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From the Chief Author's Desk

We see a drastic momentum everywhere in all fields now a day. Which in turns, say a lot to everyone to excel with all possible way. The need of the hour is to pick the right key at the right time with all extras. Citing the computer versions, any automobile models, infrastructures, etc. It is not the result of any preplanning but the implementations of planning.

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This Global Journal is like a banyan tree whose branches are many and each branch acts like a strong root itself.

Intentions are very clear to do best in all possible way with all care.

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Immunomodulatory Activity of Ayurvedic Plant Aparajita (Clitoria Ternatea L.) In Male Albino Rats

GJSFR Classification - G (FOR) 0702,1114,1115

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***Abstract*-the present study was undertaken to investigate immunomodulatory activity of Clitoria ternatea seed and root extracts. Effects on humoral immune response were investigated in SRBCs-sensitized rats. Effects on cell medicated immunity were studied by measuring delayed type hypersensitivity (DTH) response in SRBC-sensitized rats. Neutrophil recruiting and phagocytosis were measured by studying neutrophil adhesion and carbon clearance method respectively. Further the effects on hematological parameters were also studied. *C. ternatea* seed and root extracts showed significant immunosupressive effects as evident from significant decrease in primary and secondary antibody titers in SRBCs-sensitized rats, paw thickness in DTH response, and neutrophil adhesion and *In vitro* Phagocytosis. The immunomodulatory effects of *C. ternatea* on humoral, cell mediated and non-specific immune response could be attributed to decreased immune cell sensitization, immune cell presentation and phagocytosis. The anti-inflammatory and antioxidant properties of plant might be playing major role in immunomodulatory activity. The present study provided evidence for the traditional uses of the plants in Indian system of medicine.**

Keywords-DTH response, Immunosuppressive, Neutrophil index, Phagocytosis, Primary antibody titer.

I. INTRODUCTION

Clitoria ternatea L. belonging to family 'Fabaceae', is popularly known as a "Butterfly pea" in western countries and as "Aparajita" in the traditional Ayurvedic system of medicine. Clitoria ternatea (CT) is one of the important plants of Ayurvedic system of medicine and is official in the Ayurvedic Pharmacopoeia of India (Anonymous, 2003). It is reported to have brain tonic activity, and is popularly known as 'shankhapushpi' (Upadhye & Kumbhojkar, 1993) in southern India. In traditional system of medicine, it is employed against different disease conditions such as cathartic, purgative, demulcent, emetic and anti-inflammatory in swollen joints (Kirtikar & Basu, 1976; Chopra et al. 1956). Ayurvedic system prescribed various part of the plant in inflammation, hepatic disorders and as a brain tonic (Anonymous, 2003). Various parts of CT have been reported to have nootropic activity, anxiolytic activity, tranquilizing property, anti-inflammatory and analgesic activity, antipyretic, and antimicrobial activity (Mukherjee et al., 2008). It is also reported to have immunomodulatory activities in alloxan-induced diabetic rats (Daisy et al. 2004). The plant is found to possess antibacterial activity (Malabadi et al. 2005). The flavonol glycoside present in roots is reported to have antibacterial activity (Yadava & Verma, 2003). CT has been reported to contain kaempferol and related glycosides, aparajitin, anthocyanins (Shrivastava & Pande, 1977), and anthoxanthins (Gupta & Lal, 1968). However, no study had been reported on immunomodulatory activities especially of seeds and roots in animal models. Hence, we conducted the present study to evaluate an immunomodulatory activity of seeds and roots of CT in male albino rats.

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II. MATERIALS AND METHODS

A. Plant collection and Identification

The plant is available in two varieties – blue flowered and white flowered. It is climbing vine found on road side and field sides throughout India. Since, the blue variety is medicinally more important, we used only blue variety for the present investigation. The plant was collected in the month of March (2007) from the fields and road side of the Charotar region of the Gujarat state, India. The pods were allowed to dry sufficiently under shade, and finally seeds were collected manually. The plant was botanically identified by Dr. G. C. Jadeja, Professor and Head of Agricultural Botany Department, B. A. College of Agriculture, Anand Agricultural University, Anand, India. The specimens of the sample were stored in the museum of the department (specimen no. 0701). The quality of plant was ascertained as per Ayurvedic Pharmacopoeia of India by determining foreign matters, total ash, acid insoluble ash, alcohol soluble extractive, and water soluble extractive values (Anonymous, 1999)

B. Preparation of extracts

The dry powdered (40#) seeds (1kg) were extracted with petroleum ether by percolation until the percolate was free of green color. The residues were extracted with 50% v/v alcohol by heating on boiling water bath under reflex for 3 h. The solvents were evaporated to have pasty mass, referred as CT seed extract. The dry powdered (40#) roots were directly extracted with 50% v/v alcohol by heating on the boiling water bath under reflex for 4 h. The solvents were evaporated at room temperature to have pasty mass, referred as CT root extract.

C. Preliminary phytochemical screening

CT seed and root extracts were screened for phytochemicals using the method specified by Kokate, (2003).

D. Chemicals and reagents

All the chemicals used in the present study, were of analytical grade and purchased from S. D. Fine chemicals Limited, Mumbai. Dexamethsone (DMS) was obtained from Cadila Zydus Research Centre, Ahmedabad.

E. Pharmacological evaluation

Animals

Male albino rats (Wistar strain) weighing 150-200 g were divided into different groups, each consisting of six animals. Animals were maintained on a commercial chew diet (Pranav Agro Industries Ltd., Sangli, Maharastra, India) and water *at libidum* throughout the study period. This study was approved by the institutional animal ethics committee in accordance with the guidelines of Committee for the Purpose of Supervision and Control of Experiments on Animals (CPCSEA) (CPCSEA, 2003). For each experiment outlined below, rats were randomized into various groups that received CT seed extract, CT root extract, vehicle, or dexamethasone (DMS) as a reference immunosuppressive drug.

F. Treatment regimens

CT seed and root extracts were suspended in distilled water using 1% w/v gum acacia. DMS was suspended at a concentration of 0.8 µg/mL in distilled water using 1% w/v CMC. In the studies herein unless elsewise indicated, treatment rats received CT seed and root extracts at 500 mg/kg body weight (BW) in 1 mL doses daily by gavage. The control group rats received vehicle, i.e., a single 2 mL bolus bearing 1 mL each of the 1% w/v gum acacia and 1% w/v CMC solutions, in parallel daily. The rats in the reference drug group received DMS at the dose of 0.25 mg/kg BW in 1 mL volume, daily by gavage.

G. Antigen preparation

Fresh blood was collected from sheep sacrificed in the local slaughter house, and placed in Alsever's solution. During the experiment, adequate amount of stock solution of sheep red blood cells (SRBC) stored in Alsever's solution, was taken and allowed to stand at room temperature. It was washed three times with normal saline. The settled SRBC were then suspended in normal saline. The SRBC of this suspension were adjusted to a concentration of $5x10^9$ SRBC/mL for immunization and challenge (Bafna and Mishra, 2005).

H. Acute toxicity study

Animals were treated with different doses 250, 500, 750 and 1000 mg/kg, p.o. of each extracts. After single dose administration, animals were observed for death or any other deformities up to 72 h.

I. SRBC–induced humoral antibody (HA) titer

The method described by Atal et al. (1986) was utilized to examine the rats provided CT seed and root extracts once daily by gavage, starting 7 days prior to sensitization and continuing up to the second time of challenge (i.e., Day –7 up to and through Day +14; for a total of 21 d). Control and DMS–treated rats received vehicle or the drug, respectively, in parallel each day.

To specifically assess effects on antibody formation, groups of six rats per treatment were immunized with 20 µL of SRBC suspension ($5x10^9$ SRBC/mL) injected subcutaneously into right hind foot pad. The day of immunization was referred to as Day 0. Seven days later (Day +7), the rats were challenged by injecting 20 µl of SRBC suspension ($5x10^9$ SRBC/mL) intradermally into the left hind foot pad. Blood samples were collected from all the animals separately by retro–orbital puncture under light ether anesthesia on Day +7 (after challenge) for assessment of primary antibody titer and on Day +14 (after challenge) for measures of secondary antibody tire. Antibody levels were determined by the method described by Shinde et al. (1999). After allowing the collected blood to clot, serum was isolated and 25 µL was placed into one well of a 96–well microtiter plate. Serial two–fold dilutions of the serum were made using 25 µL of normal saline each time of transfer across the plate. To the 25 µL of diluted serum in each well was then added 25 µL of 1% v/v SRBC suspension in normal saline. The microtiter plate was maintained at room temperature for 1 h and then well contents examined for haemagglutination i.e., until control wells showed unequivocally negative patterns. The value of the highest serum dilution showing haemagglutination was defined as the antibody titer for the given rat.

J. SRBC–induced delayed–type hypersensitivity (DTH) Response

The method of Lagrange et al. (1974) was used to analyze effects on DTH responses in the treated rats. Daily treatment with CT seed and root extracts (500 mg/kg, by gavage) began 14 days prior to the challenge i.e., starting on the dame day as immunization with SRBC. Control and DMS–treated rats received vehicle or the drug, respectively, in parallel each day.

