Existence Of $Z = (z\text{-}t)$ Type Plane Gravitational Waves Carrying Some Energy In Six-Dimensional Space-Time

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Abstract **: This paper, which asserts the existence of Z=(z - t) type plane gravitational waves carrying some energy and momentum in the direction of their propogation in six dimensional space-time.**

Keywords **:** Metric tensor, Plane Gravitational Waves, Energy Momentum Pseudo tensor of Einstein and energy momentum tensor of Landau and Lipschitz.

I. Introduction

In general relativity one of the problem is to be solved is whether or not there exist gravitational waves carrying energy. H.Takeno [1] mathematically investigated two types of purely plane gravitational waves of Z=(z - t) and $Z = \left(\frac{t}{z}\right)$ type and obtained the line elements for both waves respectively in four dimensional space time V_4 as,

$$ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - dz^2 + dt^2 , \qquad (1.1)$$

and

$$ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - Z^2(C-E)dz^2 - 2ZEdzdt + (C+E)dt^2 . \qquad (1.2)$$

Furthermore, by calculating non vanishing components of energy momentum pseudo tensors of Einstein and Landua and Lipschitz, Takeno [1], conclude that both the waves carries some energy and momentum in the direction of their propogation. In this paper we propose to obtain the existence of Z=(z - t) type plane gravitational waves carrying some energy and momentum in the direction of their propogation in the sense of Takeno [1] by extending to Six-dimensional space-time deduced by Adhav and Karade[3].

II. Definition

A plane wave g_{ij} is defined as non flat solution of the field equation

$$R_{ij} = 0 , \quad i, j = 1,2,3,4,5,6. \qquad (2.1)$$

In an empty region of space-time with,

$$g_{ij} = g_{ij}(Z),\ \ Z = Z(x^i) \ \text{ where } \ x^i = (x, y, u, v, z, t) \tag{2.2}$$

In some suitable co-ordinate system such that,

$$g^{ij} Z_{,i} Z^{,j} = 0 \quad \left(Z_{,i} = \frac{\partial Z}{\partial xi} \right), \tag{2.3}$$

$$Z = Z(z,t), \quad Z_{;5} \neq 0, \quad Z_{,6} \neq 0 \tag{2.4}$$

The signature convention adopted is,

$$g_{aa} < 0 \quad \begin{vmatrix} g_{aa} & g_{ab} \\ g_{ba} & g_{bb} \end{vmatrix} > 0, \quad \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix} < 0$$

$$\begin{vmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{vmatrix} > 0, \quad \begin{vmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35} \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{45} \\ g_{51} & g_{52} & g_{53} & g_{54} & g_{55} \end{vmatrix} < 0, \quad g_{66} > 0\ , \tag{2.5}$$

(Not summed for *a* & *b*; *a*,*b*=1,2,3,4,5) ,

and accordingly

$$g = det(g_{ij}) < 0. \tag{2.6}$$

III. THE PLANE WAVE METRIC

We adopt the space-time deduced by Adhav and Karade [3] for *Z*=(*z*-*t*) type plane gravitational waves in six-dimensional space-time,

$$ds^2 = -dl^2 - (C - E)dz^2 + 2Edzdt + (C + E)dt^2, \tag{3.1}$$

$$dl^2 = \left(-Adx^2 - 2Ddxdy - 2Gdxdu - 2Idxdu - 2Idxdv - Bdy^2 \right.$$
$$\left. - 2Hdydu - 2Idydv - Edu^2 - 2Kdudv - Ldv^2\right) .$$

It is known that E in (3.1) can be transformed away by suitable transformation of co-ordinate, so in new system.

$$ds^2 = -dl^2 - Cdz^2 + Cdt^2\ . \tag{3.2}$$

By using transformation $z^1 + t^1 = z + t$ and $z^1 + t^1 = \int C(z)dz$, (3.2) can be transformed into simpler form

$$ds^2 = -dl^2 - dz^2 + dt^2 \ . \tag{3.3}$$

The components of metric tensor g_{ij} and g^{ij} for (3.3) are as follows

$$[g_{ij}] = \begin{bmatrix} -A & -D & -G & -I & 0 & 0 \\ -D & -B & -H & -J & 0 & 0 \\ -G & -H & -F & -K & 0 & 0 \\ -I & -J & -K & -L & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{3.4}$$

