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highlights

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- Superposed Squeezed Coherent States
- Totally Umbilical Hypersurfaces
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From the Chief Author's Desk

We see a drastic momentum everywhere in all fields now a day. Which in turns, say a lot to everyone to excel with all possible way. The need of the hour is to pick the right key at the right time with all extras. Citing the computer versions, any automobile models, infrastructures, etc. It is not the result of any preplanning but the implementations of planning.

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An Experimental Comparison Of Combustion, Performance And Emission In A Single Cylinder Thermal Barrier Coated Diesel Engine Using Diesel And Biodiesel

GJSFR Classification – D (FOR) 090201,090205,020304

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Abstract- **The use of methyl esters of vegetable oil known as biodiesel are increasingly popular because of their low impact on environment, green alternate fuel and most interestingly it's use in engines does not require major modification in the engine hardware. Use of biodiesel as sole fuel in conventional direct injection diesel engine results in combustion problems, hence it is proposed to use the biodiesel in low heat rejection (LHR) diesel engines with its significance characteristics of higher operating temperature, maximum heat release, higher brake thermal efficiency (BTE) and ability to handle the lower calorific value (CV) fuel. In this work biodiesel from Jatropha oil called as Jatropha oil methyl ester (JOME) was used as sole fuel in conventional diesel engine and LHR direct injection (DI) diesel engine. The low heat rejection engine was developed with uniform ceramic coating of combustion chamber (includes piston crown, cylinder head, valves and cylinder liner) by partially stabilized zirconia (PSZ) of 0.5 mm thickness. The experimental investigation was carried out in a single cylinder water-cooled LHR direct injection diesel engine. In this investigation, the combustion, performance and emission analysis were carried out in a diesel and biodiesel fueled conventional and LHR engine under identical operating conditions. The test result of biodiesel fueled LHR engine was quite identical to that of the conventional diesel engine. The brake thermal efficiency (BTE) of LHR engine with biodiesel is decreased marginally than LHR engine operated with diesel. Carbon monoxide (CO) and Hydrocarbon (HC) emission levels are decreased but in contrast the Oxide of Nitrogen (NOX) emission level was increased due to the higher peak temperature. The results of this comparative experimental investigation reveals that, some of the drawbacks of biodiesel could be made as advantageous factors while using it as a fuel in the LHR diesel engine. In the final analysis, it was found that, the results are quite satisfactory.**

I. INTRODUCTION

Diesel engines are the dominating one primarily in the field of transportation and secondarily in agricultural machinery due to its superior fuel economy and higher fuel efficiency. The world survey explicit that the diesel fuel consumption is several times higher than that of gasoline fuel. These fuels are fossil in nature, leads to the depletion of fuel and increasing cost. It has been found that the chemically treated vegetable oil often called as biodiesel is a promising fuel, because of their properties are similar to that of diesel fuel (DF) and it is a renewable and can be easily produced. Compared to the conventional DI diesel engine the basic concept of LHR engine is to suppress the heat rejection to the coolant so that the useful power output can be increased, which in turn results in improved thermal efficiency. However previous studies are revealing that the thermal efficiency variation of LHR engine not only depends on the heat recovery system, but also depends on the engine configuration, operating condition and physical properties of the insulation material (1–3).

The drawback of an LHR engine has to be considered seriously and effort has to be taken to reduce the increased heat loss with the exhaust and increased level of NOx emission. The potential techniques available for the reduction of NOx from diesel engines are exhaust gas recirculation (EGR), water injection, slower burn rate, reduced intake air temperature and particularly retarding the injection timing (4–6). It is strongly proven that the increasing thickness of ceramic coatings arrest the heat loss from the engine cylinder, in contrast decreases the power and torque. The optimized coating thickness can be identified through the simulation techniques (7). One of the viable significance of LHR engine is utilizing the low calorific value fuel such as biodiesel. Studies have revealed that, the use of biodiesel under identical condition as that for the diesel fuel results in slightly lower performance and emission levels due to the mismatching of the fuel properties mainly low calorific value and higher viscosity. The problems associated with the higher viscosity of biodiesel in a compression ignition (CI) engines are pumping loss, gum formation, injector nozzle coking, ring sticking and incompatibility with lubricating oil (8-12). The above identified problems with the use of biodiesel in conventional diesel engine can be reduced in LHR engines except for the injection problem.

The present investigation involves the comparison of combustion, performance and emission levels of diesel and biodiesel (Jatropha based) in conventional and LHR DI diesel engines.

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II. Fuel Preparations and Characterization

The vegetable oil was transesterified-using methanol in the presence of NaOH as a catalyst. The parameter involved in the above processing includes the catalyst amount, molar ratio of alcohol to oil, reaction temperature and reaction time (13-15). The parameters for the biodiesel production are optimized such as Catalyst amount, Molar ratio (Alcohol to Oil), Reaction temperature, and Reaction time.

The raw biodiesel obtained was brought down pH to a value of 7. This pure biodiesel was measured on weight basis and the important physical and chemical properties were determined as per the BIS standards (14).It is evident that, the dilution or blending of vegetable oil with other fuels like alcohol or diesel would bring the viscosity close to the specification range for a diesel engine (16-17). The important physical and chemical properties of the biodiesel thus prepared and given in table 1.

Table 1 Properties of the diesel and biodiesel fuel

Characteristics	Diesel Fuel	B100
Density @ 15°C(kg/m^3)	837	880
Viscosity @ 40°C(cSt)	3.2	4.6
Flash point (°C)	65	170
Cetane number	47	50
Calorific Value (MJ/kg)	42	39.5

III. Development of Test Engine

The engine combustion chamber was coated with partially stabilized zirconia (PSZ) of 0.5 mm thickness, which includes the piston crown, cylinder head, valves, and outside of the cylinder liner. The equal amount of material has been removed from the various parts of the combustion chamber and PSZ was coated uniformly. After PSZ coating, the engine was allowed to run about 10 hours, then test were conducted on it.

IV. Experimental Procedure

The experimental setup and the specification of the test engine are shown in Fig.1 and table 3 respectively.

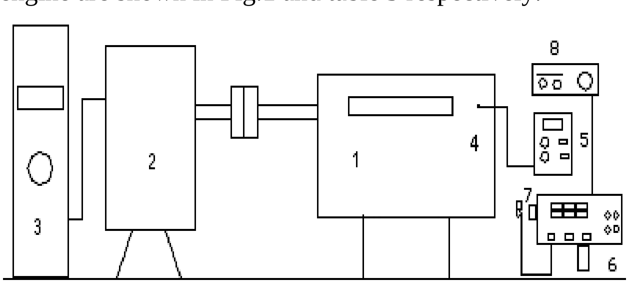

Fig.1 Experimental setup

i. Test engine
ii. Dynamometer
iii. Dynamometer controller
iv. Piezo electric pressure transducer
v. Charge amplifiers
vi. Data acquisition system
vii. Magnetic pickup
viii. Computer

The engine was coupled with an eddy current dynamometer for performance and emission testing. A piezoelectric transducer was mounted through an adopter in the cylinder head to measure the in-cylinder pressure. Signal from the pressure transducer was fed to charge amplifier. A magnetic shaft encoder was used to measure the TDC and crank angle position. The signals from the charge amplifier and shaft encoder were given to the appropriate channels of a data acquisition system.

The analyzer used to measure the engine exhaust emission was calibrated before each test. Using the appropriate calibration curve, the measurement error for each analyzer was reduced as per the recommendation by the exhaust analyzer manual. The emission values obtained in the form of ppm and percentage was expressed in term of specific mass basis (g/kWh). The NOx measurements were corrected for humidity following the procedure recommended by the Society of Automotive Engineers (SAE, 1993). Exhaust gas temperature was measured using an iron-constantan thermocouple and mercury thermometer was used to measure the cooling water temperature. Diesel and biodiesel was used in the conventional diesel engine and the PSZ coated LHR engine. Cylinder pressure data was recorded and the other desired datas were processed.

The experiments were carried out in a single cylinder, naturally aspirated, constant speed, water-cooled direct injection diesel engine with the following specifications

Table 2 Specification of test engine

No of stroke	Four stroke
No of cylinder	One
Bore, mm	87.5
Stroke, mm	110
Compression ratio	17. 5: 1
Rated power output	4.4 kW @ 1500 rpm
Injection Pressure, bar	200
Injection timing	24° BTDC

The test procedure was adopted from Beareu of Indian Standards BIS –10000(year 1985).

V. Combustion and Heat Release Analysis

The combustion parameters can be studied through the analysis of heat release rate obtained from the pressure crank angle diagram. It is assumed that the mixture is homogeneous and uniform pressure and temperature at each instant of time during the combustion process. The heat release rate can be calculated from the first law of thermodynamics.

$$\frac{dU}{dt} = \frac{dQ}{dt} + \sum m_i h_i - p\frac{dV}{dt} \qquad (1)$$

Where

$\frac{dQ}{dt}$ - Heat transfer rate

$p\frac{dV}{dt}$ - Work done by the system

m_i - Mass of flow in to the system

h_i - Enthalpy of flow in to the system

p - Pressure at any crank angle

V - Volume at any crank angle

U - Internal energy at any crank angle

By neglecting the crevice volume and its effect, the equation (1) is reduced to

$$\frac{dU}{dt} = \frac{dQ}{dt} = m_f h_f - p\frac{dV}{dt} \qquad (2)$$

Where

m_f - Fuel flow rate

h_f - Enthalpy of the fuel

This equation (2) can be further reduced to

$$\frac{dQn}{dt} = \frac{dQch}{dt} - \frac{dQht}{dt} = p\frac{dV}{dt} + mCv\frac{dT}{dt} \qquad (3)$$

Where

$\frac{dQn}{dt}$ - Net heat release rate

$\frac{dQch}{dt}$ - Gross heat release rate

$\frac{dQht}{dt}$ - Heat transfer rate to the wall

From the ideal gas relation PV=mRT this equation (3) is further modified in to

$$\frac{dQn}{dt} = \frac{n}{n-1}p\frac{dV}{dt} + \frac{1}{n-1}V\frac{dP}{dt} \qquad (4)$$

The pressure at any angle obtained form the pressure crank angle diagram makes it possible to find out the heat release at any crank angle.

VI. Results and Discussions

A. Cylinder Pressure

In a CI engine the cylinder pressure is depends on the fuel-burning rate during the premixed burning phase, which in turn leads better combustion and heat release. Figure 2 shows the typical variation of cylinder pressure with respect to crank angle. The cylinder pressure in the case of biodiesel fueled LHR engine is about 7.5 % lesser than the diesel fueled LHR engine and higher by about 2.75 % and 8.33% than conventional engine fueled with diesel and biodiesel. This reduction in the incylinder pressure may be due to lower calorific value and slower combustion rates associated with biodiesel fueled LHR engine. However the cylinder pressure is relatively higher than the diesel engine fueled with diesel and biodiesel.

It is noted that the maximum pressure obtained for LHR engine fueled with biodiesel was closer with TDC around 2 degree crank angle than LHR engine fueled with diesel The fuel-burning rate in the early stage of combustion is higher in the case of biodiesel than the diesel fuel, which bring the peak pressure more closer to TDC.

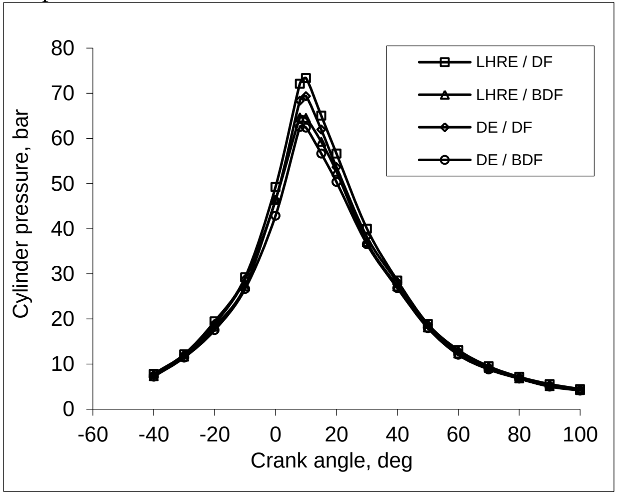

Fig. 2 Variation of cylinder pressure with respect to crank angle at full load

B. Heat Release Rate

Figure 3 shows the variation of heat release rate with respect to crank angle. It is evident from the graph that, diesel and biodiesel fuel experiences the rapid premixed combustion followed by diffusion combustion. The premixed fuel burns rapidly and releases the maximum amount heat followed by the controlled heat release. The heat release rate during the premixed combustion is responsible for the cylinder peak pressure.

The maximum heat release of LHR engine with biodiesel is lower about 8.1% than LHR engine fueled with diesel and higher about 2.4% and 7.02% respectively than conventional engine fueled with diesel and biodiesel. It was found that, premixed combustion in the case of biodiesel fuel starts earlier than the diesel fuel and it may be due to excess oxygen available along with higher operating temperature in the fuel and the consequent reduction in delay period than that of diesel fuel. It may be expected that high surrounding temperature and oxygen availability of fuel itself (bio diesel) reduce the delay period. However higher molecular weight lower calorific value and slightly higher value of viscosity bring down the peak heat release during the premixed combustion period. The heat release is well advanced due to the shorter delay period and early burning of the biodiesel. It is found that, the heat release rate of biodiesel, normally accumulated during the delay period follows the similar trends like diesel fuel.

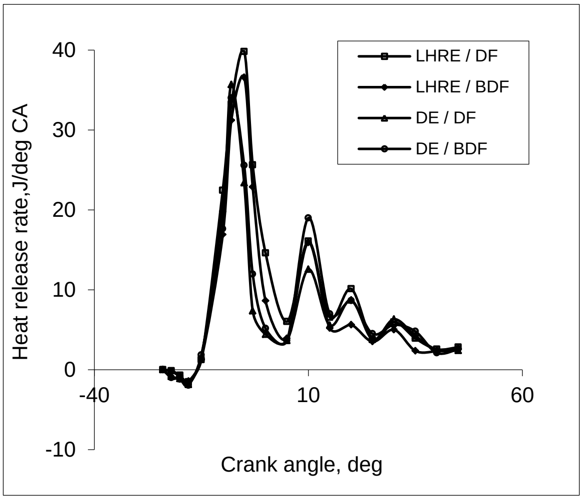

Fig. 3 Variation of heat release rate with respect to crank angle at full load

C. Cumulative Heat Release Rate

Figure 4 shows the variation of cumulative heat release with respect to crank angle. In general, the availability of oxygen in the biodiesel fuel itself enhances the combustion and thus increases the net heat release. In this investigation at full load, the net heat release for LHR engine fueled with biodiesel is lower by about 8.34% than LHR engine fueled with diesel and higher by about 2.35% and 7.04% respectively than LHR engine with biodiesel and conventional diesel engine fueled with diesel and biodiesel.

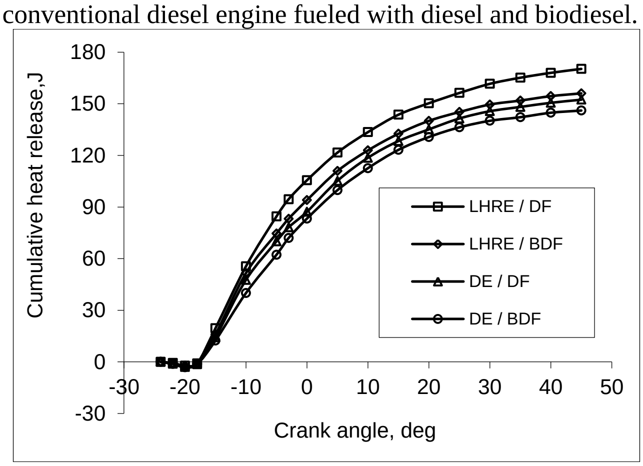

Fig.4 Variation of cumulative heat release with respect to crank angle at full load

D. Brake Thermal Efficiency

Figure 5 shows the variation of brake thermal efficiency with engine power output. The maximum efficiency obtained in the case of LHR engine fueled with biodiesel at full load was lower by about 2.92% than LHR engine fueled with diesel and higher by about 1.77% and 5.6% respectively than conventional diesel engine fueled with diesel and biodiesel. In overall, it is evident that, the thermal efficiency obtained in the case of LHR engine fueled with biodiesel is substantially good enough within the power output range of the test engine.

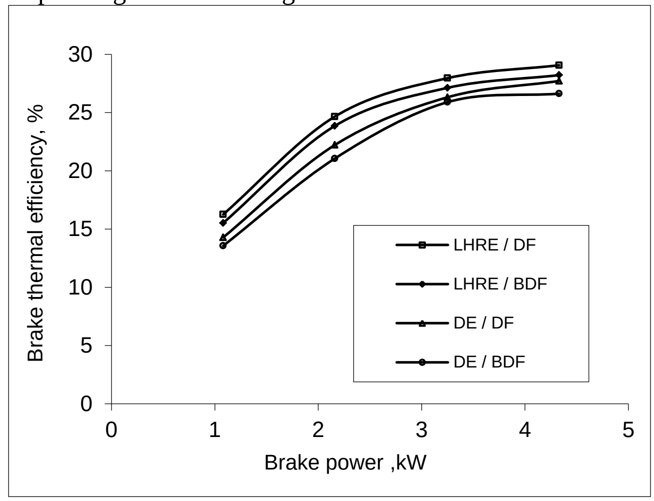

Fig. 5 Variation of brake thermal efficiency with engine power output

E. Specific Fuel Consumption

The variations of brake specific fuel consumption (SFC) with engine power output for different fuels are presented in figure 6. At maximum load the specific fuel consumption of LHR engine fueled with biodiesel is higher by about 6.27% than LHR engine fueled with diesel and lower by about 3.77% and 11.14% respectively than conventional engine fueled with diesel and biodiesel.

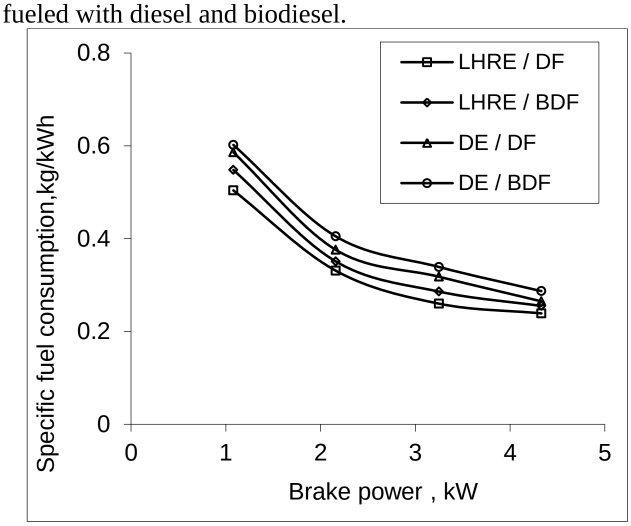

Fig. 6 Variation of Specific fuel consumption with engine power output

This higher fuel consumption was due to the combined effect of lower calorific value and high density of biodiesel. The test engine consumed additional biodiesel fuel in order to retain the same power output.

F. Specific Energy Consumption

Figure 7 shows the variation between specific energy consumption (SEC) and engine power output. The heat input required to produce unit quantity of power is proportionately varying with SFC. Higher the energy required at low load and decreases by increasing the load. It is found that the specific energy consumption of LHR engine with biodiesel is higher by about 1.27% than the LHR engine with diesel fuel and lower by about 3.78% and 11.15% respectively for conventional diesel engine with diesel and biodiesel

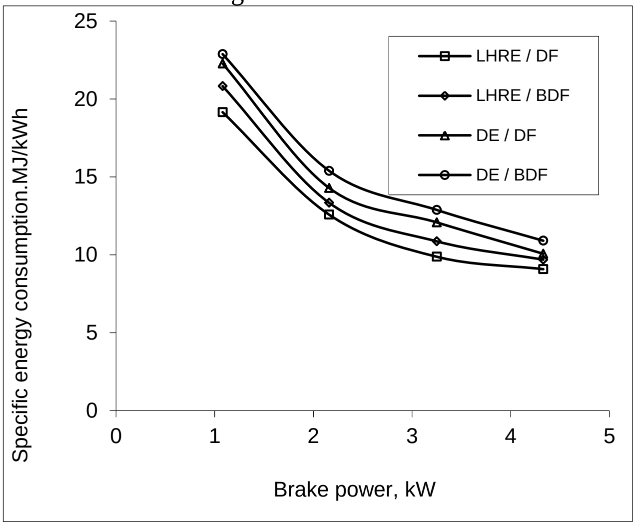

Fig.7 Variation of specific energy consumption with engine power output

G. Exhaust Gas Temperature

Figure 8 shows the variation of exhaust gas temperature with engine power output. At full load, the exhaust gas temperature of LHR engine fueled with biodiesel gives lower value by about 2.52% than LHR engine fueled with diesel and higher by about 2.83% and 6.13% respectively than conventional engine with diesel and biodiesel. The higher operating temperature of LHR engine is responsible for the higher exhaust temperature. The exhaust gas temperature of biodiesel varying proportionately with engine power output as in the case of diesel fuel. It may be due to the heat release rate by the biodiesel during the expansion is comparatively lower than diesel.

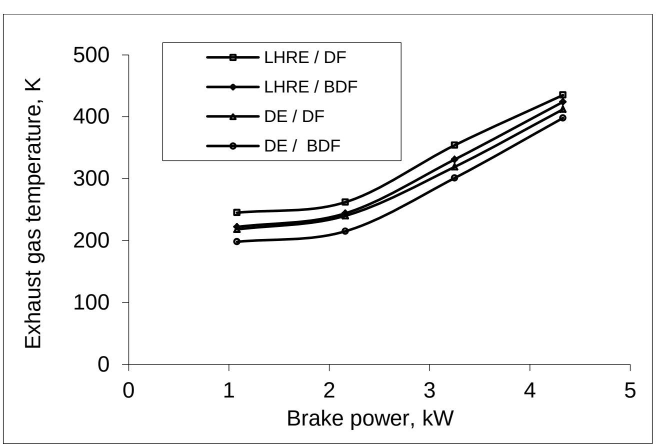

Fig.8 Variation of exhaust gas temperature with engine power output

H. Carbon Monoxide

The variation of carbon monoxide (CO) with engine power output is presented in figure 9. The fuels are producing higher amount of carbon monoxide emission at low power outputs and giving lower values at higher power conditions. Carbon monoxide emission decreases with increasing power output. At full load, CO emission for LHR engine with biodiesel fuel is lower by about 10.73%, 26.82% and 31.89% respectively than LHR engine with diesel, conventional engine fueled with biodiesel and diesel. With increasing biodiesel percentage, CO emission level decreases. Biodiesel itself has about 11% oxygen content in it and it may helps for the complete combustion. Hence, CO emission level decreases with increasing biodiesel percentage in the fuel.

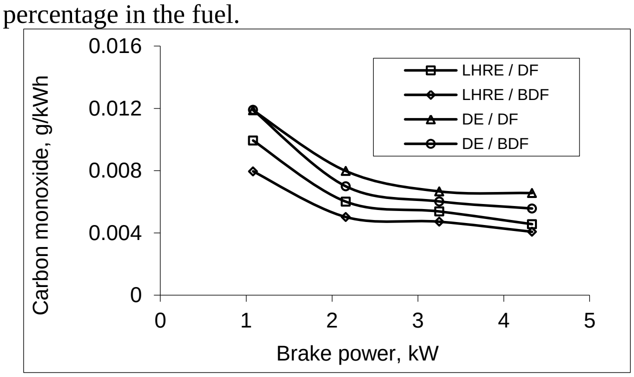

Fig. 9 Variation of carbon monoxide with engine power output

I. Unburned Hydrocarbon

The variation of hydrocarbon (HC) with respect to engine power output for different fuels are shown in figure 10.The high operating temperature in LHR engine makes the combustion nearly complete than the limited operating temperature condition as in the case of diesel engine. At full load hydrocarbon emission levels are decreases for LHR engine fueled with biodiesel than LHR engine fueled with diesel and diesel engine fueled with diesel and biodiesel such as 9.8%, 17.21% and 21.31% respectively. The air fuel mixture, which was accumulated in the crevice volume, was reduced due to the high temperature and availability of oxygen, which in turn leads to reduction in unburned hydrocarbon emissions.

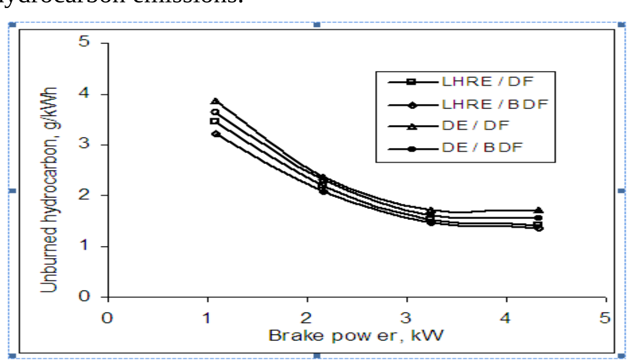

Fig. 10 Variation of hydrocarbon with engine power output

J. Oxides Of Nitrogen

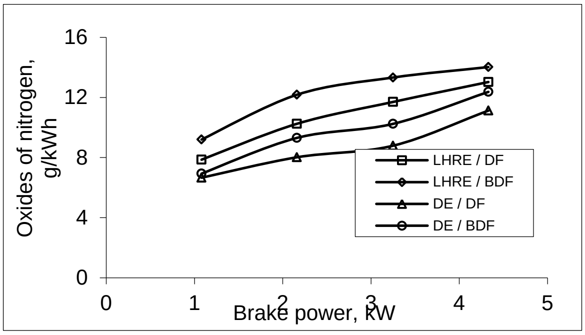

Fig. 11 Variation of oxides of nitrogen with engine power output

Figure 11 shows the variation of oxides of nitrogen with engine power output. The main reason for the formation of oxides of nitrogen in an IC engines are high temperature and availability of oxygen. At maximum load, NOx emission for LHR engine with biodiesel fuel is higher about 7.09%, 13.35 and 20.37% respectively than LHR engine fueled with diesel and conventional diesel engine with biodiesel and diesel. In LHR engine, the operating conditions are in favor of NO species and such as the availability of oxygen in the fuel itself other than the oxygen available in the air and high temperature due to insulation coating, which enhance the NO species formation.

K. Particulate Matter

Fig.12 shows the variation of particulate matter with engine power output. The particulate matter of LHR engine with biodiesel fuel is higher about 3.1% than LHR engine with diesel and lower by about 3% and 11.1% respectively than conventional diesel engine fueled with diesel and biodiesel. It is clearly found that, the particulate matter for biodiesel was higher irrespective of the engine used compared with the diesel fuel. This is due to the incomplete combustion of biodiesel fuel.

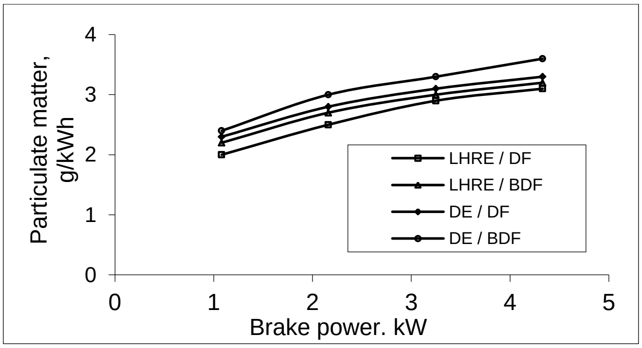

Fig. 12 Variation of particulate matter with sengine power output

VII. Conclusion

The biodiesel produced from Jatropha oil by transesterification process reduces the viscosity of the oil in order to match the suitability of diesel fuel. The diesel engine is modified in to LHR engine by means of partially stabilized zirconia (PSZ) coating. The various combustion parameters such as cylinder pressure, rate of heat release, cumulative heat releases were analyzed and the following conclusions were arrived it.

i. At full load condition, the cylinder pressure in the case of biodiesel fueled LHR engine was lower than that of the diesel fueled LHR engine. Even though this reduction under identical condition is substantial. The absolute value of this cylinder peak pressure is well within operating limits of the test engine.
ii. The final analysis of the heat release shows that, the value of net heat release in the case of biodiesel fueled LHR engine is substantially good enough for the effective work done of the test engine.

The performance characteristics such as brake thermal efficiency, specific fuel consumption and specific energy consumption and various emission characteristics were compared and summarized as follows.

i. The maximum efficiency obtained in the case of LHR engine fueled with biodiesel was lower than the LHR engine operated with diesel fuel. However the efficiency of the LHR engine with biodiesel fuel is well within the expected limits.
ii. The exhaust gas temperature of LHR engine fueled with biodiesel was lower than LHR engine fueled with diesel throughout the operating condition. The low exhaust gas temperature indicates the heat release rate during the late combustion was comparatively lower than diesel fuel.
iii. The specific fuel consumption of LHR engine with biodiesel was higher than LHR engine fueled with diesel. The higher consumption of fuel due to low calorific value and high viscosity. Even though it could be expected to the offset by the cost of biodiesel.
iv. The specific energy consumption of LHR engine with biodiesel was higher than LHR engine fueled with diesel fuel.
v. It was found that, CO and HC emissions for LHR engine with biodiesel was considerably lower than LHR engine fueled with diesel. This reduction of emissions due to excess oxygen availability along with higher operating temperature.
vi. NO emission for LHR engine with biodiesel fuel was higher than LHR engine fueled with diesel. The operating conditions of LHR engine were favorable to NO formation. However this increase in emission level was within the acceptable limits.
vii. The particulate matter of LHR engine with biodiesel fuel is higher than LHR engine fueled with diesel due to incomplete combustion.

The above comparative study clearly reveals the possibility of using the biodiesel in LHR direct injection diesel engine. The combustion, performance and emission characteristics show the suitability of biodiesel in LHR engine.

VIII. REFERENCE

1) Bryzik WR, Kamo R," TACOM/ Cummins adiabatic engine program', SAE paper No. 830314, February 1983.
2) Woschni G, Spindler W, Kolesa K," Heat insulation of combustion chamber walls—a measure to decrease the fuel combustion of I.C. engines," SAE paper No.850359.
3) Kamo R, Mavinahally NS, Kamo L, Bryzik W, Schwartz E," Injection characteristics that improve performance of ceramics coated diesel engines", SAE paper No.1999-01-0972.
4) Aman CA," Promises and challenges of the low-heat-rejection diesel", J Eng Gas Turb Power 1988; 110:475–81.
5) Herzog PL, Burgler L," NOx reduction strategies for DI Diesel engines", SAE No. 920470, 1992.
6) Maramatsu G, Abe A, Yosdiha K," Catalytic reduction of NOx in Diesel exhaust", SAE paper No. 930135, 1993.
7) P.Tamil Porai, "Simulation and Analysis of combustion and heat transfer in low heat rejection diesel engine using two zone combustion model and different heat transfer models". Ph.D Thesis, Anna University, Feb 1998.
8) Pryde EH," Vegetable oil as diesel fuels: overview. Papers from the symposium on vegetable oils as diesel fuels", Presented at the 73rd AOCS annual meeting, Toronto, Canada. J. Am. Oil Chem. Soc. 1983; 60(8).
9) Ryan TW, Dodge LG, Callahan TJ," The effects of vegetable oil properties on injection and combustion in two different diesel engines," J. Am. Oil Chem. Soc. 1984; 61(10): 1610–9.
10) Ziejewski M, Kaufman KR," Laboratory endurance test of sunflower oil blend in a diesel engine", J. Am. Oil Chem. Soc. 1983; 60(8): 1567–73.
11) Korus RA, Jaiduk J, Peterson CL," A rapid engine test to measure injector fouling in diesel engines using vegetable oil fuels", J. Am. Oil Chem. Soc. 1985; 62(11): 1563–4.
12) Rewolinski C, Shaffer DL," Sunflower oil diesel fuel: lubrication system contamination", J. Am. Oil Chem. Soc. 1985; 61(7): 1120–4.
13) Fangrui Maa, Milford A. Hannab:" Biodiesel production: a review" , Bioresource Technology, 70 (1999) 1-15.
14) P. Chitra, P. Venkatachalam and A. Sampathrajan:, "Optimisation of experimental conditions for biodiesel production from alkali-catalysed transesterification of Jatropha curcus oil", Energy for sustainable development ,Volum eIX No.3,September 2005.
15) L.C. Meher a, Vidya S.S. Dharmagadda b, S.N. Naik a, "Optimization of alkali-catalyzed transesteriWcation of Pongamia pinnata oil for production of Biodiesel ", Bioresource Technology, 97 (2006) 1392–1397
16) Agarwal AK, "Vegetable oils verses diesel fuel: development and use of biodiesel in a compression ignition engine", TIDE 1998; 8(3): 191–204.
17) Sinha S, Misra NC," Diesel fuel alternative from vegetable oils", Chem. Engg World 1997; 32(10): 77–80.
18) B.Rajendra Prasath, P.Tamil Porai, Mohd. F. Shair "Theoretical Modeling and Experimental Study of Combustion and Performance Characteristics of Biodiesel in Turbocharged Low Heat Rejection D.I Diesel Engine", World Academy of Science, Engineering and Technology 61 2010.
19) B.Rajendra Prasath, P.Tamil Porai, Mohd. F. Shair "Simulation and Analysis of Combustion, Performance and Emission Characteristics of Biodiesel Fueled Low Heat Rejection Direct Injection Diesel Engine", SAE Paper No. 2007-32-0094.
20) Mohd. F. Shair, P.Tamil Porai, B.Rajendra Prasath " Analysis of Combustion, Performance and Emission Characteristics of Turbocharged LHR Extended Expansion DI Diesel Engine" World Academy of Science, Engineering and Technology 61 2010.
21) Mohd. F. Shair, P.Tamil Porai, B.Rajendra Prasath "Analysis of expanded cycle and internal EGR for LHR DI diesel engines", Journal of future Engineering and Technology, Vol.5, No.2, pp.32-41.

Nomenclature

LHRE	-	Low heat rejection engine
BDF	-	Biodiesel fuel
DE	-	Diesel engine
DF	-	Diesel fuel
DI	-	Direct injection
PSZ	-	Partially stabilized zirconia
BTE	-	Brake thermal efficiency, %
CV	-	Calorific value, MJ/kg
CO	-	Carbon monoxide, g/kWh
HC	-	Hydrocarbon, g/kWh
NO	-	Nitric oxide, g/kWh
NO_x	-	Oxides of nitrogen, g/kWh
EGR	-	Exhaust gas recirculation
LHR	-	Low heat rejection
SEC	-	Specific energy consumption, kJ/kWh
SFC	-	Specific fuel consumption, kg/kWh
TDC	-	Top dead center
BTDC	-	Before top dead center
BIS	-	Bureau of Indian standards
U	-	Internal energy, kJ/kg K
Q	-	Heat transfer rate, kJ/s
m	-	Mass of fuel. KJ/s
h	-	Enthalpy, kJ/kg
p	-	Pressure, bar
V	-	Volume, m^3
t	-	Time, s
T	-	Temperature, K
C_v	-	Specific heat at constant volume, kJ/kgK
λ	-	Equivalence ratio

Jordan Ideal In Non-Commutative Rings

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GJSFR Classification – F (FOR)
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Abstract- **In this paper we have defined Jordan ideal in a non-commutative ring with the properties that every ideal is a Jordan in a ring with char 2. If P is a prime ideal then R / P is a prime ring for any non-commutative ring R. Later we have proved that the Jordan ideal is a Quasi as well as Bi-ideal while the converse is not true.**

Keywords- Prime Rings, Jordan Ideal, Non-commutative Ring.

I. INTRODUCTION

W. E Baxter [10] has defined the closed Jordan ideal in topological rings. I.N.Herstein [1] has defined the Jordan derivation in prime rings and proved that any Jordan derivation of a prime ring R such that char R ? 2 is a derivation of R. Later Ram Awtar [2] extended this result on Lie ideals. The main purpose of this paper is to describe the structure of Jordan ideal in non-commutative rings. Throughout in this paper we assume that R is a non-commutative ring and the Jordan ideal is denoted by J.

Definition 1.1- A ring R is said to be prime ring if R is non-commutative ring with the property that aRb = (0) implies a=0 or b=0 for all a, b $\in$ R.

Definition 1.2-A non-empty subset J of R is said to be the Jordan ideal of R if J is an additive subgroup of R with the property that JR+RJ $\subseteq$ J implies $xr_1 + r_2y \in J$, for all x, y $\in$ J, $r_1, r_2 \in$ R.

Remark 1.3-For any x in ring R with unity, x ? J if and only if xr + rx ? J.

Example 1.4-This is the example of a prime ring with unity and char 2.

Let $Q = \{a+\hat{i}b+\hat{j}c+\hat{k}d / a,b,c,d \in ?_2\}$, where $?_2$ is the ring of modulo 2

and $\hat{i}^2=\hat{j}^2=\hat{k}^2=-1$, $\hat{i}.\hat{j}==-\hat{j}.\hat{i}=\hat{k}$, $\hat{j}.\hat{k}=-\hat{k}.\hat{j}=\hat{i}$, $\hat{k}.\hat{i}=-\hat{i}.\hat{k}=\hat{j}$.

Definition 1.5- An Ideal P in a ring R is said to be prime ideal if aRb$\in$P implies a$\in$P or b$\in$P.

Example 1.6-Let R = Z. Then P = {23n/nÎ Z} is a prime ideal of R.

Theorem 1.7-Let R be a non-commutative ring and P be a prime ideal of R then R/P is a prime ring.

Proof-Since R is a non-commutative and associative, therefore R/P is noncommutative and associative. For showing that R/P is prime ring, we consider

$(x+P)\frac{R}{P}(y+P)=P$. Thus $(x+P)(r+P)(y+P)=P$, for all x$\in$R implies that x r y $\in$P, for all r$\in$R. But P is a prime ideal, therefore we have x$\in$P or y$\in$P.

Example 1.8-Let R

$$\left\{\begin{pmatrix} a & b & c \\ x & y & z \\ 0 & 0 & 0 \end{pmatrix} \Big/ a,b,c,x,y,z \in Z\right\}$$

R is a non-commutative ring with respect to matrix addition and matrix multiplication.

$$\text{Let } B=\left\{\begin{pmatrix} 0 & 0 & x \\ 0 & 0 & y \\ 0 & 0 & 0 \end{pmatrix} \Big/ x,y \in Z\right\}$$

Then B is the Jordan ideal of .

Proof-Clearly B is an ideal of R. Obviously; B is an additive subgroup of R.

$$\text{Let } X=\begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{pmatrix} \in R,\ Y=\begin{pmatrix} 0 & 0 & x \\ 0 & 0 & y \\ 0 & 0 & 0 \end{pmatrix} \in B,$$

Then

$$XY+YX=\begin{pmatrix} 0 & 0 & ax+by \\ 0 & 0 & dx+ey \\ 0 & 0 & 0 \end{pmatrix} \in B$$

Hence B is the Jordan ideal of R. The following example shows that Jordan ideal need not be an ideal.

Example 1.9-Let R = $\left\{\begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix} \Big/ x \in Z\right\}$ be ring under usual matrix operation and

$J = \left\{\begin{pmatrix} 0 & y \\ 0 & 0 \end{pmatrix} \Big/ y \in Z\right\}$Then J is a Jordan ideal but not an deal.

Proof-Let X = $\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} \in R$, $Y=\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \in J$. Then $XY+XY=\begin{pmatrix} 0 & ab \\ 0 & 0 \end{pmatrix} \in J$

Clearly J is a left ideal but not a right ideal. Hence J is not an ideal but J is the Jordan ideal of R.

Theorem 1.10-If R is a prime ring with unity then Jordan ideal is a subprime ring of a ring R.

Proof-Let J be a Jordan ideal of R. Then $xr_1 + r_2y \in$ J, for all x, y$\in$J, $r_1, r_2 \in$ R Clearly, J is a subring of R. So it remains to show that J is a prime ring.

Let x, y$\in$J such that xry= 0, for all r$\in$R. J is the Jordan ideal of R. Therefore x, y $\in$R. R is a prime ring therefore x=0 or y = 0

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Hence J is a sub-prime ring of a ring R.

Theorem 1.11-Let J be a Jordan ideal of R. Then

$[R:J]=\{x\in R \,/\, rx\in J \,,\forall x\in R\}$ is the Jordan ideal of R containing J.

Proof-Let $0\in R$ such that $r.0 = 0\in J$, for all $r\in R$.

Therefore $0\in[R: J]$. Clearly $[R: J] \subseteq R$.

Now, assume that a, b $\in$ [R: J]. Then [R: J] is an additive subgroup of R. Let $a\in[R: J]$ and $r_1\in R$. Then $ra\in J$, for all $r\in R$. Which implies that $rar_1 + r_1ra \in J$, for all $r\in R$.

By taking $r = r_1$, we get $ar + ra \in [R: J]$. Now to show that $J \subseteq [R: J]$. Let $y\in J$ be any element. Then $r_1y + yr_2 \in J$, for all $y\in J$, $r_1,r_2\in R$. By taking $r_2=0$, $r_1=r$, we get $ry\in J$. Hence $y\in[R: J]$.Thus $J \subseteq R$.

II. Algebraic Properties of Jordan Ideal

Theorem 2.1-Sum of two Jordan ideal is Jordan.

Let x, y $\in J_1+J_2$. Then $x = p_1+p_2$, $y = k_1+k_2$, $p_1, k_1 \in J_1$, $p_2, k_2\in J_2$

And, $x-y \in J_1+J_2$

Consider $x r_1+ r_2y$, for $r_1,r_2 \in R$.

$$x r_1+ r_2y = (p_1+p_2) r_1+r_2 (k_1+k_2)$$

$$= p_1r_1+r_2k_1+p_2r_1+r_2k_2\in J_1+J_2$$

Proof-Let J1 and J2be two Jordan ideals.

Theorem 2.2-Intersection of two Jordan ideals is Jordan.

Proof-Let J1 and J2 be two Jordan ideals.

Let x, y $\in J_1 \cap J_2$. Then $x-y \in J_1\cap J_2$

Let $r_1, r_2\in R$, $x,y\in J_1\cap J_2$.. Then $xr_1 +r_2y \in J_1$ and $xr_1 + r_2y \in J_2$.

Thus $xr_1+r_2y \in J_1\cap J_2$..Hence $J_1\cap J_2$.is Jordan ideal.

Similarly, we can prove the following theorem.

Theorem 2.3- An arbitrary intersection of Jordan ideals of a ring R is Jordan ideal of R.

Theorem 2.4-Union of two Jordan ideals need not be Jordan is shown by the following Example.

Let $R =\{(x, y)\in Z \times Z\}$ be a ring under the following operations

(a,b)+(c, d) = (a+c, b+d) and (a,b) (c,d)= (ac,bd)

Let $=\{(x,0)\in Z\times Z\}$, $=\{(0, y)\in Z\times Z\}$

J_1 and J_2 are Jordan ideal but

$J_1 \cup J_2 = \{(x,0),(0,y) \in Z\times Z\}$ is not Jordan.

Definition 2.5-If J1 and J2 are two Jordan ideals, then

$$J_1J_2 = \left\{\sum_{i,j=1}^{n} a_ib_j \Big/ a_i \in J_1, b_j \in J_2\right\}$$

Theorem 2.6-Let R be a ring of char 2. Then if J1 and J2 are Jordan, J1 J2 is also A Jordan ideal of R.

Proof-Let J_1 and J_2 be two Jordan ideals of a ring R of char 2, then $0 \in J_1$ and J_2.

Thus, $0 = 0.0 \in J_1 J_2$ implies $J_1J_2 \neq \phi$.

Let x, y be any two element of $J_1 J_2$.

So $x = \sum_{i,j=1}^{n} u_iv_j$, $y = \sum_{i,j=1}^{n} u_i'v_j'$ $\quad u_i,u_i' \in J_1$, $v_j,v_j' \in J_2$

Then $x-y \in J_1 J_2$

Let xÎ J1 J2 and r = r1 = r2 be any element

Consider, $xr +rx = \sum_{i,j=1}^{n} u_iv_j r + r\sum_{i,j=1}^{n} u_iv_j \in J_2 J_2 + J_1 J_2 \subseteq J_1 J_2$

Hence J1 J2 is Jordan ideal of R.

III. Jordan Ideal in Non-Commutative Ring R with Char 2

Definition3.1-An additive mapping D from a ring R to R is called a Jordan derivation if [3].

$$D(x^2) = xD(x) + D(x)\, x, \text{ for all } X\in R$$

Similarly [2], $D(x^3) = x^2 D(x) +xD(x)x + D(x) x^2$, for all $x\in R$.

Theorem 3.2-Let η be the collection of all the Jordan derivation of a ring R with unity. Then η is a Jordan ideal.

Proof-Let $D(x)\in\eta$ and $X\in R$. The $D(x)x +xD(x) \in \eta R + R\eta$ and $D(x)x+xD(x)=D(x^2) \in\eta$ [2].

Lemma 3.3-Let R be a ring with unity and char 2, then for a, b in R, ab $\in$ J implies that,

(i) ab (r-1) + (r-1) ab $\in$ J, for all r $\in$R,

(ii) r (ab-1) + (ab-1) r $\in$ J, for all r$\in$R.

Proof-Since ab$\in$J and J is the Jordan r deal, therefore $abr_1+ r_2ab\in J$, for all $r_1, r_2\in J$

Taking $r_1 = r_2 = r$, we get

abr + rab $\in$ J, r $\in$ R

(ii). Similarly, abr + rab $\in$ J, for all r$\in$R

Hence (ab-1) r+r(ab-1) $\in$ J, for all r$\in$R.

Lemma 3.4-Let R be a ring with unity and char 2, then for a, b $\in$R, aRb $\subseteq$J implies ab $\in$J.

Proof-Since R is a ring with unity, therefore for 1 ÎR, abÎJ.

Definition 3.5-An additive subgroup U of R is said to be Lie ideal of R, if [u,r] = ur-ru $\in$ U, for all u$\in$U, r$\in$R.

Theorem 3.6-In a ring R with char 2, every Lie ideal is a Jordan ideal and conversely.

Proof-Let R be a ring with char 2. Then x+x = 0, for all x$\in$R. Let J be the Lie ideal of R. Then [y, r] = yr-ry$\in$J, for all y $\in$J, r$\in$R. Since yr-ry$\in$J and J $\subseteq$ R and R is A ring with char 2. Therefore yr + ry – ry- ry $\in$J implies yr + ry $\in$J. Hence J is Jordan ideal of R. Similarly, we can prove that the Jordan ideal of a ring R with char 2 is a Lie ideal.

The following result holds trivially.

Remark3.7-Every ideal is a Jordan ideal in a ring R.

IV. Structure of Jordan Ideal in Non-Commutative Ring

Definition 4.1-[6] A non-empty subset Q of a ring R is said to be Quasi ideal if

(i). Q is additive subgroup,

(ii). $QR \cap RQ \subseteq Q$

Theorem 4.2: Jordan ideal of a ring R is a Quasi ideal.

Proof-Let J be the Jordan ideal of a ring R. then RJ + JR $\subseteq$ J.JR$\cap$RJ $\subseteq$ (JR+RJ) $\cap$ (JR+RJ) = JR + RJ$\subseteq$ J.Converse of above theorem need not be true.

Example 4.3-Let R be the ring of 2x2 matrices over integers.

Proof-Let Q = $\left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \Big/ a,b \in Z \right\}$

$$\text{Let } x = \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \in Q,\ r = \begin{pmatrix} r_1 & r_2 \\ r_3 & r_4 \end{pmatrix} \in R.$$

Then

$$xr \cap rx \in Q. \text{ But } xr + rx = \begin{pmatrix} ar_1 + r_1 a + r_2 b & ar_2 \\ br_1 + r_3 a + r_4 b & br_4 \end{pmatrix} \notin Q.$$

Hence Q is the Quasi ideal but not Jordan.

Definition 4.6-[6] A subring V of a ring R is said to be Bi-ideal if VRV$\subseteq$V.

Theorem 4.7-Every Jordan ideal is a Bi-ideal in a ring R.

Proof-J being the Jordan ideal of R is the subring of R. We have to show that JRJ$\subseteq$J.

Let x$\in$JRJ such that x $\notin$J. Since J is Jordan xr + rx $\notin$ J. which implies that (yry)r + r (yry) $\notin$J. Thus y (ryr) +ryr) y $\notin$ J. Hence $yr_1 + r_1 y \notin$ J, where r_1=ryr which is A contradiction as x = yry$\in$JRJ, for all y$\in$J, r$\in$R.

Remark 4.8-Converse of above result need not be true.

Example 4.9-Since every quasi ideal is a Bi-ideal, therefore an example 4.3 is biideal but not Jordan.

Example 4.10-Let R =

$$\left\{ \begin{pmatrix} a & b & c \\ g & h & i \\ k & l & m \end{pmatrix} \Big/ a,b,c,g,h,i,k,l,m \in Z \right\}$$

be the ring under usual matrix operation.

Then $Q = \left\{ \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Big/ a \in Z \right\}$ is Quasi, bi-ideal but not Jordan.

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VI. References

1) I.N. Hersterin: Jordan derivation of prime rings, Proc. Math. Soc. 8(1957), 1104-1110.
2) Ram Awtar: Lie ideals and Jordan derivations of prime rings, Proc. of the Amer. Math. Soc. Vol.90, No.1 (1982), 9-14.
3) Ram Awtar: Jordan derivations and Jordan homomorphisms on prime rings of char 2, Acta Math. Hung, 51 (1-2), 1988, 61-63.
4) Sarita Agarwal: A note on direct product ring, Acta Ciencia Indica, Vol. XXXIIM, No.4, 1387 (2006).
5) Sarita Agarwal and M.P. Chaudhary: R- Factor in Direct product rings, South East Asian J. Math. and Math. Sc. Vol.4, No.3 (2006), pp. 55-58.
6) Sarita Agarwal: Ideals in direct –product ring, Acta Ciencia Indica, Vol.XXXII M, No.4, 1525 (2006).
7) Sarita Agarwal: Some applications on ideals in direct-product ring, Applied Science Periodical, Vol. XI, No.3, 2009.
8) Sarita Dewan: Quasi-relation on ternary semi group, Indian J. Pure Appl.Math., 28(6) 753-766 (1997).
9) V.N. Dixit and Sarita Dewan: A note on Quasi and Bi-ideals in Ternary semigroups, Internet J. Math and Math. Sci. Vol. 18, No.3 (1995), 501-508.
10) W.E.Baxter, On Rings with proper involution, Pacific Journal of Mathematics.Vol.27,No.1,1968

Squeezing Properties Of Measurement Phase Operator In Superposed Squeezed Coherent States

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***Abstract-* The squeezing properties in the superposition of coherent States and squeezed state are investigated by means of the measurement phase operator introduced by Barnett and S M and Pagg D T.**

Keywords- A new kind of odd and even coherent states; measurement operator; squeezing.