On Day 0, all rats were immunized with20 µL SRBC solution (5 x 109 SRBC/mL) injected subcutaneously into their right hind footpad. After 14 days of gavage treatment, the thickness of each rat's left footpad was measured just before the challenge; using a Schnelltaster caliper (H.C. Kroplin Hessen, Schluchtern, Germany) that could measure to a minimum unit of 0.01 mm. The rats were then challenged by injecting 20 µL SRBC solution (5 x 109 SRBC/mL) intradermally into their left hind footpad (deemed time 0). Foot thickness was the re–measured after 24 h. The difference between the thicknesses of left foot just before and 24 h after challenge (in mm) was taken as a measure of DTH (Doherty, 1981)

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K. Neutrophil adhesion test

The method described by Wilkonson (1978), was used for evaluating the effect of CT seed and root extracts on neutrophil adhesion. After 14 days of gavage treatment, blood samples were collected from rats in each group by retro–orbital puncture under light ether anesthesia in heparinized vials and subjected to total as well as differential leukocyte count. After performance of the initial counts, the each blood sample was incubated with 80 mg/mL of nylon fibers at 37˚C for 15 min. The incubated samples were again analyzed for total and differential leukocyte count. The product of total leukocyte count and the percentage (%) neutrophil (known as neutrophil index) was determined for each rat of the respective groups (Fulzele et al., 2002). The % neutrophil adhesion for each of the test rat was then calculated as "% Neutrophil Adhesion" =100 x (NIu – NIt)/ NIu, Where 'NIu' is the neutrophil index of the blood samples before nylon fiber treatment and 'NIt ' the index after nylon fiber treatment.

L. Carbon clearance test

The method of Biozzi *et al*. (1953) was used to analyze phagocytic activity among the white blood cells in the rats. For each treatment regimen, a total of 6 rats were utilized. Daily treatment with CT seed extract (500 mg/kg, by gavage) occurred for 5 day prior to the assessment of *in situ* phagocytic activity. Control and dexamethasone – treated rats received vehicle or the drug, respectively, in parallel each day. A colloidal carbon ink suspension was injected via the tail vein into each rat 48 h after the final treatment. From each rat, blood samples (25 µL) were then withdrawn from the retro–orbital plexus under mild ether anesthesia, immediately after the injection and then 5, 10, and 15 min thereafter. Each blood sample was lysed with 2 mL of 0.1% acetic acid and the absorbance of the resulting solution evaluated at 675 nm (Damre et al., 2003). A graph of absorbance vs. time post–injection was prepared for each animal and the *in situ* phagocytic index calculated using following formula, "Phagocytic Index (PI)" = K_{sample} / $K_{standard}$, wherein K_{sample} represents the slope of the absorbance vs. time curve of blood samples from rats in the extract – treated or Dexamethasone – treated group and $K_{standard}$ represents the slope of the absorbance vs. time curve of blood samples for the rats in the control group.

M. Hematological profile

After 8 days of the repeated gavage treatment, blood was collected from each rat via their retro–orbital plexus under light ether anesthesia. Various parameters such as total white blood cell (WBC), differential WBC, red blood cell (RBC), platelet counts, as well as hemoglobin (Hb) levels were then evaluated using a Sysmax XS800i automated hematology analyzer (TOA Medical Electronic Co., Tokyo, Japan).

N. Statistical analysis

Statistical analysis was carried out using one way ANOVA followed by Tukey's test, using the SigmaState™ 2.03 software and computer with Intel Pentium® dual core™ processor. A value of $p < 0.05$ was considered a statistically significant difference between analyzed groups.

III. RESULTS

In the present study, the immunomodulatory effect of CT seed and root extracts were investigated using various experimental models. The effect on humoral immunity was estimated by measuring primary and secondary antibody titers in SRBC sensitized rats. Effect on innate or cell mediated immunity was studied against delayed type of hypersensitivity (DTH) response. Further, neutrophil recruiting and phagocytic activity of the reticuloendothelial system were measured by neutrophil adhesion and removal of carbon particles from the blood circulation.

Acute toxicity study showed that CT seed and root extracts were safe up to the dose of 1000 mg/kg, p.o. The preliminary phytochemical screening showed presence of glycosides, tannins, saponins, phenolics, flavonoids, proteins, and carbohydrates.

A. SRBC–induced antibody (HA) titer

In SRBC-sensitized rats, the primary titer was significantly decreased by CT seed (0.05 ± 0.01) and root (0.03 ± 0.01) extracts on day 14 and secondary titer was significantly decreased by both the extract (0.06 ± 0.01 and 0.03 ± 0.01) respectively on day 21 when compared with the control group (3.52 ± 0.76) on day 14 and (5.00 ± 0.76) on day 21. Reference immunosupressive drug dexamethasone showed significant decrease in primary titer (0.15 ± 0.02) and secondary titer (0.19 ± 0.05) (table 1).

Table: 1 Effects on antibody formation by SRBC-sensitized rats.

Treatment	Primary titer	Secondary titer
Control	3.52 ± 0.76	5.00 ± 0.76
Dexamethasone	0.15 ± 0.02*	0.19 ± 0.05*
CT seed extract	0.05 ± 0.01*	0.06 ± 0.01*
CT root extract	0.03 ± 0.01*	0.03 ± 0.01*

*All values represent mean ± SEM; n = 6 per treatment group. Statistical analysis was carried out using One Way ANOVA followed by Tukey's multiple range test. *: Value significantly different (p < 0.05) compared with the control group. [a]CT: C. ternatea, treatments began in period starting 7 d prior to sensitization and continuing up to time of challenge (i.e., Day -7 up to and through Day +7)*

B. SRBC-induced DTH response

The cell-mediated immune responses of CT seed and root extracts were assessed by DTH reaction, i.e. foot pad reaction. The DTH response was measured as difference in thickness of hind paw before and after the challenge with SRBC solution. CT seed (0.31 ± 0.01) and root (0.40 ± 0.02) extracts produced significant ($p < 0.001$) decrease in the DTH response when compared with the control group (0.85 ± 0.02). These effects were comparable with that of reference immunosuppressant drug – dexamethasone (0.36 ± 0.01) (table 2)

C. *Neutrophil adhesion test*

The % neutrophil adhesion was significantly ($p < 0.01$) decreased by CT seed (14.32 ± 1.09) and root (8.71 ± 0.81) extracts at the dose of 500 mg/kg, p.o., when compared with the control group (23.33 ± 1.02). These effects were comparable with the reference drug - dexamethasone (14.00 ± 3.07). The finding suggested possible immunosuppressive or immunoinhibitory action of both the extracts (table 3).

Table: 2 Effects on DTH response and the phagocytic index

Treatment	DTH Response	Phagocytic index[b]
Control	0.85 ± 0.02	1.00
Dexamethasone	0.36 ± 0.01* (-57.64%)	0.24 ± 0.005*
CT seed extract	0.31 ± 0.01* (-63.53%)	0.31 ± 0.005*
CT root extract	0.40 ± 0.02* (-52.94%)	0.40 ± 0.004*

All values represent mean ± SEM; n = 6 per treatment group. Statistical analysis was carried out using One Way ANOVA followed by Tukey's multiple range test. *: Value significantly different ($p < 0.05$) compared with the control group. CT: C. ternatea. [a] Value in parentheses indicate decrease or increase in DTH response relative to control rat value. [b] Control value set to 1.00 for comparative purposes.

D. *Carbon clearance test*

The in vivo phagocytic activities were measured by carbon clearance method. CT seed (0.31 ± 0.01) and root (0.40 ± 0.01) extracts produced significant decrease in phagocytic index. The phagocytic index of the control group was considered as unite. The dexamethasone produced significant decrease in the phagocytic index (0.24 ± 0.01) (table 2).

E. *Effects on hematological profile*

The CT seed and root extracts significantly decreased blood lymphocyte, and RBC counts, as well as Hb content when compared with the control group. The reference drug dexamethasone significantly decreased blood total WBC, neutrophil, RBC counts, and Hb content (table 4).

Table: 3 Effect of CT seed and root extracts on neutrophil index and neutrophil adhesion.

Group	TLC (10^3/mm^3) (X)		% Neutrophil (Y)		Neutrophil Index (X x Y) x10^3		% Neutrophil Adhesion
	UnB	FTB	UnB	FTB	UnB	FTB	
Control	6.38 ± 0.57	6.10 ± 0.60	14.34 ± 2.55	11.41 ± 1.85	88.18 ± 10.82	67.35 ± 7.86	23.33 ± 1.02
DMS	1.42 ± 0.34*	1.43 ± 0.31*	3.51 ± 0.09*	3.10 ± 0.08*	5.46 ± 1.32*	4.62 ± 1.09*	14.00 ± 3.07*
CT seed extract	6.83 ± 0.66	6.52 ± 0.64	32.83 ± 6.66	30.60 ± 6.55	23.18 ± 6.66*	21.10 ± 6.72*	14.32 ± 1.09*
CT root extract	4.82 ± 0.43	5.26 ± 0.57	17.60 ± 0.63	14.93 ± 0.91	8.36 ± 4.37*	7.62 ± 3.79*	8.71 ± 0.81*

All values represent mean ± SEM; n = 6 per treatment group. Statistical analysis was carried out using One Way ANOVA followed by Tukey's multiple range test. $P < 0.05$ was considered statistically significant. *: significant when compared with the control group. CT: *C. ternatea*, DMS: dexamethasone. TLC: total leukocytes count; UnB: untreated blood; FTB: nylon fiber-treated blood.

Table: 4 Effects of CT seed and root extracts on hematological parameters.

