to take place. Stokes (14) in 1845 developed the first successful theory offering amechanism by which sound waves are attenuated. Kirchhoff's (15) in 1868 utilized the macroscopic property of thermal conductivity to develop a theory, which provides a second mechanism for explaining sound absorption in fluids. As polyvinyl butyral is an industrially important polymer, it is decided to study the molecular interaction of solutions of polyvinyl butyral in acetic acid by measuring the ultrasonic velocity in the system. In order to observe the influence of heat on molecular interaction of the present system, experiments were carried out at different concentration and at different temperature. Ultrasonic velocity and related data of liquid, liquid mixtures and solutions are found to be most powerful tool in investigation of molecular interaction and characterization. By measuring the ultrasonic velocity in liquids, their density and viscosity, it is easy to estimate different parameters and physical properties associated with liquid mixtures and solutions. The parameters derived from ultrasonic velocity measurements such as adiabatic compressibility, acoustic impedance, relaxation time ultrasonic attenuation, stiffness constant, etc. provide a better insight into molecular environment in liquids, mixtures and solutions.In the present investigation we have measured the density, viscosity and ultrasonic velocity of polyvinyl butyral (PVB), at different concentration and temperatures. Using the measured values of density, viscosity and ultrasonic velocity, we have computed different acoustic parameters of PVB like adiabatic compressibility, acoustic impedance, relaxation time ultrasonic attenuation and stiffness constant.

II. EXPERIMENTAL TECHNIQUE

Solutions of polyvinyl butyral of different concentration were prepared by adding known weight of polyvinyl butyral to a fixed volume of solvent (acetic acid) and then stirring under reflex until a clear solution was obtained. The concentration range studied in the solution at which the investigation is carried out is 0.5, 1.0%, 1.5%, 2% 2.5% and 3% (m/v) respectively. Ultrasonic velocities in the polymer solutions were measured by using ultrasonic interferometer at a frequency of 1MHz at the temperatures 35^{o}C, 40^{o}C, 45^{o}C, 50^{o}C, 55^{o}C at above said concentration. The accuracy of ultrasonic velocity determination in the solution is 0.001%. The temperature of the measuring cell was maintained constant by circulating water from a thermostatically controlled water bath with an accuracy of $\pm 0.1^o$C. The densities of the solutions were measured to an accuracy of ± 3 parts in 10^3 using a specific gravity bottle of 10 ml at different temperature. The viscosities of the polymer solutions were measured to an accuracy of 1%, using an Ostwald's

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viscometer at above said range of temperature by immersing the viscometer in the thermostatically controlled water bath. The viscometer was already calibrated with a standard liquid. Single pan macro balance with an accuracy of 0.001gm has been employed for mass measurements. Using the measured values of ultrasonic velocity, density and viscosity of the solutions for concentration range 0.5% - 3% at various temperatures, the related ultrasonic parameters are calculated and presented in tables and figures.

III. Computation

From the measured values of ultrasonic velocity, density and viscosity for the polymer solutions with concentration from 0.5% to 3% at the temperature 35°C to 55°C, the acoustic parameters such as adiabatic compressibility, acoustic impedance, relaxation time, ultrasonic attenuation and stiffness constant of the polymer are computed using the following relation.

Adiabatic compressibility $(\beta) = \frac{1}{v^2\rho}$

Acoustic impedance $(Z) = \rho\, v$

Relaxation time $(\tau) = \frac{4\eta}{3\rho v^2}$

Ultrasonic attenuation $(\frac{\alpha}{f^2}) = \frac{8\pi^2\eta}{3\rho v^3}$

Stiffness constant $(c) = v^2\rho$

Where,

v = ultrasonic velocity of the medium,

ρ = density of the medium,

η = viscosity of the medium,

ω = 2πf (angular frequency of ultrasonic wave)

λ = wavelength of the ultrasonic wave, measured by micrometer

f = frequency of ultrasonic wave.

c = stiffness constant of the medium or solution.

IV. Result and Discussion

The experimentally measured values of density(ρ), viscosity(η) and ultrasonic velocity(v) at different concentration and temperature, at 1MHz frequency are listed in Table 1, 2 & 3 and the variation with temperature and concentration are represented graphically in fig. 1, 2, 3, 4, 5 & 6 respectively. The variation of density with temperature and concentration is shown in table-1 and fig. 1 & 2. It is observed from table 1 and fig.1 & 2 that density decreases with increase in temperature and increases with increases in concentration. Table-2, presents the change in viscosity with temperature and concentration, the variation is shown in fig. 3& 4. It is evident from table-2 and fig 3& 4, that the viscosity decreases with increase in temperature and it increases with increase in concentration of polyvinyl butyral. The variation of ultrasonic velocity with temperature and concentration is shown in table-3 and fig. 5 & 6 respectively. It is evident from table-3 and from fig 5 & 6 that ultrasonic velocity decreases with increase in temperature and it increase with increasing concentration. The increase in ultrasonic velocity with solute concentration may be due to the association between the solute and solvent molecules, i.e., PVB and CH_3COOH. As concentration increases, one macromolecule may influence another indirectly by mutual interactions with other molecules. When these molecules are of solvent, the phenomenon is referred to as hydrodynamic screening and significantly determines the viscous flow properties of rather dilute solutions. Alternatively, in more concentrated solutions and bulk polymers, direct segment to segment interaction occur (16). They cause association between the PVB and acetic acid molecules and this may be responsible for the increase in ultrasonic velocity.

The values of adiabatic compressibility are given in table 4. Fig. 7 & 8 represents the variation of adiabatic compressibility with temperature and concentration. It is observed from table 4 and fig 7 & 8 that adiabatic compressibility increases with increase in temperature and decreases with increase in concentration. The rapid decrease of adiabatic compressibility with increase in concentration in the system indicates the formation of a more number of tightly bound system. Since the velocity increases with concentration and the density does so, the compressibility must decrease with concentration. The variation of acoustic impedance with temperature and concentration is shown in table-5 and fig. 9 & 10. It is seen from fig. 9 &10, that acoustic impedance decreases with increase in temperature and it increases with increase in concentration. It shows the trend of linear increase of specific acoustic impedance with concentration at a given temperature. As the strength of the intermolecular attraction increases, the ultrasonic velocity also increases, consequently the acoustical impedance value also increases. The variation of relaxation time with temperature and concentration is shown in table-6 and fig 11 & 12 respectively. It is observed from table-6 and fig 11&12, that relaxation time decreases with increase in temperature and increases with increasing concentration. It is directly proportional to adiabatic compressibility and viscosity. The variation indicates the adiabatic compressibility plays a dominant role in the system. Variation in viscosity with concentration and temperature are also found to affect the relaxation process. The variation of ultrasonic attenuation with temperature and concentration are shown in table-7 and fig.13 & 14 respectively. It is seen from table 7 and fig.13&14 that ultrasonic attenuation of the solutions is found to decrease with increase in temperature and increases with increasing concentration, which shows a similar trend to that of acoustical relaxation time. In table 8 and fig. 15 &16 the variation of stiffness constant with temperature and concentration of polyvinyl butyral is shown. It is found that it decreases with increases in temperature and it increases with increase in the concentration. This is due to a modification in the nature of the molecular interaction. The measurements of ultrasonic attenuation and relaxation time seem to indicate that viscosity contributes in a significant way to the absorption. The above behavior is obvious as per kinetic theory of fluids, ultrasonic velocity, viscosity, density studies in PVB solutions revealed the presence of solute-solvent interaction.

V. Conclusion

The ultrasonic parameters such as adiabatic compressibility, acoustical impedance, relaxation time, ultrasonic attenuation and stiffness constant are calculated for polyvinyl butyral in acetic acid. These derived acoustical parameters provide information on nature of molecular interaction, and it is found that there is a molecular association in all the system.

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Table1. Density (x 10^3 Kgm^{-3}) at different temperature and concentration, at 1MHz for PVB

Temperature(°C)→ Concentration(%)↓	35	40	45	50	55
0.5	1.023	1.016	1.010	1.002	1.000
1.0	1.025	1.017	1.011	1.006	1.002
1.5	1.026	1.019	1.011	1.006	1.002
2.0	1.026	1.019	1.012	1.006	1.002
2.5	1.026	1.020	1.012	1.007	1.003
3.0	1.027	1.020	1.013	1.007	1.004

Table2. Viscosity (x 10^{-3} Nsm^{-2}) at different temperature and concentration, at 1MHz for PVB

Temperature(°C)→ Concentration(%)↓	35	40	45	50	55
0.5	1.082	0.977	0.912	0.852	0.802
1.0	1.243	1.158	1.086	1.006	0.929
1.5	1.579	1.424	1.307	1.212	1.136
2.0	2.048	1.798	1.600	1.459	1.348
2.5	2.379	2.058	1.903	1.745	1.616
3.0	3.290	2.920	2.615	2.422	2.248

Table3. Ultrasonic velocity (ms^{-1}) at different temperature and concentration, at 1MHz for PVB

Temperature(°C)→ Concentration(%)↓	35	40	45	50	55
0.5	1102.7	1102.2	1099.1	1091.7	1082.4
1.0	1116.0	1115.4	1110.0	1106.1	1103.4
1.5	1125.2	1121.6	1120.6	1120.0	1116.9
2.0	1134.7	1134.1	1131.2	1126.6	1125.8
2.5	1141.4	1140.4	1136.6	1135.7	1134.9
3.0	1153.3	1149.9	1148.8	1144.8	1142.3

Table4. Adiabatic compressibility (x $10^{-10}Kg^{-1}ms^{2}$) at different temperature and concentration, at 1MHz for PVB

Temperature(°C) → Concentration(%) ↓	35	40	45	50	55
0.5	8.039	8.103	8.197	8.371	8.534
1.0	7.831	7.901	8.029	8.128	8.197
1.5	7.702	7.804	7.877	7.926	7.998
2.0	7.571	7.632	7.726	7.830	7.870
2.5	7.478	7.540	7.649	7.703	7.741
3.0	7.319	7.412	7.478	7.577	7.633

Table5. Acoustic impedance (x $10^{5}Kgm^{-2}s^{-1}$) at different temperature and concentration, at 1MHz for PVB

Temperature(°C) → Concentration(%) ↓	35	40	45	50	55
0.5	11.281	11.196	11.100	10.942	10.826
1.0	11.443	11.347	11.220	11.122	11.057
1.5	11.539	11.425	11.329	11.265	11.193
2.0	11.640	11.554	11.443	11.336	11.287
2.5	11.716	11.630	11.503	11.431	11.383
3.0	11.847	11.732	11.640	11.528	11.469

Table6. Relaxation time (x $10^{-12}s$) at different temperature and concentration, at 1MHz for PVB

Temperature(°C) → Concentration(%) ↓	35	40	45	50	55
0.5	1.160	1.056	0.997	0.951	0.913
1.0	1.298	1.220	1.163	1.090	1.015
1.5	1.622	1.482	1.373	1.281	1.212
2.0	2.067	1.830	1.648	1.523	1.414
2.5	2.372	2.069	1.941	1.792	1.668
3.0	3.210	2.886	2.607	2.447	2.288

Table7. Ultrasonic attenuation (x $10^{-14}s^{2}m^{-1}$) at different temperature and concentration, at 1MHz for PVB

Temperature(°C) → Concentration(%) ↓	35	40	45	50	55
0.5	2.078	1.892	1.792	1.721	1.666
1.0	2.298	2.161	2.069	1.947	1.818
1.5	2.847	2.610	2.420	2.259	2.143
2.0	3.599	3.187	2.879	2.671	2.482
2.5	4.106	3.584	3.374	3.118	2.903
3.0	5.500	4.958	4.484	4.223	3.957

Table8. Stiffness constant (x $10^9 Kgm^{-1}s^{-2}$) at different temperature and concentration, at 1MHz for PVB

Temperature(°C)→ Concentration(%)↓	35	40	45	50	55
0.5	1.244	1.234	1.220	1.195	1.172
1.0	1.277	1.266	1.245	1.230	1.220
1.5	1.298	1.281	1.270	1.262	1.250
2.0	1.321	1.310	1.294	1.277	1.271
2.5	1.337	1.326	1.307	1.298	1.292
3.0	1.366	1.649	1.337	1.320	1.310

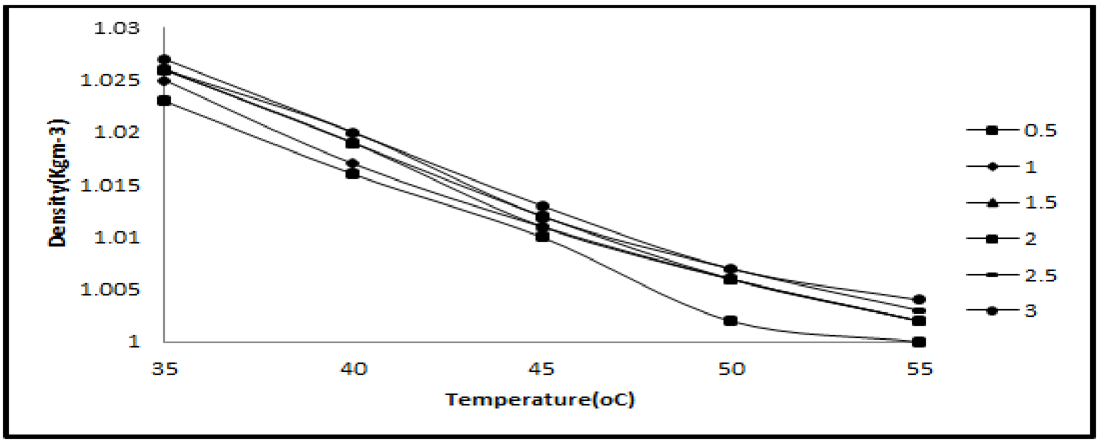

Fig. 1. Variation of Density with temperature at different concentration of PVB.

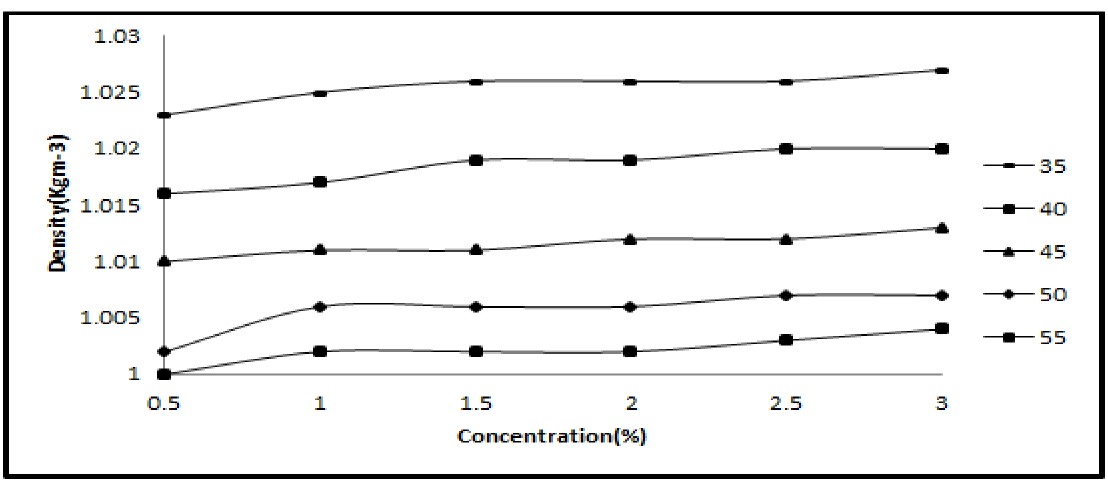

Fig. 2. Variation of Density with concentration at different temperature of PVB

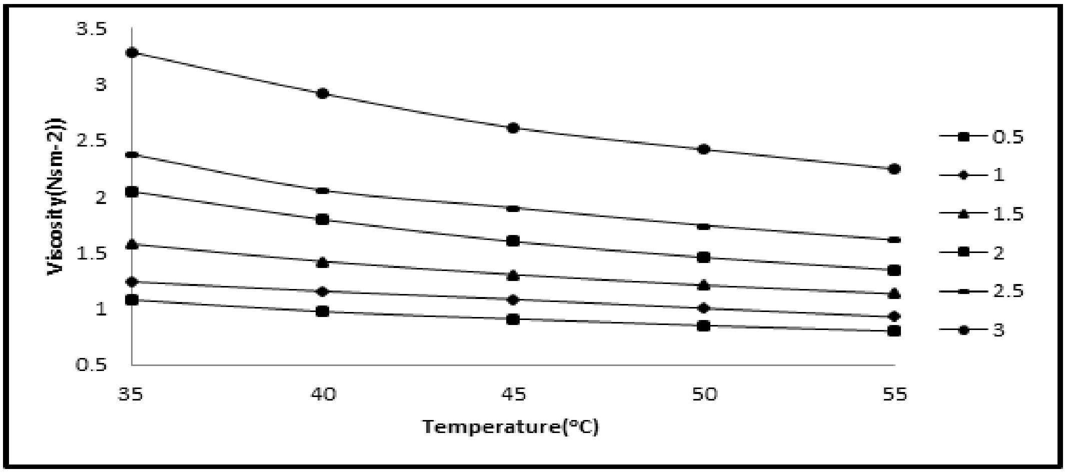

Fig. 3. Variation of Viscosity with temperature at different concentration of PVB.

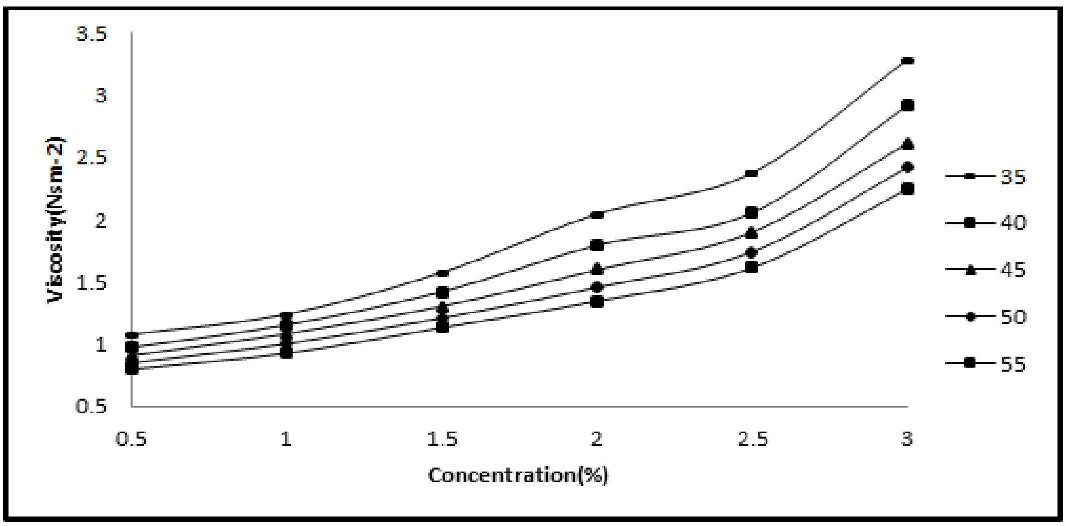

Fig. 4. Variation of Viscosity with concentration at different temperature of PVB.

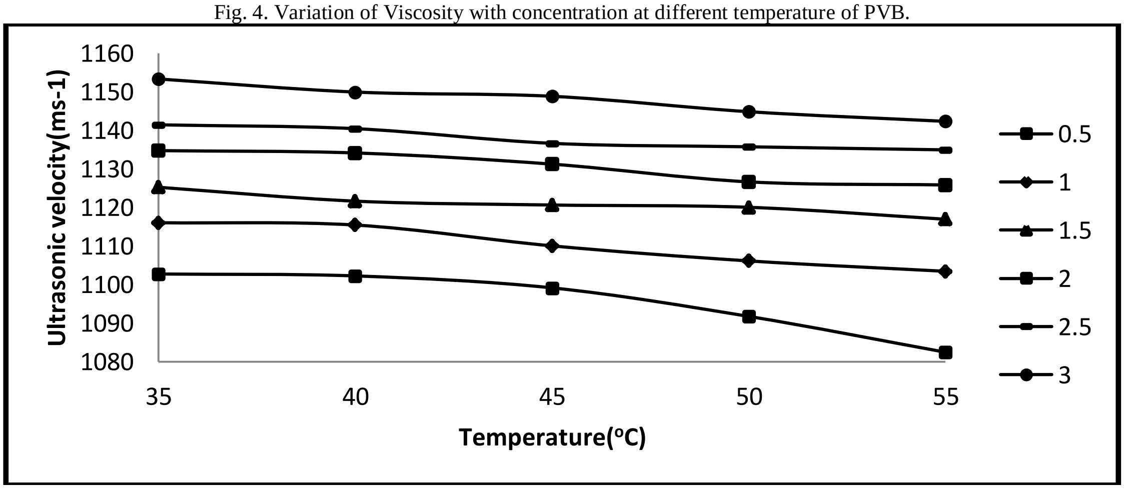

Fig. 5. Variation of Ultrasonic velocity with temperature at different concentration of PVB.

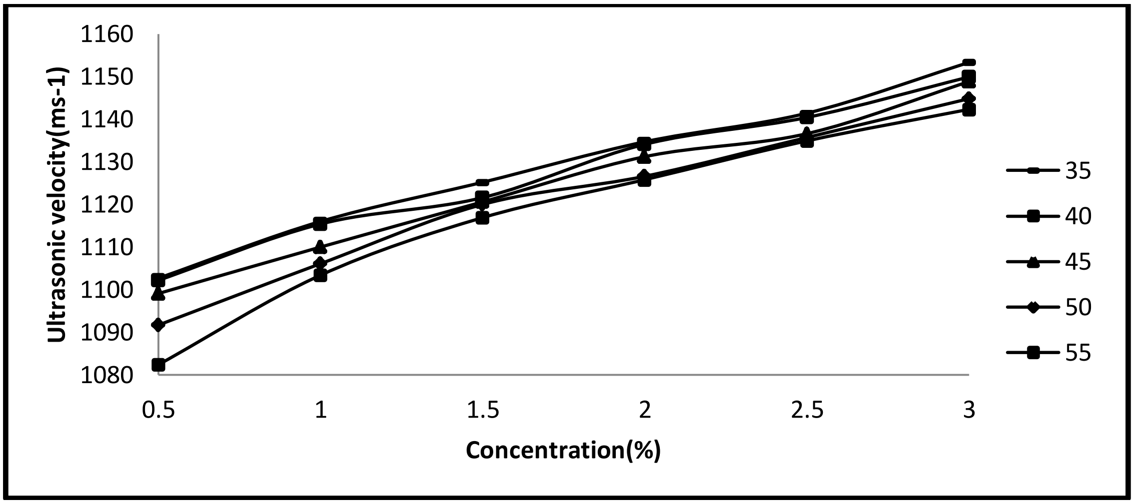

Fig. 6. Variation of Ultrasonic velocity with concentration at different temperature of PVB.

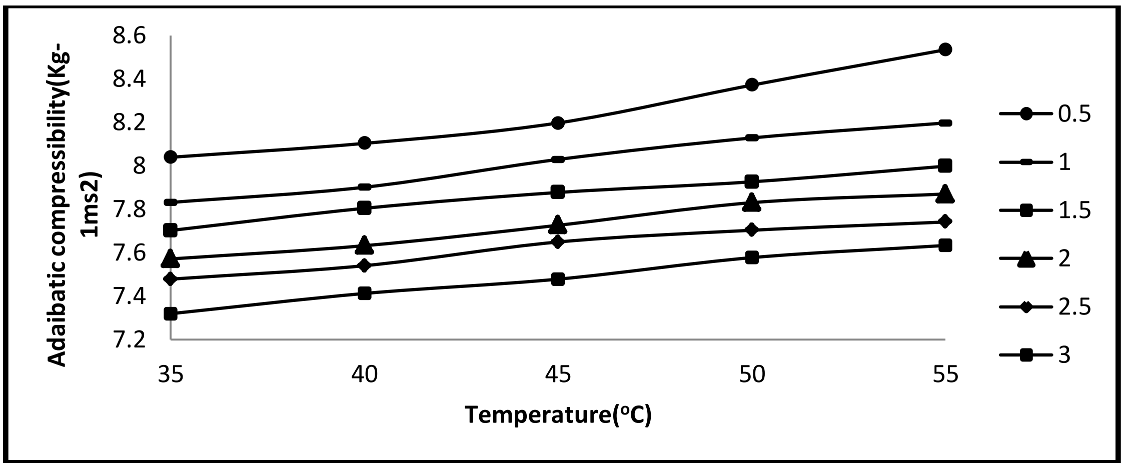

Fig. 7.Variation of Adiabatic compressibility with temperature at different concentration of PVB

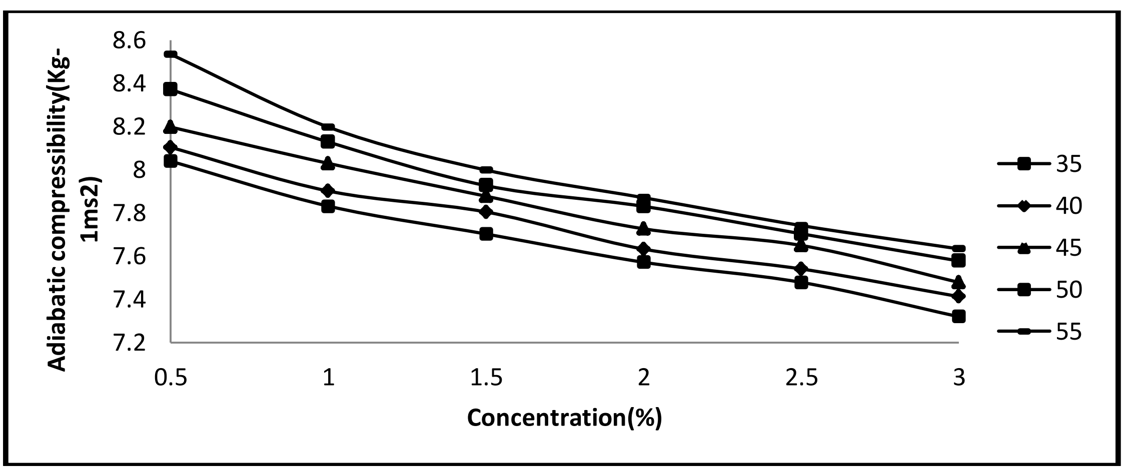

Fig. 8. Variation of Adiabatic compressibility with concentration at different temperature of PVB.

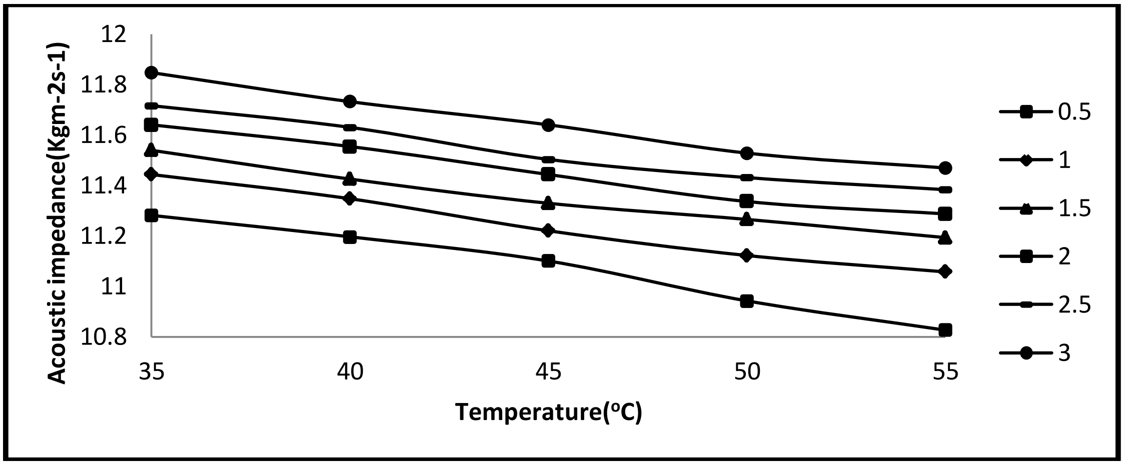

Fig. 9. Variation of Acoustic impedance with temperature at different concentration of PVB.

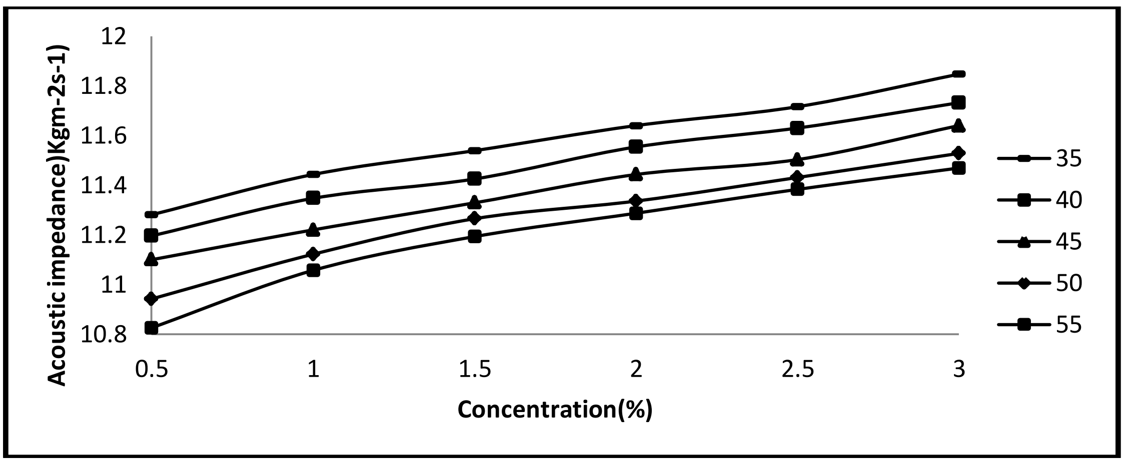

Fig.10. Variation of Acoustic impedance with concentration at different temperature of PVB.

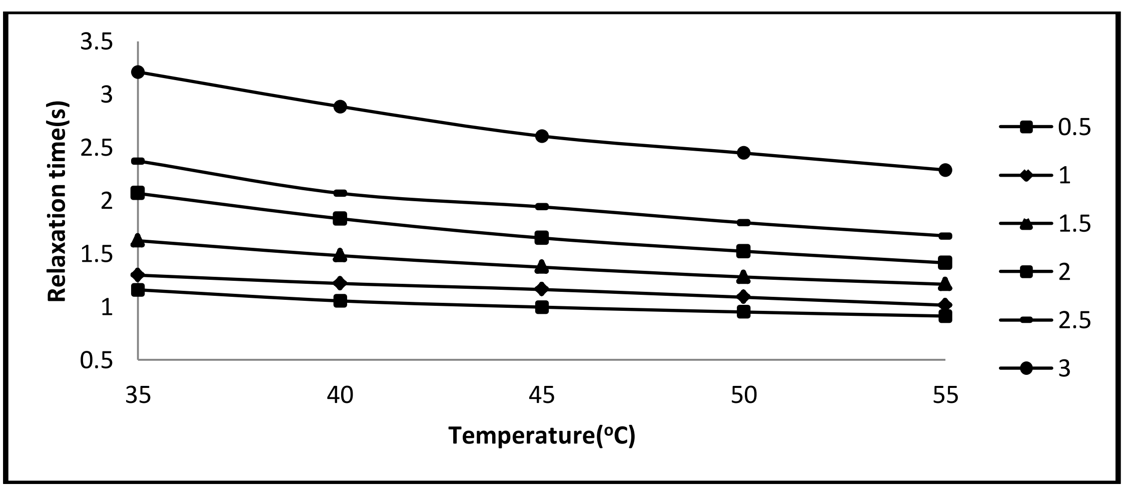

Fig. 11. Variation of Relaxation time with temperature at different concentration of PVB.

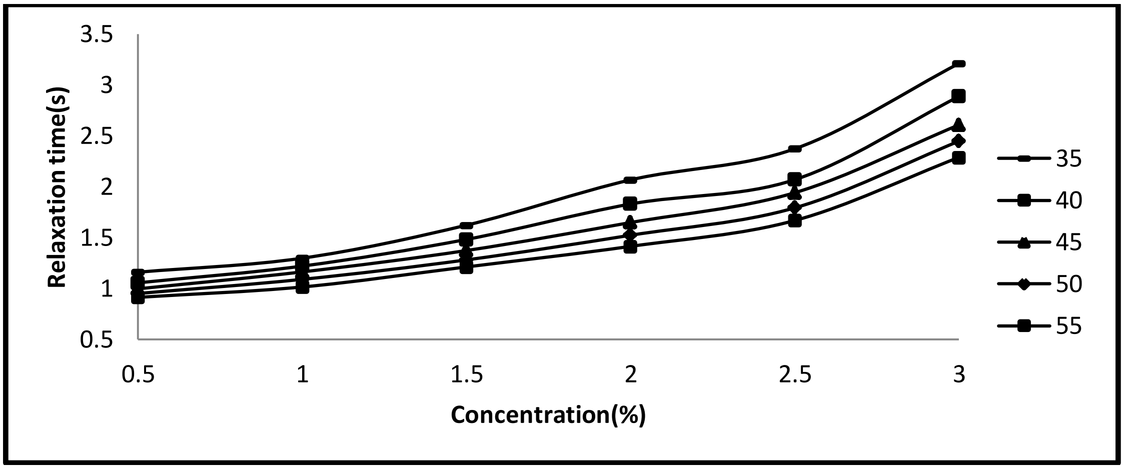

Fig.12. Variation of Relaxation time with concentration at different temperature of PVB.

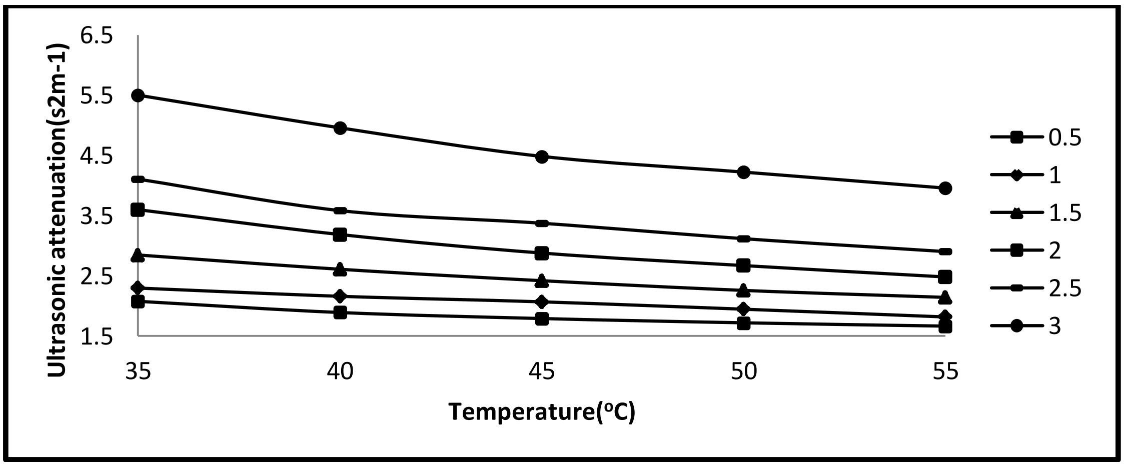

Fig. 13. Variation of Ultrasonic attenuation with temperature at different concentration of PVB.

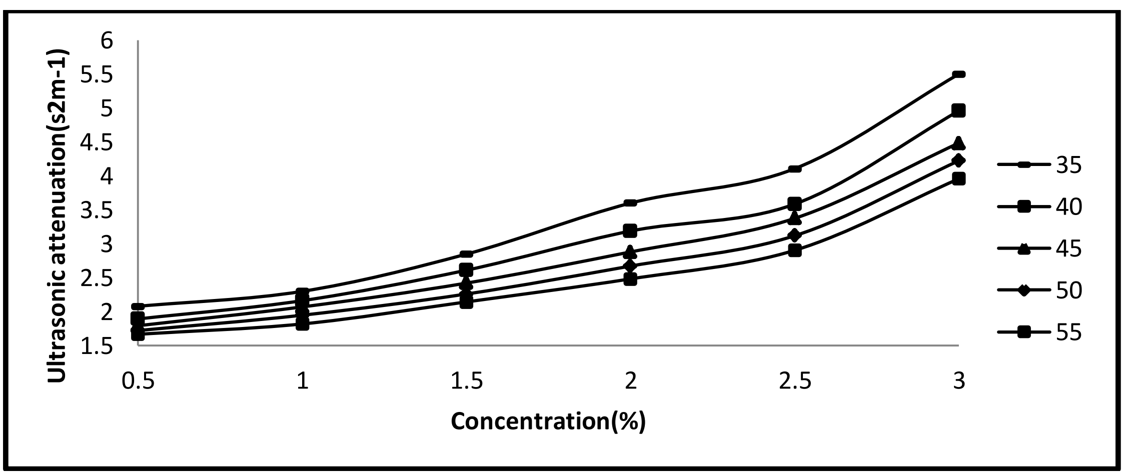

Fig. 14. Variation of Ultrasonic attenuation with concentration at different temperature of PVB.

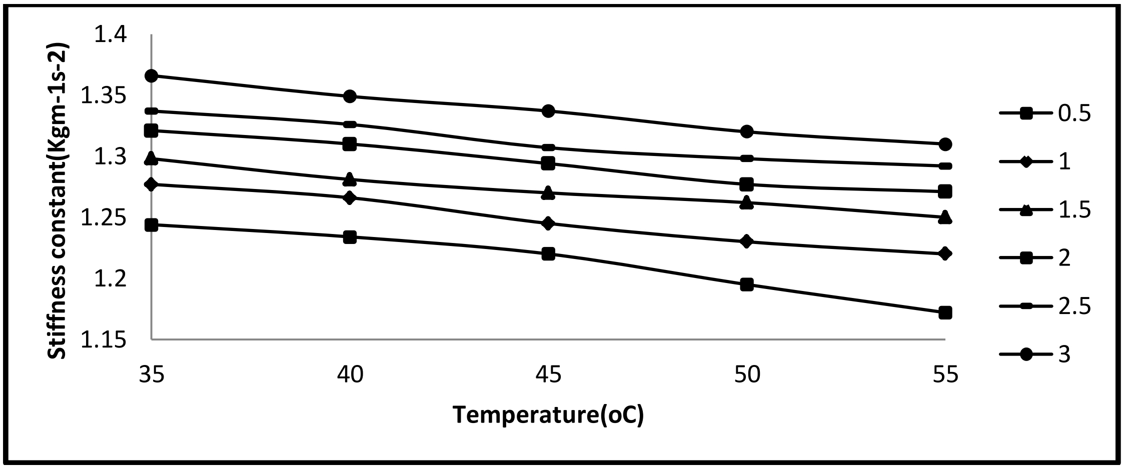

Fig. 15. Variation of Stiffness constant with temperature at different concentration of PVB.

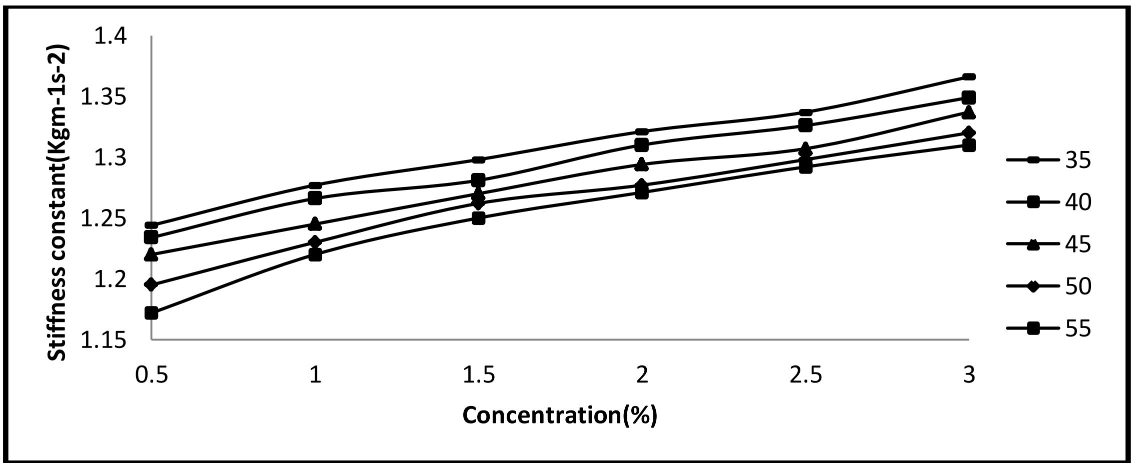

Fig. 16. Variation of Stiffness constant with concentration at different temperature of PVB.

Sum and product of k-regular fuzzy matrices

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***Abstract-In* this paper, we have determined conditions under the sum of k-regular fuzzy matrices are k-regular and the product of k-regular fuzzy matrices are k-regular.**

Keywords: Fuzzy matrices, k-regular fuzzy matrices, k-regularity.
AMS Classification: 15B33; 15A09

I. Introduction

Let $\mathcal{F}_n$ be the set of all n×n fuzzy matrices over the fuzzy algebra $\mathcal{F}$=[0,1] under the operations (+,•) defined as a+b=max{a,b} and a•b=min{a,b} for all a, b∈$\mathcal{F}$. R(A) and C(A) denotes the row space and column space of A respectively. A study of the theory of fuzzy matrices were made by Kim and Roush [2] analogous to that of Boolean matrices. Recently, a development of k-regular fuzzy matrices is made by Meenakshi and Jenita [5] analogous to that of generalized inverse of a complex matrix [1] and as a generalization of a regular fuzzy matrix [2,3]. A∈$\mathcal{F}_n$ is regular if there exists X such that AXA=A; X is called a generalized (g^-) inverse of A and is denoted as A^- [2]. A{1} denotes the set of all g-inverses of a regular matrix A.