where A > 0, B > 0, F > 0, L > 0

and

$$\begin{aligned} m = {} & ABFL - ABK^2 - AH^2L + 2AHJK = AJ^2F - D^2FL + D^2K^2 \\ & + 2DGHL - 2DHIK - 2DGJK \_ 2DFIJ - BG^2L + 2BGIK \\ & + G^2J^2 - 2GHIJ - BFI^2 + H^2I^2 \end{aligned} \tag{3.5}$$

n = -1

$$g^{ij} = \begin{bmatrix} -A' & -D' & -G' & -I' & 0 & 0 \\ -D' & -B' & -H' & -J' & 0 & 0 \\ -G' & -H' & -F' & -K' & 0 & 0 \\ -I' & -J' & -K' & -L' & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{3.6}$$

where, $A' = \dfrac{\left(-BFL + B^2K + H^2L - 2HJK + FJ^2\right)}{m}$,

$$B' = \frac{\left(-AFL + AK^2 + G^2L - 2GJK + FI^2\right)}{m},$$

$$D' = \frac{\left(DFL + DK^2 - HGL + HIK + GJK\right)}{m},$$

$$G' = \frac{\left(-DHK + DJK + BGL - BIK - GJ^2 + HIJ\right)}{m},$$

$$H' = \frac{\left(AHL - AJK - DGL + DIK + GIJ - HI^2\right)}{m},$$

$$I' = \frac{\left(DHK - DFJ - BJK + BFI + HGI - H^2 I\right)}{m},$$

$$J' = \frac{\left(-AHK + AFI + DGK - DFI - G^2 I + GHI\right)}{m},$$

$$K' = \frac{\left(ABK - AHJ - D^2 K + DHI + DGJ - BGI\right)}{m},$$

$$F' = \frac{\left(-ABL + AJ^2 + D^2 L - DIK + BI^2\right)}{m},$$

$$L' = \frac{\left(-ABF + AH^2 + D^2 F - 2GDH + BG^2\right)}{m} \ .$$

The non-vanishing components of christoffel symbols for metric (3.3) are,

$$\begin{aligned}
&\begin{Bmatrix} 1 \\ 15 \end{Bmatrix} = -\begin{Bmatrix} 1 \\ 16 \end{Bmatrix} = P \ , && \begin{Bmatrix} 3 \\ 15 \end{Bmatrix} = -\begin{Bmatrix} 3 \\ 16 \end{Bmatrix} = P' \\
&\begin{Bmatrix} 1 \\ 25 \end{Bmatrix} = -\begin{Bmatrix} 1 \\ 26 \end{Bmatrix} = Q \, , && \begin{Bmatrix} 3 \\ 25 \end{Bmatrix} = -\begin{Bmatrix} 3 \\ 26 \end{Bmatrix} = Q' \\
&\begin{Bmatrix} 1 \\ 35 \end{Bmatrix} = -\begin{Bmatrix} 1 \\ 36 \end{Bmatrix} = R \, , && \begin{Bmatrix} 3 \\ 35 \end{Bmatrix} = -\begin{Bmatrix} 3 \\ 36 \end{Bmatrix} = R' \\
&\begin{Bmatrix} 1 \\ 45 \end{Bmatrix} = -\begin{Bmatrix} 1 \\ 46 \end{Bmatrix} = S \, , && \begin{Bmatrix} 3 \\ 45 \end{Bmatrix} = -\begin{Bmatrix} 3 \\ 46 \end{Bmatrix} = S' \\
&\begin{Bmatrix} 2 \\ 15 \end{Bmatrix} = -\begin{Bmatrix} 2 \\ 16 \end{Bmatrix} = T \, , && \begin{Bmatrix} 4 \\ 15 \end{Bmatrix} = -\begin{Bmatrix} 4 \\ 16 \end{Bmatrix} = T' \qquad (3.7) \\
&\begin{Bmatrix} 2 \\ 25 \end{Bmatrix} = -\begin{Bmatrix} 2 \\ 26 \end{Bmatrix} = U \ , && \begin{Bmatrix} 4 \\ 25 \end{Bmatrix} = -\begin{Bmatrix} 4 \\ 26 \end{Bmatrix} = U' \\
&\begin{Bmatrix} 2 \\ 35 \end{Bmatrix} = -\begin{Bmatrix} 2 \\ 36 \end{Bmatrix} = V \ , && \begin{Bmatrix} 4 \\ 35 \end{Bmatrix} = -\begin{Bmatrix} 4 \\ 36 \end{Bmatrix} = V'
\end{aligned}$$

