Ellipse's Perimeter Evaluation

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Abstract*: This is a trendy problem since the time of Kepler (1609). He was asked what the length of the elliptical orbits is. Even today, thousand of mathematicians are working on this matter. Ramanujan, a super mathematician (1914) from India, is the Master of all these researchers. But, no one of them considered that the orbits are not elliptical. Their mind was blocked by the laws of Kepler for 400 years. Orbit's length and ellipse's perimeter are two different matters. In these *papers we will treat* mathematical ellipse. *Orbits are not elliptical

*Keywords***:** perimeter estimation, ellipse, other astroids, cracking Thales

I. INTRODUCTION

Thales theorem B/A=c/d is well known by everyone. No! No one of the mathematicians knows the generalized Thales theorem. Generalized Thales theorem explains the total arc length on positive Cartesian. Like Pythagoras (a^2+b^2=c^2), generalized Thales is (a^s+b^s=K^s)

So first, we must start by understanding Thales theorem. Then after we will attack the total arc length of the astroids (x/a)^r+(y/b)^r=1.When r=2 the astroid is an ellipse.

Fig.1 shows what we know and what we should know about Thales.
On the left is what we know; on the right is what we must be aware.

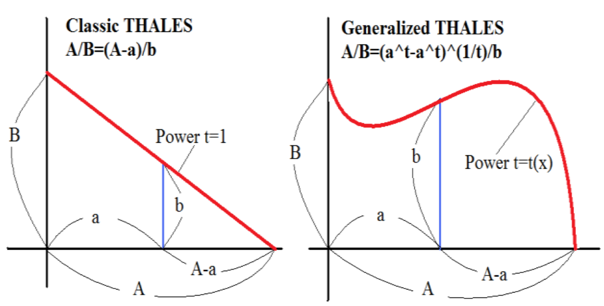


Fig.1

In fact, for an astroidal expression (x/A)^t+(y/B)^t=1 where t=t(x),variable (t) ,simply when x=a y=b is found.

As a consequence, the relation of the coordinates at the touching point P of (t variable) astroid with (r constant) astroid is shown on Fig.2

A/B=(A^t-x^t)^(1/t)/y (a)
a/b=(a^r-x^r)^(1/r)/y (b)

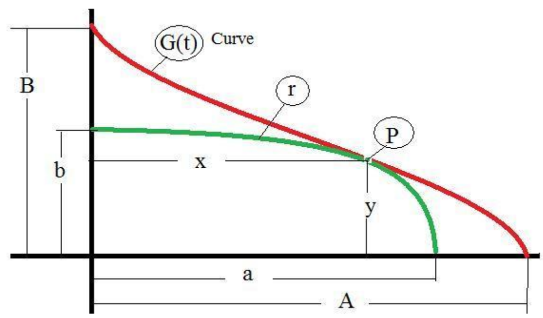


Fig. 2
And it exists an s=r*t/(r-t) which gives the relation
A/B=(A^s-a^s)^(1/s)/b. Fig.3 (c)

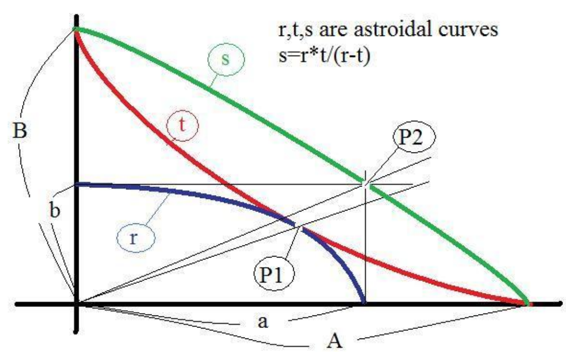


Fig. 3
That is to say: when A=B=K a^s+b^s=K^s (d)

And, when K is the length (L1) of the (r=constant) astroid (arc length on the positive Cartesian), the expression (d) is commented as

1+TAN^s=L1^s (e)
Where TAN=b/a.

Therefore the expression (d) is an **equivalent** to the elliptic integrals, with which we evaluate the total arc length on the positive Cartesian.

Then we may evaluate any astroidal value (area, length,…) without use of integrals for any (r):
Say K=total arc length on positive Cartesian, evaluate L1
Say K=total area on positive Cartesian, evaluate Areas
Say K= any elliptical subject, evaluate this subject, without use of integrals.