II. Preliminaries

Definition 2.1[5]:

A matrix A∈$\mathcal{F}_n$, is said to be right k-regular if there exists a matrix X∈$\mathcal{F}_n$ such that $A^kXA=A^k$, for some positive integer k. X is called a right k-g-inverse of A.

Let $A_r\{1^k\}=\{X/\ A^kXA=A^k\}$.

Definition 2.2[5]:

A matrix A∈$\mathcal{F}_n$, is said to be left k-regular if there exists a matrix Y∈$\mathcal{F}_n$ such that $AYA^k=A^k$, for some positive integer k. Y is called a left k-g-inverse of A.

Let $A_\ell\{1^k\}=\{Y/\ AYA^k=A^k\}$.

In general, right k-regular is different from left k-regular. Hence a right k-g-inverse need not be a left k-g-inverse.

Lemma 2.1[2]:

For A, B∈$\mathcal{F}_n$, R(B)⊆R(A) ⇔ B=XA for some X∈$\mathcal{F}_n$, C(B) ⊆ C(A) ⇔ B=AY for some Y∈$\mathcal{F}_n$.

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Lemma 2.2[5]:

For A, B∈$\mathcal{F}_n$, and a positive integer k, the following hold.
(i) If A is right k-regular and R(B)⊆R(A^k) then B=BXA for each right k-g-inverse X of A .
(ii) If A is left k-regular and C(B) ⊆ C(A^k) then B=AYB for each left k-g-inverse Y of A .

Theorem 2.1[5]:

For A, B∈$\mathcal{F}_n$, with R(A)=R(B) and R(A^k)=R(B^k) then, A is right k-regular ⇔ B is right k-regular.

Theorem 2.2[5]:

For A, B∈$\mathcal{F}_n$, with C(A)=C(B) and C(A^k)=C(B^k) then, A is left k-regular ⇔ B is left k-regular.

III. K-regularity of sum of k-regular fuzzy matrices

In this section, we discuss the k-regularity of the sum of k-regular fuzzy matrices.

Lemma 3.1:

For A, B∈$\mathcal{F}_n$, we have the following.
(i) if R(A)= R(B) and AB=BA then R(A^k)= R(B^k).
(ii) if C(A)= C(B) and AB=BA then C(A^k)= C(B^k).

Proof:

(i) By Lemma (2.1), R(A)⊆R(B) ⇒ A=XB for some X∈$\mathcal{F}_n$.
⇒A^2=XBA
⇒ A^2=XAB
⇒A^2=X(XB)B
⇒$A^2=X^2B^2$

Thus proceeding we get,
R(A)⊆R(B)⇒$A^k=X^kB^k$ ⇒ R(A^k)⊆R(B^k)
Therefore, R(A)⊆R(B) and AB=BA ⇒ R(A^k)⊆R(B^k).
Similarly, we can prove that R(B)⊆R(A) and AB=BA ⇒ R(B^k)⊆R(A^k).
Therefore, R(A^k)= R(B^k).
Hence the proof.
(ii) Proof of (ii) is similar to that of (i) and hence omitted.

Theorem 3.1:

For A, B∈$\mathcal{F}_n$, if R(B)⊆R(A)⊆R(A+B) and AB=BA then, A is right k-regular ⇔ A+B is right k-regular.

Proof:

Since AB=BA,
$(A+B)^k=A^k+A^{k-1}B+\dots\dots+AB^{k-1}+B^k$.

Since $R(B)\subseteq R(A)$ by Lemma (2.1), B=XA for some $X\in\mathcal{F}_n$.
$B=XA \Rightarrow A+B=A+XA=(I+X)A \Rightarrow R(A+B)\subseteq R(A)$.
Since $R(A)\subseteq R(A+B)$, we get $R(A)=R(A+B)$.
Therefore by Lemma (3.1), $R(A^k)=R((A+B)^k)$.
Therefore by Theorem (2.1), it follows that, A is right k-regular $\Leftrightarrow$ A+B is right k-regular.

Theorem 3.2:

For $A, B\in\mathcal{F}_n$, if $C(B)\subseteq C(A)\subseteq C(A+B)$ and AB=BA then, A is left k-regular $\Leftrightarrow$ A+B is left k-regular.
Proof:
Proof is similar to Theorem (3.1) and hence omitted.
Remark 3.1:

For k=1, the above theorems reduce to the following:

Theorem 3.3[4]:

For $A, B\in\mathcal{F}_n$, if $R(B)\subseteq R(A)\subseteq R(A+B)$ $(C(B)\subseteq C(A)\subseteq C(A+B))$ then, A is a regular fuzzy matrix $\Leftrightarrow$ A+B is a regular fuzzy matrix.

Theorem 3.4:

For $A, B\in\mathcal{F}_n$, if A, B are right k-regular, R(A)=R(B) and AB=BA with $A_r\{1^k\}\cap B_r\{1^k\}\neq\phi$ then, A+B is right k- regular.
Proof:
Let X be a right k-g-inverse of A and B. That is, $A^kXA=A^k$ and $B^kXB=B^k$ ____(3.1).
Since AB=BA____________________(3.2),
$(A+B)^k=A^k+A^{k-1}B+\ldots+AB^{k-1}+B^k$.
By Lemma (2.1), $R(B)\subseteq R(A) \Rightarrow$
B=VA __________________(3.3)
for some $V\in\mathcal{F}_n$ and $R(A)\subseteq R(B) \Rightarrow$
A=UB______________________(3.4)
for some $U\in\mathcal{F}_n$.
We claim that X is a right k-g-inverse of A+B.
$(A+B)^kX(A+B) = (A^k+A^{k-1}B+\ldots+AB^{k-1}+B^k)(XA+XB) = (A^kXA+A^{k-1}BXA+\ldots+AB^{k-1}XA+B^kXA) + (A^kXB+A^{k-1}BXB+\ldots+AB^{k-1}XB+B^kXB)$
By using Equations (3.1) to (3.4), we have
$(A+B)^kX(A+B) = A^k+A^{k-1}B+\ldots+AB^{k-1}+B^k = (A+B)^k$.
Hence the theorem.
Remark 3.2:
The converse of the above theorem need not to be true. This is illustrated in the following .**Example 3.1:** Let

$$A+B=\begin{pmatrix}1 & 0.5 & 0.5\\ 0.5 & 0 & 0.5\\ 0.5 & 0.5 & 0\end{pmatrix}.\ \text{For } X=\begin{pmatrix}1 & 0 & 0.5\\ 0.5 & 0 & 0.5\\ 0.5 & 0.5 & 0.5\end{pmatrix},$$

$(A+B)^2X(A+B)=(A+B)^2$.
Hence A+B is 2-regular, X is a right 2-g-inverse of A+B.

$$A=\begin{pmatrix}1 & 0.5 & 0.5\\ 0.5 & 0 & 0\\ 0.5 & 0.5 & 0\end{pmatrix} \text{ and}$$

$$B=\begin{pmatrix}1 & 0.5 & 0\\ 0 & 0 & 0.5\\ 0.5 & 0 & 0\end{pmatrix} \text{ such that}$$

$$A+B=\begin{pmatrix}1 & 0.5 & 0.5\\ 0.5 & 0 & 0.5\\ 0.5 & 0.5 & 0\end{pmatrix}.$$

For $X=\begin{pmatrix}1 & 0 & 0.5\\ 0.5 & 0 & 0.5\\ 0.5 & 0.5 & 0.5\end{pmatrix}$ $A^2XA=A^2$ and $B^3XB=B^3$ holds.
Thus A is 2-regular and B is 3-regular also AB=BA but $R(B)\not\subset R(A)$.

Theorem 3.5:

For $A, B\in\mathcal{F}_n$, if A, B are left k-regular, C(A)=C(B) and AB=BA with $A_\ell\{1^k\}\cap B_\ell\{1^k\}\neq\phi$ then, A+B is left k-regular.
Proof:
This can be proved as that of Theorem (3.4) and hence omitted.

IV. K-REGULARITY OF PRODUCT OF K-REGULAR FUZZY MATRICES

In this section, we discuss the k-regularity of the product of k-regular fuzzy matrices.
Lemma 4.1:
For $A\in\mathcal{F}_n$, the following hold.
(i) if $R(A)= R(A^2)$ then $R(A)=R(A^k)$.
(ii) if $C(A)= C(A^2)$ then $C(A)= C(A^k)$.
Proof:
(i) By Lemma (2.1), $R(A)\subseteq R(A^2) \Rightarrow A=XA^2$ for some $X\in\mathcal{F}_n$.

$$\Rightarrow A^2=XA^3$$
$$\Rightarrow R(A^2)\subseteq R(A^3)$$

By Lemma (2.1), $R(A^2)\subseteq R(A) \Rightarrow A^2=YA$ for some $Y\in\mathcal{F}_n$.

$$\Rightarrow A^3=XA^2$$
$$\Rightarrow R(A^3)\subseteq R(A^2)$$

Hence $R(A^2)=R(A^3)$. Thus proceeding in this way, we get $R(A)= R(A^2) \Rightarrow R(A)=R(A^k)$.
(ii) Proof of (ii) is similar to that of (i) and hence omitted.

Lemma 4.2:
For $A\in\mathcal{F}_n$, if $R(A)=R(A^k)$ then the following statements are equivalent:
(i) A is regular
(ii) A is right k-regular

(iii) A^k is regular
(iv) A^k is right k-regular.

Proof:

If A is regular, then AXA=A for some X in $\mathcal{F}_n$, for $k\geq1$, premultiplying by A^{k-1} on both sides, we get $A^kXA = A^k$. Therefore A is right k-regular for all $k\geq1$. Thus (i)⇒(ii).

Since $R(A) \subseteq R(A^k)$, by Lemma (2.1) ,

$A=YA^k$ _________(4.1)

for some $Y\in\mathcal{F}_n$.

If A is right k-regular then,

$A^kXA = A^k$ ______________ (4.2)

for some $X\in\mathcal{F}_n$.

Premultiplying by Y on both sides in (4.2) and using (4.1), we get $YA^kXA = YA^k \Rightarrow AXA=A$. Therefore A is regular. Thus (ii)⇒(i).

If A^k is regular then,

$A^kZA^k = A^k$ _________ (4.3)

for some $Z\in\mathcal{F}_n$.

This can written as $A^kWA = A^k$ where $W=ZA^{k-1}$, hence A is right k-regular and W is a right k-g-inverse of A. Thus (iii)⇒(ii).

By using (4.1) and (4.2), $A^kXYA^k = A^k \Rightarrow A^kVA^k = A^k$ where V=XY. Therefore A^k is regular. Thus (ii)⇒(iii). Hence (i)⇔(ii)⇔(iii).

Next, let us prove that (iii)⇔(iv).

Premultiplying by $(A^k)^{k-1}$ on both sides in (4.3), we get $(A^k)^k ZA^k = (A^k)^k$. Thus A^k is right k-regular. Hence (iii)⇒(iv). If A^k is right k-regular then $(A^k)^k UA^k = (A^k)^k$ _______________(4.4)

for some $U\in\mathcal{F}_n$.

By using (4.1) and (4.4), we get $A^k UA^k = A^k$. Thus (iv)⇒(iii).

Hence the theorem.

Lemma 4.3:

For $A\in\mathcal{F}_n$, if $C(A)=C(A^k)$ then the following statements are equivalent:

(i) A is regular
(ii) A is left k-regular
(iii) A^k is regular
(iv) A^k is left k-regular.

Proof:

This can be proved as that of Lemma (4.2) and hence omitted.

Theorem 4.1:

For $A, B\in\mathcal{F}_n$, if $R(A)\subseteq R(AB)$ and AB=BA then, A is right k-regular ⇔ AB is right k-regular.

Proof:

Since $R(A)\subseteq R(AB)$ and AB=BA, R(A)=R(AB) .

Therefore by Lemma (3.1), $R(A^k)=R((AB)^k)$.

Therefore by Theorem (2.1), it follows that, A is right k-regular ⇔ AB is right k-regular.

Theorem 4.2:

For $A, B\in\mathcal{F}_n$, if $C(A)\subseteq C(AB)$ and AB=BA then, A is left k-regular ⇔ AB is left k-regular.

Proof:

This can be proved as that of Theorem (4.1) and hence omitted.

Remark 4.1:

For k=1, the above theorems reduce to the following:

Theorem 4.3[4]:

For the fuzzy matrices $A, B\in\mathcal{F}_n$, if R(A)=R(AB)(C(A)=C(AB)) then, A is a regular fuzzy matrix ⇔ AB is a regular fuzzy matrix.

Remark 4.2:

In particular for A=B, the Theorem (4.1) and (4.2) reduces to the following:

For $A\in\mathcal{F}_n$, the following hold.

(i) if $R(A)= R(A^2)$ then, A is right k-regular ⇔ A^2 is right k-regular.

(ii) if $C(A)= C(A^2)$ then, A is left k-regular ⇔ A^2 is left k-regular.

The condition $R(A)= R(A^2)$ is a necessary condition in (i) and the condition $C(A)= C(A^2)$ is a necessary condition in (ii). This is illustrated in the following:

Example 4.1:

Let us consider $A=\begin{pmatrix} 1 & 0.5 & 0 \\ 0 & 0 & 0.5 \\ 0.5 & 0 & 0 \end{pmatrix}$. For this A, $A^2 = \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 \end{pmatrix}$, $A^3=\begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0.5 \end{pmatrix}$.

For $X=\begin{pmatrix} 1 & 0 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 \end{pmatrix}$, $A^3XA=A^3$ holds. Therefore A is 3-regular. $A^5= A^4$. $R(A) \not\subset R(A^2)$.

$(A^2)^2XA=(A^2)^2$. Thus A^2 is 2-regular. Hence A^2 is right 2-regular but A is not a right 2-regular. Therefore the condition $R(A)= R(A^2)$ is necessary.

Remark 4.3:

By Lemma (4.1), $R(A)= R(A^2) \Rightarrow R(A)=R(A^k)$.

Therefore by lemma (4.2), A is regular ⇔ A^k is right k-regular ⇔ A is right k-regular ⇔ A^2 is right k-regular. Thus for $R(A)=R(A^2)$, k-regularity coincides with regularity.

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Synthesis And Characterization Of 2, 4-Dihydroxy Substitted Chalcones Using PEG-400 As A Recyclable Solvent

GJSFR- B Clasification FOR 030503,030404,030305,030306

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***Abstract-* A novel method for the synthesis of 2,4-dihydroxy substituted chalcones via Claisen-Schmidt is introduced using recyclable PEG-400 as an alternative reaction solvent. The reaction is clean with excellent yield, shorter reaction time and reduces the use of volatile organic compounds (VOCs). The structures of the synthesized compounds were confirmed by IR, mass spectroscopy and elemental analysis.**

Keywords- Chalcone, Claisen-Schmdit condensation, PEG-400, IR, Mass and Elemental spectral analysis

I. Introduction

Chalcones are well known intermediates for synthesizing various heterocyclic compounds. The compounds with the backbone of chalcones have been reported to possess various antimicrobial[1], anti-inflammatory[2], analgesic[3], antiplatelet[4], antiulcerative[5], antimalarial[6], anticancer[7], antiviral[8], antileishmanial[9], antioxidant[10], antitubercular[11], antihyperglycemic[12], immunomodulatory[13], inhibition of chemcical mediators release[14], inhibition of leukotriene B_4[15], inhibition of tyrosinase[16] and inhibition of aldose reductase[17] activities. The presence of a reactive α,β - unsaturated ketone function in chalcones is found to be responsible for their antimicrobial activity.Several strategies for the synthesis of these system, based on the formation of carbon-carbon bond have been reported. Among them the direct Aldol condensation and Claisen-Schmidt condensation still occupy prominent positions. The main method for the synthesis of chalcones is the classical Claisen-Schmidt condensation in the presence of aqueous alkaline bases,[18] $Ba(OH)_2$[19] LiOH, microwave irradiation and ultrasound irradiation.[20] They are also obtained via Suzuki reaction,[21] Witting reaction, Friedel-Crafts acylation with cinnamoyl chloride, or Photo-Fries rearrangement of phenyl cinnamates. In aldol condensation the preparation of chalcones requires at least two-steps aldol formation and dehydration. Since aldol addition is reversible, mukaiyama or Claisen-Schmidt condensation approach of using enol ether has emerged as an alternative pathway.The aldol reaction is also performed under acidic medium,[21] using HCl, BF_3, B_2O_3, p-toluenesulfonic acid etc. Recently various modified methods for the synthesis of chalcones has been reported, such as by using $SOCl_2$,[22] natural phosphate, lithium nitrate,[23] amino grafted zeolites,[24] zinc oxide, water,[25] Na_2CO_3,[26], PEG_{400},[27] silicasulfuric acid,[28-29] $ZrCl_4$ and ionic liquid[30] etc. Jhala et al. synthesized chalcone using basic alumina under micro wave irradiation. However, many of these methods suffered from harsh reaction condition, toxic reagents, strong acidic or basic conditions, prolonged reaction-times, poor yields and low selectivity. Although, several modifications have been made to counter these problems. There is still a need for the development of selective and better strategies for the synthesis of α, β-unsaturated carbonyl compounds.However, many of these methods suffered from harsh reaction condition, toxic reagents, strong acidic or basic conditions, prolonged reaction times, poor yields and low selectivity. Although, several modifications had been made to counter these problems. There is still a need for the development of selective and better strategies for the synthesis of α,β-unsaturated carbonyl compounds. Herein for the first time we describe a simple and convenient method for the synthesis of chalcones using poly ethylene glycol (PEG) has been found to be an interesting solvent system. In continuation of own work on chalcones as precursors in the synthesis of various heterocycles[31], we have planned to synthesize a series of novel hetero chalcones by applying the principles of green chemistry, using PEG400 as an alternative reaction medium[32]. PEG is an environmentally benign reaction solvent, is it non-toxic, inexpensive, potentially recyclable and water soluble, which facilitates its removal from the reaction product. The synthetic pathway is presented in Scheme 1 and physicochemical data and Spectroscopic data for the synthesized compounds are given Table (1-3).

II. Materials and methods

All the products were synthesized and characterized by their spectral analysis chemicals, 2 4-dihydroxy acetophenon, 2-chloro benzaldehydes, 4-chloro benzaldehydrs, 3-nitro benzaldehydes, S.D. fine Chemicals (India). Melting points were determined in an open capillary tube and or uncorrected. IR spectra were recorded in KBr on a JASCO FT IR-5300.. The mass spectra were recorded on LCMS-2010 DATA REPORT SHIMADZU. Elemental analysis was carried out on a FLASH EA 1112 SERIES CHN REPORT THERMO FINNIGAN. Chalcones were synthesized by clasien- Schmidt condensation[33] using PEG-

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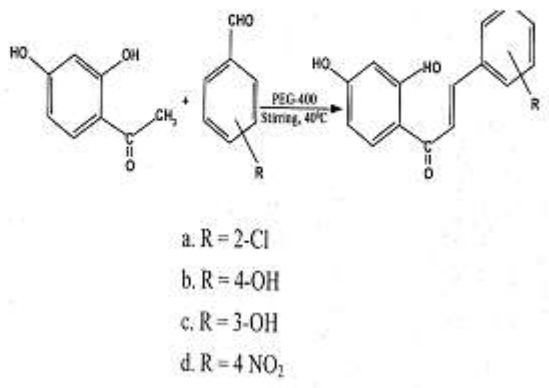
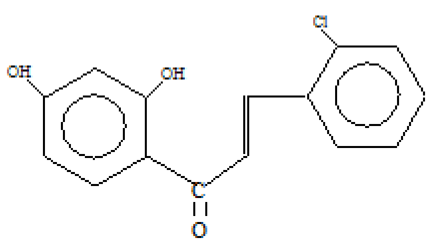
400 as reaction solvent. The chemicals and solvents used were of laboratory grade and were purified completion of the reaction was monitored by thin layer chromatography on precoated sheets of silica gel-G (Merck, Germany) using iodine vapour for detection. The synthetic pathway is presented in Scheme 1 and physicochemical data and spectroscopic data for the synthesized compounds are given Table (1-3)



Scheme 1: Synthetic diagram of 2,4 dihydroxy Substituted chalcones

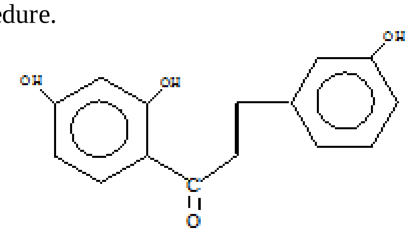
1) Synthesis of 3-(2-chlorophenyl)-1-(2,4-dihydroxyphenyl) prop-2-en-1-one



An equimolar mixture of 2,4-hydroxy acetophenone (2gms), aromatic aldehydes (2.1gms) and KOH (5ml) was stirred in PEG-400 (15 ml) at 40°C for 1 hour. after the completion of the reaction (monitored by TLC), the crude mixture was worked up in ice-cold water (100 ml). The product which separated out was filtered. The filtrate was evaporated to remove water leaving PEG behind. The same PEG was utilized to synthesize further chalcone.

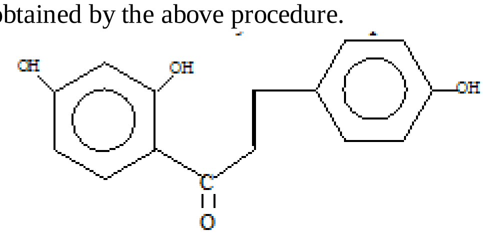
2)Synthesis of 1-(2,4-dihydroxyphenyl)-3-(4-hydroxyphenyl) prop-2-en-1-one

Reaction with 2,4-dihydroxy acetophenone (2 gms) and 4-hydroxy benzal- dehyde (2.1 gm), (2,4-dihydroxyphenyl)-3-(4-hydroxyphenyl) prop-2-en-1-one was obtained by the above procedure.



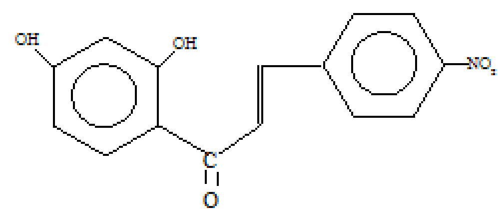
3) Synthesis of 1-(2, 4-dihydroxyphenyl)-3-(3hydroxyphenyl) prop-2-en-1-one

A mixture of 2,4-dihydroxy acetophenone (2 gm) in PEG (15ml)and 3-hydroxy benzaldehyde (2.1 gm); 1-(2-4-dihydroxyphenyl)-3-(3-hydroxyphenyl)prop-2-en-1-one was obtained by the above procedure.



4) Synthesis of 1-(2,4-dihydroxyphenyl)-3-(4-nitrophenyl)prop-2en-1-one

1-(2,4-dihydroxyphenyl)-3-(4-nitrophenyl)prop-2en-1-one was obtained by the above procedure except that starting material used was 2,4-dihydroxy acetophenone(2.0gm) in ethanol (15ml) and 4 nitro benzaldehyde. (2.2 gm)



III. RESULTS AND DISCUSSIONS

The Claisen-Schmidt condensation (34) is an important C-C bond formation for the synthesis of 1,3-diaryl-2-propen-1-ones (chalcones). It is generally carried out of the use of strong bases such as Na OH or KOH in polar solvents (MeOH or DMF). The aim of the present study was to develop an efficient protocol using PEG-400 as a recyclable reaction solvent to obtain 1,3-diaryl-2-propen-1-ones with good to excellent yields in a short span of time without formation of any side product. Synthesis of chalcone is a single step method. The synthesized chalcone derivatives were undergone physicochemical characterization and the obtained results are given in Table.2. The yields of the synthesized compounds were found to be significant. The structure of the synthesized compounds was confirmed by IR, Mass and elemental analysis. Elemental analysis showed that the percentage of the nitrogen, hydrogen and carbon was found experimentally is equivalent to the calculated values in all compounds. All the compounds give the characteristic IR peak that proved that the presence of particular functional group (Table 3) and mass spectroscopy helps to find the molecular weight of the synthesized compounds (Table 4). The Chalcone derivatives showed that the molecular ion peak that equivalent to the molecular weight of proposed compound. Hence m/z value confirms the molecular weight of the respective synthesized compound.3-(2-chlorophenyl)-1-(2,4dihydroxyphenyl)prop-2-en-1-one of $C_{15}H_{11}ClO_3$ with molecular ion peak at (274 M^{+2}) showed that m/z is equivalent to molecular weight of proposed compound. Hence m/z value confirms the molecular weight of the compound. The IR peak at 1691 cm^{-1} suggesting the presence of (C=O) group. The IR peak at

1595 cm^{-1} indicates that the presence of (C=C) group. IR peak at 3,232 cm^{-1} indicates presence of (-OH). Melting point of the compound is 180°C which is uncorrected.

1-(2,4-dihydroxyphenyl)-3-(4-hydroxyphenyl) prop-2-en-1-one have mole-cular formula C_{15} $H_{12}O_4$ and the molecular weight of the compound is equivalent to the molecular ion peak at (256 M^+H) of the compound. Hence m/z value confirms the molecular weight of compound. The IR peak at 1672 cm^{-1} suggesting the presence of (C=O) group. The IR peak at 1633 cm^{-1} indicates that the presence of (C=C) group. The IR peak at 3177 cm^{-1} indicates presence of (-OH) group. Melting point of the compound is 182 °C which is uncorrected. The molecular formula of 1-(2, 4-dihydroxyphenyl)-3-(3-hydroxyphenyl) prop-2-en-1-one is $C_{15}H_{12}O_4$ molecular ion peak at (256 M^-H) that m/z is equivalent to molecular weight of proposed compound Hence m/z value confirms the molecular weight of compound. The IR peak at 1668 cm^{-1} suggesting the presence of C=O group. The IR peak at 1581 cm^{-1} indicates that the presence of C=C group. IR peak at 3194 cm^{-1} indicates presence of (-OH) group. Melting point of the compound is 178 °C which is uncorrected.The obtained molecular ion peak of 1-(2,4-dihydroxyphenyl) -3-(4-nitrophenyl) prop-2-en-1-one (molecular formula, ($C_{15}H_{11}NO_5$) at 285 (M^+) that m/z is equivalent to molecular weight of proposed compound. Hence m/z value confirms the molecular weight of compound. The IR peak at 1693 cm^{-1} suggesting the presence of (C=O) group. The IR peak at 1601 cm^{-1} indicates that the presence of (C=C) group. IR peak at 3113 cm^{-1} indicates presence of (-OH) group. Melting point of the compound is 179°C which is uncorrected.

IV. Conclusions

In conclusion, our protocol is a practical approach which uses PEG as a commercially available, low-cost, recyclable non-ionic solvent. In most cases, reaction proceeded smoothly to produce the corresponding 1,3-diaryl-2-propen-1-ones. The reaction was clean and the products were obtained in excellent yields without formation of any side products. The synthesized compounds were characterized by TLC, melting point, IR spectroscopy, elemental analysis and mass spectroscopy. The results obtained form this study confirmed that the product has formed. Henceforth viewing these characteristic properties more compounds can be synthesized and subjected to pharmacological evaluation. These Chalcone derivatives may have variety synthesis and characterization of some new chalcone derivatives.

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Unsteady Flow Of A Dusty Viscous Fluid In A Rotating Channel In The Presence Of Transverse Magnetic Field.

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***Abstract*-The unsteady flow of a viscous, incompressible, electrically conducting, two phase fluid through a channel with time varying axial pressure gradient , in the presence of transverse magnetic field in a rotating frame of reference is studied. The governing equations of motion are derived, nondimensionalized and closed form solutions are obtained .Numerical evaluations of exact solutions are performed and graphical results for flow rate of both the phases are presented .The tables of skin friction coefficients for both the cases of pressure gradients (constant and time varying) are drawn .The observations are investigated to demonstrate the effect of the magnetic field on the physical parameters governing the flow and the results are summarized.**

Keywords-Magnetic field , rotation parameter , mass concentration , two phase fluid , skin friction .

I. Introduction

Fluid systems under rotation have been studied in principle since the days of Newton and Laplace.But in the previous century, theoretical work of Taylor and Proudman supported by Taylor'sexperiments , the peculiar effects of rotation on a fluid system have been generally appreciated.However the stimulus for research has come mainly from geophysical applications to the atmosphereand oceans and to motions within the Earth's core.Similarly the flow of a dusty and electrically conducting fluids in the presence of transversemagnetic field is encountered in a variety of applications such as magneto hydrodynamic generators,pumps, accelerators and flow meters . In these devices the solid particles in form of ash or soot aresuspended in the conducting fluid as a result of the corrosion and wear activities and / or thecombustion processes in MHD generators and plasma MHD accelerators. The consequent effect of thepresence of solid particles on the performance of such devices has led to the studies of particulatesuspensions in conducting fluids in the presence of externally applied magnetic fields. Mutual particle interaction leads to higher particle – phase viscous stresses and can be accounted for byendowing the particle phase by the so called particle – phase viscosity.There have been many articles dealing with theoretical modeling and experimental measurements of the particle-phase viscosity in a dusty fluid [[1] – [4]]. The viscosity of the particle.

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phase is needed to account for the energy dissipation between the solid particles due to their interaction Similarly the flow of conducting fluid has been investigated by many authors for instance Gadiraju Peddieson etal [5]investigated steady two phase vertical flow in a pipe. Dube and Sharma [6] reported solutions for unsteady dusty gas flow in a circular pipe in the absence of a magnetic field and particle-phase viscous stresses.Healy and Young [[7],[8]], Debnath and Basu [9]have investigated various aspects of hydrodynamic and hydromagnetic two phase flows in a non-rotating system. L.Debnath and B.C.Ghosh have also discussed the unsteady hydro magnetic flows of a dusty visco- Elastic fluid between two moving plates [10] .All the above investigations have been carried out by the authors in a fluid system having non – rotational frame of reference. On the other hand the simultaneous influence of rotation and external magnetic field on electrically conducting two-phase fluid systems seems to be dynamically important and physically useful.Gupta [11], Datta and Jana [12] have studied the effects of rotation in the dusty fluid flows in a channel. Nag [13] studied the flow of a dusty gas past a wavy moving wall with arbitrary time varying pressure gradient in a rotating frame of reference.Depnath and Ghosh [14] have investigated the hydromagnetic stokes' flow in a rotating fluid with suspended Stokesian solid particles. In this paper , the flow of unsteady , laminar , electrically conducting , dusty viscous fluid flow driven by a pressure gradient decreasing exponentially with time , in the presence of transverse magnetic field in the rotating frame of reference is studied .The effects of hydrodynamic parameter (M) , rotation parameter (E) , dust parameters (f , a) on the velocities of fluid , dust particles and also on the shear stresses are analyzed in detail.

II. Formulation of the Problem

We consider the motion of a unsteady, laminar ,conducting ,dusty viscous fluid within a parallel platechannel Z= $\pm$ L in a rotating frame of reference in presence of external magnetic field . Both the channel andthe two-phase fluid are in solid body rotation with a uniform angular velocity Ω about the Z –axis .Thechannel is considered to be infinite, so that the physical variables are functions of Z and t only . Following Saffman's model [15] of a dusty fluid and referring to Marble [16] thegoverning equations of the unsteady motion of an incompressible electrically conducting viscous fluid withembedded identical ,inert,

, spherical particles in a rotating frame of reference under an external magnetic field can be written as

$$\text{div } \overline{q} = 0 \qquad 1)$$

$$r\left[\frac{\partial \overline{q}}{\partial t} + (\overline{q} .\nabla)\overline{q} + 2\Omega \times \overline{q}\right] = -\nabla p + g \ \nabla^2 \ \overline{q} + \overline{F}_p + \overline{F}m \qquad 2)$$

$$m\left[\frac{\partial \overline{q}_p}{\partial t} + (\overline{q}_p .\nabla)\overline{q}_p + 2\Omega \times \overline{q}_p\right] = K(\overline{q} - \overline{q}_p) \qquad 3)$$

$$\frac{\partial N}{\partial t} + \nabla .(N \overline{q}_p) = 0 \qquad 4)$$

where

$\overline{q} \equiv (u, v, w)$ and $\overline{q}_p \equiv (u_p, v_p, w_p)$ represent the velocities of fluid and dust particles respectively.

$$\overline{F}_m = \bar{\text{j}} \text{ X } \overline{B} \qquad 5)$$

Here the viscosity of the pseudo fluid of solid particles have been neglected. Here p, $\overline{F}_m$ and r are pressure, Lorentz force term (force due to magnetic field) and density of the fluid respectively, and a subscript p in them denotes corresponding entities of particle phase g, Ω and K respectively are kinematic viscosity, uniform angular velocity vector of coordinate system and Stokes resistance coefficient, m and N represent mass and number density of the dust particles respectively. j is the current density, $\overline{F}_p$ is the total fluid - particle interaction force per unit volume. If Reynolds number based on the relative velocity of particle is less than unity, then the force accelerating the particle to the fluid speed is given by Stokes law which is $6 \, p \, r_p \, m \, (\overline{q}_p - \overline{q})$ where r_p is the radius of a particle. The total interaction force per unit volume is

$$\overline{F}_p = 6\,p\,N\,r_p\,m\,(\overline{q}_p - \overline{q}) = \frac{f\,p\,(\overline{q}_p - \overline{q})}{t_m} \qquad 6)$$

$$t_m = \frac{m}{6\,p\,m\,r_p} \qquad 7)$$

is called the relaxation time during which the velocity of the particle phase relative to the

fluid is reduced to $\left(\frac{1}{e}\right)$ times its initial value m and is the mass of each particle .

The Maxwell's equations and the generalized ohm's law are given by

$$div\vec{B} = 0$$
$$curl\ \vec{B} = m_0 \vec{J}$$
$$curl\ \vec{E} = \frac{\partial \vec{B}}{\partial t}$$
$$\vec{J} = s_0\left[\vec{E} + \vec{q} \times \vec{B}\right]$$

Where the displacement currents are neglected, m_0 and s_0 are constants and E is the electric field. We assume that the magnetic r_r and t_l number $r\ m = m_0 s\ 0 c << 1$ which is plorisible for most of the electrically conducting fluids. This implies that the current is mainly due to the induced electric field so that $\vec{J} = s_0[\vec{q} \times B]$ and the electric currents flowing through the fluid .it is further assumed that the induced magnetic field produced by the motion of fluid is negligible compared to the applied magnetic field so that the Lorenz force term in $\vec{F}_m$ in 2 becomes $\frac{-s_0 B_0{}^2 \vec{q}}{r}$ .as the physical variables are functions of z and t only. The equation of continuity for the liquid phase equation 1 gives $w \equiv 0$ everywhere in the flow. The equation

of conservation of dust particles viz equation 4 is similarly satisfied by $N \equiv N_0$ a constant and $w_p \equiv 0$ .In this paper we consider the flow of unsteady, laminar, electrically conducting, incompressible dusty viscous fluid driven by a pressure gradient decreasing exponentially with time in the rotating frame of reference. The effects of hydro magnetic parameter, dust parameter and rotation parameter on the velocities of the fluid and dust particles and also on the shearing stresses are analyzed in detail.

In most of the studies of rotating dusty fluid flows under magnetic field, certain simplifying assumptions are usually made for dilute suspensions.

In this study, the following assumptions have been made:

1) The number density N (the number of dust particles per unit volume of the mixture) of particles is constant

2) The solid particles are sparsely distributed and they are non-interacting so that the pressures locally have
same velocity vector and temperature. Due to this assumption of lack of randomness in local particle motion, the pressure associated with the particle cloud is negligible. Then the fluid pressure of the mixture.

3) The dust particles are assumed to be spherical in shape, all having the same radius and mass and all having undeformable.

4) The carrier fluid is assumed to be incompressible and electrically conducting and a uniform magnetic field is applied normal to the flow direction.

5) The particle phase is assumed to be incompressible, pressure less and electrically non-conducting.

6) The Hall effect of Magneto hydrodynamics is assumed to be negligible .

7) The volume fraction of suspended particles is negligible .

Taking into account all the assumptions the governing equations for dusty conducting fluid in the presence of a transverse magnetic field of intensity B_o are reduced to

$$\frac{\partial u_1}{\partial t} = -\frac{1}{r}\frac{\partial p}{\partial x} + g\frac{\partial^2 u_1}{\partial z^2} + \Omega v_1 + \frac{K N_0}{r}\left(u_{p_1} - u_1\right) - \frac{s_0}{r} B_0^2 u_1 \qquad 8)$$

2

$$\frac{\partial v_1}{\partial t} = g\frac{\partial^2 v_1}{\partial z^2} - 2\,\Omega u_1 + \frac{K N_0}{r}\left(v_{p_1} - v_1\right) - \frac{s_0}{r} B_0^2\, v_1 \qquad 9)$$

$$\frac{\partial u_{p_1}}{\partial t} = \frac{K}{m}\left(u_1 - u_{p1}\right) + 2\,\Omega\, v_{p_1} \qquad 10)$$

$$\frac{\partial v_{p_1}}{\partial t} = \frac{K}{}\left(v_1 - v_{p1}\right) - 2\,\Omega\, u_{p_1} \qquad 11)$$

Introducing non- dimensional variables

$$u = \frac{u_1 L}{g}\;,\quad v = \frac{v_1 L}{g}\;,\quad u_p = \frac{u_{p_1} L}{g}\;,\quad v_p = \frac{v_{p_1} L}{g}$$

$$T = \frac{tg}{L^2}\;,\quad h = \frac{Z}{L}\;,\quad x = \frac{x}{L}\;,\quad p_1^* = \frac{pL^2}{r g^2} \qquad 12)$$

The equations (8) – (11) become

$$\frac{\partial u}{\partial T} = F(T) + \frac{\partial^2 u}{\partial h^2} + 2Ev + fa\left(u_p - u\right) - M^2 u \qquad 13)$$

$$\frac{\partial v}{\partial T} = \frac{\partial^2 v}{\partial h^2} - 2Eu + fa\left(v_p - v\right) - M^2 v \qquad 14)$$

$$\frac{\partial u_p}{\partial T} = a\left(u - u_p\right) + 2Ev_p \qquad 15)$$

$$\frac{\partial v_p}{\partial T} = a\left(v - v_p\right) - 2Eu_p \qquad 16)$$

where

$E = \frac{\Omega L^2}{g}$ is the rotation parameter,

$M = B_0 L\sqrt{\frac{s_0}{r g}}$ is the Hartmann number, and f = $\frac{m N_0}{r}$, $a = \frac{k L^2}{m g}$ 17) are dust parameters , f being the mass concentration and $\frac{m}{k}$, the relaxation time of the dust particles,

and $-\frac{\partial p^*}{\partial x} = F(T).$ 18)

F(T) being an arbitrary function of time representing the change in the pressure gradient which causes the motion.

The initial and boundary conditions are

$$u = v \quad u_p \quad v_p = 0 \quad \text{for} \quad T = 0,$$
$$u = v = 0 \quad \text{at} \quad h = \pm 1 \quad \text{for} \quad T \succ 0.$$
$$\frac{\partial u}{\partial h} = \frac{\partial v}{\partial h} = 0 \quad \text{at} \quad h = 0 \qquad \text{(symmetry conditions)} \qquad 19)$$

Solution of the problem:

Multiplying the equations (13) – (16) by $\cos a_n h$

where $a_n = (2n+1)\frac{p}{2}$ 20)

And then integrating between the limits 0 to 1 and using the conditions(19), we get

$$\frac{du}{dT} = \frac{(-1)^n}{a_n} F(T) + 2Ev - \left(a_n^2 + M^2\right)u + f\,a\left(u_p - u\right) \qquad 21)$$

$$\frac{dv}{dT} = -2Eu - \left(a_n^2 + M^2\right)v + f\,a\left(v_p - v\right) \qquad 22)$$

$$\frac{du_p}{dT} = a\left(u - u_p\right) + 2Ev_p \qquad 23)$$

$$\frac{dv_p}{dT} = a\left(v - v_p\right) - 2Eu_p \qquad 24)$$

where the finite cosine transforms are defined as

$$\bar{u}(n,T) = \int_0^1 u \cos a_n h \, dh, \quad \bar{v}(n,T) = \int_0^1 v \cos a_n h \, dh.$$

$$\bar{u}_p(n,T) = \int_0^1 u_p \cos a_n h \, dh, \quad \bar{v}_p(n,T) = \int_0^1 v_p \cos a_n h \, dh. \qquad 25)$$

The inversion formula for the transform

$\bar{u}(n,T) = \int_0^1 u \cos a_n h \, dh$ can be proved to be $u = 2\sum_{n=0}^{\infty} \bar{u}(n,T)\cos a_n h$ 26)

and similar expressions for v, u_p, v_p.