$$\left\{\begin{matrix}2\\45\end{matrix}\right\}=-\left\{\begin{matrix}2\\46\end{matrix}\right\}=W\,,\qquad \left\{\begin{matrix}4\\45\end{matrix}\right\}=-\left\{\begin{matrix}4\\46\end{matrix}\right\}=W'$$

$$\left\{\begin{matrix}5\\11\end{matrix}\right\}=-\left\{\begin{matrix}6\\11\end{matrix}\right\}=-\frac{\bar{A}}{2},\qquad \left\{\begin{matrix}5\\12\end{matrix}\right\}=-\left\{\begin{matrix}6\\12\end{matrix}\right\}=-\frac{\bar{D}}{2}$$

$$\left\{\begin{matrix}5\\13\end{matrix}\right\}=-\left\{\begin{matrix}6\\13\end{matrix}\right\}=-\frac{\bar{G}}{2},\qquad \left\{\begin{matrix}5\\14\end{matrix}\right\}=-\left\{\begin{matrix}6\\14\end{matrix}\right\}=-\frac{\bar{I}}{2}$$

$$\left\{\begin{matrix}5\\22\end{matrix}\right\}=-\left\{\begin{matrix}6\\22\end{matrix}\right\}=-\frac{\bar{B}}{2},\qquad \left\{\begin{matrix}5\\23\end{matrix}\right\}=-\left\{\begin{matrix}6\\23\end{matrix}\right\}=-\frac{\bar{H}}{2}$$

$$\left\{\begin{matrix}5\\24\end{matrix}\right\}=-\left\{\begin{matrix}6\\24\end{matrix}\right\}=-\frac{\bar{J}}{2},\qquad \left\{\begin{matrix}5\\33\end{matrix}\right\}=-\left\{\begin{matrix}6\\33\end{matrix}\right\}=-\frac{\bar{F}}{2}$$

$$\left\{\begin{matrix}5\\34\end{matrix}\right\}=-\left\{\begin{matrix}6\\34\end{matrix}\right\}=-\frac{\bar{K}}{2},\qquad \left\{\begin{matrix}5\\44\end{matrix}\right\}=-\left\{\begin{matrix}6\\44\end{matrix}\right\}=-\frac{\bar{L}}{2}\,.$$

Where

$$P=\frac{A'\bar{A}+D'\bar{D}+G'\bar{G}+I'\bar{I}}{2m},\qquad P'=\frac{F'\bar{G}+K'\bar{I}+G'\bar{A}+H'\bar{D}}{2m}$$

$$Q=\frac{A'\bar{D}+D'\bar{B}+G'\bar{H}+I'\bar{J}}{2m},\qquad Q'=\frac{F'\bar{H}+G'\bar{D}+H'\bar{B}+K'\bar{J}}{2m}$$

$$R=\frac{A'\bar{G}+D'\bar{H}+G'\bar{F}+I'\bar{K}}{2m},\qquad R'=\frac{F'\bar{F}+G'\bar{G}+H'\bar{H}+K'\bar{K}}{2m}$$

$$S=\frac{A'\bar{I}+D'\bar{J}+G'\bar{K}+I'\bar{L}}{2m},\qquad S'=\frac{F'\bar{K}+K'\bar{L}+H'\bar{J}+G'\bar{I}}{2m}$$

$$T=\frac{B'\bar{D}+H'\bar{G}+J'\bar{I}+D'\bar{A}}{2m},\qquad T'=\frac{L'\bar{I}+I'\bar{A}+J'\bar{D}+K'\bar{G}}{2m}$$

$$U=\frac{B'\bar{B}+D'\bar{D}+H'\bar{H}+J'\bar{J}}{2m},\qquad U'=\frac{L'\bar{J}+I'\bar{D}+J'\bar{B}+K'\bar{H}}{2m}$$

$$V=\frac{B'\bar{H}+D'\bar{G}+H'\bar{F}+J'\bar{K}}{2m},\qquad V'=\frac{L'\bar{K}+K'\bar{F}+J'\bar{H}+I'\bar{G}}{2m}$$

$$W=\frac{B'\bar{J}+D'\bar{I}+H'\bar{K}+J'\bar{K}}{2m},\qquad W'=\frac{L'L+I'\bar{I}+J'\bar{J}+K'\bar{K}}{2m}\,.$$

