Global Journal of Science Frontier Research

discovering thoughts and inventing future

Volume 10 Issue 7 Version 1.0

ISSN: 0975-5896

November 2010

highlights

- Analysis of SubspaceLDA
- Viscous Dissipation Effects
- Downy Mildew Resistant Maize
- Fractional Volterra Type Equations

9 Advances & Discoveries of Science

ENG

Global Journal of Science Frontier Research

Global Journal of Science Frontier Research

Volume 10 Issue 7 (Ver. 1.0)

Global Association of Research

Incorporation No.: 0423089
License No.: 42125/022010/1186
Registration No.: 430374
Import-Export Code: 1109007027
Employer Identification Number (EIN):
USA Tax ID: 98-0673427

Global Journals Inc.

*(A Delaware USA Incorporation with "Good Standing"; **Reg. Number: 0423089**)*

Sponsors: *Global Association of Research*
Open Scientific Standards

Publisher's Headquarters office

Global Journals Inc., Headquarters Corporate Office, Cambridge Office Center, II Canal Park, Floor No. 5th, ***Cambridge (Massachusetts)***, Pin: MA 02141 United States
USA Toll Free: +001-888-839-7392
USA Toll Free Fax: +001-888-839-7392

Offset Typesetting

Global Journals Inc., City Center Office, 25200 Carlos Bee Blvd. #495, Hayward Pin: CA 94542 United States

Packaging & Continental Dispatching

Global Journals, India

Find a correspondence nodal officer near you

To find nodal officer of your country, please email us at *local@globaljournals.org*

eContacts

Press Inquiries: *press@globaljournals.org*
Investor Inquiries: *investers@globaljournals.org*
Technical Support: *technology@globaljournals.org*
Media & Releases: *media@globaljournals.org*

Pricing (Including by Air Parcel Charges):

For Authors:
22 USD (B/W) & 50 USD (Color)
Yearly Subscription (Personal & Institutional):
200 USD (B/W) & 500 USD (Color)

Dr. Bart Lambrecht
Director of Research in Accounting and FinanceProfessor of Finance
Lancaster University Management School
BA (Antwerp); MPhil, MA, PhD (Cambridge)

Dr. Carlos García Pont
Associate Professor of Marketing
IESE Business School, University of Navarra
Doctor of Philosophy (Management), Massachusetts Institute of Technology (MIT)
Master in Business Administration, IESE, University of Navarra
Degree in Industrial Engineering, Universitat Politècnica de Catalunya

Dr. Fotini Labropulu
Mathematics - Luther College
University of ReginaPh.D., M.Sc. in Mathematics
B.A. (Honors) in Mathematics
University of Windso

Dr. Lynn Lim
Reader in Business and Marketing
Roehampton University, London
BCom, PGDip, MBA (Distinction), PhD, FHEA

Dr. Mihaly Mezei
ASSOCIATE PROFESSOR
Department of Structural and Chemical Biology, Mount Sinai School of Medical Center
Ph.D., Etvs Lornd University
Postdoctoral Training,
New York University

Dr. Söhnke M. Bartram
Department of Accounting and FinanceLancaster University Management SchoolPh.D. (WHU Koblenz)
MBA/BBA (University of Saarbrücken)

Dr. Miguel Angel Ariño
Professor of Decision Sciences
IESE Business School
Barcelona, Spain (Universidad de Navarra)
CEIBS (China Europe International Business School).
Beijing, Shanghai and Shenzhen
Ph.D. in Mathematics
University of Barcelona
BA in Mathematics (Licenciatura)
University of Barcelona

Philip G. Moscoso
Technology and Operations Management
IESE Business School, University of Navarra
Ph.D in Industrial Engineering and Management, ETH Zurich
M.Sc. in Chemical Engineering, ETH Zurich

Dr. Sanjay Dixit, M.D.
Director, EP Laboratories, Philadelphia VA Medical Center
Cardiovascular Medicine - Cardiac Arrhythmia
Univ of Penn School of Medicine

Dr. Han-Xiang Deng
MD., Ph.D
Associate Professor and Research Department Division of Neuromuscular Medicine
Davee Department of Neurology and Clinical NeuroscienceNorthwestern University
Feinberg School of Medicine

From the Chief Author's Desk

We see a drastic momentum everywhere in all fields now a day. Which in turns, say a lot to everyone to excel with all possible way. The need of the hour is to pick the right key at the right time with all extras. Citing the computer versions, any automobile models, infrastructures, etc. It is not the result of any preplanning but the implementations of planning.

With these, we are constantly seeking to establish more formal links with researchers, scientists, engineers, specialists, technical experts, etc., associations, or other entities, particularly those who are active in the field of research, articles, research paper, etc. by inviting them to become affiliated with the Global Journals.

This Global Journal is like a banyan tree whose branches are many and each branch acts like a strong root itself.

Intentions are very clear to do best in all possible way with all care.

Dr. R. K. Dixit
Chief Author
chiefauthor@globaljournals.org

Asymptotic Behavior of Solutions of Nonlinear Delay Differential Equations With Impulse

Zhang xiong

GJSFR Classification - F (FOR) 010302

Abstract **This paper studies the asymptotic behavior of solutions of the second-order nonlinear delay differential equations with impulses:**

$$(r(t)x'(t))' - p(t)x'(t) + \sum_{i=1}^{n} q_i(t)x(t-\sigma_i) + f(t) = 0, \quad t \neq t_k,$$

$$x(t_k^+) - x(t_k) = a_k x(t_k) \quad x'(t_k^+) \quad x'(t_k) \quad b_k x'(t_k) \quad k \in Z^+.$$

and some sufficient conditions are obtained.

Keywords-AsymptoticBehavior,Second-order Nonlinear Delay Differential Equation, Impulses.

I. Introduction

In [1], X.Liu studied the asymptotic behavior of solution of the forced nonlinear neutral differential equation with impulses:

$$[x(t) - p x(t-\tau)]' + \Sigma_{i=1}^{n} q_i(t) f(x(t-\sigma_i)) = h(t) \quad t, \ t_k \neq ,$$

$$x(t_k^+) - x(t_k) = b_k x(t_k) \quad , \quad k \in Z^+ .$$

In [2],the authors researched the effective sufficient conditions for the asymptotic stability of the trivial solution of impulsive delay differential equation:

$$x'(t) + \Sigma_{i=1}^{n} p_i(t) x(t-\tau_i) = 0 \ t \neq t_k ,$$

$$x(t_k^+) - x(t_k) = b_k x(t_k), k = 1, 2, \cdots.$$

In this paper,we discuss the asymptotic behavior of a class of second-order nonlinear delay differential equation with impulses. The equation is:

$$(r(t)x'(t))' - p(t)x'(t) + \sum_{i=1}^{n} q_i(t)x(t-\sigma_i) + f(t) = 0, \quad t \neq t_k, \ (1)$$

$$x(t_k^+) - x(t_k) = a_k x(t_k) \quad x'(t_k^+) \quad x'(t_k) \quad b_k x'(t_k) \quad k \in Z^+, \ (2).$$

where $0 \le t_0 < t_1 < t_2 < \cdots, \lim_{k\to+\infty} t_k = +\infty$,and $a_k, b_k, k = 1, 2, \cdots$ are constant.

$$x'(t_k) = \lim_{h\to 0^-} \frac{x(t_k+h) - x(t_k)}{h}, \quad x'(t_k^+) = \lim_{h\to 0^+} \frac{x(t_k+h) - x(t_k^+)}{h}, k = 1, 2, \cdots.$$

$$r(t), p(t), q_i(t), h(t) \in C([0,\infty), R^+), i = 1, 2, \cdots; n \le 0 < \sigma_1 < \sigma_2 < \cdots < \sigma_n$$

Let PC_{t_0} denotes the set of function $\phi : [t_0 - \sigma_n, t_0] \to R$, which is continuous in the set $[t_0 - \sigma_n, t_0] \backslash \{t_k : k = 1, 2, \cdots\}$ and may have discontinuities of the first kind and is continuous from left at the points t_k situated in the interval $(t_0 - \sigma_n, t_0]$. For any $t_0 \ge 0, \phi \in PC_{t_0}$, a function x is said to be a solution of (1) and (2) and satisfying the initial value condition:

$$x(t) = \phi(t) \quad x(t_0^+) = x_0 \quad x'(t) = \phi'(t) \quad x'(t_0^+) = x_0' \quad t \in [t_0 - \sigma_n, t_0] \ (3) ,$$

(Dept.of Mathematics,shaanxi Institute of Education, Shaanxi,xi'an 710061)

in the interval $[t_0-\sigma_n,\infty)$, if $x:[t_0-\sigma_n,\infty)\to R$ satisfies (3) and

(i)for $t\in(t_0,\infty), t\neq t_k, t\neq t_k+\sigma_i, i=1,2,\cdots,n, k=1,2,\cdots, x(t), x'(t)$ is continuously differential and satisfies (1);

(ii)for $t_k\in[t_0,\infty), x(t_k^+), x'(t_k^+), x(t_k^-)$ and $x'(t_k^-)$ exist, $x(t_k^-)=x(t_k), x'(t_k^-)=x'(t_k)$ and satisfies (2).

Because (1)can be transformed to one-order differential equations with impulses, so the existence and sole of solutions of (1) can be deduced by [3].

A solution of (1)and (2) is called eventually positive (negative) if it is positive (negative) for all t sufficiently large,and it is called oscillatory if it is neither eventually positive nor eventually negative. Otherwise,it is called nonoscillatory.

II. Main Lemmas

Throughout this paper, we assume that the following conditions hold:

$$(H_1)\quad r(t)\ge r, \int_0^\infty p(t)dt\le p, q_i(t)\le q_i, i=1,2,\cdots,n, r,p,q_i\in R^+.$$

(H_2) for all $t\in[0,\infty)$, the intergration $H(t)=\int_t^\infty f(s)ds$ converges; $\Sigma_{k=1}^\infty b_k^+<\infty$ where $b_k^+=\max\{b_k,0\}$; ,

$$(H_3)\quad \lim_{n\to\infty}\sum_{m=0}^{n-1}\prod_{k=m+1}^{n-1}\prod_{l=0}^{m}(a_k+1)(b_l+1)\int_{t_m}^{t_{m+1}}\frac{1}{r(u)}\exp[\int_{t_0}^{u}\frac{p(s)}{r(s)}ds]du=+\infty.$$

$$(H_4)\quad \prod_{k=0}^{n-1}(b_{j+k}+1)\frac{r(t_j)}{r(t_{j+n})}\exp[-\int_{t_j}^{t_{j+n}}\frac{p(s)}{r(s)}ds]>1.$$

Lemma 1.Suppose that $x(t)$ is a solution of equations (1) and (2), and there exists $T\ge t_0$ such that $x(t)>0, t\ge T$, If (H_3) hold,then $x'(t_k)>0, x'(t)>0$, where $t\in(t_k,t_{k+1}], k=1,2,\cdots$.

Proof. First, we prove $x'(t_k)>0$, for all $t_k\ge T$. Otherwise,there exists some j such that $t_j\ge T, x'(t_j)<0$, then $x'(t_j^+)=(1+b_j)x'(t_j)$ from (1),we get

$$[r(t)x'(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]]'=-\sum_{i=1}^{n}q_i(t)x(t-\sigma_i)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]-f(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]$$

$$=[-\sum_{i=1}^{n}q_i(t)x(t-\sigma_i)-f(t)]\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]<0.$$

Hence, $r(t)x'(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]$ is decreasing on $(t_j,t_{j+1}]$ and

$$r(t_{j+1})x'(t_{j+1})\exp[-\int_{t_j}^{t_{j+1}}\frac{p(s)}{r(s)}ds]\le r(t_j)x'(t_j^+)\le r(t_j)(b_j+1)x'(t_j).$$

$$x'(t_{j+1})\le(b_j+1)\frac{r(t_j)}{r(t_{j+1})}x'(t_j)\exp[\int_{t_j}^{t_{j+1}}\frac{p(s)}{r(s)}ds]$$

on $(t_{j+1},t_{j+2}]$,

$$x'(t_{j+2})\le(b_{j+1}+1)\frac{r(t_{j+1})}{r(t_{j+2})}x'(t_{j+1})\exp[\int_{t_j}^{t_{j+2}}\frac{p(s)}{r(s)}ds]$$

$$\le(b_{j+1}+1)\frac{r(t_{j+1})}{r(t_{j+2})}(b_j+1)\frac{r(t_j)}{r(t_{j+1})}x'(t_j)\exp[\int_{t_j}^{t_{j+2}}\frac{p(s)}{r(s)}ds]$$

$$=(b_{j+1}+1)(b_j+1)\frac{r(t_j)}{r(t_{j+2})}x'(t_j)\exp[\int_{t_j}^{t_{j+2}}\frac{p(s)}{r(s)}ds].$$

By induction,we have,for all $n\ge 2$.

$$x'(t_{j+n})\le\prod_{k=0}^{n-1}(b_{j+k}+1)\frac{r(t_j)}{r(t_{j+n})}x'(t_j)\exp[\int_{t_j}^{t_{j+n}}\frac{p(s)}{r(s)}ds]$$

Because $r(t)x'(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]$ is decreasing on $(t_j,t_{j+1}]$, so,

$$x'(t)\le(b_j+1)\frac{r(t_j)}{r(t)}x'(t_j)\exp[\int_{t_j}^{t}\frac{p(s)}{r(s)}ds],\quad t\in(t_j,t_{j+1}].$$

Integrating the above inequality from s to t, we have

$$x(t)\le x(s)+(b_j+1)r(t_j)x'(t_j)\int_s^t\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du,\quad t_j<s<t\le t_{j+1},$$

Let $t\to t_{j+1}, s\to t_j^+$, we get

$$x(t_{j+1})\le x(t_j^+)+(b_j+1)r(t_j)x'(t_j)\int_{t_j}^{t_{j+1}}\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du$$

$$\le(a_j+1)x(t_j)+(b_{j+1}+1)r(t_j)x'(t_j)\int_{t_j}^{t_{j+1}}\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du$$

$$x(t_{j+2})\le(a_{j+1}+1)(a_j+1)x(t_j)+(a_{j+1}+1)(b_j+1)r(t_j)x'(t_j)\int_{t_j}^{t_{j+1}}\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du$$

$$+(b_{j+1}+1)(b_j+1)r(t)x'(t_j)\int_{t_{j+1}}^{t_{j+2}}\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du.$$

By induction, we get, for all n

$$x(t_{j+n})\le\prod_{k=0}^{n-1}(a_{j+k}+1)x(t_j)+r(t_j)x'(t_j)(\sum_{m=0}^{n-1}\prod_{k=m+1}^{n-1}\prod_{l=0}^{m}(a_{j+k}+1)(b_{j+l}+1)\int_{t_{j+m}}^{t_{j+m+1}}\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du).$$

because of $x(t)>0, x'(t_j)<0(t_j\ge T)$, it is contraction to the condition (H_3). Hence, $x'(t_k)>0$ for all $t_k\ge T$ and $r(t)x'(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]$ is decreasing on $(t_j,t_{j+1}]$, thus,

$$r(t)x'(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]\ge r(t_{j+1})x'(t_{j+1})\exp[-\int_{t_j}^{t_{j+1}}\frac{p(s)}{r(s)}ds]\ge 0.$$

therefore, $x'(t)\ge 0, t\in(t_k,t_{k+1}]$. The proof is complete.

Theorem 1.Let $(H_1)-(H_3)$ hold.Suppose that

$$\sum_{i=1}^{n}q_i(t+\sigma_i)\ge 0,\qquad \int_0^{\infty}\sum_{i=1}^{n}q_i(s+\sigma_i)ds=\infty,\ (4)$$

and there exists constant $\lambda>0$ such that for sufficiently large t

$$\sum_{i=1}\int_{t-\sigma_i}^{t-r}q_i(s+\sigma_i)ds\le\lambda<r+p.\ (5)$$

where $r\in[0,\sigma_n], q_i^+(t)=\max\{q_i(t),0\}, q^-(t)=\max\{-q_i(t),0\}$. Thenevery nonoscillatory solution of (1) and (2) tends to zero as $t\to\infty$.

Proof: Choose a positive integer N such that (5)holds for $t\ge t_N$ and $\sum_{k=N}^{\infty}b_k^+<r-p-\lambda$. let $x(t)$ be a nonoscillatory solution of (1) and (2).We will assume that $x(t)$ is eventually positive, the case where $x(t)$ is eventually negative is similar andomitted.Let $x(t)>0$ for $t\ge t_N$, By **Lemma 1,** we know that $x'(t)>0$, for $t\ge t_N$. Define

$$y(t)=r(t)x'(t)-\int_{t_N}^{t}p(s)x'(s)ds-\sum_{i=1}^{n}\int_{t-\sigma_i}^{t-r}q_i(s+\sigma_i)x(s)ds-H(t)-\sum_{t_N<t_k\le t}b_k^+x'(t_k).\ (6)$$

Then for $t\ne t_k, t\ne t_k+\sigma_i, i=1,2,\cdots,n; k=1,2,\cdots.$

$$y'(t) = -\sum_{i=1}^{n} q_i(t - r + \sigma_i)x(t - r) \quad (7)$$

and $y(t_k^+) - y(t_k) = (b_k - b_k^+)x'(t_k) \le 0, k = N, N+1, \cdots$.

Thus, $y(t)$ is nonincreasing on $[t_N, \infty)$. Set $L = \lim_{t\to\infty} y(t)$, we claim that $L \in R$.

Otherwise, $L = -\infty$, then $x'(t)$ must be unbounded by virtue of (H_1) and (4).Hence, it is possible to choose $t^* > t_N + \sigma_n$ such that $y(t^*) + H(t^*) < 0$ and

$x'(t^*) = \max\{x'(t) : t_N \le t \le t^*\}$. Thus, we have:

$$0 > y(t^*) + H(t^*)$$

$$\ge r(t^*)x'(t^*) \int_{t_N}^{t^*} p(s)x'(s)ds - \sum_{i=1} \int_{t^*-\sigma_i}^{t^*-r} q_i(s+\sigma_i)x(s)ds - \sum_{t_N < t_k \le t^*} b_k^+ x'(t_k)$$

$$\ge x'(t^*)(r - p - \lambda - \sum_{k=N}^{\infty} b_k^+) > 0,$$

which is a contradiction and so $L \in R$. By integrating both sides of (7) from t_N to t, we have:

$$\int_{t_N}^{t} \sum_{i=1}^{n} q_i(s - r - \sigma_i)x(s - r)ds = -\int_{t_N}^{t} y'(s)ds$$

$$= y(t_N^+) + \sum_{t_N < t_k \le t} [y(t_k^+) - y(t_k)] - y(t) < y(t_N^+) - L.$$

which, together with (4) implies that $x(t) \in L^1([t_N, \infty), R)$ and so $\lim_{t\to\infty} x(t) = 0$. The proof is then complete.

Lemma 2. Let $x(t)$ be an oscillatory solution of equation (1) and (2), suppose that there exists some $T \ge t_0$, if (H_4) hold, then $|x'(t_k)| \ge |x(t_k)|, |x'(t)| \ge |x(t)|$, where $t \in (t_k, t_{k+1}], k = 1, 2, \cdots$.

Proof: From the result of **Lemma 1**,we know that,if $x(t) > 0$ then,

$x'(t_k) > 0, x'(t) > 0$, where, $t \in (t_k, t_{k+1}]$. we will assume that when $x(t) > 0$ we have $x'(t_k) \ge x(t_k), x'(t) \ge x(t), t \in (t_k, t_{k+1}]$, the case $x(t)$ is negative is similar andomitted.From **Lemma 1**,we have $x'(t_k) > 0, x'(t) > 0, t \in (t_k, t_{k+1}]$, then the $x(t)$ is increased. We also obtained

$$[r(t)x(t)\exp[-\int_{t_j}^{t} \frac{p(s)}{r(s)}ds]]' < [r(t)x'(t)\exp[-\int_{t_j}^{t} \frac{p(s)}{r(s)}ds]]' < 0.$$

Hence, $r(t)x(t)\exp[-\int_{t_j}^{t} \frac{p(s)}{r(s)}ds]$ is decreasing on $(t_j, t_{j+1}]$ and

$$x(t_{j+1}) \le (b_j + 1)\frac{r(t_j)}{r(t_{j+1})} x(t_j)\exp[-\int_{t_j}^{t_{j+1}} \frac{p(s)}{r(s)}ds],$$

for all n ,we obtain

$$x(t_{j+n}) \le \prod_{k=0}^{n-1} (b_{j+k} + 1)\frac{r(t_j)}{r(t_{j+n})} x(t_j)\exp[-\int_{t_j}^{t_{j+n}} \frac{p(s)}{r(s)}ds].$$

By the condition (H_4) , we get $x(t_{j+n}) < x(t_j)$, which is a contraction.The proof is complete.

Theorem 2. Let $(H_1), (H_2)$ and (H_4) holds.Suppose that

$$\sum_{k=1}^{\infty} |b_k| < \infty, \quad (8)$$

and there exists positive constant λ and $r \in (0, \sigma_n]$ such that

$$\limsup_{t\to\infty} Q_1(t) + \limsup_{t\to\infty} Q_2(t) \le \lambda < r - 2p, (9)$$

$$\sum_{i=1}^{n} q_i(t+\sigma_i) \neq 0, \qquad for\, large \quad t, (10)$$

where

$$Q_1(t) = \sum_{i=1}^{n} \int_{t-\sigma_i}^{t} q_i(s+\sigma_i)ds, (11)$$

$$Q_2(t) = \sum_{i=1}^{n} \int_{t-r}^{t-\sigma_i} sgn(r-\sigma_i) q_i(s+\sigma_i)ds, (12)$$

Then every oscillatory solution (1)and (2)tends to zero as $t \to \infty$.

Proof:Let $x(t)$ be an oscillatory solution of (1) and (2). We first show that $x'(t)$ and $x(t)$ are bounded. Otherwise, $x'(t)$ is unbounded which implies that there exists positive integer N such that $\lim_{t\to\infty}\sup_{t_N \le s\le t} |x'(s)| = \infty$ and

$$\sup_{t_N+\sigma_n \le s \le t} |x'(s)| = \sup_{t_N \le s \le t} |x'(s)|, \qquad t \ge t_N + \sigma_n,$$

and

$$\sum_{k=N}^{\infty} |b_k| < \frac{r - 2|p| - \lambda}{2}. (13)$$

Set

$$y(t) = r(t)x'(t) - \int_{t_N}^{t} p(s)x'(s)ds - \sum_{i=1}^{n} \int_{t-\sigma_i}^{t-r} q_i(s+\sigma_i)x(s)ds - H(t) - \sum_{t_N < t_k \le t} b_k^+ x'(t_k),$$

where $b_k^+ = \max\{b_k, 0\}$. Then (7) holds. For $t \ge t_N + \sigma_n$, using**Lemma 2** we have

$$|y(t)| \ge r|x'(t)| - p|x'(t)| - \sum_{i=1}^{n} \int_{t-\sigma_i}^{t-r} q_i(s+\sigma_i)|x(s)|ds - |H(t)| - \sum_{t_N \le t_k \le t} |b_k x'(t_k)|$$

$$\ge (r-p)|x'(t)| - (Q_2(t) + \sum_{k=N}^{\infty} |b_k|) \sup_{t_N \le s \le t} |x'(s)| - |H(t)|,$$

which implies

$$\sup_{t_N+\sigma_n \le s \le t} |y(s)| \ge (r - p - \sup_{t_N \le s \le t} Q_2(t) - \sum_{k=N}^{\infty} |b_k|) \sup_{t_N \le s \le t} |x'(s)| - \sup_{t_N+\sigma_n \le s \le t} |H(s)|. (14)$$

Hence, $\limsup_{t\to\infty} |y(t)| = \infty$. From (7)we notice that $y'(t)$ is oscillatory, we see that there is a $\xi' \ge t_N + 2\sigma_n$ such that $|y(\xi')| = \sup_{t_N+\sigma_n \le s \le t} |y(s)|$ and $y'(\xi') = 0$. From (7) and (10), we get $x(\xi' - r) = 0$ by **Lemma 2.** We know that $x'(t)$ is oscillatory, hence, there is a $\xi > \xi' + r$ such that $x'(\xi - r) = 0$. Integrating both sides of (7) from $\xi - r$ to ξ, we obtain

$$y(\xi)=y(\xi-r)-\int_{\xi-r}^{\xi}\sum_{i=1}^{\xi}q_i(s-r+\sigma_i)x(s-r)d\quad s$$

$$=-\int_{t_N}^{\xi-r}p(s)x'(s)ds+\sum_{i=1}^{n}\int_{\xi-2r}^{\xi-r-\sigma_i}q_i(s+\sigma_i)x(s)ds+H(\xi-r)-\sum_{t_N\le t_k\le\xi-r}b_kx'(t_k)$$

$$-\int_{\xi-r}^{\xi}\sum_{i=1}^{n}q_i(s-r+\sigma_i)x(s-r)d\quad s$$

$$=\int_{t_N}^{\xi-r}p(s)x'(s)ds+H(\xi-r)-\sum_{i=1}^{n}\int_{\xi-r-\sigma_i}^{\xi-r}q_i(s+\sigma_i)x(s)ds-\sum_{t_N\le t_k<\xi-r}b_kx'(t_k),$$

which implies that

$$|y(\xi)|\le(p+Q_1(\xi-r)+\sum_{k=N}^{\infty}|b_k|)\sup_{t_N\le s\le\xi}|x'(s)|+|H(\xi-r)|.(15)$$

From (14) and (15),we have

$$-r+2p+(Q_1(\xi-r)+\sup_{t_N\le s\le\xi}Q_2(s))+2\sum_{k=N}^{\infty}|b_k|+(\sup_{t_N+\sigma_n\le s\le\xi}H(s)+|H(\xi-r)|)(\sup_{t_N\le s\le\xi}|x'(s)|)^{-1}\ge 0.$$

Let $\xi\to\infty$ and noting that $\lim_{\xi\to\infty}\sup_{t_N\le s\le\xi}|x'(s)|=\infty$, we have

$$-r+2p+\lambda+2\sum_{k=N}^{\infty}|b_k|\ge 0,$$

by (9), which contradicts (13) and so $x'(t)$ is bounded.By **Lemma 2**, we know that $x(t)$ is bounded.

Next we will prove that $\mu=\limsup_{t\to\infty}|x'(t)|=0$. To this end, we define

$$z(t)=r(t)x'(t)-\int_{t_N}^{t}p(s)x'(s)ds+\sum_{i=1}^{n}\int_{t-r}^{t-\sigma_i}q_i(s+\sigma_i)x(s)ds+H(t)+\sum_{t_k\ge t}b_kx'(t_k)\ (16),$$

then $z(t)$ is bounded and for sufficiently large t,

$$|z(t)|\ge r|x'(t)|-p|x'(t)|-Q_2(t)\sup_{t-\sigma_n\le s<t}|x'(s)|-|H(t)|-\sum_{t_k\ge t}|b_kx'(t_k)|,$$

thus, by (H_2) and (8)

$$\beta=\limsup_{t\to\infty}|z(t)|\ge(r-p)\mu-\mu\limsup_{t\to\infty}Q_2(t)$$

$$=\mu[r-p-\limsup_{t\to\infty}Q_2(t)].\qquad(17)$$

on the other hand, we have by (16) for $t\ne t_k,t\ne t_k+\sigma_i,k=1,2,\cdots.i=1,2,\cdots,$

$$z'(t)=-\sum_{i=1}^{n}q_i(t-r+\sigma_i)x(t-r)\ (18)$$

From this we see that $z'(t)$ is oscillatory. Hence there exists a sequence $\{\xi_m'\}$ such that $\lim_{m\to\infty}\xi_m'=\infty,\lim_{m\to\infty}|z(\xi_m')|=\beta,z'(\xi_m')=0.$ and $x(\xi_m'-r)=0,m=1,2,\cdots.$ similar to (15) we can obtain by (16)and (18),there is a $\xi_m>\xi_m'$, such that

$$|z(\xi_m)|\le(p+Q_1(\xi_m-r))\sup_{\xi_m-2\sigma_n\le s\le\xi_m}|x'(s)|+|H(\xi_m-r)|+\sum_{t_k\ge\xi_m-r}|b_kx(t_k)|,$$

which implies by (8) and (H_2) that

$$\beta \le \mu[p + \limsup_{t\to\infty} Q_1(t)].$$

This,together with (17),yields

$$\mu[-r + 2p + \limsup_{t\to\infty} Q_1(t) + \limsup_{t\to\infty} Q_2(t)] \ge 0.$$

Therefore,by (9)we have

$$\mu(-r + 2p + \lambda) \ge 0,$$

which implies $\mu = 0$ by (9)and so, $\lim_{t\to\infty} x'(t) = 0$. Hence we can obtain that $\lim_{t\to\infty} x(t) = 0$. Thus, the proof is completed.

III. References

[1] X.Liu,J.Shen,asymptotic behavior of solutions of impulsive neutral differential equations,J,Appl.Math.Lett. 12(1999) 51-58.
[2] A.Zhao,J.Yan,asymptotic behavior of solutions of impulsive delay differential equations,J.Math.Anal.Appl,201(1996), 943-954.
[3] L.Wen,Y.Chen,Razumikhin type theorems for functional differential equations with impulsive,Dynamics of continuous and impulsive systems,6(1999),389-400.

Simple Proof of Two Identities of Ramanujan

Bhaskar Srivastava

GJSFR Classification -F, A (FOR) 010599

Abstract-**Using two simple theta function identities we give simple proof of two identities of Ramanujan, one of which leads to the famous eight square theorem of Jacobi..**

Key words and phrases: Theta functions, q-hypergeometric series

I. INTRODUCTION

In a fragment published with his lost notebook [5, pp. 353-355], Ramanujan provided a list of twenty identities. These are immediately followed by another fragment on pages 356 and 357 with another list of twenty one identities. Most of these, but not all, can be found in Ramanujan's second notebook [4]. For more details? see Andrews and Berndt [1, pp. 395-396]. The numbering are those given by Ramanujan in the fragments. The two identities mentioned in the title of the paper are

$$\varphi^8(q) = 1 + 16\sum_{n=1}^{\infty} \frac{n^3 q^n}{1-(-q)^n}, \qquad (1.1)$$

writing $-q$ for q and then setting q^2 for q , we have

$$\varphi^8(-q^2) = 1 + 16\sum_{n=1}^{\infty} (-1)^n \frac{n^3 q^{2n}}{1-q^{2n}} \qquad (1.2)$$

and

$$\psi^2(q^4) = \sum_{n=0}^{\infty} (-1)^n \frac{q^{2n}}{1-q^{4n+2}}. \qquad (1.3)$$

The identity in (1.1) is Entry 18.2.3 [1, p. 397] which leads to Jacobi's famous formula

$$r_8(n) = 16(-1)^n \sum_{d|n} (-1)^d d^3,$$

which is Jacobi's eight square theorem. We shall, however, be proving the identity written in the form in (1.2).The second identity (1.3) is Entry 18.2.4 [1, p. 397].In the present paper I give simple proof of these two identities (1.2) and (1.3), using the following simple identities [3, eq. (8.1), p. 117] and [6, p. 480], respectively :

$$\cot^2 y - \cot^2 x + 8\sum_{n=1}^{\infty} \frac{nq^n}{1-q^n}(\cos 2nx - \cos 2ny)$$
$$= \theta_1'(0\,|\,q)^2 \frac{\theta_1(x-y\,|\,q)\theta_1(x+y\,|\,q)}{\theta_1^2(x\,|\,q)\theta_1^2(y\,|\,q)}, \qquad (1.4)$$

there is a slight misprint which I have corrected,
and

$$\frac{\theta_1'}{\theta_1}(z\,|\,q) = \cot z + 4\sum_{n=1}^{\infty} \frac{q^n}{1-q^n} \sin 2nz. \qquad (1.5)$$

Department of Mathematics and Astronomy Lucknow University Lucknow.

II. SOME BASIC RESULTS

We shall use the following standard q-notation,(q<1 :

$$(a;q^k)_n = (1-a)(1-aq^k)........(1-aq^{k(n-1)}), \quad n \geq 1$$

$$(a;q^k)_0 = 1. \tag{2.1}$$

Jacobi θ -function is defined as follows, see [6, p. 464]

$$\theta_1(z \mid q) = -iq^{\frac{1}{8}} \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{n(n+1)}{2}} e^{(2n+1)iz} \tag{2.2}$$

$$= 2q^{\frac{1}{8}} \sum_{n=0}^{\infty} (-1)^n q^{\frac{n(n+1)}{2}} \sin(2n+1)z \ , \tag{2.3}$$

where$q$= $e^{2\pi i\tau}$ and Im$(\tau) > 0.$.