I. Introduction

Recently, the discussion on phase of quantum optical field has given rise to a great many interest. It is known that in the quantum optical, the amplitude of field is directly proportional to square root of optic-quantum number operator and its phase is described by Susskind-Glogower phase operator $\exp(\pm i\varphi)$, however Susskind-Glogower phase operator are possessed of unitary trait, hence hermit operator isn't constructed by it. Though two hermit operator below

$$\cos\varphi = \frac{1}{2}[\exp(i\varphi) + \exp(-i\varphi)] \qquad (1)$$

$$\sin\varphi = \frac{1}{2}[\exp(i\varphi) - \exp(-i\varphi)] \qquad (2)$$

are set , they have classical property because $\langle \cos^2\varphi \rangle + \langle \sin^2\varphi \rangle \neq 1$. On purpose to overcome the difficulty, unitary exponential phase operator and measurement phase operator in optical field have defined by Pegg and Branett, and then the phase essence of optical field is studied in progress [1−2] . In some laboratories, the measurement phase operator usually corresponds to the measurement of phase, therefore the measurement phase operator has raised extensive concerns. Some classical properties are researched in detail for squeezing states, quasi-optical coherent states, squeezing optical number states, odd and even coherent states, Schrödinger cat states and so on.

In the quantum optics, coherent state and squeezed state are two very important states in the quantum optics [3~7] . Coherent state is introduced by Schrödinger at first as most classical state in pure quantum state, which is eigenstate of annihilation operator $\hat{a}$, whose two variances of orthogonal amplitude of vibration are equal and they satisfy with

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amplitude of vibration are equal and they satisfy with minimal uncertainty relation. Squeezed state is squeezing transition vacuum state, whose one orthogonal amplitude variance is less than coherent state, however the other is more than coherent state. Squeezed state has very important application in interference measurement in high degree of accuracy, photo-communication as well as detection of gravitation wave and microwave signal [6~7] . According to superposition principle, a great many of new quantum states are constructed through superposition of arbitrary state and many favorable works have done. But these superposition states are usually composed of the same sort state, such as superposition of odd-even coherent state or squeezed odd-even coherent state. Some of their non-classical properties on measurement operator have discussed in detail. Recently the new superposition state composed of coherent state and squeezed state is studied and its quantum effect is obtained. In the paper, the squeezed properties of the superposition state on measurement operator are researched by mean of method in the paper [6] .

II. The Superposition of Squeezed Coherent States

Superposed coherent states is defined

$$|\beta,\delta> = N[|\beta> + e^{i\delta}|-\beta>], \qquad (3)$$

where $|\beta>$ and $|-\beta>$ are the coherent states, $N = \{2[1+\cos\delta\exp(-2|\beta|^2)\}^{-1/2}$ is normal parameter.

When $\delta = 0, \pi$ there are below two equations

$$|\beta,0> = N[|\beta> + |-\beta>],\ |\beta,\pi> = N[|\beta> - |-\beta>] \qquad (4)$$

They are even coherent and odd coherent states respectively.

After squeezing operator $S(\xi) = \exp[\frac{1}{2}(\xi^* a^2 - \xi a^{+2})]$ acts on coherent $|\beta>$, it becomes squeezed coherent states $|\beta>_g$, and there is

$$|\beta>_g = S(\xi)|\beta> = \sum_{n=0}^{\infty}(un!)^{-1/2}(\frac{v}{2u})^{n/2}H_n(\frac{\beta}{\sqrt{2uv}})\exp(-\frac{|\beta|^2}{2}+\frac{v^*\beta^2}{2u}) \qquad (5)$$

Where $\beta = |\beta|e^{i\phi}$, $\xi = re^{i\theta}$, $u = \cosh r$, $v = e^{i\theta}\sinh r$, $|\beta|$ and ϕ denotes the intensity and phase of coherent $|\beta>$, r is squeezing factor, θ is squeezing angle, $H_n(z)$ is hermite polynomial.

By the same method, squeezing operator $S(\xi)$ act on (3), and then obtain squeezed coherent $|\beta,\delta>_g$

$$|\beta,\delta>_g = N[|\beta>_g + e^{i\delta}|-\beta>_g] \tag{6}$$

When $\delta = 0$ and π, (6) become squeezed even odd coherent states

$$|\beta,0>_g = N[|\beta>_g + |-\beta>_g] \tag{7}$$

$$|\beta,\pi>_g = N[|\beta>_g - |-\beta>_g] \tag{8}$$

The operator $\hat{b}$ is defined below

$$\hat{b} = u\hat{a} + v\hat{a}^+, \hat{b}^+ = u^*\hat{a}^+ + v^*\hat{a}, \tag{9}$$

where $\hat{a}$ and $\hat{a}^+$ are Bose annihilation and creating operator, operator $\hat{b}$ and $\hat{b}^+$ have the same as commutation relation $\hat{a}$ and $\hat{a}^+$, i.e. $[\hat{a},\hat{a}^+] = [\hat{b},\hat{b}^+] = 1$, and

$$\hat{b}|\beta,\delta>_g = \beta|\beta,\delta>_g. \tag{10}$$

Above expression illustrates that $|\beta,\delta>_g$ is eigenstates of $\hat{b}$. For next requirement, sign is introduced

$$|\beta,\delta>_{g,t} = N[|\beta>_g - e^{i\delta}|-\beta>_g] \tag{11}$$

the equations are derived (m is integral number) from (6) 、(10) and (11)

$$\hat{b}^{2m+1}|\beta,\delta>_g = \beta^{2m+1}|\beta,\delta>_{g,t}, \quad \hat{b}^{2m}|\beta,\delta>_g = \beta^{2m}|\beta,\delta>_g. \tag{12}$$

$$_g<\beta,\delta|\beta,\delta>_g = 1, \tag{13}$$

$$_g<\beta,\delta|\beta,\delta>_{g,t} = -2iN^2\sin\delta\exp(-2|\beta|^2), \tag{14}$$

$$_{g,t}<\beta,\delta|\beta,\delta>_{g,t} = \frac{1-\cos\delta\exp(-2|\beta|^2)}{1+\cos\delta\exp(-2|\beta|^2)} = C, \tag{15}$$

III. ORTHOGONAL MEASUREMENT PHASE OPERATOR

Pegg and Barnnet have defined orthogonal measurement phase operator below

$$\cos_M\varphi = \lambda X_1, \quad \sin_M\varphi = \lambda X_2 \tag{16}$$

Where $X_1 = \frac{1}{2}(\hat{a}+\hat{a}^+)$, $X_2 = \frac{1}{}(\hat{a}-\hat{a}^+)$ According to the non-classical requirement of $<\cos^2_M\varphi|\sin^2_M\varphi>=1$ $\lambda = (\bar{n}+1/2)^{-1/2}$, where $\bar{n}$ is average photon number in considered states in the paper, $\bar{n} = <\hat{a}^+\hat{a}> = <N>$.

By means of the relation of measurement phase operator and optical number operator, the important equations are got

$$[\cos_M\varphi, \sin_M\varphi] = \frac{i}{2}\lambda^2, \tag{17}$$

$$[\cos_M\varphi, N] = i\sin_M\varphi, \tag{18}$$

$$[\sin_M\varphi, N] = -i\cos_M\varphi. \tag{19}$$

IV. SQUEEZING OF MEASUREMENT PHASE OPERATOR

In quantum-optical field, when two operators Y_1 and Y_2 don't satisfy reciprocal relation, its accuracy of measurement is restricted by measurement indeterminacy principle below

$$<(\Delta Y_1)^2><(\Delta Y_2)^2> > \frac{1}{4}|[Y_1,Y_2]|. \tag{20}$$

If the inequality is right

$$<(\Delta Y_i)^2> < \frac{1}{2}|[Y_1,Y_2]|, \quad (i=1,2) \tag{21}$$

then there is squeezing effect in optical field component Y_i.

With the view of description degree of squeezing, S_i is defined

$$S_i = <(\Delta Y_i)^2> - \frac{1}{2}|[Y_1,Y_2]|, \quad (i=1,2) \tag{22}$$

When $S_i < 0$, it indicates that there is squeezing in the Y_i.

From the expression (15), (16), the squeezing degrees of measurement phase operator are

$$\begin{aligned} S_1^{CS} &= <(\Delta\cos_M\varphi)^2> - \frac{1}{4}|\lambda|^2 = \frac{\lambda^2}{4}<\hat{a}^2+\hat{a}^{+2}> - \frac{\lambda^2}{4}<\hat{a}+\hat{a}^+>^2 + \frac{\lambda^2}{2}<\hat{a}^+\hat{a}> \\ &= \frac{\lambda^2}{4}[2<\hat{a}^+\hat{a}> + <\hat{a}^2+\hat{a}^{+2}> - <\hat{a}+\hat{a}^+>^2] \end{aligned} \tag{23}$$

$$\begin{aligned} S_2^{CS} &= <(\Delta\sin_M\varphi)^2> - \frac{1}{4}|\lambda|^2 = -\frac{\lambda^2}{4}<\hat{a}^2+\hat{a}^{+2}> + \frac{\lambda^2}{4}<\hat{a}-\hat{a}^+>^2 + \frac{\lambda^2}{2}<\hat{a}^+\hat{a}> \\ &= \frac{\lambda^2}{4}[2<\hat{a}^+\hat{a}> + <\hat{a}-\hat{a}^+>^2 - <\hat{a}^2+\hat{a}^{+2}>] \end{aligned} \tag{24}$$

$$\begin{aligned} S^{CN} &= <(\Delta\cos_M\varphi)^2> - \frac{1}{2}|<\sin_M\varphi>| \\ &= \frac{\lambda^2}{4}<\hat{a}^2+\hat{a}^{+2}> - \frac{\lambda^2}{4}<\hat{a}+\hat{a}^+>^2 - \frac{|\lambda|}{4}|<\hat{a}-\hat{a}^+>| + \frac{\lambda^2}{2}<\hat{a}^+\hat{a}> + \frac{\lambda^2}{4} \\ &= \frac{\lambda^2}{4}[2<\hat{a}^+\hat{a}> + 1 + <\hat{a}^2+\hat{a}^{+2}> - <\hat{a}+\hat{a}^+>^2] - \frac{|\lambda|}{4}|<\hat{a}-\hat{a}^+>| \end{aligned} \tag{25}$$

$$\begin{aligned} S^{SN} &= <(\Delta\sin_M\varphi)^2> - \frac{1}{2}|<\cos_M\varphi>| \\ &= -\frac{\lambda^2}{4}<\hat{a}^2+\hat{a}^{+2}> + \frac{\lambda^2}{4}<\hat{a}-\hat{a}^+>^2 - \frac{|\lambda|}{4}|<\hat{a}+\hat{a}^+>| + \frac{\lambda^2}{2}<\hat{a}^+\hat{a}> + \frac{\lambda^2}{4} \\ &= \frac{\lambda^2}{4}[2<\hat{a}^+\hat{a}> + 1 - <\hat{a}^2+\hat{a}^{+2}> + <\hat{a}-\hat{a}^+>^2] - \frac{|\lambda|}{4}|<\hat{a}+\hat{a}^+>| \end{aligned} \tag{26}$$

In order to calculate above four values, $\hat{a}$ and $\hat{a}^+$ are denoted by $\hat{b}$ and $\hat{b}^+$

$$\hat{a}=u^{*}\hat{b}-v\hat{b}^{+},\quad \hat{a}^{+}=u\hat{b}^{+}-v^{*}\hat{b} \tag{27}$$

$$\hat{a}+\hat{a}^{+}=(u^{*}-v^{*})\hat{b}+(u-v)\hat{b}^{+}$$
$$=(\cosh r-e^{-i\theta}\sinh r)\hat{b}+(\cosh r-e^{i\theta}\sinh r)\hat{b}^{+} \tag{28}$$

$$\hat{a}-\hat{a}^{+}=(u^{*}+v^{*})\hat{b}-(u+v)\hat{b}^{+}$$
$$=(\cosh r+e^{-i\theta}\sinh r)\hat{b}-(\cosh r+e^{i\theta}\sinh r)\hat{b}^{+} \tag{29}$$

$$\hat{a}^{+}\hat{a}=(|u|^{2}+|v|^{2})\hat{b}^{+}\hat{b}-u^{*}v^{*}\hat{b}^{2}-uv\hat{b}^{+2}+|v|^{2}$$
$$=\cosh 2r\hat{b}^{+}\hat{b}-\frac{1}{2}e^{-i\theta}\sinh 2r\hat{b}^{2}-\frac{1}{2}e^{i\theta}\sinh 2r\hat{b}^{+2}+\sinh^{2}r \tag{30}$$

$$\hat{a}^{2}=u^{*2}\hat{b}^{2}+v^{2}\hat{b}^{+2}-2u^{*}v\hat{b}^{+}\hat{b}-u^{*}v \tag{31}$$

$$\hat{a}^{+2}=u^{2}\hat{b}^{+2}+v^{*2}\hat{b}^{2}-2uv^{*}\hat{b}^{+}\hat{b}-uv^{*} \tag{32}$$

$$\hat{a}^{2}+\hat{a}^{+2}=(u^{*2}+v^{*2})\hat{b}^{2}+(u^{2}+v^{2})\hat{b}^{+2}-2(u^{*}v+uv^{*})\hat{b}^{+}\hat{b}-(u^{*}v+uv^{*})$$
$$=(\cosh^{2}r+e^{-2i\theta}\sinh^{2}r)\hat{b}^{2}+(\cosh^{2}r+e^{2i\theta}\sinh^{2}r)\hat{b}^{+2} \tag{33}$$
$$-2\sinh 2r\cos\theta\hat{b}^{+}\hat{b}-2\sinh 2r\cos\theta$$

V. THE DISCUSSION OF SQUEEZING EFFECT OF MEASUREMENT PHASE OPERATOR

Through calculating, the expressions below are obtained

$$\hat{a}^{+}\hat{a}=(|u|^{2}+|v|^{2})\hat{b}^{+}\hat{b}-u^{*}v^{*}\hat{b}^{2}-uv\hat{b}^{+2}+|v|^{2}$$
$$=\cosh 2r\hat{b}^{+}\hat{b}-\frac{1}{2}e^{-i\theta}\sinh 2r\hat{b}^{2}-\frac{1}{2}e^{i\theta}\sinh 2r\hat{b}^{+2}+\sinh^{2}r$$
$$\bar{n}=<\hat{a}^{+}\hat{a}>={}_{g}<\beta,\delta|(ch2r\hat{b}^{+}\hat{b}-\frac{1}{2}e^{-i\theta}sh2r\hat{b}^{2}-\frac{1}{2}e^{i\theta}sh2r\hat{b}^{+2}+sh^{2}r)|\beta,\delta>_{g}$$
$$=|\beta|^{2}ch2r_{g,t}<\beta,\delta|\beta,\delta>_{g,t}-\frac{1}{2}e^{-i\theta}\beta^{2}sh2r_{g}<\beta,\delta|\beta,\delta>_{g}$$
$$-\frac{1}{2}e^{i\theta}\beta^{*2}sh2r_{g}<\beta,\delta|\beta,\delta>_{g}+sh^{2}r_{g}<\beta,\delta|\beta,\delta>_{g}$$
$$=C|\beta|^{2}ch2r-\frac{1}{2}e^{-i\theta}\beta^{2}sh2r-\frac{1}{2}e^{i\theta}\beta^{*2}sh2r+sh^{2}r$$
$$=C|\beta|^{2}ch2r-\frac{1}{2}|\beta|^{2}sh2r(e^{-i\theta+i2\phi}+e^{i\theta-i2\phi})+sh^{2}r$$
$$=C|\beta|^{2}ch2r-|\beta|^{2}sh2r\cos(\theta-2\phi)+sh^{2}r$$
$$\bar{n}=C|\beta|^{2}\cosh 2r-|\beta|^{2}\sinh 2r\cos(\theta-2\phi)+sh^{2}r \tag{34}$$

$$\hat{a}+\hat{a}^{+}=(u^{*}-v^{*})\hat{b}+(u-v)\hat{b}^{+}$$
$$=(chr-e^{-i\theta}shr)\hat{b}+(chr-e^{i\theta}shr)\hat{b}^{+}$$
$$<\hat{a}+\hat{a}^{+}>={}_{g}<\beta,\delta|(\hat{a}+\hat{a}^{+})|\beta,\delta>_{g}$$
$$={}_{g}<\beta,\delta|[(chr-e^{-i\theta}shr)\hat{b}+(chr-e^{i\theta}shr)\hat{b}^{+}]|\beta,\delta>_{g}$$
$$=(chr-e^{-i\theta}shr)_{g}<\beta,\delta|\hat{b}|\beta,\delta>_{g}+(chr-e^{i\theta}shr)_{g}<\beta,\delta|\hat{b}^{+}|\beta,\delta>_{g}$$
$$=\beta(chr-e^{-i\theta}shr)_{g}<\beta,\delta|\beta,\delta>_{g,t}+\beta^{*}(chr-e^{i\theta}shr)_{g,t}<\beta,\delta|\beta,\delta>_{g}$$
$$=\beta(chr-e^{-i\theta}shr)(-2iN^{2}\sin\delta\exp(-2|\beta|^{2}))+\beta^{*}(chr-e^{i\theta}shr)(2iN^{2}\sin\delta\exp(-2|\beta|^{2}))$$
$$=2iN^{2}|\beta|chr\sin\delta\exp(-2|\beta|^{2})(e^{-i\phi}-e^{i\phi})+2iN^{2}|\beta|shr\sin\delta\exp(-2|\beta|^{2})(e^{-i(\theta-\phi)}-e^{i(\theta-\phi)})$$
$$=-2iN^{2}|\beta|chr\sin\delta\exp(-2|\beta|^{2})(e^{i\phi}-e^{-i\phi})-2iN^{2}|\beta|shr\sin\delta\exp(-2|\beta|^{2})(e^{i(\theta-\phi)}-e^{-i(\theta-\phi)})$$
$$=4N^{2}|\beta|chr\sin\phi\sin\delta\exp(-2|\beta|^{2})+4N^{2}|\beta|shr\sin(\theta-\phi)\sin\delta\exp(-2|\beta|^{2})$$
$$=4N^{2}|\beta|\sin\delta[chr\sin\phi+shr\sin(\theta-\phi)]\exp(-2|\beta|^{2})$$
$$<\hat{a}+\hat{a}^{+}>=4N^{2}|\beta|\sin\delta[\cosh r\sin\phi+\sinh r\sin(\theta-\phi)]\exp(-2|\beta|^{2}) \tag{35}$$

$$\hat{a}-\hat{a}^{+}=(u^{*}+v^{*})\hat{b}-(u+v)\hat{b}^{+}$$
$$=(chr+e^{-i\theta}shr)\hat{b}-(chr+e^{i\theta}shr)\hat{b}^{+}$$
$$<\hat{a}-\hat{a}^{+}>={}_{g}<\beta,\delta|(\hat{a}-\hat{a}^{+})|\beta,\delta>_{g}$$
$$={}_{g}<\beta,\delta|[(chr+e^{-i\theta}shr)\hat{b}-(chr+e^{i\theta}shr)\hat{b}^{+}]|\beta,\delta>_{g}$$
$$=(chr+e^{-i\theta}shr)_{g}<\beta,\delta|\hat{b}|\beta,\delta>_{g}-(chr+e^{i\theta}shr)_{g}<\beta,\delta|\hat{b}^{+}|\beta,\delta>_{g}$$
$$=\beta(chr+e^{-i\theta}shr)_{g}<\beta,\delta|\beta,\delta>_{g,t}-\beta^{*}(chr+e^{i\theta}shr)_{g,t}<\beta,\delta|\beta,\delta>_{g}$$
$$=\beta(chr+e^{-i\theta}shr)(-2iN^{2}\sin\delta\exp(-2|\beta|^{2}))-\beta^{*}(chr+e^{i\theta}shr)(2iN^{2}\sin\delta\exp(-2|\beta|^{2}))$$
$$=-2iN^{2}|\beta|chr\sin\delta\exp(-2|\beta|^{2})(e^{i\phi}+e^{-i\phi})-2iN^{2}|\beta|shr\sin\delta\exp(-2|\beta|^{2})(e^{-i(\theta-\phi)}+e^{i(\theta-\phi)})$$
$$=-4iN^{2}|\beta|chr\cos\phi\sin\delta\exp(-2|\beta|^{2})-4iN^{2}|\beta|shr\cos(\theta-\phi)\sin\delta\exp(-2|\beta|^{2})$$
$$=-4iN^{2}|\beta|\sin\delta[chr\cos\phi+shr\cos(\theta-\phi)]\exp(-2|\beta|^{2})$$
$$<\hat{a}-\hat{a}^{+}>=-4iN^{2}|\beta|\sin\delta[\cosh r\cos\phi+\sinh r\cos(\theta-\phi)]\exp(-2|\beta|^{2}) \tag{36}$$

$$\hat{a}^{2}+\hat{a}^{+2}=(u^{*2}+v^{*2})\hat{b}^{2}+(u^{2}+v^{2})\hat{b}^{+2}-2(u^{*}v+uv^{*})\hat{b}^{+}\hat{b}-(u^{*}v+uv^{*})$$
$$=(ch^{2}r+e^{-2i\theta}sh^{2}r)\hat{b}^{2}+(ch^{2}r+e^{2i\theta}sh^{2}r)\hat{b}^{+2}-2sh2r\cos\theta\hat{b}^{+}\hat{b}-sh2r\cos\theta$$
$$<\hat{a}^{2}+\hat{a}^{+2}>={}_{g}<\beta,\delta|\hat{a}^{2}+\hat{a}^{+2}|\beta,\delta>_{g}$$
$$={}_{g}<\beta,\delta|[(ch^{2}r+e^{-2i\theta}sh^{2}r)\hat{b}^{2}+(ch^{2}r+e^{2i\theta}sh^{2}r)\hat{b}^{+2}-2sh2r\cos\theta\hat{b}^{+}\hat{b}-sh2r\cos\theta]|\beta,\delta>_{g}$$
$$=(ch^{2}r+e^{-2i\theta}sh^{2}r)_{g}<\beta,\delta|\hat{b}^{2}|\beta,\delta>_{g}+(ch^{2}r+e^{2i\theta}sh^{2}r)_{g}<\beta,\delta|\hat{b}^{+2}|\beta,\delta>_{g}$$
$$-2sh2r\cos\theta_{g}<\beta,\delta|\hat{b}^{+}\hat{b}|\beta,\delta>_{g}-sh2r\cos\theta\,{}_{g}<\beta,\delta||\beta,\delta>_{g}$$
$$=\beta^{2}(ch^{2}r+e^{-2i\theta}sh^{2}r)+\beta^{*2}(ch^{2}r+e^{2i\theta}sh^{2}r)-sh2r\cos\theta(2C|\beta|^{2}+1)$$
$$=2|\beta|^{2}ch^{2}r\cos 2\phi+2|\beta|^{2}sh^{2}r\cos(2\theta-2\phi)-sh2r\cos\theta(2C|\beta|^{2}+1)$$
$$<\hat{a}^{2}+\hat{a}^{+2}>=2|\beta|^{2}\cosh^{2}r\cos 2\phi+2|\beta|^{2}\sinh^{2}r\cos(2\theta-2\phi)-\sinh 2r\cos\theta(2C|\beta|^{2}+1) \tag{37}$$

Through calculating, the expressions below are obtained

$$S_1^{CS} = <(\Delta\cos_M\varphi)^2> - \frac{1}{4}|\lambda|^2 = \frac{\lambda^2}{4}<\hat a^2+\hat a^{+2}> - \frac{\lambda^2}{4}<\hat a+\hat a^+>^2 + \frac{\lambda^2}{2}<\hat a^+\hat a>$$

$$= \frac{\lambda^2}{4}[2<\hat a^+\hat a> + <\hat a^2+\hat a^{+2}> - <\hat a+\hat a^+>^2]$$

$$N = \{2[1+\cos\delta\exp(-2|\beta|^2)\}^{-1/2}$$

$$_{g,t}<\beta,\delta|\beta,\delta>_{g,t} = \frac{1-\cos\delta\exp(-2|\beta|^2)}{1+\cos\delta\exp(-2|\beta|^2)} = C$$

$$\bar n = C|\beta|^2\cosh 2r - |\beta|^2\sinh 2r\cos(\theta-2\phi) + sh^2 r \quad (34)$$

$$<\hat a+\hat a^+> = 4N^2|\beta|\sin\delta[\cosh r\sin\phi+\sinh r\sin(\theta-\phi)]\exp(-2|\beta|^2) \quad (35)$$

$$<\hat a-\hat a^+> = -4iN^2|\beta|\sin\delta[\cosh r\cos\phi+\sinh r\cos(\theta-\phi)]\exp(-2|\beta|^2) \quad (36)$$

$$<\hat a^2+\hat a^{+2}> = 2|\beta|^2\cosh^2 r\cos 2\phi + 2|\beta|^2\sinh^2 r\cos(2\theta-2\phi) - \sinh 2r\cos\theta(2C|\beta|^2+1) \quad (37)$$

Put (19) into (17)

$$\begin{aligned}S_1^{CS} &= \frac{1}{4}[C|\beta|^2 ch2r - |\beta|^2 sh2r\cos(\theta-2\phi) + sh^2r + 1/2]^{-1}\\ &\{2|\beta|^2 ch^2r\cos 2\phi + 2|\beta|^2 sh^2r\cos(2\theta-2\phi) - sh2r\cos\theta(2C|\beta|^2+1)\\ &-[4N^2|\beta|\sin\delta(chr\sin\phi + shr\sin(\theta-\phi))\exp(-2|\beta|^2)]^2\\ &+2(C|\beta|^2 ch2r - |\beta|^2 sh2r\cos(\theta-2\phi) + sh^2r)\}\end{aligned}$$

$$\begin{aligned}S_1^{CS} &= \frac{1}{4}[C|\beta|^2 ch2r - |\beta|^2 sh2r\cos(\theta-2\phi) + sh^2r + 1/2]^{-1}\\ &\{2|\beta|^2 ch^2r\cos 2\phi + 2|\beta|^2 sh^2r\cos(2\theta-2\phi) - sh2r\cos\theta(2C|\beta|^2+1)\\ &-[4N^2|\beta|\sin\delta(chr\sin\phi + shr\sin(\theta-\phi))\exp(-2|\beta|^2)]^2\\ &+2(C|\beta|^2 ch2r - |\beta|^2 sh2r\cos(\theta-2\phi) + sh^2r)\}\end{aligned}$$

$$\begin{aligned}S_1^{CS} &= \frac{1}{4}[C|\beta|^2 ch2r - |\beta|^2 sh2r\cos(\theta-2\phi) + sh^2r + 1/2]^{-1}\\ &\{2|\beta|^2[ch^2r\cos 2\phi + sh^2r\cos(2\theta-2\phi) - sh2r\cos(\theta-2\phi)]\\ &+(2sh^2r - sh2r\cos\theta) + 2C|\beta|^2[ch2r - sh2r\cos\theta]\\ &-[4N^2\exp(-4|\beta|^2)]4N^2|\beta|^2\sin^2\delta[chr\sin\phi + shr\sin(\theta-\phi)]^2\}\end{aligned}$$

$$\begin{aligned}S_1^{CS} &= \frac{1}{4}[C|\beta|^2 ch2r - |\beta|^2 sh2r\cos(\theta-2\phi) + sh^2r + 1/2]^{-1}\\ &\{2|\beta|^2 ch^2r\cos 2\phi + 2|\beta|^2 sh^2r\cos(2\theta-2\phi) - sh2r\cos\theta(2C|\beta|^2+1)\\ &-[4N^2|\beta|\sin\delta(chr\sin\phi + shr\sin(\theta-\phi))\exp(-2|\beta|^2)]^2\\ &+2(C|\beta|^2 ch2r - |\beta|^2 sh2r\cos(\theta-2\phi) + sh^2r)\}\end{aligned}$$

$$\begin{aligned}S_1^{CS} &= \frac{1}{4}[C|\beta|^2 ch2r - |\beta|^2 sh2r\cos(\theta-2\phi) + sh^2r + 1/2]^{-1}\\ &\{2|\beta|^2[ch^2r\cos 2\phi + sh^2r\cos(2\theta-2\phi) - sh2r\cos(\theta-2\phi)]\\ &+(2sh^2r - sh2r\cos\theta) + 2C|\beta|^2[ch2r - sh2r\cos\theta]\\ &-[4N^2\exp(-4|\beta|^2)]4N^2|\beta|^2\sin^2\delta[chr\sin\phi + shr\sin(\theta-\phi)]^2\}\end{aligned}$$

By means of numerical calculation technique, some of figures which indicate squeezing degree S_i^{CS} $(i=1,2)$ 、 S^{CN} and S^{SN} varying with β 、 r 、 δ 、 ϕ and θ are drawn. In these figures, Solid line denotes squeezing degree S_1^{CS} and S^{CN} curve varying with β 、 r 、 δ 、 ϕ and θ, dot line denotes S_2^{CS} and S^{SN} curve varying with β 、 r 、 δ 、 ϕ and θ. If $S_i^{CS}(i=1,2)$ is negative, it indicates that there is a kind of squeezing properties of CS . If S^{CN} or S^{SN} is negative, it indicates that there is a kind of squeezing properties of CN or SN .

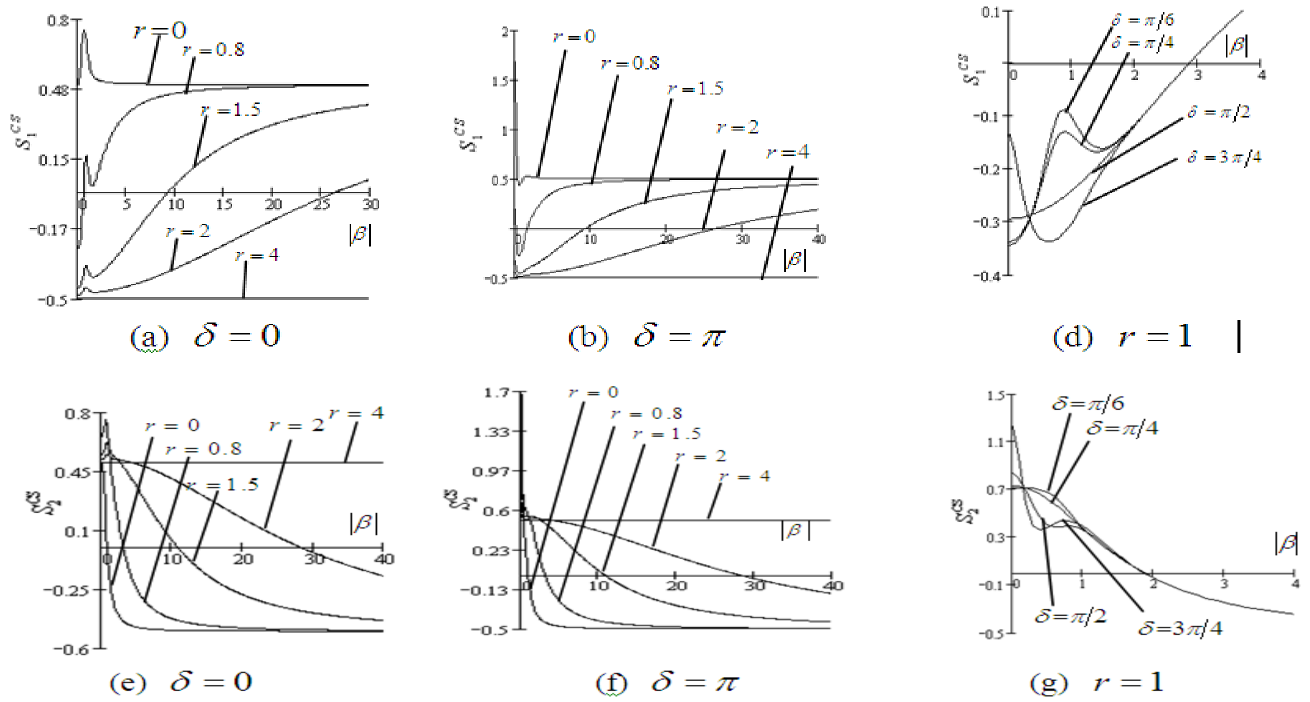

Fig 1. The curve of degree of squeeze S_1^{CS} and S_2^{CS} varying with β、 r and δ at $\phi = \theta = 0$

In fig (1), there are squeeze in squeeze odd-even coherent from fig (a)、(b)、(e) and (f) under different r at $\phi = \theta = 0$.

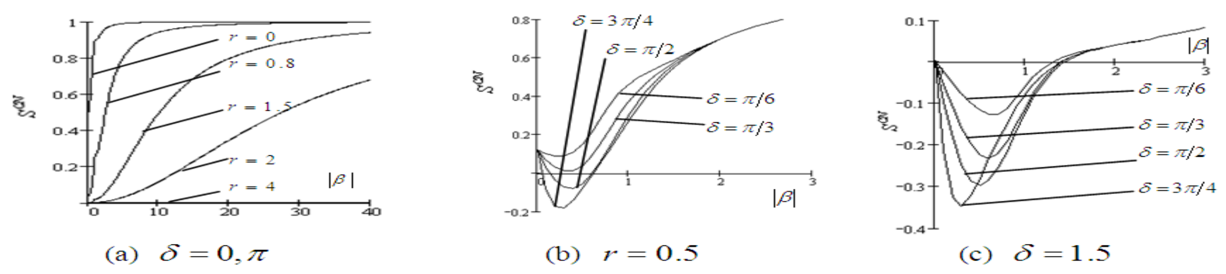

Fig 2. The curve of degree of squeeze S^{CN} varying with β、 r and δ at $\phi = \theta = 0$

In the fig 2, fig (a) indicates that there are not CN squeeze in the squeezed odd-event coherent under different r at $\phi = \theta = 0$. Fig (b) and (c) indicate there are CN squeeze under different δ, and the length of squeeze interval rise when r increase. There is not SN squeeze.

VI. Conclusion

In the paper, the squeeze properties of measurement phase operator are investigated in Superposition of Coherent State and Squeezed state.

VII. References

1) PEGG D T, BARNETT S M. Phase properties of single-mode electromagnetic field [J]. Phys Rev A, 1989, a39: 1665
2) BARNETT S M, PEGG D T. Phase in quantum optics [J]. Phys (A) Math Gen, 1986, 19: 3849.
3) LYNCH R. Phase fluctuations in a squeezed state using measured phase operators [J]. J Opt Soc Am (B), 1987, 4 : 1723.
4) NATH R, KUMER P. Quasi-photon phase states [J]. J Mod Opt, 1991, 38: 263.
5) TU HSITEH, GONG CHANGDE. Properties of the measured phase operators in squeezed number states [J]. *J Mod Opt*, 1993, 40: 57.
6) LOUDON R, KNIGHT P L. Squeezed light [J]. J Mod Opt, 1987, 34(6-7): 709-759.
7) WALL D F. Squeezed States of Light [J]. Nature, 1983, 306(10): 141-146.

Nutritional Value And Oil Content Of Indian Horse-Chestnut Seed

GJSFR Classification – G (FOR) 070306,070601,070304

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Abstract- **The nutritional value of the seeds of the Indian Horse-chestnut determined revealed that the seeds contain a good amount of various mineral elements viz. nitrogen (1.15%), crude protein (7.18%), potassium (0.79%), phosphorus (0.18%), sulphur (0.07%), calcium (0.08%), iron (159 ppm), copper (41.2 ppm), zinc (25.6 ppm) and manganese (6.95 ppm).The oil content of the seeds determined was 2.02 per cent. The nutritional value and oil content determined from seeds of Indian horse-chestnut concluded that the seeds can be used as nutritional supplement for cattle provided there are no, antinutritional factors present in the seed.**

Keywords- Aesculus indica, Seeds, Nutritive value, Oil content, Trace elements.

I. Introduction

Indian Horse-chestnut (Aesculus indica colebr; family : Hippocastanaceae) locally known as Hanudun is found growing in shady ravines of the valley and has multifarious uses as described elsewhere (Troup, 1921; Pearson and Brown, 1932). The tree is fast growing and produced timber used for packing cases, water troughs, planking, tea-boxes, mathematical instruments, shoe heels etc (Anonymous, 1985). The shoots and leaves are used as fodder. This tree has not been included as an agroforestry tree species in Kashmir Valley. This is mainly because almost no information is available with respect to its utility, propagation, cultural practices etc under Kashmir conditions.

The seeds of Indian Horse-chestnut tree have edible uses. They have been used as food during the times of famine by various tribes of North and North-Eastern India. The seeds can be ground into powder and used as a gruel (Hedrick, 1972; Singh and Kachroo, 1976). The crushed seeds if fed to cattle are reported to improve the quality and quantity of milk. There are reports that the seeds are also given to horses to cure colic disorder (Anonymous, 1985).

Oils and fats are essential and indispensable ingredients of human and animal diet. In addition to this they are also required in numerous industries such as paints and varnishes, soaps, cosmetics etc. since the year 1976 and 1977, substantial quantities of oils and fats have been imported every year. During the year 1983 and 1984 the import was as high as 16.34 lac tones (Rao, 1987).

Crushed seeds along with salt are used to cure loose motion.

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Seed coat is reported to contain several triterpeniods (Sati and Rana, 1987). The extract of seeds is considered to be active against P-388 lymphocytic leukaemia and human epidermoid carcinoma of nasopharynx (Anonymous, 2000). Extract of stem bark is reported to possess fungicidal properties and roots are used to cure leucorrhoea (Anonymous, 1985). The flower and leaf extracts of Aesculus indica controlled significantly the insect pests of rice and sugarcane (Anwar and Jabbar, 1987).

The seeds of Aesculus indica contain an oil which is used to cure rheumatism and also applied to wounds. Nine most important plants have been reported by Sharma (1991) for treating rheumatic pains in Jammu and Kashmir and among them Aesculus indica tree finds the prominent place with respect to medicinal value of its seed oil.

In a very recent finding, Singh et al(2004) have reported an effective antiviral composition β-aescin obtained from Aesculus indica seeds. The antiviral is more effective (upto 94%) than any other known antiviral agents in controlling the growth of cucumber mosaic virus(cmv) affecting the plants like vegetable plants, fruit plants, seeds, leaves, ornamental plants and other plants like cucumber, beans, spinach, crucifers, peppers, melons etc.

Seeds of Aesculus indica Colebr are used to extract oil which is among prominent nine oil yielding tree species used to cure Rheumatic pains in Kashmir (Sharma, 1991).

The fatty acid composition of oil is as follows: arachidic acid, 5. 67; myristic, 1.3; palmatic, 8.77; oleic, 57.49; linoleic, 16.91 and linolenic, 7.92 per cent. The oil shows antifungal and moderate antibacterial activity against phytopathogenic and human pathogenic bacteria, respectively (Anonymous, 2000).

The study was undertaken with following objectives:

i. To determine nutritional value of Horse-chestnut seed.
ii. To oil content of Horse-chestnut seed.

II. Materials & Methods

Faculty of Forestry Shalimar, the experimental site is located between 74.89^{o} E longitude and 34.08^{o} N latitude at an altitude of about 1587 m above mean sea level. It is roughly 15 km's north of the Srinagar city.

The climate in general is temperate; with severe winter extending from December to March. The region faces a wide temperature range from a minimum of -8.0 ^{o}C in winter to a maximum of 33 ^{o}C in the summers. Winter frost is common and medium to heavy snowfall is also witnessed. The area receives an annual precipitation of 676-1193 mm, with an average of 944.6 mm.

The seed samples were collected from healthy trees during the month of Nov-Dec 2006.The collected samples were washed with distilled water and oven dried at 60 degree celcius to remove excess moisture. The dried samples were ground to powder form in the electric grinder.

Nutritional value was determined for nitrogen, phosphorus, sulphur, potassium, calcium and trace elements (Zn, Cu, Fe and Mn) by various methods in the plant tissue-testing laboratory of SKUAST-K, Shalimar. Nitrogen was determined by Kjeldhal method, Phosphorus by Vanadomolybdate phosphoric yellow colour method (Jackson, 1973), Sulphur by turbitimetric method (Chesnin and Yien, 1950) and Potassium & Calcium with the help of Flame photometer method (Champman and Pratt, 1961).The micro nutrients i.e.Zn, Cu, Fe, Mn were determined by atomic absorption spectrophotometer (Isaac and Kerber, 1971).

Oil extraction was done with the help of Soxhlet apparatus in the laboratory-2 of Faculty of Forestry, SKUAST-K, Shalimar and per cent oil content was determined.

III. Results and discussion

The nutritional value was determined with respect to nitrogen, potassium, phosphorus, sulphur, calcium, iron, copper, zinc, manganese and oil content. The results obtained have been presented in Table-1 and revealed that Indian Horse-chestnut seeds contain nitrogen 1.15 percent (crude protein 7.18 per cent), followed by potassium 0.79 per cent, phosphorus, 0.18 per cent, sulphur, 0.07 per cent; calcium, 0.08 per cent; iron, 159 ppm; copper, 41.2 ppm; zinc, 25.6 ppm and manganese, 6.95 ppm.

The oil content determined by Soxhlet apparatus was found to be 2.02 per cent. .The oil content though less, but considering the big size of seed and large quantity of harvest (40-60 kg/tree) can be extracted and utilised for various purposes. Considering the nutritional value of Indian Horse-chestnut seeds, they can be used as a nutritional supplement for cattle provided there are no antinutritional factors present in the seed.

The studies revealed that the seeds of Indian Horse-chestnut have enough nutritional value. The oil content though less, but considering the big size of seed and quantity of harvest (40-60 kg/tree) can be extracted and utilized for various purposes.

The present studies vary with the results of Parmar and Kaushal (1982) who have reported the various mineral elements in the seeds of *Aesculus indica* seed as follows : phosphorus, 0.124; potassium, 0.733; calcium, 0.0495 and iron 0.0084 per cent. This might be due to temporal and spatial variation of the seed source.

IV. Conclusion

The nutritional value of the seeds of Indian Horse-chestnut determined revealed that the seeds contain a good amount of various mineral elements viz. nitrogen (1.15 %), crude protein (7.18 %), potassium (0.79 %), phosphorus (0.18 %), sulphur (0.07 %), calcium (0.08 %), iron (159 ppm), copper (41.2 ppm), zinc (25.6 ppm) and manganese (6.95) ppm). The oil content of Indian Horse-chestnut seeds determined was 2.02 per cent. The nutritional value and oil content determined from seeds of Indian Horse-chestnut concluded that the seeds can be used as nutritional supplement for cattle provided there are no anti-nutritional factors present in the seed.

V. Acknowledgement

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VI. References

1) Anonymous, 1985. The Wealth of India. Raw Materials, vol. I : A. Publication and Information Directorate, CSIR, Hillside Road, New Delhi.
2) Anonymous, 2000. The Wealth of India. Raw Materials, vol. I : A-Ci. National Institute of Science Communication, Council of Scientific & Industrial Research, Dr. K.S. Krishnan Marg, Pusa, New Delhi.
3) Anwar, T. and Jabbar, A. 1987. Studies on the pesticidal activity of Aesculus indica and Dryopteris ramose against rice and sugarcane pests. Proceedings of Pakistan Congress of Zoology 7 : 87-93.
4) Champman, H. D. and Pratt, P. F. 1961. Methods of analysis for soils, plants and water. University of California, Berkeley.
5) Chesnin, L. and Yien, C. H. 1950. Turbedetimetric determination of available sulphates.Soil Sci. Soc. Amer. Proc.15 :149-151.
6) Hedrick, U.P. 1972. Sturtevant's Edible Plants of the World. Dover Publications [cf : http://www.ibiblio.org/pfaf/cgi-bin/arr_html?Aesculus+indica].
7) Isaac, R. A. and Kerber, J.D. 1971.Atomic absorption and flame photometer techniques and uses in soil, plant and water analysis.In : International Methods for Analysis of Soil and Plant Tissue (Ed; Walker, L. M.).Soil Sci. Soc. America Madison,Wisconsin.
8) Jackson, M. L. 1973. Soil Chemical Analysis. Prentice Hall of India Pvt. Ltd.; New Delhi. 103 P.
9) Nautiyal, M.C. and A.N. Purohit 2000. Cultivation of Himalayan Aconites under polyhouse conditions. Current Science. 81 (5): 1062-1063.
10) Parmar, C. and Kaushal, M. K.1982. Aesculus indica, pp 6-9. In :Wild Fruits. Kalyani Publishers, New Delhi, India.
11) Pearson, R.S. and Brown, H.P. 1932. Commercial timbers of India 2 Calcutta. A.I. Reprints Agency. Volume 1. 24 B/5 Desh Bandhu Gupta Road, Karol Bagh, New Delhi. India. pp 114-116.

12) Rao, M.V. 1987. Key notes address. In : Proceedings of a Symposium on "Advances in the Technology of Oils and Allied Products", held at H.B.T.I., Kanpur.
13) Sati, O.P. and Rana, U. 1987. Triterpenoids of Aesculus indica. Pharmazie 42 : 141
14) Sharma, P.K. 1991. Herbal remedies for treating rheumatic pains in Jammu and Kashmir. Indian Journal of Forestry 14(3) : 206-210.
15) Singh, G. and Kachroo, P. 1976. Forest Flora of Srinagar [cf : http://www. ibiblio.org/pfaf/cgi-bin/arr_html?Aesculus+indica].
16) Singh, B.K., Meenu, R., Raja, Z. and Aijaz, A. 2004. A new antiviral agent from Indian horse chestnut Aesculus indica [cf : http://www.freepatentsonline. com/EPI1489910.html].
17) Troup, R.S. 1921. The silviculture of Indian trees. Oxford University Press, Oxford, vol. 111. C Poredon Press Oxford.

S. No.	Mineral element	Content
1.	Nitrogen	1.15 %
	Crude protein	7.21 %
2.	Potassium	0.79 %
3.	Phosphorus	0.18 %
4.	Sulphur	0.07 %
5.	Calcium	0.08 %
6.	Iron	159 ppm
7.	Copper	41.2 ppm
8.	Zinc	25.6 ppm
9.	Manganese	6.95 ppm
10.	Oil	2.02 %

Table-1: Nutritional value and oil content of Indian Horse-chestnut (Aesculus indica) seeds

Estimation Of Persistent Organochlorine Pesticide Residues In Selected Fruit

GJSFR Classification – B (FOR)
030403,070501,070306

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Abstract- **Organochlorine pesticide residues analysis of mango fruit sample during 2008-2009 was carried out on Gas chromatograph –ECD with capillary column .The sample was extracted in acetone and make up in hexane. Using proposed method, two laboratories (both from Kanpur) analysed mango fruit samples to evaluate some methodologies and to obtain consensus values for selected OCPs. The values of above fruit was below the maximum residues limit. The limit of detection was 0.1 to 0.5 µg / kg; where as limit of quantification 0.3 to 1.5 µg/kg were found respectively .However qualitative analysis of the standards were also carried out using kovats index. Fruit tested in our analysis do not contain any quantities of pesticide residues representing & hazard to the humans.**

Keywords-Pesticide analysis, fruits, gas chromatograph, capillary column, maximum residues limit.

I. INTRODUCTION

During the last twenty five year numerous research efforts to identified and adverse health affect the environment fate of persistent organic populants (Pops) have resulted in a variety of field method for the sampling and analysis of fruits. .In facts, Canadian Food Inspection Agency, Canada has reported finding substantial concentration of pesticide in apple fruits (Jian Wang and David Wotherspoon2007). As the production off various fruits all over the worlds under a variety of regulation, a method capable of detecting residues is very important. Organochlorine (OC) pesticide are widely used in agriculture as insecticides and leaves residues to varying extents in agriculture produce such as fruits .The current development in analytical technologies to detects pesticide residues in fruits has mostly focused on the simplification ,miniaturization and improvement of the samples extraction and clean up methods with universal micro extraction procedure(Betran and Kataoka 2000).Solid phase microextration (SPME) is a solvent free extraction techniques (Dimitra et al .2003) Although pesticide residues that remain in the food supply could pose a potential risk for human health because of their sub acute and chronic toxicity (Fong et al .1990).In India fruits and vegetables are tested and regulated for pesticide residues under national insecticide board. This monitoring program also provides insecticide board. The role of pesticide has been very vital in public health and agriculture production in developing countries including India. Further, the enormous uses of pesticide in developing countries have been of serious concern because of there persistent in nature. Large amount of pesticide are used in agriculture sector and public health programmers every year (Pesticide Manual 1997, Ciess 1998).Continuous use of pesticide lead to there presence in water, soil, air, crop plant and biological tissue. Although, pesticide residue analysis have been done in several food/product (Raizada, et al. 1998, Bhattacharya 2003). The present studies deals with the analysis of pesticide residues in above products to about evaluate the presence of these commonly used pesticide. The presence of organochlorine pesticide OCPs in terrestrial and aquatic environment may leaves to toxicological implication (Aruda et al, 1988, Cochieri and Arnese 1988: Sarkar and Gupta 1988, Dikshit, et al 1989). Due to there toxic property and potential risk to consumer, there residuals in food commodities is an issue of public concern and controlled by legislation.