Existence of (s) : proof (dated 1959.03.24 Istanbul/Turkey)

Consider Fig.4

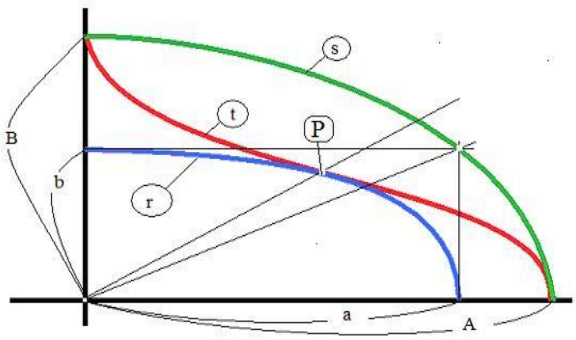


Fig . 4

The astroid family
(x/a)^r+(y/b)^r=1 (1)

where r=Constant, is enveloped by
(x/A)^t+(y/B)^t=1 where t=t(x) (2)

We search for a relation f(a,b,r,A,B,t)=0
At the touching point (P) of the graphs we write
-the slopes are equal
-the coordinates are equal

For the coordinates we write
y=b/a*(a^r-x^r)^(1/r)=B/A*(A^t-x^t)^(1/t) (3)

For the slope of the enveloped astroid we write
dy/dx=-(b/a)^r*(x/y)^(r-1)
dy/dx=-(b/a)*((x^r)/(a^r-x^r))^((r-1)/r) (4)

For the slope of the envelope itself :
say (x/A)^t=U ; (y/B)^t=V then,
U+V=1 (5)

dU+dV=0 (6)

we have
t*ln(x/A)=lnU
t*ln(y/B)=lnV (7)

When we differentiate (7), we write

dt*ln(x/A)+t*(dx/x)=dU/U
dt*ln(y/B)+t*(dy/y)=dV/V (8)

and there from,

dU=U*(dt*ln(x/A)+t*(dx/x))
dV=V*(dt*ln(y/B)+t*(dy/y)) (9)

Considering (7), the expression (6) is written as

U*(dt*1/t*lnU+t*dx/x)+V*(dt*1/t*lnV+t*dy/y)=0 (10)

and there from

V*t*dy/y=-U*(dt/t*lnU+t*dx/x)-V*(dt/t*lnV (11)

dy/dx=y/(V*t)*U*((dt/dx*1/t*lnU+t/x))+V*dt/dx*1/t*lnV) (12)

taking (U and t/x) out of the parenthesis

dy/dx=U/V*y/x*(1+dt/dx*x/t^2*1/U*(U*lnU+V*lnV))
is written (13)

say
N=(1+dt/dx*x/t^2*1/U*(U*lnU+V*lnV)) (14)

dy/dx=-U/V*y/x*N is written (15)

For the equality of the slopes, we write (15)=(4). Considering also (3), we write

(b/a)*((x^r)/(a^r-x^r))^((r-1)/r)=U/V*1/x*(B/A)*(A^t-x^t)^(1/t)*N (16)

Replacing (4) and (5) in (16) we write

(b/a)^r*(x/y)^(r-1)=(B/A)^t*(x/y)^(t-1)*N (17)

say (B/A)=E (18)

Use (3), we write (17) as follows

(b/a)^r=E^t*(x/(E*(A^t-x^t)^(1/t))^(t-r)*N (19)
and there from

b^r=E^r*a^r*(x^t/(A^t-x^t))^((t-r)/t)*N (20)

is written

using (3) and (20), the expression (1) is written as follows (x/a)^r+((A^t-x^t)^(r/t)*(A^t-x^t)^((t-r)/t))/a^r*x^(t-r)*N=1 (21)

x^t*N+A^t-x^t=a^r*x^(t-r)*N (22)

A^t=a^r*x^(t-r)*N-x^t*(N-1) (23)

Then, from (23) we get

x=((A^t-x^t*(1-N))/a^r*N)^(1/(t-r)) (24)

a^r*x^(t-r)*N=A^t-x^t*(1-N) (25)

using (25) in (20)

b^r=E^r*(A^t-x^t*(1-N))/(A^t-x^t)^(t-r)/t)
A^t=(b/E)^r*(A^t-x^t)^(t-r)/t)+x^t*(1-N) (26)
is written