Combining (21), (22), we can write

$$\frac{dH}{dT} = \frac{(-1)^n}{a_n} F(T) - \left\{\left(a_n^2 + M^2\right) + 2iE\right\} H - fa\,(H - h). \qquad 27)$$

and similarly (23) - (24) can be grouped as

$$\frac{d\,h}{d} = a\,(H - h) - 2iE\,h \qquad 28)$$

where

$$H = \bar{u} + i\bar{v}, \quad h = \bar{u}_p + + i\bar{v}_p \qquad 29)$$

Taking Laplace transforms of (27) and (28), we get

$$\left\{s + \left(a_n^2 + M^2\right) + 2iE\right\}\bar{H} = \frac{(-1)^n}{a_n}\bar{F} - f\,a\left(\bar{H} - \bar{h}\right). \qquad 30)$$

$$\left(s + a + 2iE\right)\bar{h} = a\,\bar{H}. \qquad 31)$$

where $\bar{H} = \int_0^{\infty}\left(He^{-sT}\right)dT$, $\bar{h} = \int_0^{\infty}\left(he^{-sT}\right)dT$, $\bar{F} = \int_0^{\infty}\left(Fe^{-sT}\right)dT$ 32)

From (30) and (31)

$$\overline{H} = \frac{(-1)^n \, (s + a + 2iE)}{a_n (s - a)(s - b)} \overline{F} \quad 33)$$

$$\overline{h} = \frac{(-1)^n \, a}{a_n (s - a)(s - b)} \overline{F} \quad 34)$$

where

$$a = -\frac{1}{2}\left[\left(a_n^2 + M^2 + a + f\,a + 4iE\right) - \left\{ \left(a_n^2 + M^2 + a + f\,a\right)^2 - 4a\left(a_n^2 + M^2\right)\right\}^{\frac{1}{2}} \right]$$

$$b = -\frac{1}{2}\left[\left(a_n^2 + M^2 + a + f\,a + 4iE\right) + \left\{ \left(a_n^2 + M^2 + a + f\,a\right)^2 - 4a\left(a_n^2 + M^2\right)\right\}^{\frac{1}{2}} \right] \quad 35)$$

Using Convolution theorem for obtaining inverse Laplace transforms of (33) and (34) we get

$$H = \frac{(-1)^n}{a_n} \frac{1}{b - a} \int_0^T F(T - m)\left[(b + a + 2iE) e^{bm} - (a + a + 2iE) e^{am} \right] dm \quad 36)$$

$$h = \frac{(-1)^n}{a_n} \frac{1}{b - a} \int_0^T (e^{bm} - e^{am}) F(T - m) \, dl \quad 37)$$

Applying inversion formula (26) for the cosine transforms, equations(36) , (37) become

$$u + iv = 2\sum_0^{\infty} \frac{(-1)^n}{a_n} \frac{1}{(b - a)} \left[\int_0^T F(T - m)\left[(b + a + 2iE) e^{bm} - (a + a + 2iE) e^{am} \right] dl \right] \cos a_n h$$

38)

$$u_p + i v_p = 2\sum_0^{\infty} \frac{(-1)^n}{a_n} \frac{a}{(b - a)} \left[\int_0^T F(T - l)(e^{bl} - e^{al}) \, dl \right] \cos a_n h \quad 39)$$

Let $F(T) = A e^{-l^2 T}$ 40)

The velocities given by (38) and (39) are being discussed in detail for two particular cases of initial pressure gradient :

(i) constant pressure gradient ($l = 0$)

(ii) pressure gradient decreasing with time exponentially ($l \neq$).

Substituting (40) in (38) and (39) , Integrating and separating into real and imaginary parts we have

$$u = 2A\sum_{0}^{\infty}\frac{(-1)^{n}}{a_{n}}\frac{1}{(b-a)}\left\{\begin{matrix}\dfrac{b+a}{(l^{2}-b)^{2}+4E^{2}}\left[\begin{matrix}(l^{2}-b)(1+e^{(b-l^{2})T}\cos 2ET)- \\ 2E\,e^{(b^{2}-l^{2})T}\sin 2ET\end{matrix}\right] \\ -\dfrac{a+a}{(l^{2}-a)^{2}+4E^{2}}\left[\begin{matrix}(l^{2}-a)(1+e^{(a-l^{2})T}\cos 2ET)- \\ 2E\,e^{(a^{2}-l^{2})T}\sin 2ET\end{matrix}\right]\end{matrix}\right\}\cos a_{n}h$$

41)

$$v = 2A\sum_{0}^{\infty}\frac{(-1)^{n}}{a_{n}}\frac{1}{(b-a)}\left\{\begin{matrix}\dfrac{b+a}{(l^{2}-b)^{2}+4E^{2}}\left[(b-l^{2})\sin 2ET\,e^{(b-l^{2})T}-2E\left(1+e^{(a-l^{2})T}\cos 2ET\right)\right] \\ -\dfrac{a+a}{(l^{2}-a)^{2}+4E^{2}}\left[(a-l^{2})\sin 2ET\,e^{(a-l^{2})T}-2E\left(1+e^{(a-l^{2})T}\cos 2ET\right)\right]\end{matrix}\right\}$$

$$.\ \cos a_{n}h$$

42)

$$u_{p} = 2Ae^{-l^{2}T}\sum_{0}^{\infty}\frac{(-1)^{n}a}{a_{n}(b-a)}\left[\begin{matrix}\left\{(b+l^{2})\cos 2ET+2E\sin 2ET\right\}\dfrac{e^{(b+l^{2})T}}{(b+l^{2})^{2}+4E^{2}} \\ -\dfrac{(b+l^{2})}{(b+l^{2})^{2}+4E^{2}}-\left\{(a+l^{2})\cos 2ET+2E\sin 2ET\right\}\dfrac{e^{(a+l^{2})T}}{(a+l^{2})^{2}+4E^{2}} \\ -\dfrac{(a+l^{2})}{(b+l^{2})^{2}+4E^{2}}\end{matrix}\right].(\cos a_{n}h)$$

43)

$$v_p = 2Ae^{-l^2 T} \sum_0^{\infty} \frac{(-1)^n a}{a_n(b-a)} \left[\begin{array}{l} \left\{ \frac{e^{(b+l^2)T}}{(b+l^2)^2+4E^2} - \frac{e^{(a+l^2)T}}{(a+l^2)^2+4E^2} \right\} 2E\cos 2ET \\ -\left\{ \frac{(b+l^2)e^{(b+l^2)T}}{(b+l^2)^2+4E^2} - \frac{(a+l^2)e^{(a+l^2)T}}{(a+l^2)^2+4E^2} \right\} \sin 2ET \\ -2E\left\{ \frac{1}{(b+l^2)^2+4E^2} - \frac{1}{(a+l^2)^2+4E^2} \right\} \end{array} \right] (\cos a_n h)$$

44)

The non- dimensional shear stress on the wall $h = 1$ in x and y directions are given respectively by

$$t_x = -2A \sum_{n=0}^{\infty} \frac{1}{b-a} \left[\begin{array}{l} \frac{b+a}{(l^2-b)^2+4E^2} \left\{ \begin{array}{l} (l^2-b)\left(1+e^{(b-l^2)T}\cos 2ET\right) \\ -2E\, e^{(b-l^2)T}\sin 2ET \end{array} \right\} - \\ \frac{a+a}{(l^2-a)^2+4E^2} \left\{ \begin{array}{l} (l^2-a)\left(1+e^{(a-l^2)T}\cos 2ET\right) \\ -2E\, e^{(a-l^2)T}\sin 2ET \end{array} \right\} \end{array} \right] \quad 45)$$

$$t_y = -2A \sum_{n=0}^{\infty} \frac{1}{b-a} \left[\begin{array}{l} \frac{b+a}{(l^2-b)^2+4E^2} \left\{ \begin{array}{l} (b-l^2)\left(e^{(b-l^2)T}\sin 2ET\right) \\ -2E\,(e^{(b-l^2)T}\cos 2ET+1) \end{array} \right\} - \\ \frac{a+a}{(l^2-a)^2+4E^2} \left\{ \begin{array}{l} (a-l^2)\left(e^{(a-l^2)T}\sin 2ET\right) \\ -2E\,(1+e^{(a-l^2)T}\cos 2ET \end{array} \right\} \end{array} \right] \quad \text{------}(46)$$

The non – dimensional resultant shear stress t_s on the wall $h = 1$ is given by

$$t_s = \sqrt{(t_x^2 + t_y^2)} \quad \text{-----------}(47)$$

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Discussion:-

The closed form solutions reported here are numerically evaluated and graphically plotted to elucidate the effects of the various physical parameters on the solutions. The influence of Hartmann number M , the mass concentration parameter of the particle, rotation parameter E , on the fluid phase flow rate (u ,v)and the particle phase flow rate (u_{p}, v_{p}) , are presented in figures 1 through 10 for both the cases : constant pressure gradient and pressure gradient decreases exponentially with time t. Initially both the phases are at rest and suddenly they are set to motion by constant pressure gradient. As a result , the shear stress on the surface of the channel increases. The application of transverse magnetic field has a general tendency to retard the flow of both phases causing their average velocities and the wall shear stress to decrease.

Figures 1 – 4 represent the velocity profiles of both the phases in the absence of Magnetic field, when the flow is driven by constant pressure gradient. It is interesting to note from figure1 that in the range $0 = E = 4$, the secondary velocity of the fluid increases , the particle velocity decreases and when $E > 4$, the decrease of the both velocities is seen , with higher decrease in particle velocity . Decrease in the primary velocity of both phases with increase in rotation parameter is seen in figure 2. Reversal flow of particles takes place at $E = 8$.

When pressure gradient decreases with time (figure 4), when $2 < E < 4$, fluid velocity increases, and when $E > 4$, Particle and fluid velocities are decreasing. In this case also reversal particulate flow takes place at $E = 8$.

From figures 5 and 6 ($M = 5$, $E = 2$), we find that increase of mass concentration parameter causes increase in fluid velocity and decrease in primary velocity of particles.

The same trend is seen in secondary velocities but no reversal flow of particles is observed.

For $M = 2$ and for different values of E [3 , 4 , 5 and 8] the velocity profiles of both the phases are drawn in figure 7 for constant pressure gradient and in figure 8 for time varying pressure gradient. As the pressure gradient decreases with time , a small increase in fluid velocity takes place but that decreases as E increases under low magnetic field, whereas the same decreasing trend in particle velocity is seen whether or not pressure gradient is constant.

The secondary velocity profiles are drawn for both the phases when pressure gradient is constant (figure 9). When $2 < E < 6$, fluid velocity increases and when $E > 6$, it decreases whereas a steady increase in particle velocity is able to be seen as E increases. When pressure gradient decreases with time same trend for fluid velocity continues while sudden fall in particulate velocity is observed even when there is a small increase in E.

The non-dimensional skin friction t_s at the wall h 1 corresponding to constant pressure gradient and exponentially decreasing pressure gradient (with time for various values of M , E , f , a) are furnished in tables 1 and 2 respectively.

From table1 , we find that at constant pressure gradient, when there is no magnetic field , skin friction decreases as rotation or dust parameter increases. Also at $M = 5$, The skin friction is found to be oscillatory for increasing values of a .

The same decreasing trend is seen in skin friction for higher values of M ($M = 10$) as E increases , and a slight increase is observed with increase of a .

From table 2 , We observe that when Magnetic field is not applied, there is an increase in skin friction for low rotation parameter ($E = 2$)and a decrease for $E = 4$ and for higher values of E ($E = 6 , 8$), it again increases.

As in the case of constant pressure gradient , when the flow is driven by pressure gradient (that exponentially decays with time) skin friction is found to be oscillatory for moderate increase of M , decrease for higher values of M and as E increases skin friction further decreases. The comparisons of the results reported in this paper with results reported in (5) by setting M and ? to be zero lend confidence in the correctness of the solutions in this paper.

RESULTS :

(1) The application of transverse magnetic field has got a general tendency to retard the flow of both fluid and particle phases causing their average velocities and wall shear stress to decrease.

(2) Reversal particle flow is caused mainly by rotation parameter.

(3) Increase in rotation parameter causes further decrease in skin friction .

(4) When there is no magnetic field , and the flow being driven by constant pressure gradient ,

(a) Increase in rotation parameter causes rise in secondary velocity of the fluid and fall in the same when $E > 4$.

(b) Reversal particle flow takes place at $E = 8$.

(5) Increase in mass concentration parameter in presence of Magnetic field causes increase in the fluid velocity and decrease in particle velocity.

(6) A slight fall in pressure gradient with time causes a small rise in fluid velocity , having no effect on particle velocity.

Conclusion :

The transient flow of electrically conducting two phase fluids (carrying inert suspended Stokesian solid particles) with an applied transverse magnetic field is studied analytically. The governing equations of motion are derived , non-dimensionalised and solved in closed form . Exact solutions are numerically evaluated , graphical results for the velocity of both the phases are presented and skin friction coefficients are tabulated and discussed to demonstrate the effect of magnetic field on these physical parameters. Comparisons with previously published theoretical work was performed . Neglecting the volume fraction of particles allowed the governing equations to be solved analytically. However it is hoped that the results reported herein will serve as a stimulas for experimental work on this problem and will be useful in verifying numerical schemes used to solve more complex (that is realistic problems) of this type .

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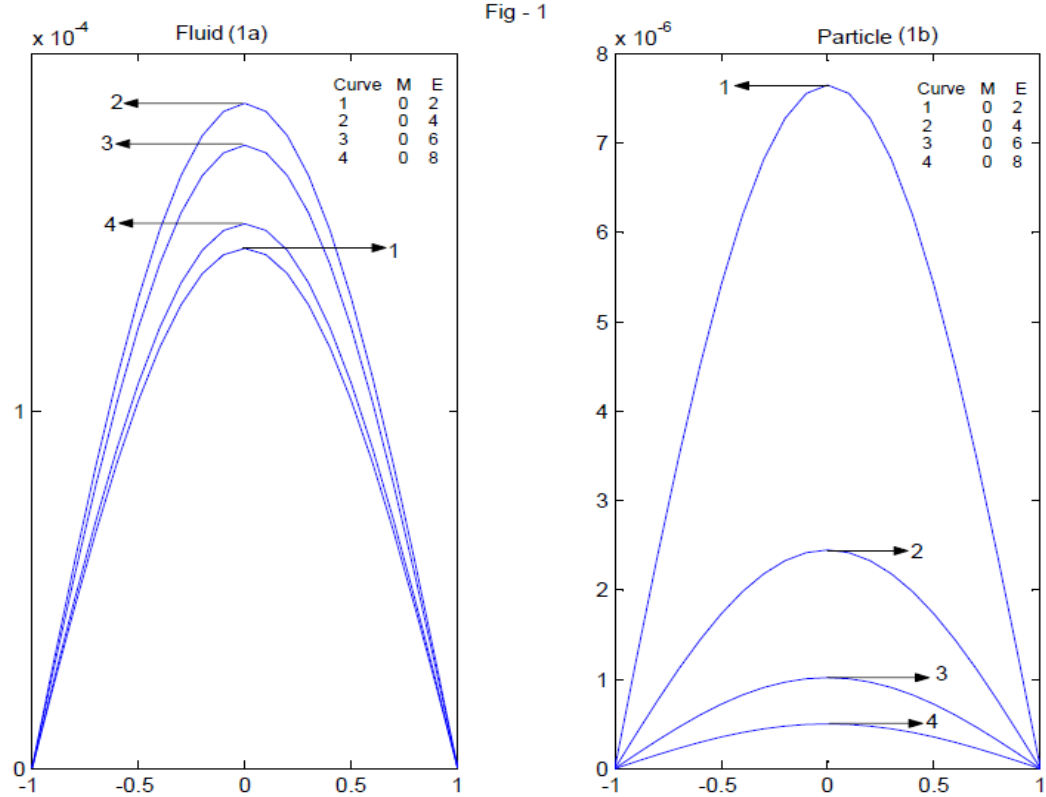
The Non-dimensional resultant skin friction for *T*= 0 and for different values of *F*, a, *E* ,*M*

		F \ E	2.0	4.0	6.0	8.0
M=0	α = 0.1	0.1	0.7092	0.4425	0.2030	0.1046
		0.2	0.5481	0.3553	0.1673	0.0972
		0.3	0.4369	0.2917	0.1407	0.0906
	α = 0.2	0.1	0.4608	0.3548	0.1749	0.0909
		0.2	0.3559	0.2814	0.1388	0.0795
		0.3	0.2822	0.2278	0.1123	0.0705
	α = 0.3	0.1	0.3406	0.2968	0.1563	0.0810
		0.2	0.2830	0.2343	0.1228	0.0680
		0.3	0.2081	0.1886	0.0982	0.0582
M=5	α = 0.1	0.1	0.7056	0.6573	0.6147	0.5775
		0.2	0.6728	0.6136	0.5680	0.5337
		0.3	0.6347	0.5716	0.5259	0.4949
	α = 0.2	0.1	2.2863	1.7613	1.4084	1.1975
		0.2	1.8969	1.4761	1.1823	1.0085
		0.3	1.5699	1.2400	1.0022	0.8626
	α = 0.3	0.1	5.0580	3.6518	2.6449	2.0374
		0.2	3.2766	2.4758	1.8665	1.4869
		0.3	2.3839	1.8582	1.4400	1.1748
M=10	α = 0.1	0.1	0.1918	0.1910	0.1901	0.1891
		0.2	0.1885	0.1876	0.1865	0.1855
		0.3	0.1853	0.1841	0.1830	0.1819
	α = 0.2	0.1	0.3889	0.3848	0.3802	0.3758
		0.2	0.3849	0.3786	0.3716	0.3656
		0.3	0.3791	0.3710	0.3124	0.3551
	α = 0.3	0.1	0.6423	0.6232	0.6023	0.5840
		0.2	0.6641	0.6356	0.6042	0.5772
		0.3	0.6653	0.6320	0.5951	0.5633

The Non-Dimensional Resultant Skin Friction For *T*=1 And For Different Values Of *F*, A ,*E* ,*M*

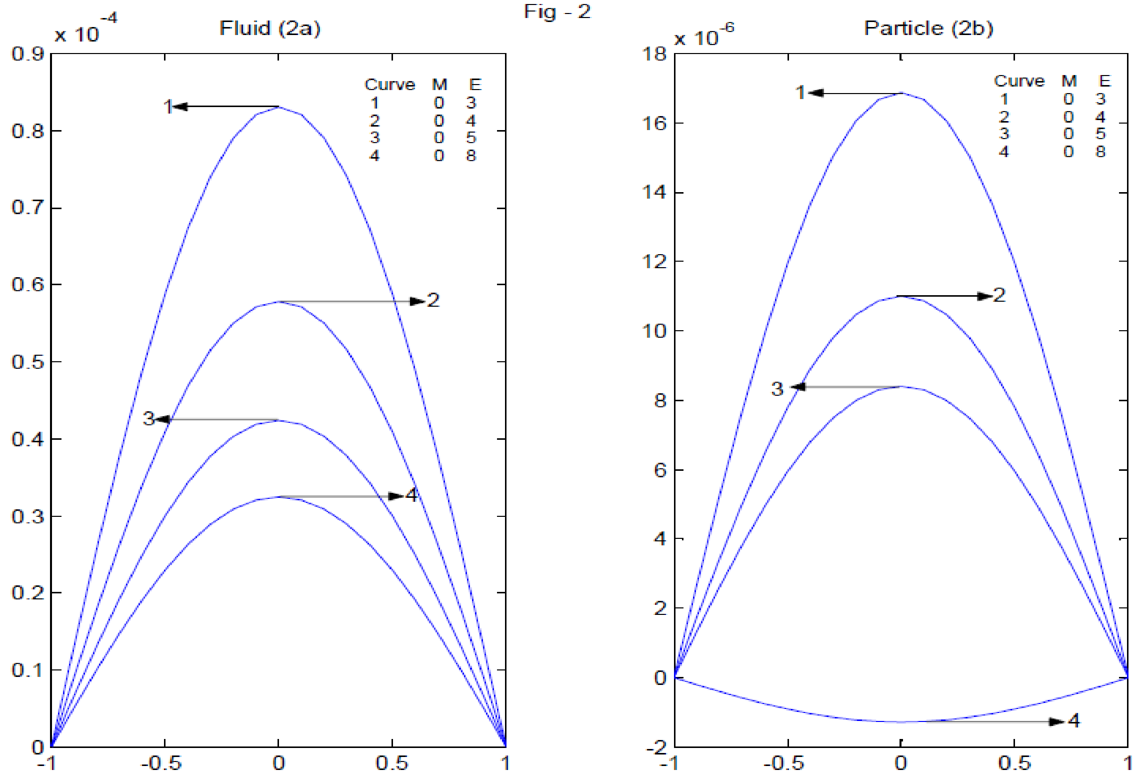
		$F \backslash E$	2.0	4.0	6.0	8.0
M=0	$\alpha = 0.1$	0.1	0.7093	0.4319	0.2157	0.1296
		0.2	0.5505	0.3489	0.1800	0.1171
		0.3	0.4411	0.2885	0.1532	0.1067
	$\alpha = 0.2$	0.1	0.4704	0.3481	0.1868	0.1151
		0.2	0.3647	0.2776	0.1507	0.0991
		0.3	0.2904	0.2259	0.1239	0.0866
	$\alpha = 0.3$	0.1	0.3506	0.2916	0.1667	0.1041
		0.2	0.2716	0.2311	0.1327	0.0869
		0.3	0.2156	0.1866	0.1077	0.0739
M=5	$\alpha = 0.1$	0.1	0.6890	0.6420	0.6008	0.5650
		0.2	0.6619	0.6033	0.5580	0.5240
		0.3	0.6278	0.5649	0.5188	0.4872
	$\alpha = 0.2$	0.1	2.3020	1.7727	1.4127	1.1961
		0.2	1.9223	1.4952	1.1936	1.0128
		0.3	1.5969	1.2610	1.0159	0.8692
	$\alpha = 0.3$	0.1	5.1496	3.7238	2.6979	2.0710
		0.2	3.3514	2.5356	1.9086	1.5139
		0.3	2.4463	1.9079	1.4759	1.1982
M=10	$\alpha = 0.1$	0.1	0.1900	0.1893	0.1884	0.1874
		0.2	0.1869	0.1860	0.1849	0.1839
		0.3	0.1839	0.1827	0.1815	0.1804
	$\alpha = 0.2$	0.1	0.3861	0.3820	0.3773	0.3729
		0.2	0.3831	0.3766	0.3695	0.3632
		0.3	0.3779	0.3697	0.3609	0.3533
	$\alpha = 0.3$	0.1	0.6401	0.6208	0.5994	0.5808
		0.2	0.6645	0.6354	0.6033	0.5756
		0.3	0.6674	0.6334	0.5956	0.5630

Fig - 1



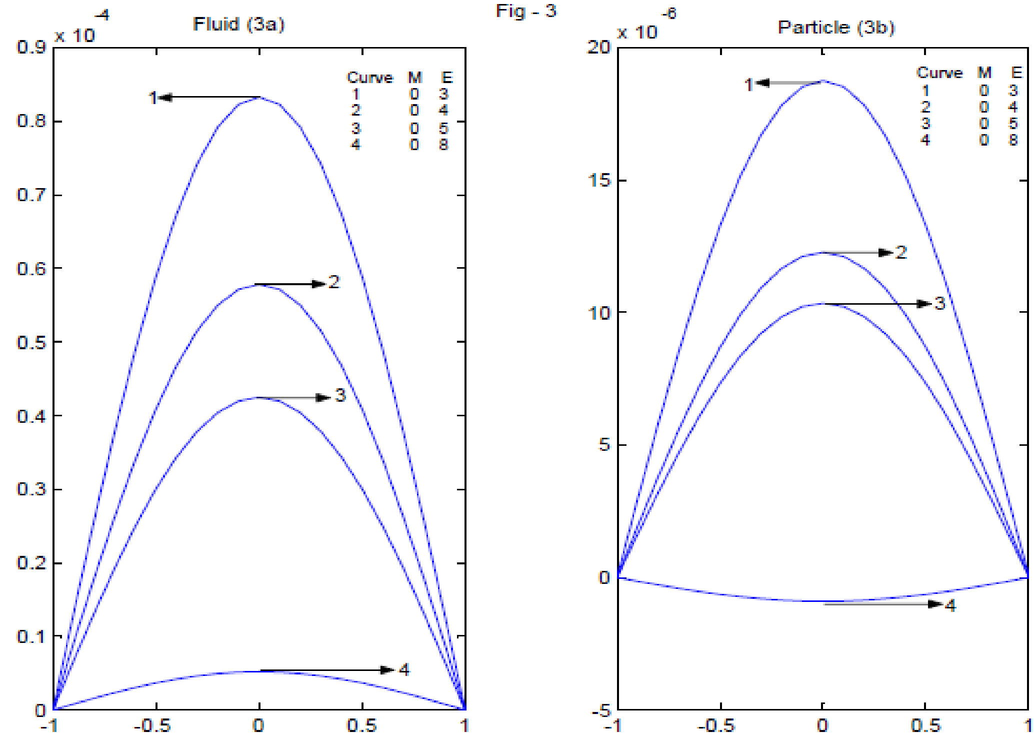
Secondary Velocity Profiles of Fluid and Particles when the Pressure Gradient is constant for different values of E at T=0.2 , for f =0.2 , α = 0.3

Fig - 2



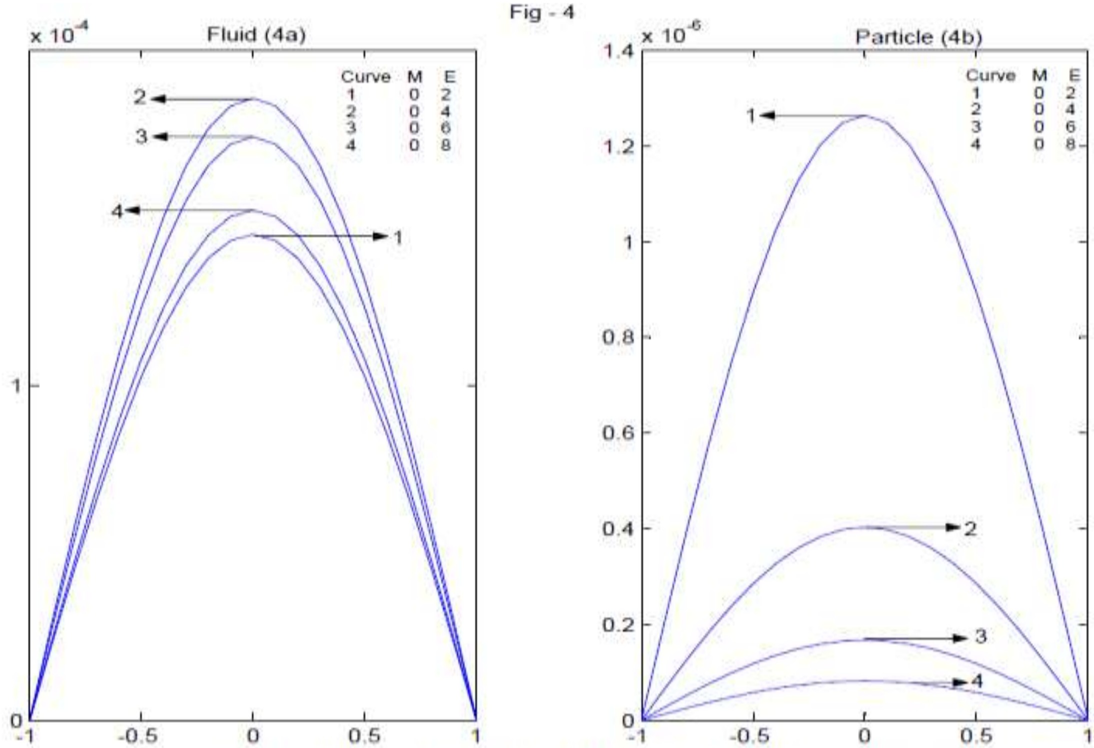
Primary Velocity Profiles of Fluid and Particles when the Pressure Gradient varies for different values of E at T = 0.2 , for f = 0.2 , α= 0.3

Fig - 3



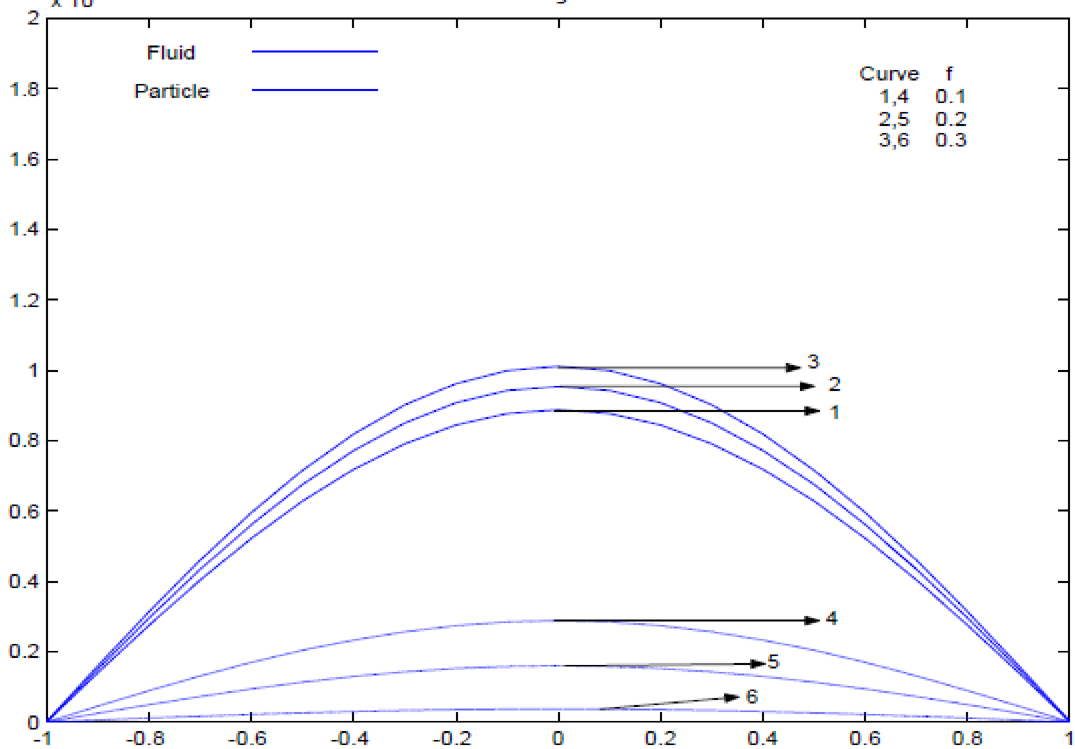
Primary Velocity Profiles of Fluid and Particles when the Pressure Gradient varies exponentially for different values of E at T=0.2 , for f = 0.2 , α = 0.3

Fig - 4



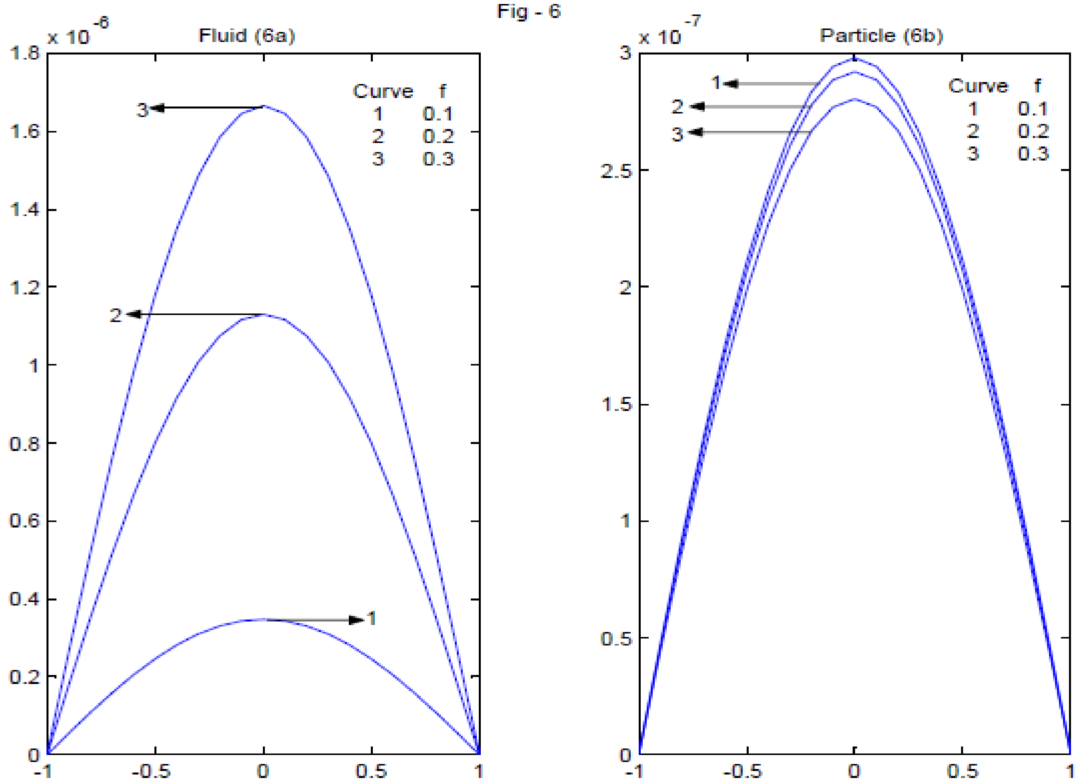
Secondary Velocity Profiles of Fluid and Particles when the PressureGradient varies exponentially for different values of E at T=0.2 , for f = 0.2 , α = 0.3

Fig - 5



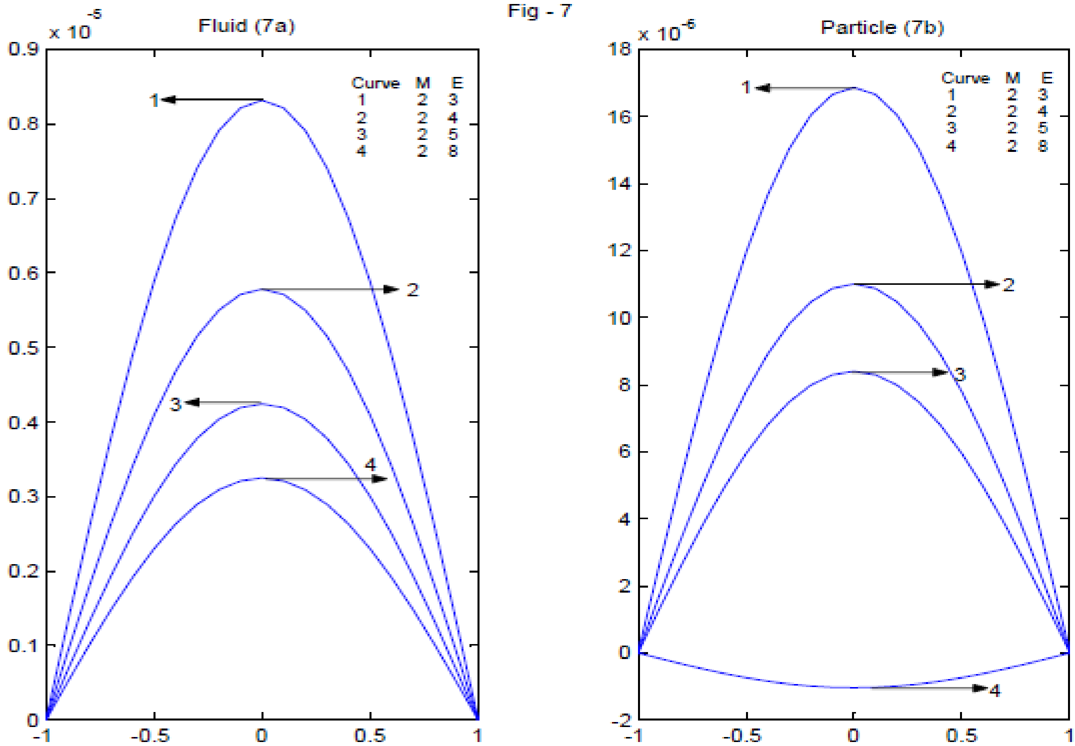
Primary Velocity Profiles of Fluid and Particles when the Pressure Gradient is constant for different values of f at T=0.2 , for α = 0.3, M= 5, E= 2

Fig - 6



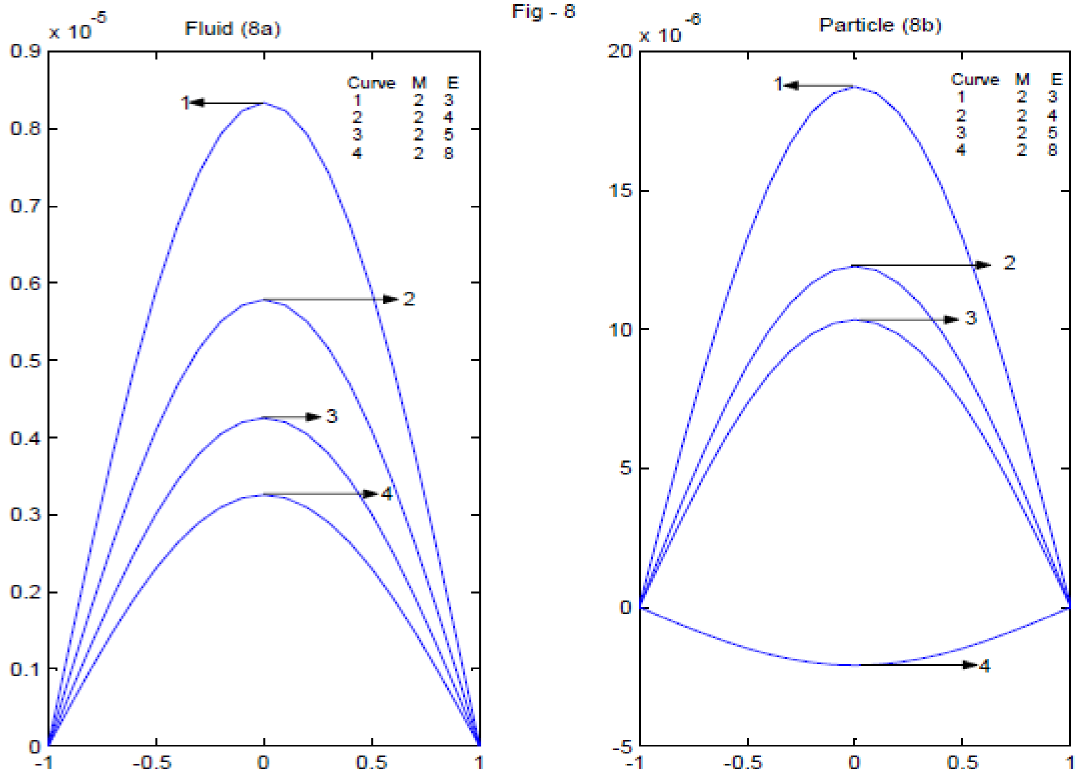
Secondary Velocity Profiles of Fluid and Particles when the Pressure Gradient is costant for different values of f at T=0.2 , for E =2 , α = 0.3, M = 5

Fig - 7



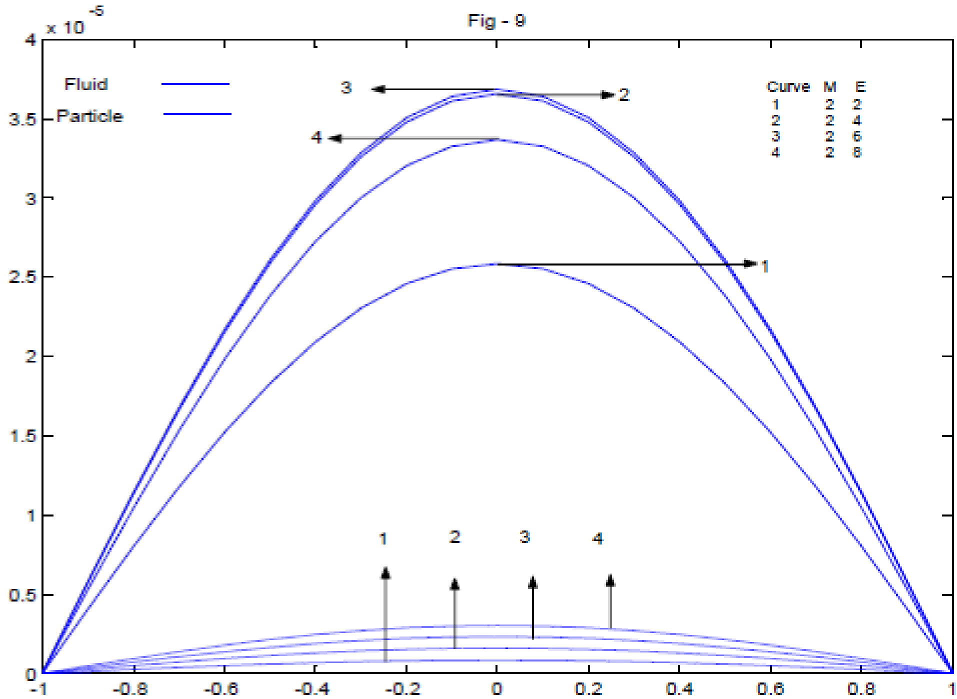
Primary Velocity Profiles of Fluid and Particles when the Pressure Gradient is constant for different values of E at T=0.2 , for f =0.2 , α = 0.3

Fig - 8



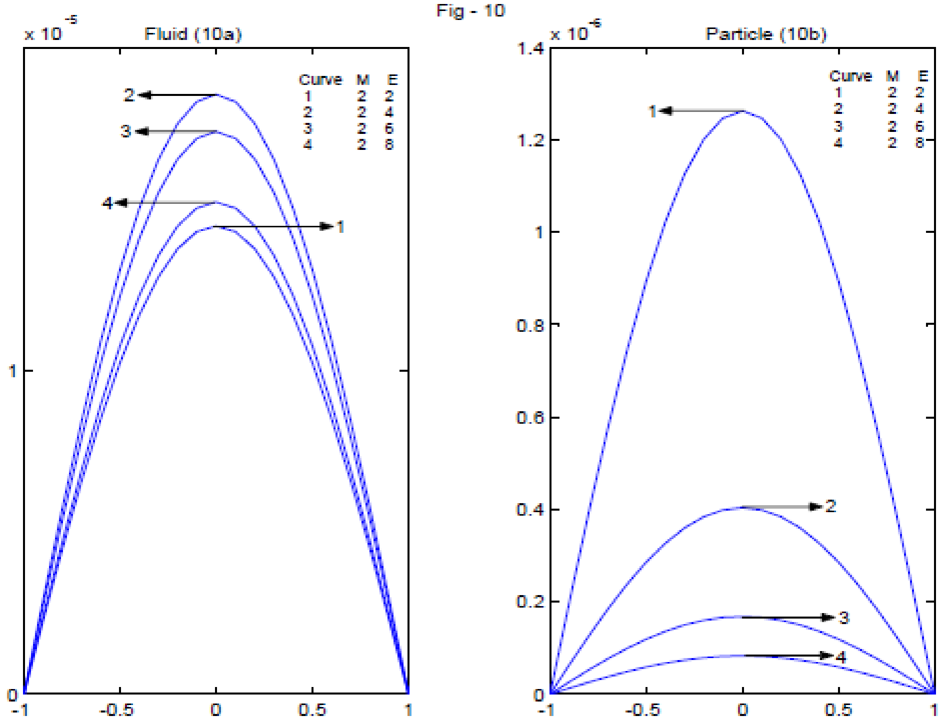
Primary Velocity Profiles of Fluid and Particles when the Pressure Gradient varies exponentially for different values of E at T=0.2 , for f = 0.2 , α = 0.3

Fig - 9



Secondary Velocity Profiles of Fluid and Particles when the Pressure Gradient is constant for different values of E at T=0.2 , for f =0.2 , α = 0.3

Fig - 10



Secondary Velocity Profiles of Fluid and Particles when the Pressure Gradient varies exponentially for different values of E at T=0.2 , for f = 0.2 , α = 0.3

A Subset Of The Space Of The Orlicz Space Of Gai Sequences

GJSFR- F Clasification FOR 010103,40A05 , 40C05 , 40D05

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Abstract
Let ~M denote the space of all Orlicz space gai sequences. Let M denote the space of all Orlicz space of analytic sequences. This paper is devoted to a study of the general properties of sectional space (~M)s of ~M.
Keywords: Sectional sequences, gai sequences, analytic sequences, Orlicz sequences.
2000 Mathematics subject classification: 40A05,40C05,40D05.