The function $\theta_1(z \mid q)$ can also be expressed in terms of an infinite product

$$\theta_1(z \mid q) = 2q^{\frac{1}{8}} \sin z \ (q;q)_\infty (qe^{2iz};q)_\infty \ (qe^{-2iz};q)_\infty , \tag{2.4}$$

$$= iq^{\frac{1}{8}} e^{-iz} \ (q;q)_\infty (e^{2iz};q)_\infty \ (qe^{-2iz};q)_\infty . \tag{2.5}$$

$$\theta_1'(0 \mid q) = 2q^{\frac{1}{8}} \ (q;q)_\infty^3 . \tag{2.6}$$

Ramanujan's theta functions $\varphi(q)$ and $\psi(q)$ are defined as [1, p. 11]

$$\varphi(q) = \sum_{n=-\infty}^{\infty} q^{n^2} = (-q;q^2)_\infty^2 (q^2;q^2)_\infty = \frac{(-q;-q)_\infty}{(q;-q)_\infty} \quad , \quad |q| < 1 \tag{2.7}$$

and

$$\psi(q) = \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} \quad , \quad |q| < 1 . \tag{2.8}$$

We shall require the following identities of Ramanujan :

$$\varphi(-q) = \frac{(q;q)_\infty}{(-q;q)_\infty}, \ [2, \text{eq. } (22.4), \text{p. } 37] \tag{2.9}$$

$$\varphi(q)\varphi(-q) = \varphi^2(-q^2) , \text{Entry 25 (iii) } [2, \ \text{p. } 40] \tag{2.10}$$

and

$$\varphi^2(q) = 1 + 4\sum_{n=0}^{\infty}(-1)^n \frac{q^{2n+1}}{1-q^{2n+1}}. \text{ Entry 8 (i) } [2, \text{ p. } 114]. \qquad (2.11)$$

Adding (v) and (vi) of Entry 25 [2, p. 40], we have

$$\varphi^2(q) - \varphi^2(q^2) = 4q\,\psi^2(q^4) \qquad (2.12)$$

Putting $z = \frac{\pi}{4}$ in (1.5), we have

$$\frac{\theta_1'}{\theta_1}\left(\tfrac{\pi}{4} \mid q\right) = 1 + 4\sum_{n=1}^{\infty}\frac{q^n}{1-q^n}\sin\frac{n\pi}{2}$$

$$= 1 + 4\sum_{n=0}^{\infty}(-1)^n \frac{q^{2n+1}}{1-q^{2n+1}}$$

$$= \varphi^2(q) \text{ by (2.11).} \qquad (2.13)$$

III. PROOF OF (1.2)

Differentiating (1.4) partially with respect to x and then putting $y = x$, we have

$$2\cot x \operatorname{cosec}^2 x - 16\sum_{n=1}^{\infty}\frac{n^2 q^n}{1-q^n}\sin 2nx = \frac{\theta_1'(0 \mid q)^3\,\theta_1(2x \mid q)}{\theta_1^4(x \mid q)}.$$

Differentiating again the above expression partially with respect to x and then x $z = \frac{\pi}{4}$ putting we finally get

$$1 + 16\sum_{n=1}^{\infty}(-1)^n \frac{n^3 q^{2n}}{1-q^{2n}} = \theta_1'(0 \mid q)^3 \frac{\theta_1(\frac{\pi}{2} \mid q)}{4\theta_1^4(\frac{\pi}{4} \mid q)} \frac{\theta_1'}{\theta_1}\left(\tfrac{\pi}{4} \mid q\right)$$

$$= \frac{(q;q)_\infty^6(-q;q)_\infty^2}{(-q^2;q^2)_\infty^4}\varphi^2(q), \quad \text{by} \qquad (2.14)$$

$$= \frac{(q;q)_\infty^2(q^2;q^2)_\infty^4}{(-q;q)_\infty^2(-q^2;q^2)_\infty^4}\varphi^2(q)$$

$$= \varphi^2(-q)\varphi^4(-q^2)\varphi^2(q), \quad \text{by (2.9)}$$

$$= \varphi^8(-q^2), \quad \text{by (2.10)}$$

which proves (1.2).

IV. PROOF OF (1.3)

Writing (1.5) as

$$\frac{\theta_1^{'}}{\theta_1}(z\,|\,q) = \cot z + 4\sum_{n=1}^{\infty}\frac{q^n(1+q^n)}{1-q^{2n}}\sin 2nz$$

$$= \cot z + 4\sum_{n=1}^{\infty}\left[\frac{q^n}{1-q^{2n}} + \frac{q^{2n}}{1-q^{2n}}\right]\sin 2nz \qquad (4.1)$$

and setting $z=\frac{\pi}{4}$ in (4.1), we obtain

$$\frac{\theta_1^{'}}{\theta_1}(\tfrac{\pi}{4}\,|\,q) = 1 + 4q\sum_{n=0}^{\infty}(-1)^n\frac{q^{2n}}{1-q^{4n+2}} + 4\sum_{n=0}^{\infty}(-1)^n\frac{q^{4n+2}}{1-q^{4n+2}}. \qquad (4.2)$$

Or

$$4q\sum_{n=0}^{\infty}(-1)^n\frac{q^{2n}}{1-q^{4n+2}} = \frac{\theta_1^{'}}{\theta_1}(\tfrac{\pi}{4}\,|\,q) - 4\sum_{n=0}^{\infty}(-1)^n\frac{q^{4n+2}}{1-q^{4n+2}} - 1$$

$$= \varphi^2(q) - \varphi^2(q^2),$$

here we have used (2.11) and (2.13).
Finally using (2.12) on the right hand side of (4.2), we have (1.3).

V. REFERENCES

1. G.E. Andrews and B.C. Berndt, *Ramanujan's Lost Notebook, Part* I, Springer-Verlag, New York, 2005.
2. B.C. Berndt, *Ramanujan's Notebooks Part* III, Springer- Verlag, New York, 1991.
3. Z-G Liu, Some Eisenstein series identities related to modular equation of the seventh order, *Pacific J. Math*., **209** (2003), 103-130.
4. S. Ramanujan, *Notebooks (2 volumes)*, Tata Institute of Fundamental research, Bombay, 1957.
5. S. Ramanujan, *The Lost Notebook and other unpublished papers*, Narosa, New Delhi, 1988.
6. E.T. Whittaker and G.N.Watson, *A Course of Modern Anaysis*, 4th ed., Cambridge University Press, Cambridge, 1966.

Trends In Precipitation Features As An Index of Climate Change In The Guinea Savanna Ecological Zone of Nigeria: Its Implications on Crop Production

GJSFR Classification - H (FOR) 040104, 050101

Olanrewaju Rhoda Mojisola

***Abstract*-This study examined the risk level of some major food crops of the guinea savanna ecological zone of Nigeria to vagaries in rainfall features. The period of study spanned between 1956 – 2005. The method of Walter (1967) as modified by Olaniran (1988) was adopted in calculating rainfall onset, cessation and Length of Growing Season (LGS) and partitioned on pentade basis to provide a quantitative summary of variations observed. Correlation and regression analyses were employed. The result confirmed a change in the climate of Kwara State towards aridity which manifest in high frequency of late onset, early cessation and reduction in LGS. Also the result showed a positive relationship which is significant at 1% level of confidence for maize and rice and significant at 5% level of confident for yam but negative relationship which is not significant is established between LGS and Sorghum and cassava. Farmers were advised on how to adapt to the changes climate is bringing to the agricultural industry of the ecological zone**

I. Introduction

Nigeria is presently not self-sufficient in the production of food crops. Agricultural production is a mirror image of population growth. The dwindling in agricultural production and shrinkages in the belts of food crops in Nigeria have been noticed by Olanrewaju (2003) and Adefolalu (2004) among others, For instance, Olanrewaju (2003) noticed that the climatic situation in Kwara State is no longer supporting the growth of melon and therefore suggested the shift of melon cultivation to the southern guinea ecological zone of Nigeria where it may thrive better. Similarly Adefolalu (2004) associated the decline in production of groundnut (to 30% of the pre-1960 production) with climate change.Rainfall has been described as the most critical agro meteorological factors of agricultural production in the tropics (Ayoade 2004). Important rainfall features crucial for agricultural activities in the tropics are the rainfall onsets, cessation and the length of rainy season (Adefolalu 2006) which is synonymous with the Length of Growing Season (LGS). This represents the time lags between the start and the end of the growing season. The amount of rainfall received is not as important as its spread.To solve the problem of food shortage in Nigeria, facts are needed on risk level of agricultural activities to variations in rainfall features enumerated above. Thus this present study focuses on producing such facts and risk levels for quick intervention that will bring a lasting solution to agricultural production in Kwara State and in Nigeria as a whole.

Department of Geography University of Ilorin, Kwara State Nigeria.
E-mail: mojisola_lanrewaju@yahoo.com

II. Rainfall Features and Agriculture.

Efficient crop production in the tropics is equated with the onset of rain and length of rainy season (Oyegade 2004). This is because, onset, cessation and duration of the rainy season form important components of moisture resources status for determining the production potential of various crops (Olanrewaju 2006). According to Fortunata (2003), the uncertainties which characterize the start and the lengths of the growing season are giving a lot of concern to farmers in Uganda. Similarly Odofin et al(2003) linked up reduced agricultural productivity and low income to the Nigeria farmers with fluctuation in moisture pattern .Significant reduction in the LGS results in a decline in crop production in Southern Nigeria (Owolabi and Adebayo 2003). Thus, this present study focuses rainfall features of onset, cessation and LGS on food crop production in the guinea savanna ecological zone of Nigeria.

III. The Study Area.

The study area is kwara State, Nigeria located within latitude 8° and 10° 15’ north of the equator and between longitude 2° 45’ and 6° 15’ east of the Greenwich Meridian. It shares a common boundary with Niger, Kogi, Oyo, Ekiti, Oshun States and Republic of Benin (Figure 1)

Figure 1: A map of the Study Area.

Source: State Survey Ilorin Kwara State, Nigeria.

The climate of Kwara State exhibits a marked wet and dry seasons. The dry season spans between November to March while wet season last from April to October. River Niger and its tributaries drain the area. The Vegetation is savanna. These above characteristics make farming the major occupation of the people with special focus on food crop production

IV. METHODOLOGY.

Rainfall data was collected on monthly basis from the archives of Nigerian Meteorological Agency, Ilorin International Airport Kwara State and from Kwara State Agricultural Development Project (KADP) Ilorin for the period of fifty years which spanned between 1956 - 2005. Mean annual rainfall was calculated from the above. Annual crop yield data were sourced from (KADP) Ilorin for the period of fifteen years which spread from1991 when KADP was created to year 2005. Rainfall onset, cessation and LGS were also calculated from monthly values using Walter (1967) method as modified by Olaniran (1988). The formula is stated below.

Days in the month X
(51-accumulated rainfall total of previous month)
Total rainfall for the month

The growing season begins when a location has received an accumulated rainfall of 51mm (Walter 1967). However, if this planting data is followed by a prolonged dry spell, such planting data is disregard and new planting data is re-established using Walter's method (Olaniran 1988). To calculate the rainfall cessation, similar formula is applied but monthly rainfall value is accumulated from December backward. The month that the accumulated total exceeds 51mm becomes the end of the rainy season. The time lags between the onset and cessation marked the length of growing season (LGS).Different onsets and cessations for guinea savanna ecosystem as put forward by Olanrewaju (2003) were identified. For rainfall onset, Olanrewaju (2003) described the periods between early and mid April as normal onset, March downward as early onset and late April upward as late onset. Last week in October is described as normal rainfall cessation, September to third week in October as early cessation and the period of November to December as late cessation.Both data (crop and rainfall features) were partitioned on pentade (five years interval) basis to provide a quantitative summary of variations observed. Ten pentade identified and designated first to tenth were the periods of 1956 - 1960, 1961 - 1965, 1966 - 1970, 1971 - 1975,1976 -1980, 1981 - 1985, 1986 - 1990, 1991-1995, 1996-2000 and 2001-2005 respectively. Crop data and LGS were subject to correlation and simple regression analyses to show the control of these features on each crop. The result of the above analyses brought out facts on risk level of crop production to variation in rainfall features in Kwara State.

V. RESULTS AND DISCUSSION

a) Pattern of Rainfall Features.

Table 1 below displays variation observed in rainfall (onset, cessation and LGS) for the study period

.

Year	*Start*	*End*	*LGS*
1956	8/ 3	22/ 10	222
1957	1/ 3	29/ 11	273
1958	22/ 3	24/ 11	247
1959	13/ 3	6/ 10	207
1960	25/ 3	4/ 10	193
1961	8/4	18/9	161
1962	5/4	21/11	230
1963	11/3	23/10	226
1964	**1/ 4**	11/2	254

1965	1/ 4	23/10	205
1966	¾	11/10	191
1967	13/4	10/12	270
1968	2/4	16/10	197
1969	6/4	13/11	221
1970	3/3	5/10	188
1971	25/4	22/10	180
1972	27/4	4/10	160
1973	1/7	22/9	83
1974	29/4	17/10	171
1975	7/4	11/ 10	187
1976	14/3	30/10	230
1977	16/4	22/10	189
1978	26/3	25/10	210
1979	17/3	24/10	222
1980	1/5	15/10	177
1981	13/3	26/10	216
1982	6/4	27/10	203

1983	11/4	14/10	169
1984	18/3	28/9	210
1985	30/3	25/10	182
1986	14/2	16/10	253
1987	12/5	29/9	188
1988	15/3	18/10	197
1989	2/3	24/10	230
1990	19/4	23/10	188
1991	10/ 3	3/ 11	227
1992	2/ 5	8/ 10	185
1993	4/ 5	22/ 10	157
1994	4/ 5	26/ 10	201
1995	4/4	17/ 11	227
1996	5/5	6/10	154
1997	29/5	22/10	146
1998	8/4	23/10	197
1999	16/3	30/10	197
2000	1/5	29/9	151
2001	21/4	3/9	151
2002	16/6	9/10	115

2003	10/4	8/10	181
2004	2/4	12/0	180
2005	2/3	6/10	218

Source: Author's Computation 2010.

Early onset of rain occurred between 1956 – 1960,1963, 1970, 1976, 1978 -1979,1981, 1984-1986, 1988- 1989, 1991, 1999 and 2005. The periods of 1971 – 1974, 1977, 1980, 1987, 1990, 1992 – 1994, 1996 – 1997 and 2000 – 2002 that witnessed delayed onset also recorded short LGS with an exception of 1995 that observed late cessation of rain to compensate for the late onset. The least LGS of 83 days was observed in year 1973 while the year 1957 witnessed the longest LGS of 273 da

b) Various Rainfall Onsets.

The frequency of different onsets of rain during each pentade of the study period is displayed in table 2 below. The frequency of early onsets fluctuates between zero and five

Table 2: Frequency of different Onsets of Rain for the Period of 1956 – 2005.

	Different Pentades of the Period 1956 – 2005.										
Onsets of rain	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	
	1956-1960	1961-1965	1966-1970	1971-1975	1976-1980	1981-1985	1986-1990	1991-1995	1996-2000	2001-2005	Total
Early	5	1	1	0	3	3	3	1	1	1	19
Normal	0	4	4	1	0	2	0	1	1	2	5
Late	0	0	0	4	2	0	2	3	3	2	16

Source: Author's Field work 2010

The frequency of early onset was very high during the first pentade (1956-1960) but declined consistently during the 2nd and 3rd pentades (1961 – 1965, 1966 – 1970) and reached its minimum between 1971 – 1975 that represents the 4th pentade. The early onset of rain rose consistently during the 5th, 6th and 7th pentades but declined and remains consistently low throughout the last pentade. Fluctuation in the frequency of both normal and late onset also exists with the latter on the higher side. Delayed onset characterized the latter part of the study period while the early pentade experienced none. The period of 1971 – 1975 (4th pentade) was worse affected. Onset was normal between 1961 – 1965, 1966 – 1970 which represent the 2nd and 3rd pentade.

c) Pattern of Rainfall Cessations.

The Pattern of the frequency of various cessations shown on table 3 below.

Table 3: Frequency of Different Cessations of Rain 1956 – 2005.Different Pentades of the Period 1956 – 2005.

Cessation	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	
	1956-1960	1961-1965	1966-1970	1971-1975	1976-1980	1981-1985	1986-1990	1991-1995	1996-2000	2001-2005	Total
Early	2	1	3	4	1	2	2	1	2	5	23
Normal	1	2	0	1	4	3	3	2	3	0	19
Late	2	2	2	0	0	0	0	2	0	0	8

Source: Author's Field work 2010.

Fluctuation exists in the distribution of early cessation between the 1st (1956 – 1960) pentade and the 7th (1986 – 1990) pentade. Thereafter a geometric progression in the frequency of cessation is observed from the 8th through the 10th pentades. The frequency of late onset of rain was low consistently during the first three pentades. The remaining pentades except one (8th pentade) did not record late onset of rain. The occurrence of normal onset of rain did not take any pattern. Normal onset was low during the period 1956 – 1960 and 1970 – 1975 while the 3rd and 10th pentades observed normal rainfall cessation as the frequency recorded was zero. The period 1976 – 1980 (5th) pentade was characterized with normal cessation. In summary close to half of the period under study (23 years) experienced early rainfall cessation, 19 years enjoyed normal rainfall cessation while 8 years out of 50 years witnessed late cessation of rain.

d) *Comparison of Different Onsets and Cessations of Rainfall.*

Different rainfall onsets and cessations are compared in figures 2 – 6 below.

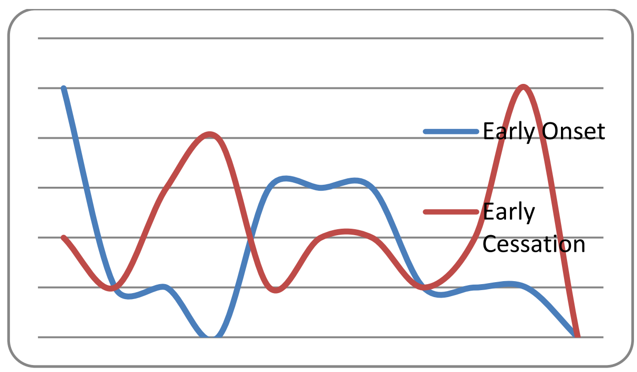


Figure 2: Frequency of early onset and early cessations of rainfall

Source ; Author's fieldwork 2010

The period of 1956 – 1960 recorded the highest frequency of early onset. During this pentade, the occurrence of early onset is greater than that of early cessation. For instance while the frequency of early onset was five that of early cessation was two. This represents the wettest period of the ten pentade considered. The frequency of early onset declined with early cessation during the second pentade (1961 – 1965). The condition described above was reversed during the periods of 1966 – 1970 and 1971 – 1975 (3rd and 4th pentade) with high frequency of early cessation but low occurrence of early onsets. Early onset reached its barest minimum of zero during the 4th pentade. (I.e. the period of five years 1971 – 1975 was characterized by delayed onset of rain) This period could be described as the driest pentade of the periods under study. Thereafter a milder scenario of the first two pentade was re-established during the periods 1976 – 1980, 1981 – 1985 and 1986 – 1990 depicting a period of lesser wetness. However, the period of 1991 –

1995, 1996 – 2000 and 2001 – 2005 which represent the last three pentade experienced a sporadic increase in the occurrence of early cessation with consistently low early onsets of rain.

e) Frequency of Normal onsets and Normal Cessations of Rain Between 1956 – 2005.

Figure 3 below showed the pattern of fluctuation that exists between normal onsets and normal cessation of rain in the ten pentade of the period 1956 – 2005.

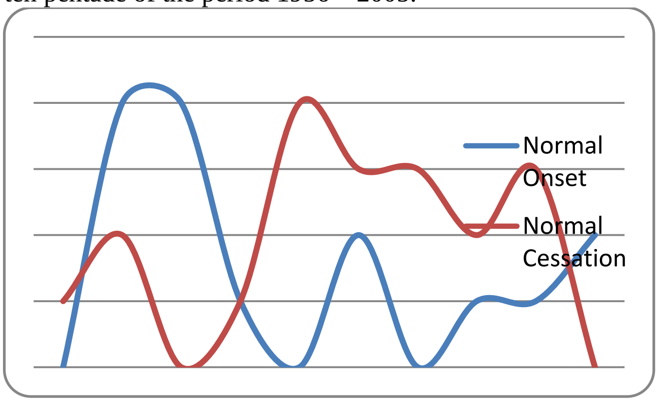


Figure 3: Normal Onset and Normal Cessation of Rainfall

Source: Author's Fieldwork 2010.

The frequency of normal onset of rain was zero during the 1st pentade. Thereafter, normal onset became higher than the cessation during the second and third pentade but the reverse is the case from the periods of 5th to 9th pentade. The last pentade witnessed a rise in normal onset with a sharp decline in the normal cessation.

f) Frequency of Late Onset and Cessation between 1956 - 2005.

Figure 4 reflects the pattern of late onset with its corresponding late cessation of rain between 1956 and 2005 in the guinea savanna ecological zone of Nigeria.

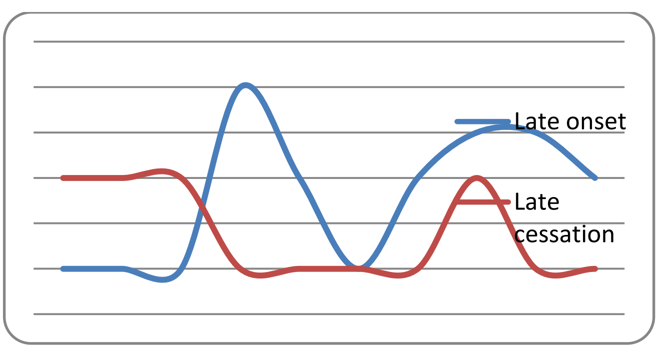


Figure 4: Late Onset and Late Cessation of Rainfall

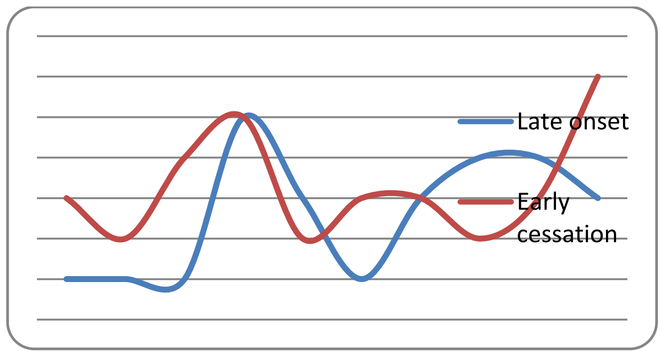
Source: Author's Fieldwork 2010.

The first three pentades enjoyed a consistently high occurrence of late cessation (rainfall extended beyond the normal date) with low late onset. This scenario represents the best condition for crop production in the tropic where agriculture is rain fed. The pattern changed during the next pentades of 1971-75, and 1976-80 when there were delayed onsets without corresponding extension of cessation dates. Situation described above could bring crop survival to crisis level. The pattern of late onset and late cessation fluctuate together during the period 1986-90, 1991-1995, 1996-2000 and 2001-2005 with higher occurrences of late onset over late cessation of rain. Delayed in onset is compensated for by delayed in rainfall cessation. This condition might be suitable for the production of yam crop. Yam seed can withstand late onset of rain by lying dormant for some months and sprout at the onset of rain.

g) Late Onset and Early Cessation

Figure 5 showed the pattern of various distributions of late onset and early cessation of rain for the ten pentades of between 1956 and 2005.

Figure 5: Late Onset and Early Cessation of Rainfall.



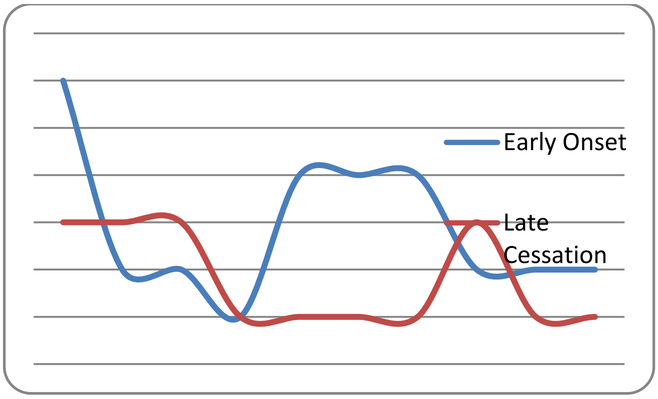
Source: Author's Fieldwork 2010.

This represents the shortest length of rainy season. The period of 1971-1975 has the highest frequency of both late onset and early cessation. The implication is that this pentade has the shortest length of rainy season among pentade considered. This further corroborates the earlier findings and further supports the assertion of Oguntoyinbo and Richard who recognized this as period of low average rainfall in sub Sahara West Africa. Late onset is a mirror image of early cessation during the period 1981-1985 through 1996-2000. High frequency of late cessation was compensated for during the period 1981-1985 and vice versa between the periods of 1986-1990 and 1996-2000. Highest frequency of late cessation which was not well compensated for was experienced during the last pentade. This makes this pentade drought prone but with lesser severity than the one experienced during the period between 1971-1975.

h) *Early Onset and Late Cessation of Rain During the Period of 1956 – 2005s.*

The pattern of early onset and late cessation is showed in figure 6 below.

Figure 6: Early Onset and Late Cessation of Rainfall



Source: Author Fieldwork 2010.

This represent the period of the longest LGS. Early onset is a perfect mirror image of late cessation throughout the period of study except during the 1st pentade. The pattern of fluctuation of the two rainfall features is the same but opposite each other i.e. as early onset increases late cessation decreases and vice-versa. The implication of this is that condition of long LGS is becoming difficult to attain in the study area for instance only one pentade (1956 – 1960) out of the ten considered attained this status and to a certain extent. The scenario of long LGS as described during the 1st pentade declined progressively throughout periods of study.The comparison of different onsets and cessations discussed above reflects a progressive decline in rainfall onsets and a sporadic increase in the frequency of rainfall cessation. This is an indication of a change in climate towards aridity in the guinea savanna ecological zone of Nigeria.

i) Different LGS of the Ten Pentade within the Period 1956 - 2005.

The frequency of different lengths of growing season is displayed in table 4 below

.

Table 4: Pattern of the Different LGS.

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	
LGS (Days)	1956-1960	1961-1965	1966-1970	1971-1975	1976-1980	1981-1985	1986-1990	1991-1995	1996-2000	2001-2005	Total
100-150(Short)	0	0	0	2	0	0	0	0	1	2	5
151-200(Medium)	1	1	3	3	2	2	3	2	4	2	23
201-250(Long)	4	4	2	0	3	3	2	3	0	1	22

Source: Author's Field work 2010.

The period of 1956 – 1960 and 1961 – 1965 (1st and 2nd Pentades) which enjoyed the highest frequency of long LGS, did not experience short LGS. Followed the above pattern closely were the periods 1976 – 1980, 1981 – 1985(5th and 6thpentades) and 1991 – 1995(8th pentade) respectively. Pentades that did not record long LGS were the 4th(1971 -1975) and the 9th (1996 – 2000) while the frequency of short LGS progressed with a decline in long LGS through the last two pentades. The above findings typified shrinkages in the LGS over the study area thus confirming the assertion of Olanrewaju (2003) and Adefolalu (2004) that the climate of Kwara State has changed and the trend is towards aridity.

Table 5 shows variation in yields of some selected crops in Kwara State Nigeria for the period of fifteen years (1991 – 2005).

j) Implications on Crop Production.

Table 5 shows variation in crop yields for kwara state during the study period.

Table 5: Crop Yield (tons '000) in Kwara State between 1991 – 2005

Year	Maize	Sorghum	Millet	Rice	Cowpea	Yam	Cassava
1991	2.22	1.21	0.92	2.08	0.65	11.82	10.10
1992	1.10	1.10	1.20	0.74	0.40	11.16	10.73
1993	1.19	1.21	1.41	1.94	0.74	11.18	9.43
1994	1.55	1.38	1.07	2.07	0.67	12.96	10.27
1995	1.80	1.20	1.41	2.60	0.76	13.40	11.17
1996	1.19	1.11	1.41	1.60	0.76	13.40	11.16
1997	0.88	0.29	1.10	1.62	0.64	11.76	8.78
1998	1.14	1.30	2.08	1.84	1.15	11.98	12.90
1999	1.25	1.62	1.98	2.70	0.94	11.0	13.40
2000	1.20	1.62	1.88	2.62	0.94	11.86	12.45
2001	1.16	1.63	1.60	1.43	0.42	10.79	12.96
2002	1.39	1.70	1.03	2.00	0.69	10.73	12.94
2003	1.33	1.27	1.04	2.28	0.72	10.86	12.56
2004	1.33	1.42	1.75	2.30	0.46	11.70	12.21
2005	1.55	1.27	2.30	2.36	0.47	11.93	12.46

Source: (1) Kwara State Agricultural Development Project 2010.

A drop in yield is observed during the years that witnessed short LGS and delayed rainfall onset. Examples of such years are 1992, 1997, 2001 – 2002.

Table 6: Results of Correlation between Selected Crops and LGS.

Rainfall Feature	Maize	Sorghum	Millet	Rice	Cowpea	Yam	Cassava
LGS	0.72***	-0.11	0.15	0.68***	0.20	0.64**	-0.16

*** Significant at 1% Confidence level.
** Significant at 5% Confidence level.
Source: Author's Computation 2010.

All the selected food crops exhibit a positive relationship with LGS except Sorghum and Cassava. The strength of this relationship varies. Strong positive relationship which is significant at 1% confidence level exists between LGS and Maize and rice but significant at 5% confidence level with yam. Weak positive relationship which is not significant exists between LGS and millet and cowpea while sorghum and cassava displayed a weak negative relationship which is not significant as well. The implication is that short length of rainy season put production of crops such as maize, rice and yam at a risk in the guinea savanna ecological zone of Nigeria. However it appears cassava, millet, sorghum and cowpea can stand short LGS.

The result of regression analysis showed that LGS alone explained 51%, 47% and 41% of the factors that affect maize, rice and yam yield respectively(see table 7 below).

Table 7; Model Summary of Regression Analysis.

CROPS	R	R - Square	Adjusted R-Square	Std. Error of the Estimate.
Maize	717[a]	0.518	477	23989
Rice	685[a]	0.469	429	41392
Yam	639[a]	0.408	363	59028

Source; Author's Computation 2010.

VI. Summary and Conclusion.

The change in climate of Kwara State is toward aridity. This manifests in high frequency of late onset of rain, early cessation and short LGS which in turn account for reduction in food crop yield.Thus farmers in this ecological zone are presented with the following suggestions as means of adaptation to the impact of climate change in the area. Focus more on the cultivation of drought resistant crops such as cassava, sorghum, millet and cowpea.Limit the cultivation of yam, rice and maize to fadama areas where they can get enough moisture to thrive.

VII. References

1) Adefolalu, D.O. (2004),"Climate Change: Part 1 – Pre and Post – Glaciations Period (10,000 – 1,000 BC). A paper Presented at the NMS Conference on climate and Water Resources in the 21st Century: Change for food security and health held at University of Agriculture, Abeokuta.
2) Fortunate, L. (2003), "Impacts of climate change on start, length and dry spells of growing season in Uganda". A paper presented at the NMS regional conference on climate change and food sustainability in the 21st century held at Oshodi Lagos.
3) Odofin, J.J. etal, (2003) "Socio-economic Implication of Change on Agriculture Production" (Ditto as above).
4) Olaniran, O.J (1988), "Climate and the Planning of Agricultural land use in Nigeria: The NRBDA Area as a Case Study. J.Agr.Met 43(4): 2855 – 294.
5) Olanrewaju, R.M (2003), "The Preliminary Study of Climate Variable on the Growth of Melon in Kwara State". Journal of the Nigerian Meteorological Society. Vol.5 Number 1, Pp 1 – 7.(2004),
6) "The Preliminary Assessment of Climate and incidence of Maize Downy Mildew in Ilorin and its

Environ". Ilorin Journal of Business and Social Sciences: Vol. 9 Number 1 and 2, Pp. 19 – 28.(2006), "Implication of the late Onset of Rains in a Coastal Ecological Area: The case Study of Lagos and its Environ." Geo-Studies Forum an International Journal of Environmental and Policy Issues: A publication of the Department of Geography University of Ilorin. Volume 3, number 1 & 2, Pp 83 – 90.

7) Owolabi, J.T and Adebayo, W. (2003), "The Impact and Changes in the length of growing season on crop yield in Ekiti State". A Paper presented at the NMS Regional Conference on Climate Change and Food Sustainability in the 21st Century at Oshodi, Nigeria.

8) Walter, M.W (1967), "The length of the Rainy Season in Nigeria.Nigeria Geog.Journal 10. 123 - 12

Pyrimidine Amines As Inhibitors For The Acid Corrosion of Mild Steel

GJSFR Classification - E (FOR) 861103

K.Parameswari[1], S.Chitra[2], J.Rajpriya[3], S.Kavitha[4] ,A.Selvaraj[5]

***Abstract*-Three Pyrimidine amines (PA1, PA2 & PA3) have been synthesized and investigated for corrosion inhibition activity towards mild steel in 1M H_2SO_4. Weight loss, potentiodynamic polarisation and electrochemical impedance spectral techniques have been used. Thermodynamic parameters [$\Delta G°_{ads}$, $\Delta H°$ and $\Delta S°$] have been evaluated. The compounds act through physisorption mechanism on the mild steel surface and obeyed Langmuir adsorption isotherm. Polarisation studies showed that the compounds behave as cathodic type.**

Keywords: Pyrimidine amines, Mild steel, Corrosion inhibition, Potentiodynamic polarisation, Electrochemical impedance, Adsorption.

I. Introduction

The corrosion of iron and steel and methods of reducing the rate at which corrosion of these materials takes place have been the subject of much investigation. A large number of organic compounds particularly those containing nitrogen, oxygen, or sulphur in a conjugated system are known to be applied as inhibitors to control acid corrosion of iron and steel. Schram et al have systematically investigated the efficiency of large number of aliphatic and aromatic amines in inhibiting the corrosion of mild steel by acid [1]. These inhibitors form readily ionizable onium compounds and get adsorbed on the cathodic areas and reduced the corrosion rate. In this work an attempt has been made to synthesize three pyrimidine amines and to evaluate their corrosion inhibition characteristics for mild steel in 1M H_2SO_4.

II. Experimental Inhibitors

The pyrimidine amines have been synthesized by the reported procedure[2] . They were purified by crystallisation.The compounds were characterised by IR spectra.

III. Weight Loss Measurements

Mild steel containing 0.08% C, 0.36% Mn, 0.12% Si, 0.02% P, 0.02% Cr and the remainder Fe, were used in all experiments. For weight loss method, rectangular specimens having size 5cm x 2cm x 0.05cmwere used. The specimens were washed, dried and polished successively using emery sheets of 1/0, 2/0, 3/0 and 4/0 grades to remove adhering impurities, degreased with trichloroethyleneand dried.

Pre-weighedmild steel specimen (in triplicate) were suspended for 3hrs in 1M H_2SO_4 with and without the inhibitors in different concentrations.After 3hrs the specimens were removed from the test solution, washed with running water, dried well and then weighed.The percentage inhibition efficiency for various concentration of the inhibitors were calculatedas,

$$\text{Efficiency of Inhibitor} = \frac{(\text{Weight loss without inhibitor}) - (\text{Weight loss with inhibitor})}{\text{Weight loss without inhibitor}} \text{X } 100$$

The effect of temperature on the inhibitory action of the inhibitor was determined by weight loss method at 1mM concentration at different temperatures (303K, 313K, 323K and 333K) after immersing for 1hr.

IV. Polarization and Impedance Measurements

Electrochemical measurements were carried out using potentiostat (1280 B Solartron, U.K).Measurements were carried out in a glass cell with a capacity of 100ml. A platinum electrode and a saturated calomel electrode (SCE) were used as counter electrode and reference electrode respectively. Mild steel rod having the same composition, embedded in Teflon with an exposed area of 0.785cm^2 was used as working electrode. The polarisation measurements were performed at a scan rate of 1mv/sec in the potential range of -200mV to +200mV vs corrosion potential of the working electrode measured against SCE. The impedance measurements were done in the frequency range 10kHz to 0.01 Hz with a signal amplitude of 10mV at OCP after a stabilisation period of 30 minutes.The AC impedance measurements are shown as Nyquist plots and polarization data as Tafel plots.The I_{corr}, E_{corr}, R_t and C_{dl} values were obtained from the data using the corresponding "corr view" and "Z view" softwares.