Pawar et.al [5] investigated coexistence of $Z= (z - t)$ type plane gravitational waves with electromagnetic waves in six-dimensional space-time and obtained exact solution of field equation (2.1) as,

$$N = 8\pi\{\sigma^2 A' - 2\sigma\rho D' - 2\eta G' - 2\sigma\xi I' + \rho^2 B' + 2\rho\eta H' + 2\rho\xi J' + \eta^2 F' + 2\eta\xi K' + \xi^2 L'\} \quad . \tag{3.8}$$

IV. SOME USEFUL FORMULAE

For metric (3.3) and from (3.7) we made following formulae which are useful to calculate the components of pseudo tensors.

$$\sqrt{-g} = \sqrt{m}, \quad \left(\sqrt{-g}\right)_{,i} = \left(0,0,0,0,\frac{\overline{m}}{2m},-\frac{\overline{m}}{2m}\right) \tag{4.1}$$

$$\begin{Bmatrix} 1 \\ 1i \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2i \end{Bmatrix} = \begin{Bmatrix} i \\ 3i \end{Bmatrix} = \begin{Bmatrix} i \\ 4i \end{Bmatrix} = 0, \quad \begin{Bmatrix} i \\ 5i \end{Bmatrix} = -\begin{Bmatrix} i \\ 6i \end{Bmatrix} = \frac{\overline{m}}{2m}, \tag{4.2}$$

$$\begin{Bmatrix} b \\ a5 \end{Bmatrix} = -\begin{Bmatrix} a \\ b5 \end{Bmatrix}, \quad g^{pq}\begin{Bmatrix} r \\ pq \end{Bmatrix} = 0, \quad g^{pq}\begin{Bmatrix} r \\ ps \end{Bmatrix}\begin{Bmatrix} s \\ qr \end{Bmatrix} = 0$$

$$\begin{Bmatrix} p \\ ac \end{Bmatrix}\begin{Bmatrix} d \\ bp \end{Bmatrix} = 0, \quad \begin{Bmatrix} b \\ 5a \end{Bmatrix}\begin{Bmatrix} a \\ 5b \end{Bmatrix} = -\begin{Bmatrix} b \\ 5a \end{Bmatrix}\begin{Bmatrix} q \\ 6b \end{Bmatrix} = \begin{Bmatrix} b \\ 6a \end{Bmatrix}\begin{Bmatrix} a \\ 6b \end{Bmatrix}, \tag{4.3}$$

$$\begin{Bmatrix} q \\ 5p \end{Bmatrix}\begin{Bmatrix} p \\ 5q \end{Bmatrix} = -\begin{Bmatrix} q \\ 5p \end{Bmatrix}\begin{Bmatrix} p \\ 6q \end{Bmatrix} = \begin{Bmatrix} q \\ 6p \end{Bmatrix}\begin{Bmatrix} p \\ 6q \end{Bmatrix} \quad .$$

Where,

$$\overline{m} = \{-A'\overline{A} - 2D'\overline{D} - 2G'\overline{G} - 2I'\overline{I} + B'\overline{B} - 2H'\overline{H} - J'\overline{J} - F'\overline{F} - 2K'\overline{K} - L'\overline{L}\}$$

and $i = 1,2,3,\ldots\ldots,6$ $a, b, c, d = 1, 2, 3, 4$; $p, q, r, s = 5,6$ and summation convention is used with respect to these indices.

V. PSEUDO-TENSOR OF EINSTEIN

We calculate the components of energy-momentum Pseudo-tensor t_i^j introduced by Einstein which satisfies the conservation relation in the form

$$\left\{\sqrt{-g}\left(T_i^{\,j}+t_i^{\,j}\right)\right\}_{,j}=0 \quad \left(,j=\frac{\partial}{\partial x^j}\right), \quad (i, j = 1, 2, 3, \ldots, 6). \tag{5.1}$$

When $T_i^{\,j}$ is energy momentum tensor of matter coincides with $E_i^{\,j}$

According to Einstein theory the $t_i^{\,j}$ expresses the energy momentum due to gravitational field and is given by

$$\begin{aligned} 16\pi\sqrt{-g}\,t_i^{\,j} &= \left\{ {j \atop mn} \right\}\left(\sqrt{-g}\,g^{mn}\right)_{,i} - \left(\log\sqrt{-g}\right)_{,m} + \\ &\delta_i^j\left[\left\{ {h \atop mk} \right\}\left\{ {k \atop nh} \right\} g^{mn}\sqrt{-g} - g^{mn}\left\{ {h \atop mn} \right\}\left(\sqrt{-g}\right)_{,h}\right] . \end{aligned} \tag{5.2}$$