1 Introduction

A Complex sequence, whose kth term is xk is denoted by {xk} or simply x. Let ϕ be the set of all finite sequences. A sequence x = {x_k} is said to be analytic if $\sup_k |x_k|^{1/k} < \infty$. The vector space of all analytic sequence will be denoted by Λ. χ was discussed in Kamthan [13]. Matrix transformation involving ~ were characterized by Sridhar [21] and Siraji-udeen [22]. A sequence x is called entire sequence if $\lim_k !1 (k! |x_k|)^{1/k} = 0$. The vector space of all gai sequences will be detnoted by χ. Kizmaz [19] defined the following difference squence spaces

$$Z(\Delta) = \{x = (x_k) : \Delta x \in Z\}$$

for $Z = \ell_\infty, c, c_0$, where $\Delta x = (\Delta x)_{k=1}^{\infty} = (x_k - x_{k+1})_{k=1}^{\infty}$ and showed that these are Banach space with norm $\|x\| = |x_1| + \|\Delta x\|_\infty$. Later on Et and Colak [20] generalized the notion as follows : Let $m \in \aleph$

$Z(\Delta^m) = \{x = (x_k) : \Delta^m x \in Z\}$ for $Z = \ell_\infty, c, c_0$ where $m \in \aleph$, $\Delta^0 x = (x_k)$, $\Delta x = (x_k - x_{k+1})$, $\Delta^m x = (\Delta^m x_k)_{k=1}^{\infty} = (\Delta^{m-1} x_k - \Delta^{m-1} x_{k+1})_{k=1}^{\infty}$.
The generalized difference has the following binomial representation:

$$\Delta^m x_k = \sum_{\gamma=0}^{m} (-1)^\gamma \binom{m}{\gamma} x_{k+\gamma},$$

They proved that these are Banach spaces with the norm

$$\|x\|_\Delta = \sum_{\gamma}^{m} |x_i| + \|\Delta^m x\|_\infty$$

Orlicz [1] used the idea of Orlicz function to construct the space (L^M). Lindenstrauss and Tzafriri [2] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to $\ell_p (1 \le p < \infty)$. Subsequently different classes of sequence spaces were defined by Parashar and Choudhary[3], Mursaleen et al.[4], Bektas and Altin[5], Tripathy et al.[6], Rao and subramanian[7],and many others. The Orlicz sequence spaces are the special cases of Orlicz spaces studied in [8]. An Orlicz function is a function $M : [0, \infty) \to [o, \infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by $M(x + y) \le M(x) + M(y)$, then this function is called modulus function, introduced by Nakano[18] and further discussed by Ruckle[9] and Maddox[10], and many others.

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An Orlicz function M is said to satisfy Δ_2- condition for all values of u, if there exists a constant $K > 0$, such that $M(2u) \leq KM(u)(u \geq 0)$.The$\Delta_2$ condition is equivalent to $M(\ell u) \leq K\ell M(u)$, for all values of u and for $\ell > 1$. Lindenstrauss and Tzafriri[2] used the idea of Orlicz function to construct Orlicz sequence space

$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \, for some \, \rho > 0 \right\}. \quad (1)$$

where $w = \{all\, complex\, sequences\}$. The space ℓ_M with the norm

$$\|x\| = \left\{ inf \quad \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\} \quad (2)$$

becomes a Banach space which is called an Orlicz sequence space.For $M(t) = t^p$, $1 \leq p < \infty$, the space ℓ_M coincide with the classical sequence space ℓ_p.
Given a sequence $x = \{x_k\}$ its n^{th} section is the sequence $x^{(n)} = \{x_1, x_2, ..., x_n, 0, 0, ...\}$ $\delta^{(n)} = (0, 0, ..., 1, 0, 0, ...)$, 1 in the n^{th} place and zero's else where and $s^k = \{0, 0, 0, \cdots, 1, -1, 0, 0, \cdots\}$, 1 in the n^{th} place and -1 in the $(n+1)^{th}$ place and zero's else where.
If X is a sequence space, we define
(i)$X' =$ the continuous dual of X.
(ii)$X^\alpha = \{a = (a_k \sum_{k=1}^{\infty} |a_k x_k| < \infty, \, for each \, x \in X\}$;
(iii)$X^\beta = \{a = (a_k) : \sum_{k=1}^{\infty} a_k x_k \, is \, convergent, \, for each \, x \in X\}$;
(iv)$X^\gamma = \left\{a = (a_k) : \sup_n \left|\sum_{k=1}^{n} a_k x_k\right| < \infty, \, for each \, x \in X\right\}$;
(v)Let X be an FK-space$\supset \phi$. Then $X^f = \{f(\delta^{(n)}) : f \in X'\}$.
$X^\alpha, X^\beta, X^\gamma$ are called the $\alpha-$(or Kö the-T öeplitz)dual of X, $\beta-$ (or generalized Kö the-T öeplitz)dual of X, $\gamma-$dual of X. Note that $X^\alpha \subset X^\beta \subset X^\gamma$.
If $X \subset Y$ then $Y^\mu \subset X^\mu$, for $\mu = \alpha, \beta, or \, \gamma$.
An FK-space(Frechet coordinate space) is a Frechet space which is made up of numerical sequences and has the property that the coordinate functionals $p_k(x) = x_k$ $(k = 1, 2, ...)$ are continuous. We recall the following definitions[see [14]]. An FK-space is a locally convex Frechet space which is made up of sequences and has the property that coodinate projections are continuous. An metric space (X, d) is said to have AK (or sectional convergence) if and only if $d(x^{(n)} x) \to 0$ as n $\to \infty$[see 14]

The space is said to have AD or be an AD space if ϕ is dense in X. We note that AK implies AD by [11].

2 Definitions and Prelimiaries

Throughout the paper w, χ_M and Λ_M denote the spaces of all, Orlicz space of gai sequences and Orlicz space of bounded sequence respectively. Let w denote the set of all complex sequences $x = (x_k)_{k=1}^{\infty}$ and $M : [0, \infty) \longrightarrow [0, \infty)$ be an Orlicz function, or a modulus function. Let t denote the sequence with $t_k = |x_k|^{1/k}$ for all $k \in \aleph$. Define the sets

$$\chi_M = \left\{ x \in w : \left(M \left(\frac{k!\, t_k}{\rho} \right) \right) \longrightarrow 0\, as\, k \longrightarrow \infty\, for some\, \rho > 0 \right\}$$

$$\Lambda_M = \left\{ x \in w : sup_k \left(M \left(\frac{t_k}{\rho} \right) \right) < \infty\, for some\, \rho > 0 \right\}$$

The space χ_M is a metric space with the metric

$$d(x,y) = inf \left\{ \rho > 0 : sup_k \left(M \left(\frac{(k!\,|x_k - y_k|)^{1/k}}{\rho} \right) \right) \leq 1 \right\}$$

The space Λ_M is a metric space with the metric

$$d(x,y) = inf \left\{ \rho > 0 : sup_k \left(M \left(\frac{|x_k - y_k|^{1/k}}{\rho} \right) \right) \leq 1 \right\}$$

Let $(\chi_M)_s = \{x = x_k : \xi = \xi_k \in \chi_M\}$,where $\xi_k = 1!x_1 + 2!x_2 + \cdots + k!x_k$ for $k = 1, 2, 3, \cdots$ and $(\Lambda_M)_s = \{y = y_k : \eta = \eta_k \in \Lambda_M\}$,where $\eta_k = y_1 + y_2 + \cdots + y_k$ for $k = 1, 2, 3, \cdots$. Then $(\chi_M)_s$ is a metric spaces with the metric

$$d(x,y) = inf \left\{ \rho > 0 : sup_k \left\{ \left(M \left(\frac{|k!(\xi_k - \eta_k)|^{1/k}}{\rho} \right) \right) : k = 1, 2, 3, \cdots \right\} \leq 1 \right\}$$

and $(\Lambda_M)_s$ is a metric spaces with the metric

$$d(x,y) = inf \left\{ \rho > 0 : sup_k \left\{ \left(M \left(\frac{|\xi_k - \eta_k|^{1/k}}{\rho} \right) \right) : k = 1, 2, 3, \cdots \right\} \leq 1 \right\}$$

Let $\sigma(\chi_M)$ denote the vector space of all sequences $x = (x_k)$ such that $\frac{\xi_k}{k})$ is an Orlicz space of gai sequence. We recall that cs_0 denotes the vector space of all sequences $x = (x_k)$ such that (ξ_k) is a null sequence.

3 Lemma

(see (14, Theorem 7.2.7)) Let X be an FK-space$\supset \phi$. Then (i) $X^{\gamma} \subset X^{f}$. (ii) If X has AK, $X^{\beta} = X^{f}$. (iii) If X has AD, $X^{\beta} = X^{\gamma}$.
We note that $\chi_M^{\alpha} = \chi_M^{\beta} = \chi_M^{\gamma} = \chi_M^{f} = \Lambda$ for all $x = \{x_k\}$ and $y = \{y_k\}$ in χ_M.

4 Remark

$x = (x_k) \in \sigma(\chi_M) \Leftrightarrow \left\{\frac{\xi_k}{k}\right\} \in \chi_M \Leftrightarrow M\left(\left|\frac{|\xi_k|^{1/k}}{\rho k}\right|\right) \rightarrow 0$ as $k \rightarrow \infty. \Leftrightarrow M\left(\frac{|\xi_k|^{1/k}}{\rho}\right) \rightarrow 0$ as $k \rightarrow \infty$, because $k^{1/k} \rightarrow 1$ as $k \rightarrow \infty \Leftrightarrow x = (x_k) \in (\chi_M)_s$. Hence $(\chi_M)_s = \sigma(\chi_M)$, the cesáro space of order 1.

5 Propostion

$(\chi_M)_s \subset \chi_M$
proof: Let $x \in (\chi_M)_s$
$\Rightarrow \xi \in \chi_M$

$$M \quad \frac{|\xi_k|^{1/k}}{\rho}\Big)\Big) \rightarrow 0\, as\, k \rightarrow \infty \tag{3}$$

But $x_k = \xi_k - \xi_{k-1}$. 4
Hence $\left(M\left(\frac{(k!|x_k|)^{1/k}}{\rho}\right)\right) \leq \left(M\left(\frac{|\xi_k|^{1/k}}{\rho}\right)\right) + \left(M\left(\frac{|\xi_{k-1}|^{1/k}}{\rho}\right)\right)$
$\leq \left(M\left(\frac{|\xi_k|^{1/k}}{\rho}\right)\right) + \left(M\left(\frac{|\xi_{k-1}|^{1/k}}{\rho}\right)\right) \rightarrow 0\, as\, k \rightarrow \infty\, by\, using\, 3.$
Therefore $\left(M\left(\frac{(k!|x_k|)^{1/k}}{\rho}\right)\right) \rightarrow 0\, as\, k \rightarrow \infty.$
$\Rightarrow x \in \chi_M$. Hence $(\chi_M)_s \subset \chi_M$.
Note: The above inclusions is strict. Take the sequence $\delta^{(1)} \in \chi_M$. We have
$M\left(\frac{\xi_1}{\rho}\right) = 1$
$M\left(\frac{\xi_2}{\rho}\right) = 1 + 0 = 1$
$M\left(\frac{\xi_3}{\rho}\right) = 1 + 0 + 0 = 1$
$\vdots$
$M\left(\frac{\xi_k}{\rho}\right) = 1 + 0 + 0 + \cdots + = 1$
$\rightarrow k - terms \rightarrow$
and so on. Now $M\left(\frac{|\xi_k|^{1/k}}{\rho}\right) = 1$ for all k. Hence $\left\{M\left(\frac{|\xi_k|^{1/k}}{\rho}\right)\right\}$ does not tend to zero as $k \rightarrow \infty$. So $\delta^{(1)} \notin (\chi_M)_s$. Thus the inclusion $(\chi_M)_s \subset \chi_M$ is strict. This completes the proof.

6 Proposition

$(\chi_M)_s$ has AK-property
Proof: Let $x = (x_k) \in (\chi_M)_s$ and take $x^{[n]} = (x_1, x_2, \cdots x_n, 0, \cdots)$ for $n = 1, 2, 3, \cdots$.

Hence $d\;\; x, x^{[n]}) = inf\left\{\rho > 0 : sup_k\left\{M \quad \frac{\left|\xi_k - \xi_k^{(n)}\right|^{1/k}}{\rho}\right)\right\} \leq 1\right\}$
$= inf\left\{\rho > 0 : sup_k\left\{M\left(\frac{|\xi_{n+1} - \xi_n|^{1/n+1}}{\rho}\right), M\left(\frac{|\xi_{n+2} - \xi_n|^{1/n+2}}{\rho}\right), \cdots\right\} \leq 1\right\}$

$\rightarrow 0\, as\, n \rightarrow \infty$

Therefore $x^{[n]} \rightarrow x$ as $n \rightarrow \infty$ in $(\chi_M)_s$. Thus $(\chi_M)_s$ has AK. This completes the proof.

7 Proposition

$(\chi_M)_s$ is a linear space over the field C of complex numbers.
Proof: It is easy. Therefore omit the proof.

8 Proposition

$(\chi_M)_s$ is solid

Proof: Let $|x_k| \leq |y_k|$ with $y = (y_k) \in (\chi_M)_s$. So,$|\xi_k| \leq |\eta_k|$ with $\eta = (\eta_k) \in \chi_M$. But χ_M is solid. Hence $\xi = (\xi_k) \in \chi_M$. Therefore $x = (x_k) \in (\chi_M)_s$. Hence $(\chi_M)_s$ is solid. This completes the proof.

9 Proposition

$\Lambda \subset (\Gamma_M)_s \subset \Lambda_M(\Delta)$

Proof: **Step 1.** By Proposition 5., we have $(\chi_M)_s \subset \chi_M$. Hence $(\chi_M)^\beta \subset [(\chi_M)_s]^\beta$. But $(\chi_M)^\beta = \Lambda$. Therefore

$$\Lambda \subset [(\chi_M)_s]^\beta \tag{4}$$

Step2: Let $y = (y_k) \in [(\chi_M)_s]^\beta$. Consider $f(x) = \sum_{k=1}^{\infty} x_k y_k$ with $x \in (\chi_M)_s$. Take $x = \delta^n - \delta^{n+1} = (0, 0, 0, \cdots, 1, -1, 0, 0, \cdots)$

$n^{th}(n+1)^{th} place$

where, for each fixed $n = 1, 2, 3, \cdots; \delta^{(n)} = (0, 0, \cdots 1, 0, \cdots), 1$ in the n^{th} place and zero's elsewhere. Then $f(\delta^n - \delta^{n+1}) = y_n - y_{n+1}$. Hence

$M\left(\frac{|y_n - y_{n+1}|}{\rho}\right) = M\left(\frac{\left|f\left(\delta^n - \delta^{n+1}\right)\right|}{\rho}\right) \leq \|f\|\, d\left(\left(\delta^n - \delta^{n+1}\right), 0\right) \leq \|f\| \cdot 1.$

So, $M\left\{\left(\frac{y_n - y_{n+1}}{\rho}\right)\right\}$ is bounded. Consequently $M\left\{\left(\frac{y_n - y_{n+1}}{\rho}\right)\right\} \in \Lambda_M$. That is $M\left(\frac{y_n}{\rho}\right) \in \Lambda_M(\Delta)$. But $y = (y_n)$ is originally in $[(\chi_M)_s]^\beta$. Therefore

$$[(\chi_M)_s]^\beta \subset \Lambda_M(\Delta) \tag{5}$$

From (4) and (5) we conclude that $\Lambda \subset [(\chi_M)_s]^\beta \subset \Lambda_M(\Delta)$. This completes the proof.

10 Proposition

The $\beta-$ dual space of $(\chi_M)_s$ is Λ
Proof: Step1. Let $y = (y_k)$ be an arbitray point in $[(\chi_M)_s]^\beta$. If y is not in Λ , then for each natural number n, we can find an index $k(n)$ such that

$$M\left(\frac{\left|y_{k(n)}\right|^{1/k(n)}}{\rho}\right) > \frac{n!}{n}, \ (n = 1, 2, 3, \cdots)$$

Define $x = (x_k)$ by

$$M\left(\frac{(k!\,x_k)}{\rho}\right) = 1/n^k, \ for\, k = k(n); \text{and}$$

$$M\left(\frac{(k!\,x_k)}{\rho}\right) = 0\,\text{otherwise}$$

Then x is in χ_M, but for infinitely many k,

$$M\left(\frac{(k!\ |y_k x_k|)}{\rho}\right) > 1 \tag{6}$$

Consider the sequence $z = \{z_k\}$, where $z_1 = 1!\,x_1 - s$ with $s = k!\sum x_k$; and $z_k = k!\,x_k\ (k = 2, 3, \cdots)$ Then z is a point of χ_M. Also $\sum M\left(\frac{z_k}{\rho}\right) = 0$. Hence z is in $(\chi_M)_s$. But by the equation(6) $\sum M\left(\frac{(k!\,z_k x_k)}{\rho}\right)$ does not converge. Thus the sequence y would not to be in $[(\chi_M)_s]^\beta$. This contradiction proves that

$$[(\chi_M)_s]^\beta \subset \Lambda \tag{7}$$

Step2. By (4) of Proposition 9, we have

$$\Lambda \subset [(\chi_M)_s]^\beta \tag{8}$$

From (7) and (8) it follows that the $\beta-$ dual space of $[(\chi_M)_s]^\beta$ is Λ This completes the proof.

11 Proposition

$[(\chi_M)_s]^\mu = \Lambda$ for $\mu = \alpha, \beta, \gamma, f$
Step1: $(\chi_M)_s$ has AK by Proposition 6. Hence by Lemma 3(i) we get $[(\chi_M)_s]^\beta = [(\chi_M)_s]^f$. But $[(\chi_M)_s]^\beta = \Lambda$. Hence

$$[(\chi_M)_s]^f = \Lambda \tag{9}$$

Step 2: Since AK implies AD. Hence by Lemma3(iii) we get $[(\chi_M)_s]^\beta = [(\chi_M)_s]^\gamma$. Therefore

$$[(\chi_M)_s]^\gamma = \Lambda \tag{10}$$

step3:$(\chi_M)_s$ is normal by Proposition 8. Hence, by [20, Proposition 2.7], we get

$$[(\chi_M)_s]^\alpha = [(\chi_M)_s]^\gamma = \Lambda \tag{11}$$

From (9),(10)and (11), we have

$$[(\chi_M)_s]^\alpha = [(\chi_M)_s]^\beta = [(\chi_M)_s]^\gamma = [(\chi_M)_s]^f = \Lambda$$

12 Proposition

The dual space of $(\chi_M)_s$ is Λ. In other words $[(\Gamma_M)_s]^* = \Lambda$

Proof: We recall that s^k has 1 in the k^{th} place, -1 in the $(k+1)^{th}$ place and zero's else where with

$$\xi = s^k, \left\{M\left(\frac{|\xi_k|^{1/k}}{\rho}\right)\right\} = \left\{\frac{M(0)^{1/1}}{\rho}, \frac{M(0)^{1/2}}{\rho}, \cdots, \frac{M(0)^{1/k}}{\rho}, \frac{M(0)^{1/k+1}}{\rho}\right\} = \left\{0, 0, \cdots, \frac{M(1)^{1/k}}{\rho}, 0, \cdots\right\}$$

which is exits. Hence $s^{(k)} \in (\chi_M)_s$. $f(x) = \sum_{k=1}^{\infty} \xi_k y_k$ with $\xi \in (\chi_M)_s$and $f \in [(\chi_M)_s]^*$, where $[(\chi_M)_s]^*$ is the dual space of $(\chi_M)_s$. Take $\xi = s^k \in (\chi_M)_s$. Then

$$|y_k| \le \|f\|\, d \quad s^k, 0) < \infty\, for all\, k \tag{12}$$

Thus (y_k) is a bounded sequence and hence an analytic sequence. In other words, $y \in \Lambda$. Therefore $[(\chi_M)_s]^* = \Lambda$. This completes the proof.

13 Lemma

[19, Theorem 8.6.1] $Y \supset X \Leftrightarrow Y^f \subset X^f$ where X is an AD-space and Y an FK-space

14 Proposition

Let Y be any FK-space $\supset \phi$. Then $Y \supset (\chi_M)_s$ if and only if the sequence $\delta^{(k)}$ is weakly analytic

Proof: The following implications establish the result.

$Y \supset (\chi_M)_s \Leftrightarrow Y^f \subset [(\chi_M)_s]^f$, since $(\chi_M)_s$ has AD and by Lemma 13.

$\Leftrightarrow y^f \subset \Lambda$, since $[(\chi_M)_s]^f = \Lambda$.

$\Leftrightarrow$ $foreach\, f \in Y'$,the topological dual of $Y. f \quad \delta^{(k)}) \in \Lambda$.

$\Leftrightarrow f \quad \delta^{(k)})$is analytic

$\Leftrightarrow \delta^k$ is weakly analytic. This completes the proof.

15 Proposition

In $(\chi_M)_s$ weak convergence does not imply strong convergence.
Proof: Assume that weak convergence implies strong convergence in $(\chi_M)_s$. They we would have $[(\chi_M)_s]^{\beta\beta} = (\chi_M)_s$.[see 19]. But $[(\chi_M)_s]^{\beta\beta} = \Lambda^\beta = \Gamma$ By
Proposition 5, $[(\chi_M)_s]$ is a proper subspace of χ_M. Thus $[(\chi_M)_s]^{\beta\beta} \neq (\chi_M)_s$. Hence weak convergence does not imply strong convergence in $[(\chi_M)_s]$. This

16 Definition

Fix $k = 0, 1, 2, \cdots$. Given a sequence (x_k),put $M\left(\frac{\xi_{k,p}}{\rho}\right) = M\left(\frac{(1+k)!\,x_{1+k}+(2+k)!\,x_{2+k}+\cdots+(p+k)!\,x_{p+k}}{\rho}\right)$ for $p = 1, 2, 3, \cdots$. Let $(\xi_{k,p} : p = 1, 2, 3, \cdots) \in \chi_M$ uniformly in $k = 0, 1, 2, \cdots$. Then we call (x_k) an "'almost Orlicz space of gai sequence"'. The set of all almost Orlicz space of gai sequences is denoted by Δ_M.

17 Proposition

$\chi_M \bigcap \sigma^\alpha (\chi_M) = \Delta_M$, where Δ_M is the set of all almost Orlicz space of gai sequences.
Proof: put $k = 0$. Then $(\xi_{0,p}) \in \chi_M \Leftrightarrow \left(\frac{1!\,x_1+2!\,x_2+\cdots+p!\,x_p}{p}\right) \in \chi_M$

$$\Leftrightarrow M\left(\frac{|1!\,x_1+2!\,x_2+\cdots+p!\,x_p|^{1/p}}{\rho p}\right) \longrightarrow 0 \text{ as } p \longrightarrow \infty$$

$$\Leftrightarrow M \quad \frac{|1!\,x_1 + 2!\,x_2 + \cdots + p!\,x_p|^{1/p}}{\rho}\Bigg) \longrightarrow 0\, as\, p \longrightarrow \infty \tag{13}$$

$$\Leftrightarrow (x_k) \in cs_0$$
$$\Leftrightarrow \Delta_M \subset cs_0$$

put $k = 1$. Then $(\xi_{1,p}) \in \chi_M \Leftrightarrow \left(\frac{2!\,x_2+3!\,x_3+\cdots+p!\,x_p}{p}\right) \in \chi_M$

$$\Leftrightarrow M\left(\frac{|2!\,x_2+3!\,x_3+\cdots+p!\,x_p|^{1/p}}{\rho p}\right) \longrightarrow 0 \text{ as } p \longrightarrow \infty$$

$$\Leftrightarrow M \quad \frac{|2!\,x_2 + 3!\,x_3 + \cdots + p!\,x_p|^{1/p}}{\rho}\Bigg) \longrightarrow 0\, as\, p \longrightarrow \infty \tag{14}$$

Simarly we get

$$M\left(\frac{3!\,x_3 + 4!\,x_4 + \cdots}{\rho}\right) = 0 \tag{15}$$

$$M\left(\frac{4!\,x_4+5!\,x_5+\cdots}{\rho}\right)=0 \qquad (16)$$

$$\vdots$$

and so on. From (13) and (14) it follows that $M\left(\frac{1!\,x_1}{\rho}\right)=M\left(\frac{1!\,x_1+2!\,x_2+\cdots}{\rho}\right)=M\left(\frac{2!\,x_2+3!\,x_3+\cdots}{\rho}\right)=0$. Simiarly we obtain $M\left(\frac{2!\,x_2}{\rho}\right)=0, M\left(\frac{3!\,x_3}{\rho}\right)=0,\cdots$ and so on. Hence $\Delta_M=\theta$ where θ denotes the sequence $(0,0,\cdots)$. Thus we have proved that $\chi_M\bigcap\sigma^\alpha\left(\chi_M\right)=\theta$ and $\Delta_M=\theta$. In other words $\chi_M\bigcap\sigma^\alpha\left(\chi_M\right)=\Delta_M$. This completes the proof.

18 Proposition

$\left(\chi_M\right)_s=\chi_M\bigcap cs_0$

Proof: By Proposition 5. $\left(\chi_M\right)_s\subset\chi_M$. Also, since every Orlicz space of gai sequence $M\left(\xi_k/\rho\right)$ is exists, it follows that $M\left(\xi_k/\rho\right)$ is exits. In other words $M\left(\xi_k/\rho\right)\in cs_0$. Thus $\left(\chi_M\right)_s\subset cs_0$. Consequently,

$$\left(\chi_M\right)_s\subset\chi_M\bigcap cs_0 \qquad (17)$$

On the other hand, if $M\left(\frac{k!\,x_k}{\rho}\right)\in\chi_M\bigcap cs_0$, then $f\left(z\right)=\sum_{k=1}^{\infty}M\left(\frac{k!x_k}{\rho}\right)z^{k-1}$ is an Orlicz space of an gai function. But $M\left(\frac{k!\,x_k}{\rho}\right)\in cs_0$. So, $f\left(1\right)=M\left(\frac{1!\,x_1+2!\,x_2+\cdots}{\rho}\right)=0$. Hence $\frac{f(z)}{1-z}=\sum_{k=1}^{\infty}M\left(\frac{\xi_k}{\rho}\right)z^{k-1}$ is also Orlicz space of an gai function. Hence $M\left(\frac{\xi_k}{\rho}\right)\in\chi_M$. So $x=\left(x_k\right)\in\left(\chi_M\right)_s$. But $\left(x_k\right)$ is arbitrary in $\chi_M\bigcap cs_0$. Therefore

$$\chi_M\bigcap cs_0\subset\left(\chi_M\right)_s \qquad (18)$$

From (17) and (18) we get $\left(\chi_M\right)_s=\chi_M\bigcap cs_0$. This completes the proof.

19 Definition

Let $\alpha>0$ be not an integer. Write $S_n^{(\alpha)}=\sum_{\gamma=1}^{n}A_{n-\gamma}^{(\alpha-1)}x_\gamma$, where $A_\mu^{(\alpha)}$ denotes the binomial coefficient $\left(\frac{(\mu+\alpha)(\mu=\alpha-1)\cdots(\alpha+1)}{\mu!}\right)$. Then $\left(x_n\right)\in\sigma^\alpha\left(\chi_M\right)$ means that $\left\{\frac{S_n^\alpha}{A_n^{(\alpha-1)}}\right\}\in\chi_M$.

20 Proposition

Let $\alpha > 0$ be a number which is not an integer. Then $\chi_M \bigcap \sigma^\alpha(\chi_M) = \theta$, where θ denotes the sequence $(0, 0, \cdots, 0)$

Proof: Since $(x_n) \in \sigma^\alpha(\chi_M)$ we have $\left\{\frac{S_n^\alpha}{A_n^{(\alpha-1)}}\right\} \in \chi_M$. This is equivalent to $\left(S_n^{(\alpha)}\right) \in \chi_M$. This, in turn, is equivalent to the assertion that $f_\alpha(z) = \sum_{n=1}^\infty s_n^{(\alpha)} z^{n-1}$ is Orlicz space of an gai function. Now $f_\alpha(z) = \frac{f(z)}{(1-z)^\alpha}$. Since α is not an integer, $f(z)$ and $f_\alpha(z)$ cannot both be integral functions, for if one is an integral function, the other has a branch at $z = 1$. Hence the assertion holds good. So, the sequence $0 = (0, 0, \cdots, 0)$ belongs to both χ_M and $\sigma^\alpha(\chi_M)$. But this is the only sequence common to both these spaces. Hence $\chi_M \bigcap \sigma^\alpha(\chi_M) = \theta$. This completes the proof.

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Inventory Production Control Model With Back-Order When Shortages Are Allowed

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***Abstract*-It is prohibited to have shortage of inventory since inventory cost is induced from the amount of product stored. This paper presents inventory control theory in production inventory problem when shortages are allowed and backorder takes place. Three assumptions are considered here on shortage and backorders and this leads to three models. The first: when demand is fixed and known, production is infinite and shortages are allowed although the cost of shortage is finite. Second when time (t) interval is fixed, replenishment is allowed and production rate is infinite. Third when production rate is finite. It makes economic sense from the applications that for any production where shortages are allowed, backorder must follows to avoid lost in sales.**

Keywords*: Inventory control, Backorder, Production, Shortage, Demand*

I. INTRODUCTION:

Due to the quest for efficiency accelerated by the so called financial crisis, inventory control is a vital function in almost all kinds of productions. Inventory models majorly focused on minimizing the total inventory cost and to balance the economics of large orders or large production runs against the cost of holding inventory and the cost of going short. The method has been efficiently and successfully applied by some researchers in many areas of operation [2, 3, 5, 6, 7, 8 and 10]. Production and inventory planning and control procedures for a target firm depends on (i). Whether production is make-to-stock or make-to-order (which in turn depends on the relation between customer promise time and production lead time.) and (ii). Whether demand is for known production or anticipated production.

II. LITERATURE REVIEW

This paper introduce some typical papers involve in the topics with different subcategories, lost sales, backorders, shortages and deterioration as well as periodic review and continuous review. One critical factor playing major roles on the inventory theory is backorders. Much of the literatures on inventory models ignore backorders. Backorders means delay in meeting demands or inability to meet it at all. Most inventory models discuss two extreme situations when items are stock out. They are: (i). All demand within shortage period is backorder. And (ii). All demand within shortage period is lost sales. In real inventory systems, demands during the period of stock out can be partially captive. If demand is fully captive, the next replenishment will fulfill unsatisfied demands during the period of backorders. On the contrary, unsatisfied demands will be completely lost if demand cannot be fully captive, yet demand rate during the period of stock out is not a fix constant if to take backorders into consideration. The recent survey of [3, 4] and many other scholars have developed inventory models on related field, and initiated the concept of demands which will be changed through time cycle into model, also included backorder status, study on [9] optimal control of production inventory system with deteriorating items and dynamic cost, and a study on optimal control of production inventory system with deterioration items using Weibull distribution [1, 11]. Also there was an inventory model of replenishing the stock after a period of backorder [13], which is that deplete cycle always started from the period of backorder. A modification of the complete backorder assumptions and proposed the concept of partial backorders [12], which assumed the backorder ratio is a constant between 0 and 1. The assumption is that usually the time scale of backorder will become consumers' main pondering factor to accept backorder. This paper looked into assumptions and models for production inventory of a single item when shortages are allowed and there is an order to meet exogenous demand at a minimum cost.

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III. METHODOLOGY

Inventory control models assumed that demand from customer are known for planning period both at present and past period. It is prohibited to have shortage of inventory since inventory cost is induced from the amount of product storage. Three assumptions were considered.

Notations and Assumptions

To develop the proposed models, the following notations and assumptions are used in this paper.

$I(t)$ = inventory level at time t.

R_t: Demand rate or the number of items required per unit time.

C_1: Holding cost per unit time

C_2: Shortage cost per unit item per time

C_3: Production Set up cost per run

t : interval between runs.

q : Number of items produced per production run

if a production is made at time interval t, a quantity q = Rt must be produced in each run. Since the stock in small time *dt is Rtdt, the stock in time period t is*

$$\int_0^t Rtdt = \frac{1}{2}Rt^2 = \frac{1}{2}qt$$

i) Assumptions 1

In this model, we assume that demand is fixed and known, production is infinite and shortages are allowed although the cost of shortage is finite. i.e.

1. The inventory system involves only one item.
2. Replenishment occurs instantaneously on ordering i.e. lead-time is zero.
3. Demand rate *R(t)* is deterministic and given by *R (t) =* ; 0 < *t* < *T.*
4. Shortages are allowed and completely backlogged.
5. The planning period is of infinite length. The planning horizon is divided into sub-intervals of length *T* units. Orders are placed at time points $t_1, and\ t_2$, *the* order quantity at each re-order point being just sufficient to bring the stock height to a certain maximum level *S*

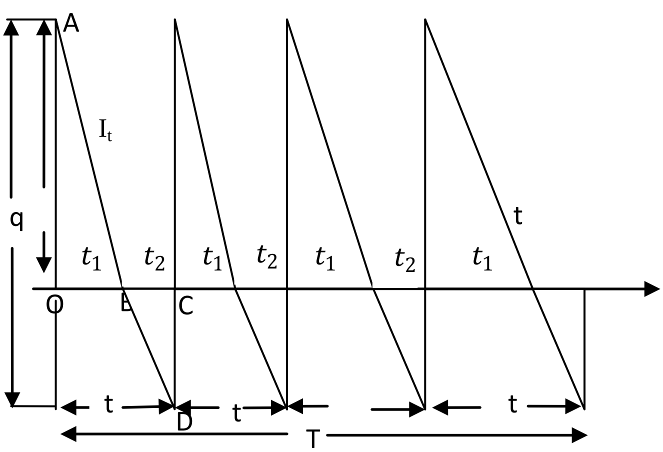

Figure 1.0 variation of inventory with

If $t = t_1 + t_2$ *then*

$\frac{t_1}{t} = \frac{I_t}{q}$ then $t_1 = \frac{I_t}{q}t$

Also,

$\frac{t_2}{t} = \frac{q - I_t}{q}$ then, $t_2 = \frac{q - I_t}{q}t$

Total inventory during time t = Area of triangle AOB

$= \frac{1}{2}I_t\ t_1$

Inventory holding cost during time t = $\frac{1}{2}C_1 I_t\ t_1$

Similarly, total shortage during time t = Area of $\Delta\ BCD$

$= \frac{q - I_t}{2}t_2$

Shortage cost during time t = $C_2\frac{(q - I_t)}{2}t_2$

Total cost during time

t = $\frac{1}{2}C_1 I_t\ t_1 + C_2\frac{(q - I_t)}{2}t_2 + C_3$

Average total cost during time

t = $\frac{1}{t}\left[\frac{1}{2}C_1 I_t\ t_1 + C_2\frac{(q - I_t)}{2}t_2 + C_3\right]$

$$C(I_t, q) = \frac{C_1 I_t^2}{2q} + \frac{C_2(q - I_t)^2}{2q} + \frac{C_3 R}{q} \qquad 1.0$$

Differentiate equation 1.0 partially w.r.t I_t, q and equate to zero to obtain optimal inventory level (I_t) and optimum lot size (q)

$$\frac{\partial C(I_t, q)}{\partial I_t} = 0$$

$$I_t = \frac{C_2\ q}{C_1 + C_2}$$

which is positive for the second derivative, it shows that the Optimal value of inventory level is

$$I_{t0} = \frac{C_2}{C_1 + C_2}q \qquad 1.1$$

Similarly, $\frac{\partial C(I_t, q)}{\partial q} = 0$

$$q = \sqrt{\frac{C_1 + C_2}{C_1 C_2}}.\sqrt{2C_3 R} \qquad 1.2$$

The optimal value of lot size q is

$$q_o = \sqrt{\frac{C_1 + C_2}{C_1 C_2}}.\sqrt{2C_3 R} = \sqrt{\frac{C_1 + C_2}{C_2}}.\sqrt{\frac{2C_3 R}{C_1}} \qquad 1.3$$

Hence, equation 1.1 can be written as

$$I_{t0} = \sqrt{\frac{C_2}{C_1(C_1 + C_2)}}.\sqrt{2C_3 R} \qquad 1.4$$

Time

Substituting the values of I_{t0} , q_o in equation 1.0, we obtain the minimum average cost per unit time i.e

$$C_0(I_{to}, q_o) = \sqrt{\frac{C_2}{(C_1 + C_2)}}.\sqrt{2C_1 C_3 R} \qquad 1.5$$

Optimum time interval between runs is given by

$$t_o = \frac{q_o}{R} = \sqrt{\frac{C_1 + C_2}{C_1 C_2}}.\sqrt{\frac{2C_3}{R}} \qquad 1.6$$

I. Assumption II.

1. Fixed time interval t.

When time interval t is fixed, it means inventory is to be replenished after every fixed time t. All other assumptions in I above hold.

Total inventory holding cost during time $t = \frac{1}{2}C_1 I_t t_1$

Total shortage cost during time t = $\frac{1}{2}C_2(q - I_t)t_2$

Set up cost C_3 and time interval t are both constant therefore, average set up cost per unit time $\frac{C_3}{t}$ is also constant. It needs not to be considered.

Total average cost per unit

$$C(I_t) = \frac{1}{t}\left[\frac{1}{2}C_1 I_t t_1 + \frac{1}{2}C_2(q - I_t)t_2\right]$$

Or

$= \frac{C_1}{2q}.I_t^2 + \frac{C_2}{2q}(q - I_t)^2$ 2.0

$\frac{\partial}{\partial I_t}(CI_t) = 0$

$I_t = \frac{C_2\, q}{C_1 + C_2}$ 2.1

Hence the minimum inventory level or order quantity given is

$I_{t0} = \frac{C_2\, q}{C_1 + C_2} \quad or \quad \frac{C_2\, Rt}{C_1 + C_2}$ 2.2

The minimum average cost per unit time from equation 2.0 is

$$C_0(I_t) = \frac{C_1}{2q}\left(\frac{C_2\, q}{C_1 + C_2}\right)^2 q^2 + \frac{C_2}{2q}\left(q - \frac{C_2}{C_1 + C_2}.q\right)^2$$

$= \frac{1}{2}.\frac{C_1 C_2}{C_1 + C_2}.q \quad or \quad \frac{1}{2}.\frac{C_1 C_2}{C_1 + C_2} Rt$ 2.3

3.. Assumption III.

Finite production / planning rate.

The model here follows the assumptions in I except that production rate is finite. With this assumption, we found that inventory is zero at the beginning. It increases at a constant rate (K-R) for time t_1 until it reaches a level I_t. No replenishment during time t_2, inventory decreases at the rate R until it reaches zero. Shortage start piling up at constant rate R during t_3 until this backlog reaches a level s. Lastly, production start and backlog is filled at a constant rate K-R during t_4 till backlog become zero. This completes cycle.

The total time taken is $t = t_1 + t_2 + t_3 + t_4$.

Holding cost $= \frac{1}{2} C_1\, I_t(t_1 + t_2)$

Shortage cost during time interval t = $\frac{1}{2} C_2\, s(t_3 + t_4)$

Set up cost = C_3

Hence, total average cost per unit time t

$C = \frac{\frac{1}{2}C_1\, I_t(t_1+t_2) + \frac{1}{2}C_2\, s(t_3+t_4) + C_3}{t_1 + t_2 + t_3 + t_4}$ 3.0

equation 3.0 *is a functions of six* (6) *variables.*

$I_t, s, t_1 + t_2 + t_3 + t_4$

Inventory level at time t_1 *is*

$I_t = (K - R)t_1$ 3.1

Also at time t_2 is

$I_t = Rt_2$ 3.2

$\therefore (K - R)t_1 = Rt_2$ 3.3

Also, S = Rt_3 , 3.4

and s = $(K - R)t_4$ 3.5

$\therefore (K - R)t_4 = Rt_3$ 3.6

Adding equation 3.3 & 3.6,

$(K - R)(t_1 + t_4) = R(t_2 + t_3)$.