Group	Total WBC (cells/µL) x 10^3	Neutrophils (cells/µL) x 10^3	Lymphocytes (cells/µL) x 10^3	RBC (cells/µL) x 10^6	Platelets (cells/µL) x 10^6	Hb (g/dL)
Control	6.14±0.84	0.46 ± 0.15	4.90 ± 0.96	9.16 ± 0.21	813.33 ± 53.48	16.13 ± 0.30
DMS	0.97 ± 0.04*	0.04 ± 0.04*	0.76 ± 0.07*	7.27 ± 0.17*	940.00 ± 47.84	13.17 ± 0.31*
CT seed extract	6.58 ± 0.59	0.33 ± 0.05	2.11 ± 0.18*	6.77 ± 0.54*	821.08 ± 58.12	14.32 ± 0.34*
CT root extract	4.63 ± 0.39	0.18 ±0.03	1.95 ± 0.12*	7.12 ± 0.18*	860.55 ± 61.72	13.76 ± 0.45*

All values represent mean ± SEM; n = 6 per treatment group. Statistical analysis was carried out using One Way ANOVA followed by Tukey's multiple range test. *: $P < 0.05$ was considered statistically significant. *: significant when compared with the control group. CT: *C. ternatea*, DMS: dexamethasone. WBC: White blood cells, RBC: Red blood cells, Hb: Hemoglobin.

IV. Discussion

In the present study, the immunomodulatory activity of CT seed and root were investigated using experimental models. The humoral response was measured as primary and secondary antibody titers in sheep red blood cells (SRBC) sensitized rats; the cell mediated immune response was measured as delayed type of hypersensitivity (DTH) response in SRBC sensitized rats. The neutrophil recruiting and Phagocytic activity of Reticuloendothelial system was measured as neutrophil adhesion and carbon clearance method. Further, the effects of CT seed and root extracts on hematological parameters were also investigated.

When animal hosts are non-intravenously sensitized with sheep red blood cells (SRBC), this 'antigen' initially becomes diffused within the extra vascular space and ultimately, via the lymphatic system, enters regional lymph nodes. Macrophages in the lymphoid tissues or lining the sinuses are then able to phagocytize the antigen, process it for presentation (in the context of surface major histocompatability Class II molecules), and become antigen-presenting cells (APC) to many cells, including lymphocytes. Another APC is the B-lymphocyte; like macro-phages, B-lymphocytes are not very effective at presenting this or other antigen to naïve T-lymphocytes. They are, however, effective in presenting antigen to memory lymphocytes, especially when antigen level is low. Once the antigen has been fragmented and processed, helper TH2 T-lymphocytes can then interact to assist/stimulate the B-lymphocytes to produce antibody against the SRBC. In general, during a first (primary) response to exposure to the SRBC/antigen, IgM is secreted initially, followed by a switch to IgG (Goldsby et al., 2003). On re-exposure to the antigen, a secondary response is elicited that is characterized by a rapid onset and highly amplified level of antibody production. Thus, Antibody molecules, a product of B lymphocytes and plasma cells, are central to humoral immune responses, IgG and IgM are the major immunoglobulins which are involved in the complement activation, opsonization, neutralization of toxins, etc. (Miller, 1991). At neutral pH, red blood cells possess negative ions cloud that makes the cells repel from one another, this repulsive force is referred to as zeta potential. Because of its size and pentameric nature, IgM can overcome the electric barrier and get cross-link red blood cells, leading to subsequent agglutination. The smaller size and bivalency of IgG, however, makes them less capable to overcome the electric barrier. This characteristic may accounted for, IgM being more effective than IgG in agglutinating red blood cells (Kuby, 1994).

In the present study, anti-SRBC antibody titers - during both primary and secondary responses -were found significantly decreased in the hydroalcoholic extracts-treated rats. The inhibition of the humoral response by CT seed and root extracts that were noted here could indicate that there was decreased responsiveness of macrophages/B-lymphocytes subsets in these hosts.

Phagocytosis represents an important innate defense mechanism against ingested particulates including whole pathogenic microorganisms. The specialized cells that are capable of phagocytosis include blood monocytes, neutrophils and tissue macrophages. Once particulate material is ingested into phagosomes, the phagosomes fuse with lysosomes and the ingested material is then digested. Thus, it is not only ingesting and removing microorganisms but also malignant cells, inorganic particles and tissue debries (Miller, 1991). In general, the rate of in situ carbon particle clearance is frequently used as a measure of reticuloendothelial system (RES) competency. Specifically, a faster removal of particles is correlated with an enhanced phagocytic activity of RES cellular components (Abbas & Litchman, 2001). In the study here, prophylactic treatment with CT seed and root extracts inhibited the rate of carbon clearance seen among control group rats.

The neutrophil, an end cell unable to divide and with limited capacity for protein synthesis is, nevertheless, capable of a wide range of responses, in particular chemotaxis, phagocytosis, exocytosis and both intracellular and extracellular killing (Dale & Foreman, 1984). Normally, a more rapid clearance of exogenous particulates from the blood by macrophages would arise from opsonization of the material with antibodies/complement C3b. The decrease in neutrophil function (i.e., adhesion activity) strongly suggests that the function in the treated rats' phagocytes was inhibited (i.e., immunoinhibited).

Cell-mediated immunity (CMI) involves effectors mechanisms carried out by T lymphocytes and their products (lymphokines). CMI responses are critical to defense against infectious organisms, infection of foreign grafts, tumor immunity and delayed-type hypersensitivity reactions (Miller, 1991). Delayed type hypersensitivity reaction is characterized by large influxes of non-specific inflammatory cells, in which the macrophage is a major participant. It is a type IV hypersensitivity reaction that develops when antigen activates sensitized TDTH cells. These cells generally appear to be a TH1 subpopulation although sometimes TC cells are also involved. Activation of TDTH cells by antigen presented through appropriate antigen presenting cells results in the secretion of various cytokines including interleukin-2, interferon-γ, macrophage migration inhibition factor and tumor necrosis factor-α (Askenase and Van Loveren, 1983). The overall effects of these cytokines are to recruits macrophages into the area and activate them, promoting increased phagocytic activity vis-a-vis increased concentration of lytic enzymes for more effective killing. Several lines of evidence suggest that DTH reaction is important in host defense against parasites and bacteria that can live and proliferate intracellularly.

In addition to the above-noted outcomes, the DTH response - the magnitude of which can be directly correlated with the competence of a host's cell-mediated immune function that was decreased in rats that received CT seed and root extracts. Apart from the key role of memory (sensitized) T-

lymphocytes in this reaction, the role of local macrophages (initially) and then recruited monocytes/other phagocytes are critical as well. From the data here, no specific conclusions about the functionality of memory T-lymphocytes can be predicted; however, decreases in anti-SRBC titers in CT seed and root extracts treated rats were suggestive of decreased activation of T-lymphocytes. The decreased phagocytic activities of local/recruited phagocytes would also be a major factor for the substantive decrease observed in DTH among extracts-treated rats.

The majority of the cells involved in the immune system are produced from common hematopoietic stem cells found in the bone marrow. This site also provides a microenvironment for antigen-dependent differentiation of B-lymphocytes (Raphael & Kuttan, 2003). Since CT seed and root extract treatments were seen here to give rise to decreased circulating antibody titers (specifically against the SRBC), it would be expected then that there should have also been decreases induced in levels of one or more of the cell types involved in the humoral response to this antigen. In the present study, the evaluations of peripheral blood of extracts-treated rats confirmed the suppression of total WBC counts. These outcomes suggested strongly that the potential effect of CT seed and root extracts was an impact on hematopoietic processes and on the bone marrow in particular.

Intensity of inflammatory immune responses is controlled by recruitment of inflammatory cells into inflammatory lesions. This process is tightly governed by expression of certain inflammatory chemokines, such as monocyte chemotectic protein 1 (MCP-1), Macrophage inflammatory protein 1a (MIP-1a), Macrophage inflammatory protein 1h (MIP-1h), and CC- Chemokine ligand 5 (CCL5) (Baggiolini & Dahinden, 1994; Kallinich et al., 2005) and adhesion molecules, such as lymphocyte function-associated antigen 1 (LFA-1), L-Selectin, and cluster of differentiation 44 (CD44), by the inflammatory cells, and inter-cellular adhesion molecule 1 (ICAM-1), and vascular cell adhesion molecule 1 (VCAM-1) by the endothelial cells (Cartier et al., 2005). Given the central role of chemokines and adhesion molecules in orchestrating the immune response, interference with the expression of these mediators substantially alter the quality of the immune response, leading to either enhancement or inhibition of the ongoing immune response. Thus, one potential mechanism that might mediate the inhibitory effect of CT on inflammatory immune responses is an alteration of trafficking of the inflammatory cells via modulating expression of chemokines and/or adhesion molecules.

Thus, the immunoinhibitory effect of CT can be explained partly by its inhibitory effects on humoral antibody formation, phagocytosis, delayed type hypersensitivity response, and immune cell activities. The anti-inflammatory activity of CT seed and root extracts against carrageenan-induced hind paw edema, pleurisy and cotton pellet granuloma model, suggesting inhibition of inflammatory components of immune response by CT.

V. Conclusion

CT seed and root extracts showed profound immunosupressive activity in male albino rat model. The antioxidant and anti-inflammatory activities of plant may be playing major role in immunoinhibition. The immunomodulatory activity might be attributed to the presence of flavonoid and phenolic compounds. The present study demonstrated and provided evidence for the traditional uses of Clitoria ternatea. Further studies might be required to determine detailed mechanisms and active phytochemicals responsible for immunomodulatory activity.