If we substitute (3.3) and (3.7) into (5.2) and using (4.1), (4.2) and (4.3) we obtain following result;

$$\begin{aligned} t_5^5 &= \frac{\beta}{16\pi m}, \\ t_5^6 = t_6^5 &= \frac{-\beta}{16\pi m}, \\ t_6^6 &= \frac{\beta}{16\pi m} . \end{aligned} \tag{5.3}$$

Where

$$\beta=\frac{1}{2}\left\{2\overline{D}\,\overline{D'}-2\overline{G}\,\overline{G'}-2\overline{I}\,\overline{I'}-2\overline{J}\,\overline{J'}-2\overline{K}\,\overline{K'}-\overline{A}\,\overline{A'}-\overline{B}\,\overline{B'}+\overline{F}\,\overline{F'}+\overline{L}\,\overline{L'}\right\}$$

And other $t_i^{\,j}=0$.

From above, we have

$$t_5^5=-t_5^6=-t_6^5=t_6^6=\phi \ , \tag{5.4}$$

where $16\pi\phi=\dfrac{\beta}{m}$. (5.5)

Here ϕ is a function of Z and does not Vanish in general.

VI. . Pseudo-tensor of Landau and Lifshitz

Next we calculate the components of the symmetric energy momentum pseudo-tensor t^{*ij} proposed by Landau and Lipshitz which satisfies the conservation relation in the form

$$\{(-g)(T^{ij} + t^{*ij})\}_{ij} = 0, \qquad (i,j=1,2,3,\ldots.6)$$

and it is given by expression

$$\begin{aligned}16\pi t^{*ij} &= \left(g^{ik}g^{jl} - g^{ij}g^{kl}\right)\left[2\begin{Bmatrix} h \\ kl \end{Bmatrix}\begin{Bmatrix} m \\ hm \end{Bmatrix} - \begin{Bmatrix} m \\ kh \end{Bmatrix}\begin{Bmatrix} h \\ lm \end{Bmatrix} - \begin{Bmatrix} h \\ kh \end{Bmatrix}\begin{Bmatrix} m \\ lm \end{Bmatrix}\right] \\ &+ g^{ik}g^{mn}\left[\begin{Bmatrix} i \\ kh \end{Bmatrix}\begin{Bmatrix} h \\ mn \end{Bmatrix} + \begin{Bmatrix} i \\ mn \end{Bmatrix}\begin{Bmatrix} h \\ kh \end{Bmatrix} - \begin{Bmatrix} j \\ nh \end{Bmatrix}\begin{Bmatrix} h \\ km \end{Bmatrix} - \begin{Bmatrix} j \\ km \end{Bmatrix}\begin{Bmatrix} h \\ nh \end{Bmatrix}\right] \\ &+ g^{jk}g^{mn}\left[\begin{Bmatrix} i \\ kh \end{Bmatrix}\begin{Bmatrix} h \\ mn \end{Bmatrix} + \begin{Bmatrix} i \\ mn \end{Bmatrix}\begin{Bmatrix} h \\ kh \end{Bmatrix} - \begin{Bmatrix} i \\ nh \end{Bmatrix}\begin{Bmatrix} h \\ km \end{Bmatrix} - \begin{Bmatrix} i \\ km \end{Bmatrix}\begin{Bmatrix} h \\ nh \end{Bmatrix}\right] \\ &- g^{hk}g^{mn}\left[\begin{Bmatrix} i \\ hm \end{Bmatrix}\begin{Bmatrix} j \\ kn \end{Bmatrix} - \begin{Bmatrix} i \\ hk \end{Bmatrix}\begin{Bmatrix} j \\ mn \end{Bmatrix}\right], \qquad \left(t^{*ij} = t^{*ji}\right) .\end{aligned} \tag{6.1}$$