V. Results and Discussion Weight Loss Studies

The results of weight loss measurements at 30°C, after 3hours of immersion of the mild steel specimen for compounds PA1 to PA3 in 1M H_2SO_4 have been reported in table-1. It is evident from the table-1 that all the compounds showed > 80% efficiency at 1mM concentration. The corrosion rate decreases and inhibition efficiency increases with increase in concentration of inhibitors.The surface coverage values θ increase with concentration. Therefore it is evident that the corrosion inhibition is due to the

1,2,3,4PSGR Krishnammal College for Women, Coimbatore, Tamil Nadu, India.
5CBM College, Coimbatore, Tamil Nadu, India.
**Email-parampps@yahoo.co.in*

adsorption of inhibitors on the metal surface forming a barrier film protecting the metal from corrosion.

VI. EFFECT OF TEMPERATURE

The influence of temperature at the optimum concentration (1mM) on inhibition efficiency was studied by weight loss method and the data are given in table-2 . It is evident that the inhibition efficiency decreases with increase in temperature which may be attributed to an increase in the solubility of the protective adsorbed film and of any reaction products precipitated on the surface of the metal that inhibit the reaction. Ergun et al[3] has attributed the decrease in inhibition efficiency with rise in temperature to be due to an enhanced effect of temperature on the dissolution process of steel in acidic media and or the partial desorption of the inhibitor from the metal surface.In an acidic solution, the corrosion rate is related to the temperature by the Arrhenius equation,

$$LogCR = \frac{-Ea}{2.303RT} + \log A$$

Activation energy (Ea) and thermodynamic data, such as change in free energy ($\Delta G°_{ads}$), enthalpy ($\Delta H°$) and entropy ($\Delta S°$) for mild steel in 1M H_2SO_4 in the presence and in the absence of the inhibitors were calculated and are listed in table-3. The activation energy at different concentration of the inhibitor in H_2SO_4 is calculated by plotting Log CR Vs 1/T [Fig-1]. Linear plots were obtained for all the inhibitors. It is clear from the table-3 that Ea values in the presence of the additives are higher than that in the absence. The higher activation energies imply a slow reaction and that the reaction is very sensitive to temperature. Similar results have been reported by Ebenso et al in the corrosion inhibition of steel by Alizarin yellow [4].The increase in activation energy in the presence of inhibitor signifies physical adsorption[5].Enthalpy and entropy of activation $\Delta H°$ and $\Delta S°$ were obtained by applying the transition state equation:

$$CR = \frac{RT}{Nh} \exp\left[\frac{\Delta S^o}{R}\right] \exp\left[\frac{-\Delta H^o}{RT}\right]$$

An endothermic adsorption process ($\Delta H°>0$) is attributed to chemisorption, an exothermic adsorption ($\Delta H°<0$) may involve either physisorption or chemisorption or a mixture of both[6]. In the present study negative $\Delta H°$ values are obtained indicating physisorption. The $\Delta S°$values presented in table-3 are negative which means that the process of adsorption is accompanied by a decrease in entropy.The negative value of $\Delta S°$ decreases in the presence of inhibitor as compared to that in their absence i.e there is an increase in entropy in the presence of inhibitor. This indicates that the adsorption process is spontaneous and there is an increase in randomness or disorder on the surface due to the adsorption process and also desorption at higher temperature. $\Delta G°$ values listed in table-3 are all negative suggesting the spontaneity of the adsorption process. But the values are all within -20KJ/mole indicating physisorption of the inhibitors on the steel surface.

VII. ADSORPTION ISOTHERMS

Surface coverage θ values for the compounds as determined by weight loss measurements for various concentrations of the inhibitors were used to find the best adsorption isotherm between those more frequently used i.e Langmuir,Temkin, and Frumkin. The best fit was obtained with Langmuir isotherm according to the equation,

$$\frac{\theta}{1-\theta} = KC$$

onrearranging it gives,

$$\frac{C}{\theta} = \frac{1}{K} + C$$

Plots of C/θ against C gives the straight line with unit slope [Fig-2]. This indicates that the adsorption of thepyrimidine amines on mild steel surface in 1M H_2SO_4 solution follows Langmuir isotherm and consequently there is no interaction between the molecules adsorbed at the metal surface.

VIII. POTENTIODYNAMIC POLARISATION STUDIES

Polarization curve for mild steel in 1M H_2SO_4 at various concentrations of PA1 is shown in fig-3. The values of corrosion current densities (I_{corr}), corrosion potential (E_{corr}), the cathodic and anodic Tafel slopes (b_a and b_c) are calculated from the curves and are given in table-4. The data reveals that the corrosion current I_{corr} decreases substantially and inhibition efficiency increases with the inhibitor concentration. It is also evident that the inhibitor adsorption shifted the corrosion potential E_{corr} in the negative direction with respect to the blank signifying that the suppression of the cathodic reation is the main effect of these inhibitors. Moreover the presence of these inhibitors enhanced the value of cathodic Tafel slopes showing that the inhibitors behave as cathodic type getting adsorbed on the cathodic sites to decrease the evolution of hydrogen.

IX. IMPEDANCE STUDIES

Fig-4show the Nyquist plot for mild steel specimen immersed in 1M H_2SO_4 containing various concentrations of inhibitors PA1. The equivalent circuit parameters obtained are tabulated in table-5. The Nyquist plots are all semicircles and correspond to a capacitive loop. The diameter of the semicircle increases with increasing concentration of inhibitors. The R_t values increase with concentration of inhibitor. Since $R_t$is inversely proportional to the corrosion current and it can be used to calculate the inhibition efficiency from the relation

$$\text{Inhibition efficiency (\%)} = \frac{R_t(\text{inh}) - R_t(\text{blank})}{R_t(\text{inh})} \times 100$$

The C_{dl} value decreases with increase in concentration which indicates the formation of a surface film on the metal and thereby decreasing the dissolution reation.

X. MOLECULAR STRUCTURE AND INHIBITION EFFICIENCY

Organic inhibitors contain atleast one polar group with an atom of N,S or O each of them might be an adsorption centre. The inhibitive properties of such compounds depend on the electron densities around the adsorption centre, the higher the electron densities around the adsorption centre, the more effective the inhibitor is.

The pyrimidine amines are organic bases and in acid solution they get protonated at the NH_2 group. The adsorption of these compounds on the mild steel surface occurs by two ways

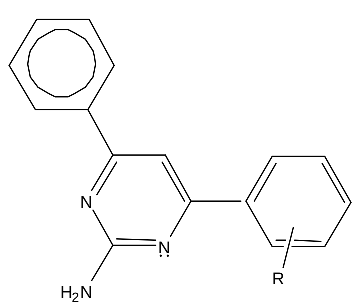
a) Electrostatic interaction between the protonated amines ($-NH_3^+$) and negatively charged Fe surface in acid medium – physisorption.

b) Interaction of lone pairs of electron on N as well as Πe^- of the rings with the Fe atom – chemisorption.

The physisorption mechanism has been proved to be predominant by higher temperature studies.The order of inhibition efficiency of the pyrimidine amines is found to be,

PA1>PA3> PA2.

The general structure of the inhibitors is (Table-7),



PA1 shows maximum efficiency of > 95% and PA2 shows83% efficiency. PA2has a –OH group in the ortho position, which is closer to the main adsorption centre. The lower I.E ofPA2 may be attributed to the formation of intramolecular hydrogen bonding of the –OH hydrogen with the N-atom via the lone pair of electrons. The lone pair of electrons is therefore less readily available for protonation in acid medium. This hinders the electrostatic adsorption leading to lower inhibition. This phenomenon of ortho effect has been discussed by Popova et al[7] in their work in attempting to explain adsorption of molecules with ortho, meta and para substituents by Hammet equation. PA3has substituents in the meta and para position showing higher inhibition efficiency. But the efficiency is somewhat lower than the unsubstituted compound which may be due to the ready solubility of the vanillin compounds in aqueous acid medium.

XI. SYNERGISTISM INFLUENCE OF HALIDE IONS

It is reported that the surface of iron is found to be positively charged in sulphuric acid media and the protonated inhibitor species would be less strongly adsorbed on to the metal surface resulting in lesser inhibition efficiencies. However the addition of halide ions in acid media has been found to increase the adsorption of amines and hence inhibition. This method is therefore effective to get better performance and to decrease the amount of usage of the inhibitor.The synthesized pyrimidine amines show 80-90% inhibition at 1mM concentration and somewhat lesser inhibitory activity at lower concentration. Inorder to improve the performance of the inhibitor at lower concentration and to reduce the cost, the synergistic effect on the addition of halide ions has been attempted by weight loss method.The values of inhibition efficiency obtained by weight loss method with and without addition of 1mM KCl, KBr, & KI are given in table-6. It is apparent that inhibition efficiency increases with the addition of halide ions at all concentrations of the inhibitors tested.With the addition of iodide ion the inhibition efficiency of all the three inhibitors has been increased to > 90% even at 0.1mM concentration of the inhibitors. This synergism effect can be explained as follows.Amines in aqueous acidic solutions may exist as either neutral molecules or in the form of cations depending on the concentration of H^+ ions in the solutions.The potential of zero charge (PZC) of iron in sulphuric acid is about -650 mV Vs SCE[8]. Therefore the amines can interact through the Π electrons of benzene ring with the positively charged metal surface and through the positively charged ammonium ions on the cathodic surface and offer marginal corrosion inhibition. But the presence of halide ions show very good synergism. The synergistic effect is due to the co-adsorption of halide ions on the surface which forms oriented dipoles with their negative ends toward the solution thus increasing the adsorption of protonateorganic inhibitor ions. Fouda et al[9] have reported similar findings in their study on synergistic influence of iodide ions with some aliphatic amines on the inhibition of corrosion of carbon steel in sulphuric acid. It can also be inferred from this study that the order of synergism of halide ions with pyrimidine amine molecules is $I^- > Br^- > Cl^-$ the reason for the better synergism with iodide ions may be due to the large size and ease of polarizability of I^- ions which facilitates electron pair bonding with iron surface.

XII. CONCLUSION

1) The synthesized pyrimidine amines are a good inhibitors for the corrosion of mild steel in 1M H_2SO_4.
2) The inhibitors are an adsorption type inhibitors and adsorption follows the Langmuir adsorption isotherm.
3) Adsorption of the inhibitor on mild steel surface is exothermic and spontaneous.

4) The inhibition efficiency of the inhibitor decreases with the rise of the temperature.
5) The results obtained from the polarization study revealed that the inhibitor under study behaved as a cathodic type inhibitor.

XIII. References

1) Lloyd E.Swearingen & Alfred.F.Schram,C.A,
2) *Substituted amines as inhibitors in the acid corrosion of steel,*
3) J.Phys.Chem, Vol 55(2), (1951), 180-187.
4) Chirag sharma, Shweta sharma, Raja Ram Dangi, & G.L Talesara,
5) *Synthetic and antimicrobial studies on some ethoxyphthalimide derivatives of tetra-hydro-naptho [1,2-e] [1,5] benzo diazepine & dihydrobenzo [h] quinozoline,*
6) J.Ind.Council.Chem, Vol 26(2), (2009), 139-145.
7) U. Ergun, D. Yuzer, K.C. Emregul,
8) *The inhibitory effect of bis-2,6-(3,5-dimethylpyrazolyl) pyridine on the corrosion behavior of mild steel in HCL solution*Mater.Chem. Phys, Vol 109, (2008), 492-499.
9) E.E. Ebenso, Hailemichael Alemu, S.A. Umoren, & I.B. Obot,
10) *Inhibition of mild steel corrosion in sulphuric acid using Alizarin Yellow G G Dye and synergistic iodide additive*,Int.J.Electrochem.Sci, Vol 3, (2008), 1325-1339.
11) K.Tebbji, B. Hammouti, H.Oudda, A. Ramdani, A. Benkadour,
12) *The inhibitive effect of bipyrazolic derivatives on the corrosion of steel in hydrochloric acid solution,*
13) Appl.Surf.Sci, Vol 252, (2005), 1378-1385.
14) F .Bentiss, M. Lebrini, M. Lagrenee,
15) *Thermodynamic characterisation of metal dissolution and inhibitor adsorption processes in mild steel 2,5-bis(n-thienyl)-1,3,4-thiadiazoles/hydrochloric acid system,*
16) Corros.Sci, Vol 47, (2005), 2915-2931.
17) A.Popova, M.Christov, S.Raicheva, E.Sokolova,
18) *Adsorption and inhibitive properties of benzimidazole derivatives in acid mild steel corrosion,*
19) Corros.Sci, Vol 46, (2004), 1333-1350.
20) Xueming Li, Libin Tang, Guannan Mu, Lin Li and Guangheng Liu,
21) *The synergistic inhibition of the cold rolled steel corrosion in 0.5M sulphuric acid by the mixture of OP and bromide ion,*
22) Materials Letter, Vol 61(13), (2007), 2723-2727.
23) A.S.Fouda, H.A.Mostafa, F,El-Taib, G.Y.Elewady,
24) *Synergistic influence of iodide ions on the inhibition of corrosion of C-steel in sulphuric acid by some aliphatic amines,*
25) Corros.sci, Vol 47, (2005), 1988-2004.

Table-1. Inhibition efficiencies of selected concentrations of the inhibitors for the corrosion of mild steel in 1M H_2SO_4 obtained by weight loss measurement at 30±1°C

Name of the inhibitor	Inhibitor concentration (mM)									
	Blank	0.02	0.04	0.06	0.08	0.1	0.15	0.25	0.5	1
PA1	-	88.86	90.57	90.74	90.90	91.31	92.56	94.73	95.81	96.26
PA2	-	36.84	42.80	51.60	55.28	62.63	68.98	73.33	79.57	83.08
PA3	-	49.29	59.34	60.66	61.49	63.96	69.13	75.12	88.98	91.77

Table-2. Inhibition efficiency (%) of 1mM concentration of inhibitors for mild steel corrosion in 1M H_2SO_4 at different temperatures

Temperature (K)	Inhibition efficiency (%)		
	PA1	PA2	PA3
303	96.26	83.08	91.77
313	94.28	79.63	86.79
323	90.38	70.19	78.99
333	88.58	66.62	70.18

Table-3. Thermodynamic parameters for mild steel corrosion in 1M H_2SO_4.

Name of the inhibitor	-ΔH° (KJ/mole)	-ΔS° (KJ/mole)	E_a(kJ)	$\Delta G°_{ads}$ at various temperatures (kJ)			
				303 k	313k	323k	333 k
Blank	0.01887	0.10542	43.46	-	-	-	-
PA1	0.01390	0.15247	51.10	-18.30	-17.74	-18.89	-19.61
PA2	0.03006	0.09893	74.90	-14.12	-15.90	-16.69	-16.23
PA3	0.03350	0.08679	79.84	-16.19	-15.35	-14.33	-13.48

Table-4. Polarisation parameters for mild steel in 1M H_2SO_4 solution with different concentrations of the inhibitors.

Name of the inhibitor	Inhibitor concentration (mM)	Tafel slopes (mv/dec)		E_{corr} (mV)	I_{corr} (μAmp/cm^2)	Inhibition efficiency (%)
		b_a	b_c			
PA1	Blank	84.482	133	-474.0	500.47	-
	0.02	56	102	-501.2	143.99	71.22
	0.1	63	191	-491.1	120.16	75.99
	1	64	243	-487.3	65.06	87.0
PA2	0.02	66	193	-501.0	261.46	47.75
	0.1	71	244	-498.4	156.48	68.73
	1	62	173	-492.6	118.13	76.39
PA3	0.02	54	166	-504.7	66.92	86.62
	0.1	65	305	-484.5	62.43	87.52
	1	56	258	-488.3	47.264	90.55

Table-5. Impedance parameters for mild steel in 1M H_2SO_4 solution with different concentrations of the inhibitors

Name of the inhibitor	Inhibitor concentration (1mM)	R_t (ohmcm^2)	C_{dl} (μF/cm^2) ×10^{-6}	Inhibition efficiency (%)
PA1	Blank	10.5299	34.2381	-
	0.02	89.54	12.208	88.24
	0.1	150.29	11.977	92.99

	1	189.73	10.948	94.45
PA2	0.02	80.74	12.175	86.95
	0.1	143.40	10.875	92.65
	1	165.46	10.083	93.63
PA3	0.02	122.16	9.2132	91.38
	0.1	133.54	6.5106	92.11
	1	170.72	5.8705	93.83

Table-6. Synergistic influence of halide ions on the inhibition efficiency of the inhibitor for mild steel corrosion in 1M H_2SO_4 at 30$\pm$1°C.

Name of the inhibitor	Inhibitor concentration on mM	Inhibition efficiency (%)			
		Without KCl, KBr & KI	With 1mM KCl	With 1mM KBr	With 1mM KI
PA1	0.02	88.86	89.95	93.32	95.56
	0.06	90.74	91.30	94.59	97.77
	0.1	91.31	92.56	98.50	99.30
PA2	0.02	36.84	46.49	76.70	81.34
	0.06	51.60	73.20	84.67	88.86
	0.1	62.63	79.59	85.17	89.13
PA3	0.02	49.29	60.70	83.32	95.88
	0.06	60.66	70.33	89.91	97.65
	0.1	63.96	73.16	89.99	98.50

Table-7.Structure of the inhibitors.

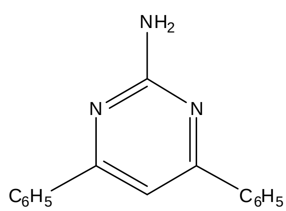
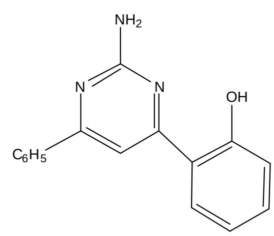
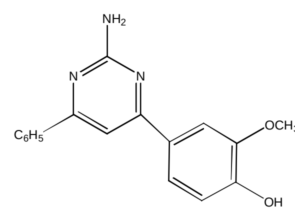
PA1	PA2	PA3
		

Figure-1.Arrhenius plot of corrosion rate of mild steel in 1M H_2SO_4 solution in the absence and presence of inhibitors.

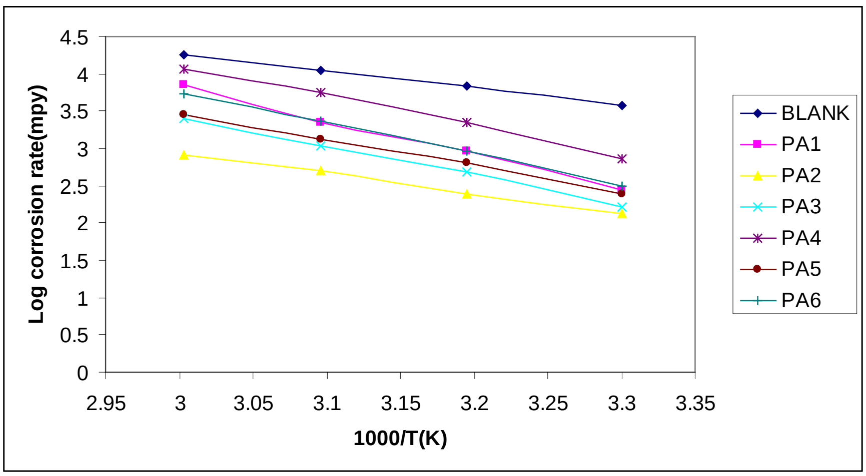


Figure -2. Langmuir plot of inhibitors for mild steel in 1M H_2SO_4

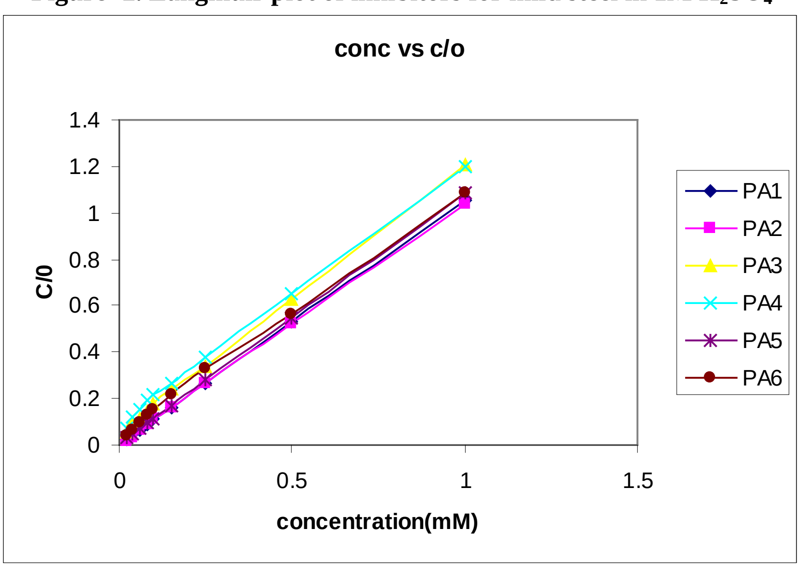


Figure-3. Polarization curves for mild steel recorded in 1M H_2SO_4 for selected concentrations of inhibitor (PA1)

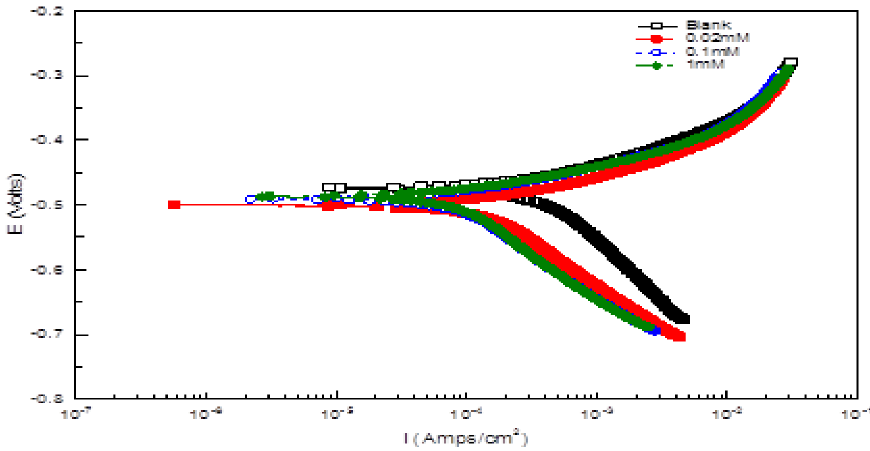
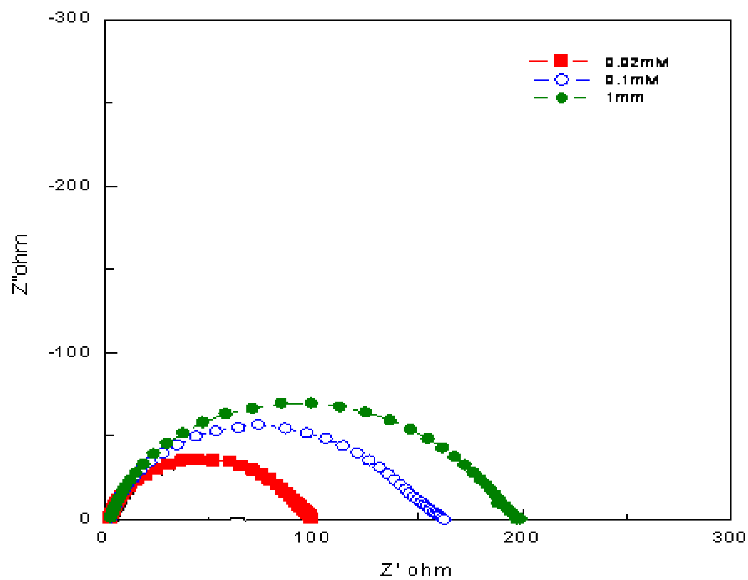


Figure-4. Nyquist diagram for mild steel in 1M H_2SO_4 for selected concentrations of inhibitor (PA1)



Solutions of Generalized Fredholm Type Integral Equations Pertaining To Product of Special Functions

GJSFR Classification - F (FOR) 010302

V.B.L. Chaurasia[1], Jagdev Singh[2]

***Abstract*- The object of this paper is to solve an integral equation whose kernel involves the product of the M-series and $\overline{\mathrm{H}}$ -functions. The solution is derived by the application of Mellin-transform. The integral equation discussed here can be used to investigate a wide class of known (may be new also) integral equations, hitherto scattered in the literature. Special cases, involving generalized Riemann zeta function, g[τ,θ,ν,ϕ;z] function and generalized Wright function are considered.**

Key Words: Fredholm integral equations, $\overline{\mathrm{H}}$ -function, generalized M-series, generalized Riemann zeta function, generalized Wright function, Mellin transform
.

I. Introduction

In the last several years a large number of Fredholm type integral equations involving various special functions or polynomials as their kernels have been studied by many researchers notably Srivastava and Bushman (1992), Srivastava and Raina (1992), Goyal and Mukherjee (2002), Gupta and Agrawal (2009), and others.

In this paper, we obtain the solution of the following Fredholm type integral equation

$$\int_0^\infty y^{-1}\, u\left(\frac{x}{y}\right) f(y)\, dy = g(x), \quad (x > 0) \tag{1}$$

where g is a prescribed function, f is an unknown function to be determined and the kernel u(x) is given by

$$u(x) = {}_k M_\ell^{\alpha,\beta}[a_1,\dots,a_k;b_1,\dots,b_\ell;\omega x^\rho]\,\overline{H}_{p',q'}^{m',n'}\left[zx^\sigma \left| \begin{matrix} (e'_j,E'_j;\alpha'_j)_{1,n'},(e'_j,E'_j)_{n'+1,p'} \\ (f'_j,F'_j)_{1,m'},(f'_j,F'_j;\beta'_j)_{m'+1,q'} \end{matrix}\right.\right]$$
$$\times \overline{H}_{p,q}^{m,n}\left[t\,x^{-\lambda} \left| \begin{matrix} (e_j,E_j;\alpha_j)_{1,n},(e_j,E_j)_{n+1,p} \\ (f_j,F_j)_{1,m},(f_j,F_j;\beta_j)_{m+1,1} \end{matrix}\right.\right]. \tag{2}$$

The generalized M-series is introduced by Sharma and Jain (2009) defined as

$$ {}_k M_\ell^{\alpha,\beta}(\omega) = {}_k M_\ell^{\alpha,\beta}(a_1,\dots,a_k;b_1,\dots,b_\ell;\omega)$$
$$= \sum_{r=0}^{\infty} \frac{(a_1)_r\cdots(a_k)_r}{(b_1)_r\cdots(b_\ell)_r}\,\frac{\omega^r}{\Gamma(\alpha r+\beta)},\ \omega,\alpha,\beta\in C,\ \mathrm{Re}(\alpha)>0. \tag{3}$$

For convergence conditions and other details of the generalized M-series see Sharma and Jain (2009).

The $\overline{\mathrm{H}}$ -function, a generalization of Fox H-function introduced by Inayat Hussain (1987) and studied by Bushman and Srivastava (1990) and others, is defined and represented in the following manner:

About[1]- Department of Mathematics, University of Rajasthan, Jaipur-302055, Rajasthan, India
Email: vblchaurasia@gmail.com
About[2]- Department of Mathematics,
Jagan Nath University, Village- Rampura, Tehsil- Chaksu, Jaipur-303901, Rajasthan
E-mail: jagdevsinghrathore@gmail.com

$$\overline{H}^{m,n}_{p,q}[z]=\overline{H}^{m,n}_{p,q}\left[z\left|\begin{matrix}(e_j,E_j;\alpha_j)_{1,n},(e_j,E_j)_{n+1,p}\\(f_j,F_j)_{1,m},(f_j,F_j;\beta_j)_{m+1,q}\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_L \overline{\phi}(\xi)z^{\xi}\,d\xi, \tag{4}$$

where

$$\overline{\phi}(\xi)=\frac{\prod_{j=1}^{m}\Gamma(f_j-F_j\xi)\prod_{j=1}^{n}\{\Gamma(1-e_j+E_j\xi)\}^{\alpha_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-f_j+F_j\xi)\}^{\beta_j}\prod_{j=n+1}^{p}\Gamma(e_j-E_j\xi)} \tag{5}$$

and the contour L is the line from C – i∞ to C + i∞, suitably indented to keep the poles of $\Gamma(f_j-F_j\xi),\ j=1,2,...,m$ to the right of the path and the singularities of $\{\Gamma(1-e_j+E_j\xi)\}^{\alpha_j}$, j = 1,2,…,n to the left of the path.

For convergence conditions and other details of the $\overline{H}$-function see Inayat Hussain (1987), Bushman and Srivastava (1990) and Gupta et al. (2007).

The series representation of $\overline{H}$-function (1987) is as follows

$$\overline{H}^{m',n'}_{p',q'}[z]=\overline{H}^{m',n'}_{p',q'}\left[z\left|\begin{matrix}(e'_j,E'_j;\alpha'_j)_{1,n'},(e'_j,E'_j)_{n'+1,p'}\\(f'_j,F'_j)_{1,m'},(f'_j,F'_j;\beta'_j)_{m'+1,q'}\end{matrix}\right.\right]$$

$$=\sum_{g=1}^{m'}\sum_{h=0}^{\infty}\frac{(-1)^h\phi(\eta_{g,h})}{h!\,F'_g}z^{\eta_{g,h}},\ldots(6)$$

where

$$\phi(\eta_{g,h})=\frac{\prod_{\substack{j=1\\j\neq g}}^{m'}\Gamma(f'_j-F'_j\eta_{g,h})\prod_{j=1}^{n'}\{\Gamma(1-e'_j+E'_j\eta_{g,h})\}^{\alpha'_j}}{\prod_{j=m'+1}^{q'}\{\Gamma(1-f'_j+F'_j\eta_{g,h})\}^{\beta'_j}\prod_{j=n'+1}^{p'}\Gamma(e'_j-E'_j\eta_{g,h})}$$

and

$$\eta_{g,h}=\frac{f'_g+h}{F'_g}.$$

II. IMELLIN TRANSFORM OF U(X)

In order to solve the integral equation (1), we require the following result

Lemma 2.1. Let U(s) = M {u(x) ;s }, where u(x) is given by (2), then

$$U(s)=\sum_{g=1}^{m'}\sum_{h,r=0}^{\infty}(-1)^h\frac{(a_1)_r\ldots(a_k)_r}{(b_1)_r\ldots(b_\ell)_r}\frac{1}{\Gamma(\alpha r+\beta)}\frac{\phi(\eta_{g,h})}{h!F'_g}\omega^r z^{\eta_{g,h}}\lambda^{-1}$$

$$\times t^{\left(\frac{s+\rho r+\sigma\eta_{g,h}}{\lambda}\right)} \bar{\phi}\left(\frac{s+\rho r+\sigma\eta_{g,h}}{\lambda}\right) \tag{7}$$

provided that

$$\max_{1\le j\le n}\left[\operatorname{Re}\left\{\alpha_j\left(\frac{e_j-1}{E_j}\right)\right\}\right]<\operatorname{Re}\left(\frac{s+\rho r+\sigma\eta_{g,h}}{\lambda}\right)<\min_{1\le j\le m}\left[\operatorname{Re}\left(\frac{f_j}{F_j}\right)\right].$$

Proof: We have

$$U(s)=M\{u(x);s\}$$

$$=M\left\{{}_kM_\ell^{\alpha,\beta}[\omega x^\rho]\bar{H}_{p',q'}^{m',n'}[z x^\sigma]\bar{H}_{p,q}^{m,n}[t x^{-\lambda}]\right\}$$

$$=\int_0^\infty x^{s-1}\,{}_kM_\ell^{\alpha,\beta}[\omega x^\rho]\bar{H}_{p',q'}^{m',n'}[z x^\sigma]\bar{H}_{p,q}^{m,n}[t x^{-\lambda}]\,dx. \tag{8}$$

Expressing the M-series and one $\bar{H}$ -function in series form defined by (3) and (6) respectively and changing the order of summations and integration (which is permissible under the conditions stated), we get

$$U(s)=\sum_{g=1}^{m'}\sum_{r,h=0}^{\infty}(-1)^h\frac{(a_1)_r\ldots(a_k)_r}{(b_1)_r\ldots(b_\ell)_r}\frac{1}{\Gamma(\alpha r+\beta)}\frac{\phi(\eta_{g,h})}{h!\,F_g'}\omega^r z^{\eta_{g,h}}$$

$$\times M\left\{x^{\rho r+\sigma\eta_{g,h}}\bar{H}_{p,q}^{m,n}[t x^{-\lambda}]\right\}. \tag{9}$$

Now, applying the known formulae (Erdélyi (1954, p.307, eq.(8))

$$M\{x^\mu f(z x^{-\lambda});s\} = \lambda^{-1} z^{\frac{s+\mu}{\lambda}} F\left(-\frac{s+\mu}{\lambda}\right) \tag{10}$$

and [9, p.114, eq. (4.1)]

$$M\left\{\bar{H}_{p,q}^{m,n}[x];s\right\}=\bar{\phi}(-s). \tag{11}$$

provided that $\Omega>0$ | arg (x) | < 1/2 $\Omega\pi$ and

$$-\min_{1\le j\le m}\left[\operatorname{Re}\left(\frac{f_j}{F_j}\right)\right]<\operatorname{Re}(s)<\min_{1\le j\le n}\left[\operatorname{Re}\left\{\alpha_j\left(\frac{1-e_j}{E_j}\right)\right\}\right]$$

in (9), we arrive at the desired result (7).