Manufacturer's rate multiply by manufacturer's time gives manufactured quantity produced

q = $Kt_1 + Kt_4 = (t_1 + t_4)K$

$(t_1 + t_4) = \frac{q}{K}$, 3.7

Adding equations 3.2 and 3.4

$I_t + s = R(t_2 + t_3)$

$I_t = R(t_2 + t_3) - s$

I_t = (K-R) $(t_1 + t_4)$ - s

$I_t = \frac{q}{K}(K - R) - s$

$I_t = \left(\frac{q}{K}\right)(K - R) - s$

$I_t = q\left(1 - \frac{R}{K}\right) - s$

From equation 3.1 & 3.2

$t_1 + t_2 = \frac{I_t}{K-R} + \frac{I_t}{R}$ 3.9

And $(t_2 + t_3) = \frac{s}{K-R} + \frac{s}{R}$

Hence, $t = t_1 + t_2 + t_3 + t_4$

$= \left(\frac{1}{K-R} + \frac{1}{R}\right)\left(q.\frac{K-R}{K}\right) = \frac{q}{R}$ 3.10

Hence, equation 3.0 becomes,

$C(q,s) = \frac{1}{2q}.\frac{K}{K-R}\left[C_1\left\{q.\frac{K-R}{K} - s\right\}^2 + C_2 s^2\right] + \frac{R}{q} C_3$ 3.11

$$\frac{\partial C(q,s)}{\partial q} = 0$$

Minimum lot size is

$q_o = \sqrt{\frac{2C_3(C_1+C_2)}{C_1 C_2}}.\sqrt{\frac{KR}{K-R}}$ 3.12

And

$\frac{\partial C(q,s)}{\partial s} = 0$, implies

$s_o = q.\frac{K-R}{K}.\frac{C_1}{(C_1+C_2)}$,

$s_o = \sqrt{2C_3.\frac{C_1}{(C_1+C_2)C_2}}.\sqrt{\frac{R(K-R)}{K}}$. 3.13

Substituting q_o and s_o into equation 3.5 above, we have the optimum shortage cost cost

$$C_o(q,s) = \sqrt{\frac{2C_1\, C_2\, C_3}{(C_1 + C_2)}}.\sqrt{\frac{R(K - R)}{K}}$$

$= \sqrt{\frac{2C_1\, C_2\, C_3 R(K-R)}{K(C_1+C_2)}}$ 3.14

Optimum time interval t_o is

$t_o = \frac{q_o}{R} = \sqrt{\frac{2KC_3(C_1 + C_2)}{C_1 C_2 R(K-R)}}$ 3.15

Optimum inventory level

$I_{t0} = q\left(1 - \frac{R}{K}\right) - s_o$

$= \sqrt{\frac{C_2}{C_1 + C_2}}.\sqrt{\frac{(K-R)}{K}}.\sqrt{\frac{2C_3 R}{C_1}}$

$= \sqrt{\frac{2\, C_2\, C_3 R(K-R)}{KC_1(C_1+C_2)}}$ 3.16

IV. NUMERICAL APPLICATIONS

Example 1

If a particular soap items has demand of 9000 units/year. The cost of one procurement is £100 and holding cost per unit is £2.40per year. The replacement is instantaneous and the cost of shortage is also £5per unit/year. We are required to determining the following:

i.) Economic lot size/Optimum lot size,
ii.) The number of orders per year ,
iii.) The time between the orders,
iv.) The total cost per year if the cost of one unit is £1.

Solution

Step I.

Demand rate R = £9000 units/year,
Holding cost, C_1= £2.40/unit/year,
Shortage cost C_2 = £5/unit/year,
Production set up cost per run
C_3 = £100/procurement.

i.) From equation (1.3),
q_o= 1,053units/run i.e. the optimum lot size / run is 1,053units.

ii.) The number of order per year = 8.55units/year
Recalled, equation (1.6)

Hence, the number of order per year is 8.55 or ≈ 9 number of times ordered per year.

iii. Time period between the order is as follows,
From equation 1.6, = 0.117year
i.e. there is approximately one month and thirteen days period between the order.

iv. From equation (1.5), the total cost per year if the cost of one unit is £1.
= £10710 per year

Hence, the total cost per year if the cost of one unit is £1 is £10,710

Example 2,

Consider an inventory system with the following data in usual Notations:

R = 20 engines/ day

$$C_2 = £10 \; per \; engines \; per \; day$$

C_1 = £12/month or $\frac{12}{30}$ = £0.4/day.

$t = 1 \; month = 30 day \; i.e. lead \; time \neq 0. (fixed).$

We now want to check for the inventory level at the beginning of each month and the optimum cost per unit.

Recall from equation (2.2)

$$I_{t0} = \frac{10}{0.4 + 10} * 20 * 30 = 577 engines/month$$

Hence the optimum inventory level at the beginning of each month is 577 engines.

Also recall from equation 2.3

$$C_0(I_t) = \frac{1}{2} \cdot \frac{C_1 C_2}{C_1 + C_2} Rt$$

$$= \frac{1}{2} \cdot \frac{0.4 * 10}{0.4 + 10} * 20 * 30$$

= £115.38/unit of engines

i.e the minimum cost of producing an engine is £115.38

Example 3.

A company has a demand of 12,000 units/year from an item and it can produce 2,000 such items per month. The cost of one set up is £400 and the holding cost/unit/month is £0.15. the shortage cost of one unit is £20 per year. Find the optimum lot size and the total cost per year, assuming the cost of one unit if £4. We can also find the maximum inventory manufacturing time and total time.

Given the following; R = 12,000
K = 2000 * 12 = 24,000/units/year,
C_1=0.15 *12 = 1.8/unit/year,
C_2 = £20/year
C_3=£400/set-up

Using equation 3.7

$$q_o = \sqrt{\frac{2 * 400 * (1.8 + 20)}{1.8 * 20}} \cdot \sqrt{\frac{24000 * 1200}{24000 - 1200}}$$

= 3,410 units

The optimum lot size is 3,410units.

The total cost per year is considered by using equation 3.9

$$C_o(q,s) = 12000 * 4 + \sqrt{\frac{2C_1 \, C_2 \, C_3 R(K-R)}{K(C_1 + C_2)}}$$

$$C_o(q,s) = 12000 * 4 + \sqrt{\frac{2 * 1.8 * 20_* 400_* 12000(24000 - 12000)}{24000(20 + 1.8)}}$$

= £50,185per year

The total cost per year is £50,185 when the cost of one item is £4

Using the equation 3.11, optimum inventory level at time t is

$$I_{t0} = \sqrt{\frac{2 * 20 * 400 * 12000 * 10}{2 * 1.8 * 10.9}}$$

= 1,564unit/production run

iii. Manufacturing time interval $t_1 + t_4$
Recall from equation 3.3,

$$(t_1 + t_4) = \frac{q}{K},$$

$$= \frac{3410}{24000}$$

Hence, the optimum inventory level at time t is = 0.1421 year

which is approximately 52 *days or*

1 *month and three weeks.*

iv. Optimum time interval t_0 is given by

v. $\frac{q}{R} = \frac{3410}{1200}$,

0.2842 years

This means that, the minimum time interval required is 103 days i.e 3 months and 8 days.

V. CONCLUSION

It can be deduced that when replenishment cost and demand rate per unit time R increase, order quantity q, and relevant total cost C will increase. An increment of inventory holding cost per unit (h), backorder cost and penalty cost will lead to the phenomenon of increasing before diminishing. This idea can induce cost items in inventory depletion period having a trade – off relationship with cost items in backorders status. Also the decision about when an order should be placed will also be based on how low the inventory should be allowed to be depleted before the order arrives. The idea is to place an order early enough so that the expected number of units demanded during the replenishment lead time will not result in stock out every often

VI. CONTRIBUTION TO KNOWLEDGE

This research work contribute to knowledge in many areas of production or daily life activities where failure to meet up with demand/supply (activities) induced a nebulous cost and pay-price or (replenishment has to be done). Many industries can benefit from this through proper implementations/ applications.

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Ellipse's Perimeter Evaluation

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Abstract*: This is a trendy problem since the time of Kepler (1609). He was asked what the length of the elliptical orbits is. Even today, thousand of mathematicians are working on this matter. Ramanujan, a super mathematician (1914) from India, is the Master of all these researchers. But, no one of them considered that the orbits are not elliptical. Their mind was blocked by the laws of Kepler for 400 years. Orbit's length and ellipse's perimeter are two different matters. In these *papers we will treat* mathematical ellipse. *Orbits are not elliptical

Keywords: perimeter estimation, ellipse, other astroids, cracking Thales

I. INTRODUCTION

Thales theorem B/A=c/d is well known by everyone. No! No one of the mathematicians knows the generalized Thales theorem. Generalized Thales theorem explains the total arc length on positive Cartesian. Like Pythagoras ($a^2+b^2=c^2$), generalized Thales is ($a^s+b^s=K^s$)

So first, we must start by understanding Thales theorem. Then after we will attack the total arc length of the astroids $(x/a)^r+(y/b)^r=1$.When r=2 the astroid is an ellipse.

Fig.1 shows what we know and what we should know about Thales.
On the left is what we know; on the right is what we must be aware.

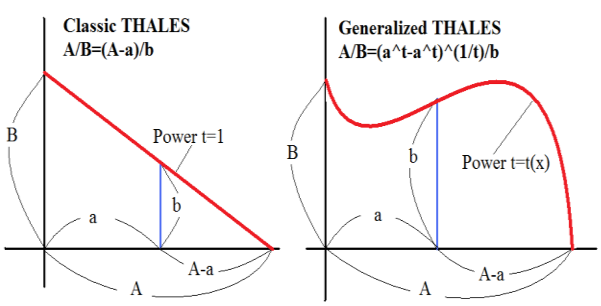

Fig.1

In fact, for an astroidal expression $(x/A)^t+(y/B)^t=1$ where t=t(x),variable (t) ,simply when x=a y=b is found.

As a consequence, the relation of the coordinates at the touching point P of (t variable) astroid with (r constant) astroid is shown on Fig.2

$A/B=(A^t-x^t)^{(1/t)}/y$ (a)
$a/b=(a^r-x^r)^{(1/r)}/y$ (b)

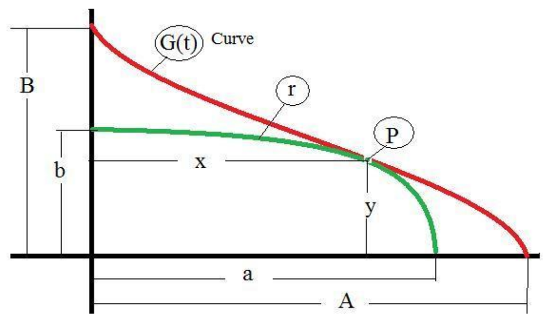

Fig. 2
And it exists an $s=r*t/(r-t)$ which gives the relation
$A/B=(A^s-a^s)^{(1/s)}/b$. Fig.3 (c)

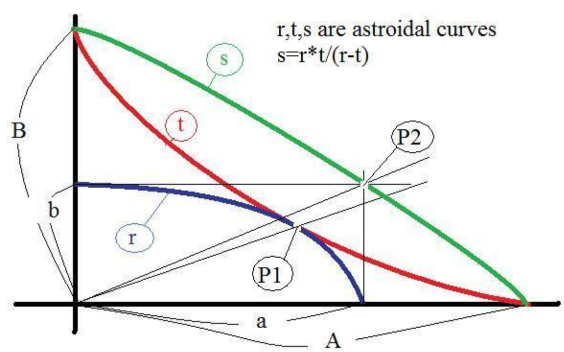

Fig. 3
That is to say: when A=B=K $a^s+b^s=K^s$ (d)

And, when K is the length (L1) of the (r=constant) astroid (arc length on the positive Cartesian), the expression (d) is commented as

$1+TAN^s=L1^s$ (e)
Where TAN=b/a.

Therefore the expression (d) is an **equivalent** to the elliptic integrals, with which we evaluate the total arc length on the positive Cartesian.

Then we may evaluate any astroidal value (area, length,…) without use of integrals for any (r):
Say K=total arc length on positive Cartesian, evaluate L1
Say K=total area on positive Cartesian, evaluate Areas
Say K= any elliptical subject, evaluate this subject, without use of integrals.

Existence of (s) : proof (dated 1959.03.24 Istanbul/Turkey)

Consider Fig.4

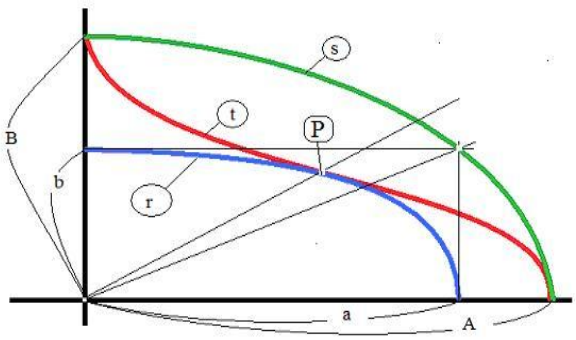

Fig . 4

The astroid family
(x/a)^r+(y/b)^r=1 (1)

where r=Constant, is enveloped by
(x/A)^t+(y/B)^t=1 where t=t(x) (2)

We search for a relation f(a,b,r,A,B,t)=0
At the touching point (P) of the graphs we write
-the slopes are equal
-the coordinates are equal

For the coordinates we write
y=b/a*(a^r-x^r)^(1/r)=B/A*(A^t-x^t)^(1/t) (3)

For the slope of the enveloped astroid we write
dy/dx=-(b/a)^r*(x/y)^(r-1)
dy/dx=-(b/a)*((x^r)/(a^r-x^r))^((r-1)/r) (4)

For the slope of the envelope itself :
say (x/A)^t=U ; (y/B)^t=V then,
U+V=1 (5)

dU+dV=0 (6)

we have
t*ln(x/A)=lnU
t*ln(y/B)=lnV (7)

When we differentiate (7), we write

dt*ln(x/A)+t*(dx/x)=dU/U
dt*ln(y/B)+t*(dy/y)=dV/V (8)

and there from,

dU=U*(dt*ln(x/A)+t*(dx/x))
dV=V*(dt*ln(y/B)+t*(dy/y)) (9)

Considering (7), the expression (6) is written as

U*(dt*1/t*lnU+t*dx/x)+V*(dt*1/t*lnV+t*dy/y)=0 (10)

and there from

V*t*dy/y=-U*(dt/t*lnU+t*dx/x)-V*(dt/t*lnV (11)

dy/dx=y/(V*t)*U*((dt/dx*1/t*lnU+t/x))+V*dt/dx*1/t*lnV) (12)

taking (U and t/x) out of the parenthesis

dy/dx=U/V*y/x*(1+dt/dx*x/t^2*1/U*(U*lnU+V*lnV))
is written (13)

say
N=(1+dt/dx*x/t^2*1/U*(U*lnU+V*lnV)) (14)

dy/dx=-U/V*y/x*N is written (15)

For the equality of the slopes, we write (15)=(4). Considering also (3), we write

(b/a)*((x^r)/(a^r-x^r))^((r-1)/r)=U/V*1/x*(B/A)*(A^t-x^t)^(1/t)*N (16)

Replacing (4) and (5) in (16) we write

(b/a)^r*(x/y)^(r-1)=(B/A)^t*(x/y)^(t-1)*N (17)

say (B/A)=E (18)

Use (3), we write (17) as follows

(b/a)^r=E^t*(x/(E*(A^t-x^t)^(1/t))^(t-r)*N (19)
and there from

b^r=E^r*a^r*(x^t/(A^t-x^t))^((t-r)/t)*N (20)

is written

using (3) and (20), the expression (1) is written as follows (x/a)^r+((A^t-x^t)^(r/t)*(A^t-x^t)^((t-r)/t))/a^r*x^(t-r)*N=1 (21)

x^t*N+A^t-x^t=a^r*x^(t-r)*N (22)

A^t=a^r*x^(t-r)*N-x^t*(N-1) (23)

Then, from (23) we get

x=((A^t-x^t*(1-N))/a^r*N)^(1/(t-r)) (24)

a^r*x^(t-r)*N=A^t-x^t*(1-N) (25)

using (25) in (20)

b^r=E^r*(A^t-x^t*(1-N))/(A^t-x^t)^(t-r)/t)
A^t=(b/E)^r*(A^t-x^t)^(t-r)/t)+x^t*(1-N) (26)
is written

(26)=(23) then,

(a/b*E)^r*x^(t-r)*N=(A^t-x^t)^((t-r)/t) (27)
is written
A^t-x^t=(a/b*E)^(r*t/(t-r))*x^t*N^(t/(t-r)) (28)
is written
A^t=x^t*(1+((a/b)*E)^((r*t)/(t-r))*N^(t/(t-r))) (29) is written
From (29) we get (x)

x=A/(1+((a/b)*E)^((r*t)/(t-r))*N^(t/(t-r)))^(1/t) (30)

(30)=(24) then,

((A^t-x^t*(1-N))/a^r*N)^(1/(t-r))=A/(1+((a/b)*E)^((r*t)/(t-r))*N^(t/(t-r)))^(1/t) (31)

(A^t-x^t*(1-N))/a^r*N=(A^(t-r)*b^r)/(b^(r*t/(t-r))+(a*E)^(r*t)/(t-r))*N^(t/(t-r)))^((t-r)/t)
(32)

We take ((t-r)/r*t) power of both sides,
We proceed, then we take the power (r*t/(r-t)) of both sides

say r*t/(r-t)=s (33)

we write

(A^t-x^t*(1-N))^(s/r)*A^t=a^s*N^(s/r)+(b/E)^s (34)

For the astroids of the same power, when b/a=TAN=Constant
dt/dx=dt/dTAN*dTAN/dx=0 and **N=1** then,
Figure 5. (35)

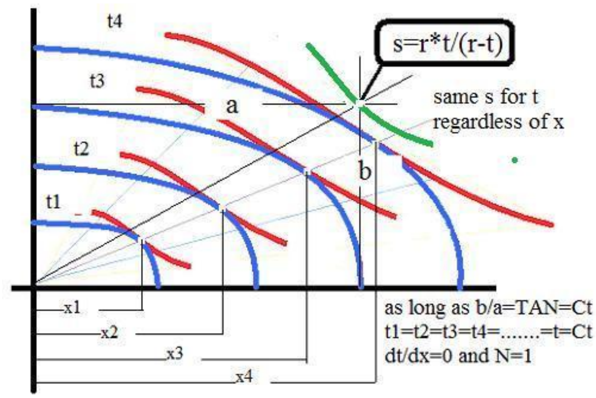

Fig.5

A^(t*s/r)*A^t=A^s=a^s+(b/E)^s (36)
is written
that is as a general expression:

(a/A)^s+(b/B)^s=1 (37)

In a symmetric case, when (A=B=K) ,
we write
K^s=a^s+b^s (38)

Say (K/a=L1) ; also say b/a=TAN

L1^s=1+TAN^s (38)L
is written
L1=(1+TAN^s)^(1/s) is proofed, found.

That is the equivalent of the astroidal finite integrals .
Insolvable integrals!!!

$$L = a.\int_0^1 (1+(b/a)^2.(k^r/(1-k^r))^{((2r-2)/r)})^{(1/2)}.dk$$

Expression (38) treats all the mathematical astroids (x/a)^r+(y/b)^r=1, ellipse included (case r=2)
Give any meaning to (K/a), crack the implicit expression (K/a)^s=1+TAN^s
Say:
(K/a) is the total arc length
evaluate L1 on the positive Cartesian
(K/a) is the total area
evaluate Area1 on the positive Cartesian
(K/a) is any astroidal subject evaluate this subject on the positive Cartesian

A cracking method:

By trend, ellipse's perimeter will be estimated. Consider Fig.6. According Thales a^s+b^s=L^s

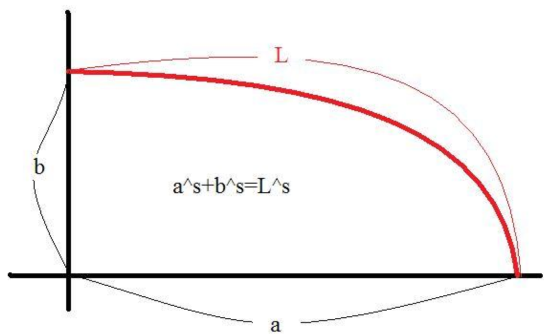

Fig.6

s=s(a,b,r)=r*t/(r-t) where (t) is variable.So (s) is variable, is an implicit power. When speaking about lengths, one should know the length (L) to evaluate (s). But if we know (L) we don't need to evaluate (s). If we know (s) we know (L). So first, we must crack Thales L^s=a^s+b^s and find an estimable expression for (s).

Various approximation steps are indicated here below to reach to the "most accurate cracking" When we say we know (L1), the unit length on the positive Cartesian, we mean (L1) evaluated by summing billion of (dL1) arc segments. Suppose, we don't know what an integral is!

Version s=Constant
we know L1exact=(Pi/2)*R, when we speak of circles.
from L1=(1+(R/R)^s)^(1/s)=(Pi/2)*R we get
s.min=ln(2)/ln(Pi/2)
s.min=1,53492853566138..

When used for the whole TAN range, this gives the error graph (Fig.7). Max.error %=0,003605937….at TAN=5,006784983..

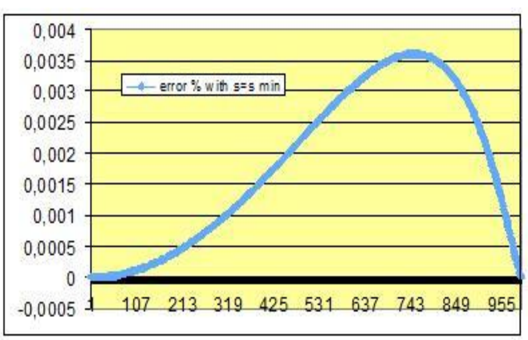

Fig.7

Error % for 1<TAN<infinity. Max.error %=0,003605937….at TAN=5,006784983..

Version s=s min +m*TAN (Linear variable)
we know L1Exact=636,62517105986… for TAN(89,91 o)=636,6192488…
from L1 max=(1+TAN.max^s)^(1/s) we get
s.max=1,71122902010321….then, according the value of TAN

s=s.min+(s.max-s.min)/(TAN.max-1)*(TAN-1)
s=1,53492853566138+0,000277355*(TAN-1) is evaluated

This gives the error graph (Fig.8)
Max.error %=0,003477761…….at TAN=4,966157118..

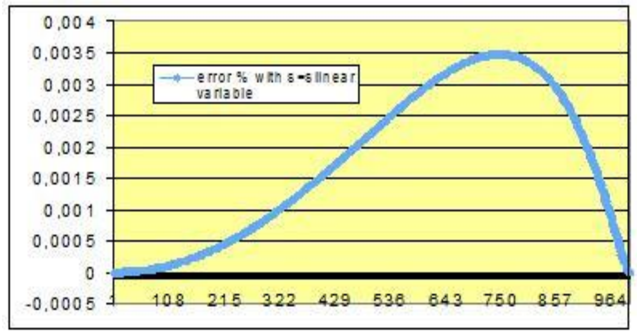

Fig.8
Error % for 1<TAN<infinity. Max.error %=0,003477761…….at TAN=4,966157118..

Version (s=sMod;a power version,with p=2,98 constant) we know all L1Exact.
an (sExact) data is obtained.Fig.9

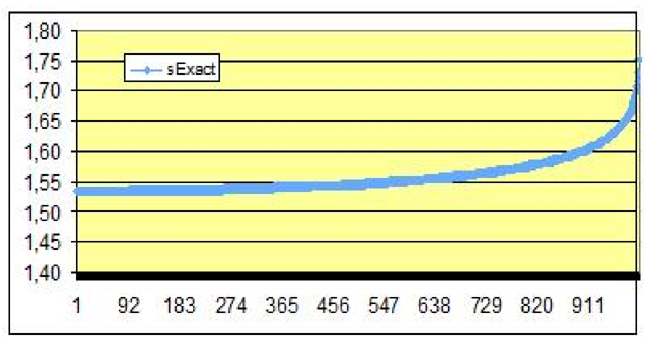

Fig.9
sReal for 1<TAN< infinity

By similarity to an astroid, we write an (s Mod) overlapping the (s Real).We start a cracking.
sMod=astroid+polynome
sMod=d1+b1*(1-(x-c1)/a1)^p)^(1/p)+(F+m1*x^v1+n1*x^w1). Proposed as a cracking method.

where ,for 1000 lines of evaluation on an excel table

Angle step	=(90o-45o)/1000
x	=(Angle-45)/Angle step
Angle o	=(45+angle step*x)
ATAN	=angle o*Pi/180
Angle o	=ATAN*180/Pi

Fig.10 shows the overlapping with the proposed parameter values of the Table I

parameters	values
	1,534928535661380

Table I

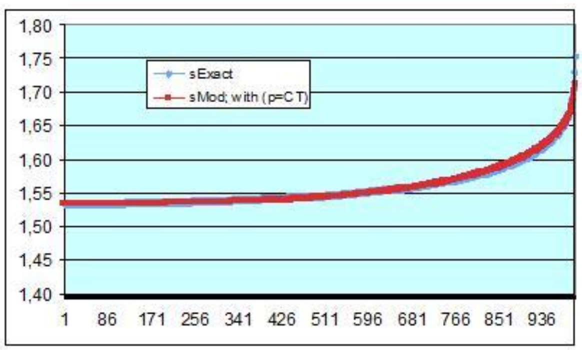

Fig.10 Overlapping of sExact&sMod with p=Constant=2,98

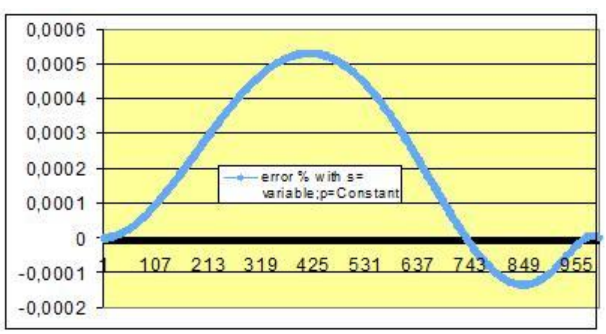

Fig.11
Error % for 1<TAN<infinity with p=2,89. Max error %=0,000529940 ..at TAN=2,0211255..

Version (s=sMod with p=pMod; power version, with p=variable)
we know all L1 exact
we know the overlapping sExact&sMod
we write error %, for this overlapping=0,this is to write: (sMod-sExact)/sMod=0
which gives a new value for (p) at every different TAN.
The graph of this new (p) looks like an astroid.
We write a pMod for this pExact data, with the following parameters of Table II

pMod=d2+b2*(1-((x-c2)/a2)^q)^(1/q)+(G+m2*x^v2+n2*x^w2)

parameters	values
a2	500
b2	0,3
c2	500
d2	0,000
q	6
G	**1,965**
m2	**0,000996000**
v2	1
n2	0
w2	1

Table. II

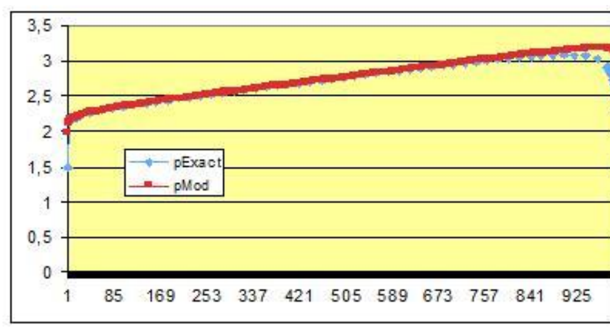

Fig.12
Overlapping of pExact&pMod. For 1<TAN<infinity

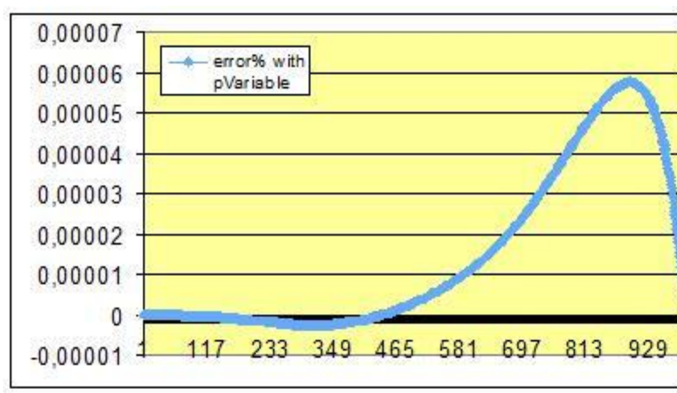

Fig.13 Error % for (1<TAN<infinity) (p variable) .Max.Error %=0,00005769…. at TAN=12,0985895…

Version (s=sMod with p=pMod; power version, with p=variable, corrected)

we know all L1 exact
we know the overlapping pExact&pMod
we write error % for this overlapping=0, that is:
(pMod-pExact)/pMod=0 and

we attack the parameter (b2,m2,G) of pMod) consecutively.
All these parameters have an astroidal looking. (This is a proposal)

parameters	values
	500 0,000000
	1,600000

Table III. b2 parameter correcting pMod

parameters	values
	1

Table IV.m2 parameter correcting pMod

parameters	values
	1,4

TableV. G parameter correcting pMod

Now, with these corrections we write
pMod=d2+b2Mod*(1-((x-c2)/a2)^q)^(1/q)+(GMod+m2Mod*x^v2+n2*x^w2)
sMod=d1+b1*(1-((x-c1)/a1)^pMod)^(1/pMod)+(F+m1*x^v1+n1*x^w1)
Error % is reduced as shown on Fig.14

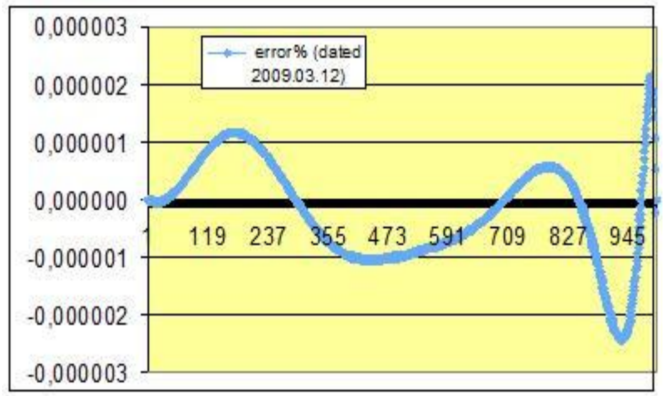

Fig.14 Error % for 1<TAN<infinity (p corrected, coarse).Max.error %=-0,0000024328…at TAN=18,43467933..

A numerical example

Evaluate the total arc length of the ellipse (a=1;b=5), on positive Cartesian
Consider 1000 evaluating lines on an excel table, for 1<TAN< infinity. Use the following designations:

	ATAN*180/Pi

We calculate:

TAN=b/a =5
ATAN =1,373400767..
Angle o =78,69006753 o
x =748,66817..

we use (x) value in b2,m2,G,in their formula ….. we get

b2Mod =0,288740099
m2Mod =0,000997983
GMod =1,948787387 then,

pMod =2,99518385
sMod =1,567203335

are evaluated

Use (sMod) in L1 =(1+TAN^sMod)^(1/sMod)
Find L1Estimated =5,25251332979251
L1 Exact =5,25251113492227 (with tools, mathematica, or ….)
Error % =0,000000418041087.. !!!

The cracking method is valid for all mathematical astroid.
(x/a)^r+(y/b)^r=1
For all (0<r<infinity)
For all (1<TAN<infinity)
Error % level is depending of the proposed math-Model.
Proposed math-Models may be cycloidal, polynomial, any function suitable for overlapping.
The equivalency formula of the elliptic finite integrals K=(1+TAN^s)^(1/s) means: total arc length, or area of the astroids, or …what else.
It is explained by the generalized Thales theorem!
It has nothing common with Hölder mean.
One may go further for better accuracy, with better proposition of math-Models

II. CONCLUSION

Fig.15 is a comparison graph of Ramanujan's and Necat's estimation
Ramanujan is Master when TAN<2,5

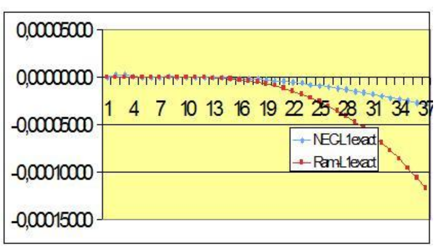

Fig.15 Field 1<TAN<10 is considered.

Ref: IAENG WCECS2008 conference book, page 944

Pathway Fractional Integral Operator Pertaining To Special Functions

GJSFR-f Classification (FOR) 010108, 010111, 26D10 45P05

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***Abstract*-The object of this paper is to study a pathway fractional integral operator associated with the pathway model and pathway probability density for certain product of special functions with general argument $x^{\xi}(x^k + c)^{-\rho}$ Our findings provide interesting unification and extension of a number of (new and known) results.**

Keywords-Pathway fractional integral operator, Fox's H-function, A general class of Polynomials, Mittag-Leffler function, Lagurre polynomial.

I. Introduction

The Pathway fractional integral operator introduced by Nair [7] is defined in the following manner

$$(P_{0_+}^{(\eta,\alpha)} f)(x) = x^{\eta} \int_0^{\left[\frac{x}{a(1-\alpha)}\right]} \left[1 - \frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{(1-\alpha)}} f(t)\,dt \tag{1}$$

where f(x) $\in L(a,b)$, $\eta \in C$, $R(\eta) > 0$, $a > 0$ and 'pathway parameter' $\alpha < 1$

The pathway model is introduced by Mathai [1] and studied further by Mathai and Haubold ([2], [3]).For real scalar α, the pathway model for scalar random variables is represented by the following probability density function (p.d.f.)W'

$$f(x) = c\,|x|^{\gamma-1}\left[1 - a(1-\alpha)|x|^{\delta}\right]^{\frac{\beta}{1-\alpha}}, \tag{2}$$

$-\infty < x < \infty, \delta > 0, \beta \geq 0, 1 - a(1-\alpha)|x|^{\delta} > 0, \gamma > 0$, where c is the normalizing constant and α is called the pathway parameter. For real α, the normalizing constant is as follows:

$$c = \frac{1}{2}\,\frac{\delta\,[a(1-\alpha)]^{\frac{\gamma}{\delta}}\,\Gamma\left(\frac{\gamma}{\delta} + \frac{\beta}{1-\alpha} + 1\right)}{\Gamma\left(\frac{\gamma}{\delta}\right)\Gamma\left(\frac{\beta}{1-\alpha} + 1\right)}, \alpha < 1, \tag{3}$$

$$= \frac{1}{2}\,\frac{\delta\,[a(1-\alpha)]^{\frac{\gamma}{\delta}}\,\Gamma\left(\frac{\beta}{1-\alpha}\right)}{\Gamma\left(\frac{\gamma}{\delta}\right)\Gamma\left(\frac{\beta}{1-\alpha} - \frac{\gamma}{\delta}\right)}, \text{ for } \frac{1}{\alpha-1} - \frac{\gamma}{\delta} > 0 \text{ ,} \alpha > 1, \tag{4}$$

$$= \frac{1}{2}\,\frac{\delta\,(a\beta)^{\frac{\gamma}{\delta}}}{\Gamma\left(\frac{\gamma}{\delta}\right)} \quad \text{for} \quad \alpha \to 1. \tag{5}$$

For $\alpha < 1$, it is a finite range density with $1 - a(1-\alpha)|x|^{\delta} > 0$ and (2) remains in the extended generalized type-1 beta family. The pathway density in (2), for $\alpha < 1$, includes the extended type-1 beta density, the triangular density, the uniform density and many other p.d.f.

For $\alpha > 1$, we have

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$$f(x) = c\,|x|^{\gamma-1}\left[1 + a\,(\alpha-1)\,|x|^{\delta}\right]^{-\frac{\beta}{\alpha-1}}, \tag{6}$$

$-\infty < x < \infty,\ \delta > 0,\ \beta \geq 0,\ \alpha > 1$, which is extended generalized type-2 beta model for real x. It includes the type-2 beta density, the F density, the Student-t density, the Cauchy density and many more.

Here it is considered only the case of pathway parameter $\alpha < 1$. For $\alpha \to 1$ (2) and (6) take the exponential form, since

$$\lim_{\alpha\to 1} c\,|x|^{\gamma-1}\left[1 - a\,(1-\alpha)\,|x|^{\delta}\right]^{\frac{\eta}{1-\alpha}} = \lim_{\alpha\to 1} c\,|x|^{\gamma-1}\left[1 + a\,(\alpha-1)\,|x|^{\delta}\right]^{-\frac{\eta}{\alpha-1}}$$

$$= c\,|x|^{\gamma-1}\,e^{-a\,\eta\,|x|^{\delta}}\ . \tag{7}$$

This includes the generalized Gamma- , the Weibull-, the Chi-square, the Laplace-, and the Maxwell- Boltzmann and other related densities.

when $\alpha \to 1_{-}$, $\left[1 - \frac{a\,(1-\alpha)\,t}{x}\right]^{\frac{\eta}{1-\alpha}} \to e^{-\frac{a\,\eta}{x}t}$ $\cup$, then operator (1) reduces to the Laplace integral transform of f with parameter $\frac{a\eta}{x}$:

$$(\,P_{0_+}^{(\eta\,,\,1)} f\,)(x) = x^{\eta}\int_0^{\infty} e^{-\frac{a\,\eta}{x}t} f(t)\,dt = x^{\eta}\ L_f\left(\frac{a\eta}{x}\right)\ . \tag{8}$$

When $\alpha = 0$, $a = 1$, then replacing η by $\eta - 1$ in (1) the integral operator reduces to the Riemann-Liouville fractional integral operator.

For the H- function [4] and a general class of polynomial [5], we establish the following Lemma

Lemma : Let $\eta \in C$, $R(\eta) > 0$, $\xi, \rho \in C$ and $\alpha < 1$, if $R(\xi) > 0$, $R(\rho) > 0$, $R\left(\frac{\eta}{1-\alpha}\right) > -1$, $k \geq 0$, then

$$P_{0_+}^{(\eta,\alpha)}\left[x^{\xi-1}\left(x^k + c\right)^{-\rho}\right]$$

$$= x^{\eta+\xi}\,\frac{1}{\Gamma(\rho)}\,\frac{\Gamma(1+\frac{\eta}{1-\alpha})}{c^{\rho}\left[a(1-\alpha)\right]^{\xi}}\ H_{2,2}^{1,2}\left[\frac{x^k}{c\left[a(1-\alpha)\right]^k}\ \middle|\ \begin{matrix}(1-\xi,k),(1-\rho,1)\\(-\xi-\frac{\eta}{1-\alpha},k),(0,1)\end{matrix}\right] \tag{9}$$

Proof: To establish Lemma , we use f(t) = $t^{\xi-1}\left(t^k + c\right)^{-\rho}$ in equation (1) and express $\left(t^k + c\right)^{-\rho}$ in terms of Mellin Barnes type contour integral with the help of Srivastava [6, p.18, eq. (2.6.4)], then we interchange the order of contour and t-integral (which is permissible under the conditions stated). Finally, on evaluating the t-integral by the following result

$$\int_0^{\frac{x}{a(1-\alpha)}}\left[1 - \frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{1-\alpha}} t^{\beta-1}dt = \frac{x^{\beta}}{[a(1-\alpha)]^{\beta}}\,\frac{\Gamma(\beta)\Gamma(1+\frac{\eta}{1-\alpha})}{\Gamma(\frac{\eta}{1-\alpha}+\beta+1)}\ ,\ \alpha < 1\ ,\ R(\eta) > 0\ ,\ R(\beta) > 0. \tag{10}$$

And reinterpreting the result thus obtained in terms of H-function, we easily arrive at the required result after a little simplification.