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II. MATERIALS

i. Fluorinated ethylene propylene (FEP) centrifuge tube, - 50 mL.
ii. Spatula / spoon and funnel: For transferring sample into centrifuge tubes.
iii. Solvent dispenser and 1-4 L solvent bottle. - For transferring 15 ml 1% of HOAc in MeCN per 15 gram samples in FEP centrifuge tubes or bottles.
iv. Centrifuge tubes (optional). - 10-15 ml graduated. For evaporation and /or dispersive –SPE.
v. Mini centrifuge tubes (optional) – 2ml for dispersive –SPE (use tubes with o – ring –sealed caps to avoid leaks).
vi. Repeating or volumetric pipettes.-Capable of accurately transferring 0.5-8ml solvents.
vii. Container.- Graduated cylinders volumetric flasks, vials ,and other general containers in which to contains samples ,extracts , solution, standards ,and reagents.
viii. Balance(s). - Capable of accurately measuring weight from 0.05-100g with in ± 0.01g.
ix. Freezer. - Capable of continuous operation <20 ^{O}C.
x. Food chopper and /or blender. - Preferable S blade vertical cutter and probe blender.
xi. 50 ml Teflon centrifuge tube with screw caps (e.g. Oak –ridge Nalgene 3414 -0050).

III. Reagents

i. Anhydrous magnesium sulphate ($MgSO_4$). – Powder form; purity > 98%.
ii. Acetonitrile (MeCN). - Quality of sufficient purity that is free of interfering compounds.
iii. Acetic acid (HOAc). - Glacial; quality of sufficient purity that is free of interfering compounds.
iv. 1% HOAc in MeCN. - Prepared on a v/v basis (e.g.10 ml glacial HOAc in a 1 L MeCN solution).
v. Anhydrous sodium acetate (NaOAc). Powder form.
vi. Primary secondary amine (PSA) sorbent. - 40 µm particle size.
vii. C-18 sorbent (Optional). - 40 mm particle size, if sample contain >1 % fats.
viii. Graphical carbon black (GCB) Sorbent (optional). - 120/400 meshes size, if no structurally planer pesticides are included among the analytes.

A. Sample Collection

Samples of mango fruits was collected from local market of Kanpur, Unnao, Lucknow, Kannuj, Fatehpur (U.P. India) for the analysis.

B. Standard Preparation

High purity reference standards of the pesticide analytes , and quality control (QC) and internal standard (ISs) obtained from M/s Phosphorous Limited Mumbai, India and prepared at highly concentrated stock solution in MeCN with 0.1% HOAc .Stored in a dark vials in the freezer .

IV. Qualitative analysis of standard

The standard mixture of mixture of pesticide and the fruits extract must be run out at least to different GC column in order to obtain reliable identification .These column should be chosen so that they have widely different characteristics' if the two column are used, the more logical choice is a column with a non polar liquid and one with polar liquid phase or one which have a special affinity with esters. The actual choice will properly depend mainly on availability in the individual laboratory, and it is not critical .The two capillary column used here having (30m×22mm.id) packed with 1.5% ov-17 P+ 1.95 Q f -1 on 100 to 120 mesh chromosorb were used. The GC is dual column with flame ionization detector and is oven temperature programmed from 50 ^{O}C at 4 ^{O}C /min .This gives excellent separation of the major constituents of the fruits extract, but analysis can be carried isothermally with slightly less good separation .In facts isothermal operation may give better reproducibility of retention time, unless the temperature programming is carried out carefully and is reproducible. It is to do qualitative analysis by using retention time only, but it is more meaningful to use one of the calculated retention indices which appear in the literature, since a retention time is characteristics of an individual instrument & set of operating condition while a retention index is independent of instrument variables and essentiality characteristic only of compounds being measured.

V. Sample preparation

Mango fruit samples accurately weight 10 gram into a 50 ml centrifuge tube (in triplicate having screw caps) were add to 10 ml of acetonitrile and shake vigorously for one minute .Add 4 g $MgSO_4$, 1g NaCl, 1g Na_3 Citrate dehydrate and 0.5 g Na_2H Citrat sesquihydrates, shake each tube directly after the salt addition shortly, shake vigorously for 1 min and then centrifuge for 5 min at 3000 U/min. For citrus fruit co –extracted wax is removed overnight in the refrigerator .5 ml of extract are transferred into a PP single use centrifuging tube, which contains 5 into 25 mg PSA and 150 mg $MgSO_4$ .Shake the mixture for 30 sec and centrifuge it for 5 min at 3000 U/min.5 ml of the extract are transferred into screw cup vial and acidified with 5 into 10 µl 5% formic acid in acetonitrile (10 µl /extract).The cleaned and acidified extract are transferred into auto sampler vials and use for the residues determination by GC techniques.

VI. Instrumentation

Analysis was carried out by using a pre calibrated GC machine(Perkin Elmer)with Ni electron capture detector .A stainless steel column (30m×22mm.id)packed with 1.5% ov-17 P+ 1.95 Q f -1 on 100 to 120 mesh chromosorb was used .Operation temperature were programmed at 220,280,300^{O}C for column, injector, detector, respectively. For specially OCPs, the GC oven temperature was programmed at an initial temperature of 80 ^{O}C with the hold time 2 min, increased to 175 ^{O}C at 16 ^{O}C /min and then further increased to 220 ^{O}C at 6 ^{O}C /min and hold for 5 min. Purified nitrogen gas was passing through silica gel and molecular sieves were used as carrier gas at flow rate of 60ml.per minute.

VII. Quality control

Each congener was identified by matching the retention time in the sample with that in the standard. Procedural blank, consisting of all reagents and glass ware used during the analysis, were periodically determine to check the cross contamination. Since no compound that interfered with is detected the sample values were not corrected for procedural blank. Recovery studies with fortified sample have indicated that overall recovery values exceeded 86%.For 10 -mg fruit samples the limit of detection (LOD) was about 0.1 to 0.5 µg/kg, whereas, limit of quantification (LOQ) was about 0.3 to1.5 µg/kg. In any case, it is highly desirable to improve the accuracy and precision for OCPs quantification in fruits samples .One of the most important way to established a common basis for accuracy measurement and quantification is the existence of a reliable certified reference material (CRM).The objective of the present analysis was to test the hypothesis that commercially available fruits samples may contains amount of POPs that can be reliably quantified.

VIII. RESULTS & DISCUSSIONS

Analytes	Laboratory 1 (DAV PG College, Kanpur)					
	Ext-1	Ext-2	Ext-3	Ext-4	Mean	SD
OCPs						
HCB	<0.3	<0.3	<0.3	<0.3	<0.3	-
Cis chlordane	<0.3	<0.3	<0.3	<0.3	<0.3	-
Transnonachlor	<0.3	<0.3	<0.3	<0.3	<0.3	-
y-HCH	Interf	3.8	4.0	4.2	4.0	0.16
p,p'- DDT	Interf	4.3	4.7	4.1	4.3	0.41
p,p'-DDE	11.0	15.8	9.7	10.0	11.6	3.1

Table 1: Concentrations (μg/kg of Mango fruit samples) of selected OCPs

The values obtained for the environmental contaminant p, p' –DDE are in perfect agreement between the two laboratories (Table 2), while no agreement or no measurements were done for other OCPs, Interestingly, HCB, *trans* - nonachlor or oxychlordane usually present in fruits samples could be evidenced , while β-HCH, the principal HCH congener present in fruits, could not be measured only.

Analytes	Laboratory 2 (DG PG College, Kanpur)					
	Ext-1	Ext-2	Ext-3	Ext -4	Mean	SD
OCPs						
HCB	2.6	2.4	2.8	2.3	2.52	0.33
Cis chlordane	1.6	1.5	1.4	1.3	1.45	0.12
Transnonachlor	1.1	1.0	0.9	0.7	0.92	0.12
β-HCH	-	-	-	-	-	
γ-HCH	-	-	-	-	-	
p,p'- DDT	-	-	-	-	-	
p, p'-DDE	11.04	10.45	10.63	9.36	10.25	0.61

Table 2: Concentrations (μg/kg of Mango fruit samples) of selected OCPs

Consensus values were calculated for the congeners for which all three or at least two of the participating laboratories agreed on the values (Table 2, 3). Mean values and standard deviations were calculated for each set of data available for the individual compounds means (6-10 individual measurements).Values were accepted only for analytes for which the RSD was lower than 15%, which was a typical value obtained for replicate analyses in each laboratory.

IX. REFERENCES

1) Aruda, J.A., Gringen, M.S., Layher, W.G. ,Kresh, G., Bever, C. (1988)Pesticide in fresh water from turtle Creek lake ,Kansas .Bull environmental contamination toxicol **41**:617-624
2) Benedzet, H.J., Matsumara, F (1973) Isomerisation of Gama BHC in Environment. Nature (London) 243: 480-481.
3) Betran, J., Lopez, F.J., Hernandez, F. (2000) "Solid phase micro extraction in pesticide residues analysis."J. of Chromatography A., 885, 389-404
4) Bhattacharya, A. (2003) Status of pesticide residues in meat, fish, and poultry .In proceedings of symposium on risk assessment of pesticide residues in water and food. By ILSI Washington DC, ITRC Luck now and ICMR New Delhi India on October 28-29, 2003, MI-8.
5) Ciers. (1998) Industrial Data Book. Centre for Industial and Economic Research .New Delhi/Thousands Oak /London. Chan A (2003) Some aspect of toxic contaminants in herbal Medicines. *Chemosphere* **52**, 1361-1371.
6) Cochieri, A., Arnese, A. (1988) Organochlorine pesticide residues in fish from southern Italian rivers .Bull Environ Contam Toxicol 40: 233-239.
7) Codex (1996) Food chemical codex .Committee on food chemical codex 4thedition.Food and Nutrition Board.Institute of Medicine, National Academy of Science. National Academy Press.Washington D.C.
8) Dikshit, T.S.S., Raizada, R.B., Kumar, S.N., Srivastava, M.K., Kulshrahtha, S.K. and Adholia ,V.N. (1989)Residues of DDT and HCH in major source of drinking water in Bhopal , India. Bull Enviorn Contam Toxicol 45:389-393.
9) Dimitra, A., Lambropoulou, Triantafyllos A. Albanis . (2003) "Headspace solid phase microextraction in combination with gas chromatography –mass spectrometry for rapid screening of organophosphorous insecticides residues in strawberriesand cherres " ,J. of Chromatography A. 993 ,197-203.
10) Fong, W.G., Moye, H.A., J.N., & Toth,J.P. (1990) Pesticide residues in foods : Method Techniques , and Regulation , John Wiley & Sons , New York ,N.Y
11) Jian Wang and David Wotherspoon (2007) Determination of pesticides in apples by liquid chromatography with eletrospray ionization tandem mass spectrometry and estimation of measurement of uncertainty. Journal of AOAC International, vol 90, no.2, 550-567
12) Kannan, K., Tanabe, S., Ramesh, A., Annamalai, S., Tatsukhawa, R (1992) Persistent organochlorine pesticide residues in foodstuff from India and their implication in human diet exposure. J. Agric Food Chem, 40:518-524.

13) Kataoka, H., Lord, H.L., and Pawliszyn ,J., (2000) "Application of solid phase microextration in food analysis".J. of Chromatography A., 880, 35-62.
14) Mukherjee, I (2003) Pesticide residues in vegetable. In, proceedings of Symposium on risk assessment of pesticide residues in water and food. By ILSI Washington DC, ITRC Lucknow and ICMR New Delhi India on October 28-29, 2003, A1-8.
15) Pesticide Manual (1997) Tenth edition, Crop Protection Publication, Published by Royal Society of Chemistry U.K.
16) Raghupathy,A. (2003).Status of pesticide residues in cereals in India.In proceedings of Symposium on risk assessment of pesticide residues in water and food. By ILSI Washington DC, ITRC Lucknow and ICMR New Delhi India on October 28-29, 2003, C 1-20.
17) Raizada, R.B., Seth, P.K., Srimal RC, (1998) Proceeding of First National Meeting of pesticide residues Component of Commonwealth Protect CREN, Published by Industrial Toxicology Research Centre Post Box 80, Lucknow India.
18) Sarkar, A., Sen Gupta, R. (1988) DDT residues in sediment from bay of bengal .Bull Environm Contam Toxicol 38: 664-669.

Electrical Conductivity Of Sugar Cane Juice

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GJSFR Classification – G (FOR) 070303,030403,030405

***Abstract-* The electrical conductivity of fresh unfiltered cane juice, filtered juice, lime added juice at 298.15 K ,318.15 K,373.15 K, taking CO-1148 cane variety have been measured .The conductivity of fresh unfiltered juice ,filtered juice and lime added juice at 298.15 K were found to be 172,180,584 ,(μS/cm) respectively .The conductivity is found to be increasing as temperature increases ,but order of magnitude of conductivity remains same across the temperature range tested .The higher observed conductivity of lime added or clarified juice ,probably have its origin in the inclusion or adsorption of impurity (non sugar, protein, colloids, mud's etc) .The value of specific conductivity of different grade of juice could be used to judge the quality of sugar. Results shows order of conductivity lime added >unfiltered >filtered were found.**

Keywords-electrical conductivity, temperature, cane juice, impurity, sugar

I. INTRODUCTION

In India sugar industry is one of the largest agro based industries. In sugar industry the sugar is manufactured by unit operation .The crushing of cane followed by its milling ,and the extracted juice undergoes clarification for desired treatment to make the juice purified and concentrated by making of crystallizable sugar [1].The convention process for the processing of the cane juice has the following limitations (a)in efficient removal of substance like gum ,ash, silica, reversible colloids during clarification which adversely effect the color of the final products (b)inversion of sucrose leading to augmentation of molasses's formation and finally (c) concentration of juice by evaporator and condensation of same before crystallization [2] A typical composition of raw sugar cane [3].

Table 1.typical composition of raw sugar cane

	Consistituents	Percentage
.1	Water	75-88
.2	Sucrose	10-21
.3	Reducing sugar	0.3-3.0
.4	Organic matter	0.5-1.0
.5	Inorganic composition	0.2-0.6
.6	Nitroenous bodies	0.5-1.0

Hence the above assessment it becomes an objective to evolve relevant parameter for better and exact control of the system .However control is based on two aspects i.e. for quality and quantity assessment .The purpose of present study is pivoted on quality of end products [4].Studies shows that the dependence of temperature is usually expressed a relative change per degree Celsius at particular temperature commonly percent / at 25 ^{O}C and thus is called the slope of solution [5].Although measurement of electrical conductivity to the relative suitability of pH for centre of end points of the first carbonation process were investigated [6].

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II. MATERIAL & METHOD

For the conductivity measurements a glass cell with platinized platinum electrode was employed. The resistance, R, of the solution was measured with an A.C. bridge(Wayne Kerr 6425)in the frequency range ω = 500 Hz-5 Hz and true resistance were obtained by extrapolation of R vs. ω-1 at infinite Frequency . The volume of the cell was 30 cm$^{3.}$ One set of experiment was performed under isothermal condition at 298.15 K. In A second set juices concentration varied and with a experimental temperature (from 298.15 to 373.15 K). A themostated ethylene water bath was used to control the temperature of the super cooled solution with in +- 0.05K .,and a kerosene /oil bath was used to control the temperature for the measurement at 298.15Kwith in +- 0.02K.The cell constant was determined as function of temperature employing the reported electrical conductivity of solution of known concentration ,aqueous KCL as temperature above 273.15K (7) and ethanol solution of KI for Temperature below 273.15K (8) .The cell constant varied from 0.1002cm^{-1} at 248K,down to 0.0957cm^{-1} at 323K,and the value at 298.15Hwas 0.0968+- 0.0006cm^{-1} .Solution were prepared with demonized water free of CO_2 and nitrogen was bubbled into the solution to minimize CO_2 solubilization prior to the measurement. In all cases specific conductivity of pure juice in water was determined to correct the measure specific conductivity by the contribution of ionic impurities of the disaccharide or residual CO_2.

III. RESULTS AND DISCUSSION

The specific conductivity versus temperature for the different variety of juice under study are shown in Fig(1).The order of specific conductivity for unfiltered ,filtered ,lime added juice at 298.15 K were 172,180,584 (μS/cm) respectively .The higher observed conductivity for lime added juice at all temperature seems to be attribute to their higher concentration of impurity (non sugar proteins ,colloids ,mud's etc).Results reveals that filtered juice shows that minimum pH value i.e. 5.2 with a conductivity value 210 μs while unfiltered juice shows conductivity 202 μS/cm value at 5.2 pH. The raw juice was treated with the milk of lime under constant stirring to raise its pH from around 5.3 to 8.1 .The liming was carried out at 298.15 K .Experiments conducted with dry lime revealed that's around 2.5 grams of lime was required per 1000 ml of juice to raise its pH to 8.1. The treated juice was then kept unstirred for around 2 hour to facilitates the settling of the solids (mud's, collides, materials proteins etc).The clear greenish –brown colored supernatant juice at pH 8.1 shows maximum conductivity i.e.584 µS/cm (fig2) .On carry out the Fig (3) shows a slightly linear relationship between conductivity in µS/cm and % dilution (up to 5 % only).

Fig 1 showing conductivity (µS/cm) and concentration of lime added cane juice may be understood in terms of changing concentration of Ca^{++} ions .Conductivity value of lime added juice is higher with the fact that ionic concentration of Ca^{++} ions which contribute to the higher conductivity. The formation and dissociation of H_2CO_3 and its reaction with $Ca(OH)_2$ is governed by a well known mechanism where stoichiometry may be found elsewhere 21, 21.Table 2 shows removal of non sugar other than CaO is recorded, as reported by Jenkins 22.

It is evident from above discussion that conductivity data provides satisfactory evidence that the minimum is the conductivity value; higher will be the purity of cane juice; could also employed as a parameter for the judging the completion of the first carbonation reaction .Although, the fundamental reaction of first carbonation has been extensively studied in the past, but it is still offers simple scope for further investigation ,so we have therefore attempts to investigate some relevant information .The observed results of non sugar present in cane juice responsible for conductivity generation is good agreements of data published by author in cane sugar and molasses [Vikesh2009].

Clarification of cane juice is primally controlled by maintain optimum pH .It is clearly shows from Table 2 and 3 that the clarification that the deviation of pH value in cane juice from 5.1 to 8.1 .It is difficult to relies the importance of such deviation and its impact on overall clarification process .However, lesser change in pH, which is practically ignored in sugar processing .On the other hand conductivity values (Table 2 and 3) clearly indicates higher deviation 180-235 µS/cm i.e. almost three times higher than initial value, showing more reproducibility. This is due to fact that non sugar part of sugar house products gives more indication about purity of sugar than sugar part as discussed by author in recent years [Vikesh 2009].

It is evident from above discussion that electrical conductivity and pH measurement was made simultaneously during clarification [Lime adding] .It is observed from the analysis that electrical conductivity data shows a sharp change with respect to pH. Although, comparison of pH and electrical conductivity suggest that electrical conductivity is a better controlling parameter than pH, indicating the additional increase of lime in mixed juice.

Table 1

Temperature	Conductivity (µS/cm)
Unfiltered	
298.15 K	172
318.15 K	266
373.15 K	443
Filtered juice	
298.15 K	180
318.15 K	277
373.15 K	464
Lime added juice	
298.15 K	584
318.15 K	675
373.15 K	862

Table 2

pH	Conductivity (µS/cm)
Unfiltered	
5.3	202
Filtered juice	
5.2	210
Lime added juice	
8.1	584

Table 3.dilution effect on conductivity

Dilution%	conductivity (µS/cm)
Unfiltered	
1 %	172
2 %	271
3 %	390
4 %	455
5 %	543
Filtered juice	
1 %	187
2 %	282
3 %	396
4 %	458
5 %	554
Lime added juice	
1 %	593
2 %	695
3 %	792
4 %	894
5 %	1080

Fig 1 Specific conductivity vs. temperature plot for different variety of Juice

Fig 2 Specific conductivity vs. pH plot for different variety of juice

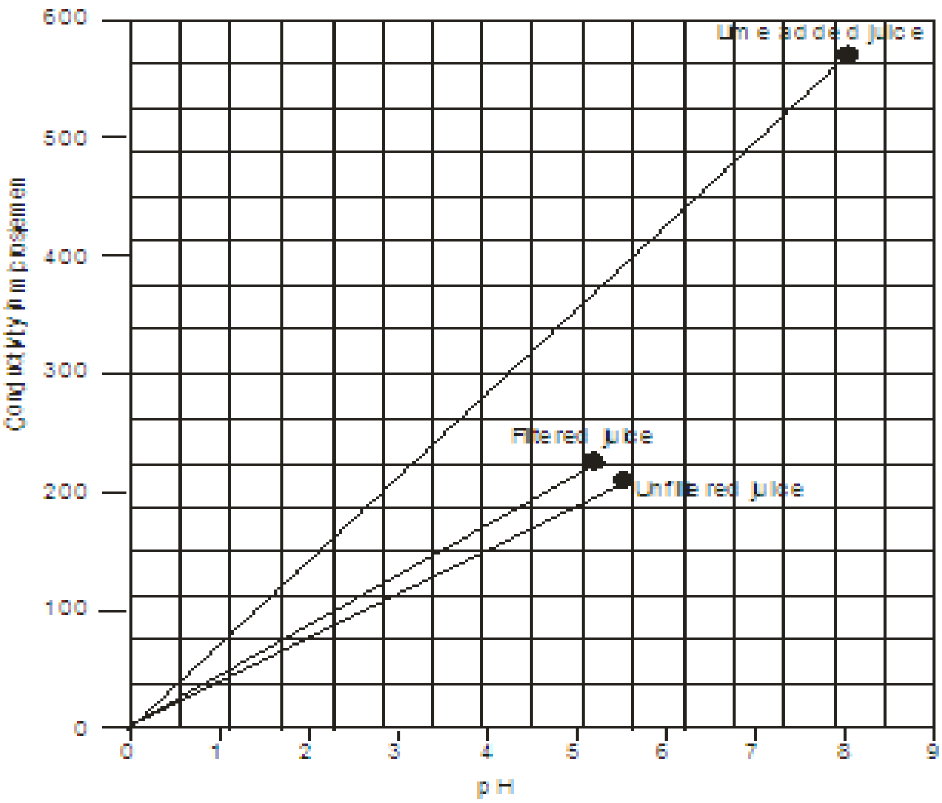
Fig 3 Dilution effect on conductivity of juices unfiltered, filtered, lime added

IV. Conclusion

Electrical conductivity measurement of juice and other sugar products could be made qualitative .Hence under the above constrain to control the process for its ongoing assessment it becomes an objective to evolve relevant parameter for better & exact control of the system.

V. References

1) C.P.Chen James ,C.-C.Chou(1993). Cane Sugar Hand Book,Wiley,New York.
2) H.Meade,G.Spencer,Cane(1963). Sugar Hand Book, Wiley, New York
3) System of technical control for cane sugar factories in India(1998).The sugar technologist association of India ,Kanpur
4) A.M. Bunyak et al. (1971).Investigation of the electrical conductivity of pulp –press water juice and syrup by means of contactsless conductometry ,Sakher, Prom, vol 45[5],p.29-32.R.
5) www.coleparmer.com
6) J.F.T.Oldfield et al.(1974): First carbonation end point control ,Brit.Sugar Corp.22nd Tech. Conf., 31pp.E.
7) Vikesh Kumar et.al (2009) Conductometric studies and its significance during cane sugar manufacture , Sugar Tech International .321-324
8) J.Barthel ,F.Feurelein,R.Neueder and R.Wachter(1983) J. Solution Chem., 9,209
9) J.Barthel ,F.Feurelein,R.Neueder and F.Strasser(1983) J. Solution Chem., 12,209



Specific conductivity vs. pH of different variety of juices

Fig. 2

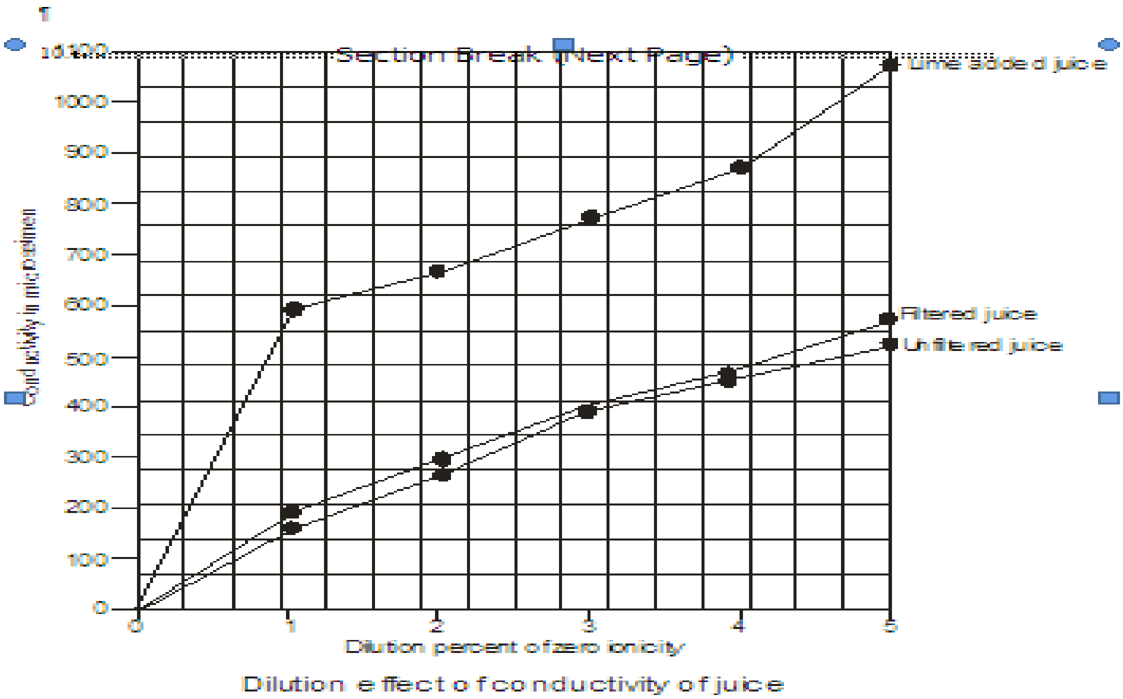
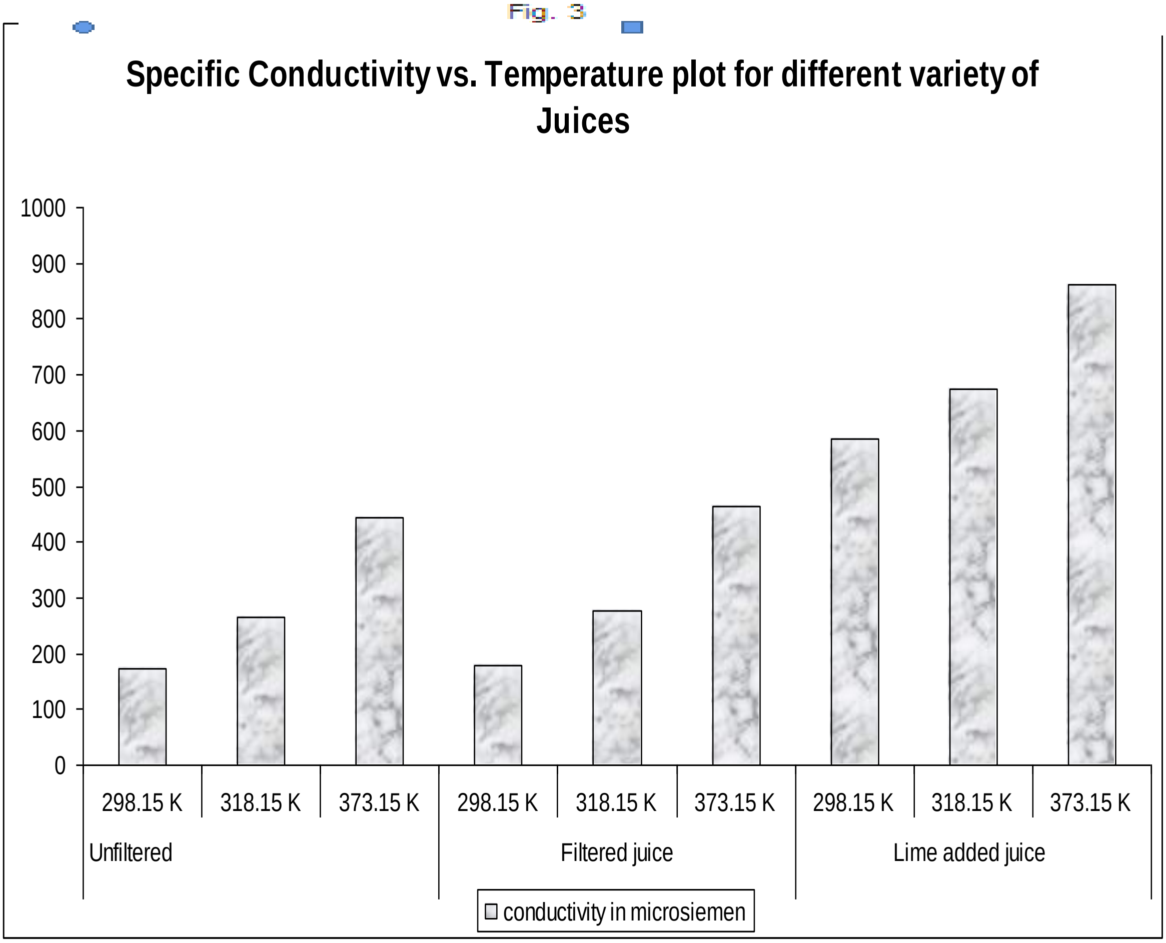

Section Break (Next Page)
Lime added juice
Filtered juice
Unfiltered juice
Conductivity in microsiemen
0
100
200
300
400
500
600
700
800
900
1000
0
1
2
3
4
5
Dilution percent of zero ionicity
Dilution effect of conductivity of juice

Fig. 3


Specific Conductivity vs. Temperature plot for different variety of Juices
1000
900
800
700
600
500
400
300
200
100
0
298.15 K
318.15 K
373.15 K
298.15 K
318.15 K
373.15 K
298.15 K
318.15 K
373.15 K
Unfiltered
Filtered juice
Lime added juice
conductivity in microsiemen

On Totally Umbilical Hypersurfaces Of Weakly Conharmonically Symmetric Spaces

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GJSFR Classification – F (FOR) 010102,010103,010204

Abstract- **The object of the present paper is to study the totally umbilical hypersurfaces of weakly conharmonically symmetric spaces.**

Keywords-. weakly conharmonically symmetric space, conharmonic curvature tensor, pseudo-conharmonically symmetric space, totally umbilical hypersurfaces, totally geodesic, mean curvature.

I. Introduction

In 1989 Tam´assy and Binh [16] introduced the notions of weakly symmetric and weakly projective symmetric spaces. A non-flat Riemannian space $V_n(n > 2)$ is called a weakly symmetric space if its curvature tensor R_{hijk} satisfies the condition

$$(1)\quad R_{hijk,l} = A_l R_{hijk} + B_h R_{lijk} + C_i R_{hljk} + D_j R_{hilk} + E_k R_{hijl},$$

where A, B, C, D and E are 1-forms (not simultaneously zero) and the ‗,‘denotes the covariant differentiation with respect to the metric of the space. The 1-forms are called the associated 1-forms of the space and an n-dimensional space of this kind is denoted by $(WS)_n$. The existence of a $(WS)_n$ is proved by Prvanovi´c [11]. Then De and Bandyopadhyay [4] gave an example of $(WS)_n$ by a metric of Roter [12] and proved that in a $(WS)_n$, B = C and D = E [4]. Hence the defining condition of a $(WS)_n$ reduces to the following form.

$$(2)\quad R_{hijk,l} = A_l R_{hijk} + B_h R_{lijk} + B_i R_{hljk} + D_j R_{hilk} + D_k R_{hijl}.$$

In this connection it may also be mentioned that although the definition of a $(WS)_n$ is similar to that of a generalized pseudosymmetric space introduced by Chaki [2], the defining condition of a $(WS)_n$ is little weaker than that of a generalized pseudosymmetric space. That is, if in (1.1) the 1-form A is replaced by 2A and E is replaced by A then the space will be a generalized pseudosymmetric space. In particular, if in (1.1) the 1-form A is replaced by 2A and B, C, D and E are replaced by A, then the space turns into a pseudosymmetric space of Chaki [1]. On the analogy of $(WS)_n$, De and Bandopadhyay [5] introduced the notion of weakly conformally symmetric spaces.

As a special subgroup of the conformal transformation group, Ishii [8] introduced the notion of conharmonic transformation under which a harmonic function transform into a harmonic function. The conharmonic curvature tensor of type (0, 4) on a Riemannian space $V_n(n > 3)$ is given by [8].

$$(3)\quad T_{hijk} = R_{hijk} - \frac{1}{n-2}\left[R_{ij}g_{hk} - R_{hj}g_{ik} + R_{hk}g_{ij} - R_{ik}g_{hj}\right],$$

which is invariant under conharmonic transformations, where R_{ij} is the Ricci tensor of the space.Recently Shaikh and Hui [14] introduced and studied the notion of weakly conharmonically symmetric spaces with the existence of such notion by several proper examples. A Riemannian space $V_n(n > 3)$ (the condition $n > 3$ is assumed throughout the paper as for $n = 3$ the conformal curvature tensor vanishes identically) is called weakly conharmonically symmetric space[14] if its conharmonic curvature tensor T is not identically zero and satisfies the condition.

$$(4)\quad T_{hijk,l} = A_l T_{hijk} + B_h T_{lijk} + C_i T_{hljk} + D_j T_{hilk} + E_k T_{hijl},$$

where A, B, C, D and E are 1-forms (not simultaneously zero). Such an ndimensional space is denoted by $(WCHS)_n$. Here 'W' stands for the word 'weakly' and 'CH' represents the 'conharmonic curvature tensor'. It is shown that [14] in a $(WCHS)_n$ the associated 1-forms B = C and D = E and hence the defining condition (1.4) of a $(WCHS)_n$ reduces to the following form:

$$(5)\quad T_{hijk,l} = A_l T_{hijk} + B_h T_{lijk} + B_i T_{hljk} + D_j T_{hilk} + D_k T_{hijl},$$

where A,B and D are 1-forms (not simultaneously zero). If in (1.4), the 1-form A is replaced by 2A and B, C, D and E are replaced by A, then the space reduces to a pseudo conharmonically symmetric space [1]. In [15] Shaikh and Hui also studied the decomposability of weakly conharmonically symmetric spaces with the existence by several non-trivial examples. Recently ¨ Ozen and Altay [9] studied the totally umbilical hypersurfaces of weakly and pseudosymmetric spaces. Again ¨ Ozen and Altay [10] also studied the totally umbilical hypersurfaces of weakly concircular and pseudo concircular symmetric

About-Absos Ali Shaikh, Indranil Roy And Shyamal Kumar Hui spaces. The object of the present paper is to study totally umbilical hypersurfaces of a $(WCHS)_n$, and it is shown that such a hypersurface with Codazzi type Ricci tensor is a pseudo-conharmonically symmetric space or a totally geodesic hypersurface.

II. PRELIMINARIES

Let $\{e_i : i = 1, 2, \ldots, n\}$ be an orthonormal basis of the tangent space at any point of the manifold. Then from (1.3), we have the following [14]:

$$(1) \qquad g^{hk}T_{hijk} = -\frac{R}{n-2}g_{ij} = g^{hk}T_{ihkj},$$

where R is the scalar curvature of the manifold and

$$(2) \qquad g^{jk}T_{hijk} = g^{hi}T_{hijk} = 0.$$

Also from (1.3) it follows that [14]

$$(3) \quad \begin{array}{ll} (i) & T_{hijk} = -T_{ihjk}, \\ (ii) & T_{hijk} = -T_{hikj}, \\ (iii) & T_{hijk} = T_{jkhi}, \\ (iv) & T_{hijk} + T_{ijhk} + T_{jhik} = 0. \end{array}$$

Definition 2.1-In a Riemannian manifold $V_n(n > 3)$ the Ricci tensor is said to be of Codazzi type ([7], [13]) if

$$(4) \qquad R_{ij,k} = R_{kj,i}.$$

Proposition 2.1. [14] In a Riemannian manifold $V_n(n > 3)$the relation

$$(5) \qquad T_{hijk,l} + T_{iljk,h} + T_{lhjk,i} = 0.$$

holds if and only if the Ricci tensor is of Codazzi type.

Proposition 2.2. [14] If a $(WCHS)_n$ is of non-zero constant scalar curvature, then its associated 1-forms A and B are related by A = 2B.

The above results will be used in section 3.

III. TOTALLY UMBILICAL HYPERSURFACES OF WEAKLY AND PSEUDO CONHARMONICALLY SYMMETRIC SPACES

Let $(\bar{V}, \bar{g})$ be an (n+1)-dimensional Riemannian space covered by a system of coordinate neighbourhoods $\{U, y^\alpha\}$

Let (V, g) be a hypersurface of $(\bar{V}, \bar{g})$ defined in a locally coordinate system by means of a system of parametric equation $y^\alpha = y^\alpha(x^i)$, where Greek indices take values 1, 2, · · · , n and Latin indices take values 1, 2, · · · , (n + 1). Let N^α be the components of a local unit normal to (V, g). Then we have

$$(3.1) \qquad g_{ij} = \bar{g}_{\alpha\beta}y_i^\alpha y_j^\beta,$$

$$(3.2) \qquad \bar{g}_{\alpha\beta}N^\alpha y_j^\beta = 0, \quad \bar{g}_{\alpha\beta}N^\alpha N^\beta = e = 1,$$

$$(3.3) \qquad y_i^\alpha y_j^\beta g^{ij} = \bar{g}^{\alpha\beta} - N^\alpha N^\beta, \quad y_i^\alpha = \frac{\partial y^\alpha}{\partial x^i}.$$

The hypersurface (V, g) is called a totally umbilical hypersurface ([3],[6]) of $(\bar{V}, \bar{g})$ if its second fundamental form Ω_{ij} satisfies

$$(3.4) \qquad \Omega_{ij} = Hg_{ij}, \quad y_{i,j}^\alpha = g_{ij}HN^\alpha,$$

where the scalar function H is called the mean curvature of (V, g) given by $H = \frac{1}{n}\sum g^{ij}\Omega_{ij}$ If, in particular, H = 0, i.e.,

$$(3.5) \qquad \Omega_{ij} = 0,$$

then the totally umbilical hypersurface is called a totally geodesic hypersurface of (V, g).

The equation of Weingarten for (V, g) can be written as $N_{,j}^\alpha = -\frac{H}{n}y_j^\alpha$. The structure equations of Gauss and Codazzi ([3],[6]) for (V, g) and $(\bar{V}, \bar{g})$ are respectively given by

$$(3.6) \qquad R_{ijkl} = \bar{R}_{\alpha\beta\gamma\delta}B_{ijkl}^{\alpha\beta\gamma\delta} + H^2G_{ijkl},$$

$$(3.7) \qquad \bar{R}_{\alpha\beta\gamma\delta}B_{ijk}^{\alpha\beta\gamma}N^\delta = (H_{,i})g_{jk} - (H_{,j})g_{ik},$$

Where R_{ijkl} and $\bar{R}_{\alpha\beta\gamma\delta}$ are curvature tensors of (V, g) and $(\bar{V}, \bar{g})$ respectively, and

$$B^{\alpha\beta\gamma\delta}_{ijkl} = B^{\alpha}_{i} B^{\beta}_{j} B^{\gamma}_{k} B^{\delta}_{l}, \quad B^{\alpha}_{i} = y^{\alpha}_{i}, \quad G_{ijkl} = g_{il}g_{jk} - g_{ik}g_{jl}.$$

Also we have ([3],[6])

$$\text{(3.8)} \qquad \bar{R}_{\alpha\delta} B^{\alpha}_{i} B^{\delta}_{j} = R_{ij} - (n-1)H^2 g_{ij},$$

$$\text{(3.9)} \qquad \bar{R}_{\alpha\delta} N^{\alpha} B^{\delta}_{i} = (n-1)H_{,i}.$$

From (1.3), (3.6) and (3.8), we obtain

$$\text{(3.10)} \qquad T_{ijkl} = \bar{T}_{\alpha\beta\gamma\delta} B^{\alpha\beta\gamma\delta}_{ijkl} - \frac{n}{n-2} H^2 G_{ijkl}.$$

Also from (1.3), (3.7) and (3.9), we have

$$\text{(3.11)} \qquad \bar{T}_{\alpha\beta\gamma\delta} B^{\alpha\beta\gamma}_{ijk} N^{\delta} = -\frac{1}{n-2} H^2 [(H_{,i}) g_{jk} - (H_{,j}) g_{ik}].$$

By virtue of Proposition 2.1, we can state the following:

Proposition 3.1. In a Riemannian space $\bar{V}_n (n > 3)$ the relation

$$\text{(3.12)} \qquad \bar{T}_{bcde,a} + \bar{T}_{bcea,d} + \bar{T}_{bcad,e} = 0.$$

holds if and only if the Ricci tensor is of Codazzi type. Let $(\bar{V}, \bar{g})$ be a weakly conharmonically symmetric space with Codazzi type Ricci tensor. Then we have

$$\text{(3.13)} \qquad \bar{T}_{bcea,d} = A_d \bar{T}_{bcea} + B_b \bar{T}_{dcea} + B_c \bar{T}_{bdea} + D_e \bar{T}_{bcda} + D_a \bar{T}_{bced},$$

where A,B and D are 1-forms (not simultaneously zero). Permutating A, B and D in (3.13) and adding these equations and using (3.12), we obtain

$$\text{(3.14)} \qquad A_d \bar{T}_{bcea} + A_e \bar{T}_{bcad} + A_a \bar{T}_{bcde} + B_b (\bar{T}_{acde} + \bar{T}_{dcea} + \bar{T}_{ecad}$$
$$B_c (\bar{T}_{bdea} + \bar{T}_{bead} + \bar{T}_{bade}) + 2(D_e \bar{T}_{bcda} + D_a \bar{T}_{bced} + D_d \bar{T}_{bcae}) = 0.$$

Let (V, g) be a weakly conharmonically symmetric space whose Ricci tensor is of Codazzi type. Multiplying both sides of (3.14) by B^{abcde}_{hijkr} and then using (1.5), (2.5) and (3.10),we get

$$\text{(3.15)} \qquad \frac{n}{n-2} H^2 [A_k G_{ijrh} + A_r G_{ijhk} + A_h G_{ijkr} + 2(D_r G_{ijkh} + D_h G_{ijrk} + D_k G_{ijhr})] = 0.$$

Transvecting (3.15) by $g^{ih} g^{jr}$, we get

$$\text{(3.16)} \qquad \text{either } H = 0, \quad \text{or } A_k = 2D_k \quad \text{for all } k.$$

We now suppose that $H \neq 0$ Then from (3.16), we obtain

$$\text{(3.17)} \qquad A_k = 2D_k \quad \text{for all } k.$$

If the scalar curvature R is non-zero, then from (2.4), it follows that R is A constant. Then by virtue of Proposition 2.2, we get

$$\text{(3.18)} \qquad A_k = 2B_k = 2D_k \quad \text{for all } k.$$

Thus from (17) and (18), we have either H = 0, or

$$\text{(3.19)} \qquad A_k = 2B_k = 2D_k \quad \text{for all } k.$$

Consequently, the relation (1.5) reduces to the following

$$T_{hijk,l} = 2A_l T_{hijk} + A_h T_{lijk} + A_i T_{hljk} + A_j T_{hilk} + A_k T_{hijl},$$

i.e., the space (V, g) is pseudo -conharmonically symmetric [1]. This leads to the following:

Theorem 3.1-If the totally umbilical hypersurface of a weakly conharmonically symmetric space with Codazzi type Ricci tensor and non-vanishing scalar curvature is a weakly conharmonically symmetric space with Codazzi type Ricci tensor and non-vanishing scalar curvature, then it is a pseudo-conharmonically symmetric space or a totally geodesic hypersurface.

We now consider that the hypersurface (V, g) is a totally geodesic, i.e.,H = 0. Then from (3.4), (3.10), we get

$$\text{(3.20)} \qquad \bar{T}_{bcde,a} B^{abcde}_{rhijk} = T_{hijk,r}.$$

Using (3.20) in (3.13), we have the relation (1.5). Thus we can state the following:

Theorem 3.2.-The totally geodesic hypersurface of a weakly conharmonically symmetric space is weakly conharmonically symmetric.

Corollary 3.1-The totally geodesic hypersurface of a pseudo-conharmonically symmetric space is pseudo-conharmonically symmetric.

Let (V, g) be a pseudo conharmonically symmetric space. Then we get

$$\bar{T}_{bcea,d} = 2A_d\bar{T}_{bcea} + A_b\bar{T}_{dcea} + A_c\bar{T}_{bdea} + A_e\bar{T}_{bcda} + A_a\bar{T}_{bced}. \quad (3.21)$$

Multiplying both sides of (3.21) by B^{abcde}_{hijkr} and using (3.10), we get

$$\frac{n}{n-2}H^2[2A_kG_{ijrh} + A_iG_{kjrh} + A_jG_{ikrh} + A_rG_{ijkh} + A_hG_{ijrk}] = 0. \quad (3.22)$$

Transvecting (3.22) by $g^{ih}g^{jr}$, we get

$$2\,H_{,k} + (2n+3)A_kH = 0 \ \text{ for all } k. \quad (3.23)$$

This leads to the following:

Theorem 3.3-If the totally umbilical hypersurface of a pseudo-conharmonically symmetric space is a pseudo-conharmonically symmetric space then its associated 1-form A satisfies the relation (3.23).

IV. References

1) Chaki, M. C., On pseudosymmetric manifolds, An. Sti. Ale Univ., "AL. I. Cuza" DIN IASI, 33 (1987), 53–58.
2) Chaki, M. C., On generalized pseudosymmetric manifolds, Publ. Math. Debrecen, 45 (1994), 305–312.
3) Chen, B. Y., Geometry of submanifolds, Marcel-Deker, New York, 1973.
4) De, U. C. and Bandyopadhyay, S., On weakly symmetric Riemannian spaces, Publ. Math. Debrecen, 54 (1999), 377–381.
5) De, U. C. and Bandyopadhyay, S., On weakly conformally symmetric spaces, Publ. Math.Debrecen, 57 (2000), 71–78.
6) Eisenhart, L. P., Riemannian Geometry, Princeton University Press, 1949.
7) Ferus, D., A remark on Codazzi tensors on constant curvature space, Lecture Notes Math.,838, Global Differential Geometry and Global Analysis, Springer-Verlag, New York, 1981.
8) Ishii, Y., On conharmonic transformations, Tensor N. S., 11 (1957), 73–80.
9) Ozen, F. and Altay, S., On weakly and pseudo symmetric Riemannian spaces, Indian J.Pure Appl. Math., 33(10) (2001), 1477–1488.
10) Ozen, F. and Altay, S., On weakly and pseudo concircular symmetric structures on A Riemannian manifold, Acta Univ. Palacki. Olomuc. Fac. rer. nat. Math., 47 (2008), 129–138.
11) Prvanovi´c, M., On weakly symmetric Riemmanian manifolds, Publ. Math. Debrecen, 46 (1995), 19–25.
12) Roter, W., On conformally symmetric Ricci recurrent space, Coll. Math., 31 (1974), 87–96.
13) Schouten, J. A., Ricci-Calculus, An introduction to Tensor Analysis and its Geometrical Applications, 2nd Edn., Springer-Verlag, Berlin, 1954.
14) Shaikh, A. A. and Hui, S. K., On weakly conharmonically symmetric manifolds, Tensor, N. S., 70 (2008), 119–134.
15) Shaikh, A. A. and Hui, S. K., On decomposable weakly conharmonically symmetric manifolds, Lobachevski J. Math., 29(4) (2008), 206–215.
16) Tam´assy, L. and Binh, T. Q., On weakly symmetric and weakly projective symmetric Riemannian manifolds, Coll. Math. Soc. J. Bolyai, 56 (1989), 663–670.

Reliable Classification Of Brachiopods Through Curvature Scale Space And Level-Set Method

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GJSFR Classification – F (FOR) 010302,010204,010108

***Abstract*- Brachiopods have a lateral outline which is quite important in systematic studies. It is often assessed by a qualitative evaluation and linear measurements, which are not clear enough and precise for describing the shape of the shell and its changes. In this work we shed light on a new approach based on digital image processing of these species. Our approach includes two steps. The first is to determine the contour of the species from their 2D images using the implicit active contours (Level set). The second step is to use the CSS (Curvature Scale Spaces) to index curves. This new mathematical model reported here presents a suitable morphologic descriptor which allows us to: (1) express a rigorous morphologic quantitative assessment; (2) emphasize the various differences between different shapes.**

Keywords- Brachiopods, Image, Active Contour, Level-Set, CSS.

I. Introduction

The analysis is the quantification of the organization's morphology which forms an important aspect in the paleontological studies. On the one hand, they make it possible to understand the biodiversity in its morphological dimension. On the other hand, they highlight the morphological transformations undergone during the biological evolution. Historically, the form was encircled by a purely descriptive approach based on the qualitative evaluations of the morphological change starting from simple images. This approach was replaced gradually by the biometric methods having leads to the populated design of the fossil species. The variables used in such methods are linear dimensions, angles, surfaces and ratio or combination of dimensions. But, these biometric descriptors are insufficiently informative since they give only one approximate quantitative representation of the form and its changes. An alternative approach of the quantitative description of morphology is the multivariate analysis based on the methods of the analysis of the form classified in two categories: the geometrical morphometry and analysis of contour. The first based on the location of points homologous on the organizations and the quantification with their morphological differences (Bookstein77), (Bookstein85), (Bookstein91). The second is called analysis contour subdivided in two classes:

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analyzed by Eigenshape defined initially by (Lohman83) its basic principle is to calculate the average shape of a population. This technique consists in applying the algorithm of (Zahn and Roskies72) to produce pairs of coordinates to regular intervals around a contour beginning to one some homologous point available, and to use the tangent angle of Bookstein. This method raises many difficulties of interpretation.