VI. Acknowledgement

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VII. References

1) Abbas, A.K. and Lichtman, A.H. (Eds.) (2001). Basic Immunology: Functions and Disorders of the Immune System. W. B. Saunders Company, Philadelphia.
2) Anonymous (2003). The Ayurvedic Pharmacopoeia of India. Department of Indian system of medicine & homoeopathy, Ministry of health and family welfare, Government of India, New Delhi.
3) Askenase, P.W. and Van Loveren, H. (1983). Delayed type hypersensitivity: activation of mast cells by antigen specific T cell factor initiates the cascade of cellular interactions. Immunol. Today. 4: 259-264.
4) Atal, C.K., Sharma, M.L., Kaul, A. and Khajuria, A. (1986). Immunomodulating agents of plant Origin, I: Preliminary screening. J. Ethnopharmacol. 18: 133–141.
5) Bafna, A.R. and Mishra, S.H. (2005). Immunomodulatory activity of methanol extract of roots of Cissampelos pareira Linn. Ars. Pharm. 46: 253–262.
6) Baggiolini, M. and Dahinden, C.A. (1994). CC chemokines in allergic inflammation. Immunol Today. 15(3): 127–133.
7) Biozzi, G., Benacerraf, B. and Halpern, B.N. (1953). Quantitative study of the granulopoetic activity of reticuloendothelial system (RES). II: A study of kinetics of the RES relationship between weight of organs and their activity. Br. J. Exp. Biol. 34: 441–456.
8) Cartier, L., Hartley, O., Dubois-Dauphin, M. and Krause, K.H. (2005). Chemokine receptors in the central nervous system: role in brain inflammation and neurodegenerative diseases. Brain Res. Brain Res. Rev. 48: 16–42.

9) Chopra RN, Nayar SL, Chopra IC (1956). Glossary of Indian Medicinal Plants, National institute of science and communication, New Delhi.
10) CPCSEA (2003). CPCSEA guidelines for laboratory animal facility. Indian J. Pharmacol. 35: 257–272.
11) Daisy P, Priya N, Rajathi M (2004). Immunomodulatory activity of Eugenia Jambolana, Clitoria ternatea and Phyllanthus emblica on alloxan induced diabetic rats. J. Exp. Zool. India. 7: 269–278.
12) Dale, M.M. and Forman, J.C. (Eds.) (1989). Textbook of Immunopharmacology, 2nd Edition, Blackwell Scientific Publication, Oxford, UK.
13) Damre, A.S., Gokhale, A.B., Phadke, A.S., Kulkarni, K.R. and Saraf, M.N. (2003). Studies on the immunomodulator activity of flavonoidal fraction of Tephrosia purpurea. Fitoterapia. 74: 257–261.
14) Doherty, N.S. (1981). Selective effects of immunosuppressive agents against the delayed hypersensitivity response and humoral response to sheep red blood cells in mice. Agents Actions. 11: 237–242.
15) Fulzele, S.V., Bhurchandi, P.M., Kanoje, V.M., Joshi, S.B. and Dorle, A.K. (2002). Immunostimulant activity of Asthmangal ghrita in rats. Indian. J. Pharmacol. 34: 194–197.
16) Goldsby, R.A., Kindt, T.J., Osborne, B.A. and Kuby, J. (Eds.) (2003). Immunology. W. H. Freeman and company, New York.
17) Gupta RK, Lal LB (1968). Chemical components of the seeds of Clitoria ternatea Linn. Indian J. Pharm. 30(7):167–168.
18) Kallinich, T., Schmidt S, Hamelmann E, Fischer A, Qin S, Luttmann, W., Virchow, J.C. and Kroczek, R.A. (2005). Chemokine-receptor expression on T cells in lung compartments of challenged asthmatic patients Clin. Exp. Allergy. 35: 26–33.
19) Kirtikar KR, Basu, BD (1976). Indian Medicinal Plants, Periodical expert book agency, Delhi.
20) Kokate, C.K., Purohit, A.P. and Gokhale, S.B. (2003). Pharmacognosy. Nirali Prakashan, Pune.
21) Kuby, J. (1994). Immunology, 2nd ed., W.H. Freeman and Co. New York.
22) Lagrange, P.H., Mackaness, G.B. and Miller, T.E. (1974). Potential of T-cell-mediated immunity by selective suppression of antibody formation with cyclophosphamide. J. Exp. Med. 139: 1529–1539.
23) Malabadi RB, Mulgund GS, Nataraja K (2005). Screening of antibacterial activity in the extracts of Clitoria ternatea. J. Med. Arom. Plant Sci. 27: 26–29.
24) Miller, L.E., 1991. In: Ludke, H.R., Peacock, J.E., Tomar, R.H. (Eds.), Manual of Laboratory Immunology. Lea and Febiger, London.
25) Mukherjee, P.K., Kumar, V., Kumar, N.S. and Heinrich, M. (2008). The Ayurvedic medicine Clitoria ternatea—From traditional use to scientific Assessment. J. Ethnopharmacol. 120: 291–301.
26) Raphael, T.J. and Kuttan, G. (2003). Effect of naturally-occurring triterpenoids glycyrrhizic acid, ursolic acid, oleanolic acid, and nomilin on the immune system. Phytomedicine. 10: 483-489.
27) Shinde, U.A., Phadke, A.S., Nair, A.M., Mungantiwar, A.A., Dikshit, V.J. and Saraf, M.N. (1999). Preliminary studies on the immunomodulatory activity of Cedrus deodara wood oil. Fitoterapia. 70: 333–339.
28) Srivastava BK, Pande CS (1977). Anthocyanins from the flowers of Clitoria ternatea. Planta. Med. 32:138–140.
29) Upadhye, A.S. and Kumbhojkar, M.S. (1993). Studies on the Ayurvedic drug Shankhapushpi from Western Maharashtra: Medico-Botanical Aspect. Bull. Medico-Ethano Bot. Res. 14: 64 –69.
30) Wilkonson, P.C. (1978). Neutrophil adhesion test. In: Handbook of Experimental Pharmacology, Vol. 1, (Vane, J. R., and Ferraria, S. H., Eds.), Springer-Verlag, Berlin.
31) Yadava, R.N. and Verma, V. (2003). Antimicrobial activity of a novel flavonol glycoside isolated from the roots of Clitoria ternatea Linn. Asian J. Chem. 15: 842–846.

Properties Of Nanostructured Sb_2S_3-Tls Thin Film Deposited By Chemical Bath Technique

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GJSFR Classification - C (FOR) 40103,20106,0303

***Abstract*-Multilayer thin film composed of Sb2S3-TlS was deposited on glass substrate by a simple and inexpensive chemical bath deposition technique within the pores of polyvinyl alcohol. The deposited film was subjected to post-deposited annealing at 100oC, in the oven for 1 hour. The structure was investigated by X-ray diffraction and the crystallite size was found to be 14 nm. The value of the optical band gap energy Eg, calculated from the absorption spectra is 1.0 eV.**

Keywords-Multilayer thin film, band-gap energy, optical properties, chemical bath deposition, solar light energy.

I. INTRODUCTION

Practical application of semiconductor materials in devices requires the fabrication of semiconductor with a differential doping structure. This is essentially important for efficient use of semiconductor materials in the fabrication of laser diodes, LEDs, solar cells etc. Direct band-gap materials are presently essential for laser diodes application [1]. Similarly, "tandem" cells have been shown to exhibit total conversion efficiency, which is higher than single-junction PV cells. The requirement to get appropriate band-gap energies for device application has led to the development of binary, ternary and quaternary thin films [2-6].

The chemical bath deposition technique is one of the simplest and low cost methods of thin film deposition. The method can be carried out at low temperatures and can allow large area deposition of semiconductor. Deposition of the films occurs when the substrate is maintained in contact with the chemical bath and the ionic product exceeds the solubility product. The chemical bath deposition method of thin film has the advantage of allowing one to easily control the growth factors including the film thickness, rate of deposition and crystalline quality. This is usually achieved by varying the bath concentration, temperature and the pH of the solutionIn this present report, chemical bath deposition technique was used to deposited nanostructured thin films of Sb2S3-TlS within the self-organized pores of polyvinyl alcohol. This is followed by the analysis of the band-gap energy and the optical transmission for possible use in solar cells and other applications

II. Experimental Details

To deposit the thin film of Sb2S3 on microscope glass slide, 5ml of acetone was used to dissolve 1.3g of SbCl3 in 50ml beaker. This was followed by sequential addition and stirring of 25ml of 1M Na2S2O3 and 34ml of PVA solution. The PVA solution was prepared by dissolving 0.9g of solid PVA in 450ml of distilled water at 90oC. The homogenous solution was aged until the temperature drops to 25oC. The deposition proceeded at room temperature and was completed by 150 mins. The coated substrate was removed from the bath and washed with distilled water and dried. Thin film of TlS was deposited on glass-Sb2S3 system by using 5ml of 0.2M TlNO3, 4ml of 1M C3H4(OH)(COONa)32H2O, 4ml of 1M (NH2)2CS and 34ml of PVA solution put in that order in 50ml beaker. The deposition time was 90mins. The film was again rinsed thoroughly with distilled water andallowed to dry. The deposited Sb2S3-TlS thin film was annealed in an oven at 1000C for 60mins.The sample was characterized with SEM, XRD and UV-VIS Spectrophotometer. Optical properties of chemical bath deposited Sb2S3-TlS thin film was measured at room temperature by using a double beam Perkin-Elmer UV-VIS Lambda 35 spectrometer. Optical band-gap was calculated from the absorption spectra. X-ray diffraction (XRD) is an efficient tool for the structural analysis of crystalline materials. The XRD patterns for the sample was recorded using D/max-2000 Rigaku X-ray diffractometer in the 2θ range of 200 - 800 using CuKα radiation of wavelength λ = 1.5408Å. The grain size of the deposited films was viewed by using scanning electron microscopy (SEM) technique.

III. RESULTS AND DISCUSSION

Fig. 1 shows the XRD pattern of nanostructured Sb2S3-TlS thin films deposited in this work. Peak broadening has been observed in recorded diffraction patterns, which shows the formation of crystalline thin films. Some of the recorded parameters from XRD analyses are displayed in table I.