(26)=(23) then,

(a/b*E)^r*x^(t-r)*N=(A^t-x^t)^((t-r)/t) (27)
is written
A^t-x^t=(a/b*E)^(r*t/(t-r))*x^t*N^(t/(t-r)) (28)
is written
A^t=x^t*(1+((a/b)*E)^((r*t)/(t-r))*N^(t/(t-r))) (29) is written
From (29) we get (x)

x=A/(1+((a/b)*E)^((r*t)/(t-r))*N^(t/(t-r)))^(1/t) (30)

(30)=(24) then,

((A^t-x^t*(1-N))/a^r*N)^(1/(t-r))=A/(1+((a/b)*E)^((r*t)/(t-r))*N^(t/(t-r)))^(1/t) (31)

(A^t-x^t*(1-N))/a^r*N=(A^(t-r)*b^r)/(b^(r*t/(t-r))+(a*E)^(r*t)/(t-r))*N^(t/(t-r)))^((t-r)/t) (32)

We take ((t-r)/r*t) power of both sides,
We proceed, then we take the power (r*t/(r-t)) of both sides

say r*t/(r-t)=s (33)

we write

(A^t-x^t*(1-N))^(s/r)*A^t=a^s*N^(s/r)+(b/E)^s (34)

For the astroids of the same power, when
b/a=TAN=Constant
dt/dx=dt/dTAN*dTAN/dx=0 and **N=1** then,
Figure 5. (35)

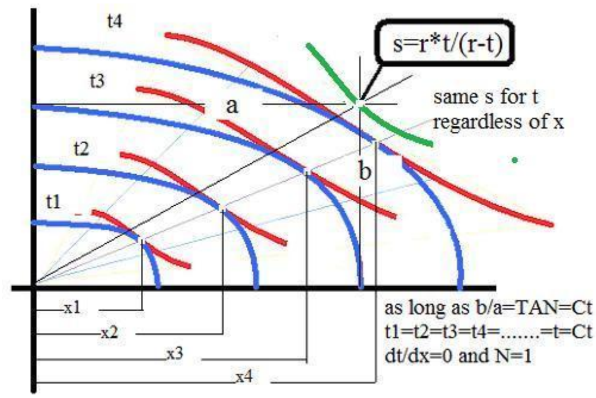


Fig.5

A^(t*s/r)*A^t=A^s=a^s+(b/E)^s (36)
is written
that is as a general expression:

(a/A)^s+(b/B)^s=1 (37)

In a symmetric case, when (A=B=K) ,
we write
K^s=a^s+b^s (38)

Say (K/a=L1) ; also say b/a=TAN

L1^s=1+TAN^s (38)L
is written
L1=(1+TAN^s)^(1/s) is proofed, found.

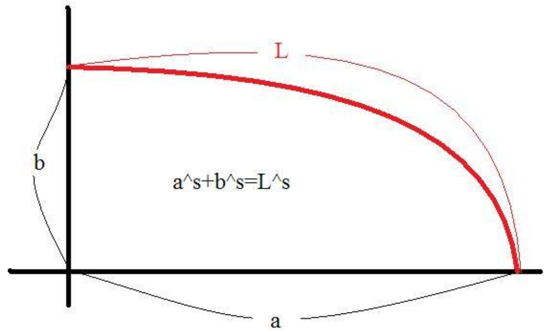
That is the equivalent of the astroidal finite integrals .
Insolvable integrals!!!

$$L = a.\int_0^1 (1+(b/a)^2.(k^r/(1-k^r))^{((2r-2)/r)})^{(1/2)}.dk$$

Expression (38) treats all the mathematical astroids (x/a)^r+(y/b)^r=1, ellipse included (case r=2)
Give any meaning to (K/a), crack the implicit expression (K/a)^s=1+TAN^s
Say:
(K/a) is the total arc length
evaluate L1 on the positive Cartesian
(K/a) is the total area
evaluate Area1 on the positive Cartesian
(K/a) is any astroidal subject evaluate this subject on the positive Cartesian

A cracking method:

By trend, ellipse's perimeter will be estimated. Consider Fig.6. According Thales a^s+b^s=L^s



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g.6

s=s(a,b,r)=r*t/(r-t) where (t) is variable.So (s) is variable, is an implicit power. When speaking about lengths, one should know the length (L) to evaluate (s). But if we know (L) we don't need to evaluate (s). If we know (s) we know (L). So first, we must crack Thales L^s=a^s+b^s and find an estimable expression for (s).