- Analysis of Fourier which consists in approaching the form by a goniometrical function defined by a sum of terms of sine and cosine. This function is broken up into a series of amplitude of harmonics and phases or into a series of coefficient of Fourier being useful like variables for the quantitative analyses. The first application of such a method for the morphological study of the fossils is that of (Kaesler and Waters72). Since, many applications succeeded but this method is valid right for the forms regular (convex external format), indeed when morphologies become complex, it is not more possible to use such descriptors see (Bachnou99).

We present in this work a new method for classification of fossil the species known as Brachiopods. This method comprises two stages: The first is the detection of the external contour of the Brachiopod starting from their image 2D by one of the techniques of active contour, and more precisely the technique of the whole of level (Level-Sets), the latter is based on the equation of wave propagation of the face, whose deformation depends on the geometry of the structure. This technique was developed by Osher and Sethian in (Osher and Sethian88) and was used for the segmentation of image which constitutes a great step in the treatment and the interpretation of the images. The second stage is the characterization of contour by the descriptor CSS (Curvature Scale Space) which has undeniable advantages of invariance, compactness, effectiveness and stability.

This article is organized in the following way: In the first part we approached the concept of active contours and more precisely the model of active contours based areas. We thus defined the functional calculus of energy to be minimized. The minimization of energy by the method of the level lines (Level-Set) is treated in the second section, after having made a recall on this method. In the third part we implemented the point on the setting of the descriptor CSS. We tested this method on real images of the fossils, this model gives results which are very satisfactory and encouraging ones, in terms of quality of image and the possibility to answer a request.

II. THE MODEL OF ENERGY

The segmentation of image is an operation of picture treatment that has as an objective to reassemble pixels between them according to of the meadow-definite criteria. Pixels are regrouped thus in regions, that constitute a paving or a partition of the picture. It can be for example about separating objects of the bottom. If the number of classes is equal to two. The segmentation is a primordial stage in treatment of picture. To this day, it exists numerous methods of segmentation, that one few citers:

1) Active contours based contours
2) Active contours based regions

The first method using the active contours is based contours exclusively (Kass et al.88). It doesn't permit a spatial information utilization intern and external to the contour. The method based region permits to introduce the global terms characterizing regions. It is therefore this method that we decided to use here. In the two cases, the active contour method consists in choosing a functional to minimize to make evolve a contour. Let's take the example of the segmentation of a picture in two regions Ω_{in} : the region containing objects to segment and Ω_{out} , the region of the bottom. The interfacing between these two regions is noted $C = \partial\Omega_{in}$. The two regions form a partition of the image, one has therefore $\Omega_{in} \cup \Omega_{out} \cup C = \Omega_{I}$. Who is the domain of the image and $\Omega_{in} \cap \Omega_{out} = \varnothing$. One can introduce a functional then general to minimize for the partition of a picture in two regions:

$$E(\Omega_{in},\Omega_{out},C) = \int_{\Omega_{in}} k_{in}(x,\Omega_{in})dx + \int_{\Omega_{out}} k_{out}(x,\Omega_{out})dx + \int_{C} k_b(x)da$$

In this criterion, k_{in} designate the descriptor of objects to segment, k_{out} the descriptor of the region of the bottom and k_b the descriptor of the contour. In our study we considered descriptors proposed by Chan and Vese in (Chan and Vese2001) for the segmentation of a image in two regions in addition terms of regularization of Munford and Shah (Munford and Shah89). So we introduce the energy function:

$$E(C,c_1,c_2) = \mu Length(C) + \nu\, Area(inside(C)) + \lambda_1 \int_{inside(C)} |u_0(x,y) - c_1|^2 dxdy + \lambda_2 \int_{outside(C)} |u_0(x,y) - c_2|^2 dxdy$$

Where u_0 is the image and $\mu \geq 0,\ \nu \geq 0,\ \lambda_1 > 0,\ \lambda_2 > 0$ are fixed parameters. So our goal is to find C, c_1 , c_2 such that $E(C,\ c_1,\ c_2)$ is minimized.

III. LEVEL SET METHOD

A. Definition

The method level-set is an analytic work setting working on the geometric evolution of objects. Instead of to use a representation classic Lagrangienne to describe geometries, the method level-set described them through a scalar function ϕ , in the formalism of level set, the curve, C is not parametric, but implicitly definite by the bay of a superior dimension function ϕ :

$$\phi : \begin{array}{l} \Omega \subset N^2 \rightarrow \Re \\ (x,y) \rightarrow \phi(x,y) \end{array}$$

The contour to one instant t is defined then by the curve of level zero of the function ϕ to this instant.

The main advantage of this method is the ability to automatically manage the change of topology of the curve evolution. Curve C can be divided into two or three curves, conversely several curves can merge and become a single curve.

Another advantage is that geometric properties of the curve are easily determined from a particular level set of the surface. For example, the normal vector for any point on the curve C is given by:

$$N = \frac{\nabla\phi}{|\nabla\phi|} \quad (1)$$

And the curvature K is obtained from the divergence of the gradient of the unit normal vector to the front:

$$K = div(\frac{\nabla\phi}{|\nabla\phi|}) \quad (2)$$

Finally, another advantage is that we are able to evolve curves in dimensions higher than two. The above formulae can be easily extended to deal with higher dimensions. This is useful in propagating a curve to segment volume data.

B. Equation Of Evolution

It has been shown in (Sethian99) that $C(t)$) evolves according to equation $\frac{\partial C((x,y),t)}{\partial t} = F.N$ then the implicit representation respects the equation: $\frac{\partial\phi((x,y),t)}{\partial t} = F.|\nabla\phi((x,y),t)|$

Where de set $\{(x,y),\ \phi_0(x,y)=0\}$ defines the initial contour, and F is the speed of propagation. Function level set initial is gotten by the utilization of the distance signed to curve initial:

$$\phi_0 = \begin{cases} -d((x,y),C_0)\ if\,(x,y) \in \text{ inside(C)} \\ +d((x,y),C_0)\ if\,(x,y) \in \text{ ouside(C)} \end{cases} \quad (3)$$

C. Level-Set Reformulation Of The Model

For the level set formulation of the variation active contour model, we replace the unknown variable C by the unknown variable, and using the Heaviside function H, and the one dimensional Dirac measure defined respectively by:

$$H(z) = \begin{cases} 1, \text{ if } z \geq 0 \\ 0, \text{ if } z<0 \end{cases} \qquad \delta = \frac{d}{dz}H(z)$$

In order to compute the associated Euler- lagrange equation for unknown function φ, our numerical simulations involve regularized version of H and δ, denoted here by H_ε and δ_ε, as $\varepsilon \to 0$. In this paper, we approximate the regularization of Heaviside by:

$$H_\varepsilon = \frac{1}{2} + \frac{1}{\pi}\ \arctan(\frac{z}{\varepsilon}) \quad \text{and} \quad \delta_\varepsilon = \frac{dH_\varepsilon}{dz}$$

We express the terms in the energy E in the following way:

$$\begin{aligned} E(C,c_1,c_2) = \mu \int_\Omega & \delta_\varepsilon(\phi(x,y))\left|\nabla\phi(x,y)\right| dxdy \\ & + \nu \int_\Omega H_\varepsilon(\phi(x,y))dxdy \\ & + \lambda_1 \int_\Omega \left|u_0(x,y) - c_1\right|^2 (1 - H_\varepsilon(\phi(x,y)))dxdy \\ & + \lambda_2 \int_\Omega \left|u_0(x,y) - c_2\right|^2)H_\varepsilon(\phi(x,y)dxdy \end{aligned}$$

The constants $c_{1,}\ c_2$ are the averages of u_0 in $\phi < 0$ and $\phi \geq 0$ respectively. So they are easily computed as:

$$c_1(\phi) = \frac{\int_\Omega u_0(x,y)(1 - H_\varepsilon(\phi(x,y)))dxdy}{\int_\Omega (1 - H_\varepsilon(\phi(x,y)))dxdy} \quad (4)$$

$$c_2(\phi) = \frac{\int_\Omega u_0(x,y) H_\varepsilon(\phi(x,y))dxdy}{\int_\Omega H_\varepsilon(\phi(x,y))dxdy} \quad (5)$$

Minimizing $E(C)$ with respect to ϕ fields the following Euler- Lagrange equation for ϕ, parameterize the descent direction by time, t>0. Equation in $\phi(t,x,y)$ with $\phi(0,x,y) = \phi_0(x,y)$ is :

$$\begin{cases} \dfrac{\partial\phi}{\partial t} = \dfrac{dH_\varepsilon}{dz}(\phi)[\ \ \mu div\left(\dfrac{\nabla\phi}{|\nabla\phi|}\right) - \nu - \lambda_1(u_0 - c_1)^2 \\ \qquad\qquad + \lambda_2(u_0 - c_2)^2\ \ ] \\ \dfrac{dH_\varepsilon}{dz}(\phi)\dfrac{\partial\phi}{\partial n} = 0 \quad if \quad t > 0 \ \ and \ \ (x,y) \in \partial\Omega \\ \phi(0,x,y) = \phi_0(x,y) \quad if \ \ (x,y) \in \Omega \end{cases} \quad (6)$$

We suggest the following approximating schemes:

$$\frac{\phi^{n+1}{}_{i,j} - \phi^n{}_{i,j}}{\Delta t} = \frac{dH_\varepsilon}{dz}(\phi^n{}_{i,j})\left[\mu K(\phi^n) - \nu - \lambda_1(u_{0,i,j} - c_1(\phi^n))^2 + \lambda_2(u_{0,i,j} - c_2(\phi^n))^2\right] \quad (7)$$

Finally, the principal steps of the algorithm are:

Choose epsilon

Initialize ϕ^0 by ϕ_0 , n=0

Repeat until $\int_\Omega (\phi^{n+1}(x,y) - \phi^n(x,y))^2 dxdy < epsilon$

Compute $c_1(\phi^n)$ and $c_2(\phi^n)$ by (4) and (5)

Compute curvature term $K(\phi^n)$ by (2)

Compute ϕ^{n+1} by (7)

End

IV. APPLICATION

To study the performance of the Level-Set method, we apply the algorithm developed to detect the outer contour of the Brachiopod from their two-dimensional images. The

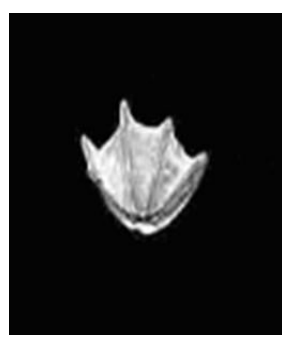

Fig.1. Initial Image

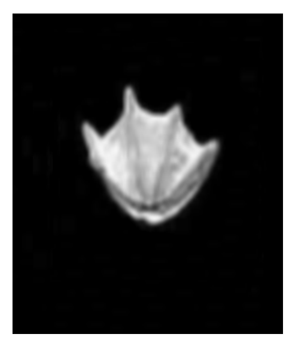

Fig.2. Filtred Image

image to segment contains several regions, so it is necessary to go through a preprocessing to adapt the image model of energy used here. For that reason we applied to the original image a medium filter.

After the filtering operation, we apply the image to the Level-Set method described above in order to extract the outer contour of the brachiopod. The initial contour used in our case is a circle containing the object as shown in Figure (3):

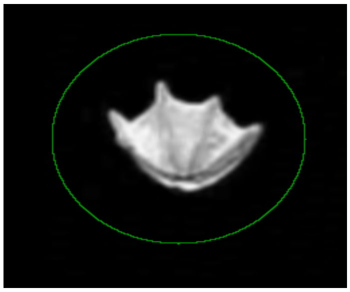

Fig.3. Initial contour

Figures (4) and (5) are two intermediate steps, and figure (6) gives the results obtained by the method of Level-Set. We can easily notice that the contour of the initial curve is perfectly shaped in the form of the outer contour of brachiopod.

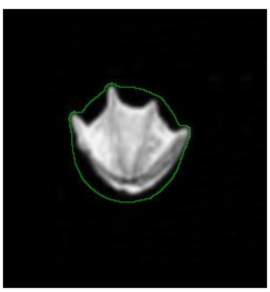

Fig.4. Intermediate step

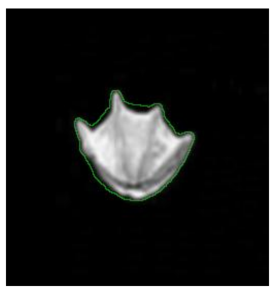

Fig.5. Intermediate step

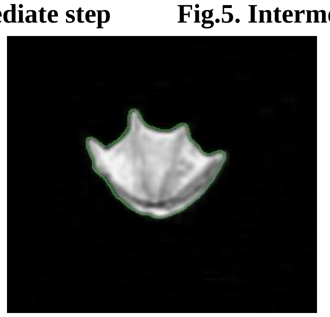

Fig.6. Segmentation by Level-Set

V. CLASSIFICATION OF BRACHIOPOD

Once the contour of the Brachiopod is determined we think of as characterizing a specific descriptor. Note that if the shape is convex Fourier analysis gives a stunning result, which is to approximate the shape by a trigonometric function defined by a sum of terms of sinus and cosines. This function is decomposed into a series of Fourier coefficients used as variables for quantitative analysis. By cons, if the shape is concave, as if here we chose the handle CSS (Curvature Scale Space) to index the shapes.

The CSS descriptor was originally proposed by Mokhtarian in (Mokhtarian92) and selected for the MPEG-7. Its basic principle is based on the description of a concave curve and its subsequent filtering in order to monitor these concavities. The role of filtering is to smooth the curve and thus gradually eliminating its ventricles. The shape of the brachiopod is described by a closed curve representing the contour. Each item is identified by its curvilinear abscissa on this standard curve $C = \{x(u),\ y(u);\ u \in [0,1]\}$.

The function C then undergoes filtering by a Gaussian kernel and it looked for points of inflection of the curve to characterize their shape. These inflection points are obtained when the curvature at different scales is equal to 0.

With

$$k(u,\sigma) = \frac{X_u(u,\sigma)Y_{uu}(u,\sigma) - X_{uu}(u,\sigma)Y_u(u,\sigma)}{(X_u(u,\sigma)^2 + Y_u(u,\sigma)^2)^2}$$

Where

$$X(u,\sigma) = x(u) * g(u,\sigma) \text{ and}$$

$$Y(u,\sigma) = y(u) * g(u,\sigma)$$

According to the properties of convolution, the derivatives of every component can be calculated easily:

$$X_u(u,\sigma) = x(u) * g_u(u,\sigma)$$

and

$$Y_u(u,\sigma) = y(u) * g_u(u,\sigma)$$

$g(u,\sigma)$ is a Gaussian kernel of with σ.

These turning points are used to iteratively build an image (image called CSS) by stacking successive layers of points where each layer represents the inflection points found in a number of filters given. So the CSS image is obtained, the horizontal axis, the curvilinear abscissa and ordinate, the number of filters. Figure (7) shows the CSS image obtained to describe the contour of brachiopods.

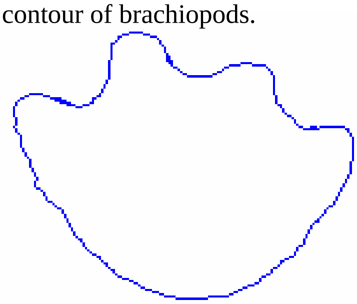

Fig.7. Contour

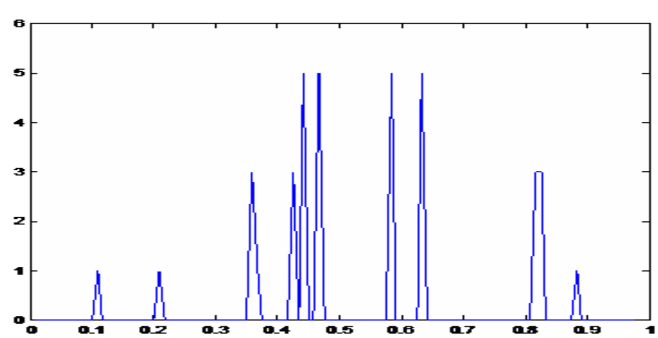

Fig.8. CSS image

Each peak represents a concave contour. The curvilinear abscissa ensures invariance in translation and scale. The rotational invariance is also guaranteed by a lateral shift of the peaks in the CSS image. Small peaks obtained in the CSS image correspond to the noise in the contour.

VI. Conclusion

In this work, we could determine the external contour of the Brachiopods with the help of the level set techniques which seem more powerful than the other techniques used previously in (Sasaki91), (Crampton92), (Renaud el al.96), (Bachnou99).

When with the classification of these species, we adopted the technique based on the CSS which makes it possible to define a descriptor of these species which have a non-convex form, the thing which was impossible by Fourier's analysis.

VII. References

1) A.Bachnou, Modélisation des contours fermés: «An-Morph» outil mis au point pour maîtriser les composantes des profils latéraux des ostracodes. Perspective d'application systématique. Géobios 32 (5), pp.733--742, 1999.
2) F. L. Bookstein, The study of shape transformation after D'Arcy Thompson. Mathematical Biosciences, pp.177-219, 1977.
3) F. LBookstein , B. Chernoff, R.L. Elder, J.M. Humphries, G.R. Smith and R.E. Strauss, Morphometrics in Evolutionary Biology, 15. The Academy of Natural Sciences of Philadelphia Special Publication, p. 277, 1985.
4) F.L. Bookstein, Morphometric tools for landmark data. Geometry and Biology. Cambridge University Press, p.435, 1991.
5) T. Chan and L. Vese, Active contours without edges, IEEE Trans. Image Processing, 10(2), pp.266-277, 2001.
6) J.S. Crampton, Morphometric description of cosmopolitan Early Cretaceous inoceramid bivalves, In: Abstracts with program Fifth North American Paleontological Convention. In: Lidgard, S., Crane, P.R. (Ed), Paleontological Society, Special Publication, 6, p. 72, 1992.
7) Kass, A .Witkins, and D. Terzopoulos, Snakes: Active contour models, International Journal of Computer Vision, 1(4), pp.321--331, 1988.
8) R.L. Kaesler and G.A. Waters, Fourier analysis of ostracode margin, Geological Society of America bulletin 83, pp.1169-1178, 1972.
9) G.P. Lohmann, Eigenshape analysis of microfossils: a generalmorphometric procedure for describing changes in shape, Mathematical Geology 15, pp.659--672, 1983.
10) F. Mokhtarian and Alan K. Mackworth, A Theory of Multiscale, Curvature-Based Shap Representation for Planar Curves, IEEE PAMI, Vol. 14, August, pp.789-805, 1992.
11) Munford and J. Shah, Optimal approximation by piecewise smooth functions and associated variational problems, Commun. Pure Appl. Math, 42, pp.577-685, 1989.
12) S. Osher and J. A. Sethian, Fronts propagating with curvaturedependent speed: algorithms based on Hamilton-Jacobi formulations,.J. Comput. Phys, vol. 79, no. 1, pp.12-49, 1988.
13) S. Renaud, J. Michaux, J.J. Jaeger and J.C. Auffray, Fourier analysis applied to Stephanomys (Rodentia, Muridae) molars: non progressive evolutionary pattern in a gradual lineage. Paleobiology 22, pp.255--265, 1996.
14) O. Sasaki, Discrimination of two new Andara species using shellgrowth parameters and Fourier descriptors. Transactions and Proceedings of the Paleontological Society of Japan, New Series 161, pp. 697--713, 1991.

Heuristics Algorithms For Job Sequencing Problems

GJSFR Classification – D (FOR)
091307,091305,010303

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Abstract **- In this paper, we propose a new heuristics technique called Time deviation method to obtain a sequence of jobs for solving job sequencing problem so that its total elapsed time is minimum. Here first applied the new method for 2 machines n jobs problems and extended the same for 3 machine n job problems. Then apply the same technique for m machines and n jobs problems after converting m machines n job problems in to 2 machines n job problems by using existing technique.**

Keywords- Time deviation method, Total elapsed time, Optimal sequence, Idle time, Processing time.

I. Introduction

Sequencing is the selection of an appropriate order for a number of different tasks to be performed on a finite number of service facilities in order to make effective use of available facilities and achieve greater output. Suppose if the time that each job requires at each machine is also given, then sequencing deals in finding the sequence for processing the jobs so that the total elapsed time for all the jobs will be minimum. The problem of sequencing may have some restrictions placed on it, such as time for each task, the order of processing each job on each machine, availability of resources (men, machine, material and space) etc. In sequencing problem, the effectiveness can be measured in terms of minimized costs, maximized profits, minimized elapsed time and meeting due dates etc.

Several research articles [5, 6, 7, 8, 9] deals for framing optimal job sequences in job sequencing problem under various circumstances. The paper [1] studies the single job lot streaming problem in a two stage hybrid flow shop that has m identical machines at the first stage and one machine at the second stage to minimize the make span and the article[10] consider the single machine bicriterion scheduling problem of enumerating the pareto – optimal sequences with respect to the total weighted completion time and the maximum lateness objectives. The paper [2] deals with the classical problem of minimizing the make span in a two machine flow shop. When the job processing times are deterministic, the optimal job sequence can be determined by applying Johnson's rule. When they are independent and exponential random variables, Talwar's rule yields a job sequence that minimizes the make span stochastically. Assuming that the job processing times are independently and Weibull distributed random variables, a new job sequencing rule is presented that includes both Johnson's and Talwar's rules as special cases. The paper[3] addresses a problem of continuous batch scheduling arising in the heating process of blooms in steel industry and the article[4] presented a simple, elegant, algorithm for finding an optimal solution to a general min – max sequencing problem. In the article [11] the problem of sequencing n jobs in a two machine reentrant shop with the objective of minimizing the maximum completion time is considered. The shop consists of two machines M_1 and M_2 and each job has the processing route (M_1, M_2, M_1). An O(n log n) time heuristic is presented which generates a schedule with length at most 4/3 times that of an optimal schedule, thereby improving the best previously available worst case performance ratio 3/2. The article [12] is concerned with the problem of scheduling n jobs with a common due date on a single machine so as to minimize the total cost arising from earliness and tardiness and the paper[13] gives computer program for solving n job m machine sequencing problems which is an extension of the well known graphical procedure for solving 2 jobs m machine sequencing problems and it is equivalent to finding the shortest path between two nodes in finite network. The effectiveness in giving an optimal solution or a good feasible solution is investigated..

In this paper, we introduce a new heuristics method called Time deviation method to frame a sequence of jobs in job sequencing problems for processing the n jobs so that the total elapsed time for the jobs will be (considerably) minimum.

II. Preliminaries

The general sequencing problem may be defined as follows. Let there be n jobs to be performed one at a time on each of m machines. The order of the machines in which each job should be performed is given. The actual or expected time required by the jobs on each of the machines is also given. The general sequencing problem is to find the sequence out of $(n!)^m$ possible sequences which minimize the total elapsed time between the start of the job in the first machine and the completion of the last job on the last machine(Under usual assumptions observed for job sequencing problems).

A. Processing N Jobs Through Two Machines

Let there be n jobs, each of which is to be processed through two machines, say M1 and M2 in the order M1- M2. If the machine M2 is not free at the moment for processing the same job then job has to wait in a waiting line for its turn on

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machine M2. Let t_{ij} (i =1,2 j = 1, 2, …,n) be the time required for processing i th job on the j th machine. Since passing is not allowed, therefore machine M1 will remain busy in processing all the n jobs one by one while machine M2 may remain idle after completion of one job and before starting of another job. The sequencing problem of n jobs through two machines can be written as follows

Jobs/ Machines	J1	J2	J3	……….	Jn
M1	T_{11}	t_{12}	t_{13}	……..	t_{1n}
M2	T_{21}	t_{22}	t_{23}	…….	t_{2n}

where t_{ij} is the time duration taken jth job by ith machine. The objective is to minimize the idle time of the second machine. Let X_{2j} be the time for which machine M2 remains idle after finishing (j – 1) th job and before starting processing the j th job (j = 1,2, .., n). The total elapsed time T is given by $T = \sum_{j=1}^{n} t_{2j} + \sum_{j=1}^{n} X_{2j}$ where some of the X_{2j} 's may be zeros. The problem is to find the sequence for n jobs on two machines which will give minimized total elapsed time.

III. New Method

A. *Time Deviation Vector Method For Processing N Jobs Through Two Machines*

We now define the following terms which will be used in the proposed new method called Time deviation method for determining an optimal sequence of jobs in job sequencing problems.

The row deviation of a cell in the Time duration table is the value which is equal to the time duration of the cell minus the minimum time duration of the corresponding row. The column deviation of a cell in the time duration table is the value which is equal to the time duration of the cell minus the minimum time duration of the corresponding column. Let r_i be the minimum time of the i th row and s_j be the minimum time of the j th column. The row time deviation of the (i, j)th cell denoting as p_{ij} is defined as $p_{ij} = t_{ij} - r_i$. Similarly the column time deviation of the (i, j)th cell denoting as c_{ij} is defined as $c_{ij} = t_{ij} - s_i$. The Time deviation vector table for the sequencing problem for two machines is as follows.

Jobs/ Machines	J1	J2	J3	……….	………	Jn
M1	(p11, c11)	(p12, c12)	(p13, c13)	…….	……	(p1n, c1n)
M2	(p21, c21)	(p22, c22)	(p23, c23)	…..	…..	(p2n, c2n)

The required sequence of jobs is obtained from the Time deviation table which is based on the value of row and column deviation. The algorithm for the time deviation vector follows.

i. *Heuristics Algorithm To Frame A Sequence Of Jobs In N Jobs Two Machines Problems*

1) Construct the time deviation vector table for the given sequencing problem.
2) Find the cell which have time deviation vectors both are zero for machine M1 and perform the corresponding job firstly.
3) Suppose more than one cell has both deviation vectors are zero then obtain sum deviations of the corresponding columns. Perform the job first which cell have the largest sum deviation and perform next job which have next largest sum deviation and so on.
4) Similarly look the cell which has time deviation vectors both are zero for machine M2 and perform the corresponding job lastly.
5) Suppose more than one cell has both deviation vectors are zero then obtain sum deviations of the corresponding columns. Perform the job last which cell have the largest sum deviation and perform next job which have next largest sum deviation prior to last and so on.
6) If we get the sequence involving all jobs for the job sequencing problem, then stop the process. Otherwise go to next step.
7) Form the reduced time duration table which contains only non assigned jobs.
8) Continue the above steps from (i) to (vi) for the reduced time duration table. But perform the jobs next to the previously assigned jobs for the cells which have both time deviation vectors are zero for M1 machine and perform jobs previous to the last assigned jobs for the cells which have both time deviation vectors are zero for M2 machine.
9) This will be continued till we get the required order of sequence of jobs for job sequencing problems which will give minimum total elapsed time.

ii. *Problem Involving Processing N Jobs Through Two Machines*

Example-Determine the (optimal) sequence of jobs that minimizes the total elapsed time required to complete the following jobs on machines M1 and M2 in the order M1-M2.

Jobs/ Machines	J1	J2	J3	J4	J5	J6	J7	J8	J9
M1	2	5	4	9	6	8	7	5	4
M2	6	8	7	4	3	9	3	8	11

Solution: The time deviation table for the given time duration table for jobs on M1 and M2 is follows.

Jobs/ Machines	J1	J2	J3	J4	J5	J6	J7	J8	J9
M1	(0, 0)	(3,0)	(2,0)	(7, 5)	(4, 3)	(6,0)	(5, 4)	(3, 0)	(2, 0)
M2	(3, 4)	(5, 3)	(4, 3)	(1, 0)	(0, 0)	(6, 1)	(0, 0)	(5, 3)	(8,7)

The job sequencing for the above time deviation table is

J1							J5	J7

The reduced Time duration table for the remaining job sequence follows

Jobs/ Machines	J2	J3	J4	J6	J8	J9
M1	5	4	9	8	5	4
M2	8	7	4	9	8	11

The Time deviation table for the reduced time duration table follows

Jobs/ Machines	J2	J3	J4	J6	J8	J9
M1	(1, 0)	(0, 0)	(5, 5)	(4, 0)	(1, 0)	(0, 0)
M2	(4, 3)	(3, 3)	(0, 0)	(5, 1)	(4, 3)	(7, 7)

The new revised job sequencing becomes

J1	J9	J3				J4	J5	J7

The reduced time duration table for the remaining job sequence follows

Jobs/ Machines	J2	J6	J8
M1	5	8	5
M2	8	9	8

The Time deviation table for the reduced time duration table follows

Jobs/ Machines	J2	J6	J8
M1	(0, 0)	(3, 0)	(0, 0)
M2	(0, 3)	(1, 1)	(0, 3)

The new revised job sequencing becomes

J1	J9	J3	J2	J8	J6	J4	J5	J7

which is the required sequence for this sequencing problem. By using this sequence, we apply the usual procedure to get the minimum total elapsed time is 61 hours and idle times on M1 and M2 are 11 hrs, 2 hrs respectively.

B. Time Deviation Vector Method For Processing Of N Jobs Through Three Machines

Let there be n jobs, each of which is to be processed through three machines, say M1, M2 and M3 in the order M1- M2-M3. Let t_{ij} (i =1,2,3 j = 1, 2, …,n) be the time required for processing i th job on the j th machine. The Time duration table follows.

Jobs/ Machines	J1	J2	J3	……….	Jn
M1	T_{11}	t_{12}	t_{13}	……..	t_1n
M2	T_{21}	t_{22}	t_{23}	…….	t_2n
M3	T_{31}	t_{32}	t_{33}	…….	t_3n

i. *Heuristics Algorithm To Obtain A Sequence Of Jobs In N Jobs Three Machines Problems*

1) Construct the Time deviation vector table for the given sequencing problem.
2) Find the cell which has time deviation vectors both are zero column wise from first column to last column.
3) If the first job (J1) has both Time deviation vectors are zero for the machine M1, then perform the job J1 first. If J1 has both Time deviation vectors are zero for the machine M3, then perform the job J1 last in the order of sequence and if it has Time deviation vectors are zero for the machine M2 then find the sum of deviation vectors separately for the above and below of the cell which have both deviation vectors are zero. Compare the both deviations.
4) Perform the job J1 first if the sum of deviation vectors above the zero cell is less than the other sum of deviation vectors. Otherwise perform the job J1 last. If both deviations are same then perform the corresponding job either in first or last. Continue the same procedure for the remaining jobs J2, J3, ….
5) If we get the required sequence for the job sequencing problem, then stop the process. Otherwise go to next step.
6) Form the reduced Time duration table which contains only non assigned jobs.
7) Continue the above steps from (i) to (vi) for the reduced time duration table. But perform the jobs next to the previously assigned jobs for the cells which have both Time deviation vectors are zero for M1 machine, perform jobs previous to the last assigned jobs for the cells which have both Time deviation vectors are zero for M3 machine and perform the jobs next to previously assigned jobs or previous to the last assigned jobs based on the sum of deviations for M2 machine..
8) If any of the job has Time deviation vectors are zero vector for M1 and M2, then perform the corresponding job very first which will reschedule the order of sequence. If any of the job has Time deviation vectors are zero vector for M2 and M3, then perform the corresponding job very last which will also reschedule the order of sequence.
9) This will be continued till we get the order of sequence of jobs for job sequencing problems which will give minimum total elapsed time.

ii. Problem Involving Processing N Jobs Through Three Machines

Example- Determine the sequence that minimizes the total elapsed time required to complete the following tasks on the machines in the order M1 – M2 – M3. Find also minimum total elapsed time and the idle times on the machines.

Jobs/Machines	J1	J2	J3	J4	J5	J6	J7
M1	3	8	7	4	9	8	7
M2	4	3	2	5	1	4	3
M3	6	7	5	11	5	6	12

Solution-The Time deviation table for the given time duration table for jobs on M1, M2 and M3 is follows.

Jobs/Machines	J1	J2	J3	J4	J5	J6	J7
M1	(0, 0)	(5, 5)	(4, 5)	(1, 0)	(6, 8)	(5, 4)	(4, 4)
M2	(3, 1)	(2, 0)	(1, 0)	(4, 1)	(0, 0)	(3, 0)	(2, 0)
M3	(1, 3)	(2, 4)	(0, 3)	(6, 7)	(0, 4)	(1, 2)	(7, 9)

The job sequencing for the above Time deviation table is

J1						J5

The reduced Time duration table for the remaining job sequence follows

Jobs/Machines	J2	J3	J4	J6	J7
M1	8	7	4	8	7
M2	3	2	5	4	3
M3	7	5	11	6	12

The Time deviation table for the reduced time duration table follows

Jobs/Machines	J2	J3	J4	J6	J7
M1	(4, 5)	(3, 5)	(0, 0)	(4, 4)	(3, 4)
M2	(1, 0)	(0, 0)	(3, 1)	(2, 0)	(1, 0)
M3	(2, 4)	(0, 3)	(6, 7)	(1, 2)	(7, 9)

The job sequencing for the above time deviation table is

J1	J4				J3	J5

The reduced Time duration table for the remaining job sequence follows

Jobs/Machines	J2	J6	J7
M1	8	8	7
M2	3	4	3
M3	7	6	12

The Time deviation table for the reduced Time duration table follows

Jobs/Machines	J2	J6	J7
M1	(1, 5)	(1, 4)	(0, 4)
M2	(0, 0)	sy (1, 0)	(0, 0)
M3	(1, 4)	(0, 2)	(6, 9)

The job sequencing for the above Time deviation table is

J1	J4	J7	J6	J2	J3	J5

which is the required sequence for this sequencing problem. By using this sequence, we apply the usual procedure to get the minimum total elapsed time is 59 hours and the idle time on the machines M1, M2 and M3 are 13hrs, 37 hrs, 7 hrs respectively.

C. Time Deviation Vector Method For Processing Of N Jobs Through M Machines

Let there be n jobs, each of which is to be processed through m machines, say M1, M2, M3, … Mm in the order M1- M2- M3-….– Mm. Let t_{ij} (i =1,2,3, … m, j = 1, 2, …,n) be the time required for processing i th job on the j th machine.

Jobs/ Machines	J1	J2	J3	………	Jn
M1	t_{11}	t_{12}	t_{13}	……..	t_1n
M2	t_{21}	t_{22}	t_{23}	…….	t_2n
M3	t_{31}	t_{32}	t_{33}	…….	t_3n
……	………	…..	……..	…….	……
Mn	t_{m1}	t_{m2}	t_{m3}	……	t_{mn}

i. Heuristics Algorithm to obtain sequence of jobs in n jobs m machines problems

1) Convert the m machines problems in to two

machines problem by checking the either of the conditions (i) Minimum of M1 ≥ Maximum of (M2, M3,…. Mm-1),

2) (Minimum of Mm ≥ Maximum of (M2, M3,…. Mm-1) or both holds and by using the procedure already exists.
3) Apply the time deviation vector algorithm for the reduced two machines job sequencing problem obtained from step (i) and get the required job sequence.

ii. Problem involving processing n jobs through m machine

Example-Determine (an optimal or very near to optimal) sequence for the following job sequencing problem. There are four jobs, each of which must go through five machines in the prescribed order(passing is not allowed) processing times in hours are given below.

Jobs/Machines	J1	J2	J3	J4
M1	7	6	5	8
M2	5	6	4	3
M3	2	4	5	3
M4	3	5	6	2
M5	9	10	8	6

Solution -Convert the five machines problems in to two machines problems.

Jobs/Machines	J1	J2	J3	J4
G	17	21	20	16
H	19	25	23	14

The Time deviation table for the converted time duration table for jobs on G and H is follows.

Jobs/Machines	J1	J2	J3	J4
G	(1, 0)	(5, 0)	(4, 0)	(0, 2)
H	(5, 2)	(11, 4)	(9, 3)	(0, 0)

The job sequencing for the above time deviation table is

			J4

The reduced Time duration table for the remaining job sequence follows

Jobs/Machines	J1	J2	J3
G	17	21	20
H	19	25	23

The Time deviation table for the reduced Time duration table for jobs on G and H is follows.

Jobs/Machines	J1	J2	J3
G	(0, 0)	(4, 0)	(3, 0)
H	(0, 2)	(6, 4)	(4, 3)

The job sequencing for the above Time deviation table is

J1			J4

The reduced Time duration table for the remaining job sequence follows

Jobs/Machines	J2	J3
G	21	20

H	25	23

The Time deviation table for the reduced Time duration table for jobs on G and H is follows.

Jobs/Machines	J2	J3
G	(1, 0)	(0, 0)
H	(2, 4)	(0, 3)

The required sequence for the given job sequencing problem is

J1	J3	J2	J4

IV. CONCLUSION

In this paper, we have introduced a new heuristics method called Time deviation method, to frame a sequence of n jobs two machines and n jobs three machines for solving job sequencing problem and in near future we extend the new heuristics method to determine an optimal sequence of n jobs through two and more than two machines without using existing conditions.

V. REFERENCES

1) Christos koulamas, GeorgeJ.kyparisis, "Single machine and two machine flow shop scheduling with general functions", European Journal of Operation Research Vol. 178, pp. 402-407, 2007.
2) Kalczynski.P.J., Kamburowski.J., "A heuristic for minimizing the expected makespan in two machine flow shops with consistent co-efficient of variation ", European Journal of Operational Research, Vol. 169, pp. 742 – 750, 2006..
3) Zhao.Y.F., Tang. L.X., "Scheduling a single continuous batch processing machine to minimize total completion time ", Tien Tzu Hsuch pao/ Acta Electronica Sinica, Vol. 36, pp. 367 – 370, 2008.
4) Pan.Y., Shi.L., Zhang.H., " A new solution approach to the general min – max sequencing problem ", Proceedings of the World Class on Intelligent Control and Automation(WCICA), Vol. 1., pp. 1359 – 1364, 2006.
5) Corlier. J., Pinston. E., "A practical use of Jackson's preemptive scheduling for solving the job shop problem", Annals of Operation Research, Vol. 26, pp.269 – 287, 1990.

job shop problem", Annals of Operation Research, Vol. 26, pp.269 – 287, 1990.

6) Elmaghraby, S.E., Thoney.K.A., " The two machine stochastic flow shop problem with arbitrary processing time distributions", IIE Transactions, Vol.31, pp.467 – 477, 1999.
7) Cai. X., Zhou. S., "Sequencing jobs with random processing times to minimize weighted completion time variance", Annals of Operation Research, Vol. 70, pp. 241 – 260, 1997.
8) Levin.V.I., Miretskii I.Yu, "Optimal job scheduling for flow shops ", Automation and Remark control, Vol. 57, pp. 773 – 793, 1996.
9) Srinivas. J., Subbaiah.K.V., Chandramouli K.V.V., "An optimal sequencing approach for job – shop production ", Journal of Scientific and Industrial Research, Vol.63, pp. 458 – 461, 2004
10) George Steiner, Paul Stephenson, "Pareto optima for total weighted completion time and maximum lateness on a single machine", Discrete Applied Mathematics, Vol. 155, pp. 2341 – 2354, 2007.
11) Drobouchevitch.I.G., Strusevich.V.A, "A heuristics algorithm for two machines reentrant shop scheduling ", Annals of operations research, Vol. 86, pp. 417 – 439, 1999.
12) Cai. X., Lum.V.Y.S, Chan.J.M.T., "Scheduling about a common due date with job dependent asymmetric earliness and tardiness penalties ", European Journal of Operational Research, Vol. 98, pp. 154 – 168, 1997.
13) Vachajitpan. P., "A computer program for solving job sequencing problem", Computers and Industrial Engineering, Vol. 7, pp. 173 – 185, 1983.

T*-Semi Generalized Continuous Maps In Topological Spaces

GJSFR Classification – F (FOR)
010112,010108,010111

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Abstract- **In this paper, we introduce new class of maps called τ*-Semi Generalized Continuous maps in topological spaces and study its relationship with some existing maps. 2000 Mathematics Subject Classification: 54A05.**

Keywords -τ*-sg-continuous maps

I. Introduction

In 1970, Levine[7] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Using generalized closed sets, Dunham [4] introduced the concept of the closure operator cl^* and a topology τ^* and studied some of its properties. Pushpalatha, Eswaran and Rajarubi [10] introduced and studied τ^*-generalized closed sets and τ^*-generalized open sets. Using τ^*-generalized closed sets, Eswaran and Pushpalatha [5] introduced and studied τ^*-generalized continuous maps. Several authors have introduced and studied various maps in topological spaces.

The purpose of this paper is to introduce and study the concept of a new class of maps, namely τ*-semigeneralized continuous maps.

Throughout this paper X and Y are topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A of a topological space X, cl(A), $cl^*(A)$, scl(A), spcl(A), $cl_\alpha(A)$ and A^c denote the closure, closure*, semi-closure, semi-preclosure, α-closure and complement of A respectively.

II. Preliminaries

We recall the following definitions:

Definition 2.1- For the subset A of a topological X, the generalized closure operator cl*[4] is defined by the intersection of all g-closed sets containing a.

Definition 2.2- For the subset A of a topological X, the topology τ* [4] is defined by $\tau^* = \{G : cl^*(G^C) = G^C\}$

Definition 2.3-A subset A of a topological space X is called τ^*-generalized closed set (briefly τ^*-g-closed) [10] if $cl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is τ^*-open. The complement of τ^*-generalized closed set is called the τ^*-generalized open set (briefly τ^*-g-open).

Definition 2.4-The τ* generalized closure operator cl_τ^* [10] for a subset A of a topological space (X, τ*) is defined by the intersection of all τ*-g-closed set containing A. That is $cl_\tau^*(A) = \cap\{G : A \subseteq G \text{ and } G \text{ is } \tau^*\text{-g-closed}\}$

Definition 2.5- A map $f : X \to Y$ from a topological space X into a topological space Y is called :

1) continuous if the inverse image of every closed set (or open set) in Y is closed (or open) in X.
2) generalized continuous[2](g-continuous) if the inverse image of every closed set in Y is g-closed in X.
3) strongly sg-continuous [12] if the inverse image of each sg-open set of Y is open in X.
4) strongly gs-continuous [12] if the inverse image of each gs-open set of Y is open in X.
5) semi continuous [13] if the inverse image of each closed set of Y is semi-closed in X.
6) sg-continuous [11] if the inverse image of each closed set of Y is sg-closed in X.
7) gs-continuous [14] if the inverse image of each closed set of Y is gs-closed in X.
8) gsp-continuous [3] if the inverse image of each closed set of Y is gsp-closed in X.
9) αg- continuous [6] if the inverse image of each closed set of Y is αg-closed in X.
10) pre-continuous [8] if the inverse image of each open set of Y is pre-open in X.
11) α- continuous [9] if the inverse image of each open set of Y is α-open in X.
12) sp-continuous [1] if the inverse image of each open set of Y is semipreopen in X.
13) weakly sg-continuous [12] if the inverse image of each sg-open set of Y is semiopen in X.
14) weakly gs-continuous [12] if the inverse image of each gs-open set of Y is semiopen in X.
15) sg*-continuous [12] if the inverse image of each semiopen set of Y is sg-open set in X
16) gs*-continuous [12] if the inverse image of each semiopen set of Y is gs-open set in X.
17) τ*-g-continuous [5] if the inverse image of each g-closed set in Y is τ*-g-closed in X.

Remark 2.6- In [10] it has been proved in Theorem 3.2 that every closed set is τ*-g-closed.

Remark 2.7-In [10] it has been proved in Theorem 3.4 that every g-closed set is τ*-g-closed.

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III. SEMI GENERALIZED CONTINUOUS MAPS IN TOPOLOGICAL SPACES

S.Eswaran and A.Pushpalatha [5] introduced and studied a map called τ*-generalized continuous map in topological space. In this section, we introduce a new class of map namely τ*-semi generalized continuous map in topological spaces and study some of its properties and relationship with some existing mappings.

Definition 3.1-A map $f : X \to Y$ from a topological space X into a topological space Y is called τ*-semi generalized continuous map (briefly τ*-sg-continuous) if the inverse image of every sg-closed set in Y is τ*-g-closed in X.

Theore 3.2.-Let $f : X \to Y$ be a map from a topological space (X, τ*) into a topological space (Y, σ*).

i. The following statements are equivalent:
   a. f is τ*-sg-continuous.
   b. The inverse image of each sg-open set in Y is τ*-g-open in X.

ii. If $f : X \to Y$ is τ*-sg-continuous, then $f(cl_\tau^*(A)) \subseteq cl(f(A))$ for every subset A of X.

Proof -1- Assume that $f : X \to Y$ is τ*-sg-continuous. Let G be sg-open in Y. Then G^C is sg-closed in Y. Since f is τ*-sg-continuous, $f^{-1}(G^C)$ is τ*-g-closed in X. But $f^{-1}(G^C) = X - f^{-1}(G)$. Thus $X - f^{-1}(G)$ is τ*-g-closed in X and so $f^{-1}(G)$ is τ*-g-open in X. Therefore (a) ⇒ (b).
Conversely assume that the inverse image of each sg-open set in Y is τ*-g-open in X. Let F be any sg-closed set in Y. Then F^C is sg-open in Y. By assumption, $f^{-1}(F^C)$ is τ*-g-open in X. But $f^{-1}(F^C) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is τ*-g-open in X and so $f^{-1}(F)$ is τ*-g-closed in X. Therefore f is τ*-sg-continuous. Hence (b) ⇒ (c). Thus (a) and (b) are equivalent.
2-Assume that f is τ*-sg-continuous. Let A be any subset of X. Then cl(f(A)) is sg-closed set in Y. Since f is τ*-sg-continuous, $f^{-1}(cl(f(A)))$ is τ*-g-closed in X and it contains A. But $cl_\tau^*(A)$ is the intersection of all τ*-g-closed set containing A. Therefore $cl_\tau^*(A) \subseteq f^{-1}(cl(f(A)))$ and so $f(cl_\tau^*(A)) \subseteq cl((f(A))$.

Theorem 3.3-If a map $f : X \to Y$ from a topological space X into a topological space Y is continuous then it is τ^*-sg-continuous but not conversely.

Proof -Let $f : X \to Y$ be continuous. Let V be a closed set in Y. Since *f* is continuous, $f^{-1}(V)$ is closed in X. Since every closed set is sg-closed , Y is sg-closed. Also by Remark 2.6, $f^{-1}(V)$ is τ^*-g-closed. Thus, f is τ^*-sg-continuous.
The converse of the theorem need not be true as seen from the following example.

Example 3.4-Let X=Y= {a, b, c}, τ = {X, ϕ, {a,}, {b}, {a, b}, {a, c}}and σ = {Y, ϕ, {a}, {b, c}}. Let $f : X \to Y$ be an identity map. Then f is τ*-sg-continuous. But f is not continuous. Since for the closed set F = {c} in Y, $f^{-1}(F)$ = {c} is not closed in X.

Theorem 3.5.-If a map $f : X \to Y$ from a topological space X into a topological space Y is g-continuous then it is τ^*-sg-continuous but not conversely.

Proof -Let $f : X \to Y$ be g-continuous. Let V be a closed set in Y. Since f is g-continuous, $f^{-1}(V)$ is g-closed in X. Since every closed set is sg-closed, V is sg-closed. Also by Remark 2.7, $f^{-1}(V)$ is τ^*-g-closed. Thus, f *is* τ^*-sg-continuous.
Converse of the theorem need not be true as seen from the following example.

Example 3.6-Let X=Y= {a, b, c}, τ = {X, ϕ, {a,}, {b}, {a, b}, {b, c}}and σ = {Y, ϕ, {c}}. Let $f : X \to Y$ be an identity map. Then f is τ*-sg-continuous. But f is not g-continuous. Since for the closed set F = {a, b} in Y, $f^{-1}(F)$ = {a, b} is not g-closed in X.

Theorem 3.7-If a map $f : X \to Y$ from a topological space X into a topological space Y is strongly sg-continuous then it is τ^*-sg-continuous but not conversely.

Proof -Let $f : X \to Y$ be strongly sg-continuous. Let F be a sg-closed set in Y. Then F^C is sg-open in Y. Since f is strongly sg-continuous, $f^{-1}(F^C)$ is open in X. But $f^{-1}(F^C) = X - f^{-1}(F)$. Therefore $f^{-1}(F)$ is closed in X. By Remark 2.6, $f^{-1}(F)$ is τ*-g-closed in X. Thus, f is τ*-sg-continuous.
Converse of the theorem need not be true as seen from the following example.

Example 3.8.-Let X=Y= {a, b, c}, τ = {X, ϕ, {c}}, σ = {Y, ϕ, {a,}, {b}, {a, b}, {a, c}}. Let $f : X \to Y$ be an identity map. Then f is τ*-sg-continuous. But f is not strongly sg-continuous. Since for the sg-open set V = {a, b} in Y, $f^{-1}(V)$ = {a, b} is not open in X.

Theorem 3.9.-If a map $f : X \to Y$ from a topological space X into a topological space Y is strongly gs-continuous then it is τ^*-sg-continuous but not conversely.

Proof - Let $f : X \to Y$ be strongly gs-continuous. Let V be a sg-closed set in Y. Since every sg-closed set is gs-closed, we have V is gs-closed set in Y. Since f is strongly gs-continuous, $f^{-1}(V^C)$ is open in X. But $f^{-1}(V^C) = X - f^{-1}(V)$. Therefore $f^{-1}(V)$ is closed in X. By Remark 2.6, $f^{-1}(V)$ is τ*-g-closed in X. Thus, f is τ*-sg-continuous.
Converse of the theorem need not be true as seen from the following example.

Example 3.10-Let X=Y= {a, b, c}, τ = {X, ϕ, {a}, {b, c}} σ = {Y, ϕ, {a,}, {a, b}, {a, c}}. Let $f : X \to Y$ be an identity map. Then f is τ*-sg-continuous. But f is not strongly gs-continuous. Since for the gs-open set V = {a, b} in Y, $f^{-1}(V)$ = {a, b} is not open in X.