III. SOLUTION OF THE INTEGRAL EQUATION

In this section we will investigate the solution of the Fredholm type integral equation (1). The result is given in the form of the following theorem

Theorem 3.1. Let the Mellin transform F(s), G(s) and U(s) ≠ 0 of the functions f(x), g(x) and u(x) defined by (2) exist and are analytic in some infinite strip $s_1 < \operatorname{Re}(s) < s_2$ of complex s-plane. Also suppose that for a fixed C ∊ (s_1, s_2), $u^*(x)$ is defined by

$$u^*(x)=M^{-1}\{U^*(s);x\}=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}x^{-s}\,U^*(s)\,ds, \tag{12}$$

where

$$U^*(s)=\left[\frac{\mu^{\xi}\Gamma(-s/\mu)}{\Gamma[-(s+\mu\xi)/\mu]}\sum_{g=1}^{m'}\sum_{h,r=0}^{\infty}(-1)^h\frac{(a_1)_r\ldots(a_k)_r}{(b_1)_r\ldots(b_\ell)_r}\right.$$

$$\times\frac{1}{\Gamma(\alpha r+\beta)}\frac{\phi(\eta_{g,h})}{h!\,F_g'}\omega^r z^{\eta_{g,h}}\lambda^{-1}t^{\left(\frac{s+\mu\xi+\gamma+\rho r+\sigma\eta_{g,h}}{\lambda}\right)}$$

$$\left.\times\bar{\phi}\left(\frac{s+\mu\xi+\gamma+\rho r+\sigma\eta_{g,h}}{\lambda}\right)\right]^{-1} \tag{13}$$

provided that the following conditions are satisfied

(i) $|\arg(t)|<\frac{1}{2}\Omega\pi$ and $\Omega>0$

(ii) $|\arg(t)|=\frac{1}{2}\Omega\pi$ and $\Omega\geq 0$

and (a) $\varsigma\neq 0$ and contour L is so chosen that $(C\varsigma+\in+1)<0$

(b) $\varsigma=0$ and $(\in+1)<0$,

where

$$\Omega=\sum_{j=1}^{m}F_j+\sum_{j=1}^{n}\alpha_jE_j-\sum_{j=m+1}^{q}F_j\beta_j-\sum_{j=n+1}^{p}E_j$$

$$\varsigma=\sum_{j=1}^{n}\alpha_jE_j+\sum_{j=n+1}^{p}E_j-\sum_{j=1}^{m}F_j-\sum_{j=m+1}^{q}F_j\beta_j$$

$$\in=\mathrm{Re}\left(\sum_{j=1}^{m}f_j+\sum_{j=m+1}^{q}F_j\beta_j-\sum_{j=1}^{n}e_j\alpha_j-\sum_{j=n+1}^{p}e_j\right)$$

$$+\frac{1}{2}\left(\sum_{j=1}^{n}\alpha_j-\sum_{j=m+1}^{q}\beta_j+p-m-n\right)$$

(iii) $\mu\neq 0$, ξ is a non-negative integer and $\min\{\rho,\sigma,\lambda\}>0$

(iv) $$\max_{1\leq j\leq n}\left[\mathrm{Re}\left\{\alpha_j\left(\frac{e_j-1}{E_j}\right)\right\}\right]<\mathrm{Re}\left(\frac{s+\mu\xi+\gamma+\rho r+\sigma\eta_{g,h}}{\lambda}\right)$$

$$<\min_{1\leq j\leq m}\left[\mathrm{Re}\left(\frac{f_j}{F_j}\right)\right].$$

Then, the solution of integral equation (1) is given by

$$f(x)=x^{-\mu\xi-\gamma}\int_0^{\infty}y^{-1}u^*\left(\frac{x}{y}\right)(y^{\mu+1}Dy)^{\xi}[y^{\gamma}g(y)]dy \tag{14}$$

provided that the integral on the right hand side of (14) exists.

Proof: Applying the Mellin transform on integral equation (1) and using the convolution theorem for Mellin transform (Erdélyi(1954)), we get

$$U(s)\,F(s)=G(s) \tag{15}$$

where U(s) is given by (7) and F(s) and G(s) are Mellin transforms of f(x) and g(x) respectively.

Now replacing s in (15) by s+ $\mu\xi+\gamma$ ($\mu\neq 0$and ξ is non-negative integer), we have

$$F(s+\mu\xi+\gamma) = U^*(s)\,\mu^{\xi}\left(-\frac{s+\mu\xi}{\mu}\right)_{\xi} G(s+\mu\xi+\gamma) \tag{16}$$

and using the formula [14]

$$M\{(x^{\xi+1}D_x)^n f(x);s\} = \xi^n\left(-\frac{s+\xi n}{\xi}\right)_n F(s+\xi n), \tag{17}$$

we get

$$F(s+\mu\xi+\gamma) = U^*(s)\,M\{(y^{\mu+1}D_y)^{\xi}[y^{\gamma}g(y)];s\}. \tag{18}$$

Further using the elementary result

$$M\{x^{\mu} f(x);s\} = F(s+\mu) \tag{19}$$

and well known convolution theorem for Mellin transform in (18), we get

$$M\{x^{\mu\xi+\gamma} f(x);s\} = M\left\{\int_0^{\infty} y^{-1} u^*\left(\frac{x}{y}\right)(y^{\mu+1}D_y)^{\xi}[y^{\gamma}g(y)]\,dy\,;s\right\}. \tag{20}$$

Finally, inverting both sides of (20) by using well known Mellin inversion theorem, we arrive at the required result (14).

IV. SPECIAL CASES

Corollary 4.1. Reducing generalized M-series to unity and one $\overline{H}$ -function to generalized Riemann zeta function $\phi(z,p',\eta')$ (Gupta and Soni (2005, p.159, eq.(5)) in the integral equation (1), it takes the following form

$$\int_0^{\infty} y^{-1} u_1\left(\frac{x}{y}\right) f(y)\,dy = g(x), \tag{21}$$

where

$$u_1(x) = \phi[z x^{\sigma}, p', \eta']\,\overline{H}^{m,n}_{p,q}[t\,x^{-\lambda}] \tag{22}$$

has its solution given by

$$f(x) = x^{-\mu\xi-\gamma}\int_0^{\infty} y^{-1} u_1^*\left(\frac{x}{y}\right)(y^{\mu+1}D_y)^{\xi}[y^{\gamma}g(y)]\,dy, \tag{23}$$

provided that integral in (23) exists, $u_1^*(x)$ is the Mellin inverse transform of

$$U_1^*(s) = \left[\frac{\mu^{\xi}\Gamma(-s/\mu)}{\Gamma[-(s+\mu\xi)/\mu]}\sum_{h=0}^{\infty}\frac{1}{(\eta'+h)\,p'} z^h \lambda^{-1} t^{\left(\frac{s+\mu\xi+\gamma+\sigma h}{\lambda}\right)}\,\overline{\phi}\left(\frac{s+\mu\xi+\gamma+\sigma h}{\lambda}\right)\right]^{-1} \tag{24}$$

and the set of conditions of the theorem modified appropriately are satisfied.

Corollary 4.2. Reducing generalized M-series to unity and one $\overline{H}$ -function to $g[\tau,\theta,\nu,\phi;z]$ in the theorem, we easily observe that the integral equation

$$\int_0^{\infty} y^{-1} u_2\left(\frac{x}{y}\right) f(y)\,dy = g(x), \tag{25}$$

where

$$u_2(x) = g[\tau,\theta,\nu,\phi; z x^{\sigma}]\,\overline{H}^{m,n}_{p,q}[t\,x^{-\lambda}] \tag{26}$$

has its solution given by

$$f(x)=x^{-\mu\xi-\gamma}\int_0^\infty y^{-1}u_2^*\left(\frac{x}{y}\right)(y^{\mu+1}D_y)^\xi[y^\gamma g(y)]dy, \quad (27)$$

provided that integral in (27) exists, $u_2^*(x)$ is the Mellin inverse transform of

$$U_2^*(s)=\left[\frac{\mu^\xi\Gamma(-s/\mu)}{\Gamma[-(s+\mu\xi)/\mu]}\frac{K_{d-1}2^{-\phi-2}\Gamma(\phi+1)B\left(\frac{1}{2},\frac{1}{2}+\frac{\nu}{2}\right)}{\pi}\right.$$

$$\left.\times\sum_{h=0}^\infty\frac{\left(\tau-\frac{\nu}{2}\right)_h(\tau)_h(\theta+h)^{-(1+\phi)}}{\left(1+\frac{\nu}{2}\right)_h}\frac{z^h}{h!}\lambda^{-1}t^{\left(\frac{s+\mu\xi+\gamma+\sigma h}{\lambda}\right)}\bar{\phi}\left(\frac{s+\mu\xi+\gamma+\sigma h}{\lambda}\right)\right]^{-1} \quad (28)$$

and the conditions of validity follow easily from those mentioned with the theorem.

Corollary 4.3. Finally, if we reduce generalized M-series to unity and one $\bar{H}$ -function to generalized Wright hypergeometric function ${}_{p'}\Psi_{q'}[z]$, then the integral equation

$$\int_0^\infty y^{-1}u_3\left(\frac{x}{y}\right)f(y)\,dy=g(x), \quad (29)$$

where

$$u_3(x)={}_{p'}\Psi_{q'}[zx^\sigma]\ \bar{H}^{m,n}_{p,q}[t\,x^{-\lambda}] \quad (30)$$

has the solution given by

$$f(x)=x^{-\mu\xi-\gamma}\int_0^\infty y^{-1}u_3^*\left(\frac{x}{y}\right)(y^{\mu+1}D_y)^\xi[y^\gamma g(y)]dy, \quad (31)$$

provided that integral in (31) exists, $u_3^*(x)$ is the Mellin inverse transform of

$$U_3^*(s)=\left[\frac{\mu^\xi\Gamma(-s/\mu)}{\Gamma[-(s+\mu\xi)/\mu]}\sum_{h=0}^\infty\frac{\prod_{j=1}^{p'}\{\Gamma(e_j'+E_j'h)\}^{\alpha_j'}}{\prod_{j=1}^{q'}\{\Gamma(f_j'+F_j'h)\}^{\beta_j'}}\frac{z^h}{h!}\lambda^{-1}\right.$$

$$\left.\times t^{\left(\frac{s+\mu\xi+\gamma+\sigma h}{\lambda}\right)}\bar{\phi}\left(\frac{s+\mu\xi+\gamma+\sigma h}{\lambda}\right)\right]^{-1} \quad (32)$$

and the set of conditions of the theorem modified appropriately are satisfied.The importances of our results lie in their manifold generality. In view of the generality of the $\bar{H}$ -functions and the generalized M-series, on specializing the various parameters and variables, we can obtained from our results, several integral equations and solutions involving a remarkably wide variety of useful functions (or product of several functions), which are expressible in terms of H-functions, G-functions, Mittag-Leffler functions and their various special cases. Thus, the results presented in this paper would at once yield a very large number of results involving a large variety of special functions occurring in the problems of mathematical analysis, mathematical physics and engineering.

V. References

1) Bushman, R.G. and Srivastava, H.M. 1990, The $\overline{\mathrm{H}}$-function associated with a certain cases of Feynmann Integrals, J. Phys. A. Math. Gen. 23, 4707-4710.
2) Erdélyi, A. et al. 1954, Tables of Integral Transforms, Vol. I, Mc-Graw Hill Book Co., New York (1954).
3) Goyal, S.P. and Mukherjee, R. 2002, Inversion of an integral involving a product of general class of polynomials, generalized polynomial set and Fox's H-function as kernel, J. Rajasthan Acad. Phy. Sci. 1(3), 189-198.
4) Gupta, K. and Agrawal, V. 2009, On the solution of generalized Fredholm type integral equations, J. Appl. Math., Stat. and Inf. 5, 29-38.
5) Gupta, K.C., Jain, R. and Agrawal, R. 2007, On Existence Conditions for Generalized Mellin-Barnes Type Integral, Natl. Acad. Sci. Lett. 30 (5 & 6), 169-172.
6) Gupta, K.C. and Soni, R.C. 2005, On a basic formula involving the product of $\overline{\mathrm{H}}$-function and Fox H-function, J. Rajasthan Acad. Phy. Sci. 4(3), 157-164.
7) Inayat-Hussain, A.A. 1987, New properties of hypergeometric series derivable from Feynman integrals : II. A generalization of the H-function, J. Phys. A : Math. Gen. 20, 4119-4128.
8) Mathai, A.M. and Saxena, R.K. 1978, The H-function with Applications in Statistics and Other Disciplines, Wiley Eastern Limited, New Delhi.
9) Saxena, R.K., Ram, C. and Kalla, S.L. 2002, Application of Generalized H-function in Bivariate distribution, Rev. Acad. Can. Cienc., XIV (Nums. 1-2), 111-120.
10) Sharma, M. and Jain, R. 2009, A note on a generalized M-series as a special function of fractional calculus, Fract. Calc. Appl. Anal. 12, 449-452.
11) Srivastava, H.M. 1972, A contour integral involving Fox's H-function, Indian Indian J. Math. 14, 1-6.
12) Srivastava, H.M. and Bushman, R.G. 1992, Theory and applications of Convolution integral equations, Kluwer Academic Publishers, Dordrecht/Boston/London.
13) Srivastava, H.M. and Raina, R.K. 1992, On certain methods of solving class of integral equations of Fredholm type, J. Austral. Math. Soc. (Series A) 52, 1-10.
14) Srivastava, R. 1994, The inversion of an integral equation involving a general class of polynomials, J. Math. Anal. Appl. 186, 11-20.

Quantification of Piperine In *P. Chaba* By HPLC And Its Bio-Potentials

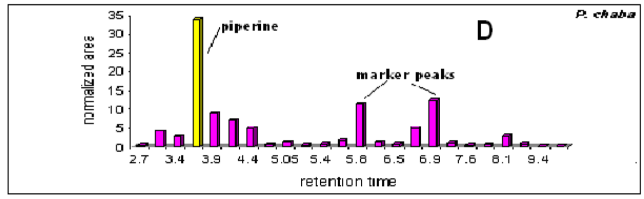
GJSFR Classification - G (FOR) 270899p

Ekta Menghani[1], Naveen Hemrajanian[2],Shalini Rajawat[3]

***Abstract*-Plants are by far the most important source of natural therapeutics, and their role in enhancing the longevity and quality of life is gaining prominence throughout the world and still the plant is often the most neglected part of plant-based medicines. Although, millions of consumers purchase medicinal plant preparations on the basis of anecdotal and scientific evidence of efficacy but very little is known about the factors that make medicinal plants different from other species. It is, therefore, necessary to standardize the medicinal plants widely used throughout the world. In view of the current importance of and interest in herbal drugs, it is necessary to prepare an International Codex containing the details of such plants so that their sale and utilization could be controlled judiciously. Therefore, in present investigations attempts have been made to isolate piperine from *P. chaba* and its quantification to evaluate the percentage of piperine for herbal validation and standardization. Further, antimicrobial, antioxidant and anti-HIV efficacy of piperine were also screened to prove its bio-potentials as bioavailability enhancer.**

HPLC analysis of pet.ether extract of *P. chaba*, exhibited a prominent peak of piperine at rt 3.642 min which was further ascertained by varying the concentration (1, 2, 5 and 10 mg/ml) of the extract, In the assessment of linearity, two calibration curves were plotted in the ranges 1.0 –5.0 and 5.0-10.0 mg/ml. Three replicates of each range were analyzed. The assay value of piperine was found to be 3.18%. The correlation coefficients for standard curves were 0.9933 and 0.9997. Standard deviation 8.38% and the coefficient of variation (cv) among the two curves was 5.77%. Validation of analytical method exhibited the cv of analysis less than 6%. The composite linear equations obtained from the regression analysis were y = 25210.62 x + 884438 and y = 83410 x - 2042764, where y is the area of I and x is the amount of the extract injected. Piperine possess appreciable efficacy as antimicrobial, antioxidant and anti-HIV agents but due to least toxicity it can be used as additive to toxic potent principles as bio-potent agents. Conclusively, piperine can be safely used for identification and herbal validation of *P. chaba* and as a vehicle for various biopotents.

ABSTRACT OUTLAY



About[1]-Department of Biotechnology, Mahatma Gandhi Institute of Applied Sciences, JECRC Campus,
Jaipur -302022. INDIA;
About[2]-Department of Computer Science and Engineering, Suresh Gyan Vihar University, Jaipur

Key words: *Piper chaba*, piperine, hplc quantification, biopotentials

I. Introduction

A medicinal plant is any plant in which one or more of its part contains substances that can be used for therapeutic purposes or which are precursors for chemopharmaceutical semisynthesis. Plants have been used in traditional medicine for several thousand years (Abu-Rabia, 2005). The knowledge of medicinal plants has been accumulated in the course of many centuries based on different medicinal systems such as Ayurveda, Unani and Siddha. In India, it is reported that traditional healers use 2500 plant species and 100 species of plants serve as regular sources of medicine (Pei, 2001). During the last few decades there has been an increasing interest in the study of medicinal plants and their traditional use in different parts of the world (Lev, 2006; Gazzaneo 2005; Al-Qura'n, 2005; Hanazaki *et al.*, 2000; Rossato *et al.*, 1999). Documenting the indigenous knowledge through ethnobotanical studies is important for the conservation and utilization of biological resources.Today according to the World Health Organization (WHO), as many as 80% of the world's people depend on traditional medicine for their primary healthcare needs. There are considerable economic benefits in the development of indigenous medicines and in the use of medicinal plants for the treatment of various diseases (Azaizeh *et al.*, 2003). Due to less communication means, poverty, ignorance and unavailability of modern health facilities, most people especially rural people are still forced to practice traditional medicines for their common day ailments. Most of these people form the poorest link in the trade of medicinal plants (Khan, 2002). A vast knowledge of how to use the plants against different illnesses may be expected to have accumulated in areas where the use of plants is still of great importance (Diallo *et al.*, 1999).In the developed countries, 25 per cent of the medical drugs are based on plants and their derivatives (Principe, 1991). A group of World Health Organization (WHO) experts, who met in Congo Brazzaville in 1976, sought to define traditional African medicine as 'the sum total of practices, measures, ingredients and procedures of all kinds whether material or not, which from time immemorial has enabled the African to guard against diseases, to alleviate his/her suffering and to cure him/herself'(Busia, 2005). Traditional medical knowledge of medicinal plants and their use by indigenous cultures are not only useful for conservation of cultural traditions and biodiversity but also for community healthcare and drug development in the present and future (Pei, 2001).Plants are by far the most important source of natural therapeutics, and their role in enhancing the longevity and quality of life is gaining prominence throughout the world and still the plant is often the most neglected part of plant-based medicines. Although, millions of consumers purchase medicinal plant preparations on the basis of anecdotal and scientific evidence of efficacy but very little is known about the factors that make medicinal plants different from other species.Current problems with medicinal plant products that compromise the quality and safety of medicinal plant products have included contamination with biological and environmental pollutants, adulteration with misidentified species, and the unsustainable harvest resulting in quantitative and qualitative variations in bioactive compounds.It is, therefore, necessary to standardize the medicinal plants widely used throughout the world. In view of the current importance of and interest in herbal drugs, it is necessary to prepare an International Codex containing the details of such plants so that their sale and utilization could be controlled judiciously.*Piper*species are known to be a rich source of *Piper* amides and their derivatives, as a result of which the plant species carry potent pharmaceutical properties like: diuretic, carminative, stimulant, etc. (Charaka Samhita, 1949; Chopra *et al*., 1956, 1969; Nadkarni and Nadkarni, 1954).Significant attention has been paid by the workers on the study of these compounds in *Piper* species (Miyakado *et al.*, 1979; Sengupta and Ray, 1987; Parmar *et al.*, 1998; Siddiqui *et al.*, 2005 a, b), but very little work on their adulteration has been carried out (Madan *et al*., 1996; Paradkar *et al*., 2001).The prime objective of this work is to study and set up certain fundamental diagnostic standards for the identification and authentication of a few important drugs such as *Piper chaba* used in the Ayurvedic System of Medicine. Efforts have been made to detect all the major and minor market adulterants with special reference to their analytical, chemical and biological screening.

II. MATERIALS AND METHODS

a) *Plant material*

Authenticated samples of (Badi pippali)*Piper chaba*(from Suttind Seeds Pvt. Ltd., Delhi) and their market samples were collected and used in the presentstudy.

b) *HPLC Analysis*

The HPLC analysis was performed using a Shimadzu Model-VP 135P2 equipped with a UV spectrophotometric detector set at 254nm, column:Luna 5µC_{18}(2) 100Å (250 x 4.6 mm; 5 particle diameter), flow rate: 1ml/min, injection volume 20µl in methanol (HPLC grade).

c) *Extraction and isolation*

The fruits of *Piper chaba*and its adulterant *Piper longum* were individually extracted with ethanol for 36 hr, filtered and concentrated to dryness. Later from each, 10 mg extractof *P. chaba* and its adulterant was dissolved in 5 ml MeOH separately and used for HPLC analysis.

d) *Quantification of piperine in P. chaba by HPLC*

Pet. ether extract (piperine-rich fraction) of *P. chaba* was weighed (10, 20, 50 and 100 mg) and dissolved in 10 ml methanol (hplc grade) to prepare a concentration of 1, 2, 5 and 10 mg/ml. 200 µl of each concentrations of *P. chaba* was injected onto HPLC and the peak which appeared at the same retention time as that of standard piperine **(I)** was recorded. This value was used to calculate the amount of **I** in the extract by using the linear equation obtained from the composite standard curve. The reproducibility of quantitative analysis was verified by carrying out three replicate injections of each extract and coefficient of variation for each determination was calculated. In the present work, various calculations were achieved by Pearson's correlation formula, which is otherwise used in many forms for correlation co-efficient (*r*) and co-efficient of variation (cv):

$$r = \frac{\Sigma XY - \frac{\Sigma X \Sigma Y}{N}}{\sqrt{\left(\Sigma X^2 - \frac{(\Sigma X)^2}{N}\right)\left(\Sigma Y^2 - \frac{(\Sigma Y)^2}{N}\right)}}$$

$$cv = \frac{\sigma}{x} \times 100$$

e) *Composite standard curve*

The area of corresponding piperine peak and concentration in*P. chaba* were plotted as composite standard curve.

III. RESULTS

The quantitative evaluation of adjoining elution curves was done by calculating the resolution (R = 2t/w_a+w_b), where t is the difference between peak of interest and preceding peak and w_a and w_b are the width of peaks respectively. An easier interpretation of the HPLC tracing, as obtained in this study, was achieved when the peak area was divided by the area of reference peak and the retention time (rt) was plotted against the respective peak area gave histograms as "normalized fingerprints".In the presentinvestigations, attempts have been made to evaluate various extracts and generate some "fingerprints as markers". In *P. chaba*and its adulterants, HPLC chromatograms showed different retention time and peak area, which are characterized as "fingerprints" of *P. chaba* and its adulterants. Similarly, overlay view clearly exhibited different peaks in market samples, and thus, indicative of adulteration. The piperine concentration was also low in market samples as compared to genuine samples, and thus an efficient marker in identification in quality control of a drug.HPLC chromatograms of extract of *P. longum* and *P. chaba* exhibited piperine at rt 3.642 and others. In *P. chaba*, two peaks at rt 5.85 and 6.98 can safely be used as marker because these peaks are absent in *P. longum* and can easily identify when adulterated with *P. chaba*. So these peaks can safely be referred as "marker peaks" (Fig. 1B and D). Further normalized fingerprints can be used as a tool for identification of the drugs.

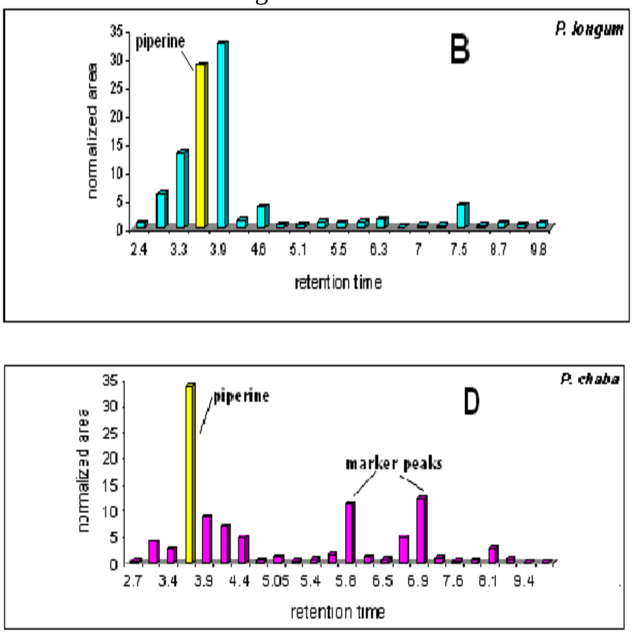


Fig. 1: HPLC Chromatograms and normalized fingerprints of alcoholic extract *P. longum* (B) and *P. chaba* (D) .

HPLC analysis of pet.ether extract of *P. chaba*, exhibited a prominent peak of piperine at rt 3.642 min which was further ascertained by varying the concentration (1, 2, 5 and 10 mg/ml) of the extract, In the assessment of linearity, two calibration curves were plotted in the ranges 1.0 –5.0 and 5.0-10.0 mg/ml (Fig. 2). Three replicates of each range were analyzed. The assay value of piperine was found to be 3.18%. The correlation coefficients for standard curves were 0.9933 and 0.9997. Standard deviation 8.38% and the coefficient of variation (cv) among the two curves was 5.77%. Validation of analytical method exhibited the cv of analysis less than 6%. The composite linear equations obtained from the regression analysis were $y = 25210.62\ x + 884438$ and $y = 83410\ x - 2042764$, where y is the area of **I** and x is the amount of the extract injected.

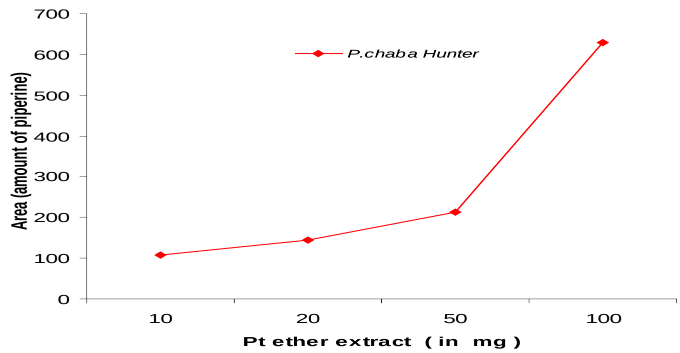


Fig. 2: The Composite standard calibration curve for quantification of piperine in *P. chaba* by HPLC

IV. DISCUSSIONS

Indian herbal medicines have undoubted efficacy but still their market value is comparatively low due to unstable quality and unsuitable approaches for quality assessment. The fingerprinting is referred to as "chemical prints" established by chromatographic and spectroscopic methods for herbal drugs as markers for standardization which is easy to monitor and judge the changes within the constituents. Such chemical fingerprinting can also be used to ensure the efficacy and safety of ISM by controlling to its constituents pattern.Similarly, *Piper chaba* was studied for "HPLC chromatograms for markers in the form of peaks at different retention time (rt). An overlay view of HPLC of *P. chaba* and *P. longum* showed the peaks at rt. 5.85 and 6.98 present in *P. chaba* but were absent in *P. longum* and its market samples, thus, indicatives of adulteration in the market samples.Simultaneously, quantification of piperine in *P. chaba* was also performed for the first time and found to be 3.18% where correlation coefficient for standard curves were 0.9933 and 0.9997 with a standard deviation 8.38% and the coefficient of variation (CV) among the two curves was 5.77%.Earlier, the use of HPLC as a tool for standardization of herbals was performed by few workers (Philipp and Isengard, 1995) but no such HPLC standardization in *Piper* species was carried out so far, and thus, it is the first report of this nature to generate HPLC chromatograms of genuine v/s adulterants.

V. REFERENCES:

1) Abu-Rabia A: Urinary diseases and ethnobotany among pastoral nomads in the Middle East.*Journal of Ethnobiology and Ethnomedicine* 2005., **1:4:**
2) Al-Qura'n S: Ethnobotanical survey of folk toxic plants in southern part of Jordan.*Toxicon* 2005, **46:**119-126.
3) Azaizeh H, Fulder S, Khalil K, Said O: Ethnomedicinal knowledge of local Arab practitioners in the Middle East Region.*Fitoterapia* 2003, **74:**98-108.
4) Busia K: Medical provision in Africa – Past and present.*Phytotherapy Research* 2005, **19:**919-923.
5) Charaka Samhita 1949. Vols 4, Shree Gulab Kunverba Ayurvedic Society, Jamnagar, India.

6) Chopra, R.N., I.C. Chopra and B.S. Varma. 1969. Supplement to Glossary of Indian Medicinal Plants, CSIR, New Delhi. p. 27.
7) Chopra, R.N., S.L. Nayar and I.C. Chopra, 1956. Glossary of Indian Medicinal Plants. CSIR, New Delhi. p. 194.
8) Diallo D, Hveem B, Mahmoud MA, Berge G, Paulsen BS, Maiga A: An ethnobotanical survey of herbal drugs of Gourma district, Mali.*Pharmaceutical Biology* 1999, **37:**80-91.
9) Gazzaneo LR, Paiva de Lucena RF, Paulino de Albuquerque U: Knowledge and use of medicinal plants by local specialists in an region of Atlantic Forest in the state of Pernambuco (Northeastern Brazil).*Journal of Ethnobiology and Ethnomedicine* 2005., **1:9:**
10) Hanazaki N, Tamashiro JY, Leitao-Filho H, Gegossi A: Diversity of plant uses in two Caicaras communities from the Atlantic forest coast, Brazil.*Biodiversity and Conservation* 2000, **9:**597-615.
11) Khan AU: History of decline and present status of natural tropical thorn forest in Punjab.*Pakistan Biological Conservation* 2002, **63:**210-250.
12) Lev E: Ethno-diversity within current ethno-pharmacology as part of Israeli traditional medicine – A review**.** *Journal of Ethnobiology and ethnomedicine* 2006., **2:4:**
13) Madan, M.M., R.S. Singhal and P.R. Kulkarni. 1996. An approach into the detection of authenticity of black pepper (*Piper nigrum* L.) oleoresin. *J. Spices Arom. Crops.***5** : 64- 67
14) Miyakado M, I. Nakagam, H. Yoshoka and N. N. Nakatani. 1979. The piperaceae amides. Structure of piperide, a new insecticidal amide from *Piper nigrum*. *Agric. Biol. Chem.* **43** :1609-1611.
15) Nadkarni, A. K. and K. M. Nadkarni. 1954. *Indian Materia Medica*. Vol. 2. Popular Book Depot, Bombay
16) Paradkar, M. M., R. S. Singhal and P. R. Kulkarni. 2001. A new TLC method to detect the presence of ground papaya seed in ground black pepper. *J. Sci. Food Agric.* **81** : 1322-1325.
17) Parmar, V.S., S.C. Jain, S. Gupta, S. Talwar, V.K. Rajwanshi, R. Kumar, A. Azim, S. Malhotra, N. Kumar, R. Jain, N.K. Sharma, O.D. Tyagi, S.J. Lawrie, W. Errington, O.W., Howarth, C.E. Olsen, S.K. Singh and J. Wengel. 1998. Polyphenols and alkaloids from *Piper*species. *Phytochemistry***49** : 1069-1078.
18) Pei SJ: Ethnobotanical approaches of traditional medicine srudies: Some experiences from Asia.*Pharmaceutical Biology* 2001, **39:**74-79.
19) Philipp, O. and H.D. Isengard. 1995. The presented HPLC method offers a new possibilities for an easy, quick and precise characterization of Lemon oils. *Z. Lebensm. Unters. Forsch* **201** : 551-554.
20) Principe P: *Monetising the pharmacological benefits of plants.*US Environmental protection Agency, Washington, D.C.; 1991.
21) Rossato SC, Leitao-Filho H, Gegossi A: Ethnobotany of Caicaras of the Atlantic forest coast (Brazil).*Economic botany* 1999, **53:**387-395.
22) Sengupta, S. and A.B. Ray. 1987. The chemistry of *Piper*species : a review. *Fitoterapia* **58** : 147-166.
23) Siddiqui, B. S., T. Gulzar, A. Mahmood, S. Begum, B. Khan, M. Rasheed, F. Afshan and R. M. Tariq. 2005a. Phytochemical studies on the seed extract of *Piper nigrum* Linn. *Nat. Prod. Res.* **19** : 703-712.
24) Siddiqui, B. S., T. Gulzar, S. Begum, F. Afshan and F.A. Sattar. 2005b. Insecticidal amides from fruits of *Piper nigrum* Linn. *Nat. Prod. Res*. **19** : 143-150.

A Study of Prevalence of Malnutrition In Government School Children In The Field Area of Azad Nagar Bangalore, India

GJSFR Classification - H (FOR) 111104

[1]Dr Izharul Hasan,[2]Mohd Zulkifle

Abstract:**Malnutrition is a problem at varying proportions in developing countries, and anthropometry is a simple tool to assess its magnitude in children. This study was aimed at identifying the prevalence of malnutrition among 500 children of govt schols of Azad Nagar, bangalore south Asia. The value of using various field based formulae and of various anthropometric indicators used for classification of malnutrition was also studied. The study was focused on children aged 8-14 years studying in class 1st to 8th in govt schools. Anthropometric data and eating practices of children were collected with the help of a pretested questionnaire and food intake diary. Selected anthropometric measurements were taken using standard techniques. Their Body Mass Index (B.M.I) for age was calculated and compared with WHO (2007) standards. Compared to WHO standards, mean BMI of school children in Azad Nagar and its surrounding area was inferior at all ages. The prevalence of malnutrition was 68%, males recorded a relatively high high rate of malnutrition 57.94% (197) than females 42.06% (143). The study reveals that the average of govt school children in Azad Nagar are underweighted. Poor nutrition of children do not only affects the cognitive development of children but also likely to reduce the work capacity in future.**

Key words: School children, BMI, Undernutrition

I. Introduction

In large areas of the world today, malnutrition, especially that affecting young children, is one of the principal public health problem in developing countries. Growing children in particular are most vulnerable to its consequences. The frequency of malnutrition cannot be easily estimated from the prevalence of commonly recognized clinical syndromes, such as Kwashiorkor and Marasmus because these constitute syndromes only, the proverbial tip of iceberg. Case with mild to moderate malnutrition are likely to remain unrecognizd because clinical criteria for their diagnosis are impercise and are difficult to interpret accurately.Anthropometry can be sensitive indicators of health, growth and development in infants and children. Anthropometry is the single most universally applicable, inexpensive and non-invasive method available to assess the size, proportion and composition of human body (W.H.O, 1995).

According to W.H.O, the ultimate intention of nutritional assessment is to improve human health (Beghin et al, 1998). Malnutrition which refers to an impairment of health either from a deficiency or excess or imbalance of nutrients is public health significance among children all over the world. There is no doubt that diet has profound influence on human health, at all ages and affecting all medical subspecialities. Defining and influencing nutritional status is, however very complicated. Adequate food and nutrition are essential for proper growth and physical development to ensure optimal work capasity, normal reproductive performance, adequate immune reactions and resistance to infections. Inadequate diet may produce severe forms of malnutrition in children, Vit A deficiency and Iodine deficiency disorders. World Health Organization (W.H.O, 1995) has recommended various indices based on anthropometry to evaluate the nutritional status of the school aged children. It has now been established that the Body Mass Index (BMI) is the most appropriate variable for nutritonal status among adolescents (WHO, 11995; Himes and Boucher, 1989; Must et al, 1991; Roland. Cachera, 1993).Several studies have investigated nutritional status of adolescents from different parts of the world (Kurz,2006; Cookson et al; Venkainsh et al, 2007; Ahmed et al, 2000). However, there is paucity of anthropometric indices based information on nutritional status of govt school children in Azad Nagar. Moreover, to date there are no studies which have dealt with sex differences in the level of malnutrition among govt school children in Azad Nagar. The present study was attempted to evaluate the overall prevalence of malnutrition, to recommened measures for correction of the nutritional deficit of the vulnerable populations group and to provide a baseline data for future research. In short, the nutritional assessment of a community aim at discovering factors and guiding action intended to improve nutrition and health. This study is carried out to estimate the prevalence of of malnutrition among children 8- 14 years and to compare the commonly used anthropometric indicators in terms of their sensitivity and specificity.