Table I: Obtained result from XRD for Sb_2S_3-TlS thin film

2θ	d-value	I/Io
26.18	3.4011	75
26.28	3.3884	100
30.30	2.9474	08
43.36	2.0851	43
43.42	2.0824	46

The diffraction peaks at 2θ values of 26.28 and 30.30 corresponds to peaks of TlS (PDF No 43-1067). Similarly, the XRD pattern at 2θ value of 26.18 is identified to be Sb2S3 (JCPDS-6-0474)

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The average crystallite size was calculated from the recorded XRD patterns using Scherrer formula:

$D = 0.89 \lambda/\beta \cos \theta$

Where D is the average crystallite size, λ is the wavelength of the incident X-ray, β is the full width at half maximum of X-ray diffraction and θ is the Bragg's angle. The average crystallite size for the thin film of Sb2S3-TlS was found to be 14nm.

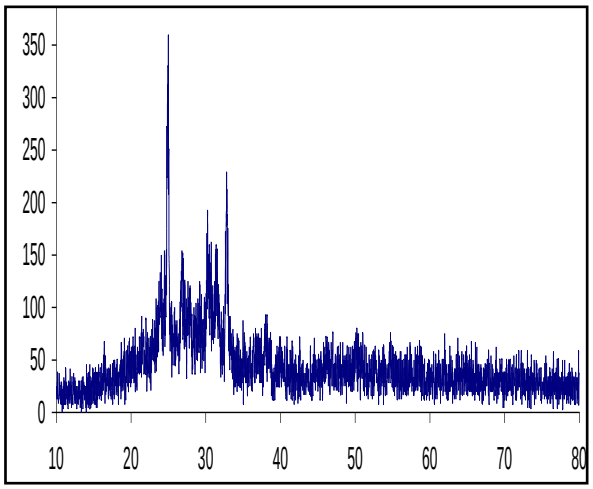

Fig.1: X-ray diffractogram of Sb_2S_3-TlS thin film

The photographical structure of the film grown in this work was studied by scanning electron microscope. The SEM of Sb_2S_3-TlS thin film is shown in fig.2. The crystalline structure is clear in the micrograph. The figure also shows that the deposited film contains pin poles and few cracks.

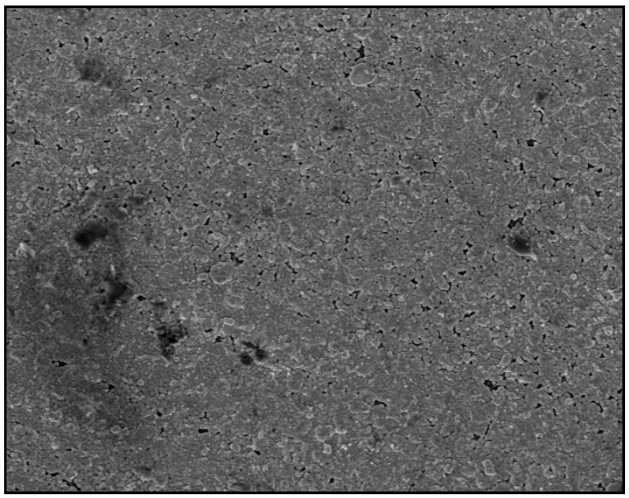

Fig. 2: SEM of Sb_2S_3-TlS

The optical absorption spectra of the films deposited onto glass substrate were studied in the range of wavelengths 400 – 1100nm. The variation of absorbance (A) and transmittance (%T) with wavelength for the sample under study are shown in fig 3 and 4 respectively

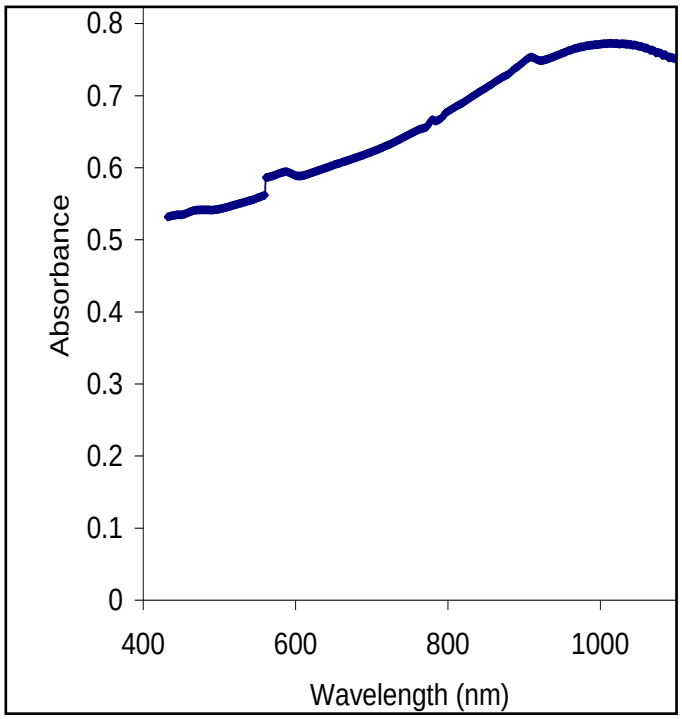

Fig.3: Plot of absorbance against wavelength for Sb2S3-TlS thin film

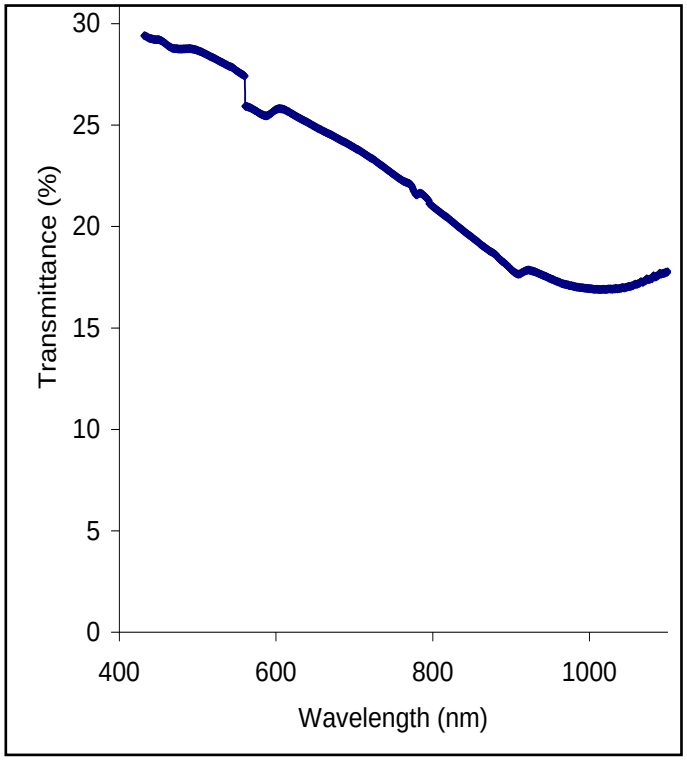

Fig.4: Plot of transmittance against wavelength for Sb_2S_3-TlS thin film

Figure 3 shows that thin film of Sb_2S_3-TlS has good absorption in both the visible spectrum and NIR region of solar radiation. The absorbance increases linearly with the wavelength. The transmittance of the film is below 40% in the visible region of solar spectrum. Human eye

is sensitive only in the range 400 – 700 nm and is peaked at 500nm (photopic vision). This is important factor in window coatings but is not met in this film. The film however is opaque in the visible region, making it unsuitable for this purpose.

The dependence of the absorption coefficient (α), on the photon energy is important in studying energy band structure and the type of transition of the electrons. The absorption coefficient was estimated by the transmittance data as shown in fig. 5

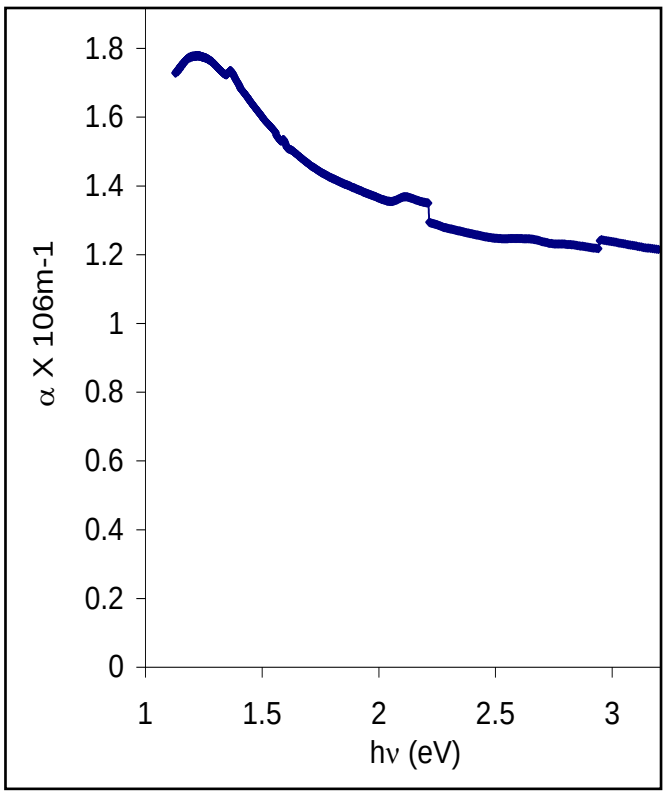

Fig.5: Plot of absorption coefficient against photon energy for Sb_2S_3-TlS thin film

The absorption spectra, which are the most direct and perhaps the simplest method for probing the band structure of semiconductors, are employed in the determination of the energy gap, E_g. The E_g was calculated using the following Tauc's relation

$$\alpha = A(h\nu - E_g)^n / h\nu,$$

Where A is a constant, hν is the photon energy and α is the absorption coefficient, while n depends on the nature of the transition. For direct transitions n = ½ or ⅔, while for indirect ones n = 2 or 3, depending on whether they are allowed or forbidden, respectively. The usual difficulty in applying this concept to polycrystalline thin films with nanometer-scale crystalline grains is the size distribution of grains and consequent variation in the band gap due to quantum confinement effects. Thus the straight-line portion may not extend beyond a few tenths of an electronvolt, and hence value of the band gap could turn out to be very subjective [8].