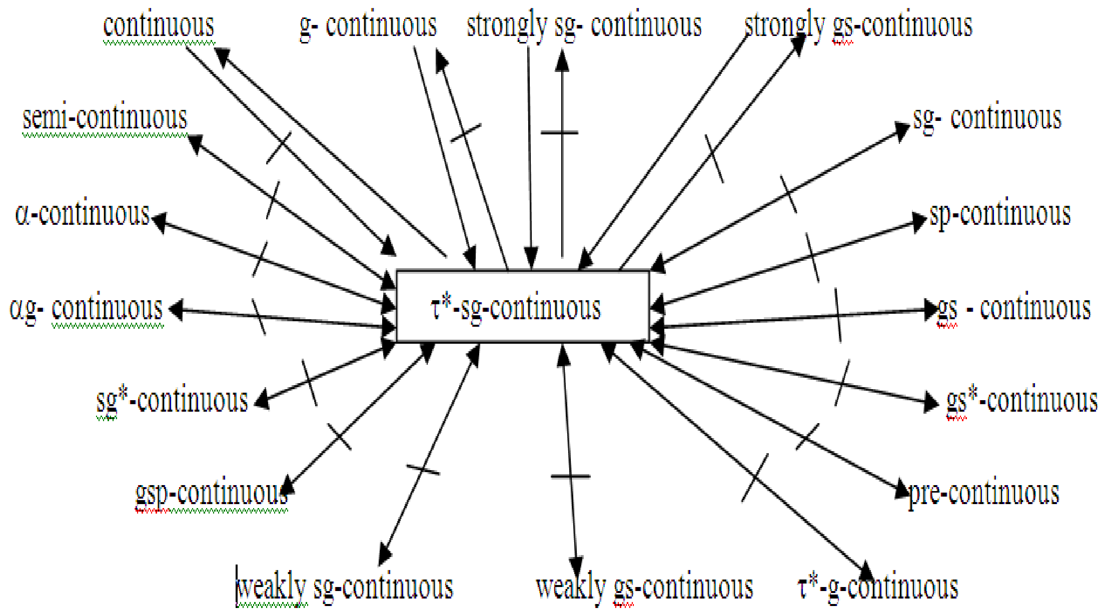
Remark 3.11-Following example shows that τ*-sg-continuous map is independent from the following mappings.

Let X = Y = {a, b, c}. Let $f : X \rightarrow Y$ be an identity map.

1) Let τ = {X, ϕ, {a}, {a, c}} and σ = {Y, ϕ, {a}, {a, b}, {a, c}}. Then f is semi-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {c} in Y, $f^{-1}(V)$ = {c} is not τ*-g-closed in X.
2) Let τ = {X, ϕ, {c}} and σ = {Y, ϕ, {b}, {c}, {b, c}}. Then f is τ*-sg-continuous But it is not semi-continuous. Since for the closed set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not semi-closed in X.
3) Let τ = {X, ϕ, {a}, {b}, {a, b}, {a, c}} and σ = {Y, ϕ, {a}, {a, c}}. Then f is sg-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a, b} in Y, $f^{-1}(V)$ = {a, b} is not τ*-g-closed in X.
4) Let τ = {X, ϕ, {c}} and σ = {Y, ϕ, {b}, {c}, {b, c}}. Then f is τ*-sg-continuous But it is not sg-continuous. Since for the closed set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not sg-closed in X.
5) Let τ = {X, ϕ, {a, b}} and σ ={Y, ϕ, {a}, {b}, {a, b}}. Then f is gs-continuous. But it is not τ*-sg-continuous. Since for the gs-closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not τ*-g-closed in X.
6) Let τ = {X, ϕ, {a}}, σ ={Y, ϕ, {b}, {c}, {b, c}, {a, c}} Then f is τ*-sg-continuous But it is not gs-continuous. Since for the closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not gs-closed in X.
7) Let τ = {X, ϕ, {a, c}} and σ ={Y, ϕ, {a}, {c}, {a, c}, {a, b}}. Then f is gsp-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {c} in Y, $f^{-1}(V)$ = {c} is not τ*-g-closed in X.
8) Let τ = {X, ϕ, {c}}, σ ={Y, ϕ, {a}, {b}, {a, b}, {b, c}}. Then f is τ*-sg-continuous But it is not gsp-continuous. Since for the closed set V = {c} in Y, $f^{-1}(V)$ = {c} is not gsp-closed in X.
9) Let τ = {X, ϕ, {a, b}} and σ ={Y, ϕ, {b}, {a, b}}. Then f is αg-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not τ*-g-closed in X.
10) Let τ = {X, ϕ, {b}}, σ ={Y, ϕ, {c}, {a, c}}. Then f is τ*-sg-continuous But it is not αg -continuous. Since for the closed set V = {b} in Y, $f^{-1}(V)$ = {b} is not αg-closed in X.
11) Let τ = {X, ϕ, {a, b}} and σ ={Y, ϕ, {a}, {b}, {a, b}}. Then f is pre-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not τ*-g-closed in X.
12) Let τ = {X, ϕ, {a}}, σ ={Y, ϕ, {c}, {a, c}, {b, c}}. Then f is τ*-sg-continuous But it is not pre-continuous. Since for the open set V = {b, c} in Y, $f^{-1}(V)$ = {b, c} is not pre-open in X.
13) Let τ = {X, ϕ, {c}, {b, c}} and σ ={Y, ϕ, {c}, {a, c}, {b, c}}. Then f is α-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {b} in Y, $f^{-1}(V)$ = {b} is not τ*-g-closed in X.
14) Let τ = {X, ϕ, {a}, {b, c}}, σ ={Y, ϕ, {a}, {c}, {a, c}, {b, c}}.. Then f is τ*-sg-continuous But it is not α-continuous. Since for the open set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not α-open in X.
15) Let τ = {X, ϕ, {a}, {a, c}} and σ ={Y, ϕ, {a}, {a, b}, {a, c}}. Then f is sp-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {c} in Y, $f^{-1}(V)$ = {c} is not τ*-g-closed in X.
16) Let τ = {X, ϕ, {c}}, σ ={Y, ϕ, {b}, {c}, {b, c}, {a, c}}. Then f is τ*-sg-continuous. But it is not sp-continuous. Since for the open set V = {b} in Y, $f^{-1}(V)$ = {b} is not sp-open in X.
17) Let τ = {X, ϕ, {c}, {b, c}} and σ ={Y, ϕ, {c}, {a, c}, {b, c}}. Then f is weakly sg-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {b} in Y, $f^{-1}(V)$ = {b} is not τ*-g-closed in X.
18) Let τ = {X, ϕ, {a}, {b, c}} and σ ={Y, ϕ, {c}, {b, c}}. Then f is τ*-sg-continuous. But it is not weakly sg-continuous. Since for the sg-open set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not semiopen in X.
19) Let τ = {X, ϕ, {c}, {a, c}} and σ ={Y, ϕ, {c}, {a, c}, {b, c}}.Then f is weakly gs-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not τ*-g-closed in X.
20) Let τ = {X, ϕ, {a}, {b, c}} and σ ={Y, ϕ, {c}, {a, c}, {b, c}}. Then f is τ*-sg-continuous. But it is not weakly gs-continuous. Since for the gs-open set V = {c} in Y, $f^{-1}(V)$ = {c} is not semiopen in X.
21) Let τ = {X, ϕ, {c}, {a, c}} and σ ={Y, ϕ, {c}, {a, c}, {b, c}}. Then f is sg*-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not τ*-g-closed in X.
22) Let τ = {X, ϕ, {b}} and σ ={Y, ϕ, {a}, {a, b}, {a, c}}. Then f is τ*-sg-continuous. But it is not sg*-continuous. Since for the semiopen set V = {a} in Y, $f^{-1}(V)$ = {a} is not sg-open in X.
23) Let τ = {X, ϕ, {a}, {a, b}, {a, c}} and σ ={Y, ϕ,{a, b}}. Then f is gs*-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not τ*-g-closed in X.
24) Let τ = {X, ϕ, {b}, {a, b}} and σ ={Y, ϕ, {a}}. Then f is τ*-sg-continuous. But it is not gs*-continuous. Since for the semiopen set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not gs-open in X.
25) Let τ = {X, ϕ, {a}, {a, b}} and σ ={Y, ϕ, {a}, {c}, {a, c}}. Let $f : X \rightarrow Y$ be defined by f(a) = c, f(b) = a and f(c) = b . Then f is τ*-g-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a} in Y, $f^{-1}(V)$ = {b} is not τ*-g-closed in X.

26) Let τ = {X, ϕ, {b}, {c}, {b, c}, {a, b}} and σ ={Y, ϕ, {b}, {a, b}}. Then f is τ*-sg-continuous. But it is not τ*-g-continuous. Since for the g-closed set V = {b, c} in Y, $f^{-1}(V)$ = {b, c} is not τ*g-closed in X.

Remark 3.12-From the above discussion, we obtain the following implications.



A ⟶ B means A implies B, A ⟶| B means A does not imply B and A ⟷| B means A and B are independent

IV. References

1) M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud, β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ.12(1)(1983),77-90.
2) K.Balachandran, P.Sundaram and J.Maki, On generalized continuous maps intopological spaces, Mem. Fac. Sci. Kochi Univ. (Math.) 12 (1991), 5-13. Monthly, 70(1963) ,36-41.
3) J.Dontchev, On generalizing semipreopen sets, Mem. Fac. Sci. Kochi Uni. Ser A, Math., 16 (1995), 35-48.
4) W.Dunham, A new closure operator for non-T1 topologies, Kyungpook Math.J. 22 (1982), 55-60
5) S.Eswaran and A.Pushpalatha,τ*-generalized continuous maps in topological spaces, International J. of Math Sci & Engg. Appls. (IJMSEA) ISSN 0973-9424 Vol. 3, No. IV, (2009), pp. 67 – 76
6) Y.Gnanambal, On generalized preregular sets in topological spaces, Indian J Pure Appl. Math.(28)3(1997),351-360.
7) N.Levine , Generalized closed sets in topology, Rend.Circ. Mat.Palermo, 19, (2) (1970), 89-96.
8) A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb , On precontinuous and weak precontinuous functions,Proc. Math. Phys. Soc.Egypt 53(1982),47-53.
9) A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb, On α-continuous and α-open mappings, Acta. Math.Hunga.41 (1983), 213-218.
10) A.Pushpalatha, S.Eswaran and P.Rajarubi, τ*-generalized closed sets in topological spaces, Proceedings of World Congress on Engineering 2009 Vol II WCE 2009, July 1 – 3,2009, London, U.K., 1115 – 1117
11) P.Sundaram, H.Maki and K.Balachandran, sg-closed sets and semi-$T_{1/2}$spaces, Bull.Fukuoka Univ.Ed.Part III,40(1991),33-40.
12) A note on gs-closed sets and sg-closed sets Unpublished.
13) Semi open sets and semicontinuity in topological spaces, Amer. Math.,
14) Semigeneralized closed and generalized closed maps, Mem.Fac.Sci.Kochi.

Hexavalent Chromium Removal By MANDELIC ACID In The Presence And Absence Of Surfactants

Jayashree Mandal, Ankita Basu, Rumpa Saha, Sumanta Kr Ghosh and Bidyut Saha*

GJSFR Classification – B (FOR) 030405,030607,030603

Abstract- **The chromic acid oxidations of dl-mandelic acid (MA) in the presence and absence of bipyridine (bpy) as catalyst have been studied in aqueous micellar media under the kinetic condition, [dl-mandelic acid]$_T$ >> [Cr(VI)]$_T$ at various temperatures. The unanalyzed path is first order with respect to [H^+], [dl-mandelic acid]$_T$ and [Cr(VI)]$_T$. The bpy catalyzed path gives zero order dependency on [H^+]. This path also shows a first order dependence on [bpy]$_T$. $HCrO_4^-$ has been found to be kinetically active in the absence of bpy while in the bpy catalyzed path, a Cr(VI)-bpy complex was considered to be the active oxidant. In this path the Cr(VI)-bpy complex undergoes a nucleophilic attack by the mandelic acid to form a ternary complex which subsequently experiences a redox decomposition involving 3e transfer leading to benzoyl radical, CO_2 and corresponding Cr(VI)-bpy complex. Benzoyl radical is then oxidized to form benzoic acid. In the unanalyzed path, the Cr(VI)-substrate ester undergoes an acid-catalyzed redox decomposition through 3e-transfer as the rate-determining step. It is striking to note that unanalyzed paths show a first order dependence on [H^+] and catalysed path shows zero order dependence on [H^+] and boh paths also show a first order dependence on [dl-mandelic acid]$_T$ and [Cr(VI)]$_T$.The bpy catalyzed path is first order in [bpy]$_T$. All these patterns remain unaltered in the presence of externally added surfactants. The effects of an anionic surfactant, sodium dodecyl sulphate (SDS), on both the unanalyzed and catalyzed paths were studied.**

Keywords- kinetics, dl-mandelic acid, chromium(VI), bipyridine, sds.

I. Introduction

Chromium toxicity among workers in tanneries and other chromium-based industries have been known for a long time. The workers are reported to suffer from ulcers, allergic dermatitis, lung cancer, renal insufficiency, and liver necrosis. Chromium pollution in groundwater as a result of leaching of Cr (VI) from spent solid and liquid wastes at chromium-based industries has currently become a serious problem at the global level [1]. The increased usage of chromium has led to great growth in chromium-based industries. This growth, however, has severely affected the ecoenvironment on one hand and has reduced the forest cover considerably on the other. Hexavalent chromium is the principal species in surface waters and aerobic soils. It forms a number of stable oxyacids and anions, including $HCrO_4^-$ (hydrochromate), $Cr_2O_7^{2-}$ (dichromate), and CrO_4^{2-} (chromate). The chromate ion has a large ionic potential and tetrahedral coordination and is both a strong acid and an oxidizing agent. Cr (VI) is not readily adsorbed to surfaces. Most of Cr (VI) salts are soluble in water, and hexavalent chromium is very mobile. Cr (VI) has a long residence time in surface water and groundwater. The high oxidizing potential, high solubility and ease of permeation of biological membranes make Cr (VI) more toxic than Cr (III) [1]. Various methods used for removal of Cr(VI) ions include chemical reduction and precipitation, reverse osmosis, ion exchange and adsorption on activated carbon or similar material [2]. But all these methods suffer from severe constraints, such as incomplete metal removal, high reagent or energy requirements, generation of toxic sludge or other waste products that require safe disposal. Some of the treatment methods involve high operating and maintenance cost. The high cost of the chemical reagents and the problems of secondary pollution also make the above physico-chemical methods rather limited in application. There is, therefore, a need for some alternative technique, which is efficient and cost-effective. The process of heavy metal removal by biological materials is known as biosorption and the biological materials used are called biosorbents. Various biosorbents like bacteria, fungi, yeasts, agricultural byproducts, industrial wastes, etc have been used for biosorption. In this regard, considerable attention has been focused in recent years upon the field of biosorption for the removal of heavy metal ions from aqueous solutions [3]. Recently Park et al established that for chromium(VI) biosorption, chromium(VI) is first reduced to chromium(III) and then it is adsorbed as chromium(III) in the biosorbent [4]. The effective biosorbents contain hydroxyl & carboxyl groups [5]. Understanding of mechanism of chromium(VI) reduction to chromium(III) by some compound containing hydroxyl & carboxyl group is important in this context. In this respect mandelic acid is ideal one. The present investigations have been carried out in micro-heterogeneous systems to substantiate the proposed reaction mechanism as we did in many cases [6-12].

Bipyridine (bpy) is an efficient chelating agent to catalyse Cr(VI) oxidation reactions. Because of the structural similarity between 2,2'- bipyridine and picolinic acid that shows an extraordinary catalytic effect on chromic acid oxidation, the catalytic effect of 2,2'- bipyridine on these oxidations is of interest. Picolinic acid (PA) and bipyridine (bpy) are never co-oxidised along with the substrate [13-17]. It was reported by Rocek et al. [18,19] that the uncatalysed chromic acid oxidation of mandelic acid proceeds through a three electron transfer at the rate determining step, involving the simultaneous rupture of C—C and C—H bonds within a

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cyclic transition state. Among the α-hydroxy acids, mandelic acid is especially interesting because, in chromic acid oxidation, it experiences a 3e-transfer as the rate-determining step while most other α-hydroxy acids undergo 2e-transfer [20-24] under comparable conditions. A 2e-transfer process in the chromic acid oxidation of mandelic acid was proposed by Panigrahi et al. [25] Thus in the literature, there is a confusion regarding the behaviour of mandelic acid in chromic acid oxidation. We carry out a detailed investigation under different conditions, namely the uncatalysed reaction and catalysed reaction. The effects of anionic micelles on both the catalysed and bpy-catalysed reactions have been studied to substantiate the proposed reaction mechanism.

II. Experimental and Methods

A. *Materials And Reagents*

dl-mandelic acid (AR, SRL), $K_2Cr_2O_7$ (AR, BDH),sodium dodecyl sulphate, SDS (AR, SRL) and all other chemicals used were of highest purity available commercially. The solutions were prepared in doubly distilled water.

B. *Procedure And Kinetic Measurements*

Under the kinetic conditions, solutions of the oxidant and mixtures containing the known quantities of the substrate (s) (i.e.,dl-mandelic acid)(under the conditions [S] >> $[Cr(VI)]_T$),acid and the other necessary chemicals were separately thermo stated(± 0.1ºC).The reaction was initiated by mixing the requisite amounts of the oxidant with the reaction mixture. Progress of the reaction was followed by monitoring the decay of oxidant [Cr(VI)] at 415nm at different time intervals with a UV-VIS spectrophotometer. Quartz cuvettes of path length 1cm were used. The observed pseudo-first-order rate constants [$k_{obs}(s^{-1})$] were determined from the linear part of the plots of $\ln(A_{415})$ versus time (t). Reproducible results giving first-order plots (co-relation co-efficient,r≥0.998) were obtained for each reaction run. A large excess (≥15-fold) of reductant was used in all kinetic runs. No interference due to other species at 415 nm was verified. Under the experimental conditions, the possibility of decomposition of the surfactants by Cr(VI) was investigated and the rate of decomposition in this path was found to be kinetically negligible.

C. *Product Analysis And Stoichiometry*

Product analysis was also carried out by the 2,4-dinitrophenylhydrazine (DNP) test [28]. The reaction product solution was treated with an excess of saturated Under the kinetic conditions ($[S]_T$>>$[Cr(VI)]_T$), dl-mandelic acid in both the bpy-catalysed and uncatalysed reactions yields carbon di oxide, benzaldehyde and benzoic acid and a very small amount of phenylglyoxylic acid. In the reaction mixture, benzaldehyde was detected and confirmed by a spot test [25] using the reagent solution prepared by dissolving 0.2 ml of 37% formaldehyde in 10 ml conc H_2SO_4. The product was extracted in a non-aqueous solvent like dichloromethane. One drop of the non-aqueous final test solution containing the oxidized product of dl-mandelic acid was treated with two drops of the reagent solution in a spot plate. After a few minutes, the colour of the resultant solution appeared red, which indicated the presence of benzaldehyde. The same observation was noted using an authentic sample of benzaldehyde.

Product was also analysed by the 2,4-dinitrophenylhydrazine (DNP) test [28]. The reaction product solution was treated with an excess of saturated solution of 2,4-dinitrophenylhydrazine in dil HCL medium.The precipitated 2,4-dinitrophenylhydrazone was filtered off, dried and recrystalised from ethanol. The melting point of DNP derivative was found to be somewhat less than the melting point (239ºC) of the DNP derivative of authentic benzaldehyde. The slight lowering of the melting point was due to the presence of the DNP derivative of phenylglyoxylic acid produced as a byproduct in a small amount. Thus the product analysis agreed with the reports of Rocek et al. Moreover phenylglyoxylic acid was detected and confirmed by spot tests [27] using a reagent solution subjected to esterification by treating with ethyl alcohol and conc. H_2SO_4. A drop of the esterified solution was placed in a micro test tube and evaporated to dryness in a water bath. The residue was dissolved in three drops of conc. H_2SO_4 and treated with two drops of thiophen solution. A characteristic red colour appeared within fifteen minutes. A similar observation was noted by using an sample of phenylglyoxylic acid.

In the reaction mixture, benzoic acid was detected and confirmed by a sot test [27]. The product was extracted into anonaqueous solvent like benzene. One drop of the benzene solution was treated with a colourless saturated solution of Rhodamine B and an intense pink colour appeared. This colour was intensified when shaken with an aquous solution of uranyl salts. The same identification was noted by using an authentic sample of benzoic acid.

Carbon dioxide was detected qualitatively [29] under the kinetic conditions, by purging di nitrogen through the reaction solution and passing out coming gas through a narrow tube containing $Ca(OH)_2$.

Thus the Stoichiometry of the reaction is:

$$2C_6H_5CH(OH)CO_2H + 2HCrO_4^- + 8H^+ \rightarrow C_6H_5CO_2H + C_6H_5CHO + 2CO_2 + 2Cr(III) + 7H_2O \quad (1)$$

III. Results and Discussion

A. *Dependence on $[Cr(VI)]_T$*

Under the experimental conditions, [mandelic $acid]_T$>>$[bpy]_T$>>$[Cr(VI)]_T$, the rate of disappearance of Cr(VI) shows a first -order dependence on Cr(VI) both in the presence and absence of bipyridine (bpy). This first-order dependence on Cr(VI) is also observed in the presence of surfactant SDS. From the linear plot of $\log[Cr(VI)]_T$ versus time(t) the pseudo first-order rate constants (k_{obs}) have been calculated.

B. Dependence on [bpy]T

The plots of k_{obs} versus $[bpy]_T$ are linear (r>0.99) with positive intercepts measuring the contribution of the relatively slower uncatalysed path (see fig2). The pseudo first-order rate constants ($k_{obs(u)}$) directly measured in the absence of bpy under the same conditions agree well with those obtained from the intercepts of the plots of k_{obs} versus $[bpy]_T$.

C. Dependence on $[S]_T$ i.e. $[\text{mandelic acid}]_T$

From the plot of k_{obs} versus $[\text{mandelic acid}]_T$ (see fig3), it has been established that both the catalysed and uncatalysed paths show a first-order dependence on $[S]_T$,i.e.

$$k_{obs(c)} = k_{obs(T)} - k_{obs(u)} = k_{s(c)}\,[S]_T \quad (2)$$

$$k_{obs(u)} = k_{(u)}\,[S] \quad (3)$$

The above first-order dependence on $[S]_T$ is also maintained in the presence of surfactant SDS.

D. Dependence on $[H^+]$

The acid dependence patterns for the uncatalysed and catalysed paths are the different (see fig4). From the experimental fit, the observations are:

$$k_{obs(u)} = k_{H(u)}[H^+] \quad (4)$$

$$k_{obs(c)} = k_{obs(T)} - k_{obs(u)} = k_{H(c)}[H^+]^0 \quad (5)$$

A similar dependence pattern is also observed in the presence of SDS with enhanced rate constants (see fig 6).

E. Test For Acrylonitrile Polymerisation

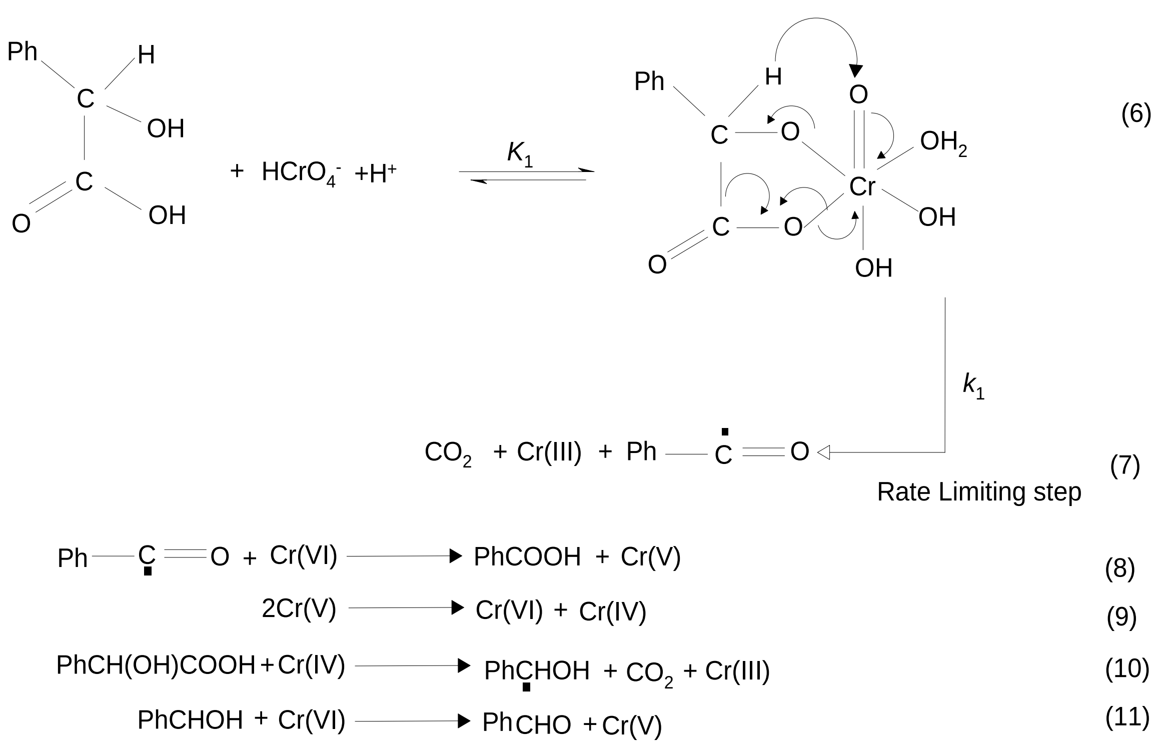
Under the experimental conditions, the polymerization of added acrylonitrile was indicated under a nitrogen atmosphere. This indicates the generation of free radicals during the course of reaction.

F. Mechanism And Interpretation

Discussion of the mechanism of the reaction can be divided in two parts: (i) uncatalysed path and (ii) catalysed path.

i. Reaction Mechanism For The Uncatalysed Chromic Acid Oxidation Of Dl-Mandelic Acid

The kinetics of the chromic acid oxidation of different α-hydroxy acids,including mandelic acid (MA), have been studied by different groups [18-24]. Of these α-hydroxy acids, mandelic acid is unique because it releases CO_2 as a product, indicating C—C bond rupture in its chromic acid oxidation.Besides the CO_2, it also produces C_6H_5CHO and C_6H_5COOH as the main products. These findings have been explained by considering a Cr(VI)—MA complex to be formed in a rapid pre-equilibrium step by the 3e-transfer in a single step within the cyclic transition state (see Scheme 1). This leads to the simultaneous rupture of C—H and C—C bond within the transition state [18,19]. Interestingly, for most of the other α-hydroxy acids, their chromic acid oxidation occurs via a 2e-transfer in the rate-determining step within the cyclic trasition state, giving rise to a keto acid [20-24].



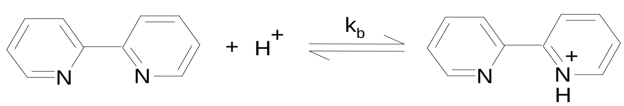
Scheme 1 Chromic acid oxidation of DL-mandelic acid.

gThe redox decomposition of the complex Cr(VI)—MA can be occurred in two parallel payhways, i.e. an acid-catalysed ($k_{H(u)}$ path) and an acid-independent path ($k_{o(u)}$ path).The acid-catalysed path is kinetically preferred. This is because the protonated complex favours electron flow towards Cr(VI) in the transition state. The 3e-transfer in a single step produces the radical C_6H_5CO which reacts subsequently with Cr(VI) in a faster step to give C_6H_5COOH and Cr(V). Mandelic acid is relatively stable [18,19] towards Cr(V) oxidation and it allows the accumulation of Cr(V) that subsequently undergoes disproportionation in a biomolecular process to generate Cr(VI) and Cr(IV). The substrate is then oxidized by Cr (IV) through C—C bond splitting to give CO_2. Here it may be pointed out that the 2e oxidation of mandelic acid without decarboxylation by Cr(IV) or Cr(V), leading to phenylglyoxylic acid under the experimental conditions, makes only a minor contribution to the total reaction.

$$C_6H_5CH(OH)COOH + Cr(IV) \rightarrow C_6H_5COCOOH + 2H^+ + Cr(VI) \quad (12)$$

ii. Reaction Mechanism For The Bpy-Assisted Chromic Acid Oxidation Of Mandelic Acid

The findings for the bpy-assisted reaction can be explained by considering the reaction mechanism outlined in scheme 2. Cr(VI)—bpy complex is the active oxidant [3,16,24, 30-32]. In the next step, the Cr(VI)—bpy complex reacts with the substrate to form ternary complexe.. This ternary complexes undergo redox decomposition through 3e-transfer within the cyclic transition state in the rate determining step involving simultaneous rupture of C—C and C—H bonds to benzoyl radical, carbon dioxide and the Cr(III)—bpy complex . Subsequently, benzoyl radical reacts in faster steps as outlined in the case of the uncatalysed path.



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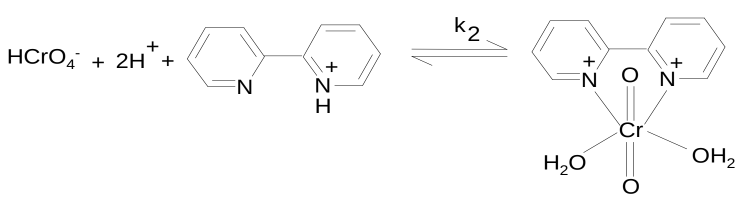

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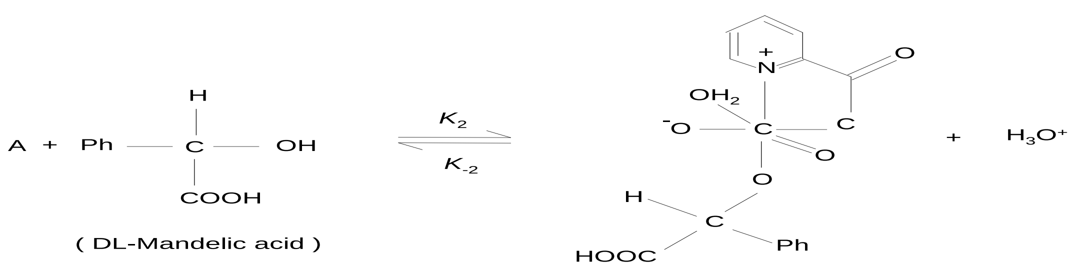

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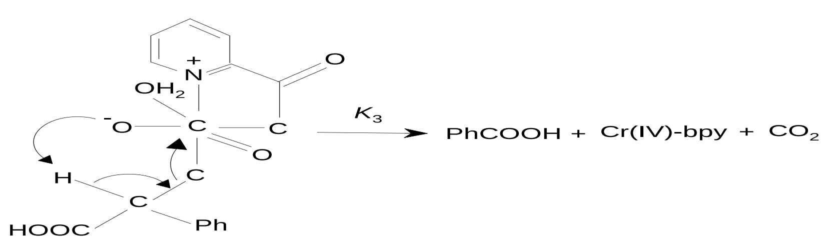

[C]

(16)

G. *Effect of SDS*

SDS (Sodium dodecyl sulphate, arepresentative anionic surfactant) accelerates both the uncatalysed and catalysed path. The rate acceleration is due to the preferential partitioning of the positively charged Cr(VI)—bpy complex by electrostatic attraction and the neutral substrate in the micellar surface (i.e. Stern layer) through H-bonding and ion-dipole interaction.Thus SDS allows the reaction to proceed in both aqueous and micellar interphases. The observed rate acceleration is due to the favoured reaction in the micellar phase where both the active oxidant and the substrate are preferentially accumulated. The plot of $k_{obs(T)}$ vs $[SDS]_T$ shows a continuous increase up to the concentration of SDS used for the bpy-catalysed reaction . An increase in $[SDS]_T$ increase the micellar counter ions (i.e. Na^+) which may displace H^+ and Ox^{+2} ions (A) out of micellar surface.

$$2Na^+_W + Ox^{2+}_M \rightleftharpoons 2Na^+_M + Ox^{2+}_W \quad (17)$$

$$Na^+_W + H^+_M \rightleftharpoons Na^+_M + H^+_W \quad (18)$$

The above equilibria lead to decrease the value of $[H_M]^+$ and $[O_xM]^{2+}$ (A) to inhibit the rate process. These two effects are opposite in nature to determine the rate of reaction. In such bpy-catalysed path, the former effect (solubilisation effect) is greater than the later effect (i.e. counter ion effect) up to the used SDS concentration.

IV. Conclusions

The Cr(VI)—bpy complex, acationic species, has been found to act as the active oxidant in the bpy-assisted path.In this path, the Cr(VI)—bpy complex experiences nucleophilic attack by the dl-mandelic acid to form a ternary complex which subsequently undergoes redox decomposition involving a three- electron transfer, leading to oxidative decarboxylation through C—C bond cleavage along with the C—H bond cleavage. Benzoyl radical, CO_2 and Cr(III)—bpy complex are produced in this rate determining step. Benzoyl radical is subsequently oxidized to benzoic acid in the fast step. The reaction have been carried out in aquous micellar media. The effect of anionic surfactant, SDS on both the catalysed and uncatalysed paths have been studied. SDS accelerates the reaction rate for both paths but in different ways.

V. Acknowledgements

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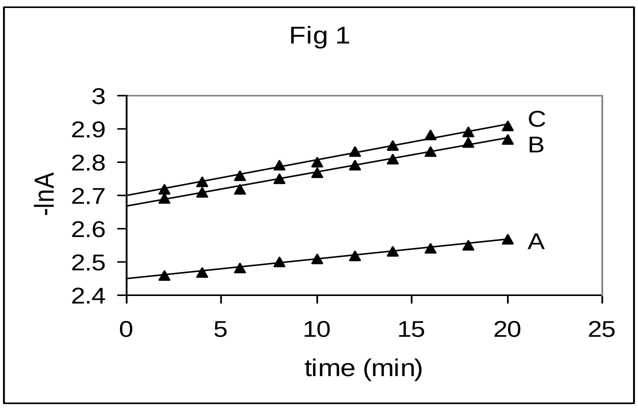
VI. References

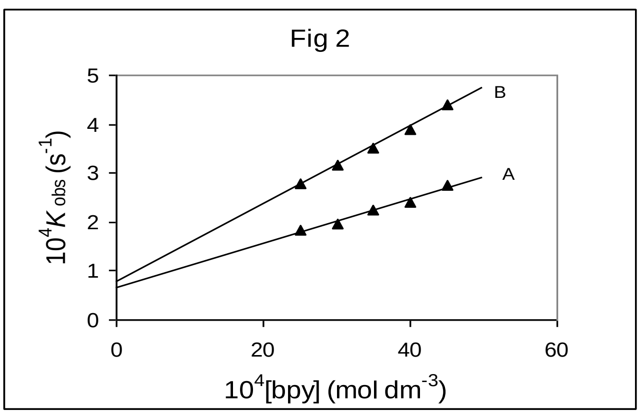
1) (a) J. D. Nriagu and E. Nieboer,(Eds) Advances in Environmental Science and Technology John milley and Sons, NY 20, 1 (1988) (b) T Sorahan, D. C Burges, L Hamilton, J. M. Harrington. Occup Environ Med., 55, 236 (1998) (c) T. F Mancuso Am J Ind Med., 31, 129 (1997). (d) S Langardt. Am J Ind Med., 17,189 (1990).
2) S. S. Ahluwalia, D. Goyal. *Bioresour. Technol*., **98**, 2243 (2007).
3) Volesky, Z. R. Holan. Biotechnol. Progress., 11, 235 (1995)
4) Park, Yeoung-Sang Yun, J. Y. Kim, J M. Park. Chem. Eng. J., 136, 173 (2008).
5) Murphy, H. Hughes, P. McLoughlin. J. Hazard. Mater., 166, 318 (2009)
6) A.K. Das, A. Roy, B. Saha, R.K. Mohanty, M. Das. J. Phys. Org. Chem., 14, 333 (2001).
7) Saha, M. Das, R.K. Mohanty, A.K. Das. J. Chin. Chem. Soc., 51, 399, (2004).
8) A.K. Das, S.K. Mondal, D. Kar, M. Das. J. Chem. Res(S)., 574, (1998).
9) A.K. Das, S.K. Mondal, D. Kar, M. Das. Int. J. Chem. Kinet., 33, 173, (2001).
10) Saha, M. Islam, A.K. Das. Inorg. React. Mech., 6, 141, (2006).
11) J. Mandal, K.M. Chowdhury, K.K. Paul, B. Saha. J. Coord. Chem. 63(1), 99, (2010).
12) K.M. Chowdhury, J. Mandal, B. Saha. J. Coord. Chem., 62, 1871 (2009).
13) P.O.I. Vitranen, R. Lindroos-Heinanen, E.Oikarinen, J. Vaskuri. J. Carbohydr. Res., 167, 29, (1987).
14) P.O.I. Vitranen, S. Pohjola, Finn. Chem. Lett., 155-158 (1984).
15) P.O.I. Vitranen, S. Kurkisno, H. Nevala, S. Pohjola., Actta Chem. Scand., Sect.A, 40, 200, (1986).
16) A.K. Das, A. Roy, B.Saha. Trans. Met. Chem., 26, 630, (2001).
17) T.Y. Lin, Y.L. Mao, C.M. Chuo. J. Chin. Chem. Soc., 38(2), 167, (1991).
18) I.P. Dominic, J. Rocek. J. Am. Chem. Soc., 44(2), 312, (1979).
19) I.P. Dominic, J. Rocek. J. Am. Chem. Soc., 101, 6311, (1979).
20) K.K. Sengupta, A.K. Chatterjee, T. Sarkar, S. Sengupta. Indian. J. Chem., 13, 1024, (1975).
21) K.K. Sengupta, A.K. Chatterjee, S.P Moulik. Bull. Chem. Soc. Jpn., 43, 3841, (1970).
22) K.K. Sengupta, A.K. Chatterjee, J.K. Chakladar. Indian. J. Chem., 10, 493, (1972).
23) P.C. Samal, B.B. Pattnaik, S.C. Rao Dharma, S.N Mahapatra. Tetrahedron, 39(1), 143, (1983).
24) B.Saha, M. Islam,. Inorg. React. Mech., 6, 141, (2006).
25) G.P. Panigrahi, S.K. Sahu, Indian. J. Chem., 35A, 660, (1996).
26) Lyons. A practical guide to data analysis for physical science students, Cambridge University Press.
27) Feigl (1956) Spot tests in organic analysis. Elsevier, New York, 5th edn. P. 135 (for

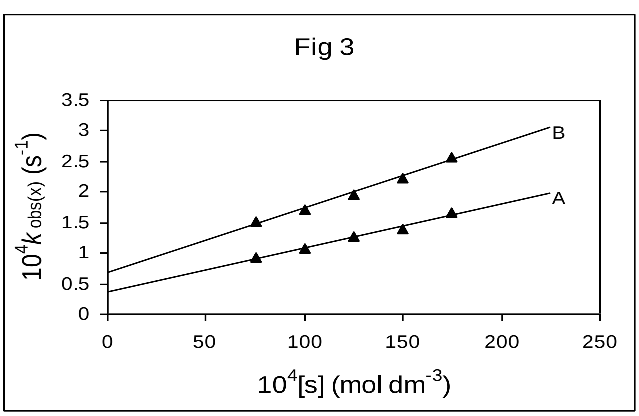
benzaldehyde); p. 204 (for phenylglyoxylic acid) p. 121 (for benzoic acid).

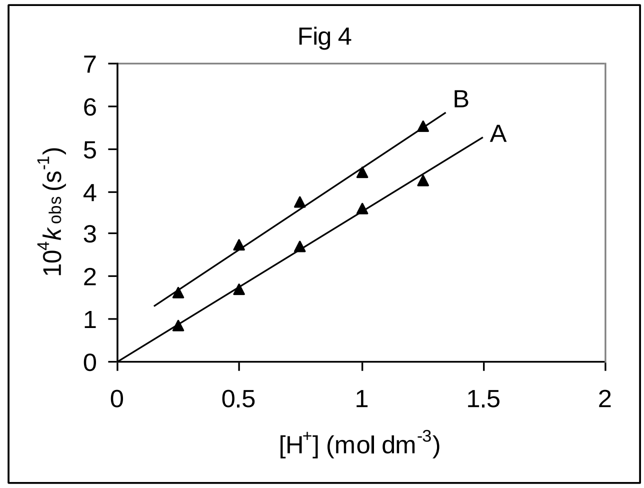
28) Vogel, in "Elementary Practical Organic Chemistry" Part-III, Quantitative Organic Analysis, ELBS and Longman Group Ltd., London, P-739(1958).
29) A.K. Das, M. Das. J. Chem. Soc. Dalton Trans.589, (1994).
30) A. K. Das, Coord. Chem. Rev., 248 , 81, (2004)[33] A. K. Das. Inorg. React. Mech., 1, 161, (1999).
31) Saha, M. Das, R.K. Mohanty, A.K. Das. J. Chin. Chem. Soc., 51, 399, (2004).
32) B.Saha, M. Das, A. K. Das. J. Chem. Res. (S)., 658, (2003).

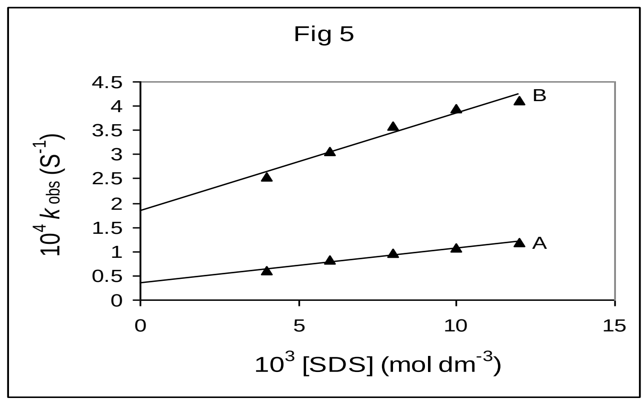
FIGURES

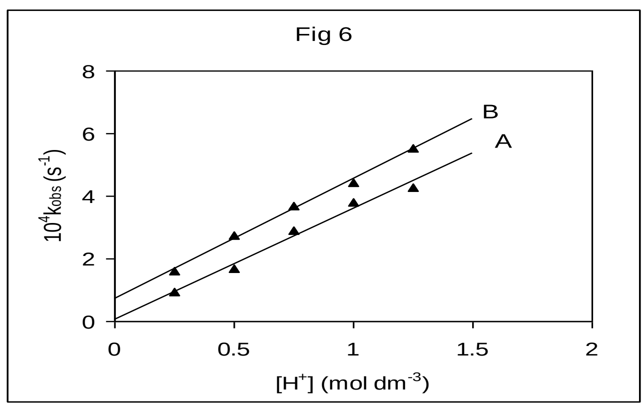












Legends to the Figures

***Fig.* 1** -Dependence of absorbance of Cr(VI) on time (min) for the oxidation of dl-mandelic acid at 25°C. $[Cr(VI)]_T$ = 5×10^{-4}mol dm^{-3}.[dl-mandelic $acid]_T$ = 75×10-4mol dm-3,[H2SO4] =.50(M). A ([bpy]T = 0 mol dm-3, $[SDS]_T$ = 0 mol dm-3),B ([bpy] = 30×10^{-4} dm^{-3}), $[SDS]_T$ = 0 mol $dm^{-3)}$, C ([bpy] = 25 mol dm^{-3} , $[SDS]_T$ = 4×10^{-3} mol dm^{-3}).

Fig. 2 -Dependence of $k_{obs(x)}$ on $[bpy]_T$for the chromium(VI) oxidation of dl-mandelic acid at 30°C. $[Cr(VI)]_T$ = 5×10^{-4}mol dm^{-3}, [H_2SO_4] = 0.5 (M), x = u or c. A 25° C, B 45° C.

Fig 3 -Dependence of k_{obs} on [dl-mandelic acid]for the chromium(VI) oxidation of dl-mandelic acid at

25°C.$[Cr(VI)]_T$ = 5×10^{-4}mol dm^{-3}, $[H_2SO_4]$ = 0.5 (M). A $[bpy]_T$ = 0 mol dm^{-3} , B $[bpy]_T$ = 25×10^{-4}mol dm^{-3} .

Fig. 4 -Dependence of $[H^+]$ on k_{obs} for the chromium(VI) oxidation of dl-mandelic acid at 25°C. A $[bpy]_T$ = 0 mol dm^{-3} , B [bpy] = 25 ×10^{-4}mol dm^{-3} . $[Cr(VI)]_T$ = 5×10^{-4}mol dm^{-3}, [dl-mandelic acid]$_T$ = 75×10^{-4}mol dm^{-3}, $[H_2SO_4]$ =0.5 mol dm^{-3}.

Fig. 5 -Dependence of $[SDS]_T$ on k_{obs} for the chromium(VI) oxidation of dl-mandelic acid at 25^0C.[Cr(VI)]= 5×10^{-4}mol dm^{-3}, [dl-mandelic acid]$_T$=75×10^{-4}mol dm^{-3}, $[H_2SO_4]$= 0.5 mol dm^{-3}. A $[bpy]_T$ = 0 mol dm^{-3} , B $[bpy]_T$ = 25 mol dm^{-3}.

Fig. 6-Dependence of $[H^+]$ on k_{obs} for the chromium(VI) oxidation of dl-mandelic acid at 25^0C. .[Cr(VI)]= 5×10^{-4}mol dm^{-3}, [dl-mandelic acid]=75×10^{-4}mol dm^{-3},$[H_2SO_4]$ = 0.5 (M) . A ([bpy] = 0 mol dm^{-3}, [SDS] = 4×10^{-3} mol dm^{-3}) , B ([bpy] = 25×10^{-4} mol dm^{-3} , [SDS] = 4×10^{-3} mol dm^{-3}).

On Generalized Nearly R-Cosymplectic Manifold

GJSFR Classification – F (FOR) 010101,010206,010111

Mohd. Nazrul Islam Khan

***Abstract-* In present paper it is shown that the generalized nearly r-cosymplectic submanifold of Kaehlar manifold is nearly r-cosymplectic manifold iff it is totally geodesic .Also , the submanifold of a Kaehlar manifold admits generalized almost r-contact pseudo-normal metric manifold.**

Keywords- Kaehlar manifold ,nearly r-cosymplectic, geodesic, pseudo-normal ,submanifold

I. INTRODUCTION

An even –dimensional differentiable manifold M on which these are defined a tensor field F of type (1,1) and a metric tensor G ,satisfying for arbitrary vector fields $\tilde{X},\tilde{Y}\varepsilon M$

$$F^2\tilde{X} = -\tilde{X} \qquad \textbf{(1.1)}$$

And

$$F(\tilde{X},\tilde{Y}) = G(F\tilde{X},\tilde{Y}) = -'F(\tilde{Y},\tilde{X}) \qquad \textbf{(1.2)}$$

is called an almost Hermite manifold with the almost Hermit structure {F,G}.Let ∇ be the Riemannian connection on M then M is said to be Kaehlar manifold [4] if

$$(\nabla_{\tilde{X}}F)(\tilde{Y}) = 0 \qquad \textbf{(1.3)}$$

Let $\overline{M}$ be an almost r-contact metric manifold with almost r-contact metric structure (ϕ,ξ,η,g) [2] that is , ϕ is a (1,1) tensor field, ξ is a vector field, η is a 1-form and g is a Riemannian metric on $\overline{M}$, such that

$$\phi^2 X = -X + \sum_{p=1}^{r}\eta^p(X)\xi_p, \qquad \textbf{(1.4)}$$

where

$$\text{(i)} \quad \phi\xi_p = 0,$$
$$\text{(ii)} \quad \eta^p \circ \phi = 0, \qquad \textbf{(1.5)}$$
$$\text{(iii)} \quad \eta^p(\xi_q) = 1$$

where p,q=1,2,…..r.

$$g(\phi X,\phi Y) = g(X,Y) - \sum_{p=1}^{r}\eta^p(X)\eta^p(Y) \qquad \textbf{(1.6)}$$

$$\phi(X,Y) = g(\phi X,Y) = -g(X,\phi Y) \qquad \textbf{(1.7)}$$

X,Y are arbitrary vector field.

An almost r-contact manifold is called a nearly r-cosymplectic manifold if [1]

$$(E_X\phi)(Y,Z) + (E_Y\phi)(X,Z) = 0 \qquad \textbf{(1.8)}$$

where E is the Riemannian connection. An almost r-contact metric manifold is called a generalized nearly r-cosymplectic manifold if [5]

$$(E_X\phi)(Y,Z) - \eta^p(Y)\{(E_X\eta^p)(\phi Z) + \eta^p(Z)\{(E_X\eta^p)(\phi Y) + (E_Y\eta^p)(\phi X)\} \qquad \textbf{(1.9)}$$

$$= (E_Y\phi)(Z,X) + \eta^p(X)(E_Y\eta^p)(\phi Z)$$

a generalized almost r-contact pseudo-normal metric manifold if [5]

$$(E_{\phi X}\phi)(\phi Y,Z) + (E_X\phi)(Y,Z) - \eta^p(Z)\{(E_{\phi X}\eta^p)(Y) - (E_X\eta^p)(\phi Y) - \eta^p(Y)(E_X\eta^p)(\phi Z) = 0 \qquad \textbf{(1.10)}$$

II. SUBMANIFOLD OF KAEHLAR MANIFOLD

Let M be a submanifold of a Riemannian manifold $\overline{M}$ with a Riemannian metric g. Consider the Gauss and Weingarten equations in the form

$$\text{(i)}\ \nabla_{BX}BY = BE_XY + \sum_{p=1}^{r}{}'\overset{p}{H}(X,Y)\underset{p}{N}$$

$$\text{(ii)} \quad \nabla_{BX}N = -B\overset{p}{H}Y + LX\underset{p}{N} \qquad \textbf{(2.1)}$$

G(BX,BY)$\circ b$=g(X,Y)

(iii) G(BX,BY)$\circ b$=g(X,Y)

where $\underset{p}{N}$ (p=1,2,…….r) unit normal vectors , $\overset{p}{H}$ (p=1,2,…….r) second fundamental tensor, D induced Riemannian connection and $\overset{p}{H}$ is associate to $'\overset{p}{H}$;

$$'\overset{p}{H}(X,Y) = g(\overset{p}{H}X,Y) = '\overset{p}{H}(Y,X) \qquad \textbf{(2.2)}$$

Let us express the transformations of BX and $\underset{p}{N}$ by F as

$$\text{(i)} \quad FBX = B\phi X + \sum_{p=1}^{r}\eta^p(X)\underset{p}{N} \qquad \textbf{(2.3)}$$

$$\text{(ii)} \quad F\underset{p}{N} = -B\xi_p + \sum_{p=1}^{r}\theta_p^q\underset{p}{N}$$

The submanifold of a Kaehlar manifold is an almost r-contact metric manifold satisfying

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$$\text{(i)}\ (E_X\phi)Y = \eta^p(Y)\overset{p}{H}(X) - \sum_{p=1}^{r}{}'\overset{p}{H}(X,Y)\xi_p$$

$$\text{(ii)}\quad (E_X\eta^p)Y = -{}'\overset{p}{H}(X,\phi Y)$$

$$\text{(iii)}\quad (E_X\xi_p)Y = \phi\overset{p}{H}X \qquad \textbf{(2.4)}$$

$$\text{(iv)}\quad g(\overset{p}{H}X) = {}'\overset{p}{H}(X,\xi_p)$$

Theorem (2.1)

The generalized nearly r-cosymplectic submanifold of a Kaehler manifold is nearly r-cosymplectic iff it is totally geodesic.