II. Material and Methods

The present study was carried out between January and June 2010. The data were collected from govt higher primary schools of Azad Nagar and its surrounding areas. Necessary approval was obtained from the school authorities prior to the commencement of the research. A total of 500 pupils (382 boys and 118 girls) aged 8-14 years participated in the study. This study was cross sectional in nature and the subjects were selected through random sampling

About[1]-MD Unani in Preventive and Social Medicine Under the guidance of Dr Mohd Zulkifle Dept of Preventive and Social Medicine National Institute of Unani Medicine Bangalore 560091PG Scholar, Dept. of PSM National Institute of Unani Medicine Bangalore, India
About[2]-HOD and Reader Dept of PSM, NIUM Bangalore, India 560091

procedures, aimed at know the prevalence of malnutrition and nutritional status in govt higher primary schols children in the field area of Azad Nagar, and to create awareness among higher primary school children, school teachers and their parents regarding childhood malnutrition, its complications and preventions.

III. DATA COLLECTION

The data were collected by visiting govt higher primary schools on different visits and a particular day was fixed for the investigation, school staff was requested to collect all students of one or more classes on that fixed day. A semi structured, pre tested questionnaire was administered to each child to collect data on socio-demographic profile (Age, Sex, father's and mother education, profession and income). Exact date of birth was verified from the school registers.

IV. ASSESSMENT OF NUTRITIONAL STATUS BY ANTHROPOMETRY

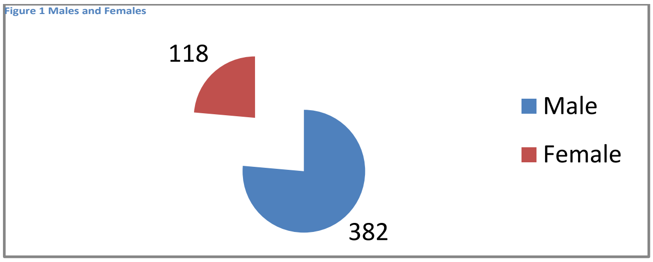
Anthopometry is the measurement of the human. It is a quantitative method and is highly sensitive to nutritional status; especially among children. Two basic variables (height and weight) and a single derived variable (Body Mass Index) have been used in the present report. All the anthropometric measurement were taken following the standard techniques recommended by (Lohmann et al,1998) and body mass index was determined by the CDC table for calculated Body Mass Index.

Table1.

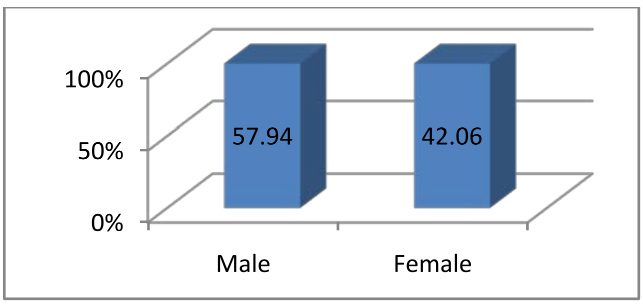
Age (years)	Gender	Height(cms)	Weight(kg) Mean+ S.D	BMI Mean + S.D	BMI Mean (NHCS/WHO Standard)
(8)	M(n=33) F(n=10)	120 + 3.12 121.40 + 4.65	19.20 + 3.12 20.04 + 3.18	13.26 + 3.78 13.68+ 3.92	15.87 15.85
(9)	M(n=61) F(n= 13)	123.12 + 5.33 122.10 + 4.84	21.10 + 4.52 20.90 + 6.12	13.94 + 4.19 14.04 + 3.70	16.68 16.32 16.66 16.90
(10)	M(n = 57) F(n = 20)	128.90 + 6.40 126.20 + 4.64	23.63 + 4.68 24.25 + 3.74	14.42 + 3.10 15.27 + 2.19	17.20 17.57
(11)	M(n = 50) F(n = 23)	131.12 + 6.44 131.90 + 3.18	26.10 + 4.70 27.04 + 3.13	15.20 + 3.42 15.78 + 2.19	17.84 17.36
(12)	M(n = 73) F(n = 20)	138.16 + 6.12 135.12 + 5.90	28.62 + 4.14 31.20 + 2.17	15.93 + 3.21 17.07 + 2.13	18.58 19.15
(13)	M(n = 58) F(n = 21)	143.10 + 4.12 35.84 + 3.10	32.70 + 4.12 35.84 + 3.10)	15.99 + 3.27 17.04 + 3.54	19.35 19.88
(14)	M(n = 50) F(n = 11)	146.19 + 4.03 145.18 + 3.13	39.10 + 3.62 41.30 + 2.17	18.34 + 3.28 18.80 + 3.97	

V. REULSTS

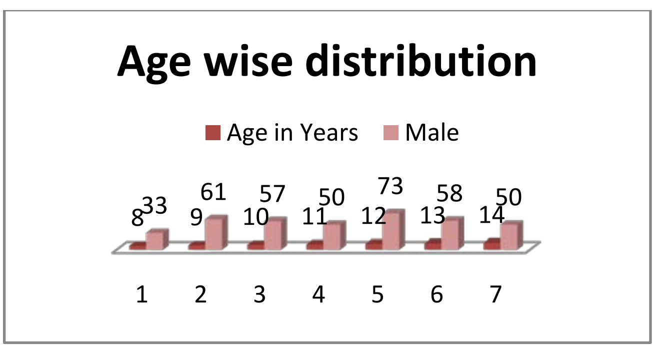
In the study population, the prevalence of malnutrition in govt schools was 68%. The findings were statistically insignificant as validated by Chi square test. In the present study 382 (76.40%) were male and 118 (23.60%) children were female. In this study the prevalence of malnutrition in males and females was 57.94% and 42.06% respectively. Observed difference was not statistically significant as revealed by Chi square test. In the age wise distribution of our study population, total number of children in the age of 8 years were 41(8.20%), in the age of 9 years were 74 (14.80%), in the age of 10 years were 77 (15.40%), in the age of 11 years were 73 (14.60%), in the age of 12 years were 93 (18.60%), in the age of 13 years 79 (15.80%) and in the age of 14 years were 61 (12.20%). Among the study population, 33 (6.60%) male and 10 (2%) female children was of 8 years, 61 (12.20%) male and 13 (2.60%) female of 9 years, 57 (11.40%) male and 20 (4%) female children of 10 years, 50 (10%) male 23 (4.60%) female children of 11 years, 73 (14.60%) male and 20 (4%) female of 12 years, 58 (11.60%) males and 21 (4.20%) female children of 13 years and 50 (10%) male and 11(2.20%) female children were of 14 years. In the distribution of the studied population according to their class levels, 127 (25.40%) of class 5th, 123 (24.60%) of class 6th, 140 (28%) of class 7th and 110 (22%) were of class 8th. Among the study population, 410 (82%) children were Muslims, 60 (12%) of Hindus and 30 (6%) were Jain, Sikh and belongs to other religions



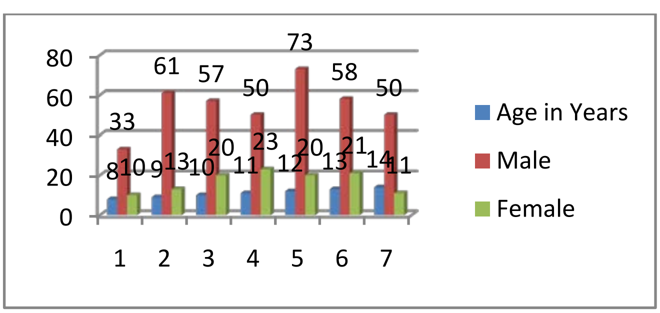
Males and Females in Study



Prevalence of malnutrition in Males and Females



Age Wise Distribution of Children



Male Female Age Wise Distribution

Considering sex variation, boys recorded a prevalence rate of 197 (57.94%) while girls recorded a prevalence rate of 143 (42.06%). However, the age groups 10,11 and 12 years recorded high prevalence of undernutrition with 6.83%, 7.00% and 11.66% respectively. There was no significant difference (X^2 = 38.72, P> 0.05) between age and sex in the prevalence of malnutrition. It is observed that both sexes and all ages are undernourished.

VI. Discussion

The use and interpretation of growth measurements differ significantly according to whether they concern the individual or entire population. The selection and use nutritional status of children in a clinical setup, in emergency situations, such as natural and man-made calamities, and in growth monitoring of children. Deficits in one or more of the anthropometric indicators are regarded as evidence of malnutrition. Each indicator has its own merits and demerits, and each indicator is best suited for a particular situation. Anthropometry, for many years, has been the indicator of choice for use during emergency situations. Indicators such as weight-for-height or height-for –age, allow malnutrition to be classified into stunting and wasting. Stunting is highly prevalent in most developing countries. The worldwide prevalence of stunting varies considerably from 5 % to 65 % in developing countries.The mean height and mean weight of the present study at all ages were found to be much inferior when compared to NCHS (National Centre for Health Statistics, USA) standard which is the reference data recommended by WHO. Mean height and Mean weight of girls is 12-14 years were better than boys of the same age groups. This could be due to the earlier onset of pubertal growth spurt in girls than boys.
The relatively high prevalence of malnutrition observed among govt school children located in Azad Nagar may be due to inadequate dietry intake of food. Alongside, the fact that most of these children are from parents of low socio-economic background mainly laborers, fisherman, auto drivers, farmers and traders, who themselves attented poor schools and live in poor houses where unhygienic living standards, unsafe drinking water, low calories food, and insanitary conditions of the immediate environmentprevail. Such environmental factors contribute to the survival of disease agents such as parasites, bacteria and viruses. After being infected by these organisms, these children loose the protein energy, iron and vitamins intake to the benefit of these disease agents which later adversely affect the growth and nutritional status of the individual. One study indicated that malnutrition in middle income or low socio-economic group children aged 6-16 years in Hydrebad was 10-13 %. Children having a body height and weight <-2 SD of the NCHS growth standards were considered as malnourished. The rate of malnutrition observed among boys (57.94%) is distinctively lower than the findings of the IRC (International Rescue Committee) in kakuma, kenya where 75% of boys were found malnourished. On the other hand, the malnutrition(42.06%) observed among the girls and (57.94%) among boys demonstrated a higher rate of malnutrition.

VII. Conclusion

Current study show a high prevalence of malnutrition even among govt schools children in Azad nagar Bangalore.

Malnutrition is a health concern in this group and these are strong indications that micronutrients deficiencies might well exists among these children. However, adequate and recent information is lacking, espeially with respect to micronutrient status, and data are only available from a number of small studies from Azad Nagar and its surrounding area that might not be representative for the general population. There is definitely a need for well-planned, large scale studies using standardized methodologies to estimate the prevalence of malnutrition and other micronutrients deficiencies. A comprehensive study including anthropometric data, biochemical data, clinical sighns and dietary intake data among the same group of children will give a better insight into situation.

In shortly, the present study provides evidence that the average govt school children in Azad Nagar state Karnataka are malnourished. The children studing in govt schools do not realize their fullgenetic potential for growth and they are considerably malnourished than their counterparts of private schools. The need for more calories, protein and micronutrients like iron and vitamins for the children of govt schools cannot be overemphasized. Giving iron tablets or micronutrient fortification are not a solution of the problem in this situation, but what they need is more food which of good nutritive value. School lunch can be an ideal vehicle to achieve this end. A protective role from the govt and communit leaders is the need of the hour.

VIII. References

1. Behrman RE, Kliegman RM, Jenson HB, editors. Nelson's Text book of pediatrics. 16th ed. Phiiladelphia: Saunders, 2000: 32-50
2. Bairagi R. On validity of some anthopometric indicators as predictors of mortality. *Am* J *Clin Nutr* 1980; 33:687-96
3. *Beghin, I., M.R. Cap and B. Dujardan*, 1998, A guide to nutritional assessment. Geneva, Switzerland: World Health Organization.
4. Chen LC, Chowdhury AKMA, Huffman SL. Anthropometric assessment of energy protein malnutrition and subsequent risk of mortality among school aged children. *Am J Clin Nutr* 1980; 33: 1836-47.
5. Cookson, S.T., B.A. Woodruff and L. Slutsker, Prevalence of anemia and low body mass index among dolescents 10-19 of age in refugee camps in Dadaab District, Kenya. Atlanta, G.A.: Centers for Disease Control and Prevention.
6. De Onis M, Monterio C, Akre J, Glugston G. The worldwide magnitude of protein energy malnutrition: an overview from the WHO global database on child growth. *Bull World Health Organ* 1993; 71: 703-12
7. Gorstein J. Assessment of nutritional status: effect of different methods to determine age on the classification of undernutrition. *Bull World Health Organ* 1989; 67: 143-50
8. Himes, J.H. and C. Boucher, 1989. Validity of anthropometry in classifying youths as obese. Int. J. Obes., 13: 183-193.
9. Lohman, T.G., A.F. Roche and R. Martore, 1998. Anthropometric Standardization Reference Manual. Chicago: Human kinetics.
10. Rolland-Cachera, M.F., 1993. Body composing during adolescence: methods, limitations and determinants. Hormone Res., 39: 25-40
11. Trowbridge FL, Sommer A. Nutritional anthropometry and mortality risk. *Am J clin Nutr* 1981; 34-35:2591-92
12. Waterlow JC. Classification and definition of protein calorie malnutrion. *Br Med J* 1972; 3: 566-70.
13. World Health Organization. Physical status: use and interpretation of anthropometry; report of a WHO Expert Committee. Geneva: World Health Organization, 1995. 452. (WHO technical report series no.854)

Integral Solutions To Heun's Di Erential Equation Via Some Rational Transformation

A. Anjorin

GJSFR Classification - F (FOR) 230107p

***Abstract*-The present work determines the integral form of solutions obtained from the transformation of Heun's equation to hypergeometric equation by rational substitution. All relevant solutions are provided.**

AMS Subject Classification: 33CXX
Key Words : Hypergeometric functions, Heun's functions.

I INTRODUCTION

The hypergeometric equation has three regular singular points. Heun's equation has four singular points. The problem of conversion from Heun's equation to hypergeometric equation has been teated in the works of K. Kuiken [1]. The purpose of this work is to derive some integrated forms of solutions to the Heun's equation via some rational transformation as stated in [1]. The steps taken shall be the conversion of Heun's functions to the hypergeometric functions then taken the integration, and through a push and pull back process we arrive back to a new Heun's functions di erent from the original Heun's function.Every homogenous linear second order di erential equation with four regular singularities can be transformed into[3

$$\frac{d^2u}{dt_2}+\left(\frac{\gamma}{t}+\frac{\delta}{t-1}+\frac{\epsilon}{t-d}\right)\frac{du}{dt}+\frac{\alpha\beta t-q}{t(t-1)(t-d)}u=0, \qquad (1.1)$$

where $\{\alpha, \beta, \gamma, \delta, \epsilon, d, q\}$ $(d \neq 0, 1)$ are parameters, generally complex and arbitrary, linked by the Fuschian constraint $\alpha+\beta+1=\gamma+\delta+\epsilon$. This equation has four regular singular points at $\{0, 1, a, \infty\}$, with the exponents of these singularities being respectively, $\{0, 1-\gamma\}$, $\{0, 1-\delta\}$, $\{0, 1-\epsilon\}$, and $\{\alpha, \beta\}$. The equation (1.1) is called Heun's equation[3]

The hypergeometric equation[3]

$$z(1-z)\frac{d^2y}{dz^2}+[c-(a+b+1)z]\frac{dy}{dz}-aby=0, \qquad (1.2)$$

has three regular singular points. In the works of [1], it has been shown that these two equations above can be trans formed to one another via six rational polynomials $z=R(t)$, where $R(t)=t^2, 1-t^2, (t-1)^2, 2t-t^2, (2t-1)^2, 4t(1-t)$. The following parameter relations were deduced[1].

For the polynomial $R(t)=t^2$

- $\alpha+\beta=2(a+b)$, $\alpha\beta=4ab$, $\gamma=-1+2c$, $\delta=1+a+b-c$, $\epsilon=\delta$, $q=0$ and $d=-1$.

For the polynomial $R(t)=1-t^2$

- $\alpha+\beta=2(a+b)$, $\alpha\beta=4ab$, $\gamma=1-2c+2a+2b$, $\delta=c$, $\epsilon=\delta=c$, $q=0$ and

Department of Mathematics, Lagos State University(LASU) PMB 1087 Apapa Lagos Nigeria.
Email and Phone number: anjomaths@yahoo.com, +2348033809538 and +2348025818386

For the polynomial $R(t) = 2t - t^2$

- $\alpha + \beta = 2(a + b)$, $\alpha\beta = 4ab$, $\gamma = c$, $\delta = 1 - 2c + 2a + 2b$, $\epsilon = \delta = c$, $q = 4ab$ and $d = 2$.

For the polynomial $R(t) = (2t - 1)^2$

- $\alpha + \beta = 2(a + b)$, $\alpha\beta = 4ab$, $\gamma = -1 + a + b - c$, $\delta = \gamma$, $\epsilon = \delta = -1 + 2c$, $q = 2ab$ and $d = 1/2$.

For the polynomial $R(t) = 4t(1 - t)$

- $\alpha + \beta = 2(a + b)$, $\alpha\beta = 4ab$, $\gamma = c$, $\delta = \gamma$, $\epsilon = 1 - 2c + 2a + 2b$, $q = 2ab$ and $d = 1/2$.

Assuming $H(d, q; \alpha, \beta, \gamma, \delta, \epsilon; t)$ and ${}_2F_1(a, b; c; z = R(t))$ are representative forms of the solutions of (1.1) and (1.2) respectively, toghether with the parametres above relations can de extablished between these two forms via the polynomials data given above. We provide an answer to this in this paper. In deed, we provide that the integral of the solution of **GHE** can be expressed in terms of another **GHE** solution.

Main Results

II Integral solutions

In this section we shall apply the relations above in deriving the integral form of solutions via these polynomial transformations. Let $\mathcal{I} = \int_C$ be a integral operator defined over a compact interval C. Since $(a)_{n-1} = \frac{(a-1)_n}{a-1}$, we have $\mathcal{I}{}_2F_1(a, b; c; z = R(t)) = \frac{R^*(t)(c-1)}{(a-1)(b-1)}{}_2F_1(a - 1, b - 1; c - 1; z = R(t))$, where $R^*(t)$ is a polynomial factor derived from the integrand and through a push and pull-back processes we have the following possible solutions;

1. For polynomial $R(t) = t^2$

   (a) Using $c = (\gamma + 1)/2$, we obtain

$$\mathcal{I}H(-1, 0; \alpha, \beta, \gamma, \delta, \epsilon; t) = \frac{2(\gamma-1)t^3}{3(\alpha-2)(\beta-2)}{}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \frac{\gamma-1}{2}; R(t) = t^2)|_C$$
$$= \frac{2(\gamma-1)t^3}{3(\alpha-2)(\beta-2)}H(-1, 0; \alpha-2, \beta-2, \gamma-2, \frac{\alpha+\beta-\gamma-1}{2}, \frac{\alpha+\beta-\gamma-1}{2}; t))|_C. \tag{2.1}$$

   (b) Using $c = 1 - \delta + a + b$, we get

$\mathcal{I}H(-1, 0; \alpha, \beta, \gamma, \delta, \epsilon; t)$

$$= \frac{4(\alpha+\beta-2\delta)t^3}{3(\alpha-2)(\beta-2)} {}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \alpha+\beta-2\delta; R(t)=t^2)|_C$$
$$= \frac{4(\alpha+\beta-2\delta)t^3}{3(\alpha-2)(\beta-2)}$$
$$\times H(-1,0;\alpha-2,\beta-2,2(\alpha+\beta-2\delta)-1, \frac{4\delta-(\alpha+\beta)-2}{2}, \frac{4\delta-(\alpha+\beta)-2}{2}; t))|_C. \tag{2.2}$$

2. For polynomial $R(t) = 1 - t^2$

   (a) Using $c = \delta$, we have

$$\mathcal{I}H(-1,0;\alpha,\beta,\gamma,\delta,\epsilon;t)$$
$$= \frac{4(\delta-1)(3t-t^3)}{3(\alpha-2)(\beta-2)} {}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \delta-1; R(t)=1-t^2)|_C$$
$$= \frac{4(\delta-1)(3t-t^3)}{3(\alpha-2)(\beta-2)} H(-1,0;\alpha-2,\beta-2,\alpha+\beta-2\delta-1,\delta-1,\delta-1;t))|_C. \tag{2.3}$$

   (b) Using $c = \epsilon$, we have

$$\mathcal{I}H(-1,0;\alpha,\beta,\gamma,\delta,\epsilon;t)$$
$$= \frac{4(\epsilon-1)(3t-t^3)}{3(\alpha-2)(\beta-2)} {}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \epsilon-1; R(t)=1-t^2)|_C$$
$$= \frac{4(\epsilon-1)(3t-t^3)}{3(\alpha-2)(\beta-2)} H(-1,0;\alpha-2,\beta-2,\alpha+\beta-2\epsilon-1,\epsilon-1,\epsilon-1;t))|_C. \tag{2.4}$$

   (c) Using $c = (1-\gamma+2a+2b)/2$, we arrive at

$$\mathcal{I}H(-1,0;\alpha,\beta,\gamma,\delta,\epsilon;t)$$
$$= \frac{2(\alpha+\beta-\gamma-1)(3t-t^3)}{3(\alpha-2)(\beta-2)} {}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \frac{\alpha+\beta-\gamma-1}{2}; R(t)=1-t^2)|_C$$
$$= \frac{2(\alpha+\beta-\gamma-1)(3t-t^3)}{3(\alpha-2)(\beta-2)}$$
$$\times H(-1,0;\alpha-2,\beta-2,\gamma-2, \frac{\alpha+\beta-\gamma-1}{2}, \frac{\alpha+\beta-\gamma-1}{2}; t)|_C. \tag{2.5}$$

3. For polynomial $R(t) = 2t - t^2$

   (a) Using $c = (\delta+1)/2$, we obtain

$$\mathcal{I}H(2,\alpha\beta;\ \alpha,\beta,\gamma,\delta,\epsilon;t)$$
$$= \frac{2(\delta-1)t^2(3-t^2)}{3(\alpha-2)(\beta-2)} {}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \frac{\delta-1}{2}; R(t)=2t-t^2)|_C$$
$$= \frac{2(\delta-1)t^2(3-t^2)}{3(\alpha-2)(\beta-2)}$$
$$\times H(2,(\alpha-2)(\beta-2);\alpha-2,\beta-2, \frac{\alpha+\beta-\delta-1}{2}, \delta-2, \frac{\alpha+\beta-\delta-1}{2}; t)|_C. \tag{2.6}$$

(b) Using $c = 1 + a + b - \gamma$, we get

$$\mathcal{I}H(2, \alpha\beta; \beta \ , \alpha, \gamma, \delta, \epsilon; t) = \frac{2(\alpha+\beta-2\gamma)t^2(3-t^2)}{3(\alpha-2)(\beta-2)} {}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \gamma+1; R(t) = 2t - t^2)|_C$$
$$= \frac{2(\alpha+\beta-2\gamma)t^2(3-t^2)}{3(\alpha-2)(\beta-2)}$$
$$\times H(2, (\alpha-2)(\beta-2); \alpha-2, \beta-2, \frac{\gamma-2}{2}, \alpha+\beta-\gamma-1, \frac{\gamma-2}{2}; t))|_C. \quad (2.7)$$

4. For polynomial $R(t) = (t-1)^2$

(a) Using $c = (1 - \delta + 2a + 2b)/2$, we get

$$\mathcal{I}H(2, \alpha\beta, \alpha, \beta, \gamma, \delta, \epsilon; t)$$
$$= \frac{2(\alpha+\beta-\delta-1)(t-1)^3}{3(\alpha-2)(\beta-2)} {}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \frac{\alpha+\beta-\delta-1}{2}; R(t) = (t-1)^2)|_C$$
$$= \frac{2(\alpha+\beta-\delta-1)(t-1)^3}{3(\alpha-2)(\beta-2)}$$
$$\times H(2, (\alpha-2)(\beta-2); \alpha-2, \beta-2, \frac{\alpha+\beta-\delta-1}{2}, \frac{\alpha+\beta-\delta-1}{2}, \frac{\alpha+\beta-\delta-1}{2}; t))|_C. \quad (2.8)$$

(b) Using $c = \gamma$, we have

$$\mathcal{I}H(2, \alpha\beta; \alpha, \beta, \gamma, \delta, \epsilon; t)$$
$$= \frac{2(\gamma-1)(t-1)^3}{3(\alpha-2)(\beta-2)} {}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \gamma-1; R(t) = (t-1)^2)|_C$$
$$= \frac{2(\gamma-1)}{(\alpha-2)(\beta-2)}$$
$$\times H(2, (\alpha-2)(\beta-2); \alpha-2, \beta-2, \gamma-1, \alpha+\beta-2\gamma-1, \alpha+\beta-2\gamma-1; t))|_C. \quad (2.9)$$

(c) By changing γ to ϵ in (2.9), similar relation can be obtained.

5. For polynomial $R(t) = (2t-1)^2$

(a) Using $c = (\epsilon+1)/2 = (\delta+1)/2$

$$\mathcal{I}H(1/2, \alpha\beta/2; \alpha, \beta, \gamma, \delta, \epsilon; t)$$
$$= \frac{2(\epsilon-1)(2t-1)^3}{6(\alpha-2)(\beta-2)} {}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \frac{\epsilon-1}{2}; R(t) = (2t-1)^2)|_C$$
$$= \frac{2(\epsilon-1)(2t-1)^3}{6(\alpha-2)(\beta-2)}$$
$$\times H(1/2, \frac{(\alpha-2)(\beta-2)}{2}; \alpha-2, \beta-2, \frac{\alpha+\beta-\epsilon-5}{2}, \frac{\alpha+\beta-\epsilon-5}{2}, \epsilon-2; t)|_C. \quad (2.10)$$

By changing ϵ to δ a similar expression can be obtained.

(b) Using $c = -1 + a + b - \gamma$, we obtain

$$\mathcal{I}H(1/2, \alpha\beta/2; \alpha, \beta, \gamma, \delta, \epsilon; t)$$
$$= \frac{2(\alpha+\beta-2(\gamma+2))(2t-1)^3}{6(\alpha-2)(\beta-2)}{}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \frac{\alpha+\beta-2\gamma-4}{2}; R(t) = (2t-1)^2)|_C$$
$$= \frac{2(\alpha+\beta-2(\gamma+2))(2t-1)^3}{6(\alpha-2)(\beta-2)}$$
$$\times H(1/2, \frac{(\alpha+2)(\beta+2)}{2}; \alpha-2, \beta-2, \gamma-1, \gamma-1, \alpha+\beta-2\gamma-5; t)|_C. \quad (2.11)$$

6. For polynomial $R(t) = 4t(1-t)$

(a) Using $c = \gamma$, we have

$$\mathcal{I}H(1/2, \alpha\beta/2; \beta, \alpha, \gamma, \delta, \epsilon, ; t)$$
$$= \frac{4(\gamma-1)2t^2(3-2t)}{3(\alpha-2)(\beta-2)}{}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \gamma-1; R(t) = 4t(1-t))|_C$$
$$= \frac{4(\gamma-1)2t^2(3-2t)}{3(\alpha-2)(\beta-2)}$$
$$\times H(1/2, \frac{(\alpha-2)(\beta-2)}{2}; \alpha-2, \beta-2, \gamma-1, \gamma-1, \alpha+\beta-2\gamma-1; t)|_C. \quad (2.12)$$

(b)

$$\mathcal{I}H(1/2, \alpha\beta/2; \beta, \alpha, \gamma, \delta, \epsilon, ; t)$$
$$= \frac{4(\delta-1)2t^2(3-2t)}{3(\alpha-2)(\beta-2)}{}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \delta-1; R(t) = 4t(1-t))|_C$$
$$= \frac{4(\delta-1)2t^2(3-2t)}{3(\alpha-2)(\beta-2)}$$
$$\times H(1/2, \frac{(\alpha-2)(\beta-2)}{2}; \alpha-2, \beta-2, \delta-1, \delta-1, \alpha+\beta-2\delta-1; t)|_C. \quad (2.13)$$

(c) Using $c = (1 - \epsilon + 2a + 2b)/2$

$$\mathcal{I}H(1/2, \alpha\beta/2; \alpha, \beta, \gamma, \delta, \epsilon, ; t)$$
$$= \frac{2(\alpha+\beta-\epsilon-1)2t^2(3-2t)}{3(\alpha-2)(\beta-2)}{}_2F_1(\frac{\beta-2}{2}, \frac{\alpha-2}{2}; \frac{\alpha+\beta-\epsilon-1}{2}; R(t) = 4t(1-t))|_C$$
$$= \frac{2(\alpha+\beta-\epsilon-1)2t^2(3-2t)}{3(\alpha-2)(\beta-2)}$$
$$\times H(1/2, \frac{(\beta-2)(\alpha-2)}{2}; \alpha-2, \beta-2, \frac{\alpha+\beta-\epsilon-1}{2}, \frac{\alpha+\beta-\epsilon-1}{2}, \epsilon-2; t)|_C. \quad (2.14)$$

III. CONCLUDING REMARKS AND SUGGESTIONS

In this paper, we have shown that the parameter relations obtained in the works of K. Kuiken[1] leads to some integral forms of solutions to the general Heun's equation. The multiple choise of close form solutions arises from the papameter relations. For example, consider the quadratic equation arising from the relations + = 2(a+b) and = 4ab leads to the parameter choise a = /2 and b = /2 or a = /2 and b = /2. The first leads to all the relations above while the later repeates all the relations described above by changing to . This method has being extended in the works of [2] to the work of Robert Maier [11] pp 15.

IV REFERENCES

1) [1]K. Kuiken, Heun's equations and the Hypergeometric equations, S. I. A. M. J. Math. Anal. 10(3), (1979), 655-657.
2) [2]M. N. Hounkonnou, A. Ronveaux and A. Anjorin, Derivative of Heun's equation from some properties of hypergeometric ;;,kfunction. Proceeding of International Workshop on geometry and physics Marseille France (In press, 2007).
3) [3]A. Ronveaux, Heun Di erential equation (Oxford University press, Oxford, 1995).
4) [4]A. O. Smirinov, C.R.M. Proceedings and Lecture notes 32, (2002), 287.
5) [5]N. H. Christ and T. D. Lee, Phys. Rev. D 12, (1975), 1606;
6) D. P. Jatker, C. N. Kumar, and A. Khare, Phys. Lett. A 142, (1989), 200;
A. Khare and B. P. Mandal, Phys. Lett. A 239, (1998), 197.
7) [6]R. K. Bhadari, A. Khare, J. Law, M. V. N. Murthy and D. Sen, J. Phys. A: Math. Gen. 30, (1997), 2557.
8) [7]M. Suzuki, E. Takasugi and H. Umetsu, Prog. Theor. Phys. 100, (1998), 491.
9) [8]K. Takemura, Commun. Math. Phys. 235, (2003), 467; J. Nonlinear Math. Phys. 11, (2004), 21.
10) [9]P. Dorey, J. Suzuki and R. Tateo, J. Phys. A 37, (2004), 2047.
11) [10]A. Ambramowitz and J. A. Stegun, Handbook of mathematical functions (New york, Dover, 1965).
12) [11]R. S. Maier, Heun-to-hypergeometric transformations, contribution to the conference of Foundations of Computational Mathematics 02 (2002); downloadable from http://www.math.umn.edu/Ìfocm/c−/Maier.pdf
13) [12]E. S. Cheb-Terrab, New closed form solutions in terms of pFq for families of the general, Confluent and Bi-Confluent Heun di erential equations, J. Phys. A: Math Gen. 37, (2004), 9923
14) [12]E. S. Cheb-Terrab, New closed form solutions in terms of pFq for families of the general, Confluent and Bi-Confluent Heun di erential equations, J. Phys. A: Math Gen. 37, (2004), 9923.
15) [13]A. Ronveaux, Factorisation of the Heun di erential operator, J. App. Math. Comp. 141, (2003), 177-184.
16) [14]H. Exton, A new solution of the biconfluent Heun equation, Rendiconti di Mathematica Serie VII 18, (1998), 615.
17) [15]A. Decarreau, M. C. I. Dumont-Lepage, P. Maroni, A. Robert, and A. Ronveaux, Formes canoniques des ´equations confluentes de l'quation de Heun, Ann. Soc. Sci. Bruxelles T. 92 I-II , (1978), 151-189.
18) [16]A. Ishkhanyan and K-A. Souminen, New solutions of Heun general equation, J. Phys. A: Math. Gen 36, (2003), L81-L85.
19) [17]G. Vallent, International Conference on Di erential Equations, Special Functions and Applications, Munich, (in press) 2005

A Survey on Theoretical and Applied Mathematics under the Efficiency

GJSFR Classification - F (FOR) 010299

Vasile Postolica

***Abstract*-This research paper is focused on the common concepts of the efficiency and set-valued map. After a short introduction, we propose some questions regarding the notion of efficiency and we emphasize the Pareto optimality as one of the first finite dimensional illustrative examples. We present the efficiency and the multifunctions in the infinite dimensional ordered vector spaces following also our recent results concerning the most general concept of approximate efficiency, as a natural generalization of the efficiency, with implications and applications in vector optimization and the new links between the approximate efficiency, the strong optimization - by the full nuclear cones - and Choquet's boundaries by an important coincidence result. In this way, the efficiency is strong related to the multifunctions and Potential theory through the agency of optimization and conversely. Significant examples of Isac's cones and several pertinent references conclude this study**.

Mathematics Subject Classification (2000): 91 – 02, 90 C 48.

Keywords : Efficiency, Multifunction, Pareto optimality, Optimization, Approximate Efficiency, Isac's (nuclear or supernormal) Cone,Choquet boundary.