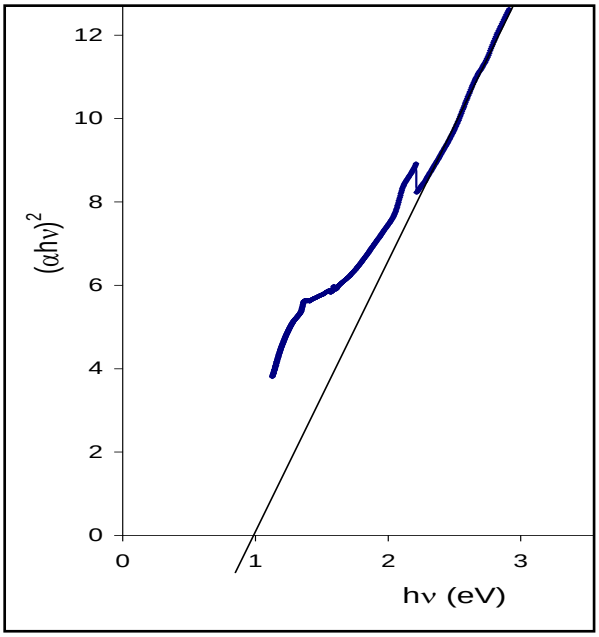

Fig.6: Direct band gap plot for Sb_2S_3-TlS thin film

The best fit of the experimental curve to a band gap semiconductor absorption function was obtained for n = ½. Hence, the calculated values of the direct energy band gap, from fig.6 is 1.0eV

A material with a direct band gap of 1 - 1.5eV with a high solar optical absorption 10^4 - $10^5 cm^{-1}$ in the wavelength range of 350 – 1000 nm has been regarded as a promising absorber for thin film photovoltaic applications [9, 10]. The low band gap value exhibited by this film together with high absorbance in the VIS - NIR makes the film suitable for use as absorber material in solar cell application. For laser diode application, the band gap energies should essentially lie in the range of 0.9 to 1.5eV. While band-to-band radiative recombination is favored in direct band gap materials, the band gap energy controls the emission wavelength: $\lambda \approx 1.2 / E_g$. [1]. Hence these films could also be used for fabrication of laser diodes.

IV. Conclusion

Chemical bath deposition technique has been successfully used to deposit multilayer thin film of the form Sb_2S_3-TlS. The optical band gap of 1.0 eV is in the desired interval for the film to be used as solar absorber materials for solar cell fabrication.

V. Reference

1) H. Kressel; Topics in Applied Physics, Springer-Verlag 39 (1987).
2) M.T.N. Nair, Y. Para, J. Compos, V.M. Garcia, P.K. Nair, Chemically deposited Sb2S3 and Sb2S3-

CuS thin films, J. Electrochem. Soc.145, 6 (1998) 2113-2120.

3) Y. Rodriguez-Lazcano, M.T.S. Nair, P.K. Nair, CuSbS2 thin film formed through annealing chemically deposited Sb2S3-CuS thin films, J of Crystal Growth 223 (2001) 399-406.
4) P.K. Vidyadharan Pillai, K.P. Vijayakumar, Characterization of CuInSe2/CdS thin-film solar cells prepared using CBD, Solar Energy Materials and Solar Cells 51 (19998) 47-54
5) Asogwa, P.U. (2009), Growth and Characterization of CuS-Sb2S3 Heterojunction Thin Films. The Pacific J. of Sc. and Tech. 10(2).
6) Ezugwu, S.C., Ezema, F.I., Osuji, R.U., Asogwa, P.U., Ekwealor, A.B.C., Ezekoye, B.A. (2009). Effect of Deposition Time on the Band Gap and Optical Properties of Chemical Bath Deposited CdNiS Thin Films. Optoelectronic and Advanced Materials-Rapid Communications, 3(2).
7) Ezema, F.I., Asogwa, P.U., Ekwealor, A.B.C., Ugwuoke, P.E., Osuji, R.U. (2007). Growth and Optical Properties of Ag2S Thin Films Deposited by Chemical Bath Deposition Technique. J. of the University of Chemical Technology and Metallurgy, 42(2).
8) V. Estrella, M.T.S. Nair, P.K. Nair, Crystalline structure of chemically deposited thallium sulfide thin films, Thin Solid Films 414 (2002) 289-295
9) K. Bindu, J. Campos, MTN. Nair, A. Sanchez, P.K. Nair, Semiconducting AgSbSe2 thin film and its application in a photovoltaic structure, Semicond. Sci. Technol. 20 (2005) 496-504
10) J. Poortmans and V. Arkhipov, Thin film Solar Cells Fabrication, Characterization and Applications. John Wiley and Sons, Ltd, The Atrium, Southern Gate, Chichester, West Sussex, England. XIX (2006)

Preliminary Studies On Biogas Scrubbing System For Family Sized Biogas Digester

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GJSFR Classification - G (FOR) 0304.070703.0701

Abstract-**This paper discusses the results of preliminary studies conducted on a locally designed and fabricated biogas digester with improvised biogas scrubbing system for the purification of the generated biogas at the national Centre for energy Research and Development, University of Nigeria, Nsukka.. The scrubbing system consists of three units, namely: the H2S scrubbing unit packed with iron fillings, the CO2 scrubbing unit packed with sodium hydroxide and a configured cylinder for trapping any condensate. The two scrubbing units made of steel are perforated at two opposite ends and connected with pipes. Generated raw biogas from plant was directly fed in turn through the iron fillings and NaOH units. Samples of the gas mixture were taken before and after scrubbing and analyzed with UNIGAS 3000 Btu automatic gas analyze. Results indicate methane content of the scrubbed biogas was raised from 62.0% to 74.01% due to removal of the contaminants, especially CO2 and H2S. CO2 was reduced by over 52% whereas H2S was reduced from 1.20% to 0.4%. Methane loss of 0.2% was however recorded. The increased energy content achieved with the scrubbed gas evidenced in the reduction of over 20% time required to raise 500ml of water to boiling is significant.**

Keywords-Biogas, scrubbing, condensate, contaminants, methane

I. INTRODUCTION

Although biogas is a mixture of gases consisting of methane CH4, carbon (iv) oxide CO2, hydrogen sulfide H2S, ammonia NH3, Chlorinated hydrocarbon, Nitrogen N2 oxygen O2 and water vapor depending on the feedstock, the energy value is mainly dependent upon the level of methane content. Increase in the level of CH4 in biogas would amount to increased energy content. According to IEA task 37, 1Nm3 of biogas upgraded to 97% CH4 is equivalent to 9.7 KWh which is comparable to 1 liter of petrol which has 9.1 KWh. The percentage of methane in biogas can be substantially increased by optimizing the production processes or by removing substantial amounts of the contaminants. A lot of processes have been developed for increasing the methane content of raw biogas. These processes involve the removal of significant amount of CO2 and H2S. Most of these processes have been developed for use in the natural gas and petroleum industries as a result of which some of these methods may not be suitable for biogas process unless high flow rates are involved. Commonly CO2 removal process also removes H2S (Kapid et al 2004). Suitable processes for upgrading biogas to high CH4 content are: absorption into liquids, which may be physical or chemical, adsorption on solid surface, membrane separation, cryogenic separation and chemical conversion (Viyaj et al 2006). Most of these methods are made to operate in large scale in advanced countries. This requires a centralized biogas productionand, or a central collection of biogas produced at different location through network of pipes. Developing countries are bereft of expertise and network of pipelines. Besides, the huge fund required for large scale plants are not easily come by. With the avalanche of biomass resources which are relatively evenly distributed, scrubbing biogas at family sized plant level appears to be a suitable alternative. Scrubbing of biogas is an important environmental and health issue especially in developing countries where it is reported by World Health Organization that emissions from burning of unprocessed biomass indoors cause the death of 1.6 million people every year (WBGU, 2003). Biogas may contain up to 4000ppm of H2S [IEA Task 37]. Consequently, some developed countries have set allowable limit for biogas transported in gas pipelines for used for heating and cooking in homes. For instance, in Canada, biogas with H2S lower than 4.6ppm is allowed in gas pipelines for heating and cooking in homes [Navaratnasamy 2008]. As energy systems shift towards sustainability, with many advocating for biogas as cooking fuel and other thermal application, it has therefore become imperative to develop a method of purifying biogas which can be incorporated with a family sized digester plants to increase the heat value on one hand and minimize GHGs emission on the other hand. This work therefore is an attempt to develop a scrubbing system that will be capable of removing significant amount of biogas contaminant to increase the heating value and reduce the chances of indoor air pollution that could arise from burning raw biogas used as cooking fuel in homes.