In (9) when $\alpha \to 1_{-}$, $\frac{\eta}{1-\alpha} \to \infty$ and hence we may expand the gamma – functions by using sterling formula, we have

$$\lim_{\alpha\to 1_-} P_{0_+}^{(\eta,\alpha)}\left[x^{\xi-1}\left(x^k + c\right)^{-\rho}\right] \to \frac{x^{\eta+\xi}}{c^{\rho}\Gamma(\rho)[a\eta]^{\xi}}\,H_{2,1}^{1,2}\left[\frac{x^k}{c\,[a\eta\,]^{\,k}}\ \middle|\ \begin{matrix}(1-\xi,k),(1-\rho,1)\\(0,1)\end{matrix}\right]. \tag{11}$$

This is the same as the Laplace transform formula, given by

$$\lim_{\alpha\to 1_-} P_{0_+}^{(\eta,\alpha)}\left[x^{\xi-1}\left(x^k+c\right)^{-\rho}\right]$$

$$= x^{\eta}\int_0^{\infty} e^{-\frac{a\eta}{x}t}\, t^{\xi-1}(t^k+c)^{-\rho}\,dt$$

$$= \frac{x^{\eta+\xi}}{c^{\rho}\Gamma(\rho)[a\eta]^{\xi}} H_{2,1}^{1,2}\left[\frac{x^k}{c[a\eta]^k}\middle|\begin{matrix}(1-\xi,k),(1-\rho,1)\\(0,1)\end{matrix}\right]$$

MAIN RESULTS

Theorem-1: Let $(\eta,\ \xi,\xi_1,\xi_2\ \rho,\ \rho_1,\rho_2 \in C)$, $R\left(1+\frac{\eta}{1-\alpha}\right)>0$, $k\geq 0$, $\alpha<1$, $P_1\in R$, $P_2\in R$, $R(\eta,\ \xi,\xi_1,\xi_2\ \rho,\ \rho_1,\rho_2)>0$ and c is a positive number, the coefficients $A_{v,s}(v,s\geq 0)$ are arbitrary constant, real or complex, then

$$P_{0_+}^{(\eta,\alpha)}\left[x^{\xi-1}\ (x^k+c)^{-\rho}\ S_v^u\,[P_1\,x^{\xi_1}(x^k+c)^{-\rho_1}]\ H_{p,q}^{m,n}\,[P_2\,x^{\xi_2}(x^k+c)^{-\rho_2}]\right] = \frac{x^{\eta+\xi}}{c^{\rho}\,[a(1-\alpha)]^{\xi}}\Gamma\left(1+\frac{\eta}{1-\alpha}\right)$$

$$\sum_{s=0}^{[v/u]}\frac{(-v)_{us}}{s!}A_{v,s}\frac{P_1^{\,s}x^{\xi_1 s}}{c^{\rho_1 s}\,[a(1-\alpha)]^{\xi_1 s}}\ H_{2,1;p,q+1;0,1}^{0,2;\,m,n\,;1,0}\left[\begin{matrix}\dfrac{P_2x^{\xi_2}}{c^{\rho_2}\,[a(1-\alpha)]^{\xi_2}}\\ \dfrac{x^k}{c[a(1-\alpha)]^k}\end{matrix}\middle|\begin{matrix}[1-\rho-\rho_1 s;\rho_2,1]\ ,\ [1-\xi-\xi_1 s;\xi_2,k]\ ;\\ [-\frac{\eta}{1-\alpha}-\xi-\xi_1 s;\xi_2,k]\ ;\\ [a_j;\alpha_j]_{1,p}\ ;\ \text{——}\\ [b_j;\delta_j]_{1,q}\,,[1-\rho-\rho_1 s;\rho_2]\ ;\ [0,1]\end{matrix}\right]. \qquad (12)$$

The proof of the result (12) can be developed by proceeding on the lines similar to the proof of (9).
When $\alpha\to 1_-$ then (12) tends to

$$\lim_{\alpha\to 1_-} P_{0_+}^{(\eta,\alpha)}\left[x^{\xi-1}\ (x^k+c)^{-\rho}\ S_v^u\,[P_1\,x^{\xi_1}(x^k+c)^{-\rho_1}]\ H_{p,q}^{m,n}\,[P_2\,x^{\xi_2}(x^k+c)^{-\rho_2}]\right] = \frac{x^{\eta+\xi}}{c^{\rho}\,(a\eta)^{\xi}}$$

$$\sum_{s=0}^{[v/u]}\frac{(-v)_{us}}{s!}A_{v,s}\frac{P_1x^{\xi_1 s}}{c^{\rho_1 s}\,(a\eta)^{\xi_1 s}}\ H_{2,0;p,q+1;0,1}^{0,2;\,m,n\,;1,0}\left[\begin{matrix}\dfrac{P_2x^{\xi_2}}{c^{\rho_2}\,(a\eta)^{\xi_2}}\\ \dfrac{x^k}{c(a\eta)^k}\end{matrix}\middle|\begin{matrix}[1-\rho-\rho_1 s;\rho_2,1]\ ,\ [1-\xi-\xi_1 s;\xi_2,k]\ ;\\ \text{——————}\ ;\\ [a_j;\alpha_j]_{1,p}\ ;\ \text{——}\\ [b_j;\delta_j]_{1,q}\,,[1-\rho-\rho_1 s;\rho_2]\ ;\ [0,1]\end{matrix}\right] \qquad (13)$$

As it gives also the formula for Laplace transform of f (t).

Theorem-2: Let $(\eta,\xi,\xi',\ \xi_1,\xi_1'\rho\ ,\rho',\rho_1,\rho_1',\rho_2,\rho_2'\in C)$, $R\left(1+\frac{\eta}{1-\alpha}\right)>0$, $R\left(1+\frac{\eta'}{1-\alpha'}\right)>0$, $k\geq 0$, $k'\geq 0$, $\alpha<1$, $P_1\in R$, $P_2\in R$, $R(\eta)>0$ and c, c' are positive numbers, the coefficients $A_{v,s}(v,s\geq 0)$ are arbitrary constants, real or complex, then

$$P_{0_+,y}^{(\eta',\alpha')}[P_{0_+,x}^{(\eta,\alpha)}\{x^{\xi-1}y^{\xi'-1}(x^k+c)^{-\rho}(y^{k'}+c')^{-\rho'}S_v^u[P_1 x^{\xi_1}y^{\xi_1'}(x^k+c)^{-\rho_1}(y^{k'}+c')^{-\rho_1'}]$$
$$H_{p,q}^{m,n}[P_2 x^{\xi_2}y^{\xi_2'}(x^k+c)^{-\rho_2}(y^{k'}+c')^{-\rho_2'}]\}]$$

$$=\frac{x^{\eta+\xi}}{c^{\rho}[a(1-\alpha)]^{\xi}}\Gamma\left(1+\frac{\eta}{1-\alpha}\right)\frac{y^{\eta'+\xi'}}{c^{\rho'}[a'(1-\alpha')]^{\xi'}}\Gamma\left(1+\frac{\eta'}{1-\alpha'}\right)\sum_{s=0}^{[v/u]}\frac{(-v)_{us}}{s!}A_{v,s}\frac{P_1^{s}x^{\xi_1 s}y^{\xi_1's}}{c^{\rho_1 s}c'^{\rho_1's}[a(1-\alpha)]^{\xi_1 s}[a'(1-\alpha')]^{\xi_1's}}$$

$$H_{4,2;p,q+2;0,1;0,1}^{0,4;m,n;1,0;1,0}\left[\begin{array}{c|l}\frac{P_2x^{\xi_2}y^{\xi_2'}}{c^{\rho_2}c'^{\rho_2'}[a(1-\alpha)]^{\xi_2}[a'(1-\alpha')]^{\xi_2'}} & [1-\rho-\rho_1 s;\rho_2,1,0],\ [1-\xi-\xi_1 s;\xi_2,k,0], \\ \frac{x^k}{c[a(1-\alpha)]^k} & [-\frac{\eta}{1-\alpha}-\xi-\xi_1 s;\xi_2,k,0] \\ \frac{y^{k'}}{c'[a'(1-\alpha')]^{k'}} & \end{array}\right.$$

$$\left.\begin{array}{lll}[1-\rho'-\rho_1's;\rho_2',0,1],\ [1-\xi'-\xi_1's;\xi_2',0,k']; & [a_j;\alpha_j]_{1,p} & ;\ \_\_\_;\ \_\_ \\ [-\frac{\eta'}{1-\alpha'}-\xi'-\xi_1's;\xi_2',0,k'] & ;\ [b_j;\delta_j]_{1,q},[1-\rho-\rho_1 s;\rho_2]\ [1-\rho'-\rho_1's;\rho_2'] & ;\ [0,1]\ ;[0,1]\end{array}\right].$$

(14)

The result (14) can also be proved by applying the result (12) twice on two independent variables instead of one.

When $\alpha\to 1_-,\alpha'\to 1_-$ result (14) gives also the formula for Laplace transform of two variables of

$$\{x^{\xi-1}y^{\xi'-1}(x^k+c)^{-\rho}(y^{k'}+c')^{-\rho'}S_v^u[P_1x^{\xi_1}y^{\xi_1'}(x^k+c)^{-\rho_1}(y^{k'}+c')^{-\rho_1'}]$$
$$H_{p,q}^{m,n}[P_2x^{\xi_2}y^{\xi_2'}(x^k+c)^{-\rho_2}(y^{k'}+c')^{-\rho_2'}]\}\ .$$

Special Cases

If we take, $m=1,n=1,p=1,q=2,b_1=0,\delta_1=1,b_2=1-\xi,\delta_2=\xi_2,a_1=1-\gamma,\alpha_1=1$, then H- function reduces to Mittag–Leffler function in (12)

$$P_{0_+}^{(\eta,\alpha)}\left[x^{\xi-1}(x^k+c)^{-\rho}S_v^u[P_1x^{\xi_1}(x^k+c)^{-\rho_1}]E_{\xi_2,\xi}^{\gamma}[P_2x^{\xi_2}(x^k+c)^{-\rho_2}]\right]=\frac{1}{\Gamma(\gamma)}\frac{x^{\eta+\xi}}{c^{\rho}[a(1-\alpha)]^{\xi}}\Gamma\left(1+\frac{\eta}{1-\alpha}\right)$$

$$\sum_{s=0}^{[v/u]}\frac{(-v)_{us}}{s!}A_{v,s}\frac{P_1^{s}x^{\xi_1 s}}{c^{\rho_1 s}[a(1-\alpha)]^{\xi_1 s}}H_{2,1;1,3;0,1}^{0,2;1,1;1,0}\left[\begin{array}{c|ll}\frac{P_2x^{\xi_2}}{c^{\rho_2}[a(1-\alpha)]^{\xi_2}} & [1-\rho-\rho_1 s;\rho_2,1],\ [1-\xi-\xi_1 s;\xi_2,k]\ ; & \\ \frac{x^k}{c[a(1-\alpha)]^k} & [-\frac{\eta}{1-\alpha}-\xi-\xi_1 s;\xi_2,k] & ;\end{array}\right.$$

$$\left.\begin{array}{ll}[1-\gamma,1] & ;\ \_\_ \\ [0,1],[1-\xi,\xi_2],[1-\rho-\rho_1 s;\rho_2] & ;\ [0,1]\end{array}\right], \qquad (15)$$

where $(\eta,\gamma,\ \xi,\ \xi_1,\xi_2,\ \rho,\ \rho_1,\rho_2\in C)$, $R\left(1+\frac{\eta}{1-\alpha}\right)>\max[0,-R(\xi)]$, $k\geq 0$, $\alpha<1$, $P_1\in R$, $P_2\in R$, $R(\eta,\gamma,\ \xi,\ \xi_1,\xi_2,\ \rho,\ \rho_1,\rho_2),R(\gamma)>0$ and c is positive number, the coefficient $A_{v,s}(v,s\geq 0$ are arbitrary constant, real or complex, If we set u =1 and $A_{v,s} = \binom{v+\gamma}{v} \frac{1}{(v+1)_s}$ in (12), then a general class of polynomial reduce to Lagurre polynomials

$$P_{0_+}^{(\eta,\alpha)}\left[x^{\xi-1}(x^k+c)^{-\rho} L_v^{\gamma}[P_1 x^{\xi_1}(x^k+c)^{-\rho_1}]\, H_{p,q}^{m,n}[P_2 x^{\xi_2}(x^k+c)^{-\rho_2}]\right] = \frac{x^{\eta+\xi}}{c^{\rho}[a(1-\alpha)]^{\xi}} \Gamma\left(1+\frac{\eta}{1-\alpha}\right)$$

$$\sum_{s=0}^{[v]} \binom{v+\gamma}{v-s} \frac{P_1^{\,s} x^{\xi_1 s}}{s!\, c^{\rho_1 s}[a(1-\alpha)]^{\xi_1 s}}\, H_{2,1;p,q+1;0,1}^{0,2;\,m,n;1,0}\left[\begin{matrix} \dfrac{P_2 x^{\xi_2}}{c^{\rho_2}[a(1-\alpha)]^{\xi_2}} \\ \dfrac{x^k}{c[a(1-\alpha)]^k} \end{matrix}\middle| \begin{matrix} [1-\rho-\rho_1 s;\rho_2,1],\ [1-\xi-\xi_1 s;\xi_2,k]; \\ [-\frac{\eta}{1-\alpha}-\xi-\xi_1 s;\xi_2,k]; \end{matrix}\right.$$

$$\left.\begin{matrix} [a_j;\alpha_j]_{1,p} & ;\ \_\_ \\ [b_j;\delta_j]_{1,q},[1-\rho-\rho_1 s;\rho_2] & ;\ [0,1] \end{matrix}\right], \quad (16)$$

It is valid under the same conditions as given in (12).

On account of the most general nature of the functions occurring in our main findings, a number of simpler corresponding results involving simpler functions can be obtained easily merely by specializing the parameters in them.

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Strongly Minimal Generalised Closed Maps And Strongly Minimal Generalised Open Maps In Minimal Structures

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GJSFR- F Clasification FOR 010102,010108,37E05,46T20

***Abstract*-In this paper, we introduce a new class of smg-closed maps and smg-open maps and studied some of its basic properties. we obtain some characterizations of such functions.**

Keywords-m-structure, smg-closed set, smg-open mapping, smg-closed mapping

I. INTRODUCTION

The generalized closed or g-closed sets were defined by Norman Levine and their properties were investigated in [1].He defined a set A to be generalized closed if its closure belongs to every open superset of A and introduced the notion $T_{1/2}$ -spaces, which is properly placed between T_0 and T_1 -spaces. Dunham[16] proved that a topological space is $T_{1/2}$ if and only if every singleton is open or closed. S.R.Malghan(1982) introduced generalized closed maps and studied their properties in topological spaces. The concept of generalized open map was introduced by Sundaram[4]. Sundaram and Pushpalatha[8] introduced and investigated some properties of strongly generalized closed maps and strongly generalized open maps in topological spaces. Balachandran[5] introduced the concept of generalized continuous maps and gc-irresolute maps on a topological space. Chamber[2] investigated open,closed maps. Nagaveni[12] introduced wg-closed continuous maps in topological spaces. Valeriu Popa[11] introduced on m-continuous functionIn this paper, we introduce the new class of strongly minimal generalized closed maps and strongly minimal generalized open maps in minimal structures and also studied some properties of strongly minimal generalized closed sets.

II. PRELIMINARIES

In this section, we begin by recalling some definitions and properties.Definition 2.1[11]: A subfamily m_X of the power set P(X) of a non-empty set X is called a minimal structure(briefly m-structure) on X if $\phi \varepsilon m_X$ and Remark 2.2[11] :Let be a topological space. Then the families $X \varepsilon m_X$ Remark 2.2[11] :Let (X,τ) be a topological space. Then the families

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$PO(X),\alpha(X),\beta(X),\delta(X),\delta SO(X)$ and $SR(X)$ are all m-structure on X.Definition 2.3[11]: Let X be a non-empty set and m_X an m-structure on X. For a subset A of X, the m_X -closure of A and the m_X -interior of A are defined in [13] as follows:

(1) $m_X - cl(A) = \cap\{F : A \subset F, X - F \in m_X\}$

(2) $m_X - \text{int}(A) = \cup\{F : U \subset A, U \in m_X\}$

Definition 2.4[1]: Let (X,τ) be a topological space, a subset A of X is said to be g-closed if cl(A) $\subseteq$ G whenever A $\subseteq$ G and G is open in X. The complement of g-closed set is called g-open in X.Definition 2.5[9]: Let (X,τ) be a topological space, a subset A of X is said to be strongly g-closed if cl(A) $\subseteq$ G whenever A $\subseteq$ G and G is g-open in X.Definition 2.6[14]: Let (X,m_X) be an m-space. A subset A of X is said to be strongly minimal generalized closed(smg-closed) if $m_X - cl(A) \subseteq G$ whenever A $\subseteq$ G and G is mg-open. The complement of an smg-closed set is called a smg-open set in (X,m_X).Definition 2.7[3]: A map $f : X \to Y$ is called g-closed(respectively g-open) if for each closed set (respectively open set) F in X ,f(F) is g-closed(respectively g-open) in Y.Definition 2.8[8]: A map $f : X \to Y$ is called strongly generalized closed (strongly g-closed) map if for each closed set F in X, f(F) is a strongly g-closed set in Y.

III. SOME PROPERTIES OF SMG-CLOSED SETS

In this section, we introduce two properties of smg-closed sets in minimal structures.Theorem 3.1: The intersection of two smg-closed sets is an smg-closed set in (X,m_X) .Proof: Let A and B be any two smg-closed sets in (X,m_X).To prove that $A \cap B$ is smg-closed, let G be an mg-open set such that $A \cap B \subseteq G \Rightarrow A \subseteq G$ and $B \subseteq G$. Since A and B are smg-closed sets, $m_X - cl(A) \subseteq G$ and $m_X - cl(B) \subseteq G$, $\Rightarrow m_X - cl(A) \cap m_X - cl(B) \subseteq G$ Hence

$m_X - cl(A \cap B) \subseteq G$ .Corollary 3.2: If A is $m_X - closed$ and B is smg-closed in (X, m_X), then $A \cap B$ is smg-closed in (X, m_X).Proof: Let A be any $m_X - closed$ set and B is an smg-closed set. To prove that $A \cap B$ is an smg-closed set in (X, m_X), let G be an mg-open set such that $A \cap B \subseteq G \Rightarrow A \subseteq G$ *and* $B \subseteq G$. Since A is mg-closed set, $m_X - cl(A) = A$ and hence $m_X - cl(A) \subseteq G \rightarrow (1)$ .Since B is an mg-closed set, $m_X - cl(B) \subseteq G \rightarrow (2)$. From (1) and (2), we get $\Rightarrow m_X - cl(A) \cap m_X - cl(B) \subseteq G$. Hence $m_X - cl(A \cap B) \subseteq G$.

IV. STRONGLY MINIMAL GENERALIZED CLOSED MAPS AND STRONGLY MINIMAL GENERALIZED OPEN MAPS

In this section, we introduce the concepts of strongly minimal generalized (smg) closed maps and strongly minimal generalized (smg) open maps in topological spaces.Definition 4.1: A map $f : (X, m_X) \rightarrow (Y, .m_Y)$ is called strongly minimal generalized closed (strongly mg-closed or smg-closed) map if for each closed set F in X, f(F) is a strongly mg-closed set in YDefinition 4.2: A subset A of a minimal space X is called strongly minimal generalized open set(smg-open) if A^c is smg-closed. The class of all smg-open sets is denoted by by $STMGO(X, m_X)$

Theorem 4.3: If $f : (X, m_X) \rightarrow (Y, .m_Y)$ is a closed map, then it is strongly mg-closed but not conversely.Proof: Since every closed set is strongly mg-closed, the result follows.The Converse of the above theorem need not be true as seen from the following example.Example 4.4: Let $X = Y = \{a, b, c\}$, $m_X = \{\phi, X, \{b\}\}$ and $m_Y = \{\phi, Y, \{b, c\}\}$.Let f be the identity map from X onto Y.Then f is strongly minimal g-closed but not a closed map, since for the closed set {a,c} in (X, m_X) ,f({a,c})={a,c} is not a closed set in Y.

Definition 4.5: A map $f : X \rightarrow Y$ is called a strongly minimal generalized open map(strongly mg-open) map if f(U) is strongly mg-open in Y for every open set U in X.Theorem 4.6: If $f : X \rightarrow Y$ is an open map, then it is strongly mg-open but not conversely.Proof: Let $f : X \rightarrow Y$ be an open map. Let U be any open set in X. Then f(U) is an open set in Y. Then f(U) is strongly mg-open, since every open set is strongly mg-open. Therefore f is strongly mg-open.The Converse of the above theorem need not be true as seen from the following example.Example 4.7: Let $X = Y = \{a, b, c\}$ with minimal spaces $m_X = \{\phi, X, \{b\}\}$ and $m_Y = \{\phi, Y, \{a, b\}\}$ .Here STMGO $(Y, m_Y) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ .Then the identity function $f : X \rightarrow Y$ is strongly mg-open but not open, since for the open set {b} in (X, m_X) ,f({b}) = {b} is strongly mg-open but not open in (Y, m_Y).Therefore f is not an open map.

Theorem 4.8: A map $f : X \rightarrow Y$ is strongly minimal g-closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$,there is a strongly mg-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$ .Proof: Suppose f is strongly mg-closed. Let S be a subset of Y and U is an open set of X such that $f^{-1}(S) \subseteq U$ .Then $V = Y - f(X - U)$ is a strongly mg-open set containing S such that $f^{-1}(V) \subseteq U$ Conversely, suppose that F is a closed set in X. Then $f^{-1}(Y - f(F)) = X - F$ and $X - F$ is open. By hypothesis, there is a strongly mg-open set V of Y such that $Y - f(F) \subseteq V$ and $f^{-1}(V) \subseteq X - F$ .Therefore $F \subseteq X - f^{-1}(V)$

Hence $Y - V \subseteq f(F) \subseteq f(X - f^{-1}(V)) \subseteq Y - V$ which implies $f(F) = Y - V$. Since $Y - V$ is strongly mg-closed, f(F) is strongly mg- closed and thus f is strongly mg-closed map.Theorem 4.9: If $f : X \rightarrow Y$ is mg-continuous and strongly mg- closed and A is a strongly mg-closed set of X, then f(A) is strongly mg- closed in Y.Proof: Let $f(A) \subseteq O$ where O is an mg-open set of Y. Since f is mg-continuous, $f^{-1}(O)$ is an mg-open set containing A. Hence $m_X - cl(A) \subseteq f^{-1}(O)$ as A is a strongly mg-closed set. Since f is strongly mg- closed, $f(m_X - cl(A))$ is a strongly mg- closed set contained in the mg-open set O, which implies that $m_X - cl(f(m_X - cl(A))) \subseteq O$ and hence $m_X - cl(f(A)) \subseteq O$. So, f(A) is a strongly mg-closed set in Y.

Corollary 4.10: If $f : (X, m_X) \rightarrow (Y, m_Y)$ is continuous and closed and A is a strongly mg- closed set of X, then f(A) is strongly mg- closed in Y.Proof: Since every continuous map is mg-continuous and every closed map is strongly mg-closed, by above theorem the result follows.Theorem 4.11: If $f : (X, m_X) \rightarrow (Y, m_Y)$ is closed and $h : (Y, m_Y) \rightarrow (Z, m_Z)$ is strongly mg- closed, then $h \circ f : (X, m_X) \rightarrow (Z, m_Z)$ is strongly mg-closed.Proof: Let $f : (X, m_X) \rightarrow (Y, .m_Y)$ is a closed map and $h : (Y, .m_Y) \rightarrow (Z, .m_Z)$ is a strongly mg- closed map. Let V be any closed set in X. Since $f : (X, m_X) \rightarrow (Y, m_Y)$

is closed, f(V) is closed in Y and since $h:(Y,m_Y)\rightarrow(Z,m_Z)$is strongly mg- closed, h(f(V)) is a strongly mg- closed set in Z. Therefore $h\circ f:(X,m_X)\rightarrow(Z,m_Z)$ is a strongly mg-closed map.Theorem 4.12: If $f:X\rightarrow Y$ is strongly mg-closed and A is closed set in X, then $f_A:A\rightarrow Y$ is strongly mg-closed.Proof: Let V be any closed set in A. Then V is closed in X. Therefore, it is strongly mg-closed in X .By theorem 4.8, f(V) is strongly mg-closed.That is $f_A(V)=f(V)$ is strongly mg-closed in Y. Therefore, $f_A:A\rightarrow Y$ is strongly mg-closed.

Theorem 4.13: If $f:X\rightarrow Y$ is smg-closed and $A=f^{-1}(B)$ for some closed set B of Y, then $f_A:A\rightarrow Y$ is smg-closed.Proof: Let F be a closed set in A. Then there is a closed set H in X such that $F=A\cap H$ .Then $f_A(F)=f(A\cap H)=f(H)\cap B$.Since f is smg-closed, f(H) is smg-closed in Y. So $f(H)\cap B$ is smg-closed in Y, since the intersection of a smg-closed set and a closed set is a smg-closed set.Hence f_A is smg-closed.Remark 4.14: If B is not closed in Y, then the above theorem does not hold as it can be seen from the following example.

Example 4.15: Let X={a,b,c} and $m_X=\{\phi,X,\{a\},\{b\},\{a,b\}\}$, $m_Y=\{\phi,X,\{a\},\{a,b\}\}$ be the minimal structures on X. Let $f:(X,m_X)\rightarrow(X,m_Y)$ be the identity map. Take B={a,b}.Then $A=f^{-1}(B)$ and {a} is closed in A but $f_A(\{a\})=\{a\}$ is not smg-closed in Y.{a} is also not smg-closed in B

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A Hilbert-type Integral Inequality with Parameters

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Abstract.-In this paper it is shown that a Hilbert-type integral inequality can be established by introducing two parameters $\mu(\mu>-1)$ and λ (0λ>). And the constant factor expressed by -function is proved to be the best possible. And then some important and especial results are enumerated. As applications, some equivalent forms are given.

Key words: Hilbert-type integral inequality, weight function Γ function, Euler number，Catalan constant.
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1. 1ntroduction and Lemmas

Let $f(x), g(x) \in L^2(0,+\infty)$. Then

$$\int_0^\infty\int_0^\infty \frac{f(x)g(y)}{x+y}dxdy \le \pi\left\{\int_0^\infty f^2(x)dx\right\}^{\frac{1}{2}}\left\{\int_0^\infty g^2(x)dx\right\}^{\frac{1}{2}} \tag{1.1}$$

where the constant factor π is the best possible. And the equality contained in (1.1) holds if and only if $f(x)=0$, or $g(x)=0$. This is the famous Hilbert integral inequality,（see [1],[2]）. Owing to the importance of the Hilbert inequality and the Hilbert type inequality in analysis and applications, some mathematicians have been studying them. Recently, various improvements and extensions of (1.1) appear in a great deal of papers (see [3]—[11]etc.). Specially, Gao and Hsu enumerated the research articles more than 40 in the paper [6].

For convenience, we define $\left(\ln\frac{x}{y}\right)^0=1$, when $x=y$. The purpose of the present paper is to establish the Hilbert-type integral inequality of the form

$$\int_0^\infty\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu f(x)g(y)}{x^\lambda+y^\lambda}dxdy \le C\left\{\int_0^\infty \omega(x)f^2(x)dx\right\}^{\frac{1}{2}}\left\{\int_0^\infty \omega(x)g^2(x)dx\right\}^{\frac{1}{2}} \tag{1.2}$$

where $\mu>-1$ and $\lambda>0$. We will give the constant factor C and the expression of the weigh function $\omega(x)$, and prove the constant factor C to be the best possible, and then give some important and especial results, and study some equivalent forms of them. Evidently, the inequality (1.2) is an extension of (1.1). The new inequality established is significant in theory and applications.

In order to prove our main results, we need the following lemmas.

Lemma 1.1. Let a be a positive number and $\mu>-1$. Then

$$\int_0^\infty x^\mu e^{-ax}dx = \frac{\Gamma(\mu+1)}{a^{\mu+1}}.. \tag{1.3}$$

Proof. According to the definition of Γ-function, we obtain immediately (1.3). This result can be also found in the paper [12] (pp. 226, formula 1053).

Lemma 1.2. Let a be a positive number**.** Then

$$\int_0^\infty \frac{x}{\cosh ax}dx = \frac{2G}{a^2} \qquad (1.4)$$

where G is Catalan constant, i.e. $G = 0.915965594\cdots$.

Proof. Let $\mu > -1$. Expanding the hyperbolic secant function $\frac{1}{\cosh ax}$, and then using Lemma 1.1 we have

$$\int_0^\infty \frac{x^\mu}{\cosh ax}dx = 2\int_0^\infty \frac{x^\mu e^{-ax}}{1+e^{-2ax}}dx = 2\int_0^\infty x^\mu e^{-ax}\sum_{k=1}^\infty (-1)^{k+1} e^{-2(k-1)ax}dx$$

$$= 2\sum_{k=1}^\infty (-1)^{k+1}\int_0^\infty x^\mu e^{-(2k-1)ax}dx = \frac{2\Gamma(\mu+1)}{a^{\mu+1}}\sum_{k=1}^\infty \frac{(-1)^{k+1}}{(2k-1)^{\mu+1}}$$

$$= \frac{2\Gamma(\mu+1)}{a^{\mu+1}}G(\mu) \qquad (1.5)$$

where the function $G(\mu)$ is defined by

$$G(\mu) = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(2k-1)^{\mu+1}} \qquad (1.6)$$

Let $\mu = 1$.Then $\Gamma(\mu+1) = 1$. In accordance with the definition of the Catalan constant (see [12], pp.503.), i.e.

$$G = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(2k-1)^2} = 0.915965594\cdots$$

we obtain from (1.5) the equality (1.4) at once**.**

Lemma 1.3. Let $\lambda > 0$ and $\mu > -1$. Then

$$\int_0^1 t^{-\frac{2-\lambda}{2}}\left(\ln\tfrac{1}{t}\right)^\mu \tfrac{1}{1+t^\lambda}dt = \left(\tfrac{2}{\lambda}\right)^{\mu+1}\Gamma(\mu+1)G(\mu). \qquad (1.7)$$

where the function $G(\mu)$ is defined by (1.6)**.**

Proof. Substitution $x = \ln\frac{1}{t}$ it is easy to deduce that

$$\int_0^1 t^{-\frac{2-\lambda}{2}}\left(\ln\tfrac{1}{t}\right)^{\mu}\frac{1}{1+t^{\lambda}}dt=\int_0^{\infty}\frac{x^{\mu}e^{-\frac{\lambda}{2}x}}{1+e^{-\lambda x}}dx=\int_0^{\infty}\frac{x^{\mu}}{e^{\frac{\lambda}{2}x}+e^{-\frac{\lambda}{2}x}}dx=\frac{1}{2}\int_0^{\infty}\frac{x^{\mu}}{\cosh\frac{\lambda}{2}x}dx.$$

By using (1.5), the equality (1.7) follows.

Lemma 1.4. With the assumptions as Lemma 1.3, then

$$\int_0^{\infty} t^{-\frac{2-\lambda}{2}}\left|\ln\tfrac{1}{t}\right|^{\mu}\frac{1}{1+t^{\lambda}}dt=2\left(\frac{2}{\lambda}\right)^{\mu+1}\Gamma(\mu+1)G(\mu) \qquad (1.8)$$

where $G(\mu)$ is defined by (1.6).

Proof. It is easy to deduce that

$$\int_0^{\infty} t^{-\frac{2-\lambda}{2}}\left|\ln\tfrac{1}{t}\right|^{\mu}\frac{1}{1+t^{\lambda}}dt=\int_0^{1} t^{-\frac{2-\lambda}{2}}\left|\ln\tfrac{1}{t}\right|^{\mu}\frac{1}{1+t^{\lambda}}dt+\int_1^{\infty} t^{-\frac{2-\lambda}{2}}\left|\ln\tfrac{1}{t}\right|^{\mu}\frac{1}{1+t^{\lambda}}dt$$

$$=\int_0^{1} t^{-\frac{2-\lambda}{2}}\left|\ln\tfrac{1}{t}\right|^{\mu}\frac{1}{1+t^{\lambda}}dt+\int_0^{1} v^{-\frac{2-\lambda}{2}}\left|\ln v\right|^{\mu}\frac{1}{1+v^{\lambda}}dv$$

$$=\int_0^{1} t^{-\frac{2-\lambda}{2}}\left(\ln\tfrac{1}{t}\right)^{\mu}\frac{1}{1+t^{\lambda}}dt+\int_0^{1} v^{-\frac{2-\lambda}{2}}\left(\ln\tfrac{1}{v}\right)^{\mu}\frac{1}{1+v^{\lambda}}dv$$

$$=2\int_0^{1} t^{-\frac{2-\lambda}{2}}\left(\ln\tfrac{1}{t}\right)^{\mu}\frac{1}{1+t^{\lambda}}dt.$$

By Lemma 1.3, the equality (1.8) follows at once .

2. Main results

In this section, we will prove our assertions by using the above Lemmas.

Theorem 2.1. Let f and g be two real functions, $\mu>-1$ and $\lambda>0$. If $0<\int_0^{\infty}x^{1-\lambda}f^2(x)dx<+\infty$ and $0<\int_0^{\infty}x^{1-\lambda}g^2(x)dx<+\infty$, then

$$\int_0^{\infty}\int_0^{\infty}\frac{\left|\ln\frac{x}{y}\right|^{\mu}f(x)g(y)}{x^{\lambda}+y^{\lambda}}dxdy < C\left\{\int_0^{\infty}x^{1-\lambda}f^2(x)dx\right\}^{\frac{1}{2}}\left\{\int_0^{\infty}x^{1-\lambda}g^2(x)dx\right\}^{\frac{1}{2}}, \qquad (2.1)$$

where the constant factor C is defined by

$$C=2\left(\frac{2}{\lambda}\right)^{\mu+1}\Gamma(\mu+1)G(\mu) \qquad (2.2)$$

and the function $G(\mu)$ is defined by (1.6) and $\Gamma(z)$ is Γ-function**.** And the constant factor C in (2.3) is the best possible**.**

Proof. We can apply the Cauchy inequality to estimate the left-hand side of (2.1) as follows.

$$\int_0^\infty\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu f(x)g(y)}{x^\lambda+y^\lambda}dxdy \le \left(\int_0^\infty \tilde{\omega}(x) f^2(x)dx\right)^{\frac{1}{2}}\left(\int_0^\infty \tilde{\omega}(x)g^2(x)dx\right)^{\frac{1}{2}}, \qquad (2.3)$$

where $\tilde{\omega}(x)=\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda+y^\lambda}\left(\frac{x}{y}\right)^{\frac{2-\lambda}{2}}dy$.

By proper substitution of variable, and then by Lemma 1.4, it is easy to deduce that

$$\tilde{\omega}(x)=\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda\left(1+\left(\frac{y}{x}\right)^\lambda\right)}\left(\frac{x}{y}\right)^{\frac{2-\lambda}{2}}dy$$

$$=x^{1-\lambda}\int_0^\infty t^{-\frac{2-\lambda}{2}}\left|\ln t\right|^\mu \frac{1}{1+t^\lambda}du = Cx^{1-\lambda} \qquad (2.4)$$

where the constant factor C is defined by (1.8)**.**

It follows from (2.3) and (2.4) that

$$\int_0^\infty\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu f(x)g(y)}{x^\lambda+y^\lambda}dxdy \le C\left\{\int_0^\infty x^{1-\lambda} f^2(x)dx\right\}^{\frac{1}{2}}\left\{\int_0^\infty x^{1-\lambda}g^2(x)dx\right\}^{\frac{1}{2}}, \qquad (2.5)$$

If (2.5) takes the form of the equality, then there exist a pair of non-zero constants c_1 and c_2 such that

$$c_1\frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda+y^\lambda}f^2(x)\left(\frac{x}{y}\right)^{\frac{2-\lambda}{2}} = c_2\frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda+y^\lambda}g^2(y)\left(\frac{y}{x}\right)^{\frac{2-\lambda}{2}} \quad \text{a.e. on } (0,+\infty)\times(0,+\infty)$$

Then we have

$$c_1x^{2-\lambda}\ f^2(x)=c_2y^{2-\lambda}\ g^2(y)=\tilde{C}. \quad \text{(constant)} \qquad \text{a.e. on } (0,+\infty)\times(0,+\infty)$$

Without losing the generality, we suppose that $c_1\neq 0$, then

$$\int_0^\infty x^{1-\lambda}f^2(x)dx=\frac{\tilde{C}}{c_1}\int_0^\infty x^{-1}dx.$$

This contradicts that $0<\int_0^\infty x^{1-\lambda}f^2(x)dx<+\infty$. Hence it is impossible to take the equality in (2.5).

So the inequality (2.1) is valid.

It remains to need only to show that C in (2.1) is the best possible. $\forall\ 0<\varepsilon<\lambda$.

Define two functions by

$$\tilde{f}(x)=\begin{cases}0 & x\in(0,1)\\ x^{-\frac{2-\lambda+\varepsilon}{2}} & x\in[1,\infty)\end{cases} \quad \text{and} \quad \tilde{g}(y)=\begin{cases}0 & y\in(0,1)\\ y^{-\frac{2-\lambda+\varepsilon}{2}} & y\in[1,\infty)\end{cases}$$

It is easy to deduce that

$$\int_0^{+\infty} x^{1-\lambda}\tilde{f}^2(x)dx = \int_0^{+\infty} y^{1-\lambda}\tilde{g}^2(y)dy = \tfrac{1}{\varepsilon} .$$

If C in (2.1) is not the best possible, then there exists $K>0$, such that $K<C$ and

$$H(\lambda,\mu)=\int_0^\infty\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu \tilde{f}(x)\tilde{g}(y)}{x^\lambda+y^\lambda}dxdy$$

$$\le K\left(\int_0^\infty x^{1-\lambda}\tilde{f}^2(x)dx\right)^{\frac{1}{2}}\left(\int_0^\infty y^{1-\lambda}\tilde{g}^2(y)dy\right)^{\frac{1}{2}}.$$

$$= K\left(\int_1^\infty x^{1-\lambda}\tilde{f}^2(x)dx\right)^{\frac{1}{2}}\left(\int_1^\infty y^{1-\lambda}\tilde{g}^2(y)dy\right)^{\frac{1}{2}} = \frac{K}{\varepsilon}. \qquad (2.6)$$

On the other hand, we have

$$H(\lambda,\mu)=\int_0^\infty\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu \tilde{f}(x)\tilde{g}(y)}{x^\lambda+y^\lambda}dxdy = \int_1^\infty\int_1^\infty \frac{\left\{x^{-\frac{2-\lambda+\varepsilon}{2}}\right\}\left\{\left|\ln\frac{x}{y}\right|^\mu y^{-\frac{2-\lambda+\varepsilon}{2}}\right\}}{x^\lambda+y^\lambda}dxdy$$

$$=\int_1^\infty\left\{\int_1^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu y^{-\frac{2-\lambda+\varepsilon}{2}}}{x^\lambda\left(1+\left(\frac{y}{x}\right)^\lambda\right)}dy\right\}\left\{x^{-\frac{2-\lambda+\varepsilon}{2}}\right\}dx$$

$$=\int_1^\infty\left\{\int_{1/x}^\infty \frac{\left|\ln\frac{1}{t}\right|^\mu t^{-\frac{2-\lambda+\varepsilon}{2}}}{1+t^\lambda}dt\right\}\left\{x^{-1-\varepsilon}\right\}dx$$

$$=\int_1^\infty\left\{\int_{1/x}^1 \frac{|\ln t|^\mu t^{-\frac{2-\lambda+\varepsilon}{2}}}{1+t^\lambda}dt\right\}\left\{x^{-1-\varepsilon}\right\}dx+\int_1^\infty\left\{\int_1^\infty \frac{|\ln t|^\mu t^{-\frac{2-\lambda+\varepsilon}{2}}}{1+t^\lambda}dt\right\}\left\{x^{-1-\varepsilon}\right\}dx$$

$$=\int_0^1\left\{\int_{1/t}^\infty x^{-1-\varepsilon}dx\right\}\frac{|\ln t|^\mu t^{-\frac{2-\lambda+\varepsilon}{2}}}{1+t^\lambda}dt+\int_1^\infty\left\{\int_1^\infty \frac{|\ln t|^\mu t^{-\frac{2-\lambda+\varepsilon}{2}}}{1+t^\lambda}dt\right\}\left\{x^{-1-\varepsilon}\right\}dx$$

$$=\frac{1}{\varepsilon}\int_0^1 \frac{|\ln t|^\mu t^{-\frac{2-\lambda-\varepsilon}{2}}}{1+t^\lambda}dt+\frac{1}{\varepsilon}\int_1^\infty \frac{|\ln t|^\mu t^{-\frac{2-\lambda+\varepsilon}{2}}}{1+t^\lambda}dt. \qquad (2.7)$$

When ε is sufficiently small, we obtain from (2.7) that

$$H(\lambda,\mu)=\frac{1}{\varepsilon}\left(\int_0^1 \frac{|\ln t|^\mu t^{-\frac{2-\lambda}{2}}}{1+t^\lambda}dt+\circ_1(1)\right)+\frac{1}{\varepsilon}\left(\int_1^\infty \frac{|\ln t|^\mu t^{-\frac{2-\lambda}{2}}}{1+t^\lambda}dt+\circ_2(1)\right)$$

$$=\frac{1}{\varepsilon}\left(\int_0^\infty \frac{|\ln t|^\mu\, t^{-\frac{2-\lambda}{2}}}{1+t^\lambda}dt+\circ(1)\right) \qquad (\varepsilon\to 0)$$

By (2.4), we have

$$H(\lambda,\mu)=\frac{1}{\varepsilon}\big(C+\circ(1)\big). \quad (\varepsilon\to 0) \qquad (2.8)$$

Evidently, the inequality (2.8) is in contradiction with (2.6). Therefore, the constant factor C in (2.1) is the best possible. Thus the proof of Theorem is completed.

Based on Theorem 2.1, we have the following important results.

Theorem 2.2. If $0<\int_0^\infty f^2(x)dx<+\infty$ and $0<\int_0^\infty g^2(x)dx<+\infty$, then

$$\int_0^\infty\int_0^\infty \frac{\left|\ln\frac{x}{y}\right| f(x)g(y)}{x+y}dxdy < 8G\left\{\int_0^\infty f^2(x)dx\right\}^{\frac{1}{2}}\left\{\int_0^\infty g^2(x)dx\right\}^{\frac{1}{2}}. \qquad (2.9)$$

where G is the Catalan constant. And the constant factor $8G$ in (2.9) is the best possible.