Proof-

From (1.8) and (2.4), it is totally geodesic.

$$(E_X\phi)(Y,Z) + (E_Y\phi)(X,Z) = \eta^p(X)'\overset{p}{H}(Y,Z) + \eta^p(Y)\overset{p}{H}(X,Z) - 2\eta^p(Z)'\overset{p}{H}(X,Y)$$

is equivalent to

$$(E_X\phi)(Y) + (E_Y\phi)(X) = \eta^p(X)\overset{p}{H}Y + \eta^p(Y)\overset{p}{H}X - 2'\overset{p}{H}(X,Y)\xi_p \qquad \textbf{(2.5)}$$

If $\overset{p}{H}$ X=0 , then from (2.5) , we have

$(E_X\phi)Y + (E_Y\phi)X$ =0

which proves that the connection is sufficient.Conversely if $(E_X\phi)Y + (E_Y\phi)X$ =0, then from (2.5), we get

$$\eta^p(X)\overset{p}{H}Y + \eta^p(Y)\overset{p}{H}X = 2'\overset{p}{H}(X,Y)\xi_p$$

but this equation is satisfied iff $\overset{p}{H}$ X=0 hence the condition is necessary.

Theorem (2.2)

On the generalized nearly r-cosymplectic submanifold of a Kaehler manifold

$$div\phi = 0 \ \ iff\ \overset{p}{\eta}(Y)tr\overset{p}{H} - \overset{p}{\eta}(\overset{p}{H}Y)$$

Proof-

Contracting (2.5), we get

$$(div\,\phi)Y = \overset{p}{\eta}(Y)tr\overset{p}{H} - \overset{p}{\eta}(\overset{p}{H}Y)$$

which proves the statement.

Theorem (2.3)

The submanifold of a Kaehlar manifold admits generalized almost r-contact pseudo-normal metric manifold.

Proof

From (2.4)(i), we find

$$(E_X\phi)(\phi Y,Z) + (E_X\phi)(Y,Z) = -\eta^p(Z)'\overset{p}{H}(\phi X,\phi Y) + \eta^p(Y)'\overset{p}{H}(X,Z) - \eta^p(Z)'\overset{p}{H}(X,Y) \qquad \textbf{(2.6)}$$

from (2.4)(ii), we get

$$\overset{p}{H}(X,Y) = (E_X\overset{p}{\eta})(\phi Y) + \overset{p}{\eta}(Y)'\overset{p}{H}(X,\xi_p) \quad \textbf{(2.7)}$$

hence in view of (2.6) and (2.7), we have

$$E_{\phi X}\phi)(\phi Y,Z) + (E_X\phi)(Y,Z) - \eta^p(Y)\{(E_X\eta^p)(\phi Z) - \overset{p}{\eta}(Z)\{(\xi_{\phi X}\eta)(Y) - (E_X\eta^p)(\phi Y)\} = 0$$

which proves the theorem.

III. References

1) Blair, D.E. (1970)-The theory of quasi-sasakian structure, journal Diff. Geom. 4,115.
2) J. Vanzura (1972)- almost r-contact structure, Ann. Scuol. Norm. Sup. Pisa. Cl. Sci. III, Vol.26.
3) Kaehler, E. (1933)-Ubereine bemerkenswerte Hermitische metric, Abh. Math. Sem. Univ. Hamburg, 175.
4) Mishra, R.S. (1984)-Structures on a differentiable manifold and their application, Chandrama Prakashan,50A , Balrampur House, Allahabad, India.
5) Mishra, R.S. (1991)- Almost contact metric manifolds, Monograph I, Tensor Society of India, Lucknow.
6) Mishra, R.S. (1995)-Normality of the hypersurfaces of almost Hermite manifolds, Journal of Indian Math . Soc., 61(1&2) , 71.
7) Sasaki,S.(1960)-On differentiable manifolds with certain structures which are closely related to almost contact structure, Tohuku Math. J. ,12 , 456.
8) Tashiro, Y. (19630-On contact structures of hypersurfaces in complex manifolds, Tohuku Math. J. ,15, 56.

Soft Lattices

GJSFR Classification – F (FOR)
010303,010206,010105

FU Li

Abstract- **Using the soft set defines the soft Lattice, gives the properties of the soft lattices,and discusses the relation between the soft lattices and the fuzzy soft sets. GJSFR-F FOR 010303,010206,010105**

Keywords- soft set, complete
, soft lattices, soft sublattices.

I. Introduction

In 1999, Molodsov [1] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodsov in [1]. After Molodsov's work, some different applications of soft sets were studied in [2,3].Furthermore Maji, Biswas and Roy worked on soft set theory in [4]. Also Maji et al. [5] presented the definition of fuzzy soft set and Roy et al. presented some applications of this notion to decision making problems in [6]. Recently, the many authors discuss the soft set, research on the soft set theory is progressing rapidly, for example, the concepts of soft semi-ring, soft group, soft BCK/BCI-algebra, soft BL-algebra,and fuzzy soft group etc. have been proposed and investigated (see [7-11] respectively).

Many important properties of an ordered set P are expressed in terms of the existence of certain upper bounds or lower bounds of subsets of P. Two of the most important classes of ordered set defined in this way are lattices and complete lattices. Lattices theory has been applied to all kinds of fields. The study of finite distributive lattice combines algebraic, ordertheoretic and graph-theoretic idea.

The present paper first given the basic concepts of soft set and lattice, and defined the soft lattice , given some example about soft lattice in section 2, defined the operations of soft lattice in section 3 , and continues the discussion and compares the concepts of soft lattice and L-fuzzy sets and point their relationship. Lastly, section 5 is conclusion.

II. Preliminaries

Definition 1.1[1] -Let U be an initial universe set and E be a set of parameters . Let P(U) denotes the power set of U and A ⊂ E. Then a pair (F,A) is called a soft set over U, where F : A → P(U) is a mapping.

That is, the soft set is a parameterized family of subsets of the set U. Every set F(e), ∀e ∈ E, from this family may be considered as the set of e-elements of the soft set (F,E), or considered as the set of e-approximate elements of the soft set.

According to this manner, we can view a soft set (F,E) as consisting of collection of approximations: (F,E) = {f(e) | e ∈ E}.

Definition 1.2[13]-Let P be a none-empty ordered set.

i. If x ∨ y and x ∧ y exist for all x, y ∈ P, then P is called a lattice;
ii. If ∨S and ∧S exist for all S ⊆ P, then P is called a complete lattice.

III. Soft lattices

Definition 2. 1 Let triplet M = (f,X,L) , where L is a complete lattice , f : X → L is a mapping, X is a universe set, then M is called the soft lattice .

Example 1 Let X = {e_1, · · · , e_5}, in which e1 stands for "expensive", e_2 : "beautiful", e_3 : "wooden", e_4 : "cheap", e_5 : "in the green surrounding", $L =< 0, {}_1, h_1, h_2, h_3, h_4, h_5 >$.

Suppose that:

$f(e_1) = \{h_2, h_4\}$, $f(e_2) = \{h_1, h_3\}$, $f(e_3) = \emptyset$, $f(e_4) = \{h_1, h_3, h_5\}$, $f(e_5) = \{h_1\}$, its lattice structure as figure 1.

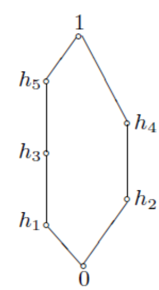

Figure 1

If we suppose that:

$f(e_1) = \{h_2, h_4\}$, $f(e_2) = \{h_1, h_3\}$, $f(e_3) = \{h_2, h_3, h_5\}$, $f(e_4) = \{h_1, h_3, h_5\}$, $f(e_5) = \{h_1\}$, it has the lattice structure as figure 2.

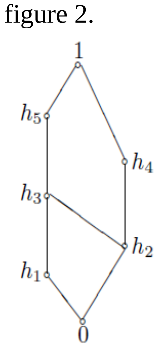

Figure 2

In fact, the lattice structure is not unique.

Example 2-Let *L = P(U)* is the power set of *U*, then the soft lattice (*f*,*X*,*L*) will be a set over *U*.

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Example 3-Let $L = [0, 1]$, then soft lattice $M = (f,X,L)$ is a fuzzy set in X.

Example 4-Let $L = \{[a, b] \mid 0 \le a \le b \le 1\}$, then L is a complete lattice according the inclusion order, and the soft lattice $M = (f,X,L)$ is a interval value fuzzy set in X.

Example 5-Let $L = \{(\mu(\cdot), \nu(\cdot)) \mid \mu(\cdot)+\nu(\cdot) \le 1, 0 \le \mu(\cdot), 0 \le \nu(\cdot)\}$, and $(\mu(\cdot), \nu(\cdot)) \bullet (s(\cdot), t(\cdot))$ if and only if $(\mu(\cdot) \bullet s(\cdot), \nu(\cdot)) \bullet t(\cdot))$, then L is a complete lattice, and the soft lattice $M = (f,X,L)$ is a intuitive fuzzy set.

Example 6-Let X be a topology space, $\delta(X)$ is A open sets lattice 0f X, that is , $L = P(\delta(X))$, then L is a complete lattice, and let $\forall x \in X, f(x) = \mu(x)$, where $\mu(x)$ is the open neighbor of x, then $M = (f,X,L)$ is a the soft lattice.

Proposition 1-Suppose that (f,X) is a soft set over L, if $\forall x \in X$, $f(x)$ is a single point subset of L, then (f,X,L) ia a soft lattice.

Definition 2.2-For two soft lattice $M = (f,X,L),N = (g, Y,L)$ over a common complete lattice, we say $M = (f,X,L)$ is a soft sublattice of $N = (g, Y,L)$, if (i) $X \subset Y$ (ii) $\forall \varepsilon \in X$, $g(\varepsilon)$ is extension of $f(\varepsilon)$.

Denoted $(f,X,L) \tilde{\subset} (g, Y,L)$.

Example 7- Consider the above example , Let $Y = \{e_1, e_2, e_3\}$, and $g(e_1) = \{h_2\}$, $g(e_2) = \{h_1, h_3\}$, $g(e_3) = \emptyset$, then (g, Y,L) is the soft sublattices of (f,X,L).

Definition 2.3- A soft lattice (f,X,L) is said to be null soft lattice, if $\forall \varrho \in X, f(\varrho) = 0$; a soft lattice (f,X,L) is said to be absolute soft lattice, if $\forall \varrho \in X, f(\varrho) = 1$, in which, 0 is the bottom element, and 1 is the top element.

Definition 2.4 -(Equality of two soft lattice) Two soft lattice $M = (f,X,L),N = (g, Y,L)$ are said to be soft equal if $M \tilde{\subset} N$ and $N \tilde{\subset} M$.

IV. The Operations of Soft Lattices

Definition 3.1- (Complement soft lattice of a soft lattice) The complement of a soft lattice $M = (f,X,L)$ is denoted by $Mc = (f,X,L)c$ and is defined as $(f,X,L)c = (fc, \neg X,Lc)$.

Furthermore, we can define the " or"," and ",""intersection", "union" as follows:

Definition 3.2- Let (f,X,L), (g, Y,L) be two soft lattices, "(f,X,L) or (g, Y,L)" is defined by: $(f,X,L)e\vee(g, Y,L) = (h,X \times Y,L)$, where $\forall(\alpha, \beta) \in X \times Y$, $h(\alpha, \beta) = f(\alpha) \cup g(\beta)$; "$(f,X,L)$ and (g, Y,L)" is defined by: $(f,X,L)e\wedge(g, Y,L) = (o,X \times Y,L)$, where $\forall(\alpha, \beta) \in X \times Y$, $o(\alpha, \beta) = f(\alpha) \cap g(\beta)$.

Example 8- Consider example 1, Let $Y = \{\alpha_1, \alpha_2, \alpha_3\}$, and $g(\alpha_1) = \{h_1, h_2\}$, $g(\alpha_2) = \{h_3, h_5\}$, $g(\alpha_3) = \{h_1, h_4, h_5\}$, for all $(e_1, \alpha_1) \in X \times Y$, having $h(e_1, \alpha_1) = f(e_1) \cup g(\alpha_1) = \{h_2, h_4\} \cup \{h_1, h_2\} = \{h_1, h_2, h_4\}$, and $o(e_1, \alpha_1) = f(e1) \cap g(\alpha_1) = \{h_2, h_4\} \cap \{h_1, h_2\} = \{h_2\}$.

Obviously,

$(f,X,L) \tilde{\vee} (g, Y,L)$ and $(f,X,L) \tilde{\wedge} (g, Y,L)$

are still the soft lattices. In fact, $\forall \varrho \in X \cap Y$, $o : X \cap Y \to L$ is a mapping ,in which, $X \cap Y$ is a set, and L is a complete lattice, so, $(o,X \cap Y,L)$ is a soft lattice, similarly, $(h,X \cup Y,L)$ is a soft lattice.

Definition 3.3- Let (f,X,L), (g, Y,L) be two soft lattices, the intersection of (f,X,L) and (g, Y,L) is the soft lattice $(h,X \cap Y,L)$, in which $\forall \varrho \in X \cap Y$, $h(\varepsilon) = f(\varrho) \cap g(\varrho)$, denoted by $(f,X,L) \sqcap (g, Y,L)$; the union of (f,X,L) and (g, Y,L) is the soft lattice $(h,X \cup Y,L)$, in which $\forall \varrho \in X \cap Y$, $h(\varepsilon) = f(\varrho) \cap g(\varrho)$, denoted by $(f,X,L) \sqcup (g, Y,L)$.

Proposition 3.1- Let (f,X,L), (g, Y,L) be two soft lattices, $(f,X,L) \sqcap (g, Y,L)$ is a soft sublattices,especially, if $X = Y$, the conclusion holds too.Proof By (f,X,L), (g, Y,L) being two soft lattices, then f, g are mapping from the set X or Y to a complete lattice L, using the set the-ory, we know $X \cap Y \subseteq X$ is still a set, and L is a complete lattice, and by the definition 3.3,we know $h : X \cap Y \to L$ is a mapping , so ,$(f,X,L) \sqcap (g, Y,L)$ is a soft sublattices.

In fact, $X = Y$ is a trivially case.

Remark 1- The operations "or", "and", "in- tersection", "union" defined on the same complete lattice L can be generated to the finite soft lattices (f_i,X_i,L), $i = 1, 2, \cdot \cdot \cdot$ n, and we can prove these operations are still soft lattices in the finite case. For example, $Vn_{\ i}=1(f_i,X_i,L) = (o,\times n_{\ i}=1X_i,L)$, where $\forall(\alpha_1, \cdot \cdot \cdot, \alpha_n) \in X_1\times \cdot \cdot \cdot \times X_n$, $o(\alpha_1, \cdot \cdot \cdot, \alpha_n) = f_1(\alpha_1) \cap \cdot \cdot \cdot \cap fn(\alpha_n)$, which is still a soft lattice; the other operations are similar.

V. Comparison of the Concepts of Soft Lattice and L-Fuzzy Set

Definition 4.1[14]- Let E be a non-empty set and L be a infinitely distributive complete lattice, then a mapping $F : E \to L$ is called an L-fuzzy set in E. Because L can be chosen in a wide scope arbitrarily, hence the concept of L- fuzzy set possesses a kind of universality so that many related concepts can be included in the same concept of L-fuzzy set.

Example 9- Let $L = [0, 1]$, then a fuzzy set in E is an L-fuzzy set in E.

Example 10- Let $L = P(U)$, then a soft set (F,A) over U is an L-fuzzy set in A .

Proposition 4.1- An L-fuzzy set is a special soft set.

Proof -Let A be an L-fuzzy subset of the universe set X, then A is a mapping $A : X \to L$.Define a soft set (A,X) over L such that $\forall x \in X,A(x)$ is a single point subset of L, then the L-fuzzy set A is a soft set.

Remark 2- If a complete lattice L in the soft lattice is the a infinitely distributive complete lattice, then soft lattice will be a L-fuzzy set.

Definition 4.2[13] -Let L be a lattice. A non-empty subset J of L is called an ideal if

i. If $a, b \in J$ implys $a \vee b \in J$;
ii. If $a \in L$, $b \in J$ and $a \le b$ imply $a \in J$.

Definition 4.3[13] -Let L be a lattice. A non-empty subset G of L is called an filter if

i. If $a, b \in G$ implies $a \wedge b \in J$;
ii. If $a \in L$, $b \in G$ and $a \le b$ imply $a \in G$.

Remark 3- If we require L is a complete lattice, we can get the definition of ideal and filter in the soft lattices. Furthermore, if we define the operation $\rightarrow$ and $\otimes$, we can discuss the filter in soft lattices as paper [11].

VI. Conclusion

In this paper, we define the soft lattices ,soft sub-lattices, discuss its properties, and study the relation of soft lattices and soft set , fuzzy soft set. Furthermore, if we define the operation (e.g. $\rightarrow$ and $\otimes$,), the we can get the soft lattice is a special soft BL-algebra. In fact ,we can discuss the relationship of soft lattices and fuzzy soft sets , rough soft sets ,soft rough sets, soft-rough fuzzy sets [14] etc.

VII. References

1) Molodtsov, Soft set theory—first results, Computers Math. Appl. 37(4/5),1931(1999).
2) Chen, E.C.C. Tsang, D.S. Yeung, X. Wang, The parameterization reduction of soft set and its applications, Computers Math. Appl. 49 (2005) 757-763.
3) P.K. Maji, A.R. Roy, An application of Soft set in decision making problem, Computers Math. Appl. 44 (2002) 1077-1083.
4) P.K.Maji, R.Biswas and A.R.Roy, Soft set theory, Computers Math. Appl.45(2003)555-562.
5) P.K.Maji, R.Biswas and A.R.Roy, Fuzzy soft sets, J. Fuzzy Mathematics,9(3)(2001)589-602.
6) P.K.Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, Journal of Fuzzy Mathematics 9 (3) (2001) 589-602.
7) A.R. Roy, P.K. Maji, A fuzzy soft set theoretic approach to decision making problems, Journal of Computational and Applied Mathematics 203 (2007) 412-418.
8) F.Feng,Y.B.Jun,X.Z.Zhao, Soft semi-rings, Computers Math. Appl.56(2008)2621-2628.
9) H.Aktas,N.Cagman,Soft sets and soft groups, Information Sciences,177(2007)2726-2735.
10) Y.B.Jun, Soft BCK/BCI-algebras, Computers Math. Appl.56(2008)1408-1413.
11) J.M.Zhan,Y.B.Jun, Soft BL-algebras, Computers Math. Appl.(In press).
12) A.Aygunoglu,H.Aygun, Introduction to fuzzy soft groups, Computers Math. Appl.58(2009)1279-1286.
13) B.A.Davey, H.A.Priestley, Introduction to lattices and order , Cambridge University Press, 1999.
14) J.A.Goguen, L-fuzzy sets,J.Math.Anal.appl.18(1967)145-174.
15) F. Feng, Changxing Li, B.Davvaz, Soft sets combined with fuzzy sets and rough sets:atentative approach, Soft Comput, Published online:27 June 2007.

Remarks On The Support Of The Generalized Mellin Transformation

GJSFR Classification – D (FOR) 091307,010506,010501

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***Abstract-A* classical theory of the Mellin transform is extended to a space of general-ized functions. The support of the generalized Mellin transform is shown to apply through a certain motivation on the Mellin differentation .**

keywords-Mellin transformation , Tempered distribution ,Multinormed space ,Tempered ultradistribution , Tempered Boehmian .

I. INTRODUCTION

The conventional Mellin transform of a suitably restricted f (x) defined on $I\ (0,\infty)$ is due to [18]

$$(\mu f)(s) = (f)_m^{\wedge}(s) = \int_0^\infty f(x)\, x^{s-1} dx, \qquad (1)$$

where $s = \delta + i\tau$ is a certain strip in the complex s-plane .
The mellin transform is essentially bilateral laplace transform and ,further,it can be expressed as an exponential fourier transform as well .Let a; b $\in \mathbb{R}$ and

$$\zeta_{a,b}(x) := \begin{cases} x^{-a}, 0 < x \leq 1 \\ x^{-b}, 1 < x < \infty \end{cases} \qquad (2)$$

then the space $\mu_{a,b}$ is defined to be the linear space of all infinitely smooth complex valued functions ϕ on $I(0,\infty)$ for which

$$y_k(\phi) := \sup_{x\in I} \left|\zeta_{a,b}(x)\, x^{k+1} D_x^k \phi(x)\right| \qquad (3)$$

is finite, *k = 0; 1; 2; :::; :*
The topology equipped with $\mu_{a,b}$ is generated by the sequence of multinorms $\{y_k\}_{k\in N_0}$.The space $\mu_{a,b}$ is in turn a complete multinormed space. The dual of $\mu_{a,b}$ is denoted by $\acute{\mu}_{a,b}$ which assigned the weak topology.
Denoting by *D(I)* the space of test functions of .nite support the space *D(I)* is a subspace of $\mu_{a,b}$ and the topology of*D(I)* is stronger than the topology induced on D(I) by $\mu_{a,b}$
Further, $\mu_{a,b}$ is dense in *E* (the space of C^∞ functions) and hence, $\acute{E}$ (the space of distributions of bounded support) is a subspace of $\acute{\mu}_{a,b}$
We say f is a Mellin transformable generalized function if it is a member of $\acute{\mu}_{a,b}$ for some real numbers a; b. Clearly, f is a member of $\acute{\mu}_{c,d}$ for every $c \geq a$ and $d \leq b$.

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We say f is a Mellin transformable generalized function if it is a member of $\acute{\mu}_{a,b}$ for some real numbers a; b. Clearly, f is a member of $\acute{\mu}_{c,d}$ for every $c \geq a$ and $d \leq b$.
Consequently, there exist real members and (possibly $= -\infty$ and $\sigma_2 = \infty$) such that σ_1 $f \in \acute{\mu}_{a,b}$ for every a > σ_1 and b < σ_2 and f $\acute{\mu}_{a,b}$ for a > σ_1 for a > σ_1 and b < σ_2.
The function x^{s-1} is a member of $\acute{\mu}_{a,b}$ for all s, $a \leq Re\ s \leq b$.
Let $\Omega_f := \{s : \sigma_1 < Re\ s < \sigma_2\}$. The generalized Mellin transform of $f \in \acute{\mu}_{a,b}$ is given by [18,p.p.107].

$$(\mu f)(s) := \left(f\right)_m^{\wedge}(s) := \left\langle f(x), x^{s-1}\right\rangle . \qquad (4)$$

(For further properties reader is referred to [18]) .
If ϕ is a function on R, the closure of the set $\{x : \phi(x) \neq 0\}$ is called the support of ϕ and denoted by sup p ϕ Let Φ be a testing function space and Φ be the dual space of Φ
A generalized function $f' \in \Phi$ is said to have a compact support in a set u if $\langle f, \phi\rangle = 0$ whenever $\phi' \in \Phi$ has a support in R/u. The smallest closed set having this property is called the support of f denoted by sup pf.

II. THE SUPPORT OF THE DISTRIBUTIONAL MELLIN TRANSFORM

In this section we include two characterization theorems investigating the support of distributions in the space $\acute{\mu}_{a,b}$
In the .rst theorem , we show that the support of a distributional Mellin transform is involved with a series of Mellin transforms of in.nite number of derivatives of distributions in $\acute{\mu}_{a,b}$ In the second theorem, the support of the distributional Mellin transform, can be predicted from a convergent sequence of certain antiderivatives . This is in short has been shown as follows :
Theorem 2.1-Let $f \in \acute{\mu}_{a,b}(0,\infty), \beta > 0$. Then the following are equivalent.
(i) *supp* $(\mu f)(s) \subseteq [-\beta, \beta]$, $a < s < b, s \in \mathbb{R}$.
(ii)*for any real number* $\eta > \beta$,

$$\underset{k\to\infty}{Lim}\, \mu\left(\frac{x^k f^{(k)}(x)}{\eta^k (s+k)^k}\right)(s) = 0,$$

Proof. (i)- (ii). Assume (i) is satis.ed. Choose *p* such that $\beta < \rho < \eta$.Let $\psi \in C^\infty(\mathbb{R})$ be such that

$$\begin{aligned} \psi &\equiv 1 \text{ in anbd of } [-\beta, \beta] \\ &\equiv 0 \text{ in anbd of } \{s \in \mathbb{R}; |s| > \rho\} \end{aligned}$$

Then , properties of di¤erentiation of the Mellin transform ,for $\phi' \in D(R)$ imply that

$$\begin{aligned} \left\langle \mu\left(\eta^{-k}(s+k)^{-k} x^k f^{(k)}(x)\right)(s), \phi(s)\right\rangle &= \left\langle (-1)^k \eta^{-k}(s+k)^{-k}\frac{\Gamma(s+k)}{\Gamma(s)}(\mu f)(s), \phi(s)\right\rangle \\ &= \left\langle (\mu f)(s), (-1)^k \eta^{-k}(s+k)^{-k}\frac{\Gamma(s+k)}{\Gamma(s)}(\psi\phi)(s)\right\rangle \end{aligned}$$

Therefore, to proof this part of the theorem we merely need to show that the series

$$(-1)^k \eta^{-k} (s+k)^{-k} \frac{\Gamma(s+k)}{\Gamma(s)} (\psi\phi)(s) \to 0$$

in the sense of D. For this end, Leibnitz.rule and integration by parts and simple calculations , for k > j , yield.

$$\left| D_s^j (-1)^k \eta^{-k} (s+k)^{-k} \frac{\Gamma(s+k)}{\Gamma(s)} (\psi\phi)(s) \right| \leq \left| D_s^j (-1)^k \eta^{-k} (s+k)^{-k} \frac{\Gamma(s+k)}{\Gamma(s)} (\psi\phi)(s) \right|$$

$$= \eta^{-k} \sum_{r=0}^{\infty} (s+k)^{-k} \left| D^r (s+k-1)(s+k-2)\dots(s+1)s \right| \left\| D^{j-r} (\psi\phi)(s) \right\|_\infty$$

$$\leq \sum_{r=0}^{j} \eta^{-k} (s+k)^{-k} (k-r) \left| s+k-1 \right|^{k-r} \left\| D^{j-r} (\psi\phi)(s) \right\|_\infty$$

$$\leq M \sum_{r=0}^{j} \eta^{-k} (s+k)^{-k} |sk|^k$$

$$< M \sum_{r=0}^{j} \left(\frac{s}{\eta}\right)^k \frac{1}{(s+k)^r}$$

$$= M \sum_{r=0}^{j} \left(\frac{s}{\eta}\right)^k$$

$$\to 0, \text{as } k \to \infty,$$

since $\frac{\rho}{\eta} < 1.$ $\qquad \eta / |s| < \eta / \rho < 1.$

Theorem 2.2.-Let $f \in \mu'_{a,b}(0,\infty), \beta > 0.$ Then ,if for any real number $\eta > \beta,$

$$\lim_{k\to\infty} \mu \left(\frac{x^k f^{(k)}(x)}{\eta^k (s+k)^k} \right)(s) = 0,$$

we have $supp\ (\mu f)(s) \subseteq [-\beta, \beta], a < s < b, s \in \mathbb{R}.$

Proof Assume $\mu \left(\frac{x^k f^{(k)}(x)}{\eta^k} \right)(s) = 0$ then $\lim_{k\to\infty} \frac{x^k f^{(k)}(x)}{\eta^k} = 0.$

We show

$$\langle (\mu f)(s), \phi(s) \rangle = 0 \qquad (5)$$

for all $\phi \in D(0,\infty$ with a property that supp $\phi \subset \{s \in \mathbb{R} : |s| > \beta\}$ The support of ϕ being compact, leads to an existence of $\rho > \beta\ (\rho > 0)$ such that supp $\phi \subset \{s \in \mathbb{R} : |s| > \beta\}$ and $\beta < \eta < \rho.$ Properties of di¤erentiation of the Mellin transform [13, P.216] imply

$$\left(\mu \left(\eta^{-k} x^k f^{(k)}(x) \right)(s) \right)_{k \geq 0} = \left((-1)^k \eta^{-k} \frac{\Gamma(s+k)}{\Gamma(s)} (\mu f)(s) \right)_{k \geq 0}$$

Write now

$$\langle (\mu f)(s), \phi(s) \rangle = \left\langle (-1)^k \eta^{-k} \frac{\Gamma(s+k)}{\Gamma(s)} (\mu f)(s), (-1)^k \eta^k \frac{\Gamma(s)}{\Gamma(s+k)} \phi(s) \right\rangle.$$

.Our objective in (5) can be fulfilled when the sequence

$$(-1)^k \eta^k \frac{\Gamma(s)}{\Gamma(s+k)} \phi(s), k = 0; 1; 2; ::$$

converges to zero in the sense of the topology of D. Leibnitz. rule and integration by parts imply

$$\left| D_s^j (-1)^k \eta^k \frac{\Gamma(s)}{\Gamma(s+k)} \phi(s) \right| \leq \eta^k \sum_{r=0}^{j} D_s^r \left| \frac{1}{(s+k-1)(s+k-2)\dots(s+1)s} \right| \left\| D^{j-r} \phi(s) \right\|_\infty \qquad (1)$$

Calculations on (6) then yield

$$\left| D_s^j (-1)^k \eta^k \frac{\Gamma(s)}{\Gamma(s+k)} \phi(s) \right| \leq \sum_{r=0}^{j} r \left(\frac{\eta}{s}\right)^{k(r+1)} \eta^{k-kr} \left\| D^{j-r} \phi(s) \right\|_\infty . \qquad (7)$$

Now, since lsl > p we have

Therefore, Expression (7) tends to zero as $k \to \infty$ for all $r = 0, 1, 2, \dots, j.$

This completes the proof of the theorem .

Theorem 2.3-

ivalent

(ii) There is a sequence (θ_k) in $\mu_{a,b}(0,\infty)$ with properties:

(a) $\theta_0 = f$

(b) $x\theta'_{k+1}(x) = \theta_k(x)$

(c) $\eta^{-k}(\mu\theta_k) \to 0,$ *as* $k \to \infty$ *for all* $\eta > \beta^{-1}$

proof -.(ii) - (i) . It is sufficiently enough to show that

$$\langle (\mu f)(s), \phi(s) \rangle = 0, \qquad (8)$$

for all $\phi \in D(\mathbb{R})$ such that supp $\phi \subset [-\alpha, \alpha], 0 < \alpha < \beta$.

From (ii) it can be easily observed that

$$\begin{aligned} f &= \theta_0 \\ &= x\theta'_1(x) \\ &= x^2 \theta_2^{(2)}(x) \\ &= \dots \\ &= x^k \theta_k^{(k)}(x) . \end{aligned} \qquad (9)$$

Since $\alpha < \beta$ i is arbitrary we may select $\beta^{-1} < \eta < \alpha^{-1} < 1.$ By virtue of properties of di¤erentiation of the transform [13, p.216], Equation(9) and Condition (c) we have

$$\begin{aligned} \langle (\mu f)(s), \phi(s) \rangle &= \left\langle \mu \left(x^k \theta_k^{(k)}(x) \right)(s), \phi(s) \right\rangle \\ &= \left\langle (-1)^k \frac{\Gamma(s+k)}{\Gamma(s)} (\mu\theta_k)(s), \phi(s) \right\rangle \\ &= \left\langle (\mu\theta_k)(s), (-1)^k \eta^k \frac{\Gamma(s+k)}{\Gamma(s)} \phi(s) \right\rangle \end{aligned} \qquad (10)$$

Since is bounded, (8) can be con.rmed by showing that the sequence

$$\left((-1)^k \eta^k \frac{\Gamma(s+k)}{\Gamma(s)} \phi(s) \right)_{k \geq 0} \to 0 \text{ in the sence of D:}$$

Once again , Leibnitz.rule and calculations show that

$$\left|D^j(-1)^k\eta^k(s+k+1)\dots(s+1)s\phi(s)\right| \leq \sum_{r=0}^{j}\left|(-1)^k\eta^k k(s+k+1)\right|\left\|D^{j-r}\phi(s)\right\|_\infty$$
$$< \sum_{r=0}^{j}\left|\eta^k k^2 s^k\right|\left\|D^{j-r}\phi(s)\right\|_\infty$$
$$= \sum_{r=0}^{j}\left|k^2(\eta s)^k\right|\left\|D^{j-r}\phi(s)\right\|_\infty \qquad (11)$$

for all values of s such that $|s| < \alpha, |s\eta| < 1$.
Hence, Expression in (11) tends to zero as $k \to \infty$
$(i) \Rightarrow (ii)$ Assume that (i) holds. De.ne the sequence $(\theta_k)_{k\geq 0}$ by

$$(\mu\theta_0)(s) = (\mu f)(s)$$

and

$$(\mu\theta_k)(s) = (-1)^k\frac{\Gamma(s+k)}{\Gamma(s)}(\mu f)(s), k\geq 1.$$

Since μf vanishes in a nbhd of zero ,the sequence (θ_k) is, indeed, well defined.
Let $\eta > \beta^{-1}$ and choose a such that

$$\eta^{-1} < \alpha < \beta. \qquad (12)$$

Let $\psi \in C^\infty$ satisfy

$$\psi(s) \equiv 1, |s| \geq \beta - \frac{\beta-\alpha}{2}$$
$$\equiv 0, |s| \leq \alpha$$

Clearly $\psi(\mu f) = (\mu f)$. Therefore

$$\langle \eta^{-k}(\mu\theta_k)(s), \phi(s)\rangle = \left\langle \eta^{-k}(-1)^k\frac{\Gamma(s+k)}{\Gamma(s)}\psi(\mu f)(s), \phi(s)\right\rangle$$
$$= \left\langle (\mu f)(s), (-1)^k\eta^{-k}\frac{\Gamma(s+k)}{\Gamma(s)}(\phi\psi)(s)\right\rangle$$

From above it is su¢ cient to show that

$$(-1)^k\eta^{-k}\frac{\Gamma(s+k)}{\Gamma(s)}(\phi\psi)(s) \to 0 \quad \text{in } D(\mathbb{R}).$$

This relation has already been shown in the proof considered in the first part of Theorem 2.1.with the assumption that (12) is being employed .
This completes the proof of the theorem.

III. References

1) Al-Omari, S.K.Q. ,Loonker D. , Banerji P. K. and Kalla, S. L. (2008).Fourier sine(cosine) transform for ultradistributions and their extensions to tempered and ultraBoehmian spaces, Integ.Trans.Spl.Funct. 19(6), 453 .462.
2) Al-Omari,S.K.Q.(2009).Certain Class of Kernels for Roumieu-Type Convolution Transform of Ultradistribution of Compact Growth, J.Concr.Appli.Math.7(4),310-316.
3) (2009).Cauchy and Poison Integrals of Tempered Ultradistributions of Roumieu and Beurling Types , J. Concr.Appli.Math.7(1),36-47.
4) Al-Omari, S.K.Q. and Al-Omari, J.M. (2009),Operations on Multipliers and Certain Spaces of Tempered Ultradistributions of Roumieu and Beurling types for the Hankel-type Transformations, J.Appl. Funct.Anal.(2010)(to Appear)
5) Banerji, P.K. and Al-Omari ,S. K.Q. (2006). Multipliers and Operators on the Tempered Ultradistribution Spaces of Roumieu Type for the Distributional Hankel-type transformation spaces ,Internat. J.Math.Math. Sci.,2006(2006), Article ID 31682 ,p.p.1-7.
6) Banerji,P.K., Alomari,S.K.and Debnath,L.(2006).Tempered Distributional Fourier Sine(Cosine)Transform,Integ.Trans.Spl.Funct.17(11),759-768.
7) Beurling,A.(1961).Quasi-analiticity and generalized distributions ,Lectures 4 and 5,A.M.S.Summer Institute, Stanford.
8) Gray,A. and Mathews,G.B. and MacRobert,T.M.(1952).A Treatise on Bessel Functions and their Applications to Physics, MacMillan, London.
9) Komatsu, H. (1973). Ultradistributions, I : Structure Theorems and a Characterization, J.Fac. Sci.Univ.,Tokyo Sect. IA Math. 20, 25-105.
10) Pathak,R.S.(1997). Integral transforms of generalized functions and their applications,Gordon and Breach Science Publishers,Australia ,Canada,India,Japan.
11) Pathak,R.S. (1976).Distributional Stieltjes Transformation ,Proc.Edinburgh Math.Soc.Ser.,20Part 1,15-22.
12) Richard D. Carmichel, Pathak, R.S. and Pilipovic, S. (1990). Cauchy and Poison integrals of ultradistributions, complex variables. 14, 85-108.
13) Roumieu,C.(1962-63).Ultradistributions de.nes sur( ) etsurcertaines classea de variétése di¤eretiables, J.d.Analyse Math.10,153-192.
14) Roumieu,C.(1960).Sur quelques extensions de la notin de distribution,Ann. Ecole Norm.Sup.77,41-121.
15) Zemanian, A.H. (1987). Generalized integral transformation, Dover Publications, Inc., New York.First published by interscience publishers, New York (1968).

Linear Operators Associated With A Certain Subclass Of Uniformly Convex Functions

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***Abstract*-Making use of certain linear operator, we define a new subclass of uniformly convex functions with negative coefficients and obtain coefficient estimates, extreme points, closure and inclusion theorems and the radii of starlikeness and convexity for the new subclass. Furthermore, results on convolution products are discussed.**

Mathematics Subject Classification: 30C4.

Keywords-Univalent, starlike, convex, uniformly convex, hadamard product.

I. Introduction

Let A denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \qquad (1)$$

which are analytic and univalent in the open unit disc U = {z : |z| < 1, z ∈ C}. Also in [7] denote by T the subclass of A consisting of functions of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n, \quad a_n \geq 0 \qquad (2)$$

Following Goodman [2, 3], Ronning [4, 5] introduced and studied the following subclasses.

Definition 1.- A function f ∈ A is said to be in the class $S_p(\alpha, \beta)$uniformly β*-starlike* functions if it satisfies the condition.

$$Re\left\{\frac{zf'(z)}{f(z)} - \alpha\right\} > \beta\left|\frac{zf'(z)}{f(z)} - 1\right|, \quad z \in U, -1 < \alpha \leq 1 \text{ and } \beta \geq 0. \qquad (3)$$

Definition 1.2- A function f ∈ A is said to be in the class UCV (α, β) uniformly β*-convex* functions if it satisfies the condition

$$Re\left\{1 + \frac{zf''(z)}{f'(z)} - \alpha\right\} > \beta\left|\frac{zf''(z)}{f'(z)} - 1\right|, \quad z \in U, -1 < \alpha \leq 1 \text{ and } \beta \geq 0. \qquad (4)$$

It follows from (1.3) and (1.4) that

$$f \in UCV(\alpha, \beta) \Leftrightarrow zf' \in S_p(\alpha, \beta). \qquad (5)$$

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For functions f ∈ A given by (1.1) and g ∈ A given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n,$$

we define the Hadamard product of f and g by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \quad z \in U. \qquad (6)$$

Let ϕ (a, c; z) be the incomplete Beta function defined by

$$\phi(a, c; z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}} z^n, \quad c \neq 0, -1, -2, .. (7)$$

where $(\lambda)_n$ n is the Pochhammer symbol defined in terms of the gamma functions, by

$$(\lambda)_n = \frac{\Gamma(\lambda + n)}{\Gamma(\lambda)} = \begin{cases} 1, & n = 0 \\ \lambda(\lambda+1)(\lambda+2)\cdots(\lambda+n-1), & n \in N \end{cases} \qquad (8)$$

Further, for f ∈ A

$$L(a, c)f(z) = \phi(a, c; z) * f(z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}} a_n z^n \qquad (9)$$

where L(a, c) is called Carlson-Shaffer operator [1].We note that

$$L(a, a)f(z) = f(z), \quad L(2, 1)f(z) = zf'(z).$$

For, $-1 \leq \alpha < 1$, $\lambda \geq 0$ and $\beta \geq 0$, we let $S(\lambda, \alpha, \beta)$ be the subclass of *A* consisting of functions of the form (1.1) and satisfying the analytic criterion.

$$Re\left\{\frac{z(L(a, c)f(z))' + \lambda z^2(L(a, c)f(z))''}{L(a, c)f(z)} - \alpha\right\}$$
$$\geq \beta\left|\frac{z(L(a, c)f(z))' + \lambda z^2(L(a, c)f(z))''}{L(a, c)f(z)} - 1\right|, \quad z \in U \qquad (10)$$

where L(a, c)f(z) is given by (1.9). We also let

$$S_T(\lambda, \alpha, \beta) = S(\lambda, \alpha, \beta) \cap T.$$

By suitably specializing the values of (a), (c) and $\lambda = 0$ the class $S(\lambda, \alpha, \beta)$ can be reduces to the class studied earlier by Ronning [4, 5]. Also choosing $\lambda = ,0$ $\alpha = 0$ and $\beta = 1,$ the class coincides with the classes studied in [9] and [10] respectively.

The main object of this paper is to study the coefficient bounds, extreme points, radius of starlikeness and convolution results for functions belong to the generalized class $S_T(\lambda, \alpha, \beta)$

II. BASIC PROPERTIES

In this section we obtain a necessary and sufficient condition for functions f(z) in the classes $S(\lambda,\alpha,\beta)$ and $S_T(\lambda,\alpha,\beta)$

Theorem 2. 1- A function f(z) of the form (1.1) is in $S(\lambda,\alpha,\beta)$

$$\sum_{n=2}^{\infty}[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]\frac{(a)_{n-1}}{(c)_{n-1}}|a_n|\le 1-\alpha,$$

$-1\le\alpha<1,\beta\ge 0$ *and* $\lambda\ge 0.$ (1)

Proof It suffices to show that

$$\beta\left|\frac{z(L(a,c)f(z))'+\lambda z^2(L(a,c)f(z))''}{(L(a,c)f(z))}-1\right| - Re\left\{\frac{z(L(a,c)f(z))'+\lambda z^2(L(a,c)f(z))''}{(L(a,c)f(z))}-1\right\}\le 1-\alpha.$$

We have

$$\beta\left|\frac{z(L(a,c)f(z))'+\lambda z^2(L(a,c)f(z))''}{(L(a,c)f(z))}-1\right| - Re\left\{\frac{z(L(a,c)f(z))'+\lambda z^2(L(a,c)f(z))''}{(L(a,c)f(z))}-1\right\}$$
$$\le(1+\beta)\left|\frac{z(L(a,c)f(z))'+\lambda z^2(L(a,c)f(z))''}{(L(a,c)f(z))}-1\right|$$
$$\le\frac{(1+\beta)\sum_{n=2}^{\infty}[n+\lambda n(n-1))-1]\frac{(a)_{n-1}}{(c)_{n-1}}|a_n|}{1-\sum_{n=2}^{\infty}\frac{(a)_{n-1}}{(c)_{n-1}}|a_n|}.$$

This last expression is bounded above by (1- a _) if

$$\sum_{n=2}^{\infty}[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]\frac{(a)_{n-1}}{(c)_{n-1}}|a_n|\le 1-\alpha.$$

Theorem 2.2-A necessary and sufficient condition for f(z) of the form (1.2) to be in the class $S_T(\lambda,\alpha,\beta),\ -1\le\alpha<1,\ \beta\ge 0$ and $\lambda\ge 0$

$$\sum_{n=2}^{\infty}[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]\frac{(a)_{n-1}}{(c)_{n-1}}a_n\le 1-\alpha.$$

(2)

Proof-In view of Theorem 2.1, we need only to prove the necessity. If $f(z)\in S_T(\lambda,\alpha,\beta)$ and z is real then

$$\frac{1-\sum_{n=2}^{\infty}n\frac{(a)_{n-1}}{(c)_{n-1}}a_nz^{n-1}-\lambda\sum_{n=2}^{\infty}n(n-1)\frac{(a)_{n-1}}{(c)_{n-1}}a_nz^{n-1}}{1-\sum_{n=2}^{\infty}\frac{(a)_{n-1}}{(c)_{n-1}}a_nz^{n-1}}-\alpha$$
$$\ge\beta\left|\frac{\sum_{n=2}^{\infty}(\lambda n(n-1)+n-1)\frac{(a)_{n-1}}{(c)_{n-1}}a_nz^{n-1}}{1-\sum_{n=2}^{\infty}\frac{(a)_{n-1}}{(c)_{n-1}}a_nz^{n-1}}\right|$$

$$\sum_{n=2}^{\infty}[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]\frac{(a)_{n-1}}{(c)_{n-1}}a_n\le 1-\alpha,$$

$-1\le\alpha<1,\beta\ge 0$ and $\lambda\ge 0.$

Letting z → 1 along the real axis, we obtain the desired inequality

Theorem 2.3-Let f(z) defined by (1.2) and g(z) defined by $g(z)=z-\sum_{n=2}^{\infty}b_nz^n$ be in the class $S_T(\lambda,\alpha,\beta)$ then the function h(z) defined by

$$h(z)=(1-\mu)f(z)+\mu g(z)=z-\sum_{n=2}q_nz^n,$$

where $q_n=(1-\mu)a_n+\mu b_n,\ 0\le\mu<1$ *is also in the class* $S_T(\lambda,\alpha,\beta)$.

Theorem 2.4-(Extreme points) Let f1(z) = z and

$$f_n(z)=z-\frac{(1-\alpha)(c)_{n-1}}{[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)](a)_{n-1}}z^n\quad for\ n=2,3,4,$$

(3)

Then $f(z)\in S_T(\lambda,\alpha,\beta)$ *if and only if* $f(z)$ *can be expressed in the form*

$$f(z)=\sum_{n=1}^{\infty}\mu_nf_n(z)\ where\ \mu_n\ge 0\ and\ \sum_{n=1}^{\infty}\mu_n=1.$$

The proof of Theorem 2.4, follows on line similar to the proof of the theorem on extreme points given in Silverman [8].

We prove the following theorem by defining fj(z) (j = 1, 2, . . ,m) of the form

$$f_j(z)=z-\sum_{n=2}^{\infty}a_{n,j}z^n\quad\text{for}\quad a_{n,j}\ge 0, z\in U.$$

(4)

Theorem 2.5 (Closure theorem) Let the functions fj(z) (j = 1, 2, . . . ,m) defined by (2.4) be in the classes $S_T(\lambda,\alpha_j,\beta)$ (j = 1, 2, . . . ,m) respectively.

Then the function h(z) defined by

$$h(z)=z-\frac{1}{m}\sum_{n=2}^{\infty}\left(\sum_{j=1}^{\infty}a_{n,j}\right)z^n$$

is in the class where

Proof-Since $f_j(z)\in S_T(\lambda,\alpha_j,\beta)\ (j=1,2,\ldots,m)$ by applying Theorem 2.2 to (2.4) we observe that

$$\sum_{n=2}^{\infty}[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]\frac{(a)_{n-1}}{(c)_{n-1}}\left(\frac{1}{m}\sum_{j=1}^{m}a_{n,j}\right)$$
$$=\frac{1}{m}\sum_{j=1}^{m}\left(\sum_{n=2}^{\infty}[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]\frac{(a)_{n-1}}{(c)_{n-1}}a_{n,j}\right)$$
$$\le\frac{1}{m}\sum_{j=1}^{m}(1-\alpha_j)\le 1-\alpha,$$

which in view of Theorem 2.2 again implies that h $h(z)\in S_T(\lambda,\alpha,\beta).$

Theorem 2.6-Let *Let* $f\in S_T(\lambda,\alpha,\beta).$ Then f is starlike of order $\delta\ (0\le\delta<1)$in the unit disc |z| < r1; that is

$$Re\left\{\frac{zf'(z)}{f(z)}\right\}>\delta,\ (|z|<r_1;0\le\delta<1),$$ where

$$r_1 = \underset{n\leq 2}{Inf}\left\{\frac{(a)_{n-1}}{(c)_{n-1}}\left(\frac{1-\delta}{n-\delta}\right)\frac{[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]}{1-\alpha}\right\}^{\frac{1}{n-1}}$$

Each of these results are sharp for the extremal function f(z) given by (2.3).