Various approximation steps are indicated here below to reach to the "most accurate cracking" When we say we know (L1), the unit length on the positive Cartesian, we mean (L1) evaluated by summing billion of (dL1) arc segments. Suppose, we don't know what an integral is!

Version s=Constant
we know L1exact=(Pi/2)*R, when we speak of circles.
from L1=(1+(R/R)^s)^(1/s)=(Pi/2)*R we get
s.min=ln(2)/ln(Pi/2)
s.min=1,53492853566138..

When used for the whole TAN range, this gives the error graph (Fig.7). Max.error %=0,003605937….at TAN=5,006784983..

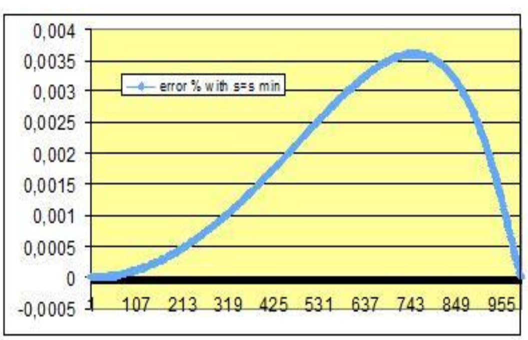

Fig.7

Error % for 1<TAN<infinity. Max.error %=0,003605937….at TAN=5,006784983..

Version s=s min +m*TAN (Linear variable)
we know L1Exact=636,62517105986… for TAN(89,91 o)=636,6192488…
from L1 max=(1+TAN.max^s)^(1/s) we get
s.max=1,71122902010321….then, according the value of TAN

s=s.min+(s.max-s.min)/(TAN.max-1)*(TAN-1)
s=1,53492853566138+0,000277355*(TAN-1) is evaluated

This gives the error graph (Fig.8)
Max.error %=0,003477761…….at TAN=4,966157118..

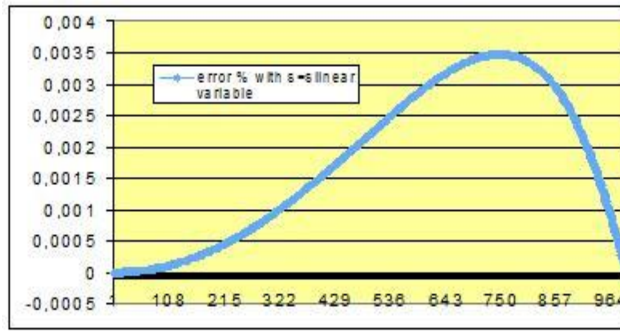

Fig.8
Error % for 1<TAN<infinity. Max.error %=0,003477761…….at TAN=4,966157118..

Version (s=sMod;a power version,with p=2,98 constant) we know all L1Exact.
an (sExact) data is obtained.Fig.9

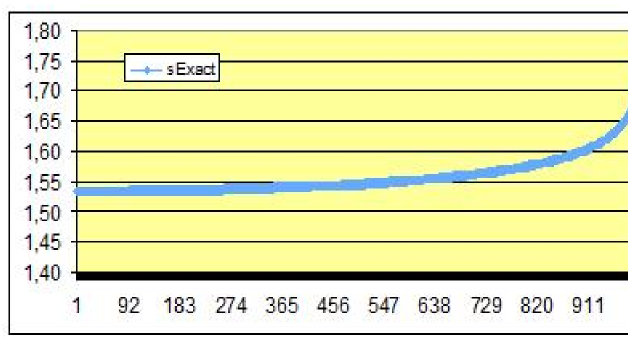

Fig.9
sReal for 1<TAN< infinity

By similarity to an astroid, we write an (s Mod) overlapping the (s Real).We start a cracking.
sMod=astroid+polynome
sMod=d1+b1*(1-(x-c1)/a1)^p)^(1/p)+(F+m1*x^v1+n1*x^w1). Proposed as a cracking method.