Theorem 2.3. Let n be a nonnegative integer and $\lambda>0$. If $0<\int_0^\infty x^{1-\lambda}f^2(x)dx<+\infty$ and $0<\int_0^\infty x^{1-\lambda}g^2(x)dx<+\infty$, then

$$\int_0^\infty\int_0^\infty \frac{\left(\ln\frac{x}{y}\right)^{2n} f(x)g(y)}{x^\lambda+y^\lambda}dxdy < \left(\left(\frac{\pi}{\lambda}\right)^{2n+1}E_n\right)\left\{\int_0^\infty x^{1-\lambda}f^2(x)dx\right\}^{\frac{1}{2}}\left\{\int_0^\infty x^{1-\lambda}g^2(x)dx\right\}^{\frac{1}{2}}, \qquad (2.10)$$

where $E_0=1$ and the $E_{n's}$ are the Euler numbers, viz. $E_1=1,\ E_2=5,\ E_3=61,\ E_4=1385,\ etc.$. And the constant factor $\left(\frac{\pi}{\lambda}\right)^{2n+1}E_n$ in (2.10) is the best possible.

Proof. We need only to show that the constant factor in (2.10) is true. When $\mu=2n$, it is known from (2.2) that

$$C=2\left(\frac{2}{\lambda}\right)^{\mu+1}\Gamma(\mu+1)G(\mu)=2\left(\frac{2}{\lambda}\right)^{2n+1}\Gamma(2n+1)G(2n)$$

$$=2\left(\frac{2}{\lambda}\right)^{2n+1}(2n)!\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^{2n+1}}.$$

According to the paper [13] (pp. 231.), we have

$$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^{2n+1}}=\frac{\pi^{2n+1}}{2^{2n+2}(2n)!}E_n.$$

where the $E_{n's}$ are the Euler numbers, viz. $E_1 = 1,\ E_2 = 5,\ E_3 = 61, E_4 = 1385,\ etc.$ Notice that

$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k-1)} = \frac{\pi}{4}$, hence we can define $E_0 = 1$. Whence we obtain $C = \left(\frac{\pi}{\lambda}\right)^{2n+1} E_n$.

In particular, for case $\lambda = 1$, based on Theorem 2.3, we get the following result at once.

Theorem 2.4. Let n be a nonnegative integer. If $0 < \int_0^\infty f^2(x)dx < +\infty$ and $0 < \int_0^\infty g^2(x)dx < +\infty$, then

$$\int_0^\infty\int_0^\infty \frac{\left(\ln\frac{x}{y}\right)^{2n} f(x)g(y)}{x+y} dxdy < \left(\pi^{2n+1}E_n\right)\left\{\int_0^\infty f^2(x)dx\right\}^{\frac{1}{2}}\left\{\int_0^\infty g^2(x)dx\right\}^{\frac{1}{2}}, \qquad (2.11)$$

where $E_0 = 1$ and the $E_{n's}$ are the Euler numbers. And the constant factor $\pi^{2n+1}E_n$ in (2.11) is the best possible.

Specially, when $n = 0$, the inequality (1.1) follows from (2.11) immediately.

Similarly, we can establish also a great deal of new inequalities. They are omitted here.

3. Some Equivalent Forms

As applications, we will build some new inequalities.

Theorem 3.1. Let f be a real function, $\mu > -1$ and $\lambda > 0$. If $0 < \int_0^\infty x^{1-\lambda} f^2(x)dx < +\infty$, then

$$\int_0^\infty y^{\lambda-1}\left\{\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda + y^\lambda} f(x)dx\right\}^2 dy < C^2\int_0^\infty x^{1-\lambda} f^2(x)dx, \qquad (3.1)$$

where C is defined by (2.2) and the constant factor C^2 in (3.1) is the best possible. And the inequality (3.1) is equivalent to (2.1).

Proof. First, we assume that the inequality (2.1) is valid. Setting a real function $g(y)$ as

$$g(y) = y^{\lambda-1}\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda + y^\lambda} f(x)dx, \qquad y \in (0, +\infty)$$

By using (2.1), we have

$$\int_0^\infty y^{\lambda-1}\left\{\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda + y^\lambda} f(x)dx\right\}^2 dy = \int_0^\infty\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda + y^\lambda} f(x)g(y)dxdy$$

$$< C\left\{\int_0^\infty x^{1-\lambda} f^2(x)dx\right\}^{\frac{1}{2}}\left\{\int_0^\infty y^{1-\lambda} g^2(y)dy\right\}^{\frac{1}{2}}$$

$$= C\left\{\int_0^\infty x^{1-\lambda} f^2(x)dx\right\}^{\frac{1}{2}}\left\{\int_0^\infty y^{\lambda-1}\left(\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda + y^\lambda} f(x)dx\right)^2 dy\right\}^{\frac{1}{2}} \qquad (3.2)$$

It follows from (3.2) that the inequality (3.1) is valid after some simplifications.

On the other hand, assume that the inequality (3.1) keeps valid, by applying in turn the Cauchy inequality and (3.1), we have

$$\int_0^\infty\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda+y^\lambda} f(x)g(y)dxdy=\int_0^\infty y^{\frac{\lambda-1}{2}}\left\{\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda+y^\lambda} f(x)dx\right\} y^{\frac{1-\lambda}{2}} g(y)dy$$

$$\le\left\{\int_0^\infty y^{\lambda-1}\left(\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^\mu}{x^\lambda+y^\lambda} f(x)dx\right)^2 dy\right\}^{\frac{1}{2}}\left\{\int_0^\infty y^{1-\lambda}g^2(y)dy\right\}^{\frac{1}{2}}$$

$$<\left\{C^2\int_0^\infty x^{1-\lambda}f^2(x)dx\right\}^{\frac{1}{2}}\left\{\int_0^\infty y^{1-\lambda}g^2(y)dy\right\}^{\frac{1}{2}}$$

$$=C\left\{\int_0^\infty x^{1-\lambda}f^2(x)dx\right\}^{\frac{1}{2}}\left\{\int_0^\infty y^{1-\lambda}g^2(y)dy\right\}^{\frac{1}{2}}. \qquad (3.3)$$

Therefore the inequality (3.1) is equivalent to (2.1).

If the constant factor C^2 in (3.1) is not the best possible, then it is known from (3.3) that the constant factor C in (2.1) is also not the best possible. This is a contradiction. Theorem is proved.

Theorem 3.2. Let f be a real function. If $0<\int_0^\infty f^2(x)dx<+\infty$, then

$$\int_0^\infty\left\{\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|}{x+y} f(x)dx\right\}^2 dy<(8G)^2\int_0^\infty f^2(x)dx \qquad (3.4)$$

where G is the Catalan constant and the constant factor $(8G)^2$ in (3.4) is the best possible. And the inequality (3.4) is equivalent to (2.9).

Its proof is similar to one of Theorem 3.1. Hence it is omitted.

Theorem 3.3. Let f be a real function, $n\in N_0$ and $\lambda>0$. If $0<\int_0^\infty x^{1-\lambda}f^2(x)dx<+\infty$, then

$$\int_0^\infty y^{\lambda-1}\left\{\int_0^\infty \frac{\left|\ln\frac{x}{y}\right|^{2n}}{x^\lambda+y^\lambda} f(x)dx\right\}^2 dy<\left(\left(\frac{\pi}{\lambda}\right)^{2n+1}E_n\right)^2\int_0^\infty x^{1-\lambda}f^2(x)dx, \qquad (3.5)$$

where $E_0=1$ and the $E_{n's}$ are the Euler numbers, viz. $E_1=1,\ E_2=5,\ E_3=61,\ E_4=1385,\ etc.$. And the constant factor $\left(\left(\frac{\pi}{\lambda}\right)^{2n+1}E_n\right)$ in (3.5) is the best possible. And the inequality (3.5) is equivalent to (2.10). Similarly, we can establish also some new inequalities. They are omitted here

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A Comparative Analysis Of Waiting Time Of Customers In Banks

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***Abstract*-This paper analyses the waiting time of customers in Guaranty Trust Bank (GTB) and Union Bank (Ado-Ekiti metropolis) for cash saving and withdrawal services. Data was collected by observation in respect of the number of people being serviced and the number of arrivals for 21days.The average arrival rate, average service rate, average time spent in the queen, among other things, for the two banks from which necessary comparisons were made.**

I. Introduction

A tree cannot make a forest, so says an adage. Thus the economy of a nation depends on some sector like the agricultural sector, the health sector, the works and housing sector and most importantly, the banking sector, among others. All these sectors work together and integrate to make the life of a nation. A breakdown in any of these sectors will definitely have untold effects on the progress and success of the nation.Banking is a sector of the economy that should be carefully handled and much attention should also be paid to it. Banking sector is one of the most important sector upon which the future of a nation depends. Any slight decline in the performance of the Banks may cause untold effect on the country's foreign exchange.Queuing in Banks has much negative consequence as apart from leading to chaos, wasting of man hours per day. Cases have been witnessed where customers while waiting to cash some money, get bombarded by armed robbers. Cases have been witnessed where armed robbers kill and collected huge sums of money from people on queues in the Banking hall.In view of all these many new generation banks were licensed with a view to reducing congestion in the old generation bank. Despite all these, we still have queues in some of these banks. It is therefore necessary to carry out appraise the performance if banks can address their queuing problems.

II. Methodology

Data on arrival times, time service begins, time service end, and departure time for customers was collected for 21days. This data will help us to obtain the average arrival time, average service rate, average time spent on the queen and the traffic intensity of the customers.

III. Model Specification

The M/M/1 Queue (single-channel Queuing system). In this queuing system, the customers arrive according to a Poisson process with rateλ.
The service rate is the number of people serviced per unit time is μ and the number of servers is c.

IV. Exponential Distribution

The continuous random variable x has the exponential distribution with parameter λ if its pfd is

$$f(x) = \lambda e^{-\lambda x},\ x \geq 0$$

Facts: $F(x) = 1- e^{-\lambda x},\ x \geq 0$

$$E(x) = 1/\lambda$$

$$Var(x) = 1/\lambda^2$$

$M_x(t) = \lambda/(\lambda-t),\quad t < \lambda$

2.0.3THEOREM

The exponential distribution has the memory less property namely, for
$s_1\ t > 0$

$$pr(x > s + t / x > t) = pr(x > s)$$

Example: if $x \sim exp(^1/_{10})$, then
$pr(x > 10\ / x > 5) = pr(x > 5)$
$= e^{-5/10}$
$= 0 .607$

Remark: The exponential is the only continuous distribution with this property.For a continuous distribution, the hazard function (or failure rate) is

$$r(t) = \frac{f(t)}{1-F(t)}$$

The hazard function is the conditional pdf that x will fail at time t (given that x made it to t).

$$r(t)dt\,\frac{f(t)dt}{1-F(t)}$$

$$\approx \frac{pr(x\varepsilon(t,t+dt))}{pr(x>t)}$$

$$= \frac{pr(x\varepsilon(t,t+dt))\quad and\quad x\ >\ t)}{pr(x>t)}$$

$= p(x\ t(t,t + dt) / x > t)$

2.0.4 THEOREM:

$X \sim exp(\lambda)$ implies that $r(t) = \lambda$. This makes sense in light of the memory less properties. In fact, the exp (λ) (uniquely) determines F (t).

$$r(t) = \frac{f(t)}{1-F(t)}$$

$$= -\frac{d}{dt}\ ln(1-F(t)$$

So that

$$F(t) = 1 - \exp\left(-\int_o^t r(s)ds\right)$$

The multi-channel queuing system is used for the analysis of this paper. The formulas developed for the multi-channel queuing system are given below:

Traffic intensity is given as

$$p = \frac{\lambda}{c\mu} \quad (1)$$

The probability of having no customers in the system

$$P_o = \frac{c!(1-\ell)}{(\ell c)^c + c(1-\ell)\left[\sum_{n=0}^{c-1} \frac{1}{n!}(\ell c)^n\right]} \quad (2)$$

The probability of not queuing on arrival

$$P_n = \frac{(\ell c)^c}{c!(1-\ell)} P_o \quad (3)$$

The average number of customers in the system

$$\bar{S} = \frac{\ell(\ell c)^c}{c!(1-\ell)^2} P_o + \ell c \quad (4)$$

The average number of customers in the queue

$$\bar{n} = \frac{\ell(\ell c)^c}{c!(1-\ell)^2} P_o \quad (5)$$

The average time a customers spends in the system

$$\bar{t} = \frac{(\ell c)^c}{c!(1-\ell)^2 c\mu} P_o + \tfrac{1}{\mu} \quad (6)$$

The average time spend in the queue

$$\bar{W} = \frac{(\ell c)^c}{c!(1-\ell)^2 c\mu} P_o \quad (7)$$

A single line will break up into shorter lines in front of each server and the number of customers on the queue at a certain time can take one of these two values:

1. All arrivals are being serviced because there is no queue, that is, $n \leq c$ (the number of customers in the system at a particular time is less than or equal to the number of servers)
2. Service demanded by customers is greater than the capability of servers and so a queue is formed, that is, $n > c$ (the number of customers in the system at a particular time is greater than the number of servers).

V. Results

The arrival times as well as the time service began and ended for customers in the Guaranty. Trust Bank and Union Bank were observed and recorded between 8:00am and 4:00pm.A total of 21 days were used for the data collection. The waiting times and service times were obtained. Therefore; we arrive at the following:

For Union Bank, c = 2

1.Average arrival rate λ = Number of arrivals/hr = 1096 = 52.19Number of days 21

Average arrival rate λ = 52.19 customers/hr:. λ = 0.87 customer/minute

2.Average service rate μ = No of departure = 1161 = 55.29 No of day 21:. μ = 0.92 customer/minute

3.Traffic Intensity $\ell = \frac{\lambda}{c\mu} = \frac{0.87}{2x0.92} = 0.47$

4.The probability of having no customers in the system is:

$$P_o = \frac{1.06}{2.94} = 0.36$$

5.The Prob: of not queuing on arrival:

$$P_n = \frac{(\ell c)^c}{c!(1-\ell)} P_o = \frac{0.8836}{1.06} \ x \ 0.36 = 0.30$$

6.The average number of customers in the system

$\bar{S} = \frac{\ell(\ell c)^c}{c!(1-\ell)^2} P_o$ =0.415292x0.36= 0.27 customer/minute 0.5618= 1.21 customers/minute

7.Average number of customers in the queue

$\bar{n} = \frac{\ell(\ell c)^c}{c!(1-\ell)^2} P_o$ =0.415292x0.36= 0.27 customer/minute 0.5618

8.Average time spent in the system is

$$\bar{t} = \frac{0.8836x0.36}{1.033712} + 1.087 = 1.40$$

9. The average time spend in the queue

$\bar{W} = \frac{(\ell c)^c}{c!(1-\ell)^2} P_o$ = 0.8836 x 0.36 0.5618 = 0.60 minute

For Guaranty Trust Bank, C = 4

1.Average arrival rate λ = total Number of arrivals/hrNo of days = 1121 = 53.38 customers/hr 21

:. λ = 0.89 customer/minute

2.Average service rate μ = Number of departure No of days= 59.67 customers/hr

:. λ = 0.99 customer/minute

3.Thus, traffic intensity P = $\frac{\lambda}{c\mu}$ = $\frac{0.89}{4 \ x \ 0.99}$ = 0.22

4.The probability of having no customer in the system P_o = 0.16

5.The Probability of not queuing on arrival

P_o

$$\frac{(\ell c)^c}{c!(1-\ell)} P_o = \frac{0.5997}{18.72} \; x \; 0.16 \cong 0.01$$

6.The average number of customer in the system is

$$\bar{S} = \frac{\ell(\ell c)^c}{c!(1-\ell)^2} P_o + \ell c \cong 0.01$$

$$= \frac{0.131934}{14.6016} + 0.16 \simeq 0.2 \text{ customers}$$

7.Average number of customer in the queue

$$\bar{n} = \frac{\ell(\ell c)^c}{c!(1-\ell)^2} P_o$$

$$= \frac{0.131934}{14.6016} \; x \; 0.16$$

0.00145 customer/minute

8.Average time spent in the system is

$$\bar{t} = \frac{(\ell c)^c}{c!(1-\ell)^2 c\mu} P_o + \frac{1}{\mu}$$

$$= \frac{0.095952}{57.822336} + \frac{1}{0.99}$$

$$\bar{t} = 1.01176$$

9.The average time spent in the queue

$$\bar{w} = \frac{(\ell c)^c}{c!(1-\ell)^2} P_o = \frac{0.5997}{14.6016} \; x \; 0.16 = 0.00657$$

$\cong$ 0.01 minute.

VI. Discussion of Results

After the whole analysis, it is observed that the traffic intensity in Union Bank is 0.47 while that of the Guaranty. Trust Bank is 0.22 which is quite low compare to that of Union Bank. The probability of no customer in the system of Union Bank is 0.36 which is very high, which the probability of no queuing in GTB is 0.16 which is very low. The average time spent in the system of Union Bank is 1munite 40 seconds while that of the GTB is 1minute. Comparing the two we can deduce that the time spent in queuing and servicing is much less GTB to that of Union Bank. The average waiting time of customers in Union Bank is 0.60 minute and that of the Guaranty Trust Bank is 0.01 minute. This means that move time was spent by the time the service was being rendered in Union Bank.

VII. Conclusion and Recommendation

The queuing theory is a useful statistical technique for solving peculiar problems. Its applications in the organization are indispensable. The queuing difficulties encountered in GTB and Union Bank is similar to what is encountered in other financial institutions across the globe. Excessive waste of time in Banks or filling stations may lead to customers' health complications.

As a result, it is recommended that the management of the Union Bank needs to employ 1 or more cashier to make a total of 3 cashiers instead of 2 or better still; workers that are less busy can be drafted to serve as cashiers. But the management of the Guaranty Trust Bank can still maintain the 4-cashiers since the waiting time is low. There is still room for improvement to reduce queuing to zero level. Therefore, y\by adopting this method it will leads to wasting of customer's time.

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On New Solutions To Lame's Differential Equation

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Abstract
We extends the works of [1] and [2] in obtaining new polynomial solutions Lame di
erential equation. We show that the derivative of solution of Lame's equation can be expressed in terms of solution di
erent from the former. However the new solutions are expressed in terms of polynomials as suggested in [1].
Keyword:Lame dierential equation, polynomial.
Subject classification: 33F10, 33C05, 33C15,33C47.

1 Introduction

The Heun's differential equation is a natural extension of the Riemann hypergeometric differential equation which can be written as [4]

$$P_3(z)\,y''(z) + P_2(z)\,y'(z) + P_1(z)\,y(z) = 0, \tag{1.1}$$

where $P_i(z)$ are arbitrary polynomials of degree $i(i = 3, 2, 1)$ in the complex variable z. Replacing the variable z by the variable x, the above equation reads as

$$\mathcal{D}^2\,y \,+\, (\frac{\gamma}{x} + \frac{\delta}{x-1} \,+\, \frac{\epsilon}{x-a})\mathcal{D}\,y \,+\, \frac{(\alpha\beta x - q)}{x(x-1)(x-a)}\,y \,=\, 0, \tag{1.2}$$

where $D = \frac{d}{dx}$, $\{\alpha, \beta, \gamma, \delta, \epsilon, a, q\}$ $(a \neq 0, 1)$ are parameters, generally complex and arbitrary, linked by the Fuschian constraint $\alpha + \beta + 1 = \gamma + \delta + \epsilon$. This equation has four regular singular points at $\{0, 1, a, \infty\}$, with the exponents of these singularities being respectively, $\{0, 1-\gamma\}$, $\{0, 1-\delta\}$, $\{0, 1-\epsilon\}$, and $\{\alpha, \beta\}$.

With the following assumptions

$$\gamma = \delta = \epsilon = \frac{1}{2},\;\; \alpha = -\frac{1}{2}v,\;\;\; \beta = \frac{1}{2}(v+1),\;\; q = -\frac{1}{4}dh, \tag{1.3}$$

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equation (1.2) above reduces to

$$\frac{d^2u}{dt^2} + \frac{1}{2}\left(\frac{1}{t} + \frac{1}{t-1} + \frac{1}{t-d}\right)\frac{du}{dt} + \frac{dh - v(v+1)z}{4t(t-1)(t-d)}u = 0. \tag{1.4}$$

This equation is called the **Lame's** equation in one of its algebraic form. The parameter v is called the order of the equation and many special features arises when v is an integer. The Heun's equation arises from the separation of variables for Schrodinger equation in certain system. This equation arises from the separation of variables in Laplace's equation, ellipsoidal or elliptic-conal coordinates. Indeed, it is the only equation of non-confluent Heun's type.

In this paper, the works in [1] and [2] are combined together to obtain polynomial solutions to the **Lame's** equation.

In [1], a newly method to solve linear differential equations was developed. Their approach to Heun equation is very transparent in obtaining solution; in particular the polynomial solutions come out very naturally as it was shown below. A single variable differential equation, after appropriate manipulation, can be cast in the form

$$\left(F(D) + P(x, \frac{d}{dx})\right) y(x) = 0, \tag{1.5}$$

where $D \equiv x\frac{d}{dx}$, and $F(D) = \sum_n a_n D^n$ is a diagonal operator in the space of monomials with a_n's being some parameters. $P(x, \frac{d}{dx})$ is an arbitrary polynomial function of x and d/dx, excluding the diagonal operator. It was shown that the following ansatz,

$$y_\lambda(x) = \sum_{m=0}^{\infty}(-1)^m \left[\frac{1}{F(D)}P(x, \frac{d}{dx})\right]^m x^\lambda \tag{1.6}$$

is a solution of the above equation provided and the coefficient of x^λ in $y(x) - x^\lambda$ is zero, in order to ensure that the solution,$y(x)$ is non - singular. The fact that D is diagonal in the space of monomials x^n, makes $1/F(D)$ well defined in the above expression.

In [2], it was shown that any differential equation (1.4), say, can be rewritten in the general form

$$P\mathcal{D}^2\, y \,+\, Q\mathcal{D}\, y \,+\, R\, y \,=\, 0, \tag{1.7}$$

where P, Q,R, are polynomial functions. Setting $\mathcal{D}y = z$, taking the derivative of (1.7), subtracting the equation (1.7) multiplied by some function $\psi(x)$, we obtain the differential equation of the form

$$P\mathcal{D}^2 z + (P' + Q - \psi(x)P)\mathcal{D}z + (Q' + R - \psi(x)Q)z + (R' - \psi(x)R)y = 0, \quad (1.8)$$

where $V' := \mathcal{D}V$. The condition which render (1.7) invariant under derivative operation, i.e, allowing to write (1.8) in the form

$$\bar{P}\mathcal{D}^2 z + \bar{Q}\mathcal{D}z + \bar{R}z = 0, \quad (1.9)$$

where $\bar{P}, \bar{Q}, \bar{R}$ being appropriate polynomials, are of great interest in the investigation of solutions of the **Lame's** equation [4]. The suitable amsatz reads $\psi = \frac{R'}{R}$. In this case, (1.7) reads

$$P\mathcal{D}^2 z + (P' + Q - \frac{R'}{R}P)\mathcal{D}z + (Q' + R - \frac{R'}{R}Q)z = 0, \quad (1.10)$$

where $R(x) = xR' + R(0)$ is a polynomial of degree 1 which can be written as

$$R(x) = R'(x - c), \qquad \text{with} \qquad c = -\frac{R(0)}{R'}. \quad (1.11)$$

Therefore, P/R and Q/R have to be polynomials defining the singularities of the equation (1.10). In general, such a transformation leads to more singular points than in the initial equations. When the singular points coincide with already existing ones, the number of singularities to that of the initial **Lame's** equation increases by one . The derived equation can be transformed to another **Lame's** equation (1.4) and the derivative of the solution to the initial **Lame's** equation can be expressed in terms of a solution to another **Lame's** equation . This property may lead to interesting series solutions to **Lame's** equation.

After appopriate manipulation, the **Lame's** equation (1.4) can be cast in form (1.10) and the polynomial $\bar{P}, \bar{Q}, \bar{R}$ identified. For any such transformed equation, the pole can be eliminated setting either $R' = 0$ or $R(0) = 0$.

Let us denote $\mathcal{L}_n(\alpha, \beta, \gamma, \delta; x)$, the corresponding solutions to **Lame's**. In the next section we proceeds to obtaining the derived polynomial solutions of the **Lame's** at the various singularities.

2 Derived solutions to Lame's Equation

Considering the Lame's equation (1.4) and (1.10), we have

$$(x) = -\frac{v(v+1)}{ah - v(v+1)},$$

leading to an extra singularity

$$z = \frac{ah}{v(v+1)}.$$

This extra singularity coincides with the singularity $z = 0$ when $ah = 0$ and with $z = 1$ when $ah = v(v+1)$, and $z = \infty$ when $v(v+1) = 0$. These conditions leads to three cases to investigate.

2.1 Case I: Singularity at $t = 0$

In this case we have the following characteristics ;

$$ah = 0, \ \psi(x) = 1/z$$

leading to the equation

$$A(z)D^2 z + B(z)Dz + C(z)z = 0, \tag{2.1}$$

where

$$A(z) = z^3 - z^2(a+1) + az, \ \ B(z) = \frac{7}{2}z^3 - 2z^2(a+1) + \frac{1}{2}az$$

and

$$C(z) = z^2(\frac{3}{2} - \frac{v(v+1)}{4}) - z\frac{ah}{4} - \frac{a}{2}$$

Applying (1.6) to (2.1), we obtain

$$a_0 = -\frac{a}{2}, \ a_1 = 1 + \frac{3}{2}a, \ a_2 = -(1+a), \ \ \lambda = 1 \text{ or } \frac{a}{2(a+1)},$$

where the λ's are the exponents of the singularity at $z = 0$ of equation (2.1).

Since $z := \mathcal{D}y$, the polynomial solution is

$$\begin{aligned} \mathcal{D}y_\lambda \ &= \ \mathcal{D}\mathcal{L}_n(\alpha,\beta,\gamma,\delta;z) = \sum_{m=0}^{\infty}(-1)^m \left[\frac{1}{(D-1)(D+\frac{a}{2(a+1)})}\right]^m \times \\ &\left[(z^3+az)\frac{d^2}{dz^2} + (\frac{7}{2}z^3 - 2z^2(a+1))\frac{d}{dz} + z^2(\frac{3}{2} - \frac{v(v+1)}{4}) - z\frac{ah}{4})\right]^m z^\lambda dz : \\ &\lambda = 1 \text{ or } \frac{a}{2(a+1)} \end{aligned} \tag{2.2}$$

2.2 Case II: Singularity at $t = 1$

When $x = 1$, the corresponding characteristics are

$$\sigma = 4p\alpha, \quad \psi(x) = \frac{1}{x-1}.$$

The corresponding differential equation is

$$x(x-1)^2 D^2 z \quad + \quad (1+\gamma+4px)(x-1)^2 + \delta x(x-1)\big) Dz$$
$$+(1+\alpha)4p(x-1)^2 - \delta)z = 0. \tag{2.3}$$

With the application of $t = x - 1$, we obtain the equation

$$(t^2+t^3)D^2z + \ (1+4p+\gamma+\delta)t^2 + 4pt^3 + \delta t\,\big) Dz + \ 4p(1+\alpha)t^2 - \delta\big)\, z = 0, \tag{2.4}$$

leading to $\lambda = -1$ or $\lambda = -\delta$. The corresponding polynomial solution is

$$\mathcal{D}y_\lambda \quad = \quad \mathcal{DC}_n(\alpha,\beta,\gamma,\delta;t) = \sum_{m=0}^{\infty}(-1)^m\left[\frac{1}{(D-1)(D+\delta)}\right]^m \times$$
$$\left[t^3\frac{d^2}{dt^2} + \ (1+4p+\gamma+\delta)t^2 + 4pt^3\big)\frac{d}{dt} + 4p(1+\alpha)t^2\right]^m t^\lambda dt \ \lambda = 1 \ \text{ or} - \delta \tag{2.5}$$

2.3 Case III: Singularity at $t = \infty$

When $x = \infty$, the corresponding characteristics are

$$4p\alpha = 0, \quad \psi(x) = 0.$$

The corresponding differential equation with the introduction of transformation $x = 1/t$ and assumption of $p = 0$ for possible solution is

$$t^2(1-t)D^2z + \ (\gamma-1)t^2 + 2t(1-t) - t(1+\gamma+\delta) \ \ Dz + (\gamma+\delta-\sigma)\, z = 0, \tag{2.6}$$

leading to)

$$\lambda = \frac{1+\gamma+\delta \pm \sqrt{(1+\delta+\gamma)^2 - 4(\gamma+\delta-\sigma)}}{2}.$$

The corresponding polynomial solution is

$$\mathcal{D}y_\lambda \quad = \quad \mathcal{DC}_n(\alpha,\beta,\gamma,\delta;t) = \sum_{m=0}^{\infty}(-1)^m\left[\frac{1}{(D-1)(D+\delta)}\right]^m \times$$
$$\left[t^3\frac{d^2}{dt^2} + \ (1+4p+\gamma+\delta)t^2 + 4pt^3\big)\frac{d}{dt} + 4p(1+\alpha)t^2\right]^m t^\lambda dt : \tag{2.7}$$

If we assume that $\sigma = 0$, we obtain $\lambda = 1$ or $\lambda = \gamma + \delta$ with the polynomial solution

$$\mathcal{D}y_\lambda = \mathcal{D}\mathcal{C}_n(\alpha,\beta,\gamma,\delta;t) = \sum_{m=0}^{\infty}(-1)^m\left[\frac{1}{(D-1)(D-(\delta+\gamma))}\right]^m \times \left[t^3\frac{d^2}{dt^2} + \ (1+4p+\gamma+\delta)t^2+4pt^3\right)\frac{d}{dt}+4p(1+\alpha)t^2\right]^m t^\lambda dt : \quad (2.8)$$

These solutions are new solutions of **Lame** equation in relation to the derivatives of the old ones.

3 Concluding remarks

Derivatives of old solutions of the **Lame** equation has been expressed in terms of new ones. The polynomial solutions obtained are from the series solutions to the derived differential equation terminated at some positive integer say m. This method of obtaining polynomial solutions is being extended to some other confluent forms of diff erential equations, in particular other various confluent forms of the Heun's differential equation.

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Determination Of Sorafenib In Spiked HumanUrine By Differential Pulse PolarographyAt Dropping Mercury Electrode

GJSFR-B Clasification FOR 030402,030403,030405,030306

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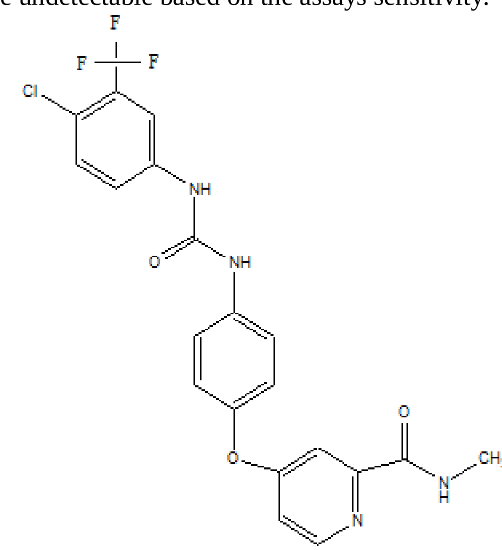
***Abstract*- The electrochemical reduction behavior and determination of sorafenib was studied by differential pulse polarography at dropping mercury electrode. A linear response was obtained over the concentration range 5.0×10^{-8} to 1.0×10^{-5}M with lower detection limits 4.2×10^{-8}M for sorafenib. Sorafenib exbits well defined cathodic waves in universal buffers over the pH range 2.0 to 6.0. The carbonyl group getting reduced to the saturated compound in a four electron processes and reduction mechanism has been proposed. The kinetic parameters such as diffusion coefficients (D), transfer coefficients (α_{na}) and heterogeneous forward rate constants (K^{0}_{fh}) are evaluated and reported. The relative standard deviation and correlation coefficient value was found to be 0.326% and 0.65 respectively. Differential pulse polarography was employed for determination of the sorafenib in trace levels using both standard addition and calibration methods.**

*Keywords***-** Sorafenib, polarography, dropping mercury electrode, formulations and urine.

I. INTRODUCTION

Sorafenib(4[4[[4chloro3(trifluoromethyl)phenyl]carbamoylamino]phenoxy]-N-methylpyridine-2-carboxamide) is a multikinase inhibitor currently approved by the FDA for the treatment of advanced renal-cell carcinoma (RCC) and unresectable hepatocellular carcinoma (HCC), and by the EMEA for the treatment of HCC and advanced RCC. Sorafenib is available as a tablet formulation. Phase I trial of sorafenib in combination with gefitinib[1] and combination with carboplatin and paclitaxel in patients in lung cancer.[2] Phase II trial of first-line treatment with sorafenib versus interferon alfa-2a in patients with metastatic renal cell carcinoma[3] and patients with metastatic or recurrent sarcomas.[4] The determination of sorafenib and sorafenib-glucuronide in mouse plasma and liver homogenate was developed.[5] The HPLC-UV method for sorafenib determination in human plasma and application to cancer patients.[6] Sorafenib triggers antiproliferative and pro-apoptotic signals in human esophageal adenocarcinoma cells.[7] Angiogenesis and signaling through the RAF/ mitogen-activated protein/ extra cellular signal-regulated kinase (ERK) kinase (MEK)/ERK cascade was reported to play important roles in the development of hepatocellular carcinomas (HCC)[8] LC-MS/MS assay for the determination of sorafenib in human plasma.[9,10] Sorafenib concentrations in the samples from the patients with hand foot skin reaction were undetectable based on the assays sensitivity.[11]



Scheme1: Structure of sorafenib

II. INSTRUMENTATION

Direct current polarography and differential pulse polarography were performed with a model 362 polarographic analyzer supplied by Elico Ltd, Hyderabad. Polarographic analyzer connected with Epson LX – 300^{+} printer. The electrode assembly consisted of a dropping mercury electrode of surface area 0.026 cm^2 as the working electrode, a saturated Ag/AgCl(S), Cl^- as the reference electrode and a platinum wire as the auxiliary electrode were used. Metrohm unit E 506 polarecord coupled with E612 VA-scanner, E 648 VA – controller and digital electronics x-y/t recorder are used for cyclic voltammetry. Dissolved air from the solutions was removed by degassing with oxygen free nitrogen for 10 – 15 minutes before polarograms taken. pH measurements were carried out with Elico digital pH meter. Potentiostat was supplied by Tec.Ino.Electronics, Lucknow, India used to perform controlled potential electrolysis.

III. REAGENTS

Sorafenib was kindly provided by Manusaktteva, India. Drugs containing sorafenib labeled to 200 mg per drug were obtained from commercial sources were used without prior

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purification. Sorafenib stock standard solutions ($1x10^{-3}$ M) were prepared daily by direct dissolution in dimethyl sulfoxide.Human urine samples were obtained from healthily volunteers. The universal buffers of pH 2.0 to 12.0 were prepared by using 0.2M boric acid, 0.05M citric acid and 0.1M trisodium orthophosphate. Triple distilled water was used throughout the experiments. All the experiments were carried out at 27^0C temperature.

IV. Recommended Procedure

From the stock solution, 1.0 ml of electrolyte solution were transferred into a polarographic cell, 9.0 ml of the supporting electrolyte of pH 4.0 were added and deoxygenated with nitrogen gas for 15 minutes. After recording the polarograms small increments (0.2 milliliter) of standard solutions were added and polarograms were recorded after each addition under the similar conditions. The optimum conditions for the determination of sorafenib at pH 2.0 were found to be a drop time 2 sec. pulse amplitude 50 mV and applied potential of -1.31V respectively. The above described analytical procedure has been employed for the determination of sorafenib in pharmaceutical formulations and urine samples.

V. Analysis of Drug

Ten tablets were weighed and powdered in an Agate Mortar. Portion equivalent to a stock solution of a concentration about $1x10^{-3}$M was accurately weighed and transferred into a 100 ml standard flask containing buffer. The content was stirring magnetically for 15 minutes to affect complete dissolution and then diluted to the mark with the selected supporting electrolyte appropriate solutions were prepared by taking suitable aliquots of the clear supernatant and diluted with a buffer solution. Each solution was transferred into a polarographic cell and polarograms were subsequently recorded following the optimized conditions. The content of the drug in tablet was determined referring to standard addition and calibration methods.

VI. Analysis of Drug in Urine Sample

Transfer 1.0 ml of human urine sample into a centrifugation tube adds aliquots of sorafenib stock solution and mix well using vortex mixer. Transfer the contents of the centrifugation tube quantitatively into a 25 ml beaker add 9.0 ml of universal buffer solution then transfer the whole contents into a polarographic cell and pass the nitrogen gas for 15 min. and the content of drug in urine sample was determined referring to the calibration graphs.

VII. Results and Discussion

a) Influence of pH effect on the polaragraphic peaks

The polarographic behavior of sorafenib was studied in universal buffer solution. The electrochemical reduction of sorafenib was giving a single well defined cathodic peak at a dropping mercury electrode in pH 2.0 to 6.0. Fig.1 shows a typical d.c.polarograpm of $2.0x10^{-5}$M solution of sorafenib at pH 4.0 and drop time 2 seconds. When the concentration of the sorafenib increases, the half wave/peak potential values are found to change to more negative values. Fig.2 shows the cyclic voltammograms of sorafenib at pH 2.0 and scan rate $40mVs^{-1}$ and concentration $2.0x10^{-5}$M. A peculiar behavior was observed i.e., absence of anodic peak in reverse scan and cathodic peak was obtained which may be due to the reduction of carbonyl compound to hydroxyl derivative. The peak height decreased with increase in pH and gave a characteristic Ep in all the buffer systems because of the decreased available of protons. Fig.3 shows the typical differential pulse polarogram of sorabenib at pH 4.0, concentration $2.0x10^{-5}$M and drop time two seconds. There was no peaks obtained in the basic medium (pH 8.0 to 12.0) due to the precipitation of the electroactive species. The simultaneous reduction of the two carbonyl groups into the corresponding hydroxyl derivative in a four electrons process. The peaks were developed in acidic medium (pH 2.0 to 6.0).

b) Nature of the electrode process

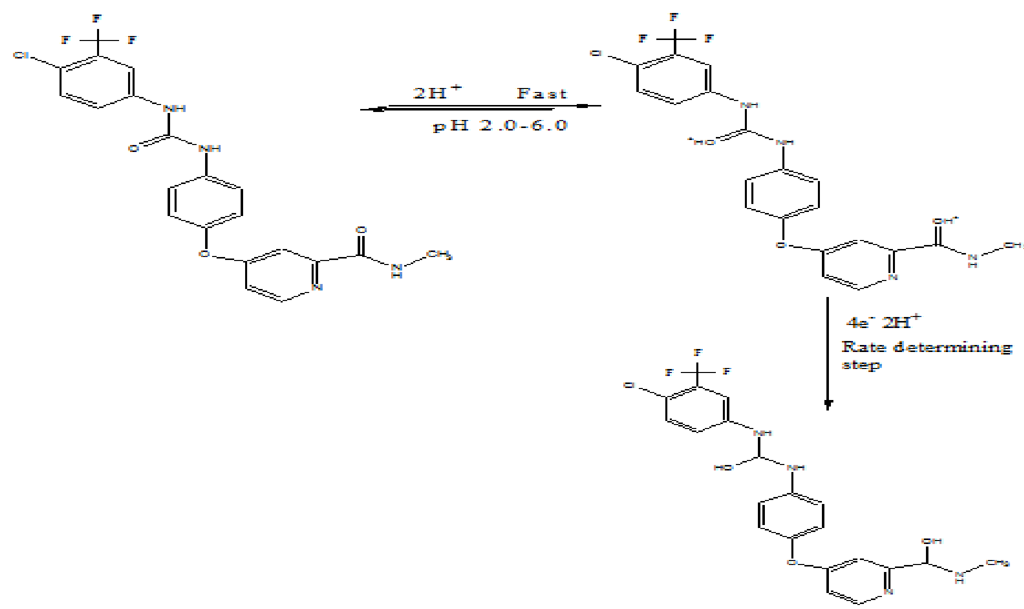
The electrode process was found to be diffusion controlled in all the buffer systems studied, as shown by the linear dependence of limiting current on $h,^{1/2}$ $v^{1/2}$ and $t.^{2/3}$ All the plots are observed to be passing through origin indicating the absence of adsorption complications. When the concentrations of the sorafenib increases, the $E_{1/2}$, E_p and E_m values are found to change to more negative values indicated the irreversibility of the electrode process. The marginal variation of peak potential (E_m) with concentration, nonlinearity in the plot of i_m vs $(1-\sigma/1+\sigma)$ in differential pulse polarography and disobedience of Tomes' criterion also confirm the irreversible nature of the electrode process. Millicoulometry employed at pH 2.0 to find out the number of electrons involved in the electrode process. The results showed the number of electrons to be four for sorafenib. From the slope of $E_{1/2}$ vs pH plot, the number of protons involved in the rate determining step of the electrode process in found to be two. Controlled potential electrolysis experiments are carried out at -0.27V vs saturated calomel electrode at pH 4.0. The isolated product was identified as hydroxyl product and confirmed by I.R. spectral method (absence of C=O stretch 1690 – 1650 cm^{-1}, O-H stretch 3420 cm^{-1}, O-H bend 1360 cm^{-1} and C-O stretch 1240 cm^{-1}). For the irreversible process, the value of α_{na} was calculated from the equation.

$$E = E_{1/2} - \frac{0.0542}{\alpha_n a} \log \left(\frac{i}{i_d - i}\right),$$

where, i was the cathodic current in µA, i_d was the cathodic diffusion current in µA and the α_{na} are illustrated in Table 1. In polarography, the theoretical equation for the maximum diffusion current obtained with a dropping mercury electrode, which was first derived by Ilkovic[12-16] was given by $i_d = 708\ nCD^{1/2}m^{2/3}t^{1/6}$ The diffusion coefficients values were obtained in a good agreement indicating the diffusion controlled and adsorption free nature of the electrode process. The variation of diffusion current with the pH of the supporting electrolyte influences the diffusion coefficient values also to vary in the same manner. The reason for slight variation in diffusion coefficient values with increase in pH may be attributed to the decrease in the availability of protons with increase in pH of the supporting electrolyte. The number of protons (Z) involved in the rate determining step of the electrode reaction is given by $\Delta E_{1/2} / \Delta pH = -0.059P/\alpha_{na}$The number of proton was determined to be 1.45, i.e. two protons were probably consumed in the rate determining step of the electrode reaction. The heterogeneous forward rate constant values (k^0_{fh}) are found to decrease with increasing pH indicating that the electrode reaction in more and more irreversible with increasing pH of the solution.

c) Electrode mechanism

Based on the experimental results obtained from all the techniques employed, a possible electrochemical reduction mechanism has been suggested on the basis of protons and electrons involved in the reduction as follows.