Proof-Given f ∈ A, and f is starlike of order δ we have

$$\left|\frac{zf'(z)}{f(z)}-1\right| < 1-\delta. \qquad (5)$$

For the left hand side of (2.5) we have

$$\left|\frac{zf'(z)}{f(z)}-1\right| \leq \frac{\sum_{n=2}^{\infty}(n-1)a_n|z|^{n-1}}{1-\sum_{n=2}^{\infty}a_n|z|^{n-1}}.$$

The last expression is less than $1-\delta$ if,

$$\sum_{n=2}^{\infty}\frac{n-\delta}{1-\delta}a_n|z|^{n-1} < 1.$$

Using the fact, that f ∈ $S_T(\lambda,\alpha,\beta)$ if and only if

$$\sum_{n=2}^{\infty}\frac{[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]}{1-\alpha}\frac{(a)_{n-1}}{(c)_{n-1}}a_n < 1.$$

We say that (2.5) is true if

$$\frac{n-\delta}{1-\delta}|z|^{n-1} < \frac{[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]}{1-\alpha}\frac{(a)_{n-1}}{(c)_{n-1}}.$$

Or, equivalently

$$|z|^{n-1} < \frac{(1-\delta)[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]}{(n-\delta)(1-\alpha)}\frac{(a)_{n-1}}{(c)_{n-1}}$$

which yields the starlikeness of the family.

Theorem *2.7*-Let f ∈ $S_T(\lambda,\alpha,\beta)$ then f is convex of order δ $(0\leq\delta<1)$ in the unit disc |z| < r2; that is $Re\left\{1+\frac{zf''(z)}{f'(z)}\right\} > \delta,\ (|z|<r_2, 0\leq\delta<1),$

Where

$$r_2 = \underset{n\leq 2}{Inf}\left\{\frac{(a)_{n-1}}{(c)_{n-1}}\frac{(1-\delta)[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]}{n(n-\delta)(1-\alpha)}\right\}^{\frac{1}{n-1}}$$

Proof Using the fact that f is convex if and only if zf′ is starlike, we can prove the Theorem 2.7 on lines similar the proof of the Theorem 2.6.

III. Modified Hadamard Products

Let the functions fj(z) (j = 1, 2) be defined by

$$f_j(z) = z-\sum_{n=2}^{\infty}a_{n,j}z^n \quad \text{for} \quad a_{n,j}\geq 0, z\in U. \qquad (1)$$

The modified Hadamard product of f1(z) and f2(z) is defined by

$$(f_1 * f_2)(z) = z-\sum_{n=2}^{\infty}a_{n,1}a_{n,2}z^n.$$

Using the techniques of Schild and Silverman [6], we prove the following results.

Theorem 3.1 For functions fj(z) (j = 1, 2) defined by (3.1), let f1(z) ∈$S_T(\lambda,\alpha,\beta)$ and $f_2(z)\in S_T(\lambda,\delta,\beta)$. Then (f1 ∗ f2)(z) ∈ where $\gamma=\gamma(\alpha,\beta,\delta)$

$$=1-\frac{(1-\alpha)(1-\delta)(1+\beta)}{(2+\beta+2\lambda(1+\beta)-\alpha)(2+\beta+2\lambda(1+\beta)-\delta)\frac{(a)}{(c)}-(1-\alpha)(1-\delta)} \qquad (3.2)$$

and $-1\leq\delta\leq 1,\ -1<\gamma\leq 1;\ z\in U.$ *The result is best possible for*

$$f_1(z) = z-\frac{(1-\alpha)}{(2+\beta+2\lambda(1+\beta)-\alpha)}\frac{(c)}{(a)}z^2$$

$$f_2(z) = z-\frac{(1-\delta)}{(2+\beta+2\lambda(1+\beta)-\delta)}\frac{(c)}{(a)}z^2$$

Proof- In view of Theorem 2.2, it suffices to prove that

$$\sum_{n=2}^{\infty}\frac{[(1+\beta)(n+\lambda n(n-1))-(\gamma+\beta)]}{1-\gamma}\frac{(a)_{n-1}}{(c)_{n-1}}a_{n,1}a_{n,2}\leq 1,\ (-1<\gamma\leq 1)$$

where is defined by (3.2). On the other hand, under the hypothesis, it follows from (2.2) and the Cauchy's-Schwarz inequality that

$$\sum_{n=2}^{\infty}[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]^{\frac{1}{2}}[(1+\beta)(n+\lambda n(n-1))-(\delta+\beta)]^{\frac{1}{2}}\times$$

$$\frac{(a)_{n-1}}{(c)_{n-1}}\sqrt{a_{n,1}a_{n,2}}\leq 1. \qquad (3)$$

Thus we need to find the largest such that

$$\sum_{n=2}^{\infty}\frac{[(1+\beta)(n+\lambda n(n-1))-(\gamma+\beta)]}{1-\gamma}\frac{(a)_{n-1}}{(c)_{n-1}}a_{n,1}a_{n,2}$$

$$\leq\sum_{n=2}^{\infty}\frac{[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]^{\frac{1}{2}}[(1+\beta)(n+\lambda n(n-1))-(\delta+\beta)]^{\frac{1}{2}}}{\sqrt{(1-\alpha)(1-\delta)}}\times$$

$$\frac{(a)_{n-1}}{(c)_{n-1}}\sqrt{a_{n,1}a_{n,2}}$$

Or, equivalently that

$$\sqrt{a_{n,1}a_{n,2}}\leq\frac{1-\gamma}{\sqrt{(1-\alpha)(1-\delta)}}\times$$

$$\frac{[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]^{\frac{1}{2}}[(1+\beta)(n+\lambda n(n-1))-(\delta+\beta)]^{\frac{1}{2}}}{[(1+\beta)(n+\lambda n(n-1))-(\gamma+\beta)]}$$

By view of (3.3) it is sufficient to find largest γ such that

$$\frac{\sqrt{(1-\alpha)(1-\delta)}}{[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]^{\frac{1}{2}}[(1+\beta)(n+\lambda n(n-1))-(\delta+\beta)]^{\frac{1}{2}}}\frac{(c)_{n-1}}{(a)_{n-1}}$$

$$\leq\frac{1-\gamma}{\sqrt{(1-\alpha)(1-\delta)}}\times$$

$$\frac{[(1+\beta)(n+\lambda n(n-1))-(\alpha+\beta)]^{\frac{1}{2}}[(1+\beta)(n+\lambda n(n-1))-(\delta+\beta)]^{\frac{1}{2}}}{[(1+\beta)(n+\lambda n(n-1))-(\gamma+\beta)]}$$

which yields

$$\gamma = 1 - \frac{(n-1)(1-\alpha)(1-\delta)(1+\beta)(1+\lambda n)}{\Phi(\lambda,\alpha,\beta,\delta,n)\frac{(a)_{n-1}}{(c)_{n-1}} - (1-\alpha)(1-\delta)}, \quad n \geq 2,$$

where $\Phi(\lambda,\alpha,\beta,\delta,n) = [(1+\beta)(n+\lambda n(n-1)) - (\alpha+\beta)][(1+\beta)(n+\lambda n(n-1)) - (\delta+\beta)]$.

Since

$$\phi(n) = 1 - \frac{(n-1)(1-\alpha)(1-\delta)(1+\beta)(1+\lambda n)}{\Phi(\lambda,\alpha,\beta,\delta,n)\frac{(a)_{n-1}}{(c)_{n-1}} - (1-\alpha)(1-\delta)}, \quad n \geq 2 \tag{4}$$

is an increasing function of n $(n \geq 2)$, for $-1 \leq \alpha < 1$, $-1 \leq \delta \leq 1$, $\lambda \geq 0$ and $\beta \geq 0$, letting $n = 2$ in (3.4), we have

$$\gamma \leq \phi(2) = 1 - \frac{(1-\alpha)(1-\delta)(1+\beta)(1+2\lambda)}{(2+\beta+2\lambda(1+\beta)-\alpha)(2+\beta+2\lambda(1+\beta)-\delta)\frac{(a)}{(c)} - (1-\alpha)(1-\delta)}$$

which completes the proof.\

Theorem 3.2-Let the functions fj(z) (j = 1, 2) defined by (3.1), be in the class $S_T(\lambda,\alpha,\beta)$ with $-1 \leq \alpha \leq 1,\ \lambda \geq 0$ and $\beta \geq 0$. Then (f1 ∗ f2)(z) ∈ $S_T(\lambda,\eta,\beta)$ where

$$\eta = 1 - \frac{(1-\alpha)^2(1+\beta)(1+2\lambda)}{(2+\beta+2\lambda(1+\beta)-\alpha)^2\frac{(a)}{(c)} - (1-\alpha)^2}$$

Proof-By taking $\delta = \alpha$ in the above theorem, the result follows.

Theorem 3.3-Let the function f(z) defined by (1.2) be in the class $S_T(\lambda,\alpha,\beta)$.

Also let g(z) = $z - \sum_{n=2}^{\infty} b_n z^n$ *for* $|b_n| \leq 1$. *Then* $(f * g)(z) \in S_T(\lambda,\alpha,\beta)$.

Proof Since

$$\sum_{n=2}^{\infty}[(1+\beta)(n+\lambda n(n-1)) - (\alpha+\beta)]\frac{(a)_{n-1}}{(c)_{n-1}}|a_n b_n|$$

$$\leq \sum_{n=2}^{\infty}[(1+\beta)(n+\lambda n(n-1)) - (\alpha+\beta)]\frac{(a)_{n-1}}{(c)_{n-1}}a_n|b_n|$$

$$\leq \sum_{n=2}^{\infty}[(1+\beta)(n+\lambda n(n-1)) - (\alpha+\beta)]\frac{(a)_{n-1}}{(c)_{n-1}}a_n$$

$$\leq 1-\alpha$$

it follows that $(f * g)(z) \in S_T(\lambda,\alpha,\beta)$, by the view of Theorem 2.2.

Theorem 3.4-Let the functions fj(z) (j = 1, 2) defined by (3.1) be in the class $S_T(\lambda,\alpha,\beta)$. Then the function h(z)

defined by h(z) = $h(z) = z - \sum_{n=2}^{\infty}(a_{n,1}^2 + a_{n,2}^2)z^n$ is in the class *where* $\xi = 1 - \frac{2(1-\alpha)^2(1+\beta)(1+2\lambda)}{(2+\beta+2\lambda(1+\beta)-\alpha)^2\frac{(a)}{(c)} - 2(1-\alpha)^2}$

Proof -By virtue of Theorem 2.2, it is sufficient to prove that

$$\sum_{n=2}^{\infty}\frac{[(1+\beta)(n+\lambda n(n-1)) - (\xi+\beta)]}{1-\xi}\frac{(a)_{n-1}}{(c)_{n-1}}(a_{n,1}^2 + a_{n,2}^2) \leq 1 \tag{5}$$

Where $f_j(z) \in S_T(\lambda,\alpha,\beta)\ (j = 1,2)$ we find from (3.1) and Theorem 2.2, that

$$\sum_{n=2}^{\infty}\left\{\frac{[(1+\beta)(n+\lambda n(n-1)) - (\alpha+\beta)]}{1-\alpha}\frac{(a)_{n-1}}{(c)_{n-1}}\right\}^2 a_{n,j}^2$$

$$\leq \sum_{n=2}^{\infty}\left\{\frac{[(1+\beta)(n+\lambda n(n-1)) - (\alpha+\beta)]}{1-\alpha}\frac{(a)_{n-1}}{(c)_{n-1}}a_{n,j}\right\}^2 \tag{6}$$

which yields

$$\sum_{n=2}^{\infty}\frac{1}{2}\left\{\frac{[(1+\beta)(n+\lambda n(n-1)) - (\alpha+\beta)]}{1-\alpha}\frac{(a)_{n-1}}{(c)_{n-1}}\right\}^2 (a_{n,1}^2 + a_{n,2}^2) \leq 1. \tag{7}$$

On comparing (3.6) and (3.7), it is easily seen that the inequality (3.5) will be satisfied if

$$\frac{[(1+\beta)(n+\lambda n(n-1)) - (\xi+\beta)]}{1-\xi}\frac{(a)_{n-1}}{(c)_{n-1}} \leq \frac{1}{2}\left\{\frac{[(1+\beta)(n+\lambda n(n-1)) - (\alpha+\beta)]}{1-\alpha}\frac{(a)_{n-1}}{(c)_{n-1}}\right\}^2$$

for $n \geq 2$. That is, if

$$\xi \leq 1 - \frac{2(n-1)(1-\alpha)^2(1+\beta)(1+n\lambda)}{[(1+\beta)(n+\lambda n(n-1)) - (\alpha+\beta)]^2\frac{(a)_{n-1}}{(c)_{n-1}} - 2(1-\alpha)^2}. \tag{8}$$

Since

$$\psi(n) = 1 - \frac{2(n-1)(1-\alpha)^2(1+\beta)(1+n\lambda)}{[(1+\beta)(n+\lambda n(n-1)) - (\alpha+\beta)]^2\frac{(a)_{n-1}}{(c)_{n-1}} - 2(1-\alpha)^2}$$

is an increasing function of n (n ≥ 2). Taking n = 2 in (3.8),we have

$$\xi \leq \psi(2) = 1 - \frac{2(1-\alpha)^2(1+\beta)(1+2\lambda)}{(2+\beta+2\lambda(1+\beta)-\alpha)^2\frac{(a)}{(c)} - 2(1-\alpha)^2}$$

which completes the proof.

IV. References

1) B.C. Carlson and S.B. Shaffer, Starlike and Prestarlike hypergeometric functions, SIAM J. Math. Anal., 15 (2002), 737-745.
2) A.W. Goodman, On uniformly convex functions, Ann. Polon. Math., 56 (1991), 87-92.
3) A.W. Goodman, On uniformly starlike functions, J.Math. Anal. & Appl. 155 (1991), 364-370.
4) F. Ronning, Uniformly convex functions and a corresponding class of starlike functions, Proc. Amer. Math. Soc., 118 (1993), 189-196.
5) F. Ronning, Integral representation for bounded starlike functions, Annal. Polon. Math., 60 (1995), 289-297.

6) Schild and Silverman, Convolution of univalent functions with negative coefficients, Ann. Univ. Marie Curie - Sklodowska Sect. A, 29 (1975), 99-107.
7) Silverman, Univalent functions with negative coefficients, Proc. Amer. Math. Soc. 51 (1975), 109-116.
8) Silverman, Partial sums of starlike and convex functions, J. Math. Anal. & Appl., 209 (1997), 221-227.
9) K.G. Subramanian, G. Murugusundaramoorthy, P. Balasubrahmanyam and H. Silverman, Subclasses of uniformly convex and uniformly starlike functions, Math. Japonica, 42(3) (1995), 517-522.
10) K.G. Subramanian, T.V. Sudharsan, P. Balasubrahmanyam and H. Silverman, Classes of uniformly starlike functions, Publ. Math. Debrecen.,53(3-4) (1998), 309-315.

When Is An Algebra Of Endomorphisms An Incidence Algebra?

GJSFR Classification – F (FOR) 010101,010105,010206

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***Abstract*-Spiegel and O'Donnell give a characterization of algebras of n×n matrices which are isomorphic to incidence algebras of partially ordered sets with n elements. We generalize this result to get a characterization of algebras of endomorphisms of a vector space which are isomorphic to incidence algebras of lower finite partially ordered sets.**

AMS Subject Classification: 16S50.

Keywords-incidence algebra, partially ordered set, lower finite, endomorphism.

I. Introduction

A partially ordered set X is said to be locally finite if, the subset Xyz = {x ∈X : $y \le x \le z$} is finite for each y, z ∈ X such that $u < z$. X is said to be lower finite if the subset $X_z = \{x \in X : x \le z\}$ is finite for each z ∈ X and is said to be upper finite if the subset $X_z = \{x \in X : x \le z\}$ is finite for each z ∈X. If a partially ordered set X is lower or upper finite then it is clearly locally finite.

The Incidence algebra *I(X,R)* of a locally finite partially ordered set *X* over the commutative ring R with identity is

$$I(X,R) = \{f : X \times X \to R \mid f(x,y) = 0 \; if \; x \not\leq y\}$$

with operations defined by

$$(f+g)(x,y) = f(x,y) + g(x,y)$$
$$(f.g)(x,y) = \sum_{x \le z \le y} f(x,z).g(z,y)$$
$$(r.f)(x,y) = r.f(x,y)$$

for all f, g ∈ I(X,R), r ∈ R and x, y, z ∈ X. The identity element of I(X,R) is

$$\text{for all } f,g \in I(X,R),\ r \in R$$
$$\delta(x,y) = \begin{cases} 1 & if\ x = y \\ 0 & otherwise \end{cases}$$

The JacobsonRadical, denoted by J(T) of a ring with identity is the intersection of all its maximal right ideals. This is always a two sided ideal and it is the largest ideal J of the ring T such that 1 − t is invertible for all t ∈J. It is proved ([1], Theorem 4.2.5) that the Jacobson Radical of an incidence algebra consists of all the functions f ∈X. So we have,

Proposition 1.- ([1], Cor.4.2.6) Let X be a locally finite partially ordered set

and R a commutative ring with identity. Then

$$I(X,R)/J(I(X,R)) \cong \prod_{x \in X} R/J(R).$$

The following result gives a relation between multiplication in an incidence algebra and matrix multiplication.

Proposition *2-([1],* Proposition *1.2.4)* Let X be a locally finite partially ordered set and R a commutative ring with identity. Then *I(X,R)* is isomorphic to a subring of $M_{|X|}(R)$, the *R*−module of all maps from *X×X to R* with pointwise addition and scalar multiplication.

Then a natural question that arises is that, which subalgebras of $M_{|X|}(R)$ are incidence algebras? For incidence algebras of finite posets over a field, we have the following characterization,

Theorem 1-([1], Theorem4.2.10) Let *K* be a field and S a subalgebra of $M_n(K)$. Then there is a partially ordered set *X* of order n such that I(*X,K*) ≅ S if and only if

i. S contains n pairwise orthogonal idempotents, and
ii. $\frac{S}{J(S)}$ is commutative.

II. A characterization of I(X,K) where X is a lower finite partially ordered set and K is a field

Theorem 2- Let V be a *K*−vector space with dimension |*X*|, for a suitable set *X*. Let S be a subalgebra of $End_K V$. Then there exists a lower finite partial ordering in X such that S ≅ *I(X,K)* if and only if,

i. 1 ∈ *S*
ii. S/J(S) is commutative
iii. For each x ∈ X, there is an Ex ∈ S of rank 1, such that $E_x.E_y = \delta_{xy}E_x$ and $\bigoplus_{x \in X} E_x(V) = V$
iv. Xy = $\{z \in X \mid E_z.S.E_y \neq 0\}$ is finite for each y ∈ X

Proof-First we prove that the conditions given are sufficient. Let S be A subalgebra of EndKV satisfying conditions (1), (2), (3) and (4). From condition (3) it is clear that we may find a basis $\{v_x \mid x \in X\}$ for V such that $E_x(v_y) = \delta_{xy}v_x$.

Define $E_{xy} \in End_K(V)$ by $E_{xy}(vz) = \delta_{yz}v_x$.

Define an order "≤" in X by $x \le y$ if and only if $E_{xy} \in S$ for all x ∈ X, ≤ is reflexive

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If $x \leq y$ and $y \leq z$, then we have $E_{xy}, E_{yz} \in S$, Hence their product $E_{xz} \in S$ which implies $x \leq z$. So $\leq$ is transitive.

Now let $x \leq y$ and $y \leq x$. That is $E_{xy}, E_{yx} \in S$. Since $S/J(S)$ is commutative w have $E_{xy}.E_{yx} + J(S) = E_{yx}.E_{xy} + J(S)$, which implies $E_{xx} + J(S) = E_{yy} + J(S)$ So, $E_{xx} - E_{yy} \in J(S)$, that is, $1 - E_{xx} + E_{yy}$ is invertible. Hence $x = y$ and $\leq$ i

antisymmetric.

Now from condition (4), it is clear that $(X, \leq)$ is a lower finite partially ordered set, since

$$X_y = \{z \in X \mid E_{zy} \in S\} = \{z \in X \mid z \leq y\}$$

We prove that $S \cong I(X, K)$ Observe that, for each $T \in End_K(V)$, we have by writing

$$T(v_y) = \sum_{x \in X} t_{x,y} v_x,$$

$$E_{xx}.T.E_{yy}(v_z) = 0 \text{ if } z \neq y \text{ and}$$

$$\begin{aligned} E_{xx}.T.E_{yy}(v_y) &= E_{xx}.T(v_y) \\ &= E_{xx}\left(\sum_{z \in X} t_{zy} v_z\right) \quad where \quad t_{zy} \in K \\ &= t_{xy} v_x \end{aligned}$$

So $E_{xx}.T.E_{yy} = t_{xy} E_{xy}$

Define $\Phi : S \to I(X, K)$ by $\Phi(T) = f_T$, where $f_T(x, y) = t_{xy}$

It is clear that Φ will preserve addition and scalar multiplication and Φ will map identity of S to identity in I(X,K). Now we will prove that Φ will preserve multiplication. Let $P(v_y) = \sum_{z \in X} p_{zy} v_z$ and $Q(v_y) = \sum_{z \in X} q_{zy} v_z$ for two elements P,Q $\in$ S.

Then,

$$\begin{aligned} E_{xx}.PQ.E_{yy}(v_y) &= E_{xx}.P.Q(v_y) \\ &= E_{xx}.P\Big(\sum_{z \in X} q_{zy} v_z\Big) \\ &= E_{xx}. \sum_{z \in X} q_{zy} P(v_z) \\ &= E_{xx}. \sum_{z \in X} q_{zy} \sum_{u \in X} p_{uz} v_u \\ &= E_{xx}. \sum_{z,u \in X} q_{zy} p_{uz} v_u \\ &= E_{xx}. \sum_{z,u \in X} p_{uz} q_{zy} v_u \\ &= \sum_{z \in X} p_{xz} q_{zy} v_x \end{aligned}$$

So that

$$\Phi(PQ) = f_{PQ}, \text{ where } f_{PQ}(x, y) = \sum_{z \in X} p_{xz} q_{zy} = (f_P.f_Q)(x, y)$$

So Φ preserves multiplication, and hence Φ is a homomorphism. If $\Phi(P) = \Phi(Q)$ then $p_{xy} = q_{xy}$ for each pair x, y $\in$ X and this implies P = Q. So is injective. Let $f \in I(X, K)$ Define T such that $T(v_y) = \sum_{x \in X} f(x, y) v_x$ (this is possible since X is lower finite). Clearly, Φ(T) = f.

Hence Φ is surjective and Φ is an isomorphism from S to I(X,K).

Now we have to prove that conditions (1), (2), (3) and (4) are necessary. Suppose that $I(X,K) \cong$ S where S is a subalgebra of EndK(V) of a K−Vector space V , with dimension |X|, where X is a lower finite partially ordered set. Let us consider the map Φ^{-1} : I(X,K) ! EndKV where Φ is the map that is defined in the proof of sufficiency part.Thus $\Phi^{-1}(f) = T_f$ where Tf is such that $T_f(v_y) = \sum_{x \leq y} f(x, y) v_x$. So $Im(\Phi^{-1})$ will be isomorphic to I(X,K) and will satisfy the conditions (1), (3), (4) clearly and (2) follows from Proposition 1 and the fact that $I(X,K)/J(I(X,K)) \cong S/J(S)$. So $S \cong I(X,K) \cong Im(\Phi^{-1})$. So S Will also have the properties specified in conditions (1), (2), (3) and (4). Hence the theorem.

III. References

1) E.Spiegel and C.J.O'Donnell ; Incidence Algebras, Monographs and Textbooks in Pure and Applied Mathematics, Vol.206, Marcel Dekker, New York, 1997.

Generalized Fractional Calculus Of Certain Product Of Special Functions

GJSFR Classification – F (FOR) 010102,010110,010103

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***Abstract*- The paper is devoted to study the generalized fractional calculus of arbitrary complex order for the $\overline{H}$ -function defined by Inayat Hussain [8]. The classical fractional integrals and derivatives of Riemann-Liouville type are treated. The considered generalized fractional integration and differentiation operators contain the Gauss Hypergeometric function as a kernel and generalize classical Riemann-Liouville, Erdelyi-Kober types ones. It is proved that the generalized fractional integrals and derivatives of $\overline{H}$ -function turn also out $\overline{H}$ -functions but of greater order. Especially, the obtained results define more precise and general ones than known. Corresponding assertion for Riemann-Liouville and Erdelyi-Kober fractional integral operators are also presented.**

Keywords- Fractional integral operator, $\overline{H}$ - function, Riemann Zeta-function, General class of polynomials.

I. Introduction

By evaluating certain Feynman integrals which arise naturally in perturbation calculations of the equilibrium properties of a magnetic model of phase transitions, Inayat-Hussain [7] derived a number of interesting properties and characteristics of hypergeometric functions of one and more variables. While presenting some further examples of the use of these Feynman integrals, Inayat-Hussain [8] was led to a novel generalization of the familiar H-function of Charles Fox [4]. This function is popularly known as $\overline{H}$ -function and contains some new special cases, such as the polylogarithm of a complex order and the exact partition function of the Gaussian model in Statistical mechanics. In terms of Mellin-Barnes contour integral, it is defined as follows:

$$\overline{H}^{M,N}_{P,Q}\left(z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right)$$

$$=\frac{1}{2\pi i}\int_L \overline{\phi}(\xi)z^{\xi}d\xi, \qquad (z\neq 0) \tag{1.1}$$

Where

$$\overline{\phi}(\xi)=\frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

Here $a_j\,(j=1,\dots,p)$ and $b_j\,(j=1,\dots,q)$ are complex parameters, $A_j>0\,(j=1,\dots,p)$ and the exponents $a_j\,(j=1,\dots,p)$ can take noninteger values. The following sufficient conditions for the absolute convergence of the defining integral for $\overline{H}$ - Function given by (1.1) have been recently given by Gupta, Jain and Agrawal [5]:

(i) $|\arg(z)|<1/2\Omega\pi$ *and* $\Omega>0$,

(ii) $|\arg(z)|=1/2\Omega\pi$ *and* $\Omega\geq 0$,

and (a) $\mu\neq 0$ and the contour L is so chosen that $(c\mu+\lambda+1)<0$,

(b) $\mu=0$ *and* $(\lambda+1)<0$,

where

$$\Omega=\sum_1^M\beta_j+\sum_1^N\alpha_jA_j-\sum_{M+1}^Q\beta_jB_j-\sum_{N+1}^P\alpha_j, \tag{1.4}$$

$$\mu=\sum_1^N\alpha_jA_j+\sum_{N+1}^P\alpha_j-\sum_1^M\beta_j-\sum_{M+1}^Q\beta_jB_j, \tag{1.5}$$

$$\lambda=\mathrm{Re}\left(\sum_1^M b_j+\sum_{M+1}^Q b_jB_j-\sum_1^N a_jA_j-\sum_{N+1}^P a_j\right)+\frac{1}{2}\left(-M-\sum_{M+1}^Q B_j+\sum_1^N A_j+P-N\right) \tag{1.6}$$

It may be noted that the conditions of validity given above are more general than those given earlier [1].

The following series representation of the $\overline{H}$ -function was given by Rathie [12].

$$\overline{H}^{M,N}_{P,Q}\left(z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right)=\sum_{\upsilon=1}^{M}\sum_{P=0}^{\infty}\overline{\theta}(S_{P,\upsilon})z^{S_{P,\upsilon}} \tag{1.7}$$

where

$$\overline{\theta}(S_{P,\upsilon})=\frac{\prod_{j=1,j\neq\upsilon}^{M}\Gamma(b_j-\beta_jS_{P,\upsilon})\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_jS_{P,\upsilon})\}^{A_j}(-1)^P}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_jS_{P,\upsilon})\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_jS_{P,\upsilon})P!\beta_\upsilon} \tag{1.8}$$

The following behavior of $\overline{H}^{M,N}_{P,Q}[z]$ for small and large value of z as recorded by Saxena [16, p.112, eqs. (2.3) and (2.4)] will be required in the sequel

$\overline{H}^{M,N}_{P,Q}[z]=O\left[|z|^{\alpha}\right]$ for small z, where

$$\alpha=\min_{1\leq j\leq M}\left[\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)\right] \tag{1.9}$$

$\overline{H}^{M,N}_{P,Q}[z] = O\left[|z|^{\beta}\right]$ for large z, where

$$\beta = \max_{1\le j\le N}\left[\mathrm{Re}\left(\frac{a_j - 1}{\alpha_j}\right)\right] \quad (1.0)$$

and the conditions (1.3) are satisfied. Also $S_V^U[x]$ occurring in the sequel denotes the general class of polynomials [17, p.1, Eq. (1)]

$$S_V^U[x] = \sum_{R=0}^{[V/U]} (-V)_{UR} A(V,R) \frac{x^R}{R!} \quad (1.11)$$

where U is an arbitrary positive integer, V=0,1,2,… and the coefficients A(V,R) $A_{V,R}\,(V,R\ge 0)$ are arbitrary constants, real or complex. On suitably specializing the coefficients $A_{V,R}$ yields a number of known polynomials as its special cases.

II. Classical and Generalized Fractional Calculas Operators

For α ∊ C (Re (α) > 0), the Riemann-Liouville left and right-sided fractional calculus operators are defined as the following ([15, sections 2.3 and 2.4)]:

$$(I_{0+}^{\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f(t)}{(x-t)^{1-\alpha}} dt, \quad (x>0) \quad (1)$$

$$(I_{0-}^{\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_x^\infty \frac{f(t)}{(t-x)^{1-\alpha}} dt, \quad (x>0) \quad (2)$$

and

$$(D_{0+}^{\alpha} f)(x) = \left(\frac{d}{dx}\right)^{[\mathrm{Re}(\alpha)+1]} \left(I_{0+}^{1-\alpha+[\mathrm{Re}(\alpha)]} f\right)(x)$$

$$= \left(\frac{d}{dx}\right)^{[\mathrm{Re}(\alpha)]+1} \frac{1}{\Gamma(1-\alpha+[\mathrm{Re}(\alpha)])}$$

$$\int_0^x \frac{f(t)}{(x-t)^{\alpha-[\mathrm{Re}(\alpha)]}} dt, \quad (x>0) \quad (3)$$

$$(D_{-}^{\alpha} f)(x) = \left(-\frac{d}{dx}\right)^{[\mathrm{Re}(\alpha)]+1} \left(I_{-}^{1-\alpha+[\mathrm{Re}(\alpha)]} f\right)(x)$$

$$= \left(-\frac{d}{dx}\right)^{[\mathrm{Re}(\alpha)]+1} \frac{1}{\Gamma 1-\alpha+\mathrm{Re}(\alpha)])}$$

$$\int_x^\infty \frac{f(t)}{(t-x)^{\alpha-[\mathrm{Re}(\alpha)]}} dt, \quad (x>0) \quad (4)$$

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respectively, where [Re(α)] is the integral part of Re (α). In particular, for real α > 0, the operators D_{0+}^{α} and D_{-}^{α} take more simple forms.

$$(D_{0+}^{\alpha} f)(x) = \left(\frac{d}{dx}\right)^{[\alpha]+1} (I_{0+}^{1-\{\alpha\}} f)(x)$$

$$= \left(\frac{d}{dx}\right)^{[\alpha]+1} \frac{1}{\Gamma(1-\{\alpha\})} \int_0^x \frac{f(t)}{(x-t)^{\{\alpha\}}} dt, \quad (x>0) \quad (2.5)$$

$$(D_{-}^{\alpha} f)(x) = \left(-\frac{d}{dx}\right)^{[\alpha]+1} (I_{-}^{1-\{\alpha\}} f)(x)$$

$$= \left(-\frac{d}{dx}\right)^{[\alpha]+1} \frac{1}{\Gamma(1-\{\alpha\})} \int_x^\infty \frac{f(t)}{(t-x)^{\{\alpha\}}} dt, \quad (x>0) \quad (2.6)$$

where [α] and {α} are integral and fractional parts of α. For α, β, η ∊ C and x > 0 the generalized fractional calculus operators [13] are defined by

$$(I_{0+}^{\alpha,\beta,\eta} f)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1}$$

$${}_2F_1(\alpha+\beta,-\eta;\alpha;1-\frac{t}{x})\, f(t)\, dt, \quad \left[\mathrm{Re}(\alpha)>0\right] \quad (2.7)$$

$$(I_{0+}^{\alpha,\beta,\eta} f)(x) = \left(\frac{d}{dx}\right)^n (I_{0+}^{\alpha+n,\beta-n,\eta-n} f)(x)$$

$$(\mathrm{Re}(\alpha)<0;\eta=[\mathrm{Re}(-\alpha)]+1); \quad (2.8)$$

$$(I_{-}^{\alpha,\beta,\eta} f)(x) = \frac{1}{\Gamma(\alpha)} \int_x^\infty (t-x)^{\alpha-1} t^{-\alpha-\beta}$$

$${}_2F_1(\alpha+\beta,-\eta;\alpha;1-\frac{x}{t}) f(t)\, dt \quad (\mathrm{Re}(\alpha)>0); \quad (2.9)$$

$$(I_{-}^{\alpha,\beta,\eta} f)(x) = \left(-\frac{d}{dx}\right)^n (I_{-}^{\alpha+n,\beta-n,\eta} f)(x)$$

$$(\mathrm{Re}(\alpha)<0;\eta=[\mathrm{Re}(-\alpha)]+1); \quad (2.10)$$

and

$$(D_{0+}^{\alpha,\beta,\eta} f)(x) \equiv (I_{0+}^{-\alpha,-\beta,\alpha+\eta}) f(x)$$

$$= \left(\frac{d}{dx}\right)^n (I_{0+}^{-\alpha+n,-\beta-n,\alpha+\eta-n} f)(x)$$

$$(\mathrm{Re}(\alpha)>0;n=[\mathrm{Re}(\alpha)]+1); \quad (2.11)$$

$$(D_{-}^{\alpha,\beta,\eta} f)(x) \equiv (I_{-}^{-\alpha,-\beta,\alpha+\eta}) f(x)$$

$$= \left(-\frac{d}{dx}\right)^n (I_{0+}^{-\alpha+n,-\beta-n,\alpha+\eta} f)(x)$$

$$(\mathrm{Re}(\alpha)>0;n=[\mathrm{Re}(\alpha)]+1) \quad (2\ 12)$$

Here ${}_2F_1(a,b,c;z)\ (a,b,c.z\in C)$ is the Gauss hypergeometric function of the series form

$$ {}_2F_1(a,b;c;z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!} \quad (2.13)$$

With

$$(a)_0 = 1, (a)_k = a(a+1)\ldots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} \left(k \in N\right) \quad (2.14)$$

where Γ(z) is the gamma function [3, Chap.I] and **N** denotes the set of positive integers.
The series in (2.13) is convergent for | z | < 1 and | z | = 1 with Re (c-a-b) > 0 and can be analytically continued into {z ∈ C : |arg (1- z) | < π} [3, Chapter II].
Since

$$ {}_2F_1(0,b;c;z) = 1 , \quad (2.15)$$

the generalized fractional calculus operators (2.7), (2.9), (2.11) and (2.12) coincide, if $\beta = -\alpha$ with the Riemann-Liouville operators (2.1) – (2.4) for Re (α) > 0:

$$(I_{0+}^{\alpha,-\alpha,\eta} f)(x) = (I_{0+}^{\alpha} f)(x),$$
$$(I_{-}^{\alpha,-\alpha,\eta} f)(x) = (I_{-}^{\alpha} f)(x) \quad (2.16)$$
$$(D_{0+}^{\alpha,-\alpha,\eta} f)(x) = (D_{0+}^{\alpha} f)(x),$$

$$(D_{-}^{\alpha,-\alpha,\eta} f)(x) = (D_{-}^{\alpha} f)(x) \quad (2.17)$$

According to the relation [3, 2.8 (4)]

$$ {}_2F_1(a,b;a;z) = (1-z)^{-b} , \quad (2.18)$$

the operators (2.7) and (2.9) coincide with the Erdelyi-Kober fractional integrals [15, § 18.1] when β = 0:

$$(I_{0+}^{\alpha,0,\eta} f)(x) = \frac{x^{-\alpha-\eta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} t^{\eta} f(t)\, dt$$

$$= (I_{\eta,\alpha}^{+} f)(x), \quad (\alpha,\eta \in \text{ Re}(\alpha) > 0), \quad (2.19)$$

$$(I_{-}^{\alpha,0,\eta} f)(x) = \frac{x^{\eta}}{\Gamma(\alpha)} \int_x^{\infty} (t-x)^{\alpha-1} t^{-\eta-\alpha} f(t)\, dt$$

$$\equiv (K_{\eta,\alpha}^{-} f)(x) \quad (\alpha,\eta \in \mathbf{C}, \text{Re}(\alpha)>0). \quad (2.20)$$

Therefore the operators (2.7), (2.9) and (2.11), (2.12) are called 'generalized' fractional integrals and derivatives, respectively. Moreover, the operators (2.11) and (2.12) are inverse to (2.7) and (2.9):

$$D_{0+}^{\alpha,\beta,\eta} = (I_{0+}^{\alpha,\beta,\eta})^{-1}, \; D_{-}^{\alpha,\beta,\eta} = (I_{-}^{\alpha,\beta,\eta})^{-1} \quad (2.21)$$

We also need following asymptotic behaviour of ${}_2F_1(a,b;c;z)$ at the point z = 1.

Lemma 1. For a, b, c ∈ **C** with Re (c) > 0 and z ∈ C, there hold the following asymptotic relations near z = 1:

$$ {}_2F_1(a,b;c;z) = 0(1) \quad (z \to 1-) \quad (2.22)$$

for Re (c – a – b) > 0 ;

$$ {}_2F_1(a,b;c;z) = 0\left((1-z)^{c-a-b}\right) \quad (z \to 1-)$$

Re (c – a – b) < 0 ; (2.23)

And

$ {}_2F_1(a,b;c;z) = 0(\log(1-z)) \quad (z \to 1-);$ for c – a – b = 0, a, b ≠ 0, -1, -2,… and |arg (z) | < π. (2.24)

III. *LEFT-SIDED GENERALIZED FRACTIONAL INTEGRATION OF THE* $\bar{\mathbf{H}}$ *-FUNCTION*

Here we consider the left sided generalized fractional integration $I_{0+}^{\alpha,\beta,\eta}$ defined by (2.7).

THEOREM 1- Let α, β, η ∈ C with Re (α) > 0, Re (β) ≠ Re (η). Let the constants aj, bj ∈ C, αj, βj > 0 (j = 1,…, P; j = 1,…,Q) and λ ∈ C, $\sigma_1, \sigma_2 > 0$ and

$$\sigma_1 \min_{1\le j\le M_1} \text{Re}\left(\frac{f_j}{F_j}\right) + \sigma_2 \min_{1\le j\le M} \text{Re}\left(\frac{b_j}{\beta_j}\right)$$

$$+ \text{Re}(\lambda) + \min[0, \text{Re}(\eta-\beta)] > 0 \quad (3.1)$$

Then the generalized fractional integral $I_{0+}^{\alpha,\beta,\eta}$ of the product of $\bar{H}$ -functions with $S_V^U[\delta t^{\rho}]$ exists and the following relation holds:

$$\left(I_{0+}^{\alpha,\beta,\eta} t^{\lambda-1} \bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1} \left| \begin{matrix} (e_j,E_j;\varepsilon_j)_{1,N_1},(e_j,E_j)_{N_1+1,P_1} \\ (f_j,F_j)_{1,M_1},(f_j,F_j;\Im_j)_{M_1+1,Q_1} \end{matrix} \right. \right] \right.$$
$$\left. \bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right] S_V^U[\delta t^{\rho}] \right)(x)$$
$$= x^{\lambda-\beta-1} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A(V,R) \delta^R x^{\rho R} \sum_{\upsilon=1}^{M_1} \sum_{p=0}^{\infty} \bar{\theta}\left(S_{p,\upsilon}\right)$$
$$\left(w_1 x^{\sigma_1}\right)^{S_{p,\upsilon}} \bar{H}_{P+2,Q+2}^{M,N+2}\left[w_2 x^{\sigma_2} \left| \begin{matrix} (1-\lambda-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1) \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right.$$
$$\left. \begin{matrix} (1-\lambda-\eta+\beta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (1-\lambda+\beta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)(1-\lambda-\alpha-\eta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1) \end{matrix} \right] \quad (3.2)$$

Proof-By (2.7), we have

$$\left(I_{0+}^{\alpha,\beta,\eta} t^{\lambda-1} \bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1} \right] \bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2} \right] S_V^U[\delta t^{\rho}] \right)(x)$$
$$= \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} t^{\omega} {}_2F_1(\alpha+\beta,-\eta;\alpha;1-\frac{t}{x})$$
$$\bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1} \right] \bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2} \right] S_V^U[\delta t^{\rho}]\, dt$$

According to (2.22), (2.23) the integrand in (3.3) for any x > 0 has the asymptotic estimate at zero

$$(x-t)^{\alpha-1} t^{\omega} {}_2F_1\left(\alpha+\beta,-\eta;\alpha;\frac{t}{x}\right)$$

$$\bar{H}^{M_1,N_1}_{P_1,Q_1}\left[w_1 t^{\sigma_1}\right]\bar{H}^{M,N}_{P,Q}\left[w_2 t^{\sigma_2}\right] S^U_V[\delta t^{\rho}]$$

$$= O\left(t^{\lambda+\sigma_1 e^*+\sigma_2 f^*+\min[0,\,\mathrm{Re}(\eta+\beta)]-1}\right)\ (t\to+0)$$

$$= O\left(t^{\lambda+\sigma_1 e^*+\sigma_2 f^*+\min[0,\,\mathrm{Re}(\eta+\beta)]-1}[\log(t)]^{N^*}\right),(t\to+0)$$

Here

$$e^* = \min_{1\le j\le M_1}\left[\frac{\mathrm{Re}(f_j)}{F_j}\right],\ f^* = \min_{1\le j\le M}\left[\frac{\mathrm{Re}(b_j)}{\beta_j}\right]$$

and N^* is the order of one of the poles $b_{j\ell} = \frac{-b_j-\ell}{\beta_j}$ (j = 1,…,M; ℓ = 0,1,2,…) to which some other poles of $\Gamma(b_j+\beta_j s)$ (j = 1,…,M) coincide. Therefore the condition (3.1) ensures the existence of the integral (3.3).

Applying (1.1), (1.7), (1.11), making the change of variable t = x τ, changing the order of summation and integration and taking into account the formula [11, § 2.21.1.11]

$$\int_0^x t^{\alpha-1}(x-t)^{c-1}\,{}_2F_1\left(a,b;c;1-\frac{t}{x}\right)dt$$

$$= \frac{\Gamma(c)\Gamma(\alpha)\Gamma(\alpha+c-a-b)}{\Gamma(\alpha+c-a)\Gamma(\alpha+c-b)}\, x^{\alpha+c-1}$$ (a, b, c, α ∈ C, Re (α) > 0, Re (c) > 0,

Re (α+c – a – b > 0). (3.4)

We obtain

$$\left(I^{\alpha,\beta,\eta}_{0+} t^{\lambda-1}\bar{H}^{M_1,N_1}_{P_1,Q_1}\left[w_1 t^{\sigma_1}\right]\bar{H}^{M,N}_{P,Q}\left[w_2 t^{\sigma_2}\right] S^U_V[\delta t^{\rho}]\right)(x)$$

$$= \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\frac{1}{2\pi i}\int_L \bar{\phi}(\xi)(w_2)^{\xi}d\xi \sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{(V,R)}\delta^R$$

$$\sum_{\upsilon=1}^{M_1}\sum_{p=0}^{\infty}\bar{\theta}(S_{P,\upsilon})(w_1)^{S_{P,\upsilon}}\int_0^x t^{\lambda+\rho R+\sigma_1 S_{p,\upsilon}+\sigma_2\xi-1}$$

$$(x-t)^{\alpha-1}\,{}_2F_1\left(\alpha+\beta,-\eta;\alpha;1-\frac{t}{x}\right)dt$$

$$= x^{\lambda-\beta-1}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left(\delta t^{\rho}\right)^R \sum_{\upsilon=1}^{M_1}\sum_{p=0}^{\infty}\bar{\theta}(S_{P,\upsilon})\left(w_1 t^{\sigma_1}\right)^{S_{P,\upsilon}}$$

$$\cdot\frac{1}{2\pi i}\int_{\alpha}\bar{\phi}(\xi)\frac{\Gamma(\lambda+\rho R+\sigma_1 S_{p,\upsilon}+\sigma_2\xi)}{\Gamma(\lambda-\beta+\rho R+\sigma_1 S_{p,\upsilon}+\sigma_2\xi)}$$

$$\frac{\Gamma(\lambda+\eta-\beta+\rho R+\sigma_1 S_{p,\upsilon}+\sigma_2\xi)}{\Gamma(\lambda+\alpha+\eta+\rho R+\sigma_1 S_{p,\upsilon}+\sigma_2\xi\eta)}x^{\sigma_2\xi}d\xi$$

and in accordance with (1.1), we obtain (3.2) which completes the proof of Theorem 1.

Corollary 1.1- Let α ∈ **C** with Re (α) > 0 and let the constants λ ∈ **C,** $\sigma_1,\sigma_2>0$ and

$$\sigma_1\min_{1\le j\le M_1}\mathrm{Re}\left(\frac{f_j}{F_j}\right)+\sigma_2\min_{1\le j\le M}\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)+\mathrm{Re}(\lambda)>0 \quad (3.5)$$

then the Riemann-Liouville fractional integral I^{α}_{0+} of the product of $\bar{H}$ -functions with $S^U_V[\delta t^{\rho}]$ exists and the following relation holds:

$$\left(I^{\alpha}_{0+} t^{\lambda-1}\bar{H}^{M_1,N_1}_{P_1,Q_1}\left[w_1 t^{\sigma_1}\right]\bar{H}^{M,N}_{P,Q}\left[w_2 t^{\sigma_2}\right] S^U_V[\delta t^{\rho}]\right)(x)$$

$$= x^{\lambda+\alpha-1}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{(V,R)}\left(\delta x^{\rho}\right)^R\sum_{\upsilon=1}^{M_1}\sum_{p=0}^{\infty}\bar{\theta}(S_{P,\upsilon})$$

$$\left(w_1 x^{\sigma_1}\right)^{S_{P,\upsilon}}\bar{H}^{M,N+1}_{P+1,Q+1}\left[w_2 x^{\sigma_2}\left|\begin{matrix}(1-\lambda-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(1-\lambda-\alpha-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\end{matrix}\right] \quad (3.6)$$

Corollary 1.2- Let α, η, ∈ **C** with Re (α) > 0 and let the constants λ ∈ **C,** $\sigma_1,\sigma_2>0$ satisfy and

$$\sigma_1\min_{1\le j\le M_1}\mathrm{Re}\left(\frac{f_j}{F_j}\right)+\sigma_2\min_{1\le j\le M}\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)$$
$$+\mathrm{Re}(\lambda)+\min[0,\mathrm{Re}(\eta)]>0 \quad (3.7)$$

then the Erdélyi-kober fractional integral $I^{+}_{\eta,\alpha}$ of the product of $\bar{H}$ -functions with $S^U_V[\delta t^{\rho}]$ exists and the following relation holds:

$$\left(I^{+}_{\eta,\alpha} t^{\lambda-1}\bar{H}^{M_1,N_1}_{P_1,Q_1}\left[w_1 t^{\sigma_1}\right]\bar{H}^{M,N}_{P,Q}\left[w_2 t^{\sigma_2}\right] S^U_V[\delta t^{\rho}]\right)(x)$$

$$= x^{\lambda-1}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{(V,R)}\left(\delta x^{\rho}\right)^R\sum_{\upsilon=1}^{M_1}\sum_{p=0}^{\infty}\bar{\theta}(S_{P,\upsilon})$$

$$\left(w_1 x^{\sigma_1}\right)^{S_{P,\upsilon}}\bar{H}^{M,N+1}_{P+1,Q+1}\left[w_2 x^{\sigma_2}\left|\begin{matrix}(1-\lambda-\eta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(1-\lambda-\alpha-\eta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\end{matrix}\right] \quad (3.8)$$

IV. RIGHT-SIDED FRACTIONAL INTEGRATION OF THE $\bar{H}$ FUNCTION

In this section we consider the right sided generalized fractional integration $I^{\alpha,\beta,\eta}_{-}$ defined by (2.9).

Theorem 2.- Let α, β, η ∈ **C** with Re (α) > 0, Re (β) ≠ Re (η). Let the constants λ ∈ **C,** $\sigma_1,\sigma_2>0$ and

$$\sigma_1\max_{1\le j\le N_1}\left[\frac{\mathrm{Re}(e_j)-1}{E_j}\right]+\sigma_2\max_{1\le j\le N}\left[\frac{\mathrm{Re}(a_j)-1}{\alpha_j}\right]$$

$+\operatorname{Re}(\lambda) < \min[\operatorname{Re}(\beta), \operatorname{Re}(\eta)]$ (4.1)

then the generalized fractional integral $I_{-}^{\alpha,\beta,\eta}$ of the product of $\bar{H}$-functions with $S_V^U[\delta t^\rho]$ exists and the following relation holds:

$$\left(I_{-}^{\alpha,\beta,\eta} t^{\lambda-1} \bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1}\right] \bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]\right)(x)$$

$$= x^{\lambda-\beta-1} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{(V,R)} \left(\delta x^\rho\right)^R \sum_{\upsilon=1}^{M_1} \sum_{p=0}^{\infty} \bar{\theta}\left(S_{p,\upsilon}\right)\left(w_1 x^{\sigma_1}\right)^{S_{p,\upsilon}}$$

$$\bar{H}_{P+2,Q+2}^{M+2,N}\left[w_2 x^{\sigma_2} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (1-\lambda+\beta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2) \end{matrix} \right. \right.$$

$$\left. \begin{matrix} (1-\lambda-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2)(1-\lambda+\alpha+\beta+\eta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2) \\ (1-\lambda+\eta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2)(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right]$$

(4.2)

***Proof*-By (2.9), we have**

$$\left(I_{-}^{\alpha,\beta,\eta} t^{\lambda-1} \bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1}\right] \bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]\right)(x)$$

$$= \frac{1}{\Gamma(\alpha)} \int_x^\infty (t-x)^{\alpha-1} t^{\lambda-\alpha-\beta-1} {}_2F_1(\alpha+\beta,-\eta;\alpha;1-\frac{x}{t})$$

$$\bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1}\right] \bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]\ dt \quad (4.3)$$

Due to (2.22) and (2.23) the integrand in (4.3) for any x > 0 has the asymptotic estimate at infinity

$$(t-x)^{\alpha-1} t^{\lambda-\alpha-\beta-1} {}_2F_1\left(\alpha+\beta,-\eta;\alpha;1-\frac{x}{t}\right)$$

$$\bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1}\right] \bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]$$

$$= O\left(t^{\lambda+\sigma_1 e_1^*+\sigma_2 f_1^*-\min[\operatorname{Re}(\beta),\operatorname{Re}(\eta)]-1}\right), (t \to +\infty)$$

$$= O\left(t^{\lambda+\sigma_1 e_1^*+\sigma_2 f_1^*-\min[\operatorname{Re}(\beta),\operatorname{Re}(\eta)]-1}[\log(t)]^{N^*}\right), (t \to +\infty)$$

Here

$$e_1^* = \max_{1\le j\le N_1}\left[\frac{\operatorname{Re}(e_j)-1}{E_j}\right],\ f_1^* = \max_{1\le j\le N}\left[\frac{\operatorname{Re}(a_j)-1}{\alpha_j}\right]$$

and N^* is the order of one of the poles $a_i k = \frac{1-a_i+k}{\alpha_i}$ (i = 1,…,N ; k = 0,1,2,…) to which some other poles of Γ(1-a_i- α_is) (i = 1,2,…,N) coincide. Therefore the condition (4.1) ensures the existence of the integral (4.3).