I. Introduction.

Throughout in our Life, the Efficiency was, it is and it will remain essential for the Existence and not only. Its mathematical models were unanimously accepted in various knowledge fields. Also, we propose it as a New Frontier in Mathematical Physics and Engineering in the World context of priorities concerning the Alternative Energies, the Climate Exchange and the Education. To it we dedicate our modest scientific contribution. The content of this research work is organized as follows: Section 2 is dedicated to some useful matters on the efficiency. We briefly present in Section 3 the Pareto Optimality as one of the main starting points for the mathematical modelling of the Economical Efficiency. In Section 4, conceived as a concise Survey, we present the most general notion of Approximate Efficiency and its particular case of the usual Efficiency, with the immediate connections to Multifunctions, in the Infinite Dimensional Ordered Vector Spaces, completed by the coincidence between Choquet's Boundaries and the Approximate Efficient Points Sets in Ordered Hausdorff Locally Convex Spaces, this conclusion being based on the first result established by us concerning such a property as this for Pareto type efficient points sets and the corresponding Choquet boundaries. Our results represent strong relationships between important Great Fields of Mathematics and the Human Knowledge: Vector and Strong Optimization, Set-valued Maps, the Axiomatic Theory of Potential together with its Applications and the Human Efficiency. Finaly, we indicate the selected bibliography which refers only to the papers which were used. Sections 2, 3 and a part of Section 4 are presented following [74]

Romanian Academy of Scientists Bacău State University, Faculty of Sciences,
Department of Mathematics and Computer Science, ROMANIA,
E-mail: vpostolica@ambra.ro

II. Some Selected Questions on the Efficiency.

First of all, we present a short survey on the efficiency.The current language defines the efficiency as "the ability to produce the desired effect in dealing with any problem". In the actual world characterized by: globalization which has generated efficiency gains, liberalisation, individualization, informatization, informalization ([78] and so on), the efficiency is perceived as follows: "working well, quickly and without waste". But, our life has its Divine efficiency and the projection in the reality deals with a lot of kind of descriptions: information efficiency, energy efficiency, eco - efficiency (see, as recent references, [42], [43], [50] and so on), home energy efficiency, water efficiency market enhancement programs, sports and efficiency, economic efficiency (agricultural and industrial efficiency, human efficiency in business, efficiency in financial and betting markets, efficiency in capital asset pricing models, etc.), efficiency in Mathematics (efficient algorithms in computational complexity, efficiency frontier in data envelopment analysis, statistics and so on, efficiency in multi – objective, stochastic and goal programming, efficiency in mathematical economics, etc.), efficiency in medicine, technical efficiency, etc., all of them being based on the fundamental question: "whose valuations do we use, and how shall they be weighted ?"[28]. Thus, the economic efficiency is characterized by the "optimal" relationships between the value of the ends (the physical outputs) and the value of the means (the physical inputs), both of them "measured" by the money. Hence, the monetary evaluations are essential for the economic efficiency which usually is associated with the sustainability defined as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs" to achieve a good society ([11], p.43). However, the goals of efficiency and sustainability might not be enough to ensure a positive social development because the economic efficiency does not include any component of it (see, for example, the distribution of goods) ([54], p.13) and the sustainability was not very clear defined [77]. In terms of the technical efficiency, a machine is considered to be useful or “more efficient than another” when it generates “more work output per unit of energy input”. But, it is not possible to speak about complete efficiency in any process since we have not developed yet workable procedures concerning the corresponding evaluations (see, for instance, the urban automobile traffic which engages different people from many points of view; in general, the transport systems are highly complex with different goals often conflict). In fact, in all the processes, we try to approximate the real efficiency by some kinds of fuzzy or relative efficiency in order to obtain several controls on it, taking into account its complexity. For example, one of the main purposes of the data envelopment analysis is to “to measure” the relative efficiency and the comparison of decision making units by the estimation of “the distance” between the evaluated real units and the virtual units (see, for instance, [42]). Another strong argument and a significant example in this direction is represented by the permanent measure and the continuous supervision of the interest rate risk to obtain the optimal efficiency balance in the management of banks’ assets and liabilities (see, for example, the pertinent asset liability management model together with the simulation analysis given in [43]). Generally speaking, from the decisional point of view, the main steps to obtain the efficiency are the following:

2.1. Wording of the problems by the managerial staff in an adequate language for mathematicians and computer scientists because the “bilateral” dialogue is absolutely necessary to solve the programs through the agency of such as these cooperations.

2.2. The elaboration of the appropriate mathematical models and the corresponding numerical processings.

2.3. The selection , by a serious study, of the best multicriteria decisions.

2.4. The application of them.

2.5. The evaluation of the efficiency.

We must remark that the multicriteria decision aid ([20], [78]) and the decision making under uncertainty viewed as an important area of decision making and recognized as “ the fact that in certain situations a person does not have the information which quantitatively and qualitatively is appropriate to describe, prescribe or predict determinastically and numerically a system, its behaviour or other characteristics”, that is,” it relates to a state of the human mind” characterized by the “lack of complete knowledge about something” [89] represents the real risk for the efficiency which, in our opinion, generates it. Following [89], we remember that the term of ”risk” was initially applied for “the situations in which the probabilities of outcomes were objectively known”. In accordance with [22] and [79] now this concept “means a possibility of something bad happening” and the uncertainty ”is applied to the problems in which real alternatives with several possible outcomes exist”. Another important matter is the efficacy defined as “the quality of being efficacious” in the sense of “producing the desired effects or results”. In our opinion, this means to be efficient step by step, that is, a discrete efficiency with appropriate links between the stages

.

III. Pareto Optimality: Anillustrative Example of Efficiency.

Not even for the market economies there exists no an universal mathematical model. Pareto efficiency or Pareto optimality, the term being named for an Italian economist Vifredo Pareto [56], is a central theory in economics with broad applications in game theory, engineering and the social sciences. Pareto efficiency is important because it provides a weak but widely accepted standard for comparing the economic outcomes.It’s a weak standard since there may be many efficient situations and Pareto’s test doesn’t tell us how to choose between them. Any policy or action that makes at least one person better off without hurting anyone is called a Pareto improvement. From the Mathematics point of view, the Pareto efficiency represents the actual finite dimensional part of the multiobjective programming in vector optimization. Thus, whenever a feasible deviation from a genuine solution S of an arbitrary multiobjective programme generates the improvement of at least one of the objectives while some other objectives degrade, any such a solution S as this is called efficient or nondominated. A system in economics and in politics is called Pareto efficient whenever “no individual can be made better off without another being made worse off” that is, a social state is economically efficient, or Pareto optimal, provided that “no person in society can become better off without anyone else becoming worse off”. This characteristic of Pareto type efficiency has been pointed out in [1] - [3], [19], [25], [56], [82] and the others. In terms of the alternative allocations this means that given a set of alternative allocations and a set of individuals, any movement from one alternative allocation to another that can make at least one individual better off, without making any other individual worse off is called a Pareto improvement or a Pareto optimization. An allocation of resources is named Pareto efficient or Pareto optimal whenever no further Pareto improvements can be made. If the allocation is strictly preferred by one person and no other allocation would be as good for everyone, then it is called strongly Pareto optimal. A weakly Pareto optimal allocation is one where anyfeasible reallocation would be strictly preferred by all agents [89]. Consequently, Pareto type efficiency is an important approximate criterion for evaluating the economic systems and the political policies, with minimum assumptions on the interpersonal comparability. We said “approximate criterion” because it asks “the ideal” which may not reflect, for example, the workings of real economics, thanks to the folowing restrictive assumptions necessary for the existence of Pareto efficient outcomes: the markets exist for all possible goods, being perfectly competitive and the transaction costs are negligible. In the political policies, not every Pareto efficient outcome is regarded as desirable (see, for instance, the strategies based on the unilateral benefits). For these reasons, Pareto optimality was accepted with some or much uncertainty and controversy, but, by the Arrow's renowned impossibility theorem given in 1951 according to which "no social preference ordering based on individual orderings only could satisfy a small set of very reasonable conditions"- the Pareto criterion being one of them, it remains "plausible and uncontroversial" [78]. Usually, the concept of efficiency replaces the notion of optimality in multiple criteria optimization because whenever the solutions of a multiple – objectives program exist in Pareto'sense, they cannot be improved following the ordering induced by the cone

IV APPROXIMATE EFFICIENCY, EFFICIENCY AND MULTIFUNCTIONS IN INFINITE DIMENSIONAL ORDERED VECTOR SPACES WITH RECENT CONNECTED RESULTS

Seemingly, the concept of efficiency is equivalent to the optimality, as we can see from the next abstract construction. In reality, the optimality represents a particular case of the efficiency, that is, "the best approximation" of all the efficient points. Let X be a real or complex ordered vector space, let K be the class of all convex cones defined on X and let A be an arbitrary, non-empty subset of X with respect to an arbitrary $K \in$ K is in the following relation with the "vectorial" minimization or maximization

$$Eff(A) \supseteq \bigcup_{K \in K} MIN_K(A) \cup \bigcup_{K \in K} MAX_K(A).$$

Clearly, any vector optimization program (which has its origin in the usual ordered Euclidean spaces programs thanks to Pareto optimality of vector-valued real functions) includes the corresponding strong optimization program and allows to be described as one of the next optimization problems. Moreover, it is possible to replace the convex cone K by any other convenient, non-empty subset of X:

$$(P_{T,K}) : MIN_K f(T) \text{ or } (P_{T,K}) : MAX_K f(T)$$

where $K \in$ K, T is a given non-empty set and $f : T \to X$ is an appropriate application. If

one denotes by $S_f(T, K)$ the corresponding set of solutions, then the announced

equivalence seems to be justified by the following relation:

$$\bigcup_{\emptyset \neq A \subseteq X} Eff(A) = \bigcup_{\substack{T \neq \emptyset \\ K \in K, f:T \to X}} S_f(T, K)$$

but, in reality, only the next inclusion is valid :

$$\bigcup_{\emptyset \neq A \subseteq X} Eff(A) \supseteq \bigcup_{\substack{T \neq \emptyset \\ K \in K, f:T \to X}} S_f(T, K)$$

thanks to the refinement of the notion of efficiency which can not be totally described for the moment by mathematical means, that is, the final agreement between "the dictionary's concept of efficiency" and "the corresponding notion in mathematics" was not signed yet.

Now, let X be a non-empty set, let E be a vector space ordered by a convex, pointed cone K and let $f : X \to E$ be a function. We consider the next Vector Optimization Problem:

$$(P) \begin{cases} \min f(x) \\ x \in X \end{cases},$$

To solve it means to identify all the efficient points $x_0 \in X$ in the sense that $f(X) \cap [f(x_0) - K] = \{f(x_0)\}$. In all these cases, x_0 is *an efficient solution* and $f(x_0)$ is called *a nondominated point* of the program (P) for Multicriteria Optimization Programs in Finite Dimensional Vector Spaces. If one replaces K by K\{0} or int(K), then x_0 is called strictly(weakly) efficient for X, respectively, and $f(x_0)$ is named strictly (weakly) nondominated. Consequently, whenever E is an usual ordered Euclidean space, the above concepts for x_0 means *Pareto optimal*, *strictly Pareto optimal* and *weakly Pareto optimal solution*, respectively, with the corresponding nondominated concepts for f (see, for a recent instance, [21]). So, concerning the practical vector - optimization problems, it is important to know when the set of all efficient points is non-empty, to establish its main properties (existence, domination, connectedness, compactness, density in varied topologies, etc.) and to extend the concepts together with the results to multicriteria optimization in infinite dimensional ordered vector spaces. The proper efficiency introduced in [47] and developped in [5] – [7], [9], [10], [18], [24], [26], [27], [37], [40], [53], etc. appears as a refined case of the efficiency, that is, the set of all properly efficient solutions and the set of all positive proper efficient solutions of problem (P) are subsets of the set containing all the efficient points and the study of them has been proposed in order to eliminate some "undesirable" efficient solutions. We recall that $x_o \in X$ is *a properly efficient solution* of (P) if it is *an efficient point* and $cl[cone(f(X) + K - \{f(x_o)\})] \cap K = \{0\}$; x_o is a positive proper efficient solution of (P) if there exists a linear continuous functional φ on E such that $\varphi(k) > 0$ for every $k \in K$ and $\varphi[f(x_o)] \leq \varphi[f(x)]$ for all $x \in X$. This section deals with a new generalization of the efficiency, named by us the approximate efficiency, in ordered Hausdorff locally convex spaces. All the elements concerning the ordered topological vector spaces used here are in accordance with [57].

Let E be a vector space ordered by a convex cone K, K_1 a non-void subset of K and A a non-empty subset of E. The following definition introduces a new concept of approximate efficiency which generalizes the well known notion of Pareto efficiency.

Definition 1.[73]. *We say that* $a_0 \in A$ *is a* K_1 *- Pareto (minimal) efficient point of* A, *in notation,* $a_0 \in eff(A, K, K_1)$ $\left(or\ a_0 \in MIN_{K+K_1}(A)\right)$ *if it satisfies one of the following equivalent conditions:*

(i) $A \cap (a_0 - K - K_1) \subseteq a_0 + K + K_1$;

(ii) $(K + K_1) \cap (a_0 - A) \subseteq -K - K_1$;

In a similar manner one defines the Pareto (maximal) efficient points by replacing $K + K_1$ with $-(K + K_1)$. Clearly,

$$A \cap (a_0 - K) \subseteq a_0 + K_1 \Rightarrow A \cap (a_0 - K - K_1) \subseteq a_0 + K + K_1 \Rightarrow A \cap (a_0 - K_1) \subseteq a_0 + K,$$

which suggests other concepts for the approximate efficiency in ordered linear spaces.

Remark 1. The notion of *approximate efficiency* in the sense of the above definition is *the most general notion* of *approximate efficiency* introduced until now. We also remark that $a_0 \in eff(A, K, K_1)$ iff it is a *fixed point* for *the multifunction* $F : A \to A$ defined by $F(t) = \{a \in A : A \cap (a - K - K_1) \subseteq t + K + K_1\}$. Consequently, for the existence of the Pareto type efficient points we can apply appropriate fixed points theorems for multifunctions (see, for instance, [13], [55], [88] and any other proper scientific papers) and we need the next usual continuity properties of multifunctions. Namely, if X and Y are two topological spaces and $f : X \to 2^Y$ is a set-valued map, then: f is *upper semicontinuous (usc)* if for any $x \in X$ and any open set $D \supseteq f(x)$ we have $D \supseteq f(x_1)$ for all x_1 in some neighbourhood $V(x)$ of x. The multifunction f is *lower semicontinuous (lsc)* if for any $x \in X$ and every open set D with $f(x) \cap D \neq \emptyset$ it follows that $f(x_1) \cap D \neq \emptyset$ for all x_1 from some neighbourhood $V(x)$ of x. The map f is *continuous* iff it is both usc and lsc. Whenever *the graph* of f defined by $gr(f) = \{(x, y) \in X \times Y : y \in f(x)\}$ is a *closed (open)* set of $X \times Y$ one says that f has a closed (open) graph. Any multifunction having a closed graph is also called *closed*. If $f(x)$ is a closed (compact) subset of Y for any $x \in X$, then f is *a closed-valued (compact-valued)* map. The set-valued map f is called *compact* if

$im(f) = f(X) = \bigcup_{x\in X} f(x)$ is contained in a compact subset of Y. A topological space is *acyclic* if all of its reduced Čech homology groups over rationals vanish (see for example, the contractible spaces; in particular, the convex sets and the star-shaped sets are acyclic). The multifunction f is *acyclic* if it is usc and $f(x)$ is non-empty, compact and acyclic for all $x\in X$. The next theorem is useful to establish the existence of the (approximate) efficient points taking into account the above mentioned connection with the fixed points of multifunctions.

Theorem 1. [55]. *If A is any non-empty convex subset of an arbitrary Hausdorff separated locally convex space and $F\ A\to 2^A$ a compact acyclic multifunction, then F has a fixed point, that is, there exists $a_0\in A$ such that $a_0\in F(a_0)$.*

Remark 2. In [52] it was proved that whenever $K_1\subset K\setminus\{0\}$, the existence of this new type of efficient points for bounded from below sets characterizes the semi -Archimedian ordered vector spaces and the regular ordered locally convex spaces.

Remark 3. When K is pointed, that is, $K\cap(-K)=\{0\}$, then $a_0\in eff(A,K,K_1)$ means that $A\cap(a_0-K-K_1)=\emptyset$ or, equivalently, $(K+K_1)\cap(a_0-A)=\emptyset$ for $0\notin K_1$ and $A\cap(a_0-K-K_1)=\{a_0\}$, respectively, if $0\in K_1$. Whenever $K_1=\{0\}$, from Definition 1 one obtains the usual concept of *efficient (Pareto minimal, optimal or admissible) point* :

$a_0\in eff(A,K)\ (or\ a_0\in MIN_K(A))$ if it fulfils *(i), (ii)* or any of the next equivalent properties:

(iii) $(A+K)\cap(a_0-K)\subseteq a_0+K$;

(iv) $K\cap(a_0-A-K)\subseteq -K$

These relations show that a_0 is *a fixed point* for at least one of the following *multifunctions*:

$$F_1:A\to A, F_1(t)=\{\alpha\in A: A\cap(\alpha-K)\subseteq t+K\},$$

$$F_2:A\to A, F_2(t)=\{\alpha\in A: A\cap(t-K)\subseteq \alpha+K\},$$

$$F_3:A\to A, F_3(t)=\{\alpha\in A:(A+K)\cap(\alpha-K)\subseteq t+K\},$$

$$F_4:A\to A, F_4(t)=\{\alpha\in A:(A+K)\cap(t-K)\subseteq \alpha+K\},$$

that is, $a_0 \in F_i(a_0)$ for some $i = \overline{1,4}$. If, in addition, *K* is pointed, then $a_0 \in A$ is an efficient point of *A* with respect to *K* if and only if one of the following equivalent relations holds:

(v) $A \cap (a_0 - K) = \{a_0\}$;

(vi) $A \cap (a_0 - K \setminus \{0\}) = \emptyset$;

(vii) $K \cap (a_0 - A) = \{0\}$;

(viii) $(K \setminus \{0\}) \cap (a_0 - A) = \emptyset$;

(ix) $(A+K) \cap (a_0 - K \setminus \{0\}) = \emptyset$.

and we notice that $eff(A,K) = \bigcap_{\{0\} \neq K_2 \subseteq K} eff(A,K,K_2)$. Moreover, $a_0 \in eff(A,K)$ iff it is *a critical (equilibrium) point* ([33], [34], [69], [72]) for the *generalized dynamical system* $\Gamma : A \to 2^A$ defined by $\Gamma(a) = A \cap (a - K), a \in A$. Thus, $eff(A,K)$ describes the moments of equilibrium for Γ and *the ideal equilibria* are contained in this set. In particular, this kind of critical points generates the special class of critical boundaries for dynamical systems

represented by Pareto type boundaries.Taking $K_1 = \{\varepsilon\}$ ($\varepsilon \in K \setminus \{0\}$), it follows that $a_0 \in eff(A,K,K_1)$ iff $A \cap (a_0 - \varepsilon - K) = \emptyset$. In all these cases, the set $eff(A,K,K_1)$ is denoted by $\varepsilon - eff(A,K)$ and it is obvious that $eff(A,K) = \bigcap_{\varepsilon \in K \setminus \{0\}} [\varepsilon - eff(A,K)]$.

Concerning existence results on the efficient points and significant properties for the efficient points sets we refer the reader to [12], [33] – [39], [48], [49], [52], [53], [61] – [72], [81], [85], [86] for a survey.The following theorem offers the first immediate connection between the strong optimization and this kind of approximate efficiency, in the environment of the ordered vector spaces.

Theorem 2. [41]. *If we denote by* $S(A,K,K_1) = \{a_1 \in A : A \subseteq a_1 + K + K_1\}$ *and* $S(A,K,K_1) \neq \emptyset$ *, then* $S(A,K,K_1) = eff(A,K,K_1)$.

Remark 4. We shall denote by $S(A,K)$ the set $S(A,K,\{0\})$. If $S(A,K,K_1) \neq \emptyset$, then $K + K_1 = K$, hence $eff(A,K,K_1) = eff(A,K)$. Indeed, let $a \in S(A,K,K_1)$. Then, $a \in a + K + K_1$ which implies that $0 \in K + K_1$. Therefore, $K \subseteq K_1 + K + K = K_1 + K \subseteq K$.

The above theorem shows that, for any non-empty subset of an arbitrary vector space, the set of all strong minimal elements with respect to any convex cone through the agency of every non-noid subset of it coincides with the corresponding set of the efficient points, whenever there exists at least a strong minimal element. Obviously, the result remains valid for the strong maximal elements and the corresponding efficient points, respectively. Using this conclusion and the abstract construction given in [59] and [66] for the splines in the H-locally convex spaces introduced in [75] as separated locally convex spaces $(X, P = \{p_\alpha : \alpha \in I\})$ with any semi- norm p_α $(\alpha \in I)$satisfying the parallelogram law:

$p_\alpha^2(x+y) + p_\alpha^2(x-y) = 2[p_\alpha^2(x) + p_\alpha^2(y)]$ whenever $x, y \in X$, linear topological spaces also studied in [45], it follows that the only best simultaneous and vectorial approximation for each element in the direct sum of any (closed) linear subspace and its orthogonal, with respect to any linear (continuous) operator between two arbitrary H-locally convex spaces, is its spline function. We also note that it is possible to have $S(A, K, K_1) = \emptyset$ and $eff(A, K, K_1) = A$. Thus, for example, if one considers $X = R^n (n \in N, n \geq 2)$ endowed with the separated H - locally convex topology generated by the semi - norms $p_i : X \to R_+$,

c

$p_i(x) = |x_i|, \forall\ x = (x_i) \in X$, $i = \overline{1, n}$, $K = R_+^n$, $K_1 = \{(0, ..., 0)\}$ and for each real number we define $A_c = \left\{(x_i) \in X : \sum_{i=1}^{n} x_i = c\right\}$, then it is clear that $S(A_c, K, K_1)$ is empty and $eff(A_c, K, K_1) = A_c$.

At the same time, in the usual real linear space of all sequences, ordered by the convex cone $K = \{(x_n) : n \in N^*, n \geq 2, x_n \geq 0, \forall\ n \geq 2\}$, for $A = \{(x_{n\alpha}) : n \in N^*, n \geq 2, \alpha > 0\}$ with $x_{n\alpha} = (n-1)^{-\alpha} - n^{-\alpha}, n \in N^*, n \geq 2, \alpha > 0$ and $K_1 = \{(0, 0, ...)\}$ we have

$$Eff(A, K, K_1) = S(A, K, K_1) = \emptyset .$$

In all our further considerations we suppose that X is *a Hausdorff Locally Convex Space* having *the topology* induced by a family $P = \{p_\alpha : \alpha \in I\}$ of *semi-norms*, *ordered* by *a convex cone* K and *its topological dual space* X^* In this framework, the next theorem contains a significant criterion for the existence of the approximate efficient points, in defined by $K^* = \{x^* \in X^* : x^*(x) \geq 0, \forall x \in K\}$ and *its attached polar cone* is $K^0 = -K^*$. The cone K is called *Isac's cone* (*supernormal or nuclear in* [33], [34]) if for every seminorm $p_\alpha \in P$ there exists $f_\alpha \in X^*$ such that $p_\alpha(k) \leq f_\alpha(k)$ for all $k \in K$ .Moreover, if $\varphi : P \to K^*$ is a function, then the convex cone

$K_\varphi = \{x \in X : p_\alpha(x) \leq \varphi(p_\alpha)(x), \forall p_\alpha \in P\}$ is the *full nuclear cone* associated to K, P and φ [39]. A characterization of the supernormality by the full nuclearity is gven in the next Remark 7.

Theorem 3. [41]. *If A is any non-empty subset of X and* K_1 *is every non-void subset of K, then* $a_0 \in eff(A, K, K_1)$ *whenever for each* $p_\alpha \in P$ *and* $\eta \in (0,1)$ *there exists* x^* *in the polar cone* K^0 *of K such that* $p_\alpha(a_0 - a) \leq x^*(a_0 - a) + \eta, \forall a \in A$.

Remark 5. The above theorem represents an immediate extension of Proposition 1.2 in [76]. Generally, the converse of this theorem is not valid at least in partially ordered separated locally convex spaces as we can see from the example considered in Remark 4. Indeed, if one assumes the contrary in the corresponding mathematical background then, taking $\eta = \frac{1}{4}$, it follows that for each $\lambda_0 \in [0,1]$, there exists $c_1, c_2 \leq 0$ such that

$|\lambda_0 - \lambda| \leq (c_1 - c_2)(\lambda_0 - \lambda) + \frac{1}{4}, \forall \lambda \in [0,1]$. Taking $\lambda_0 = \frac{1}{4}$ one obtains $|1 - 4\lambda| \leq (c_1 - c_2)(1 - 4\lambda) + 1, \forall \lambda \in [0,1]$ which for $\lambda = 0$ implies that $c_2 \leq c_1$ and for $\lambda = \frac{1}{2}$ leads to $c_1 \leq c_2$, that is, $|1 - 4\lambda| \leq 1, \forall \lambda \in [0,1]$, a contradiction.

Remark 6. If $a_0 \in A$ and for every $p_\alpha \in P$, $\eta \in (0,1)$ there exists $x^* \in K^0$ such that $p_\alpha(a_0 - a) \leq x^*(a_0 - a) + \eta$, $\forall\ a \in A$, then $K \cap (a_0 - A) = \{0\}$ even if K is not pointed. Indeed, if $x \in K \cap (a_0 - A)$, then $a_0 - x \in A$ and for each $p_\alpha \in P$ and $\eta \in (0,1)$ there exists $x^* \in K^0$ with $p_\alpha(x) = p_\alpha(a_0 - (a_0 - x)) \leq x^*(x) + \eta \leq \eta$ .Because η is arbitrarily chosen in $(0,1)$, we obtain $p_\alpha(x) = 0$ and since X is separated it follows that $x=0$. If $0 \in K+K_1$, then $K+K_1=K$ and $0 \notin K+K_1$ implies that $(K+K_1) \cap (a_0 - A) = \emptyset$. Consequently, $a_0 \in eff(A,K,K_1)$ in both cases and in this way we indicated also another proof of the theorem.The beginning and the considerations in Section 4 of [39] suggested us to consider for each function $\varphi : P \to K^* \setminus \{0\}$ *the full nuclear cone*

$K_\varphi = \{x \in X : p_\alpha(x) \leq \varphi(p_\alpha)(x), \forall p_\alpha \in P\}$ in order to give the next generalization of Theorem 7 indicated in [38].

Theorem 4. [41]. *If* $0 \in K_1$ *and there exists* $\varphi : P \to K^* \setminus \{0\}$ *with* $K \subseteq K_\varphi$, *then*

$$eff(A,K,K_1) = \bigcup_{\substack{a \in A \\ \varphi \in P \to K^* \setminus \{0\}}} S(A \cap (a - K - K_1), K_\varphi)$$

for any non-empty subset K_1 *of K.*

Remark 7. If $0 \notin K_1$, then $a_0 \in eff(A,K,K_1)$ implies that $A \cap (a_0 - K - K_1) = \emptyset$. Therefore, it is not possible to have $a_0 \in S(\emptyset, K_\varphi)$.In case of $0 \in K_1$, then $eff(A,K,K_1)=eff(A,K)$ and a $_0 \in eff(A,K)$ iff $A \cap (a_0 - K) = \{a_0\}$, so in the right member of the first proved inclusion it can be selected any convex cone, not necessary K_φ .The hypothesis $K \subseteq K_\varphi$ imposed upon the convex cone K is automatically satisfied whenever K is an *Isac's (nuclear or supernormal)* cone ([33] – [37], [39]) and it was used only to prove the inclusion

$$eff(A,K,K_1) \subseteq \bigcup_{a \in A, \varphi : P \to K^* \setminus \{0\}} S(A \cap (a - K - K_1), K_\varphi).$$

Moreover, K is an Isac's cone if and only if there exists $\varphi : P \to K^* \setminus \{0\}$ such that $K \subseteq K_\varphi$. Indeed, Lemma 5 of [39] ensures the necessity of the above inclusion condition.

Conversely, since for every seminorm $p_\alpha \in P$ there exists $\varphi(p_\alpha) \in K^* \setminus \{0\}$ and for any x $\in K \subseteq K_\varphi$ it follows that $p_\alpha(x) \leq \varphi(p_\alpha)(x)$, we conclude the nuclearity of K . When K is an arbitrary pointed convex cone, A is a non-empty subset of X and $a_0 \in eff(A,K)$, then, by virtue of (v) in Remark 3, we have $A \cap (a_0 - K) = \{a_0\}$, that is, $A \cap (a_0 - K) - a_0 = \{0\} \subset K_\varphi$. Hence, $a_0 \in S(A \cap (a_0 - K), K_\varphi)$ for every mapping $\varphi : P \to K^* \setminus \{0\}$ and the next corollary is valid.

Corollary 4.1. *For every non-empty subset A of any Hausdorff locally convex space ordered by an arbitrary, pointed convex cone K with its dual cone* K^* *we have*

$$eff(A,K) = \bigcup_{\substack{a \in A \\ \varphi : P \to K^* \setminus \{0\}}} S(A \cap (a - K), K_\varphi)$$

Remark 8. The hypothesis of Theorem 4 together with Lemma 3 of [39] involves K to be pointed. Consequently, $0\in K_1$ iff $0\in K+K_1$. If $a_0\in S(A\cap(a-K-K_1),K_\varphi)$ for some $\varphi:P\to K^*$ and $a\in A$ with $a_0=a-k-k_1$, $k\in K$, $k_1\in K_1$, then $K\cap(a_0-A)=\{0\}$ because $A\cap(a-K-K_1)\subseteq a_0+K_\varphi$ in any such a case as this. Indeed, let $x\in K\cap(a_0-A)$ be an arbitrary element. Then, $a_0-x\in A$ and $a_0-x=a-k-k_1-x\in a-K-K_1$.Therefore, $a_0-x\in a_0+K_\varphi$, that is, $x\in K_\varphi$. For every $p_\alpha\in P$ we have $p_\alpha(-x)\leq\varphi(p_\alpha)(-x)=-\varphi(p_\alpha)(x)\leq 0$. Since p_α was arbitrary chosen in P and X is a Hausdorff locally convex space, it follows that $x=0$.

Remark 9. Clearly, the announced theorem represents a significant result concerning the possibilities of scalarization for the study of some Pareto efficiency programs in separated locally convex spaces, as we can see also in the final comments of [39] for the particular cases of Hausdorff locally convex spaces ordered by closed, pointed and normal cones.

Remark 10. As an open problem, it is interesting to replace K_1 with any non-empty subset of an ordered linear space X, under proper hypotheses.

Remark 11. It is well known that the Choquet boundary represents a basic concept in the axiomatic theory of potential and its applications and the efficiency is a fundamental notion in vector optimization. The main aim of the last part in this section is to indicate the recent generalization of our coincidence result established in [12] between the set of all Pareto type minimum points of any non-empty, compact set in an ordered Hausdorff locally convex space and the Choquet boundary of the same set with respect to the convex cone of all real, increasing and continuous functions defined on the set, using our new concept of approximate efficiency. Following this line, firstly the Choquet boundary concept is revised in an original manner.

Let us consider an arbitrary Hausdorff locally space (E,τ), where τ denotes its topology and let K be any closed, convex, pointed cone in E. The usual order relation $\leq_K$ associated with K is defined by $x\leq_K y$ ($x,y\in E$) if there exists $k\in K$ with $y=x+k$. Clearly, this order relation on E is closed, that is, the set G_K given by $G_K=\{(x,y)\in E\times E : x\leq_K y\}$ is a closed subset of $E\times E$ endowed with the induced product topology. If S is any convex cone satisfying the properties:

a) $\forall x\in X, \exists s\in S, s>0$ and $s(x)<+\infty$;

b) S linearly separates $X_1 = \{x \in X : \exists s \in S \text{ with } s(x) < 0\}$, that is, for every $x, y \in X_1, x \neq y$, there exists $s, t \in S$ with real values in x and y such that $s(x)t(y) \neq s(y)t(x)$, then, on the set $M_+(X)$ of all positive Radon measures defined on X, one associates the following natural pre-order relation: if $\mu, \upsilon \in M_+(X)$, then $\mu \leq_S \upsilon$ means that $\mu(s) \leq \upsilon(s)$ for all $s \in S$.

Let S_1 be the convex cone of all lower semicontinuous and bounded from below real functions s on X having the next property: if $x \in X$ and $\mu \leq_S \varepsilon_x$, where $\varepsilon_x(f) = f(x)$ for every real continuous function f on X denotes the Dirac measure, implies that $\mu(s) \leq s(x)$.

Any non-empty subset $T \subseteq X$ will be called

S - *boundary* if, whenever $s \in S_1$ and its restriction on T denoted by $s_{/T}$ is positive, it follows that $s \geq 0$. The smallest, closed

S - boundary is usually called *the Silov boundary of X with respect to S.* A closed set $A \subseteq X$ is called S - *absorbent* if $x \in A$ and $\mu \leq_S \varepsilon_x$ implies that $\mu(X \setminus A) = 0$. The set

$\partial_S X = \{x \in X_1 : \{x\} \text{ is } S\text{ - absorbent}\}$ is named *the Choquet boundary of X with respect to S* and clearly its closure coincides with *the Silov boundary* of X with respect to S. The trace on $\partial_S X$ of the topology on X in which the closed sets coincide with X or with any of the S-absorbent subsets of X contained in X_1 is usually called *the Choquet topology of* $\partial_S X$.

Definition 2.[73]. *A real function* $f : X \to R$ *is called* $(K + K_1)$ *- increasing if*

$f(x_1) \geq f(x_2)$ *whenever* $x_1, x_2 \in X$ *and* $x_1 \in x_2 + K_1 + K$.

It is obvious that every real increasing function defined on any linear space ordered by an arbitrary convex cone K is $K+K_1$ - increasing, for each non-empty subset K_1 of K .

Now, we present the coincidence of the approximate efficient points sets and the Choquet boundaries, which generalizes the main results given in [12] and [63], respectively, and can not be obtained as a consequence of the Axiomatic Potential Theory.

Theorem 5. [74]. *If A is any non-void, compact subset of X and*

(i) K is an arbitrary, closed, convex, pointed cone in X;

(ii) K_1 *is a non-empty subset of K such that* $K + K_1$ *is closed with respect to the Hausdorff separated locally convex topology on* X.

Then, $eff(A, K, K_1)$ *coincides with the Choquet boundary of A with respect the convex cone* S_1 *of all* $K + K_1$ *- increasing real continuous functions on A. Consequently, the set*

$eff(A,K,K_1)$ endowed with the corresponding trace topology is a Baire space and, if (A,τ_A) is metrizable, then $eff(A,K,K_1)$ is a G_δ - subset of X.