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II. MATERIALS AND METHODS

A 0.2m2 fixed dome steel biodigester was designed and constructed at the National Center for Energy Research and Development (NCERD), University of Nigeria, Nsukka. The system was charged with cow dung at the ratio of 3:1 (water to cow dung) and left in open area with ambient daily average temperature of 31.5oC and monitored for a period of 14 days. After proper mixing, the digester was closed (air-tight) to ensure an anaerobic environment inside the digester and manually stirred occasionally every day, The volume of biogas generated within the period was measured by downward displacement method as described by (Varel et al, 1977). The ambient and slurry temperatures were measured using mercury in-glass thermometer and digital thermocouple respectively. Pressure was measured with a

U-tube manometer while pH of the fermenting slurry was recorded with a Jean Way 3020 model pH Meter. Volatile solids and Total Solids of the waste were determined daily by the methods described by Meynell (`1982). Flash tests were carried out to ascertain onset of flammability and the appropriate time to incorporate the scrubbing system.

III. Preparation of the Cylinder

A normal empty compressed natural Gas CNG cylinder was air–evacuated with the use of high vacuum pump. A manifold comprising of two tubes was connected to the cylinder with the first tube. The second tube was connected to the vacuum pump which was connected to the power supply. The air in the cylinder was evacuated to nearly 30 psi.

IV. Scrubbing System

The scrubbing system was set up as shown in Figure 1. It consists of three units, the hydrogen sulfide, (H_2S) scrubbing unit, Carbon (iv) oxide,(CO_2) scrubbing and condensate trapping units. The three scrubbing units were interconnected with a plastic hose. The H_2S unit consists of a packed bed of iron fillings in a steel cylinder of about 4cm diameter and 9cm long and closed at both ends. A hole of about 10mm diameter was opened at two opposite ends for incorporating the pipes. A similar steel cylinder was filled with Sodium Hydroxide for the scrubbing of carbon (iv) oxide. Thecondensate trapping unit is a modified steel cylinder with two openings and configured to about 30psi. It is meant to trap water vapor or any other condensation product(s) after the CO_2 scrubbing unit. As shown in Figure 1, the gas mixture driven by the pressure in the digester head was first passed through the iron fillings to remove the H2S. Biogas emerging from this unit continues along the pipe to the next scrubbing unit where CO2 dissolves in NaOH. Most of the water vapor dissolves here; thereby enhancing the CO2 scrubbing. With the help of a tap, the configured cylinder is opened to allow the scrubbed gas in. The vacuum in the cylinder aids the initial flow of the scrubbed biogas. Any condensate is trapped in the cylinder, thereby enhancing the purity of the biogas. The outlet tap is opened simultaneously with the tap of the digester plant. The configured cylinder is connected to the burner. Simultaneous opening of all the taps supplies scrubbed biogas to the burner.

Figure 1: Biodigester incorporated with a scrubbing system

V. Results and Discussion

The results of some of the indices of biogas digester performance efficiency monitored during the fermentation period are presented in Table 1. From the results obtained, biogas production started on the 3rd day of fermentation. There was a steady increase in gas generation till the 8th day which was the peak of production; thereafter, there was a decline in production. A corresponding pH value of 6.9 recorded on the 8th day favoured maximum biogas yield. This result is backed up by the earlier result of Hills (1979), who observed that biogas production can be impaired at pH values below 6.2 and above 7.6. Blanchard and Gills (1987) reported that a pH of less than 6.0 or greater than 8.0 rapidly inhibits methanogenesis under most operating conditions, and this apparently explains the reasons for the decline in biogas when the pH dropped sharply to 6.5 on the 9th day.

The magnitude of pressure in the biodigester increased with increase in biogas production though not linearly related. The explanation is based on the fact that biogas in the digester is always in constant motion, colliding with each other and the wall of the digester; thereby increasing the pressure within the system. Even though the highest

temperature of 32.60C was recorded on the 9th day of fermentation, that of pressure and maximum biogas generation was observed on the 8th day. The inference to be drawn from this result is that these 2 parameters-temperature and pressure interact closely in anaerobic environment.

Maximum biogas production was recorded on the 8th day with a corresponding pH of 6.9 and a temperature of 310C. This result is within the range of a recent report that methane and acid-forming bacteria can exist only in pH 6.8 – 7.0 and degrade the substrates, hence the continuous decrease in Total solids and volatile solids with increase in Retention Time.(Honemeier, 2008) . All the bacteria types have tendency to suspend their activities in case the pH level exceeds the optimum, hence, biogas production is suspended as well The initial rise in pH values of the system is attributed to the fact that at initial period of fermentation, large amount of organic acids are produced by acid-forming bacteria,, thus , the pH inside the digester can decrease to below 5 pH. This inhibits or even stops the fermentation process and accounts for the low gas yield. When methanogenic bacteria set in, the pH shifts towards neutral and more biogas is produced. Later, as the fermentation process continued, concentration of NH4 increased due to digestion of nitrogen which can increase the pH values, to above 8. Earlier research finding has shown that pH range remains buffered between 7 and 8.8 (Shoemaker and Visser, 2000). The pH values in the present report did not exceed pH 6.7.

Table 1: Indices of Biogas Generation monitored

Parameters	Time (days)										
	1	2	3	4	5	6	7	8	9	10	11
Gas yield (L)0^{-2}	0	0	28.8	64	95.6	130.1	166.4	182	170.8	162.7	140.2
pH	6.4	6.5	6.7	6.8	6.8	6.8	6.9	6.9	6.8	6.7	6.5
Pressure(mm)	0	7	9	11	12	13	14	16	15	14	13
Average Temp(^{0}C)	28	28.2	30	30	29.5	30.6	31.4	31	32.6	30	30.2
Average Total Solid (mg/L)x10^2	160.5	156.4	158.9	140.6	122.3	119	103.8	97.2	93.7	90	82.4
Average Volatile Solid (mg/L)x10^3	140.2	135.6	130	102.8	98.8	67.4	40.2	25.8	16.4	10.2	6.5

The results of the performance test on the scrubbing system to evaluate the effect on biogas constituents and the time required by different energy sources to heat 500ml of water to boiling are presented in Tables 1 and 2 respectively.

Table 1: Biogas composition before and after scrubbing

Gas Constituents	Per cent before scrubbing	Per cent after scrubbing	Per cent removed by scrubbing	Per cent reduction of constituent
H_2S	1.20	0.40	0.81	66.9
CO_2	26.40	12.5	13.9	52.65
CH_4	62.00	74.32	0.47	0.62
N_2	10.20	8.00	2.2	0.21

VI. Effect of scrubbing on heating value

In order to elucidate the impact of scrubbing on the heating value and cooking time of the scrubbed biogas, scrubbed and raw biogas were used to heat 500ml of water as shown in table 2.

Table 2: Time for boiling 500ml of water

Energy source	Time (minutes) for heating to boiling 500ml of water	*Time(minute) loss relative to scrubbed biogas
Fuel wood	8.11	2.98
Kerosene	8.50	3.33
Raw biogas	6.44	1.31
Scrubbed biogas	5.13	1.20

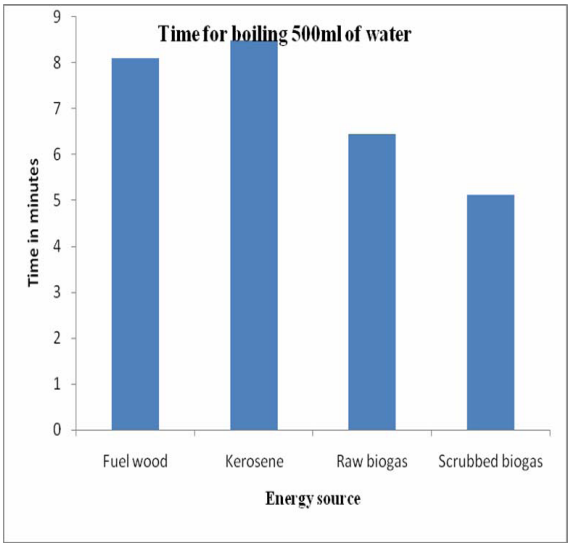

Figure 2: Variation in time for boiling500ml of water with energy source

The study showed that scrubbing was better with low pressure (low rate of gas flow). Biogas flow rate and pressure is dependent on the amount of gas allowed to accumulate in the digester headspace. Therefore, low gas flow rate can be achieved by scrubbing more frequently and storing the gas for later use. Allowing the gas to build up to high pressures will translate to high flow rates which implies less contact time between the raw biogas and the adsorbent/absorbent in the scrubbing units. This in turn would lead to low efficiency of the scrubbing unit. Furthermore, increased pressure in the headspace of the digester and therefore high flow rate will lead to increased leakages. Leakages of unscrubbed biogas involve releasing the gas mixture including methane into the atmosphere. Methane is a greenhouse gas with twenty times more impact than carbon dioxide [IEA 1997]. In addition, loss of methane is also loss of energy content.

The reduced heating time required by the scrubbed biogas means that food cooked with scrubbed biogas will retain more nutrients than those cooked with unscrubbed biogas. even though nutritional analysis of foods cooked with scrubbed biogas was not carried out in this study to corroborate this view. According to Khattak and Klopfenstein (1989) prolonged heating is known to destroy some heat-labile vitamins such as B vitamins in foods

VII. Conclusion

This work has shown that improvised scrubbing system can be successfully integrated with a family sized digester plant for cooking meals. Although the economics has not been fully explored, the system is simple to operate and the material requirements are readily available. Iron.

fillings are regenerated, and. NaOH is required to be replaced after saturation. However, with a family sized digester plant, the amount required for scrubbing is relatively small and disposal is not likely to cause any serious environmental impact. Unscrubbed biogas has a better cooking efficiency than fuel wood and even kerosene and it is equally most environmentally friendly than either of the two energy sources. Finally, the drudgery associated with cooking with fuel wood occasioned by the carcinogenic smokes, is completely eliminated while cooking with biogas

VIII. References

1) Blancard, J.P and Hills T.A. (1987). Environmental Factors Affecting Biogas Production,132 -167.
2) Biootechnology and energy Consumption. Tata mccgraw-Hill Publishers, NewYork. pp. 32-48.
3) Hills, D.T (1985). Design of ndigestion system for maximum methane production. Trans ASAE 25(1): 226-230.
4) Honemeier, B.(2008). The impact of genotype and harvest time on drh matter, biogas and methane yields on maize (Zea mayse). A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Agricultural Science to the Faculty of Agricultural science, national and Environmental management, Justus-Liebig-University, Giessen.
5) IEA (1997) Bioenergy Anaerobic Digestion Activity, Report, Systems and Markets Overview of Anaerobic Digestion
6) Kapid S.S., Viyaj V.K., Rajesh S.K., Prasad R., (2004) Biogas Scrubbing, Compression and Storage: perspective and prospectus in Indian context. http:// www.sciencedirect

.com/science?_ob=Article URL&_Udi. accessed on 28 04 2009.