Scheme 2: Electrode mechanism of sorafen

The differential pulse polarography was used for the determination of the sorafenib. Both calibration and standard addition methods are used. The polarographic peaks obtained in the pH range 2.0 to 6.0 are well resolved and reproducible. Calibration plots are linear for sorafenib in the concentration over the range from $5.0x10^{-8}$ to$1.0x10^{-5}$M, the height of the peak was a linear function of the concentration at any pH value. The lower detection limit was calculated as $4.2x10^{-8}$M using the expression dl = 3xSd/m, where Sd was the standard deviation and m was the slope of the calibration plot.

VIII. Recommended Analytical Procedure

A stock solution ($1x10^{-3}$M) was prepared by dissolution of the appropriate amount of the electroactive species in dimethyl sulfoxide due to the low solubility of the drug in water. 1.0 ml of the standard solution was transferred into polarographic cell and made up with 9 ml of the supporting electrolyte and then deoxygenated with nitrogen gas for 10 min. After recording the polarogram small increments (0.3ml) of standard solution were added, and the polarograms are recorded after each addition under similar conditions. In the present study, the best precision was obtained at pH 4.0 with a drop time of 2 sec, pulse amplitude of 50 mV, and peak potential of -1.54 V for sorafenib respectively. The relative standard deviation and correlation coefficients values are found to 0.326 and 0.65 for the respective compound for 10 replicates.

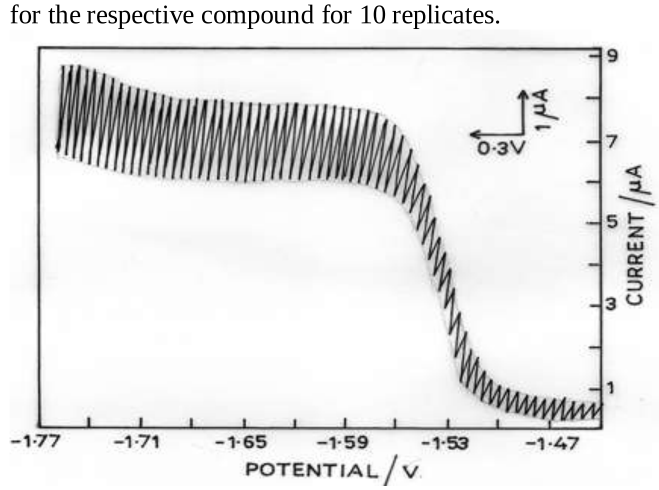


Fig.1Typical d.c.polarogram of sorafenib at pH 4.0 Concentration: $2.0x10^{-5}$ M, Drop time: 2 Sec

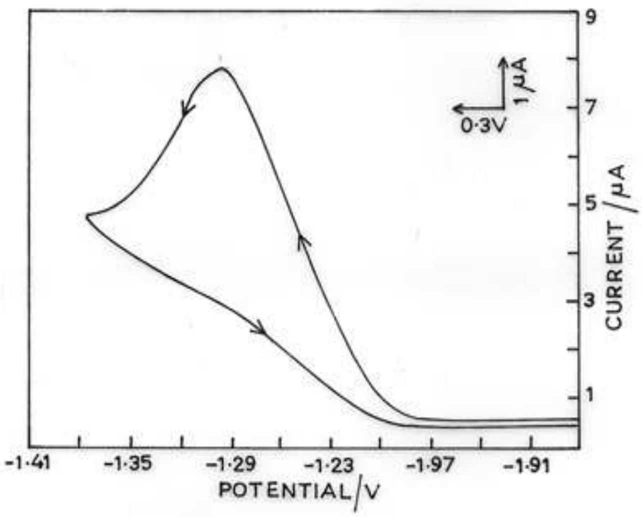


Fig. 2. Typical cyclic voltammogram of sorafenib at pH 2.0, Concentration: $2.0x10^{-5}$ M, Scan rate: $40mVs^{-1}$

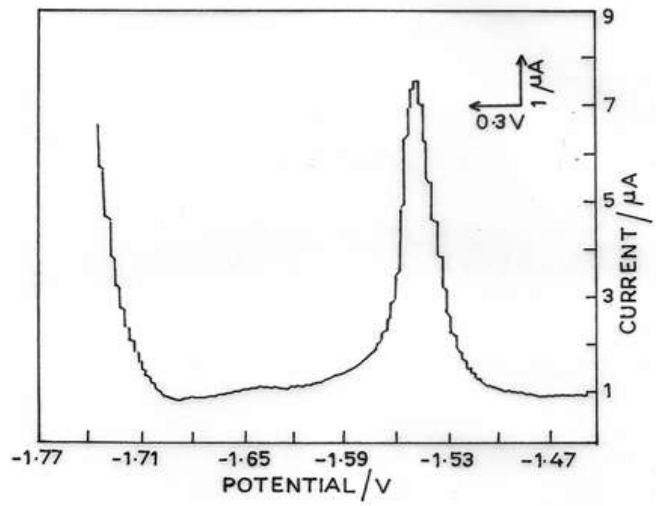


Fig. 3. Typical differential pulse polarogram of sorafenib at pH 4.0, Concentration: $2.0x10^{-5}$ M, Drop time: 2 sec, Pulse amplitude: 50 mV.

pH of the supporting electrolyte	$-E_{1/2}$/V	Id/μA	ΔpH	$\Delta E_{1/2}$	$\frac{\Delta E_{1/2}}{\Delta pH}$	α_{na}
2.0	1.32	8.40	1	0.15	0.15	0.82
3.0	1.47	7.95	1	0.08	0.08	0.77
4.0	1.55	7.30	1	0.07	0.07	0.72
5.0	1.62	6.70	1	0.05	0.05	0.67
6.0	1.67	6.10				0.58

Table 1: – Effect of pH on the polarographic behavior of sorafenib

pH of the supporting electrolyte	D.C. Polarography			Cyclic voltammetry			Differential pulse polarography		
	$-E_{1/2}$ V	D x 10^{-5} Cm^2s^{-1}	$K^0_{f,h}$ Cms^{-1}	$-E_p$ V	D x 10^{-5} Cm^2s^{-1}	$K^0_{f,h}$ Cms^{-1}	$-E_m$ V	D x 10^{-5} Cm^2s^{-1}	$K^0_{f,h}$ Cms^{-1}
2.0	1.32	2.82	$8.54x10^{-4}$	1.30	2.79	$7.26x10^{-4}$	1.31	2.86	$7.86x10^{-4}$
3.0	1.47	2.64	$6.26x10^{-4}$	1.45	2.68	$4.82x10^{-4}$	1.46	2.62	$2.48x10^{-4}$
4.0	1.55	2.47	$4.25x10^{-5}$	1.56	2.51	$5.52x10^{-5}$	1.54	2.38	$6.84x10^{-5}$
5.0	1.62	2.24	$1.62x10^{-5}$	1.64	2.10	$3.10x10^{-5}$	1.63	2.18	$2.26x10^{-5}$
6.0	1.67	1.97	$5.47x10^{-6}$	1.65	2.06	$6.26x10^{-6}$	1.66	2.02	$4.12x10^{-6}$

TABLE 2: - Typical kinetic data of sorafenib

S.No.	Sample	Type	Labeled amount mg/L	Average amount found mg/L	Recovery %	Standard deviation	%Relative Standard deviation
1	sorafenib	**Pharmaceutical formulation**	200	199.20	99.60	0.40	0.200
1	sorafenib	**Urine**	200	198.70	99.35	0.65	0.326

TABLE 3: - Polarographic assay of sorafenib by DPP in pharmaceutical formulations and in spiked human urine samples.

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A Conceptual Framework For Implementing Priority Based Segregation For Decision Support Applications

GJSFR-F Clasification FOR 010404,010401,62Cxx,90C59

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***Abstract*- In this research paper, priority is used as a heuristic. In a given problem space containing hub and spoke type networks, priority is used to identify priority networks, as those with free capacity available to service requests. A priority function is used to identify such networks, which can be used to satisfy priority service requests. The general decision making process is explained first including concept of Meta data. After that the corresponding problem model is formulated. Finally agent based architecture is constructed for implementing this framework. The Conceptual design of the agent that performs the segregation is also done.**

Keywords: Priority; priority functions; priority based segregation; priority networks; priority service requests; agents, etc.

I. INTRODUCTION

Metadata is data about data that is information about other information. e.g. libraries usually group books by subject. In that context the metadata about the book is the subject. Digital cameras allow people to take pictures, in this context the data is the picture taken and the metadata is information about the picture taken like when (the date and time), where (perhaps using information from GPS), or how it was taken. For example, the photographic exposure details are also metadata about a picture taken by a digital camera.

Theories, heuristics, concepts, Meta knowledge, facts and rules are the nature of Meta information. Now, if we partition this knowledge according to type, each sub category will form a separate field. Accordingly it can be noted that heuristics is a type of Meta information and form a distinct sub type of it. In this research paper it will be shown that Meta information can be captured in a template, which consists of distinct fields, one out of which is the heuristics field. Note that priority is a heuristic and forms a distinct field of the Meta information template. In further analysis, it will be shown that a priority variable assumes different values and according to its particular value, different inferences can be drawn.The objective of this research paper is to show how priority values can be captured in a decision support template through the use of priority function. There after a reference to this template gives the priority value, which is used thereafter for decision support in different types of situations and applications, though the generic concept remains priority based segregation, used for decision support.

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Capturing this information in a template means that the template can be deployed as an object also and subsequently queried in a distributed architecture implementation. We consider Five examples to introduce this concept – namely:
Process Scheduling.
Toll booth traffic Analysis.
OPD management in a hospital.
Computer networks.
Hub and Spoke model- used in decision support [1].

Application	Priority Variable	Significance
Process Scheduling	Process CPU burst time	Shorter jobs get priority.
Toll booth traffic analysis	Vehicle type and Average waiting time / vehicle.	Some Vehicle types get priority over others. Some counters also get Priority.
OPD management in hospital	Average waiting time/ patient and average length of waiting queue at counter	Counters can increase to reduce waiting time. Neglected patient types can get priority.
Computer networks	Throughput in network.	Which network gets priority and which process gets priority.
Hub and Spoke model	Network type	Which network is designated as a priority network? priority networks can service priority patient types (priority service requests).

Table 1: Showing application of priority in various applications.

1. Process Scheduling: As in operating systems, the highest priority process is selected and priorities can be externally assigned or it can be internal to the system that is provided by some algorithms or it can be a combination of external and internal methods. Similarly here the p field can be used to provide the priority. For example in health care systems priority can be observe between various patients according to their service requests profile or their arrival time etc. Therefore, this paper has very important application in deciding the priority.

2. Toll Collection Booth: Here we are interested in traffic segregation, identifying both traffic type and also priority service counters. Queuing model simulation is an important part of the system.

3. In Data Warehouse: Meta data provides a catalog of data in the data warehouse and the pointers to this data. In addition to locate the data, Meta data contains information about:

(i). Structure of the data
(ii). Data extraction/transformation history
(iii). Data usage statistics
(iv). Data warehouse table sizes
(v). Column aliases
(vi). Data summarization/modeling algorithms
(vii). Attribute hierarchies and their definition
(viii). Business metrics and their definition
(ix). Performance metrics/confidence levels, etc.

Data mining, heuristics and other algorithms exists to perform data abstraction.

4. Computer Networks: Although various algorithms exist for the removal of congestion in computer networks. One of the methods for regulating access to a communication network is to use a data processing system. The data processing system employs a component that can be implemented in hardware logic or software. The component regulates access to the priority queue. All access to the priority queue or transmit channel must pass through this component, thus subjecting all communication transactions to rejection or tracking by the component. The component allocates a frame size based on the information to be transmitted and the priority to assure the transmission will be completed in line with the quality of service required. The component monitors the rate and size of messages to assure that an application's actual throughput does not exceed its negotiated throughput. The component, moreover, is capable of operating in correction mode where throughput and frame size violations are prevented and reported.

5. Hub & Spoke Model: It is a DSS model that can be used in load balancing, manpower planning and equipment planning etc. Hub and Spoke model framework was proposed by Vohra [1] where we are interested in knowing which network should be a priority network.

The purpose of this research paper is to use a priority function(s) to determine which network should be a priority network and another agent to determine which service requests are the priority requests, which can be redirected to be serviced by the identified networks.

II. Review of Literature

The Hub & Spoke model for health care delivery has been described by Vohra [1]. Collaboration between Hub and Spoke type networks in the form of load transfer, using the concept of Meta data has been described by Vohra et al [2]. Priority is a well established parameter used for CPU Scheduling in design of operating systems [3]. The task of priority functions is to establish priority so that important tasks are done first and get precedence over lower priority tasks. Another task of priority functions is to manage conflicts [4]. In the research problem at hand, the priority function is tasked with identifying networks with free capacity, so that they can handle in coming patient service requests. This is done by examining the value of the priority function, as each network has a corresponding priority function value, which is computed by dynamic traversal through the network. The function value can only be one of the following: negative, zero and positive [5]. Since the networks are dynamic and change, the concept of priority functions here uses dynamic priority computation. In the case of patients, we use class based segregation, setting up a priority flag. In this case priority setting is static. An agent matches priority patient requests to priority networks [6]. Priority functions can be implemented like a normal function. This enables different code to be run at different priority levels. Different priority levels can be assigned to such functions. This application is most suitable for real time operating systems (RTOS) [7]. Priority functions can be encapsulated inside a task and a priority functions scheduler can implement time slicing [8]. Objects can be assigned different priority levels e.g. a data structure or a resource. Functions can then perform operations on various objects with various priority levels. This concept of priority objects helps to define priority resources and data in the system [9]. Network traffic is classified in VLANS by use of user priority field. The traffic classes have been defined in the IEEE 802.1D user priority class descriptions. The mapping of user priority to the number of available traffic classes can be tabulated also, by a mapping table [10]. Brandstatter et.al model and test the priority heuristic in predicting people's choices in certain game playing situations [11]. The Priority heuristic consists of priority rule, stopping rule and decision rule. While the priority rule equals traversal of the priority function values of all the networks in the problem space, the stopping rule details when to stop traversal of the problem space. The decision rule is making the actual choice [12]. Shu-hsien Liao has discussed military command and control decision support [13]. Xin Ye et.al has described architecture for an emergency decision support system [14].

III. General Decision Process of Servicing Patient Requests in a Priority Based Network Space

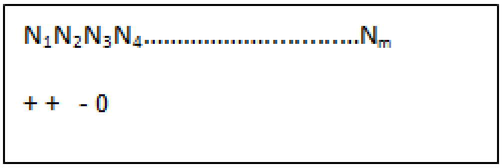
Given a problem space of Hub & Spoke type networks $N_1N_2\ldots N_m$. We are required to direct incoming patient service requests to the networks with servicing capacity i.e. those with free capacity. The decision support application requires identification of the networks with free capacity. This is done by examining the value of a priority function that is associated with each network.The networks $N_1N_2N_3\ldots.N_m$ have $P_1P_2P_3\ldots.P_m$ as the corresponding priority function values in their associated networks. Vohra et. al [2] have proposed a meta data structure for capturing meta data about each network. This structure is as follows:

Name of Hub	Names of Spokes	Free Capacity at Hub	Free Capacity at Spokes

Accordingly, a network with a positive value of free capacity will be able to service patient service requests and is a priority network for this task. Hence it can be stated that:

F > 0: p is +ve : Priority Network.
F < 0: p is –ve : Non Priority Network.
F = 0: p = 0: Non priority Network.

Where F is free capacity of the network and P is priority function value. Thus we get a list of priority networks out of the total networks in the problem space, for which P (Priority Function) has a +ive value. These are the Networks that can satisfy patient requests for service.



In the above figure we see network names in the first row with corresponding priority function signs in the second row. It can be seen that N_1 and N_2 have positive signs and are positive valued, hence these two networks can be segregated as priority networks, from the above discussion.
Now, the further segregation of patient service requests can follow two protocols:

a. All patient service requests are at par (Figure 1).
b. Patient service requests segregated by priority, by setting a priority flag (Figure 2).

In both the cases, the redirection of patient service requests to the networks identified for servicing such requests is done by an agent. We assume it to be a software agent.

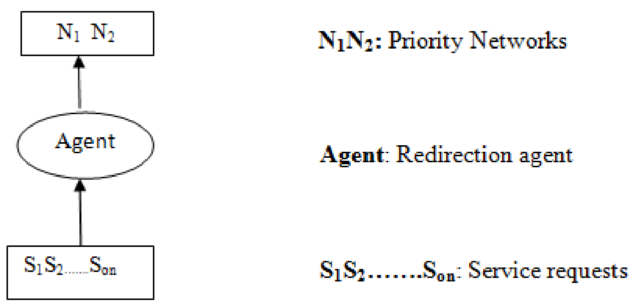


Figure 1: All patient service requests are at par

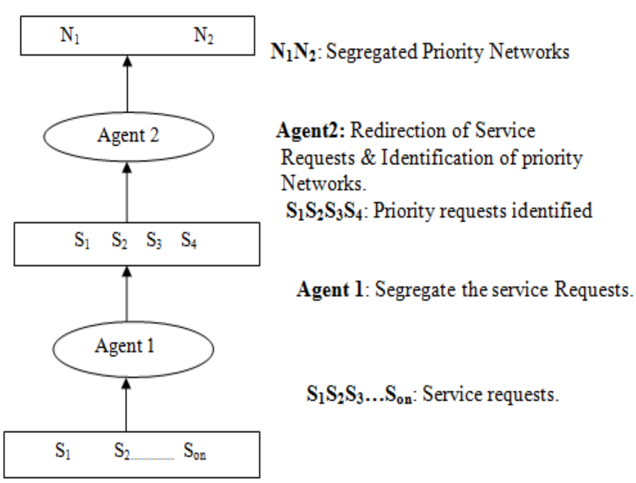


Figure 2: Patient requests being segregated by priority by setting the priority class for service requests.

Note: The priority class can be set in the user registration form of patient service requests, for a patient, depending on various criterions like emergency cases, type of patients e.g. defense personnel, government staff etc and economic criterion e.g. poor patients.In reference to figure 2 above, the design of the software agent, agent 1, responsible for segregating priority service requests is the subject matter of a full research paper. Hence its basic operation can be described at this point.

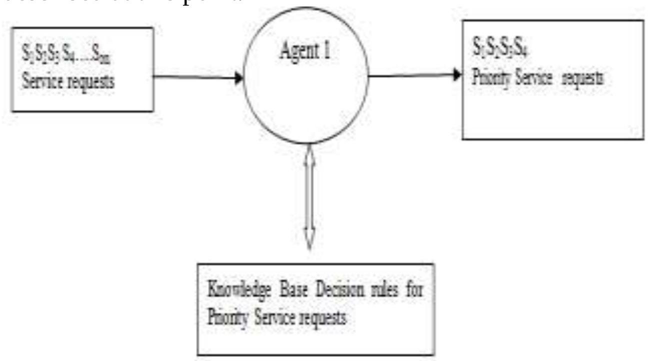


Figure 3: Operation of agent1

The basis of this segregation can be economic criterion (giving priority to lower income patients), service request profile, emergency requests and inputs obtained from queuing analysis, data mining systems and analysis of usage patterns, about which service requests should be classified as priority service requests. The choice and simulation can be made interactive.

IV. THE PROBLEM MODEL

4.1 Problem Definition: Create a conceptual framework to implement priority based segregation in A network space, using priority functions, where in service requests are to be redirected to priority networks. This redirection is performed by a software agent

4.2 Key Elements of the Problem Model

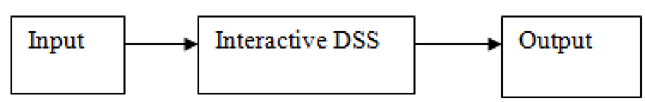


Figure 4: Key elements of problem model.

Input: Service requests, Network space
Output: Priority networks **Computation:**

- Matching priority service requests to priority networks.
- Agent based segregation and redirection of service requests to identify priority networks.
- Design of priority function to identify priority networks.
- Computation of priority class for service requests, thus getting priority service requests.
- Design of software agent with interfaces and processing logic to perform redirection of service requests to priority networks.

Decision Support Analysis:

- Identifying priority networks.
- Generation of priority function values for all networks in the problem space.
- Setting priority class (decision rules) to a service request.
- Identifying priority service requests.
- Description of networks that can satisfy incoming service requests.

V. An Agent Based Architectur

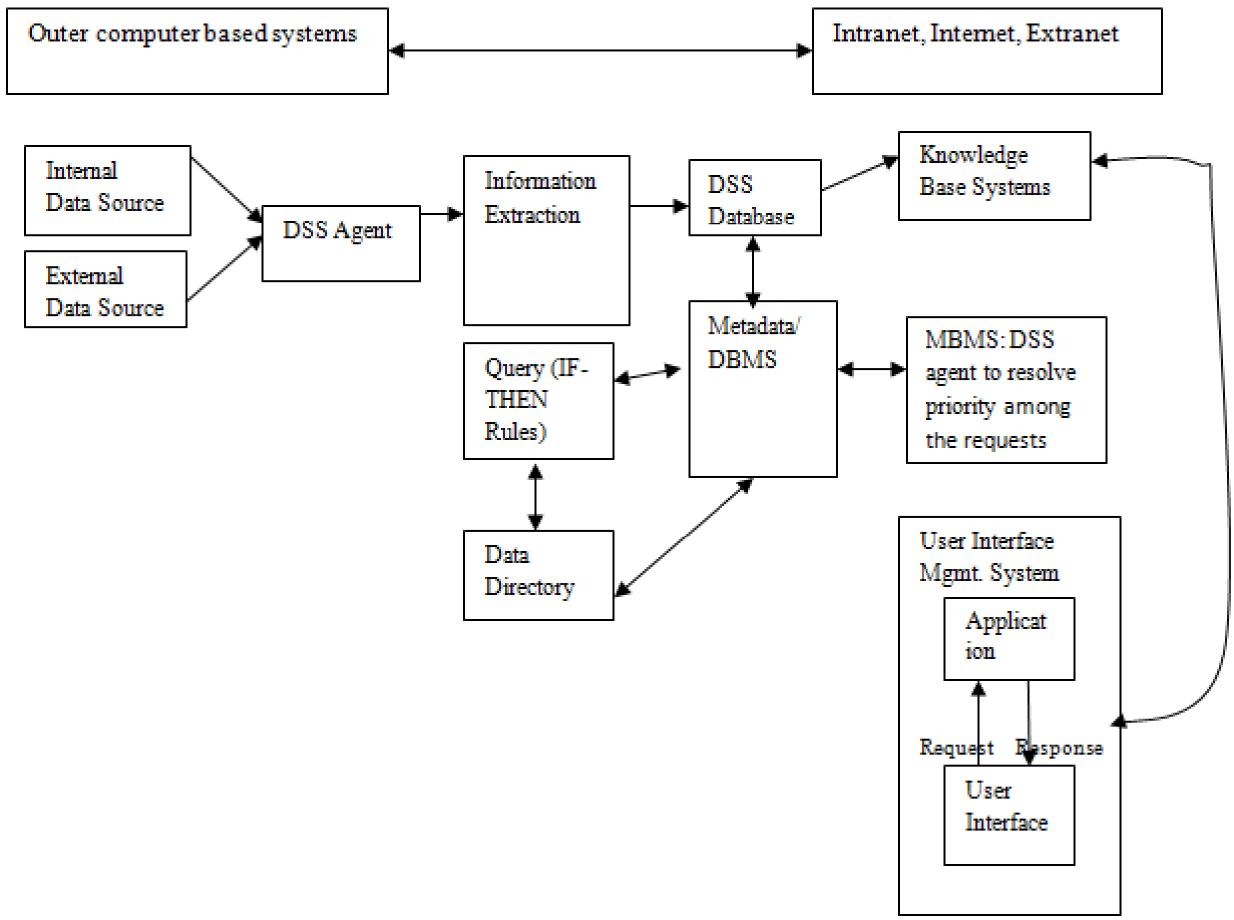


Figure 5: Model of an agent based DSS architecture

Description of Figure 5:

1. DSS Database: This stores the data of service requests, network data and Meta data for each network in the state space, patient information and priority function values etc.

2. Query Facility: Querying the system for information related to identified priority networks, segregated service requests and Meta data values for each network in the state space. *Example*: which networks are priority networks.

3 Data Dictionary: This gives the definition and description of all data elements used by the system. *Example*: name of network, attributes of a service request, patient details etc.

4. DBMS: The DBMS manages the data stated above in terms of retrieval, updating, insertion and deletion. *Example*: Viewing Meta data for a network.

5. Extraction of Data: Extracting priority service requests and identifying priority networks in the given space are examples of data extraction.

6. Model Management System: The algorithms to manage the twin agents for classifying service requests, redirection of service requests to identified priority networks form the core of the model management software. Also, managing and capturing the priority function values is a key task here. This conceptual model is manipulated to obtain priority based segregation.

7. Knowledge Base Management: The knowledge base can store service request classification rules and decision rules for identifying priority networks. This can interface with the software agents to complete the agent based architecture and framework for implementing segregation.

8. User Interface system: An interactive menu driven system for model manipulation, priority segregation, model querying, managing the service requests and overall system design using menus and other UI constructs. *Example*: View

Priority function values for each network in the given problem space

VI. CONCLUSIONS & DIRECTIONS FOR FURTHER RESEARCH

This research paper presented a conceptual agent based architectural framework to implement Priority based segregation for Decision Support applications. The concept of Priority Functions has been used to identify priority networks, while another agent consults Decision Rules to classify patient service requests into priority service requests. The functionality of twin agent architecture to implement this has been illustrated. The Design of an Agent using Priority functions to identify priority networks has been shown through a flow chart.In further work, detailed design specifications of these two agents can be done. In addition, pseudo code to implement the agents can be written. The interaction of these agents with a customized Decision support system (DSS), can be taken up for software implementation. Also, the classification rules for obtaining priority service requests can be written in software, and use of this Knowledge base made to perform this segregation.The various components needed to implement this segregation model can be integrated together and a software prototype developed for the same. In addition, Priority functions can be used for suitable applications in Computer Networking- for example how to identify a priority computer network. The concept remains valid though the algorithm and parameters vary. Real time Priority based segregation, using an Agent based architecture can also be used for segregation of vehicular traffic at Toll gateways. The design of agent1 (Fig 2), tasked with segregating priority requests can be done in a full research paper. Also simulation of different criterion for this segregation will lead to further research work.

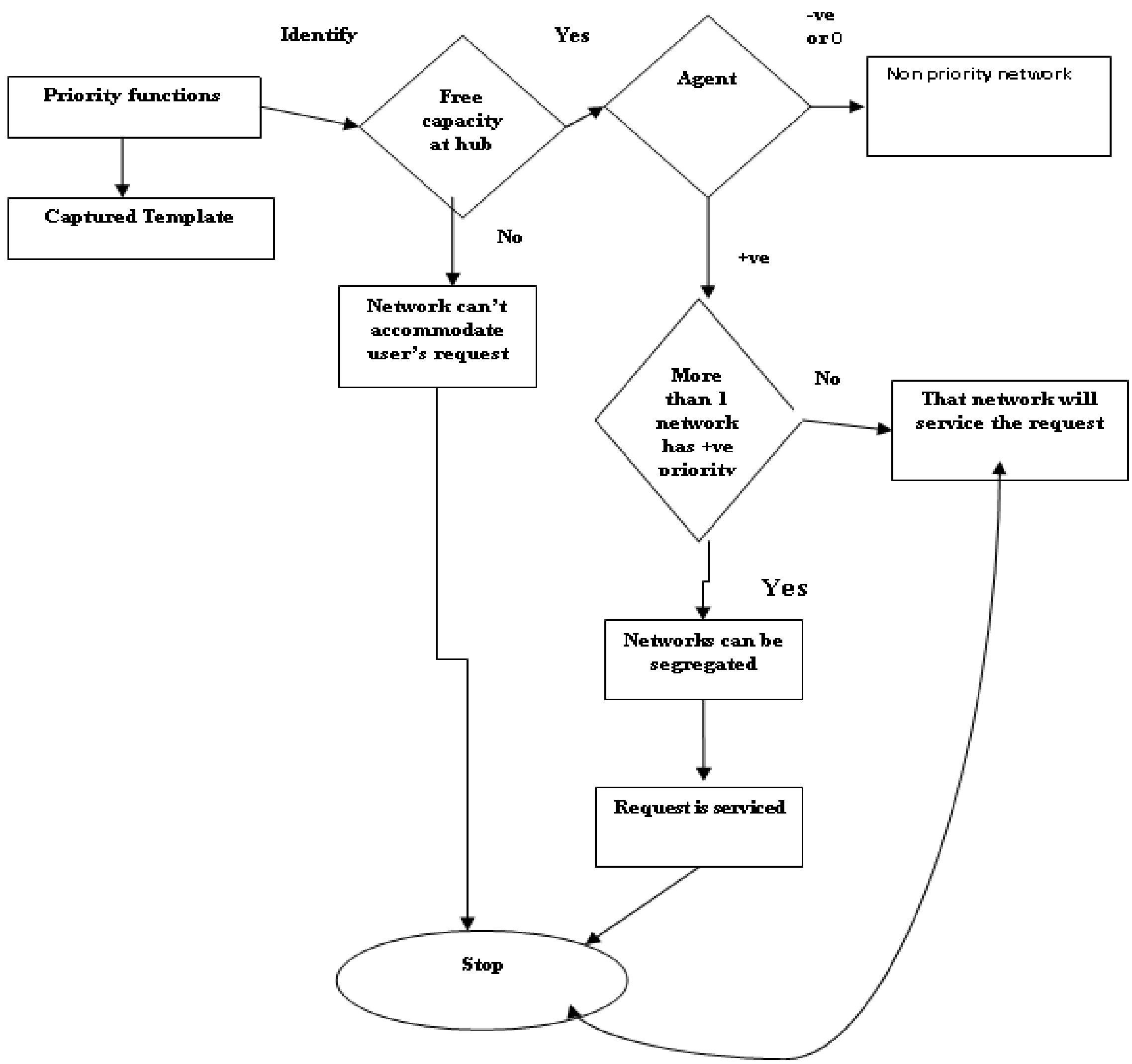


Figure 6: Operation of an agent to identify priority networks using priority functions

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Holder 不等式再推广的几种形式

Holder further promote several forms of inequality

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摘要:本文主要对原 Holder 不等式进行了进一步的推广，将原 Holder 不等式由 2 个实数列推广到 m(m>=2)个实数列，同时给出推广后的 Holder 不等式的几种不同形式并对其加以数学证明.

关键字:Holder 不等式， 推广的 Holder 不等式， 引理(Young 不等式的推广)，琴生不等式，级数，Riemann 积分.

中图分类号：O178

1．Holder 不等式

设 $a=(a_1,a_2,........,a_n)$与$b=(b_1,b_2,..........,b_n)$是两个非负实数列；$\partial$ 与 β 为两个正数，且 $\partial+\beta=1$

则：$\sum_{i=1}^{n} a_i^{\partial} b_i^{\beta} \le (\sum_{i=1}^{n} a_i)^{\partial}(\sum_{i=1}^{n} b_i)^{\beta}$

且只有当 $a_i=0$ 或 $b_i=0$ 或者存在正数 k 使得 $a_i = k b_i$ 时（$i=1,2,......n$）等号才成立.

2．推广的 Holder 不等式

设 $a_j=(a_{j1},a_{j2},.........,a_{jn}),(j=1,2,........,m)$ 是 m 个非负实数列；$\partial_1,\partial_2,......\partial_m$ 为 m 个正数，且

$\sum_{j=1}^{m}\partial_j=1$ （$m\ge 2$）则：$\sum_{i=1}^{n}\prod_{j=1}^{m} a_{ji}^{\partial_j} \le \prod_{j=1}^{m}(\sum_{i=1}^{n} a_{ji})^{\partial_j}$

(1)

且只有当 $a_{1i}=0$ 或 $a_{2i}=0$ ……或 $a_{mi}=0$ 或者存在 m 个正数 $k_1,k_2,......k_m$ 使得

$k_1a_{1i}=k_2a_{2i}=......=k_ma_{mi}$ 时（$i=1,2,......n$）等号才成立.

证明：1）引理(Young 不等式的推广)： 设 $a_j\ge 0$ （$j=1,2,......m$）；$\partial_1,\partial_2,......\partial_m$ 为 m 个正数，

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且 $\sum_{j=1}^{m}\partial_j=1$ 则：$\prod_{j=1}^{m} a_j^{\partial_j} \le \sum_{j=1}^{m}\partial_j a_j$

且只有当 $a_1 = a_2 = \ldots\ldots = a_m$ 时，等号才成立. **证明：**① 当 $a_1, a_2 \ldots\ldots a_m$ 中有一个为 0 时，不等式显然成立

② 当 $a_1, a_2 \ldots\ldots a_m$ 都大于 0 时，对不等式 $\prod_{j=1}^{m} a_j^{\partial_j} \leq \sum_{j=1}^{m} \partial_j a_j$ 两边取自然对数，可得 $\sum_{j=1}^{m} \partial_j \ln a_j \leq \ln(\sum_{j=1}^{m} \partial_j a_j)$，只需证明该不等式成立即可，这可利用琴生不等式进行证明

令函数 $f(x) = \ln x$ （$x > 0$） 显然该函数 $f(x) = \ln x$ 在 $x > 0$ 上为上凸函数，而又正数列 $\partial_1, \partial_2, \ldots\ldots \partial_m$ 满足 $\sum_{j=1}^{m} \partial_j = 1$

因此由琴生不等式可得：$\sum_{j=1}^{m} \partial_j f(a_j) \leq f(\sum_{j=1}^{m} \partial_j a_j)$ 即，$\sum_{j=1}^{m} \partial_j \ln a_j \leq \ln(\sum_{j=1}^{m} \partial_j a_j)$

且只有当 $a_1 = a_2 = \ldots\ldots = a_m$ 时，等号才成立

于是不等式：$\prod_{j=1}^{m} a_j^{\partial_j} \leq \sum_{j=1}^{m} \partial_j a_j$ 得证.

2）再证明推广的 Holder 不等式(即（1）式)

① 当 $a_{1i} = 0$ 或 $a_{2i} = 0$ ．．．．．或 $a_{mi} = 0$ 时（$i = 1,2,\ldots\ldots n$），（1）式显然成立，且等号成立

② 当 $\sum_{i=1}^{n} a_{ji} \neq 0$ （即：$\sum_{i=1}^{n} a_{ji} > 0$ ）时（$j = 1,2,\ldots\ldots m$）

记：$\sum_{i=1}^{n} a_{ji} = A_j$ （$j = 1,2,\ldots\ldots m$）

则（1）式变为：$\sum_{i=1}^{n} \prod_{j=1}^{m} a_{ji}^{\partial_j} \leq \prod_{j=1}^{m} A_j^{\partial_j}$ 即：$\dfrac{\sum_{i=1}^{n} \prod_{j=1}^{m} a_{ji}^{\partial_j}}{\prod_{j=1}^{m} A_j^{\partial_j}} \leq 1$，只需证明此不等式即可

$$\frac{\sum_{i=1}^{n}\prod_{j=1}^{m}a_{ji}^{\partial_j}}{\prod_{j=1}^{m}A_j^{\partial_j}}=\sum_{i=1}^{n}\prod_{j=1}^{m}\frac{a_{ji}^{\partial_j}}{A_j^{\partial_j}}=\sum_{i=1}^{n}\prod_{j=1}^{m}(\frac{a_{ji}}{A_j})^{\partial_j}$$

而 $\sum_{j=1}^{m}\partial_j=1$，因此由上述引理可得：

$$\sum_{i=1}^{n}\prod_{j=1}^{m}(\frac{a_{ji}}{A_j})^{\partial_j}\le\sum_{i=1}^{n}\sum_{j=1}^{m}\partial_j\cdot\frac{a_{ji}}{A_j}=\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{\partial_j}{A_j}a_{ji}$$

$$=\sum_{j=1}^{m}\frac{\partial_j}{A_j}\sum_{i=1}^{n}a_{ji}=\sum_{j=1}^{m}\frac{\partial_j}{A_j}\cdot A_j=\sum_{j=1}^{m}\partial_j=1$$

即得：$\frac{\sum_{i=1}^{n}\prod_{j=1}^{m}a_{ji}^{\partial_j}}{\prod_{j=1}^{m}A_j^{\partial_j}}\le 1$ ，于是（1）式得证

且由引理知：只有当 $\frac{a_{1i}}{A_1}=\frac{a_{2i}}{A_2}=......=\frac{a_{mi}}{A_m}$ 即 $k_1a_{1i}=k_2a_{2i}=......=k_ma_{mi}$ 时（$i=1,2,......n$）等号才成立，其中 $k_j=\frac{1}{A_j}$ （$j=1,2,......m$）.

3．推广的 Holder 不等式的级数形式

设 $a_j=(a_{j1},a_{j2},.........),(j=1,2,........,m)$ 是 m 个无穷非负实数列；$\partial_1,\partial_2,......\partial_m$ 为 m 个正数，且 $\sum_{j=1}^{m}\partial_j=1$ （$m\ge 2$）

若级数 $\sum_{i=1}^{\infty}a_{ji}$ （$j=1,2,......m$）均收敛, 则级数 $\sum_{i=1}^{\infty}\prod_{j=1}^{m}a_{ji}^{\partial_j}$ (绝对) 收敛

且：$$\sum_{i=1}^{\infty}\prod_{j=1}^{m}a_{ji}^{\partial_j}\le\prod_{j=1}^{m}(\sum_{i=1}^{\infty}a_{ji})^{\partial_j} \quad . \qquad (2)$$

证明:由上述引理(Young 不等式的推广)知: $0 \le \prod_{j=1}^{m} a_{ji}^{\partial_j} \le \sum_{j=1}^{m} \partial_j a_{ji} \le \sum_{j=1}^{m} a_{ji}$

由此可得级数 $\sum_{i=1}^{\infty}\prod_{j=1}^{m} a_{ji}^{\partial_j}$ (绝对)收敛

记数列: $S_n = \sum_{i=1}^{n}\prod_{j=1}^{m} a_{ji}^{\partial_j}$ ， $A_{jn} = \sum_{i=1}^{n} a_{ji}$ （$j = 1,2,\ldots\ldots m$）

则由（1）式知：$S_n \le \prod_{j=1}^{m} A_{jn}^{\partial_j}$ ，再由数列极限性质可得（2）式成立.

4. 推广的 Holder 不等式的 Riemann 积分形式

设函数 $f_j(x)$ （$j = 1,2,\ldots\ldots m$）均在闭区间[a, b]上 Riemann 可积，且于[a, b]上恒有 $f_j(x) \ge 0$ （$j = 1,2,\ldots\ldots m$）；$\partial_1, \partial_2, \ldots\ldots \partial_m$ 为 m 个正数，且 $\sum_{j=1}^{m}\partial_j = 1$ （$m \ge 2$）

则：
$$\int_a^b \prod_{j=1}^{m} f_j^{\partial_j}(x)\,d_x \le \prod_{j=1}^{m}\left(\int_a^b f_j(x)\,d_x\right)^{\partial_j} . \qquad (3)$$

证明：将区间[a, b]进行 n 等分，分点 $x_i = a + \frac{b-a}{n} i$， $i = 1,2,\ldots\ldots n$ 于是

$$\int_a^b \prod_{j=1}^{m} f_j^{\partial_j}(x)\,d_x = \lim_{n\to\infty}\sum_{i=1}^{n}\prod_{j=1}^{m} f_j^{\partial_j}(x_i)\frac{b-a}{n}$$

$$\int_a^b f_j(x)\,d_x = \lim_{n\to\infty}\sum_{i=1}^{n} f_j(x_i)\frac{b-a}{n} \qquad j = 1,2,\ldots\ldots m$$

由（1）式可得: $\sum_{i=1}^{n}\prod_{j=1}^{m} f_j^{\partial_j}(x_i) \le \prod_{j=1}^{m}\left(\sum_{i=1}^{n} f_j(x_i)\right)^{\partial_j}$ 而 $\sum_{j=1}^{m}\partial_j = 1$

再由极限保号性质可得（3）式成立.

注： 对 x∈[a,b]，只有当 $f_1(x)\equiv 0$或 $f_2(x)\equiv 0$..........或 $f_m(x)\equiv 0$ 或者存在 m 个常数 $k_1,k_2,......k_m$ 使得 $k_1 f_1(x)\equiv k_2 f_2(x)\equiv ....\equiv k_m f_m(x)$ 时，（3）式中等号才成立.

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22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. **Think technically:** Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.
- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.

You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.

- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Administration Rules Listed Before Submitting Your Research Paper to Global Journals Inc. (US)

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.
- Do not give permission to anyone else to "PROOFREAD" your manuscript.

Written Material: You may discuss with your guides and key sources.

- Do not copy or imitate anyone else paper. (Various Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

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by Global Journals Inc. (US)

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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