Corollary 5.1.

(i) $eff(A,K,K_1) = \{a \in A : f(a) = \sup\{f(a') : a' \in A \cap (a - K - K_1)\}$ for all $f \in C(A)\}$;

(ii) $eff(A,K,K_1)$ and $eff(A,K,K_1) \cap \{a \in A : s(a) \le 0\}$ $(s \in S)$ are compact sets with respect to Choquet's topology;

(iii) $eff(A,K,K_1)$ is a compact subset of A.

(iv) under the above hypotheses (i) and (ii) in Theorem 5, for any non-empty, compact set A in X such that (A,τ_A) is metrizable and every non-void G_δ - subset T of A there exists a convex cone S of $K + K_1$ - increasing real continuous functions on A which contains the constants and separates the points of A such that T coincides with $eff(A,K,K_1)$.

Corollary 5.2. *Under the hypotheses of the above theorem we proved that*

$$eff(A,K,K_1) = \{a \in A : f(a) = \bar{f}(a), \forall f \in C(A)\} = \partial_{S_1} A = \{a \in A : a \in F(a)\}$$

where $F : A \to A$ is the multifunction defined by

$$F(t) = \{a \in A : A \cap (a - K - K_1) \subseteq t + K + K_1\}, \forall t \in A.$$

Remark 12.

In general, $eff(A,K,K_1)$ coincides with the Choquet boundary of *A* only with respect to the convex cone of all real, continuous and $K + K_1$ - increasing functions on *A*. Thus, for example, if *A* is a non-empty, compact and convex subset of *X,* then the Choquet boundary of *A* with respect to the convex cone of all real, continuous and concave functions on *A* coincides with the set of all extreme points for *A*. But, it is easy to see that, even in finite dimensional cases, an extreme point for a compact convex set is not necessary an efficient point and conversely.

Remark 13.

As we have already specified before Theorem 1, there exists more general conditions than compactness imposed upon a non-empty set *A* in a separated locally convex space ordered by a convex cone *K* ensuring that $eff(A,K) \neq \emptyset$. Perhaps our coincidence result suggests a natural extension of the Choquet boundary at least in these cases. Anyhow, Theorem 5 represents an important link between vector optimization and potential theory and a new way for the study of the properties of efficient points sets and the Choquet boundaries. Indeed, one of the main question in potential theory is to find the Choquet boundaries. This fact is relatively easy for particular cases but, in general, it is an unsolved problem. Since in a lot of cases the efficient points sets contain dense subsets which can be identified by adequate numerical optimization methods, it is possible to determine the corresponding Choquet boundaries in all these situations. In this direction of study, an important role is attributed to the density properties of the efficient points sets with respect to varied topologies. Consequently, our coincidence result has its practical consequences at first for the axiomatic theory of potential and its applications. At the same time, by the above coincidence result, the Choquet boundaries offer important properties for the efficient points sets. In this way, the above coincidence result establishes a strong relationship between the approximate (in particular, strong) solutions for vector optimization programs in separated, ordered topological vector spaces and Choquet's boundaries of non-empty compact sets. Similar to the Choquet integral considered as an important risk measure [79], Choquet's boundary represents a very significant efficiency mathematical model. Nevertheless, even if these both concepts belong to Choquet, they are completely different at least because Choquet's boundary is defined as a non-empty set with respect to a convex cone and Choquet's integral is considered a measure. Thus, any possible connection between Choquet's boundary and Choquet's integral represents a genuine new open problem.

5. Isac's (nuclear or supernormal) cones

Throughout the research works devoted to nuclear (supernormal) cones professor Isac considered any locally convex space in the sense of the next definition.

Definition 1. *(Treves, F., 1967). A locally convex space is any couple* $(X, Spec(X))$ *which is composed of a real linear space* X *and a family* $Spec(X)$ *of seminoms on* X *such that:*

(i) $\chi p \in Spec(X), \forall \chi \in R_+, p \in Spec(X)$;

(ii) *if* $p \in Spec(X)$ *and* q *is an arbitrary seminorm on* X *such that* $q \leq p$*, then* $q \in Spec(X)$;

(iii) $\sup(p_1, p_2) \in Spec(X), \forall p_1, p_2 \in Spec(X)$ where

$$\sup(p_1, p_2)(x) = \sup(p_1(x), p_2(x)), \forall x \in X.$$

It is well known [84] that whenever such a family as this $Spec(X)$ is given on a real vector space X, there exists a locally convex topology τ on X such that (X, τ) is a topological linear space and a seminorm p on X is τ - continous iff $p \in Spec(X)$. A a non-empty subset $\mathcal{B}$ of $Spec(X)$ is a *base* for it if for every $p \in Spec(X)$ there exist $\chi \rhd 0$ and $q \in \mathcal{B}$ such that $p \leq \chi q$ and (X, τ) is a Hausdorff locally convex space iff $Spec(X)$ has a base $\mathcal{B}$, named *Hausdorff base*, with the property that $\{x \in X : p(x) = 0, \forall p \in \mathcal{B}\} = \{\theta\}$ where θ is the null vector in X. In this research paper we will suppose that the space (X, τ) sometimes denoted by X is a Hausdorff locally convex space. Every non-empty subset K of X satisfying the following properties: $K+K\subseteq K$ and $\chi K\subseteq K,\forall \chi\in R_+$ is named *convex cone*. If, in addition, $K\cap K=\{\theta\}$, then K is called *pointed*. Clearly, any pointed convex cone K in X generates an ordering on X defined by $x\le y(x,y\in X)$ iff $y-x\in K$. If X^* is the dual of X, then the *dual cone* of K is defined by $K^*=\{x^*\in X^*:x^*(x)\ge 0,\forall x\in K\}$ and its corresponding *polar* is $K^0=-K$. We recall that a pointed convex cone $K\subset(X,Spec(X))$ is *normal* with respect to the topology defined by $Spec(X)$ if it fulfils one of the next equivalent assertions:

(i) there exists at a base Ω of neighborhoods for the origin θ in X such that $V=(V+K)\cap(V-K),\forall V\in\Omega$ *;*

(ii) there exists a base B *of* $Spec(X)$ *with* $p(x)\le p(y),\forall x,y\in K,x\le y,\forall p\in B$*;*

(iii) for any two nets $\{x_i\}_{i\in I},\{y_i\}_{i\in I}\subset K$ *with* $\theta\le x_i\le y_i,\forall i\in I$ *and* $\lim y_i=\theta$ *it follows that* $\lim x_i=\theta$ *. In particular, a convex cone K is normal in a normed linear space* $(E,\|.\|)$ *iff there exists* $t\in(0,\infty)$ *such that* $x,y\in E$ *and* $y-x\in K$ *implies that* $\|x\|\le t\|y\|$
.

It is well known that the concept of *normal cone* is the most important notion in the theory and applications of convex cones in topological ordered vector spaces. Thus, for example, *for every separated locally convex space* $(X,Spec(X))$ *and any closed normal cone* $K\subset(X,Spec(X))$ *we have* $X^*=K^*-K^*$ (see, for instance, [29], [32]). Each pointed convex cone $K\subset(X,Spec(X))$ for which there exists a non-empty, convex bounded set $B\subset X$ such that $0\notin\overline{B}$ and $K=\bigcup_{\chi\ge 0}\chi B$ is called *well-based*. A cone $K\subset(X,Spec(X))$ *is well-based iff there exists a base* $B=\{p_i\}_{i\in I}$ *of* $Spec(X)$ *and a linear continuous functional* $f\in K^*$ *such that for every* $p_i\in B$ *there exists* $c_i>0$ *with* $c_ip_i(x)\le f(x),\forall x\in K$ ([29], [30]). Clearly, every well-based cone is a normal cone, but, in general, the converse is not true, as we can see in the examples below, starting from the next basic notion.

Definition 2. *(Isac, G., 1981,1983) In a Hausdorff locally convex space* $(X,Spec(X))$ *a pointed convex cone* $K\subset X$ *is nuclear (supernormal) with respect to the topolgy induced by* $Spec(X)$ *if there exists a base* $B=\{p_i\}_{i\in I}$ *of* $Spec(X)$ *such that for every* $p_i\in B$ *there exists* $f_i\in X^*$ *with* $p_i(x)\le f_i(x),\forall x\in K$.

Remark 1. For the first time, we called any such as this cone “Isac’s cone” in [74], with the permission of the regretted professor George Isac, taking into account that the above definition of locally convex spaces is equivalent with the following: Let X be a real or complex linear space and $\{P = p_\alpha : \alpha \in A\}$ a family of seminorms defined on X. For every $x \in X, \varepsilon > 0$ and $n \in N^*$ let

$V(x; p_1, p_2, ..., p_n; \varepsilon) = \{y \in X : p_\alpha(y - x) < \varepsilon, \forall \alpha = \overline{1,n}\}$, then the family

$\mathcal{V}_0(x) = \{V(x; p_1, p_2, ..., p_n; \varepsilon) : n \in N^*, p_\alpha \in P, \alpha = \overline{1,n}, \varepsilon > 0\}$

has the properties :

(V_1) $x \in V, \forall V \in \mathcal{V}_0(x)$;

(V_2) $\forall V_1, V_2 \in \mathcal{V}_0(x), \exists V_3 \in \mathcal{V}_0(x) : V_3 \subseteq V_1 \cap V_2$;

(V_3) $\forall V \in \mathcal{V}_0(x), \exists U \in \mathcal{V}_0(x), U \subseteq V$ such that $\forall y \in U, \exists W \in \mathcal{V}_0(y)$ *with* $W \subseteq V$.

Therefore, $\mathcal{V}_0(x)$ is *a base of neighborhoods* for x and taking $\mathcal{V}(x) = \{V \subseteq X : \exists U \in \mathcal{V}_0(x)$ cu $U \subseteq V\}$, the set $\tau = \{D \subseteq X : D \in \mathcal{V}(x), \forall x \in D\} \cup \{\emptyset\}$ is the locally convex topology generated by the family P. Obviously, the usual operations which induce the structure of linear space on X are continuous with respect to this topology. The corresponding topological space (X, τ) is a Hausdorff locally convex space iff the family P is sufficient, that is, $\forall x_0 \in X \setminus \{\theta\}, \exists p_\alpha \in P$ with $p_\alpha(x_0) \neq 0$. In this context, a convex cone $K \subset X$ is an Isac's cone iff $\forall p_\alpha \in P, \exists f_\alpha \in X^* : p_\alpha(x) \leq f_\alpha(x), \forall x \in K$. The best special, refined and non-trivial Isac's cones class associated to normal cones in Hausdorff locally convex spaces was introduced and studied in [39] as *the full nuclear cones* family defined as follows: if $(X, Spec(X))$ is an arbitrary locally convex space $B \subset Spec(X)$ is a Hausdorff base of $Spec(X)$ and $K \subset X$ is a normal cone, then for any mapping $\varphi : B \to K^* \setminus \{0\}$ one says that the set

$K_\varphi = \{x \in X : p(x) \leq \varphi(p)(x), \forall p \in B\}$ is *a full nuclear cone* associated to K whenever $K_\varphi \neq \{\theta\}$. Taking into account that in a real normed linear space $(E, \|.\|)$ a non-empty set $T \subset E$ is called *a Bishop-Phelps cone* if there exists y^* in the usual dual space E^* of E and $\alpha \in (0,1]$ such that $T = \{y \in E : \alpha \|y\| \leq y^*(y)\}$ and the applications of such as these cones in Nonlinear Analysis and in Pareto type optimization for vector-valued mappings, we conclude that, for Haudorff locally convex spaces, the full nuclear cones are similar and generalizations of Bishop-Phelps cones in normed vector spaces.

In the next considerations we offer significant examples and adequate remarks on the supernormal cones. The existence of the efficient points and important properties of the efficient points sets are ensured in separated locally convex spaces ordered by (weak) supernormal cones named by us “Isac’s cones”, through the agency of the (weak) completeness instead of compactness (the reader is referred to, Isac, G., 1981, 1983, 1985, 1994, 1998, Isac, G., Postolică, V., 1993, Postolică, V., 1993, 1994, 1995, 1997, 1999, 2001, 2002, 2009, Truong, X. D. H., 1994 and so on).

Theorem 1. (Bahya, A. O., 1989) A convex and normal cone K in a Hausdorff locally convex space is supernormal if and only if every net of K weakly convergent to zero converges to zero in the locally convex topology.

Let us consider some pertinent examples

1. *Any convex*, *closed* and *pointed cone* in *an arbitrary* usual *Euclidean space* R^k with k in N* is *supernormal*.

2. In *every locally convex space* any *well-based convex cone is an Isac’s cone*.

3. *A convex cone* is *an Isac’s cone* in *a normed linear space if and only if* it is *well-based*.

4. Let $n \in N^*$ be *arbitrary fixed* and let *Y* be the space of *all real symmetric (n, n) matrices ordered* by *the pointed*, *convex cone* $C = \{A \in Y : x^T Ax \geq 0, \forall x \in R^n\}$. Then, *Y* is *a real Hilbert space* with respect to *the scalar product* defined by *<A, B>= =trace (A. B)* for *all* $A, B \in Y$ and *C* is *well-based by* $B = \{A \in \mathbf{C} : \langle A, I \rangle = 1\}$ where *I denotes the identity matrix*.

5. *Every pointed*, *locally* or *weakly locally compact convex cone* in *any Hausdorff locally convex space is an Isac’s cone*.

6. *A convex cone is an Isac’s cone in a nuclear space (Pietch, A., 1972) if and only if it is a normal cone*.

7. In *any Hausdorff locally convex space* a *convex cone* is *an weakly Isac’s cone if and only if* it is *weakly normal*.

8. In $L^p([a, b])$, $(p \geq 1)$, *the convex cone* $K_p = \{x \in L^p([a, b]) : x(t) \geq 0$ *almost everywhere*$\}$ is *an Isac’s cone if and only if p=1, being well based in this case* by *the set* $B = \{x \in K_1 :$

$\int_a^b x(t)\,dt = 1\}$. Indeed, *if p>1*, then *the sequence* (x_n) *defined by*

$$x_n(t) = \begin{cases} n^{1/p}, & a \leq t \leq a+(b-a)/2n \\ 0, & a+(b-a)/2n < t \leq b \end{cases} \qquad n \in \mathbf{N}$$

converges to 0 in the weak topology but not in the usual norm topology. Therefore, by virtue of Theorem 1, K_p *is not an Isac's cone. Generally, for every p>1,* K_p *has a base* $B = \{x \in K_p : \int_a^b x(t)\,dt = 1\}$ *which is unbounded and any cone generated by a closed and bounded set*

B_t={x∈ B: $\int_a^b |x(t)|^p dt \le t$} with t≥0 is certainly an Isac's cone. A similar result holds for $L^p(R)$. Thus, if we consider a countable family (A_n) of disjoint sets which covers R such that (A_n) = 1 for all n in N, where μ is the Lebesgue measure, then the sequence (y_n) given by $y_n(t) = 1$ if $t\in A_n$ and $y_n(t) = 0$ for $t\in R\backslash A_n$ converges weakly to zero while it is not convergent to zero in the norm topology. Taking into account the above theorem, it follows that the usual positive cone in $L^p(R)$ is not an Isac's cone if p>1, that is, it is not well-based in all these cases. However, these cones are normal for every p≥1. The same conclusion concerning the non-supernormality is valid for the positive orthant of the usual Orlicz spaces.

9. In *l^p (p≥1) equipped with the usual norm* $\|\cdot\|_p$ *the positive cone*

C_p= {(x_n)∈ l^p: $x_n \in 0$ for all n∈N} is also *normal* with respect to *the norm topology*, but *it is not an Isac's cone excepting the case p = 1*. Indeed, *for every p>1, the sequence (e_n) having 1 at the nth coordinate* and *zeros elsewhere converges to zero in the weak topology, but not in the norm topology* and by *virtue of Theorem 1* it follows that *C_p is not an Isac's cone*. For *p = 1*, C_p is *well-based* by *the set* B = {x∈ C_1: $\|x\|_1 = 1$} and *Proposition 5 (Isac, G., 1983) ensures that* it is *an Isac's cone*. If we consider *in this case the locally convex topology in l^1 defined by the seminorms* $p_n((x_k)) = \sum_{k=0}^{n} |x_k|$ *for every (x_k) in l^1 and n∈N*, which is *weaker than its usual weak* topology, then *the usual positive cone remains an Isac's cone* with respect to *this topology* (now *it is normal in a nuclear space* and *one applies Proposition 6* of *(Isac, G., 1983)* but *it is not well based*. Taking into account *the concept* of *H-locally convex space* introduced by *Precupanu, T.* in *1969* and defined as *any Hausdorff locally convex space* with *the seminorms satisfying the parallelogram law* and *the property* that *every nuclear space is also a H-locally convex space* with respect to *an equivalent system* of *seminorms (Pietch, A., 1972), the above example shows* that in *a H-locally convex space a proper convex cone may be an Isac's cone without to be well-based*. Moreover, *if we consider in l^2 the H-locally convex topology induced by the seminorms*

$$\tilde{p}_n((x_k)) = \left(\sum_{i\ge n} |x_i|^2 \right)^{1/2}, \; n\in N, \;\; x_k) \in \ell^2,$$

then *the convex cone* $C_2 = \{(x_k) \in l^2 : x_k \in 0$ *for all* $k \in N\}$ *is normal* in *the H-locally convex space* $(l^2, \{\tilde{p}_n\}_{n\in N})$, *but it is not a supernormal cone* because *the same sequence* (e_k) *is weakly convergent to zero* while $(\overline{p}_n(e_k))$ *is convergent to 1 for each* $n \in N$ and *one applies again Theorem1*. Another *interesting example* of *normal cone* in *a H-locally convex space* which *is not supernormal is the usual positive cone* in *the space* $L^2{}_{loc}$ *(R) of all functions from R to C which are square integrable over any finite interval of R*, *endowed with the system of seminorms* $\{\overline{p}_n : n\in N\}$ defined by $\overline{p}_n(x) = \left(\int_{-n}^{n} |x(t)|^2 dt\right)^{1/2}$ *for every x in* $L^2{}_{loc}$ *(R)*. In *this case, the sequence* (x_k) given by:

$$x_k(t) = \begin{cases} 0, & t\in(-\infty, 0)\cup(1/k, +\infty) \\ \sqrt{k}, & t\in[0, 1/k] \end{cases}$$

converges weakly to zero, but *it is not convergent in the H-locally convex topology*. *The results follows* by *Theorem 1*. It is clear that *every weak topology is a H-locally convex topology* and, *in these cases, the supernormality of convex cones coincides* with *the normality* thanks *to the Corollary of Proposition 2* in *(Isac, G., 1983).*

10. In *the space C([a, b]) of all continuous, real valued functions defined on every non-trivial, compact interval [a, b] equipped with the usual supremum norm the convex cone K =* $\{x\in C([a, b]) : x$ *is concave,* $x(a)=x(b) = 0$ *and* $x(t)\geq 0$ *for all* $t\in[a, b]\}$ *is supernormal, being well based by the set* $\{x\in K : x(t_0) = 1\}$ *for some arbitrary* $t_0\in[a,b]$. *The hypothesis that all* $x\in K$ *are concave is essential for the supernormality.*

11. *The convex cone* of *all nonnegative sequences* in *the space* of *all absolutely convergent sequences* is *the dual* of *the usual positive cone* in *the space* of *all convergent sequences*. Consequently, *it has a weak star compact base* and hence *it is a weak star supernormal cone.*

12. In l^∞ or in c_0 *equipped with the supremum norm*, *the convex cone* consisting of *all sequences having all partial sums non-negative is not normal*, hence *it is not supernormal.*

13. In *every Hausdorff locally convex space any normal cone is supernormal* with respect to *the weak topology.*

14. In *every locally convex lattice which is a (L)-space the ordering cone is supernormal* (see also *the Example 7* given by *Isac, G*. in *1994*).

15. If we consider *the space of all locally integrable functions* on *a locally compact space Y* with respect to *a Radon measure* μ endowed with *the topology induced by the family of*

seminorms {pA} where $pA(f) = \int_A |f(x)| d\mu$ *for every non-empty* and *compact subset A* of *Y* and *every locally integrable function f*, then *the convex cone K = {f: f(x)≥0, "x∈Y} is supernormal*.

16. If *Z is any locally convex lattice ordered* by *an arbitrary convex cone K* and Z^* *is its topological dual ordered by the corresponding dual cone* K^*, then *the cone K is supernormal* with respect to *the locally convex topology defined on Z* by *the neighbourhood base at the origin* $\{[-f, f]^\circ\}_{f\in \tilde{K}}$

17 In *every regular vector space (E, K)* (that is, *the order dual* E^* *separates the points of E*) with *the property* that *E = K - K the convex cone K is supernormal* with respect to *the topology* defined in *the preceding example*.

18. Any *semicomplete cone* in *a Hausdorff locally convex space* is *supernormal* (for this concept see *the Example 11* of *Isac, G., 1994*).

Remark 2. Clearly, *if a convex cone K is supernormal in a normed space*, then *K admits a strictly positive*, *linear and continuous functional*, that is, *there exists a linear, continuous functional f* such that *f(k)>0 for all k∈K\{0}.Generally, the converse is not true even in a Banach space as we can see in the following examples*:

19. *If one considers in the usual space* l^p $(1\le p\le\infty)$ *the convex cone K* $= \ell^p_+ = \{x = (x_i)\in \ell^p : x_i \ge 0 \text{ for every } i\in N\}$ *of infinite vectors with non-negative components,*

then the functional φ defined by $\varphi(k) = \sum_{i=1}^{\infty} k_i$ *for any k =(k_i)∈* l^p *is linear, continuous and strictly positive. But, as we have seen in the above considerations (Example 9), this cone is supernormal if and only if p = 1.*

20. *Let K be the usual positive cone* $L^p_+ = \{x\in L^p([a, b]):x(t)\ge 0$ *almost everywhere} in* $L^p([a, b])$ $(1\le p\le\infty)$. *Then, the linear and continuous functional ψ on* $L^p([a, b])$ *given by* $\psi(x) = \int_a^b x(t)dt$ *for every* $x\in L^p([a, b])$ *is strictly positive on K while K is supernormal (see the above Example 8) if and only if p = 1.*

Therefore, l^1_+ and L^1_+ are *supernormal cones* with *empty topological interiors* and for *every* $p\in(1,+\infty)$ *it follows that* l^p_+ *and* L^p_+ *are normal cones with empty interiors which are not supernormal. Hence, these convex cones are not well based.* A *very simple example* of *supernormal cones having non-empty topological interior is* R_+^n *(n∈N*)*.

Remark 3. In *the order complete vector lattice B([a, b])* of *all bounded*, *real valued* f*unctions* on *a compact non-singleton interval [a, b]* endowed with *its usual norm the standard positive cone* $K = \{u \in B([a, b]): u(t) \gtrsim$ *for all* $t \in ([a, b])\}$ *is normal but it has not a base*, that is, *it is not supernormal*. However, *this cone has non-empty interior*. *If we consider the linear space* l^1 *endowed with the separated locally convex topology generated by the family* $\{p_n : n \in N\}$ *of seminorms defined by* $p_n(x) = \sum_{k=0}^{n} |x_k|$ *for every* $x = (x_k) \in l^1$, then *the convex cone* $K = \{ x = (x_k) \in l^1 : x_k \geq 0$ *whenever* $k \in N\}$ *is supernormal* but *it is not well based*.

Remark 4. *The natural context of supernormality* (nuclearity) *for convex cones* is *any separated locally convex space*. *Isac, G. introduced the concept of "nuclear cone" in 1981*, *published it* in *1983* and *he showed that in a normed space a convex cone is nuclear* if and only if it is *well based* or equivalently *iff* it is *"with plastering",* the last concept being defined by Krasnoselski, M. A. in fifties (see ,for example , Krasnoselski, M. A., 1964 and so on). Such a convex cone was initially called "nuclear cone" by Isac, G. (1981) because in every nuclear space (Pietch, A., 1972) any normal cone is a nuclear cone in Isac's sense (Proposition 6 of Isac, G., 1983). Afterwards, since the nuclear cone introduced by Isac appears as a reinforcement of the normal cone, it was called supernormal. The class of supernormal cones in Hausdorff locally convex spaces was initially imposed by the theory and the applications of the efficient (Pareto minimum type) points (especially existence conditions based on completeness instead of compactness were decisive together with the main properties of the efficient points sets), the study of critical points for dynamical systems and conical support points and their importance was very well illustrated by important results, examples and comments in the specified references and in other connected papers. It is also very significant to mention again that the concept of supernormality introduced by Isac, G. (1981) is not a simple generalization of the corresponding notion defined in normed linear spaces by Krasnoselski, M. A. and his colleagues in the fifties. Thus, for example, Isac's supernormality attached to the convex cones has his sense in every Hausdorff locally convex space identically with the well known Grothendieck's nuclearity. By analogy with the fact that a normed space is nuclear in Grothendieck's sense if and only if it is isomorphe with an usual Euclidean space, a convex cone is supernormal in a normed space if and only if it is well based, that is, it is generated by a convex bounded set which does not contain the origin in its closure.Beside Pareto type optimization, we also mention Isac's significant contributions, through the agency of supernormal cones, to the convex cones in product linear spaces and Ekeland's variational type principles(Isac.G.,2003;Isac,G.,Tammer,Chr.,2003). Therefore, the more appropriate background for Isac's cones is any separated locally convex space.

VI USEFUL SPLINES FOR THE BEST APPROXIMATION AND OPTIMIZATION IN H-LOCALLY CONVEX SPACES

We conclude this research report with some topics on the best approximation (simultaneous and vectorial) and the optimization in H-locally convex spaces. So, it is known that the concept of H-locally convex space was introduced and studied for the first time by Precupanu, T. (1969) and defined as any Hausdorff locally convex space with the seminorms satisfying the parallelogram law. At the same time, we introduced the notion of spline function in H-locally convex space (Postolică, V., 1981) and we established the basic properties of approximation and optimal interpolation for these splines. Our splines are natural extensions in H-locally convex spaces of the usual abstract splines which appear in any Hilbert space like the minimizing elements for a seminorm subject to the restrictions given by a set of linear continuous functionals.

Let $(X, P = \{p_\alpha : \alpha \in I\})$ *be a H-locally convex space* with *each seminorm* p_α *being* *induced by a scalar semiproduct* $(.,.)_\alpha(\alpha\in I)$ and *M a closed linear subspace* of *X for which there exist a H-locally convex space* $(Y,\mathcal{Q}=\{q_\alpha:\alpha\in I\})$ with *each seminorm* $q_\alpha\in\mathcal{Q}$ *generated by a scalar semiproduct* $<.,.>_\alpha(\alpha\in I)$ and *a linear* (*continuous*) *operator* $U:X\to Y$ *such that* $M=\{x\in X: (x,y)_\alpha=<Ux,Uy>_\alpha\ \forall\ \alpha\in I\}$.

The space of spline functions with respect to U was defined by Postolică, V.(1981) as the U-orthogonal of M, that is,

$M^\perp=\{x\in X: <Ux,U\zeta>_\alpha=0,\ \forall\ \zeta\in M,\ \alpha\in I\}$. Clearly, $M^\perp$ *is the orthogonal of M in the H-locally convex sense.*

Let $x_0\in X$ and *G a non-empty subset of X.*

Definition 1. (*Postolică V.,1993)* $g_0\in G$ *is said to be a best simultaneous approximation for* x_0 *by the elements of* G *with respect the family* $\mathcal{P}$ *(abbreviated* g_0 *is a* $\mathcal{P}$ *-b.s.a. of* x_0*) when*

$$p_\alpha(x_0-g_0)\le p_\alpha(x_0-g)\ \textit{for all}\ g\in G\ \textit{and}\ p_\alpha\in\mathcal{P}.$$

If, in addition, each element $x\in X$ *possesses at least one* $\mathcal{P}$ *- b.s.a. in* G*, then the set* G *is called* $\mathcal{P}$ *- simultaneous proximinal.*

Definition 2. *(Postolică V., 1993)* $g_0\in G$ *is said to be a best vectorial approximation of* x_0 *by* G *with respect to* $\mathcal{P}$ *(abbreviated* g_0 *is a* $\mathcal{P}$ *- b.v.a. of* x_0*) if*

$$(p_\alpha(x_0-g_0))\in MIN_K(\{(p_\alpha(x_0-g)): g\in G\})\ \textit{where}\ K=R_+^I.$$

The set G *is called* $\mathcal{P}$ *- vectorial proximinal whenever each element* $x\in X$ *possesses at least one* $\mathcal{P}$ *- b.v.a. in* G.

Let us consider *the direct sum* $X'=M\oplus M^\perp$ and *for every* $x\in X'$*, we denote its projection onto* $M^\perp$ *by* s_x. Then, taking into account *the Theorem 4* obtained by *Postolică, V.* in *1981,* it follows that *this spline is a best simultaneous U-approximation of x with respect to* $M^\perp$ since *it satisfies all the next conditions* : $p_\alpha(x-s_x)\le p_\alpha(x-y)\ \forall\ y\in M^\perp,\ p_\alpha\in\mathcal{P}$.

Moreover, following the definition of the approximate efficiency, the results given in Chapter 3 of (Isac, G., Postolică, V., 1993) and the conclusions obtained by (Postolică, V.,1981,1993, 1998), we have

Theorem 1.

(i) for every $x \in X'$ *the only elements of best simultaneous and vectorial approximation with respect to any family of seminorms which generates the* H*-locally convex topology on* X *by the linear subspace of splines are the spline functions* s_x.

Moreover, if M *and* $M^{\perp}$ *supply an orthogonal decomposiion for* X, *that is* $X=M\oplus M^{\perp}$,

then $M^{\perp}$ *is simultaneous and vectorial proximinal;*

(ii) if $K=R_{+}^{I}$, *then for each* $s\in M^{\perp}$, *every* $\sigma\in M^{\perp}$ *is the only solution of following optimization problem* $MIN_K(\{(q_\alpha(U(\eta-s))): \eta\in X' \text{ and } \eta-\sigma\in M\})$;

(iii) for every $x\in X'$ *its spline function* s_x *is the only solution for the next vectorial optimization problems:*

$MIN_K(\{(q_\alpha(U(\eta-x))): \eta\in M^{\perp}\})$, $MIN_K(\{(p_\alpha(x-y)): y\in M^{\perp}\})$, $MIN_K(\{(q_\alpha(Uy)): y-x\in M\})$.

Finally, let us consider *two numerical examples* in which, following *Postolică,V.,(1981), Isac, G., Postolică, V.,(1993) and Postolică, V., (1998),* we specify *the expressions of splines* and *M together with* $M^{\perp}$ *realizes orthogonal decompositions*.

Example 1. *Let* $X=H^m(R)=\{f\in C^{m-1}(R): f^{(m-1)}$ *is locally absolutely continuous and* $f^{(m)}\in L^2_{loc}(R)\}$, $m\geq 1$ *endowed with the H-locally convex topology generated by the scalar semiproducts*

$$(x,y)_k=\sum_{h=0}^{m-1}[x^{(h)}(k)\,y^{(h)}(k)+x^{(h)}(-k)\,y^{(h)}(-k)]+\int_{-k}^{k}x^{(m)}(t)\,y^{(m)}(t)dt,\ k=0,1,2,\ \dots \text{ and}$$

$Y=L^2_{loc}(R)$ *with the H-locally convex topology induced by the scalar semiproducts* $\langle x,y\rangle_k=$

$$\int_{-k}^{k}x(t)\,y(t)dt,\ k=0,1,2,\ \dots$$

If $U:X\to Y$ *is the derivation operator of order m, then*

$M=\{x\in H^m(R): x^{(h)}(\nu)=0,\ \forall\ h=\overline{0,m-1},\ \nu\in Z\}$

and $M^{\perp}=\{s\in H^m(R): \int_{-k}^{k}s^{(m)}(t)\,x^{(m)}(t)dt,\ \forall\ x\in M,\ k=0,1,2,\ \dots\}$

We proved in (Postolică, V., 1981) that

$M^{\perp}$ ={s∈ $H^m(R)$: $s_{/(\nu,\nu+1)}$ is a polynomial function of degree 2m-1 at most} and if $y=(y_\nu)$, $y'=(y'_\nu)$, $y''=(y''_\nu)$, $y^{(m-1)}=(y^{(m-1)}_\nu)$ are m sequences of real numbers, then there exists an unique spline $S\in M^{\perp}$ satisfying the following conditions of interpolation: $S^{(h)}(\nu)=y^{(h)}(\nu)$ whenever $h=\overline{0,m-1}$ and ν∈ Z. Moreover, we observed in the paragraph 3 of (Isac, G., Postolică, V., 1993) that any spline function S such as this is defined by

$$S(x)=p(x)+\sum_{h=0}^{m-1} c_1^{(h)}(x-1)_+^{2m-1}+\sum_{h=0}^{m-1} c_2^{(h)}(x-2)_+^{2m-1}+\ldots+\sum_{h=0}^{m-1} c_0^{(h)}(-x)_+^{2m-1}+\ldots$$

where $u_+=(|u|+u)/2$ for every real number u, p is a polynomial function of degree 2m-1 at most perfectly determined by the conditions $p^{(h)}(0)=y_0^{(h)}$ and $p^{(h)}(1)=y_1^{(h)}$ for all $h=\overline{0,m-1}$ and the coefficients $c_\nu^{(h)}$ ($h=\overline{0,m-1}$, ν∈ Z) are successively given by the general interpolation.

Therefore, for every function f∈ $H^m(R)$, there exists an unique function denoted by $S_f \in M^{\perp}$ such that $S_f^{(h)}(\nu)=f^{(h)}(\nu)$, ∀ $h=\overline{0,m-1}$ and ν∈ Z. Hence, in this case, M and $M^{\perp}$ give an orthogonal decomposition for the space $H^m(R)$.

Example 2. Let $X=F_m=\{f\in C^{m-1}(R): f^{(m-1)}$ is locally absolutely continuous and $f^{(m)}\in L^2(R)\}$ endowed with the H-locally convex topology induced by the scalar semiproducts

$(x,y)_\nu=x(\nu)y(\nu)+\int_R x^{(m)}(t)\, y^{(m)}(t)dt$, ν∈ Z, $Y=L^2(R)$ with the topology generated by the inner product $(x,y)_\nu=\int_R x(t)\, y(t)dt$, ν∈ Z and U:X→ Y be the derivation operator of order m.

Then, M= {x∈ F_m: x(ν)=0 for all ν∈ Z} and

$M^{\perp}=\{s\in F_m: \int_R x^{(m)}(t)\, y^{(m)}(t)dt=0$ for every x∈ M}.