If we substitute (3.3) and (3.7) into expression (6.1) and using (4.1), (4.2) and (4.3) we obtain following result :

$$t^{*55} = t^{*56} = t^{*65} = t^{*66} = \phi^*, \quad \text{other } t^{*ij} = 0, \tag{6.2}$$

where

$$16\pi\phi^* = -\frac{\left\{\beta + \dfrac{\overline{m}^2}{m}\right\}}{m}, \tag{6.3}$$

and

$$\begin{aligned}\beta = [\ &\left(\overline{A}\,\overline{B} - \overline{D}^2\right)\left(FL - K^2\right) + \left(\overline{A}\,\overline{F} - \overline{G}^2\right)\left(BL - J^2\right) + \left(\overline{A}\,\overline{L} - \overline{I}^2\right)\left(BF - H^2\right) \\ &\left(\overline{F}\,\overline{B} - \overline{D}^2\right)\left(AL - I^2\right) + \left(\overline{L}\,\overline{B} - \overline{J}^2\right)\left(AF - G^2\right) + \left(\overline{F}\,\overline{L} - \overline{K}^2\right)\left(AB - D^2\right) \\ &+ 2\left(\overline{A}\,\overline{B} - \overline{D}\,\overline{G}\right)(JK - LH) + 2\left(\overline{A}\,\overline{J} - \overline{D}\,\overline{I}\right)(JF - KH) + 2\left(\overline{A}\,\overline{K} - \overline{I}\,\overline{G}\right)(JH - BK) \\ &+ 2\left(\overline{B}\,\overline{G} - \overline{H}\,\overline{D}\right)(IK - LG) + 2\left(\overline{B}\,\overline{I} - \overline{D}\,\overline{J}\right)(GK - FI) + 2\left(\overline{B}\,\overline{K} - \overline{J}\,\overline{H}\right)(IG - AK) \\ &+ 2\left(\overline{F}\,\overline{D} - \overline{H}\,\overline{G}\right)(JI - LD) + 2\left(\overline{F}\,\overline{I} - \overline{K}\,\overline{G}\right)(JD - BI) + 2\left(\overline{F}\,\overline{J} - \overline{H}\,\overline{K}\right)(ID - AJ) \\ &+ 2\left(\overline{L}\,\overline{D} - \overline{I}\,\overline{J}\right)(GH - FD) + 2\left(\overline{L}\,\overline{H} - \overline{K}\,\overline{J}\right)(GD - AH) + 2\left(\overline{L}\,\overline{G} - \overline{I}\,\overline{K}\right)(HD - BG) \\ &+ 2\overline{H}\,\overline{I}\,(2HI - DK - JG) + 2\overline{G}\,\overline{J}\,(2GJ - DK - HI) + 2\overline{D}\,\overline{K}\,(2DK - GJ - HI)\ ].\end{aligned}$$

Again ϕ^* is not zero in general.

To compare results (5.4) with (6.2) we introduce the quantity t^{ij} in the coordinate system, as,

$$g^{ik}t_k^j = t^{ij} = t^{ij}. \tag{6.5}$$

Then we have from (5.4)

$$\cancel{t}^{55}.\quad t^{56}=t^{65}=t^{66}=\phi=\frac{-\beta}{16\pi m} \qquad (6.6)$$

Here we find that t^{ij} and t^{*ij} are composed of similar components but they are not same. When the field equations (3.8), is satisfied, we have

$$N=\frac{\overline{m}}{2m}-\frac{\overline{m}^2}{4m^2}-\frac{\beta}{2m}=8\pi\left(\sigma^2A'-2\sigma\rho D'-2\eta G'-2\sigma\xi I'+\rho^2B'+2\rho\eta H'+2\rho\xi J'+\eta^2F'+2\eta\xi K'+\xi^2L'\right) \quad . \qquad (6.7)$$

Hence we have,

$$-16\pi\phi=2\left(N-\frac{\overline{m}}{2m}+\frac{\overline{m}^2}{4m^2}\right),$$
$$and\ 16\pi\phi^*=2\left(N-\frac{\overline{m}}{2m}\right) \quad . \qquad (6.8)$$

Also when space-time is empty, we have N=0, ϕ and ϕ^* becomes respectively

$$-16\pi\phi=\frac{\left(-\overline{m}+\dfrac{\overline{m}^2}{2m}\right)}{m},$$
$$16\pi\phi^*=-\frac{\overline{m}}{2m}. \qquad (6.9)$$

Again these values are functions of Z and do not vanish in general.

VII. Conclusion

We conclude that, the energy momentum pseudo tensors (5.1) or (6.1) which expresses the energy momentum due to gravitational field hence the gravitational waves given by (3.3) carry some energy and momentum in the direction of their propogation.

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