where ,for 1000 lines of evaluation on an excel table

Angle step	=(90o-45o)/1000
x	=(Angle-45)/Angle step
Angle o	=(45+angle step*x)
ATAN	=angle o*Pi/180
Angle o	=ATAN*180/Pi

Fig.10 shows the overlapping with the proposed parameter values of the Table I

parameters	values
	1,534928535661380

Table I

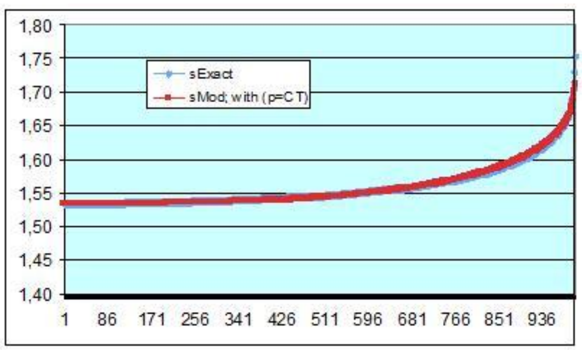

Fig.10 Overlapping of sExact&sMod with p=Constant=2,98

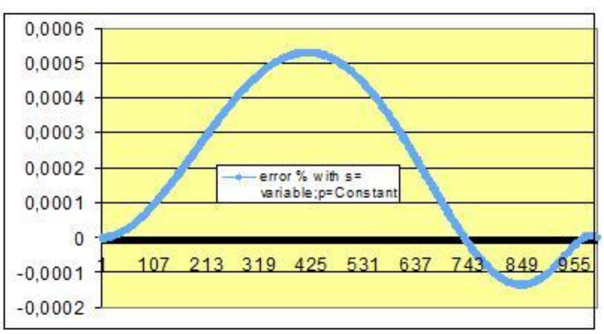

Fig.11
Error % for 1<TAN<infinity with p=2,89. Max error %=0,000529940 ..at TAN=2,0211255..

Version (s=sMod with p=pMod; power version, with p=variable)
we know all L1 exact
we know the overlapping sExact&sMod
we write error %, for this overlapping=0,this is to write: (sMod-sExact)/sMod=0
which gives a new value for (p) at every different TAN.
The graph of this new (p) looks like an astroid.
We write a pMod for this pExact data, with the following parameters of Table II

pMod=d2+b2*(1-((x-c2)/a2)^q)^(1/q)+(G+m2*x^v2+n2*x^w2)

parameters	values
a2	500
b2	0,3
c2	500
d2	0,000
q	6
G	**1,965**
m2	**0,000996000**
v2	1
n2	0
w2	1

Table. II

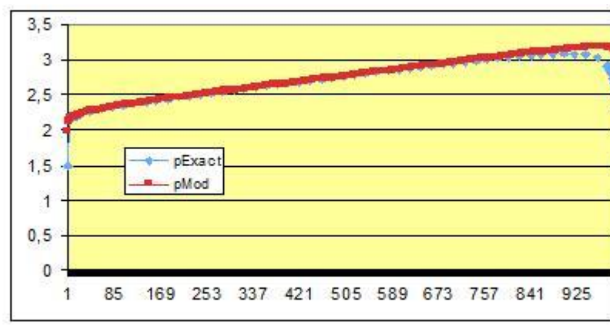

Fig.12
Overlapping of pExact&pMod. For 1<TAN<infinity

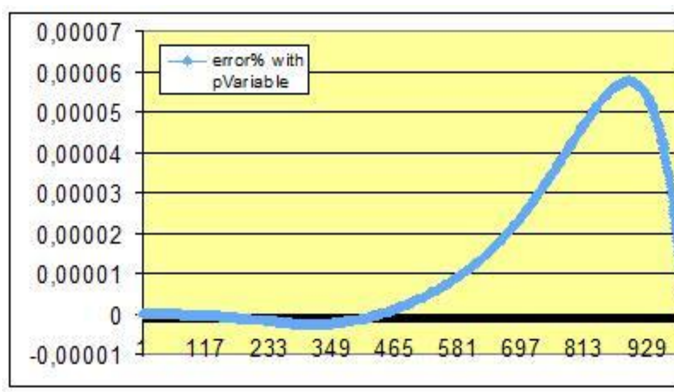

Fig.13 Error % for (1<TAN<infinity) (p variable) .Max.Error %=0,00005769…. at TAN=12,0985895…