In a similar manner as in Example1 it may be proved that $M^{\perp}$ coincides with the class of all piecewise polynomial functions of order 2m (degree 2m-1 at most) having their knots at the integer points. Moreover, for every function f in F_m there exists an unique spline function $S_f\in M^{\perp}$ which interpolates f on the set Z of all integer numbers, that is, S_f

satisfies the equalities $S_f(\nu)=f(\nu)$ *for every* $\nu\in Z$*, being defined by*

$$S_f(x)= p(x)+ a_1(x-1)_+^{2m-1}+ a_2 (x-2)_+^{2m-1}+ \dots +a_0(-x)_+^{2m-1}+ a_{-1}(-x-1)_+^{2m-1}+ \dots$$

where u_+ *has the same signification as in Example1, the coefficients* $a_\nu(\nu \in Z)$ *are successively and completely determined by the interpolation conditions* $S_f(\nu)=f(\nu)$*,* $\nu\in Z\setminus\{0, 1\}$ *and p is a polynomial function satisfying the conditions* $p(0)=f(0)$ *and* $p(1)=f(1)$*. The uniqueness of* S_f *is ensured in Theorem 2 given by (Postolică, V., 1981).*

Thus, M and $M^{\perp}$ *give an orthogonal decomposition of the space* F_m *and, as in the preceding example,* $M^{\perp}$ *is simultaneous and vectorial proximinal with respect to the family of seminorms generated by the above scalar semiproducts.*

Remark 1. *Our examples show that the abstract construction of splines can be used to solve also several frequent problems of interpolation and approximation, having the possibility to choose the spaces and the scalar semiproducts. It is obvious that for a given (closed) linear subspace of a H-locally convex space X such a H-locally convex space Y (respectively, a linear (continuous) operator* $U:X\to Y$*) would not exist. Otherwise, the problem of best vectorial approximation by the corresponding orthogonal space of any (closed) linear subspace M for the elements in the direct sum* $M\oplus M^{\perp}$ *might be always reduced to the best simultaneous approximation. But, in general, such a possibility doesn't exist. Even in a H-locally convex space it is possible that there exist best vectorial approximations and the set of all best simultaneous approximations to be empty for some element of the space. We confine ourselves to mention the following simple example.*

Example 3. *Let* $X= R^N$ *endowed with the topology generated by the family* $\mathcal{P}=\{p_i : i \in N\}$ *of seminorms defined by* $p_i(x)= |x_i|$ $(i \in N)$ *for every*

$$x= (x_i)\in X \text{ and } G=\{(x_i)\in X: x_i\geq 0 \text{ whenever } i\in N \text{ and } \sum_{i\in N} x_i=1\}$$

X is a $\mathcal{P}$*-simultaneous strictly convex (Isac, G., Postolică, V., 1993) H-locally convex space. Nevertheless, every element of G is* $\mathcal{P}$*-b.v.a. for the origin while its corresponding set of the best simultaneous approximations with respect to* $\mathcal{P}$ *is empty.*

V. Selected references

1. ARROW, K.J. (1951) – Social choise and individual values.The second edition.Yale University Press, 1963.
2. ARROW, K.J., HAHN, F.M. – General competitive analysis. Holden Day, New York, 1971.
3. AUBIN, J.P. – Optima and equilibria. An introduction to nonlinear analysis. Springer – Verlag Berlin Heidelberg, 1993.
4. BAHYA, A. O. - Ensembles coniquement bornés et cônes nucléaires dans les éspaces localement convexes separés. Théses de Doctorat de 3-ème Cycle en Mathématiques. L'école Normale Superieure - Takaddoum Rabat (Maroc), 1989.
5. BENSON, H.P. – The vector maximization problem: proper efficiency and stability. SIAM J. Appl. Math. 32, 1977, 64-72.
6. BENSON, H.P.– An improved definition of proper efficiency for vector minimization with respect to cones. J. Math. Anal. Appl. 79, 1979, 232-241.
7. BENSON, H.P. – Efficiency and proper efficiency for vector minimization with respect to cones. J. Math. Anal. Appl. 93, 1983, 273-289.
8. BOBOC, N., BUCUR, GH. - Convex cones of continuous functions on compact spaces, (in Romanian). Publishing House of Romanian Academy, Bucharest, 1976.
9. BORWEIN, J.M. – Proper efficient points for maximization with respect to cones. SIAM J. Control Optim., 15, 1977, 57 – 63.
10. BORWEIN, J.M. – On the existence of Pareto efficient points Math Oper. Res. 8, no. 1, 1983, 64 - 73.
11. BRUNDTLAND COMMISSION (WORLD COMMISSION ON ENVIRONMENT AND DEVELOPMENT) – Our common future. Oxford University Press, Oxford, 1987.
12. BUCUR, I., POSTOLICĂ, V.- A coincidence result between sets of efficient points and Choquet boundaries in separated locally convex spaces. Research Report at the 4th Workshop of the German Working Group on Decision Theory, Hotel Talblick, Holzhau,Germany, March 14-18, 1994. Optimization, 36, 1996, 231-234.
13. CARDINALI, T. and PAPALINI, F.- Fixed point theorems for multifunctions in topological vector spaces. J. Math. Anal. Appl., 186, 1994,769-777.
14. CHOQUET, G.- Ensembles et cônes convexes faiblement complets. C. R. Acad. Sci. Paris, 254, 1962, 2123-2125.
15. CHOQUET, G.- Theory of capacities. Ann. Inst. Fourier, 5, 1955.
16. CHOQUET, G., DENY, J.- Ensembles semi-reticulés et ensembles reticulés des fonctions continues. Journ. Math. Pures Appl., 36, 1957, 179-189.
17. CHOQUET, G., MEYER, P. A.- Existence et unicité des représentations dans les convexes compacts quelconques. Ann. Inst. Fourier, 13, 1963, 139-154.
18. DAUER, J. P., GALLAGHER, R. J. – Positive proper efficient points and related coneresults in vector optimization theory. SIAM J. Control Optim. 28(1), 1990, 158-172.
19. DEBREU GERARD – Theory of value. Wiley, New York, 1959.
20. DOMINIAK CEZARY – Multicriteria decision aid under uncertainty. In: Multiple Criteria Decision Making'05 (Ed.Tadeusz Trzaskalik), Publisher of The Karol Adamiecki University of Economics in Katowice, 2006, 63-81.
21. EHRGOTT, MATTHIAS – Multicriteria Optimization. Springer Berlin-Heidelberg, 2005.
22. FISHBURN, P. C. – Foundations of risk management science I. Risk or probable loss. Management Science, 30, 1984, 396 – 406.
23. GASS SAUL, I., HARRIS CARL, M. – Enciclopedia of operations research and management science. Centennial Edition. Kluwer Academic Publishers, Boston, Dordrecht, London, 2001.
24. GEOFFRION, A. M.- Proper efficiency and the theory of vector maximzation. J. Math. Anal. Appl., 22, 1968, 618 -630.
25. HANSSON, S.O. – Welfare, justice and Pareto efficiency. Ethical Theory and Moral Practise no. 7, 2004.
26. HARTLEY, R. – On cone - efficiency, cone convexity and cone – compactness. SIAM J. Appl. Math., 34(2), 1978, 211-222.
27. HENING, M. I. – Proper efficiency with respect to cones. J. Optim. Theory. Appl., 36,1982, 387-407.
28. HEYNE, PAUL – Efficiency. Research Report, University of Washington, 2000.
29. HYERS, D. H., ISAC, G., RASSIAS, T. M. – Topics in nonlinear analysis and applications. World Scientific Publishing Co, Pte. Ltd., 1997.
30. ISAC, G. – Cônes localement bornés et cones completement reguliers. Applications a l'Analyse Non-Linéaire Moderne. Université de Sherbrooke, no. 17, 1980.
31. ISAC, G. - Points critiques des systémes dinamiques. Cônes nucléaires et optimum de

Pareto, Research Report, Royal Military College of St. Jean, Québec, Canada, 1981.
32. ISAC, G. - Sur l'existence de l'optimum de Pareto, Riv. Mat. Univ. Parma, 4(9),1983,
303 - 325.
33. ISAC, G. - Supernormal cones and absolute summability, Libertas Mathematica, 5,
1985, 17 - 32.
34. ISAC, G., POSTOLICĂ, V.- The best approximation and optimization in locally convex
spaces, Verlag Peter Lang GmbH, Frankfurt am Main, Germany, 1993.
35. ISAC, G.- Pareto optimization in infinite dimensional spaces: the Importance of
nuclear cones, J. Math. Anal. Appl., 182(2), 1994, 393 - 404.
36. ISAC, G. - On Pareto efficiency. A general constructive existence principle. Research
Report (16 pp.), Department of Mathematics and Computer Science, Royal Military College
of Canada, 1998.
37. ISAC, G., BAHYA, A., O. - Full nuclear cones associated to a normal cone.
Application to Pareto efficiency. Applied Mathematics Letters, 15, 2002, 633 - 639.
38. ISAC, G., BULAVSKY, V. A., KALASHNIKOV, V. V.- Complementarity,
equilibrium, efficiency and economics. Kluwer Academic Publishers, 2002.
39. ISAC, G. – Nuclear cones in product spaces. Pareto efficiency and Ekeland type
variational principles in locally convex spaces (Preprint, Royal Military College of Canada,
2003).
40. ISAC, G., TAMMER, CHR.- Nuclear and full nuclear cones in product spaces. Pareto
efficiency and Ekeland type variational principles (Preprint, Royal Military College of
Canada and University of Hall, 2003).
41. ISAC, G., POSTOLICĂ, V.- Full nuclear cones and a relation between strong
optimization and Pareto efficiency. J. Global Optimiz., 32, 2005, 507 - 516.
42. JABLONSKY, J.– A slack based model for measuring super-efficiency in data
envelopment analysis. In: Multiple Criteria Decision Making'05 (Ed.Tadeusz Trzaskalik),
Publisher of The Karol Adamiecki University of Economics in Katowice, 2006, 101-112.
43. KORHONEN,P., LUPTÁČIK, M. – Eco-efficiency analysis of power plants : an
extension of data envelopment analysis. European Journal of Operational Research, 154,
2004, 437 - 446.
44. KOSMIDOU, K., ZOPOUNIDIS, C. – Managing interest rate risk in commercial banks
via multicriteria analysis. Research Paper presented at The 61st Meeting of The European
Working Group "Multiple Criteria Decision Aiding", University of Luxembourg, March 10
- 11, 2005.
45. KRAMAR, E. - Locally convex topological vector spaces with hilbertian seminorms.
Rev. Roum. Math Pures et Appl., Tome XXVI, no. 1, 1981, 55-62
46. KRASNOSELSKI, M. A. - Positive Solutions of Operator Equations. Gröningen,
Noordhoff, 1964.
47. KUHN, H. W., TUCKER, A. W. – Nonlinear programming. Proceedings of the Second
Berkely Sympsium on Mathematical Statistics and Probability (Neyman J. Ed.), University
of California Press, Berkely, 1950, 481-492.
48. LORIDAN, P. - e -solutions of vector minimization problems. J. Optim. Theory Appl.,
43(2), 1984, 265 - 276.
49. LUC, D. T. - Theory of vector optimization. Springer-Verlag, 1989.
50. LUPTÁČIK, M., BÖHM, B. – Measuring eco - efficiency an a Leontief input - output
model. In: Multiple Criteria Decision Making'05 (Ed.Tadeusz Trzaskalik), Publisher of The
Karol Adamiecki University of Economics in Katowice, 2006, 121-135.
51. NACHBIN, L. - Topology and order. New York, Van Nostrand, 1965.
52. NÉMETH, A. B. - Between Pareto efficiency and Pareto e - efficiency.
Optimization,10(5), 1989, 615 - 637.
53. NG., K. F., ZHENG, X. Y. - Existence of efficient points in vector
optimization and generalized Bishop-Phelps theorem. JOTA, 115, 2002,
29 - 47.
54. NYBERG, L. et all – Förslag till Transportmål för Stockholms Län (Suggested
Transport Goals for Stockholms County). Stockholm: Länsstyrelsen i Stockholms Län,
2001.
55 . PARK , S. – Some coincidence theorems on acyclic multifunctions applications.In:
Tan, K.- K(ed) Fixed Point Theory and Applications, World Sci. NJ, 1992, 248 –277.
56. PARETO, VILFREDO – Manuel d'économie politique. Girard et Brière, 1909.
57. PERESSINI, A. L. - Ordered topological vector spaces. Harper 8 Row, New York,
1967.
58. PIETCH, A. – Nuclear Locally Convex Spaces. Springer – Verlag, 1972.
59. POSTOLICĂ, V. - Spline functions in H-locally convex spaces. An. Şt. Univ. "Al. I.
Cuza", Iaşi, 27, 1981, 333-338.
60. POSTOLICĂ, V. - Existence conditions of efficient points for multifunctions with

values in locally convex spaces. Stud. Cerc. Mat., Tom 41, no. 4, 1989, 325-331.

61. POSTOLICĂ, V. - New existence results for efficient points in locally convex spaces ordered by supernormal cones, J. Global Optimiz., 3, 1993, 233-242.

62. POSTOLICĂ, V. - An extension to sets of supernormal cones and generalized subdifferentials. Optimization, 29, 1994, 131-139.

63. POSTOLICĂ, V. - Properties of Pareto sets in locally convex spaces. Optimization, 34, 1995, 223-229.

64. POSTOLICĂ, V. - Recent conditions for Pareto efficiency in locally convex spaces. Research Report at the Joint International Meeting Euro XV-INFORMS XXXIV, Barcelona, Spain, July 14-17, 1997.

65. POSTOLICĂ, V. - Properties of efficient points sets and related topics. Research Report at The Second International Conference on Multi–Objective Programming and Goal Programming, Torremolinos, Spain, May 16-18, 1996. Published in Advances in Multiple Objective and Goal Programming, Lecture Notes in Economics and Mathematical Systems, 455, (Eds.: Rafael Caballero, Francisco Ruiz, Ralph E. Steuer),1997, 201-209.

66. POSTOLICĂ, V.- A method which generates splines in H-locally convex spaces and connections with vectorial optimization. Positivity, 2 (4), 1998, 369 - 377.

67. POSTOLICĂ, V.- Conditions for Pareto efficiency in locally convex spaces. Mathematical Reports of Romanian Academy 2, 1(51), April-June, 1999, 257-269.

68. POSTOLICĂ, V., SCARELLI, A.- Some connections between best approximation, vectorial optimization and multicriterial analysis. Nonlinear Analysis Forum, 5, 2000, 111-123.

69. POSTOLICĂ, V. SCARELLI, A., VENZI, L. - On the equilibrium of a multidimensional ecosystem. Nonlinear Analysis Forum, 6(2), 2001, 321-335.

70. POSTOLICĂ, V. - New existence results of solutions for vector optimization programs with multifunctions. Optimization, vol. 50, 2001, 253-263.

71. POSTOLICĂ, V. - Pareto efficiency, Choquet boundaries and operators in Hausdorff locally convex spaces. Nonlinear Analysis Forum, 7(2), 2002, 215 – 230.

72. POSTOLICĂ, V. – A class of generalized dynamical systems in connection with Pareto efficiency and related topics. Research Paper submited to the International Conference“Complex Systems, Intelligence and Modern Technological Applications”, Cherbourg, France, September 19 - 22, 2004. Published in the coresponding Electronics Proceedings Volume.

73. POSTOLICĂ, V. - Choquet boundaries and efficiency. Computer and Mathematics with Applications, 55, 2008, 381 - 391.

74. POSTOLICĂ, V. – Approximate efficiency in infinite dimensional ordered vector spaces. International Journal of Applied Management Science (IJAMS), Vol. 1, No. 3, 2009, 300 – 314.

75. PRECUPANU, T. - Espaces linéaires à semi-normes hilbertiennes. An. Şt. Univ. “Al. I. Cuza”, Iaşi, 15, 1969, 83-93.

76. PRECUPANU, T. - Scalar minimax properties in vectorial optimization. International Series of Numerical Mathematics, Birkhäuser Verlag Basel, 107, 1992, 299 - 306.

77. ROSENCRANTZ, HOLGER – Transport policy goals, welfare concerns, and rationality. Research Report presented at The 62-nd Meeting of The European Working Group “Multiple Criteria Decision Aiding”, Borlänge, Sweden, 22-24 September, 2005.

78. ROTMANS, JAN – New trends in modelling: complexity, sustainable development, ethics. The International Centre for Integrative Studies (ICIS). Plenary Talk at the 12th Mini Euro Conference organized by the Euro Working Groups Decision Support Systems (DSS), Human Centered Processes (HCP) and Prometheus (Ethics), Vrije University of Brussels, Belgium, April 2-5, 2002.

79. SONGSAK SRIBOONCHITTA, WING-KEUNG WONG, SOMPONG DHOMPONGSA, HUNG, T. NGUYEN – Stochastic dominance and applications to finance, risk and economics. CRC Press, Taylor and Francis Group, October 2009, 456 pp., ISBN: 978-1-4200-8266-1.

80. SONNTAG, Z., ZĂLINESCU, C. - Comparison of existence results for efficient points. JOTA, 105, 2000, 161-188.

81. STERNA - KARWAT, A. - On the existence of cone maximal points in real topological linear spaces. Israel Journal of Mathematics, 54(1), 1986, 33 - 41.

82. STEWART, D. J. – Uncertainties in MCDA. In: Multi Criteria Decision Analysis (Ed. P. Greco). Springer-Verlag, 2004.

83. TAYLOR, A. D. – Mathematics and politics. Strategy, voting, power and proof. Springer – Verlag New York, Inc., 1995.

84. TREVES, F.- Locally convex spaces and linear partial differential equations. Springer – Verlag, New York, 1967.

85. TRUONG, X. D. H. - A note on a class of cones ensuring the existence of efficient points in bounded complete sets. Optimization, 31, 1994, 141-152.
86. TRUONG, X. D. H. - On the existence of efficient points in locally convex spaces. J.Global Optimiz., 4, 1994, 265-278.
87. www.wikipedia.org
88. ZHANG CONG - JUN – Generalized bi-quasi-variational inequalities and generalized quasi-variational inequalities. Acta Analysis Functionalis Applicata, 7(2), 2005, 116 - 122 .
89. ZIMMERMANN, H. – An application - oriented view of modeling uncertainty.European Journal of Operation Research, 122, 2000, 190 - 198.

Global Journals Inc. (US) Guidelines Handbook 2011

www.GlobalJournals.org

FELLOW OF INTERNATIONAL CONGRESS OF SCIENCE FRONTIER RESEARCH (FICSFR)

- 'FICSFR' title will be awarded to the person/institution after approval of Editor-in-Chief and Editorial Board. The title 'FICSFR' can be added to name in the following manner:
  e.g. Dr. Andrew Knoll, Ph.D.,FICSFR
- FICSFR can submit two papers every year for publication without any charges. The paper will be sent to two peer reviewers. The paper will be published after the acceptance of peer reviewers and Editorial Board.
- Free unlimited Web-space will be allotted to 'FICSFR 'along with subDomain to contribute and partake in our activities.
- A professional email address will be allotted free with unlimited email space.
- FICSFR will be authorized to receive e-Journals-GJFS for the Lifetime.
- FICSFR will be exempted from the registration fees of Seminar/Symposium/Conference/Workshop conducted internationally of GJFS (FREE of Charge).
- FICSFR will be an Honorable Guest of any gathering held.

ASSOCIATE OF INTERNATIONAL CONGRESS OF SCIENCE FRONTIER RESEARCH (AICSFR)

- AICSFR title will be awarded to the person/institution after approval of Editor-in-Chef and Editorial Board. The title 'AICSFR' can be added to name in the following manner:
  eg. Dr. Thomas Knoll, Ph.D., AICSFR
- AICSFR can submit one paper every year for publication without any charges. The paper will be sent to two peer reviewers. The paper will be published after the acceptance of peer reviewers and Editorial Board.
- Free 2GB Web-space will be allotted to 'AICSFR' along with subDomain to contribute and participate in our activities.
- A professional email address will be allotted with free 1GB email space.
- AICSFR will be authorized to receive e-Journal GJFS for lifetime.

ANNUAL MEMBER

- Annual Member will be authorized to receive e-Journal GJFS for one year (Journal subscription for one year).
- The member will be allotted free 1 GB Web-space along with subDomain to contribute and participate in our activities.
- A professional email address will be allotted free 500 MB email space.

PAPER PUBLICATION

- The members can publish paper once. The paper will be sent to two-peer reviewer. The paper will be published after the acceptance of peer reviewers and Editorial Board.

Process of submission of Research Paper

The Area or field of specialization may or may not be of any category as mentioned in 'Scope of Journal' menu of the GlobalJournals.org website. There are 37 Research Journal categorized with Six parental Journals GJCST, GJMR, GJRE, GJMBR, GJSFR, GJHSS. For Authors should prefer the mentioned categories. There are three widely used systems UDC, DDC and LCC. The details are available as 'Knowledge Abstract' at Home page. The major advantage of this coding is that, the research work will be exposed to and shared with all over the world as we are being abstracted and indexed worldwide. The paper should be in proper format. The format can be downloaded from first page of 'Author Guideline' Menu. The Author is expected to follow the general rules as mentioned in this menu. The paper should be written in MS-Word Format (*.DOC,*.DOCX).

The Author can submit the paper either online or offline. The authors should prefer online submission.Online Submission: There are three ways to submit your paper:

(A) (I) Register yourself using top right corner of Home page then Login from same place twice. If you are already registered, then login using your username and password.

(II) Choose corresponding Journal from "Research Journals" Menu.

(III) Click 'Submit Manuscript'. Fill required information and Upload the paper.

(B) If you are using Internet Explorer (Although Mozilla Firefox is preferred), then Direct Submission through Homepage is also available.

(C) If these two are not convenient, and then email the paper directly to dean@globaljournals.org as an attachment.

Offline Submission: Author can send the typed form of paper by Post. However, online submission should be preferred.

Preferred Author Guidelines

MANUSCRIPT STYLE INSTRUCTION **<u>(Must be strictly followed)</u>**

Page Size: 8.27" X 11'"

- Left Margin: 0.65
- Right Margin: 0.65
- Top Margin: 0.75
- Bottom Margin: 0.75
- Font type of all text should be Times New Roman.
- Paper Title should be of Font Size 24 with one Column section.
- Author Name in Font Size of 11 with one column as of Title.
- Abstract Font size of 9 Bold, "Abstract" word in Italic Bold.
- Main Text: Font size 10 with justified two columns section
- Two Column with Equal Column with of 3.38 and Gaping of .2
- First Character must be two lines Drop capped.
- Paragraph before Spacing of 1 pt and After of 0 pt.
- Line Spacing of 1 pt
- Large Images must be in One Column
- Numbering of First Main Headings (Heading 1) must be in Roman Letters, Capital Letter, and Font Size of 10.
- Numbering of Second Main Headings (Heading 2) must be in Alphabets, Italic, and Font Size of 10.

You can use your own standard format also.

Author Guidelines:

1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript's Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

1. GENERAL

Before submitting your research paper, one is advised to go through the details as mentioned in following heads. It will be beneficial, while peer reviewer justify your paper for publication.

Scope

The Global Journals Inc. (US) welcome the submission of original paper, review paper, survey article relevant to the all the streams of Philosophy and knowledge. The Global Journals Inc. (US) is parental platform for Global Journal of Computer Science and Technology, Researches in Engineering, Medical Research, Science Frontier Research, Human Social Science, Management, and Business organization. The choice of specific field can be done otherwise as following in Abstracting and Indexing Page on this Website. As the all Global

Journals Inc. (US) are being abstracted and indexed (in process) by most of the reputed organizations. Topics of only narrow interest will not be accepted unless they have wider potential or consequences.

2. ETHICAL GUIDELINES

Authors should follow the ethical guidelines as mentioned below for publication of research paper and research activities.

Papers are accepted on strict understanding that the material in whole or in part has not been, nor is being, considered for publication elsewhere. If the paper once accepted by Global Journals Inc. (US) and Editorial Board, will become the *copyright of the Global Journals Inc. (US).*

Authorship: The authors and coauthors should have active contribution to conception design, analysis and interpretation of findings. They should critically review the contents and drafting of the paper. All should approve the final version of the paper before submission

The Global Journals Inc. (US) follows the definition of authorship set up by the Global Academy of Research and Development. According to the Global Academy of R&D authorship, criteria must be based on:

1) Substantial contributions to conception and acquisition of data, analysis and interpretation of the findings.

2) Drafting the paper and revising it critically regarding important academic content.

3) Final approval of the version of the paper to be published.

All authors should have been credited according to their appropriate contribution in research activity and preparing paper. Contributors who do not match the criteria as authors may be mentioned under Acknowledgement.

Acknowledgements: Contributors to the research other than authors credited should be mentioned under acknowledgement. The specifications of the source of funding for the research if appropriate can be included. Suppliers of resources may be mentioned along with address.

Appeal of Decision: The Editorial Board's decision on publication of the paper is final and cannot be appealed elsewhere.

Permissions: It is the author's responsibility to have prior permission if all or parts of earlier published illustrations are used in this paper.

Please mention proper reference and appropriate acknowledgements wherever expected.

If all or parts of previously published illustrations are used, permission must be taken from the copyright holder concerned. It is the author's responsibility to take these in writing.

Approval for reproduction/modification of any information (including figures and tables) published elsewhere must be obtained by the authors/copyright holders before submission of the manuscript. Contributors (Authors) are responsible for any copyright fee involved.

3. SUBMISSION OF MANUSCRIPTS

Manuscripts should be uploaded via this online submission page. The online submission is most efficient method for submission of papers, as it enables rapid distribution of manuscripts and consequently speeds up the review procedure. It also enables authors to know the status of their own manuscripts by emailing us. Complete instructions for submitting a paper is available below.

Manuscript submission is a systematic procedure and little preparation is required beyond having all parts of your manuscript in a given format and a computer with an Internet connection and a Web browser. Full help and instructions are provided on-screen. As an author, you will be prompted for login and manuscript details as Field of Paper and then to upload your manuscript file(s) according to the instructions.

To avoid postal delays, all transaction is preferred by e-mail. A finished manuscript submission is confirmed by e-mail immediately and your paper enters the editorial process with no postal delays. When a conclusion is made about the publication of your paper by our Editorial Board, revisions can be submitted online with the same procedure, with an occasion to view and respond to all comments.

Complete support for both authors and co-author is provided.

4. MANUSCRIPT'S CATEGORY

Based on potential and nature, the manuscript can be categorized under the following heads: Original research paper: Such papers are reports of high-level significant original research work.

Review papers: These are concise, significant but helpful and decisive topics for young researchers.

Research articles: These are handled with small investigation and applications

Research letters: The letters are small and concise comments on previously published matters.

5. STRUCTURE AND FORMAT OF MANUSCRIPT

The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also. Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

Papers*:* These are reports of significant research (typically less than 7000 words equivalent, including tables, figures, references), and comprise:

(a)*Title* should be relevant and commensurate with the theme of the paper.

(b) A brief Summary, *"Abstract"* (less than 150 words) containing the major results and conclusions.

(c) Up to *ten keywords*, that precisely identifies the paper's subject, purpose, and focus.

(d) An *Introduction*, giving necessary background excluding subheadings; objectives must be clearly declared.

(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; *conclusions* should be summarizing.

(h) Brief Acknowledgements.

(i) References in the proper form.

Authors should very cautiously consider the preparation of papers to ensure that they communicate efficiently. Papers are much more likely to be accepted, if they are cautiously designed and laid out, contain few or no errors, are summarizing, and be conventional to the approach and instructions. They will in addition, be published with much less delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and to make suggestions to improve briefness.

It is vital, that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

Format

Language: The language of publication is UK English. Authors, for whom English is a second language, must have their manuscript efficiently edited by an English-speaking person before submission to make sure that, the English is of high excellence. It is preferable, that manuscripts should be professionally edited.

Standard Usage, Abbreviations, and Units: Spelling and hyphenation should be conventional to The Concise Oxford English Dictionary. Statistics and measurements should at all times be given in figures, e.g. 16 min, except for when the number begins a sentence. When the number does not refer to a unit of measurement it should be spelt in full unless, it is 160 or greater.

Abbreviations supposed to be used carefully. The abbreviated name or expression is supposed to be cited in full at first usage, followed by the conventional abbreviation in parentheses.

Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than 1.4 × 10-3 m3, or 4 mm somewhat than 4 × 10-3 m. Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

Structure

All manuscripts submitted to Global Journals Inc. (US), ought to include:

Title: The title page must carry an instructive title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) wherever the work was carried out. The full postal address in addition with the e-mail address of related author must be given. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining and indexing.

Abstract, used in Original Papers and Reviews:

Optimizing Abstract for Search Engines

Many researchers searching for information online will use search engines such as Google, Yahoo or similar. By optimizing your paper for search engines, you will amplify the chance of someone finding it. This in turn will make it more likely to be viewed and/or cited in a further work. Global Journals Inc. (US) have compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Key Words

A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy and planning a list of possible keywords and phrases to try.

Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

Choice of key words is first tool of tips to write research paper. Research paper writing is an art.A few tips for deciding as strategically as possible about keyword search:

- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.

- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

References follow the *Harvard scheme* of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals Inc. (US) homepage at the judgment of the Editorial Board.

The Editorial Board and Global Journals Inc. (US) recommend that, citation of online-published papers and other material should be done via a DOI (digital object identifier). If an author cites anything, which does not have a DOI, they run the risk of the cited material not being noticeable.

The Editorial Board and Global Journals Inc. (US) recommend the use of a tool such as Reference Manager for reference management and formatting.

Tables, Figures and Figure Legends

Tables: Tables should be few in number, cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g. Table 4, a self-explanatory caption and be on a separate sheet. Vertical lines should not be used.

Figures: Figures are supposed to be submitted as separate files. Always take in a citation in the text for each figure using Arabic numbers, e.g. Fig. 4. Artwork must be submitted online in electronic form by e-mailing them.

Preparation of Electronic Figures for Publication

Even though low quality images are sufficient for review purposes, print publication requires high quality images to prevent the final product being blurred or fuzzy. Submit (or e-mail) EPS (line art) or TIFF (halftone/photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Do not use pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings) in relation to the imitation size. Please give the data for figures in black and white or submit a Color Work Agreement Form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution (at final image size) ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs) : >350 dpi; figures containing both halftone and line images: >650 dpi.

Color Charges: It is the rule of the Global Journals Inc. (US) for authors to pay the full cost for the reproduction of their color artwork. Hence, please note that, if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a color work agreement form before your paper can be published.

Figure Legends: Self-explanatory legends of all figures should be incorporated separately under the heading 'Legends to Figures'. In the full-text online edition of the journal, figure legends may possibly be truncated in abbreviated links to the full screen version. Therefore, the first 100 characters of any legend should notify the reader, about the key aspects of the figure.

6. AFTER ACCEPTANCE

Upon approval of a paper for publication, the manuscript will be forwarded to the dean, who is responsible for the publication of the Global Journals Inc. (US).

6.1 Proof Corrections

The corresponding author will receive an e-mail alert containing a link to a website or will be attached. A working e-mail address must therefore be provided for the related author.

Acrobat Reader will be required in order to read this file. This software can be downloaded

(Free of charge) from the following website:

www.adobe.com/products/acrobat/readstep2.html. This will facilitate the file to be opened, read on screen, and printed out in order for any corrections to be added. Further instructions will be sent with the proof.

Proofs must be returned to the dean at dean@globaljournals.org within three days of receipt.

As changes to proofs are costly, we inquire that you only correct typesetting errors. All illustrations are retained by the publisher. Please note that the authors are responsible for all statements made in their work, including changes made by the copy editor.

6.2 Early View of Global Journals Inc. (US) (Publication Prior to Print)

The Global Journals Inc. (US) are enclosed by our publishing's Early View service. Early View articles are complete full-text articles sent in advance of their publication. Early View articles are absolute and final. They have been completely reviewed, revised and edited for publication, and the authors' final corrections have been incorporated. Because they are in final form, no changes can be made after sending them. The nature of Early View articles means that they do not yet have volume, issue or page numbers, so Early View articles cannot be cited in the conventional way.

6.3 Author Services

Online production tracking is available for your article through Author Services. Author Services enables authors to track their article - once it has been accepted - through the production process to publication online and in print. Authors can check the status of their articles online and choose to receive automated e-mails at key stages of production. The authors will receive an e-mail with a unique link that enables them to register and have their article automatically added to the system. Please ensure that a complete e-mail address is provided when submitting the manuscript.

6.4 Author Material Archive Policy

Please note that if not specifically requested, publisher will dispose off hardcopy & electronic information submitted, after the two months of publication. If you require the return of any information submitted, please inform the Editorial Board or dean as soon as possible.

6.5 Offprint and Extra Copies

A PDF offprint of the online-published article will be provided free of charge to the related author, and may be distributed according to the Publisher's terms and conditions. Additional paper offprint may be ordered by emailing us at: editor@globaljournals.org .

Before start writing a good quality Science Frontier Research Paper, let us first understand what is Science Frontier Research Paper? So, Frontier Research Paper is the paper which is written by professionals or scientists who are associated to Physics, Mathematics, Chemistry, Zoology, Botany, Bio-tech, Geology, Military Science, Environment and all Interdisciplinary & Frontier Subjects etc., or doing research study in these areas. If you are novel to this field then you can consult about this field from your supervisor or guide.

Techniques for writing a good quality Applied Science Research Paper:

1. Choosing the topic- In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Frontier Science. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: At a first glance, this point looks obvious but it is first recommendation that to write a quality research paper of any area, first draft your paper in Microsoft Word. By using MS Word, you can easily catch your grammatical mistakes and spelling errors.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

9. Use and get big pictures: Always use encyclopedias, Wikipedia to get pictures so that you can go into the depth.

10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. **Pick a good study spot:** To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. **Think technically:** Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

INFORMAL GUIDELINES OF RESEARCH PAPER WRITING

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.
- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.

You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.

- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Administration Rules Listed Before Submitting Your Research Paper to Global Journals Inc. (US)

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.
- Do not give permission to anyone else to "PROOFREAD" your manuscript.

Written Material: You may discuss with your guides and key sources.

- Do not copy or imitate anyone else paper. (Various Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

Criterion for Grading a Research Paper *(Compilation)*
by Global Journals Inc. (US)

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals Inc. (US).

Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

Index

N

O

P

Q

R

S

T

U

W

save our planet

Global Journal of Science Frontier Research

Visit us on the Web at www.GlobalJournals.org | www.ComputerResearch.org
or email us at helpdesk@globaljournals.org

ISSN 9755896