Applying (1.2) making the change $t = \frac{1}{\tau}$ and using (3.4), we obtain

$$\left(I_{-}^{\alpha,\beta,\eta} t^{\lambda-1} \bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1}\right] \bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]\right)(x)$$

$$= \frac{1}{\Gamma(\alpha)} \int_{1/x}^\infty \left(t-\frac{1}{x}\right)^{\alpha-1} t^{\lambda-\alpha-\beta-1}$$

$${}_2F_1(\alpha+\beta,-\eta;\alpha;1-\frac{1}{tx}) \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{(V,R)} \delta^R$$

$$\sum_{\upsilon=1}^{M_1} \sum_{p=0}^{\infty} \bar{\theta}\left(S_{p,\upsilon}\right)\left(w_1\right)^{S_{p,\upsilon}} \frac{1}{2\pi i} \int_L \bar{\phi}(\xi)\left(w_2 t^{\sigma_2}\right)^\xi\ d\xi\ dt$$

$$= \frac{x^{1-\alpha}}{\Gamma(\alpha)} \int_0^x (x-\tau)^{\alpha-1}\ \tau^{\alpha+\beta-\lambda-\rho R-\sigma_1 S_{p,\upsilon}-\sigma_2\xi-1}$$

$${}_2F_1(\alpha+\beta,-\eta;\alpha;1-\frac{\tau}{x})\, d\tau \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{(V,R)} \delta^R$$

$$\sum_{\upsilon=1}^{M_1} \sum_{p=0}^{\infty} \bar{\theta}\left(S_{p,\upsilon}\right)\left(w_1\right)^{S_{p,\upsilon}} \frac{1}{2\pi i} \int_L \bar{\phi}(\xi)\left(w_2\right)^\xi\ d\xi$$

$$= x^{\lambda-\beta-1} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{(V,R)} \left(\delta t^\rho\right)^R \sum_{\upsilon=1}^{M_1} \sum_{p=0}^{\infty} \bar{\theta}\left(S_{p,\upsilon}\right)$$

$$\left(w_1 t^{\sigma_1}\right)^{S_{p,\upsilon}} \frac{1}{2\pi i} \int_L \bar{\phi}(\xi)\left(w_2\right)^\xi \frac{\Gamma(1-\lambda+\beta-\rho R-\sigma_1 S_{p,\upsilon}-\sigma_2\xi)}{\Gamma(1-\lambda-\rho R-\sigma_1 S_{p,\upsilon}-\sigma_2\xi)}$$

$$\frac{\Gamma(1-\lambda+\eta-\rho R-\sigma_1 S_{p,\upsilon}-\sigma_2\xi)}{\Gamma(1-\lambda+\alpha+\beta+\eta-\rho R-\sigma_1 S_{p,\upsilon}-\sigma_2\xi)} x^{\sigma_2\xi}\, d\xi$$

and in accordance with (1.1), we obtain (4.2) which completes the proof of Theorem 2.

Corollary 2.1 -Let α ∊ **C**, with Re (α) > 0 and let the constants λ ∊ **C,** $\sigma_1, \sigma_2 > 0$ satisfy and

$$\sigma_1 \max_{1\le j\le N_1}\left[\frac{\operatorname{Re}(e_j)-1}{E_j}\right] + \sigma_2 \max_{1\le j\le N}\left[\frac{\operatorname{Re}(a_j)-1}{\alpha_j}\right]$$

$$+\operatorname{Re}(\lambda)+\operatorname{Re}(\alpha)<0 \quad (4.5)$$

Then the Riemann-Liouville fractional integral I_{-}^{α} of the product of $\bar{H}$-functions with $S_V^U[\delta t^\rho]$ exist and the following relation hold:

$$\left(I_{-}^{\alpha} t^{\lambda-1} \bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1}\right] \bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]\right)(x)$$

$$= x^{\lambda+\alpha-1} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{(V,R)} \left(\delta x^\rho\right)^R \sum_{\upsilon=1}^{M_1} \sum_{p=0}^{\infty} \bar{\theta}\left(S_{p,\upsilon}\right)$$

$$\left(w_1 x^{\sigma_1}\right)^{S_{p,\upsilon}} \bar{H}_{P+1,Q+1}^{M+1,N}\left[w_2 x^{\sigma_2} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (1-\lambda-\alpha-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2) \end{matrix} \right. \right.$$

$$\left. \begin{matrix} (1-\lambda-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2) \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right] \quad (4.6)$$

Corollary 2.2- Let α ∈ **C**, with Re (α) > 0 and let the constants λ ∈ **C**, $\sigma_1, \sigma_2 > 0$ satisfy and

$$\sigma_1 \max_{1 \le j \le N_1} \left[\frac{\mathrm{Re}(e_j) - 1}{E_j} \right] + \sigma_2 \max_{1 \le j \le N} \left[\frac{\mathrm{Re}(a_j) - 1}{\alpha_j} \right]$$

$$+ \mathrm{Re}(\lambda) + \mathrm{Re}(\eta) < 0 \qquad (4.7)$$

Then the Erdelyi-Kober fractional integral $K^{-}_{\eta,\alpha}$ of the product of $\bar{H}$ -functions with $S_V^U[\delta t^\rho]$ exist and the following relation hold:

$$\left(K^{-}_{\eta,\alpha} t^{\lambda-1} \bar{H}^{M_1,N_1}_{P_1,Q_1}\left[w_1 t^{\sigma_1}\right] \bar{H}^{M,N}_{P,Q}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]\right)(x)$$

$$= x^{\lambda-1} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{(V,R)} \left(\delta x^\rho\right)^R \sum_{\upsilon=1}^{M_1} \sum_{p=0}^{\infty} \bar{\theta}\left(S_{p,\upsilon}\right) \left(w_1 x^{\sigma_1}\right)^{S_{p,\upsilon}}$$

$$\bar{H}^{M+1,N}_{P+1,Q+1}\left[w_2 x^{\sigma_2} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P} \\ \left(1-\lambda+\eta-\alpha-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2\right) \end{matrix} \right. \right.$$

$$\left. \begin{matrix} \left(1-\lambda+\alpha+\beta+\eta-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2\right) \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right] \qquad (4.8)$$

V. Left- sided Generalized Fractional Differentition of $\bar{H}$ -Function

Now we treat the left-sided generalized fractional derivative $D^{\alpha,\beta,\eta}_{0+}$ given by (2.11).

Theorem 3-Let α, β, η ∈ **C** with Re (α) > 0, Re (α+β+η) ≠ 0. Let the constants a_j, b_j ∈ **C**, α_j, β_j > 0 (j = 1,…, P; j = 1,…,Q) and λ ∈ **C**, $\sigma_1, \sigma_2 > 0$ and

$$\sigma_1 \min_{1 \le j \le M_1} \mathrm{Re}\left(\frac{f_j}{F_j}\right) + \sigma_2 \min_{1 \le j \le M} \mathrm{Re}\left(\frac{b_j}{\beta_j}\right)$$

$$+ \mathrm{Re}(\lambda) + \min[0, \mathrm{Re}(\alpha+\beta+\eta)] > 0 \qquad (5.1)$$

Then the generalized fractional derivative $D^{\alpha,\beta,\eta}_{0+}$ of the product of $\bar{H}$ -functions with $S_V^U[\delta t^\rho]$ exists and the following relation holds:

$$\left(D^{\alpha,\beta,\eta}_{0+} t^{\lambda-1} \bar{H}^{M_1,N_1}_{P_1,Q_1}\left[w_1 t^{\sigma_1}\right] \bar{H}^{M,N}_{P,Q}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]\right)(x)$$

$$= x^{\lambda+\beta-1} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{(V,R)} \left(\delta x^\rho\right)^R \sum_{\upsilon=1}^{M_1} \sum_{p=0}^{\infty} \bar{\theta}\left(S_{p,\upsilon}\right)$$

$$\left(w_1 x^{\sigma_1}\right)^{S_{p,\upsilon}} \bar{H}^{M,N+2}_{P+2,Q+2}\left[w_2 x^{\sigma_2} \left| \begin{matrix} \left(1-\lambda-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2;1\right) \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right.$$

$$\left. \begin{matrix} \left(1-\lambda-\alpha-\beta-\eta-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2;1\right)(a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P} \\ \left(1-\lambda-\beta-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2;1\right)\left(1-\lambda-\eta-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2;1\right) \end{matrix} \right]$$

(5.2)

Proof.- Let n = [Re (α)] +1. From (2.11) we have

$$\left(D^{\alpha,\beta,\eta}_{0+} t^{\lambda-1} \bar{H}^{M_1,N_1}_{P_1,Q_1}\left[w_1 t^{\sigma_1}\right] \bar{H}^{M,N}_{P,Q}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]\right)(x)$$

$$= \left(\frac{d}{dx}\right)^n \left(I^{-\alpha+n,-\beta-n,\alpha+\eta-n}_{0+} t^{\lambda-1} \bar{H}^{M_1,N_1}_{P_1,Q_1}\left[w_1 t^{\sigma_1}\right] \bar{H}^{M,N}_{P,Q}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]\right) \qquad (5.3)$$

which exists according to Theorem 1 with α, β and η being replaced by - α + n, - β - n and α + η - n respectively. Then we obtain

$$\left(D^{\alpha,\beta,\eta}_{0+} t^{\lambda-1} \bar{H}^{M_1,N_1}_{P_1,Q_1}\left[w_1 t^{\sigma_1}\right] \bar{H}^{M,N}_{P,Q}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]\right)(x)$$

$$= \left(\frac{d}{dx}\right)^n x^{\lambda+\beta+n-1} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{(V,R)} \left(\delta x^\rho\right)^R \sum_{\upsilon=1}^{M_1} \sum_{p=0}^{\infty} \bar{\theta}\left(S_{p,\upsilon}\right)$$

$$\left(w_1 x^{\sigma_1}\right)^{S_{p,\upsilon}} \bar{H}^{M,N+2}_{P+2,Q+2}\left[w_2 x^{\sigma_2} \left| \begin{matrix} \left(1-\lambda-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2;1\right) \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right.$$

$$\left. \begin{matrix} \left(1-\lambda-\alpha-\beta-\eta-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2;1\right)(a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P} \\ \left(1-\lambda-\beta-n-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2;1\right)\left(1-\lambda-\eta-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2;1\right) \end{matrix} \right] \qquad (5.4)$$

on differentiating n times and the relation $n\Gamma(n) = \Gamma(n+1)$ imply (5.2) which completes the proof of the theorem.

Corollary 3.1- Let α ∈ C with Re (α) > 0 and let the constants and λ ∈ C, $\sigma_1, \sigma_2 > 0$ satisfy the conditions in (3.8). Then the Riemann-Liouville fractional derivative D^{α}_{0+} of the product of $\bar{H}$ -functions with $S_V^U[\delta t^\rho]$ exists and the following relation holds:

$$\left(D^{\alpha}_{0+} t^{\lambda-1} \bar{H}^{M_1,N_1}_{P_1,Q_1}\left[w_1 t^{\sigma_1}\right] \bar{H}^{M,N}_{P,Q}\left[w_2 t^{\sigma_2}\right] S_V^U[\delta t^\rho]\right)(x)$$

$$= x^{\lambda-\alpha-1} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{(V,R)} \left(\delta x^\rho\right)^R \sum_{\upsilon=1}^{M_1} \sum_{p=0}^{\infty} \bar{\theta}\left(S_{p,\upsilon}\right)$$

$$\left(w_1 x^{\sigma_1}\right)^{S_{p,\upsilon}} \bar{H}^{M,N+1}_{P+1,Q+1}\left[w_2 x^{\sigma_2} \left| \begin{matrix} \left(1-\lambda-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2;1\right) \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right.$$

$$\left. \begin{matrix} (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P} \\ \left(1-\lambda+\alpha-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2;1\right) \end{matrix} \right] \qquad (5.2)$$

VI. Right-sided Fractional Differentiation of $\bar{H}$ -Function

Theorem 4-Let α, β, η ∈ **C** with Re (α) > 0, Re (α + β + η) + Re(α) + 1 ≠ 0. Let the constants λ ∈ **C**, $\sigma_1,\sigma_2>0$ satisfy and

$$\sigma_1 \max_{1\le j\le N_1}\left[\frac{\mathrm{Re}(e_j)-1}{E_j}\right]+\sigma_2 \max_{1\le j\le N}\left[\frac{\mathrm{Re}(a_j)-1}{\alpha_j}\right]$$
$$+\mathrm{Re}(\lambda)-1+\max\left[\mathrm{Re}(\beta),\{\mathrm{Re}(\alpha)+1\}-\mathrm{Re}(\alpha+\eta)\right]<0 \tag{6.1}$$

Then the generalized fractional derivative $D_{-}^{\alpha,\beta,\eta}$ of the product of $\bar{H}$ -functions with $S_V^U[\delta t^\rho]$ exists and the following relation holds:

$$\left(D_{-}^{\alpha,\beta,\eta} t^{\lambda-1}\bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1}\right]\bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2}\right]S_V^U[\delta t^\rho]\right)(x)$$
$$=(-1)^{[\mathrm{Re}(\alpha)+1]}\, x^{\lambda+\beta-1}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{(V,R)}\left(\delta x^\rho\right)^R\sum_{\upsilon=1}^{M_1}\sum_{p=0}^{\infty}\bar{\theta}(S_{p,\upsilon})$$
$$\left(w_1 x^{\sigma_1}\right)^{S_{p,\upsilon}}\bar{H}_{P+2,Q+2}^{M+2,N}\left[w_2 x^{\sigma_2}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(1-\lambda+\alpha+\eta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\end{matrix}\right.\right.$$
$$\left.\left.\begin{matrix}(1-\lambda-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)(1-\lambda-\beta+\eta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\\(1-\lambda-\beta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right] \tag{6.2}$$

Proof.- Let n = [Re (α)] + 1. From (2.12) we have

$$\left(D_{-}^{\alpha,\beta,\eta} t^{\lambda-1}\bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1}\right]\bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2}\right]S_V^U[\delta t^\rho]\right)(x)$$
$$=\left(-\frac{d}{dx}\right)^n\left(I_{-}^{-\alpha+n,-\beta-n,\,\alpha+\eta}\,t^{\lambda-1}\right.$$
$$\left.\bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1}\right]\bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2}\right]S_V^U[\delta t^\rho]\right) \tag{6.3}$$

which exists according to Theorem 2 with α, β and η being replaced by - α + n, - β - n and α + η respectively. Then we obtain

$$\left(D_{-}^{\alpha,\beta,\eta} t^{\lambda-1}\bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1}\right]\bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2}\right]S_V^U[\delta t^\rho]\right)(x)$$
$$=\left(-\frac{d}{dx}\right)^n x^{\lambda+\beta+n-1}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{(V,R)}\left(\delta x^\rho\right)^R\sum_{\upsilon=1}^{M_1}\sum_{p=0}^{\infty}\bar{\theta}(S_{p,\upsilon})$$
$$\left(w_1 x^{\sigma_1}\right)^{S_{p,\upsilon}}\bar{H}_{P+2,Q+2}^{M+2,N}\left[w_2 x^{\sigma_2}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(1-\lambda-\beta+\eta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\end{matrix}\right.\right.$$
$$\left.\left.\begin{matrix}(1-\lambda-\beta-n-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)(1-\lambda+\alpha+\eta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}(1-\lambda-\beta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\end{matrix}\right.\right] \tag{6.4}$$

which implies the formula (6.2) in view of n times differentiation and the relation $n\Gamma(n)=\Gamma(n+1)$.

Corollary 4.1-Let α ∈ **C** with Re (α) > 0 and let the constants λ ∈ **C**, $\sigma_1,\sigma_2>0$ satisfy and

$$\sigma_1 \max_{1\le j\le N_1}\left[\frac{\mathrm{Re}(e_j)-1}{E_j}\right]+\sigma_2 \max_{1\le j\le N}\left[\frac{\mathrm{Re}(a_j)-1}{\alpha_j}\right]$$
$$+\mathrm{Re}(\lambda)+\mathrm{Re}(\alpha)<0 \tag{6.5}$$

Then the Riemann-Liouville fractional derivative D_{-}^{α} of the product of $\bar{H}$ -functions with $S_V^U[\delta t^\rho]$ exists and the following relation holds:

$$\left(D_{-}^{\alpha} t^{\lambda-1}\bar{H}_{P_1,Q_1}^{M_1,N_1}\left[w_1 t^{\sigma_1}\right]\bar{H}_{P,Q}^{M,N}\left[w_2 t^{\sigma_2}\right]S_V^U[\delta t^\rho]\right)(x)$$
$$=(-1)^{[\mathrm{Re}(\alpha)+1]}\, x^{\lambda+\beta-1}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{(V,R)}\left(\delta x^\rho\right)^R\sum_{\upsilon=1}^{M_1}\sum_{p=0}^{\infty}\bar{\theta}(S_{p,\upsilon})$$
$$\left(w_1 x^{\sigma_1}\right)^{S_{p,\upsilon}}\bar{H}_{P+1,Q+1}^{M+1,N}\left[w_2 x^{\sigma_2}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(1-\lambda+\alpha-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\end{matrix}\right.\right.$$
$$\left.\left.\begin{matrix}(1-\lambda-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\\(1-\lambda-\beta-\rho R-\sigma_1 S_{p,\upsilon},\sigma_2;1)\end{matrix}\right.\right] \tag{6.6}$$

Special cases

i. If in theorem 1, we reduce S_V^U to $L_V^\alpha(x)$ Laguerre polynomial, $\bar{H}_{P_1,Q_1}^{M_1,N_1}$ to generalized Wright hypergeometric function ${}_{P_1}\bar{\psi}_{Q_1}$ and $\bar{H}_{P,Q}^{M,N}$ to generalized Wright Bessel function with the help of results [20, p.101, Eq. (5.1.6)], [6, p.271, Eq. (7)], [18, p.271, Eq. (9)] respectively, we get the following theorem:

$$\left(I_{0+}^{\alpha,\beta,\eta} t^{\lambda-1}\,{}_{P_1}\bar{\psi}_{Q_1}\left[w_1 t^{\sigma_1}\left|\begin{matrix}(1-e_j,E_j;\varepsilon_j)_{1,P_1}\\(1-f_j,F_j;\Im_j)_{M_1+1,Q_1}\end{matrix}\right.\right]\right.$$
$$\left.\bar{J}_{\varsigma}^{\upsilon,\mu}\left[w_2 t^{\sigma_2}\right]L_V^{(\alpha')}[\delta t^\rho]\right)(x)$$
$$=x^{\lambda-\beta-1}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}\binom{V+\alpha}{V}\frac{\left(\delta x^\rho\right)^R}{(1+\alpha)_R}$$
$$\sum_{p=0}^{\infty}\frac{\prod_{j=1}^{P_1}\left\{\Gamma(1-e_j+E_j\xi)\right\}^{\varepsilon_j}}{\prod_{j=1}^{Q_1}\left\{\Gamma(1-f_j+F_j\xi)\right\}^{\Im_j}}\left(w_1 x^{\sigma_1}\right)^p\bar{H}_{2,4}^{1,2}\left[w_2 x^{\sigma_2}\left|\begin{matrix}(1-\lambda-\rho R-\sigma_1 p,\sigma_2;1)\\(0,1),(-\zeta,\upsilon;\mu)(1-\lambda+\beta-\rho R-\sigma_1 p,\sigma_2;1)\end{matrix}\right.\right.$$
$$\left.\left.\begin{matrix}(1-\lambda-\eta+\beta-\rho R-\sigma_1 p,\sigma_2;1)\\(1-\lambda-\alpha-\eta-\rho R-\sigma_1 p,\sigma_2;1)\end{matrix}\right.\right]$$

ii. Once again in theorem 1, if we reduce S_V^U polynomial to $y_V(-\beta' x, \alpha', \beta')$ Bessel polynomial, $\overline{H}_{P_1,Q_1}^{M_1,N_1}$ to generalized Riemann Zeta function $\phi(x)$ and $\overline{H}_{P,Q}^{M,N}$ to generalized hypergeometric function ${}_P\overline{F}_Q$ using results [9, p.108, Eq. (34)], [3, p.271, Eq. (1)], [6, p.471, Eq. (9)] respectively, it takes the following interesting form:

$$\left(I_{0+}^{\alpha,\beta,\eta} t^{\lambda-1} \phi\left(w_1 t^{\sigma_1}, k, r\right) {}_P\overline{F}_Q \left[w_2 t^{\sigma_2} \left| \begin{matrix} (1-a_j, A_j)_{1,P} \\ (1-b_j, B_j)_{1,Q} \end{matrix} \right. \right] y_V[-\beta'\delta t^{\rho}, \alpha', \beta'] \right)(x)$$

$$= x^{\lambda-\beta-1} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} (\alpha' + V - 1)_R \left(\delta x^{\rho}\right)^R \sum_{p=0}^{\infty} \frac{\prod_{j=1}^{Q} \{\Gamma(1-b_j)\}^{B_j}}{\prod_{j=1}^{P} \{\Gamma(1-a_j)\}^{A_j}} \frac{(w_1 x^{\sigma_1})^p}{(p+r)^k}$$

$$\overline{H}_{P+2,Q+3}^{1,P+2} \left[w_2 x^{\sigma_2} \left| \begin{matrix} (1-a_j, A_j)_{1,P} \\ (0,1)_{1,M}, (1-b_j, B_j)_{1,Q} \end{matrix} \right. \begin{matrix} (1-\lambda-\rho R-\sigma_1 p, \sigma_2; 1)(1-\lambda-\eta+\beta-\rho R-\sigma_1 p, \sigma_2; 1) \\ (1-\lambda+\beta-\rho R-\sigma_1 p, \sigma_2; 1)(1-\lambda-\alpha-\eta-\rho R-\sigma_1 p, \sigma_2; 1) \end{matrix} \right]$$

iii. If in Theorem 2, we reduce S_V^U polynomial to Hermite polynomial $H_V(x)$, $\overline{H}_{P_1,Q_1}^{M_1,N_1}$ to H-function and $\overline{H}_{P,Q}^{M,N}$ to g_1 function with the help of [20, p.106, Eq. (5.5.4)], [8, p.4125, Eq. (20)] we arrive at the following result after a little simplification:

$$\left(I_{-}^{\alpha,\beta,\eta} t^{\lambda-1} \overline{H}_{P_1,Q_1}^{M_1,N_1} \left[w_1 t^{\sigma_1} \right] g\left[r, \mu, \tau, m, w_2 t^{\sigma_2} \right] [\delta t^{\rho}]^{V/2} H_V \left[\frac{1}{2\sqrt{\delta t^{\rho}}} \right] \right)(x)$$

$$= x^{\lambda-\beta-1} \sum_{R=0}^{[V/U]} \frac{(-V)_{2R}}{R!} (-1)^R \left(\delta x^{\rho}\right)^R \sum_{\upsilon=1}^{M_1} \sum_{p=0}^{\infty} \bar{\theta}(S_{p,\upsilon}) \left(w_1 x^{\sigma_1}\right)^{S_{p,\upsilon}} \frac{\Gamma(m+1)\Gamma\left(\frac{1+\tau}{2}\right)}{\pi^{d/2} 2^{m+d} \Gamma\left(\frac{d-1}{2}\right) \Gamma(r) \Gamma\left(r-\frac{\tau}{2}\right)}$$

$$\overline{H}_{5,5}^{3,3} \left[w_2 x^{\sigma_2} \left| \begin{matrix} (1-r,1;1), \left(1-r+\frac{\tau}{2},1;1\right) \\ (0,1), \left(-\frac{\tau}{2},1;1\right)(-\mu,1;1+m) \end{matrix} \right. \begin{matrix} (1-\lambda-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2)(1-\lambda+\alpha+\beta+\eta-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2) \\ (1-\lambda+\beta-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2)(1-\lambda+\eta-\rho R-\sigma_1 S_{p,\upsilon}, \sigma_2) \end{matrix} \right]$$

where

$$\theta(S_{P,\upsilon}) = \frac{\prod_{j=1, j\neq\upsilon}^{M_1} \Gamma(b_j - \beta_j S_{p,\upsilon}) \prod_{j=1}^{N_1} \Gamma(1-a_j+\alpha_j S_{p,\upsilon})(-1)^p}{\prod_{j=M+1}^{Q_1} \Gamma(1-b_j+\beta_j S_{p,\upsilon})\} \prod_{j=N+1}^{P_1} \Gamma(a_j - \alpha_j S_{p,\upsilon}) p! \beta_\upsilon}$$

$$S_{p,\upsilon} = \frac{b_\upsilon + p}{\beta_\upsilon}$$

The results obtained by M. Saigo and A.A. Kilbas in [14] can be easily deduced from our results. If in theorem 1 and 2 we put $w=1$, reduce the polynomial S_V^U and $\overline{H}_{P_1,Q_1}^{M_1,N_1}$ to unity and $\overline{H}_{P,Q}^{M,N}$ to familiar H-function we arrive at known results recorded in [10, pp. 109-110, Eqs. (3.130), (3.131)]. Further, if in corollary (1.1) and (1.2) we reduce S_V^U and $\overline{H}_{P_1,Q_1}^{M_1,N_1}$ to unity, we get the results given by Srivastava H.M. [19, p. 97, Eqs. (2.4), (2.5)]. Also by reducing $\overline{H}_{P_1,Q_1}^{M_1,N_1}$ to unity, we at once get the results obtained by Chaurasia et al. [2].

VII. REFERENCES

1) Buschman R.G. and Srivastava H.M., The $\overline{H}$ - Function associated with a certain class of Feynman integrals, J. Phys. A: Math. Gen., 23 (1990), 4707-4710.
2) Erdélyi A., Magnus W., Oberhettinger F. and Tricomi F.G., Higher Transcendental Functions, Vol. I, Mc-Graw Hill, New York, 1953.
3) Chaurasia V. B. L. and Jain M., Genaralized Fractional calculas of the $\overline{H}$ -function, Acta Cien. Indica, V0l. XXXIII M, (4), (2007) 1653-1666.
4) Fox C, The G- and H- Functions as Symmetrical Fourier Kernels, Trans. Amer. Math. Soc. 98 (1961) 395-429.
5) Gupta KC, Jain R and Agrawal R, On Existence Conditions for Generalized Mellin-Barnes Type Integral, Nat. Acad. Sci. Lett. 30(5&6) (2007) 169-172.
6) Gupta KC, Jain R and Sharma A, A study of Unified Finite Integral Transforms with

Applications, J. Raj. Aca. Phy. Sci. 2(4) (2003) 269-282.

7) Inayat Hussain A.A., New properties of hypergeometric series derivable from Feynman integrals: I – Transformation and reduction formula, J. Phys. A: Math. Gen., 20 (1987), 4109-4117.
8) Inayat Hussain A.A., New properties of hypergeometric series derivable from Feynman integrals: II. A generalization of the H-function, J. Phys. A: Math. Gen., 20 (1987), 4119-4128.
9) Krall H.L. and Frink O., A new class of orthogonal polynomials: The Bessel polynomials, Trans. Amer. Math. Soc., 65 (1949), 100-115.
10) Mathai AM and Saxena RK and Haubold H.J., The H-function, Theory and Application, Springer, 2010.
11) Prudnikov A.P., Brychkov A. Yu and Marichev O.I., Integrals and series, Vol.3, More Special Functions, Gordon and Breach, New York, 1990.
12) Rathie AK, A New Generalization of Generalized Hypergeometric Functions, Le Matematiche Fasc.II, 52 (1997) 297-310.
13) Saigo M., A remark on integral operators involving the Gauss hypergeometric functions, Math. Rep. Kyushu Univ. 11 (1978), 135-143.
14) Saigo M.and Kilbas A.A., Generalized Fractional Calculus of the H-function, Fukuoka Univ. Sci. Reports, 29(1), 31-45 (1999).
15) Samko S.G., Kilbas A.A. and Marichev O.I., Fractional Integrals and Derivatives, Theory and Applications, Gordon and Breach, New York, 1993.
16) Saxena RK, Ram C and Kalla SL, Applications of Generalized H-Function in Bivariate Distributions, Rev. Acad. Canar. Cienc., 14(2002)111-120.
17) Srivastava H.M., A contour integral involving Fox's H-function, Indian J. Math., 14 (1972).1-6
18) Srivastava HM, Gupta KC and Goyal SP, The H-functions of One and Two Variables with Applications, South Asian Publishers, New Delhi and Madras, 1982.
19) Srivastava H.M., Lim S. and Wang P., Some Fractional-calculas results for the $\overline{H}$ -function associated with a class of Feynman integrals, Russ. J. math. Phy., 13 (1) (2007) 94-100.
20) Szegö G., Orthogonal polynomials, Amer. math. soc. colloq. publ., Vol. 23, Fourth edition, Amer. math. soc., Providence, Rhode Island.

Adoption Of Agricultural Innovations By Rice Farmers In Yola – South Local Government, Adamawa State, Nigeria

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Abstract- **Technological development and planned dissemination of ideas are necessary ingredient for the development of peasant production system and welfare. This study examined the adoption of agricultural innovation by rice farmers in Yola – South Local Government, Adamawa State. The research focused on the extent to which farmers adopt innovations, the determinants of adoption of innovations by the farmers. Structured questionnaires were distributed to one hundred respondent who were mainly rice farmers using a simple random sample technique. Data were analysed using simple descriptive statistics and chi-square tests. The study revealed that age and education level of the farmers contributed significantly to the rate of adoption of innovations. Major constraints faced by farmers which greatly impede their adoption of agricultural innovations in the study are input, economic and extension service. The study also recommended the provision of agro-service centre by government in rural areas for timely provision, delivery of farming inputs and to strengthen the provision of facilities through the rural banking scheme by making funds available for loans.**

Keywords- Adoption, Agricultural innovations, Rice Farmers, determinants, age, educational level

I. Introduction

Adoption is the decision by farmers to accept and make full use of an innovation which provides the means of achieving a sustainable increase in farm productivity and consequently leading to improved living standard of the people (Adams, 1983). These innovations include improved ideas, methods, practices and inputs, which supersedes the ones previous in use.

Agricultural innovations are defined as new ideas, practices or techniques which provide the means of achieving sustained increase in farm productivity and income (Adams, 1983). Adoption is a mental decision by farmers to make full use of new idea(s) as the best cause of action. Increase in productivity in the traditional agriculture requires an adoption of improved technologies. Efforts have been made by government to increase the productivity of the farmers through the introduction of numerous improved practices. Consequently, efforts have also been directed towards creating awareness of these improved practices and persuading farmers to adopt, even though the traditional farmers is bound to his local ideas and most often sceptical to new ideas (Auta, 1992).

Production continues to grow at a slow pace because farmers do not readily accept innovations immediately. They are still sceptical about innovations and dogmatic on their beliefs hence they need time before making any rational decision.

In recent years, agriculture in most countries has shown encouraging signs of change from traditional to modern agriculture through conversion of agricultural technology into production accomplishment (Pandey, 1989). Robert and Timothy (1995) indicated that profit is the major reason for adopting an innovation. Farmers assess an innovation based on economic factors and concepts, such as risk in past and present practices, production alternative and compelling production practices.

This study examined the extent of adoption and the determinants of adoption of agricultural innovations by farmers in Yola- South Local Government, Adamawa State.

II. Materials and Methods

The study was carried out in Yola-South Local Government Area of Adamawa state, Nigeria. It lies between latitude 9°14'N of the equator and longitude 12°28'E of the Greenwich meridian. The average annual rainfall is less that 1,000mm due to the influence of altitude. The maximum temperature can reach 40°C particularly in March and April which are the hottest periods, while the minimum temperature can also be as low as 18°C between December and January. The relative humidity around January and March is extremely low between 20 – 30% (Adebayo and Tukur, 1999).

The study area has two (2) districts with ten (10) wards – Yola-South Districts covers Toungo, Bako, Bamoi, Makama, Adarawo, Wuro Hausa, Bole and Yolde Pate Wards. While Namtari district has Namtari Yolde and Yolde Kohi Wards respectively. The major occupation of the people in the area is farming and commercial activities. The major crops grown are rice, maize, guinea corn, beans and vegetable production.

Data collected for this study were obtained mainly from primary and secondary sources. The primary data were collected through the use of structured questionnaires and interview schedule, while the secondary source of data include journals, publications, seminars/workshop papers, texts and other relevant works. A total of 100 farmers were

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sample using simple random sampling technique. Six out of the ten wards in the two districts of the local government were sampled out using a purposive sampling technique for the purpose of the study. Hover, only seventy-one questionnaires were retrieved and used in the analysis of data for the study. Data was analysed using simple descriptive statistics (tables, frequency distribution and percentages). Variables analysed included age, educational level, socio-economic status of adoptors and non-adoptors. Chi- Square test was used to compare observed frequency with those of expected frequencies related to variables such as age and education and adoption attitudes of farmers.

III. Results and Discussion

A. Extent To Which Rice Farmers In The Study Area Adopt Innovations In Rice Production

Table 1. Distribution of farmers on the basis of their adoption of agricultural innovations

Improved technology	Adoptors		Non-Adoptors	
	Frequency	Percentage (%)	Frequency	Percentage (%)
Inorganic fertilizer	67	94.4	4	5.6
Herbicide	48	67.6	23	32.4
Improved seed	43	60.6	28	39.4
Tractor	38	53.5	33	46.5
Insecticide	31	43.7	40	56.3

Table 1 shows that fertilizer was the most accepted and adopted technology by the farmers, with adoption rate of 94.4%. this was followed by herbicide, improve seed, tractor and insecticide with adoption rate of 67.6%, 60.6%, 53.5% and 43.7% respectively. The high level of adoption of inorganic fertilizer (94.4%) may be attributed to its positive effect on rice yields. Since most of the farmlands are considered infertile or degraded due to continuous cropping (Ukeje, 2002). Although the innovation of using inorganic fertilizer has been greatly adopted by the farmers, yields recorded were still low ad may be attributed to inadequate supply of the commodity due to high purchase prices, untimely supply and ineffective services to educate farmers on the use of the innovation packages efficiently in their production. Although the innovation of using inorganic fertilizer has been greatly adopted by the farmers, yields recorded were still low and may be attributed to inadequate supply of the commodity due to high purchase prices, untimely supply and ineffective extension services to educate farmers on the use of the innovation packages efficiently in their production.

The moderately high adoption rate of use of tractor (53.5%) may be due to its efficiency in pulverisation of the soil, incorporation of organic matter and in weed control which promotes plant growth.

The high level of adoption of herbicide (67.6%) may be attributed to its efficiency in the reduction of the cost of hired labour and also minimizes drudgery and fatigue associated with mechanical or manual labour. Improved seed increase yield per hectare of land and also farmers income, and may be the reason for its high adoption rate (60.6%). The low adoption of insecticide (43.7%) may be attributed to the lack of noticeable cases of insects attacks on the farmers crops.

B. Determinants of Adoption of New Technology

Table 2. Distribution of farmers on the basis of credit facilities.

Credit Source	Adoptors		Non-Adoptors	
	No.	Percentage (%)	No.	Percentage (%)
Agricultural banks	11	15.5	60	84.5
Commercial banks	1	1.4	70	98.6
Friends/relatives	4	5.6	67	94.1
Farmers society	7	9.9	64	90.1
Non-beneficiaries from any source	48	67.9	-	-

Table 2 showed that majority of the farmers (67.6%) were non-beneficiaries of credit facilities fro any of the credit sources which may be attributed to lack of collateral security, high interest rate charged by the banks, lack of awareness, complex procedures in processing loans, etc. While very few of the farmers are beneficiaries from agricultural banks, commercial banks, friends/relatives and farmers' society with 15.5%, 1.4%, 5.6% and 9.9% respectively.

Inadequate working capital hinders small-scale farmers to expand their scale of operations and/or take advantage of profitable packages of technology to boost productivity. The bulk of capital injection by this category of farmers comes from owners' equity (67.6%). The long and cumbersome bureaucratic processes might have prevented the flow of credit through the government established credit schemes and commercial banks to the farmers. Theses may invariably may affect farmers desire to adopt improved agricultural technologies.

Table 3. Distribution to farmers on the basis of their educational level and adoption of agricultural innovations.

Educational Level	Adoptors		Non-Adoptors	
	No.	**Percentage (%)**	**No.**	**Percentage (%)**
Non-formal education	2	3.8	10	52.6
Adult education	3	5.8	4	21.1
Primary education	1	1.9	3	15.8
Secondary education	12	23.1	1	5.3
Tertiary education	34	65.4	1	5.3
Total	**52**	**100**	**19**	**100**

Table 3 shows that 3.8% of the adoptors of improved technology had no formal education, while 5.8%, 1.9% and 23.1% had adult, primary and secondary education respectively. Majority of the adoptors (65.4%) attended tertiary education, while majority of the non-adoptors (52.6%) had no formal education. Among the non-adoptors 21.1%, 15.8%, 5.3% attained adult, primary, secondary and tertiary education respectively.

Low level of education of small-scale farmers who form the bulk of the agricultural labour force has remained a major constraint to the adoption of modern farming techniques and their inability to access other inputs necessary for increased productivity in the sector (Ukeje, 2002). Educated farmers can gather reliable information through such media as the newspapers, gazettes, handouts, posters, radio, televisions, etc.

Table 4. Distribution of Farmers on the Basis of Contacts with Extension Agents

Farmers visited and benefited from the visit		Farmers visited but not benefited		Farmers not visited at all	
No.	Percentage (%)	No.	Percentage (%)	No.	Percentage (%)
29	40.8	2	2.8	40	56.3

Table 4 shows that 56.3% of the farmers were not visited at all by the extension agents, while 40.8% were visited and benefited from the visit, and only 2.8% were did not benefited from the visit. Extension contact is very vital for effective adoption of technologies (Omotayo *et al.*, 1997).

Limited extension visit here may imply that farmers are not adequately informed about new agricultural innovations or are not persuaded to effectively put them into practice; this might affect their rate of adoption of improved technologies.

Table 5. Distribution of farmers on the basis of age and adoption of improved technologies

Age (yrs)	Adoptors		Non-Adoptors	
	No.	**Percentage (%)**	**No.**	**Percentage (%)**
15 – 34	19	38.8	2	9.1
35 – 54	26	53.0	6	27.3
Above 54	4	8.2	14	63.6
Total	49	100	22	100

Table 5 shows that 38.8% of the adoptors were within the age range of 15 – 34, while 8.2% were within the age range of 35 – 54 years above 54 years respectively. Conversely, 9.1% of the non-adoptors were within the age range of 15 – 34 years, while 27.3% and 63.6% were within the ranges of 35 – 54 and above 54 years respectively.

It can be concluded that the middle aged farmers had the highest percentage of adoption of agricultural innovations (53.0%) while the highest non-adoptors (63.6%) were the old aged farmers. Old aged farmers tend to be sceptical on improved innovations and are usually tied to their beliefs and norms.

Table 6. Distribution of farmers on the basis of sources of innovation packages

Sources of packages	No.	Percentage (%)
Market	66	93.0
Adamawa ADP	28	39.4
Cooperatives/Relatives	6	8.5
Non-users of innovation packages	2	2.8

Table 6 shows that 93.0% of the farmers obtained their innovation packages from the market, while 39.4% and 8.5% of the farmers obtained their innovation packages from Adamawa ADP and Cooperatives/relatives respectively. However, non-users of the innovation packages constituted only 2.8%.

Government efforts to develop efficient and effective input procurement and distribution system to farmers through the Agricultural Development Programmes (ADPs) have not been successful. The persistence of this problem has been attributed largely to the issue of subsidy and its administration. Farmers prefer to go to markets where they can really access these packages regardless of the cost implications. The implication is that utilisation of these packages may not be efficient to guarantee output maximization.

IV. Conclusion

Agricultural innovations provide the means of achieving a sustained increase in farm productivity and income and consequently leading to better living standard. It has been revealed that the extent to which farmers adopt agricultural innovations in the study area was encouraging. However, the major problems of the innovations were the complexities of their usage and high cost of packages in markets.

The chi-square results on distribution of farmers on the basis of their educational level and adoption of agricultural innovations shows that the adoption is dependent on the farmers level of education, that is, there is a correlation between level of education and adoption of new agricultural technology. This agrees with the findings of Osuji (1983). The results also indicated that the adoption of improved technologies is dependent on the farmers age and agrees with the findings of Pandey (1987) and Auta (1992) who reported that young farmers tend to be more flexible in their decisions to adopt new ideas readily, because of the anticipated longer life span within which the investment in new technology will pay off than the old age.

V. Recommendations

The approaches to fostering the adoption of agricultural innovations are recommended below: -

i. Provision of effective agro-service centres by the government in rural areas for timely provision and delivery of farming in inputs to the rural farmers.
ii. Provision of credit facilities through the rural banking scheme should be strengthened by making more funds available for loan schemes.
iii. Supply of modern farm inputs at subsidised price should be done early, since agriculture is a time specific venture.
iv. Farmers should be encouraged to form cooperative organisations in order to solve their problems collectively; this will make them share their knowledge on farm experiences.

VI. References

1) Adams, M.E. (1983). Agricultural Extension in Developing Countries. Longman Group, Longman House, UK. 39 – 180.
2) Adebayo, A.A. and Tukur, A.L. (1999). Climate (Sunshine, Temperature, Evaporation and relative Humidity). Adebayo, A.A. and Tukur, A.L. (eds) Adamawa State in Map. Paraclete Publishers, Yola, Nigeria. 3 – 5.
3) Auta, S.I., Ariyo, J.A. and Apkoko, J.T. (1992). Socio-spatial variation in the adoption of agricultural innovations in selected villages in Funtua and Jamare Local Government Areas 7 (1&2).
4) Omotayo, A.M., Chikwendu, D.O., Zaria, M.B., Yusuf, J.O. and Omensa, Z.E. (1997). Effectiveness of radio in Nigeria in dissemination of information on improved farming practices. Journal of Extension Services. 13:103 – 120.
5) Osuji, I.O. An institutional factor associated with the adoption of new farm techniques in Eastern Nigeria. Department of Agricultural Economics and Extension, University of Nigeria, Nsukka.
6) Pandey, K.N. (1989). Adoption of agricultural innovations: a study of small and marginal farmers of Varanasi. Department of Extension Education, Institute of agricultural sciences, Banara, Hindu University. Varanari-Law Publishers, Allahbad, India. 1-5, 22-26.
7) Robert, N.K. and Timothy, J.R. (1995). Factors influencing the adoption of nitrogen testing programme. Journal of Extension. 33(4).
8) Ukeje, E. (2002). Modernising smallholder agriculture to ensure food security and gender empowerment. Issues and policy. 1 – 6.

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Language: The language of publication is UK English. Authors, for whom English is a second language, must have their manuscript efficiently edited by an English-speaking person before submission to make sure that, the English is of high excellence. It is preferable, that manuscripts should be professionally edited.

Standard Usage, Abbreviations, and Units: Spelling and hyphenation should be conventional to The Concise Oxford English Dictionary. Statistics and measurements should at all times be given in figures, e.g. 16 min, except for when the number begins a sentence. When the number does not refer to a unit of measurement it should be spelt in full unless, it is 160 or greater.

Abbreviations supposed to be used carefully. The abbreviated name or expression is supposed to be cited in full at first usage, followed by the conventional abbreviation in parentheses.

Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than 1.4 × 10-3 m3, or 4 mm somewhat than 4 × 10-3 m. Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

Structure

All manuscripts submitted to Global Journals, ought to include:

Title: The title page must carry an instructive title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) wherever the work was carried out. The full postal address in addition with the e-mail address of related author must be given. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining and indexing.

Abstract, used in Original Papers and Reviews:

Optimizing Abstract for Search Engines

Many researchers searching for information online will use search engines such as Google, Yahoo or similar. By optimizing your paper for search engines, you will amplify the chance of someone finding it. This in turn will make it more likely to be viewed and/or cited in a further work. Global Journals have compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Key Words

A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy and planning a list of possible keywords and phrases to try.

Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

Choice of key words is first tool of tips to write research paper. Research paper writing is an art.A few tips for deciding as strategically as possible about keyword search:

- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.
- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

References follow the *Harvard scheme* of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals homepage at the judgment of the Editorial Board.

The Editorial Board and Global Journals recommend that, citation of online-published papers and other material should be done via a DOI (digital object identifier). If an author cites anything, which does not have a DOI, they run the risk of the cited material not being noticeable.

The Editorial Board and Global Journals recommend the use of a tool such as Reference Manager for reference management and formatting.

Tables, Figures and Figure Legends

Tables: Tables should be few in number, cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g. Table 4, a self-explanatory caption and be on a separate sheet. Vertical lines should not be used.

Figures: Figures are supposed to be submitted as separate files. Always take in a citation in the text for each figure using Arabic numbers, e.g. Fig. 4. Artwork must be submitted online in electronic form by e-mailing them.

Preparation of Electronic Figures for Publication

Even though low quality images are sufficient for review purposes, print publication requires high quality images to prevent the final product being blurred or fuzzy. Submit (or e-mail) EPS (line art) or TIFF (halftone/photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Do not use pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings) in relation to the imitation size. Please give the data for figures in black and white or submit a Color Work Agreement Form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution (at final image size) ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs) : >350 dpi; figures containing both halftone and line images: >650 dpi.

Color Charges: It is the rule of the Global Journals for authors to pay the full cost for the reproduction of their color artwork. Hence, please note that, if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a color work agreement form before your paper can be published.

Figure Legends: Self-explanatory legends of all figures should be incorporated separately under the heading 'Legends to Figures'. In the full-text online edition of the journal, figure legends may possibly be truncated in abbreviated links to the full screen version. Therefore, the first 100 characters of any legend should notify the reader, about the key aspects of the figure.

6. AFTER ACCEPTANCE

Upon approval of a paper for publication, the manuscript will be forwarded to the dean, who is responsible for the publication of the Global Journals.

6.1 Proof Corrections

The corresponding author will receive an e-mail alert containing a link to a website or will be attached. A working e-mail address must therefore be provided for the related author.

Acrobat Reader will be required in order to read this file. This software can be downloaded

(Free of charge) from the following website:

www.adobe.com/products/acrobat/readstep2.html. This will facilitate the file to be opened, read on screen, and printed out in order for any corrections to be added. Further instructions will be sent with the proof.

Proofs must be returned to the dean at dean@globaljournals.org within three days of receipt.

As changes to proofs are costly, we inquire that you only correct typesetting errors. All illustrations are retained by the publisher. Please note that the authors are responsible for all statements made in their work, including changes made by the copy editor.

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The Global Journals are enclosed by our publishing's Early View service. Early View articles are complete full-text articles sent in advance of their publication. Early View articles are absolute and final. They have been completely reviewed, revised and edited for publication, and the authors' final corrections have been incorporated. Because they are in final form, no changes can be made after sending them. The nature of Early View articles means that they do not yet have volume, issue or page numbers, so Early View articles cannot be cited in the conventional way.

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Please note that if not specifically requested, publisher will dispose off hardcopy & electronic information submitted, after the two months of publication. If you require the return of any information submitted, please inform the Editorial Board or dean as soon as possible.

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Informal Tips for writing a Science Frontier Research Paper to increase readability and citation

Before start writing a good quality Science Frontier Research Paper, let us first understand what is Science Frontier Research Paper? So, Frontier Research Paper is the paper which is written by professionals or scientists who are associated to Physics, Mathematics, Chemistry, Zoology, Botany, Bio-tech, Geology, Military Science, Environment and all Interdisciplinary & Frontier Subjects etc., or doing research study in these areas. If you are novel to this field then you can consult about this field from your supervisor or guide.

Techniques for writing a good quality Applied Science Research Paper:

1. Choosing the topic- In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Frontier Science. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: At a first glance, this point looks obvious but it is first recommendation that to write a quality research paper of any area, first draft your paper in Microsoft Word. By using MS Word, you can easily catch your grammatical mistakes and spelling errors.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

9. Use and get big pictures: Always use encyclopedias, Wikipedia to get pictures so that you can go into the depth.

10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. **Pick a good study spot:** To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. **Think technically:** Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.
- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.

- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.

You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described.
Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Administration Rules Listed Before Submitting Your Research Paper to Global Journals

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals:

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.
- Do not give permission to anyone else to "PROOFREAD" your manuscript.

Written Material: You may discuss with your guides and key sources.

- Do not copy or imitate anyone else paper. (Various Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

Criterion for Grading a Research Paper (Compilation)
by Global Journals

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals.

Topics	Grades		
	A-B	C-D	E-F
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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