Version (s=sMod with p=pMod; power version, with p=variable, corrected)

we know all L1 exact
we know the overlapping pExact&pMod
we write error % for this overlapping=0, that is:
(pMod-pExact)/pMod=0 and

we attack the parameter (b2,m2,G) of pMod) consecutively.
All these parameters have an astroidal looking. (This is a proposal)

parameters	**values**
	500 **0,000000**
	1,600000

Table III. b2 parameter correcting pMod

parameters	**values**
	1

Table IV.m2 parameter correcting pMod

parameters	**values**
	1,4

TableV. G parameter correcting pMod

Now, with these corrections we write
pMod=d2+b2Mod*(1-((x-c2)/a2)^q)^(1/q)+(GMod+m2Mod*x^v2+n2*x^w2)
sMod=d1+b1*(1-((x-c1)/a1)^pMod)^(1/pMod)+(F+m1*x^v1+n1*x^w1)
Error % is reduced as shown on Fig.14

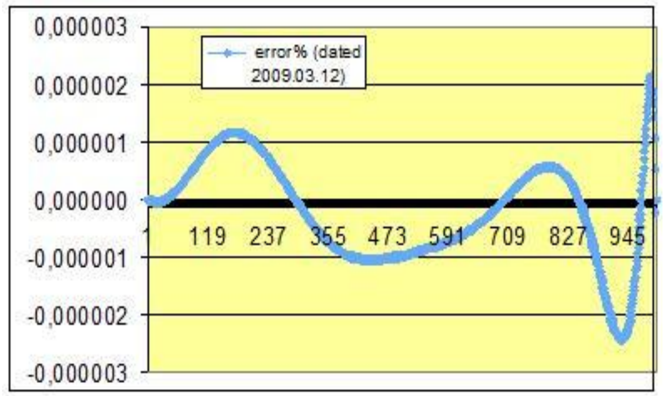

Fig.14 Error % for 1<TAN<infinity (p corrected, coarse).Max.error %=-0,0000024328…at TAN=18,43467933..

A numerical example

Evaluate the total arc length of the ellipse (a=1;b=5), on positive Cartesian
Consider 1000 evaluating lines on an excel table, for 1<TAN< infinity. Use the following designations:

	ATAN*180/Pi

We calculate:

TAN=b/a	=5
ATAN	=1,373400767..
Angle o	=78,69006753 o
x	=748,66817..

we use (x) value in b2,m2,G,in their formula ….. we get

b2Mod	=0,288740099	
m2Mod	=0,000997983	
GMod	=1,948787387	then,

pMod	=2,99518385
sMod	=1,567203335

are evaluated

Use (sMod) in L1 =(1+TAN^sMod)^(1/sMod)
Find L1Estimated =5,25251332979251
L1 Exact =5,25251113492227 (with tools, mathematica, or ….)
Error % =0,000000418041087.. !!!

The cracking method is valid for all mathematical astroid.
(x/a)^r+(y/b)^r=1
For all (0<r<infinity)
For all (1<TAN<infinity)
Error % level is depending of the proposed math-Model.
Proposed math-Models may be cycloidal, polynomial, any function suitable for overlapping.
The equivalency formula of the elliptic finite integrals K=(1+TAN^s)^(1/s) means: total arc length, or area of the astroids, or …what else.
It is explained by the generalized Thales theorem!
It has nothing common with Hölder mean.
One may go further for better accuracy, with better proposition of math-Models

II. CONCLUSION

Fig.15 is a comparison graph of Ramanujan’s and Necat’s estimation
Ramanujan is Master when TAN<2,5

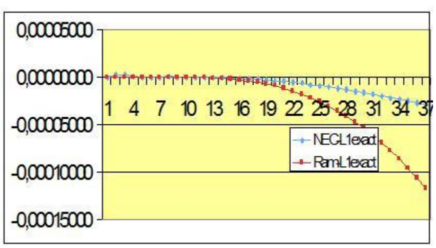

Fig.15 Field 1<TAN<10 is considered.

Ref: IAENG WCECS2008 conference book, page 944