7) Khattak A. B and Klopfenstein C. F (1989) Effects of Gamma Irradiation on the Nutritional Quality of Grain and Legumes. I. Stability of Niacin, Thamin, and Riboflavin Cereal Chem 66(3): 169-170
8) Meynell, P.J (1982). Methane: Planning a Digester. Prism Press, Stable Court, Chalmington, Dorset.
9) Navaratnasamy M. (2008) Biogas: Cleaning and Uses. Agrifacts Agdex 768-5 June., 2008.Avaliable at http://wwwl.agric.gov.ab/$department/deptdoc.nsf/all/agdex768-5. accessed on 12 07 2007
10) Varel, G.H and Byrant M. (1977) Thermophilic Methane Production Appl. Environ Microbiology. 38:298-307
11) Viyaj KV, Chandra R, Subbarao P M V and Kapid S S (2006) Biogas purification and Bottling into CNG Cylinders: Producing Bio-CNG from Biomass for Rural Automotive Applications. A paper presentation at The 2ndJoint International Conference on Sustainable Energy and Environmetnt (SEE) on 21-23 November 2006 Bangkok, Thailand.
12) WB GU, (2003) An economic assessment of anaerobic digestion system using separate programming. Unpublished M.sc Thesis, Manitoba, Wimnnipeg, Manitoba.

Temperature dependence of $< S_q^z >$ and $< S_q^x >$, collective proton frequency width, collective phonon mode frequency, in paraelectric phase for KH_2PO_4

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***Abstract*-The model Hamiltonian proposed by the [Blinc and Žekš *advances in Physics*, 29, (1972) 159] has been modified by considering the lattice anharmonicity upto fourth order for the stochastic motion of groups. The correlations appearing in the dynamic equation have been evaluated using double time thermal retarded Green's functions and Dyson's equation. The proton Green's function and phonon Green's function have been evaluated for the collective motion of the system, using model Hamiltonian. The higher order correlations in the proton Green's function have been evaluated using the symmetrical decoupling scheme, after applying the Dyson's treatment. The expressions for the proton renormalized frequency of the coupled system and collective proton wave half widths have been calculated. The higher order correlations in the phonon response function have been calculated without any decoupling and using renormalized Hamiltonian. The expressions for the renormalized phonon frequency and acoustic phonon widths and shifts have been calculated. Using expectation value of the proton collective mode components at site q [Blinc and Žekš *advances in Physics*, 29, (1972) 159] the temperature dependence of $< S_q^z >$ and $< S_q^x >$ for different values of, have been calculated. This shows the order of phase transition. In paraelectric phase the value of decreases when temperature increases from transition temperature (TC). Our theoretical results fairly agree with experimentally reported 55-77 result within experimental errors.**

Keywords-Green's function, collective proton frequency width, and proton collective mode components.

I. INTRODUCTION

The anharmonic interactions in solids are profoundly responsible for their physical properties. The study of various thermal and dynamic properties of solid is essentially a many body problem. Usual perturbation theories of the Born-type are not appropriate for solving **these problems**[1-3]. The methods of quantum field theory have been widely employed in solving many body problems in solid state physics. One of them, the thermodynamics Green's function method4 has become an invaluable tool in the study of complicated system of the interacting particles in statistical physics. This method is non-perturbative and provides a systematic approach for calculating thermally averaged correlation functions and hence physically observable quantities. Most of the physical properties of a system can be expressed in terms of these Green's functions which lead to thermally averaged observables. This provides an effective means of calculating the observable macroscopic as well as the microscope properties of a solid.

In order-disorder dielectrics, as KH_2PO_4, the transitions is associated with the tunneling of proton through a barrier between two positions of minimum potential energy in the double well potential in the hydrogen bond at the transition temperature5. In displacive phase transition is caused due to the displacement of a whole sub lattice of ions of one type relative to other sublattice, e.g., in Ba TiO_3 and most of the double oxide ferroelectrics. The atomic displacements at the transition point are small compared to the unit cell dimensions. The dynamical behaviour of two types of distortive structural phase transition (SPT) is also quite different. The order-disorder systems behave like the magnetic ones. In the order-disorder (KDP-type) systems the proton can tunnel through the barrier which separates the two minima of the potential energy in the hydrogen bond and the ground state of the system splits into two levels separated by energy 2Ω. The magnitude of 2Ω depends on the overlap of the wave functions appropriate to a proton localized in each of the two minima. However, the protons interact with one another and with other atoms. The result is to give a spectrum consisting of one branch, if there is one proton per unit cell, and the frequency $\Omega(q)$ is temperature dependent 6-8. The system of two energy levels is most conveniently discussed in terms of spin-1/2 system. An account of this approach has been given by Tokunaga and Matsubara9 and Tokunaga10 with reference to earlier work by Slater, Blinc, de Gennes and others. The spin can be through of as precessing around the direction along which the 'field' 2Ω is directed. Above T_C the excitation spectrum shows relaxation character and is centered around $\omega = 0$. Only below T_C, a mode of finite frequency $\omega \neq 0$ is found (as per spin waves in ferromagnets).

Potassium dihydrogen phosphate (KDP), KH_2PO_4, is the prototype of order-disorder type ferroelectrics. The most important contribution to an understanding of the atomistic mechanism occurring at the ferroelectrics transition of KH_2PO_4 comes from structural

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investigations by means of X-ray and neutron diffraction [11-13]. The X-ray data give strong indications as to the location of hydrogen atoms in the lattice from the interatomic distances between oxygen atoms and the neutron diffraction techniques accurately locate the structure[14-18]. The overall picture of transition occuring in KH_2PO_4 at 123^0 K, as obtained both from X-ray and neutron studies, reveals the role played by the hydrogen atoms in the co-operative phenomenon which leads to ferroelectricity.

Simpler mean field theories have been used to gain qualitative physical insights in KDP-systems. In order to go beyond the mean field approximation and to include later –Takagi short range order effects into psedospin model, Tokunaga and Matsubara[9], Matsubara and Yoshimitsu[19] and Blinc and Svetina[20] developed a four proton clusters model which takes into account the correlations governing motions of four protons surrounding by a PO_4 group.Vaks et al[21] used the model of Blinc and Svetina[20] but could not explain most of the features of KDP-system expect the difference between the Curie and Curie-Weiss temperature. Vaks and Zinenko[22] performed extensive calculations for the static thermodynamics behaviour in the four-particle cluster approximation and found satisfactory agreement with the experimental data. Similar four particle cluster approximations were made by Yoshimitsu and Matsubara [23] and Havlin and Sompolinsky[24]. Their results, however, are in good agreement with experimental results, but they could not explain the observed relaxational behavior of dielectric properties and ultrasonic attenuation explicitly, in KDP-type ferroelectrics.

In analogy with the spin wave in magnetic system, the pseudo-spin formalism suggests the presence of some proton collective motion. Tokunaga[10], Nankovic[25] and Brout *et al*[26] independently derived the proton collective modes within the random phase approximation and showed that the softening of this collective mode takes place and its frequency with the longest wavelength vanishes at a critical temperature T_C. Expressions for longitudinal and transverse susceptibilities were obtained by introducing a phenomenological damping constant for the longitudinal relaxation time between the splitted levels.

Kobayashi[5] extended the pseudospin model by including the interaction of proton mode with the lowest frequency transverse optic mode of the same symmetry K-PO_4 system. Other workers [10, 27-29] also suggested the inclusion of spin-lattice interaction term. This extended model explains very well the Raman spectroscopic data. Samara [30] used this model to explain the results of pressure effect on dielectric properties of KDP-crystals. However, the large shifts of transition temperature corresponding to the small shifts of Curie-Weiss constant on deuteration of KDP could not be explained simultaneously by Kobayashi's extended model. Houston and Bolten[31] used Kobayashi's extended model in their calculations and showed that the Curie-Weiss constant should contain the tunneling term. A number of workers [6,32-35] used the psedospin-lattice coupled mode model to elucidate the dielectric and ferroelectric behaviour of order-disorder (KDP-type) ferroelectrics.

Jhang *et al*[36] have applied undetermined constant method to pseudospin model with four spin coupling term. They have not considered phonon part in their calculations which however has a very important contribution in crystals. Some workers[37-39] have used pseudospin lattice coupled mode (PLCM) model alnogwith phonon anharmonicity upto fourth order for KDP-type crystal. Upahhyay and Semwal[40] have used cubic and quartic phonon anharmonic

interactions in the PLCM model for KDP-type crystal to study microwave dielectric tangent losses in KDP and DKDP crystals, their theoretical expressions for frequency, shift and width and soft mode frequency are compared to present study. A good agreement has been found.

In this paper, we consider short range and long range forces, the finite overlap of the protonic wave function between the two sites in a given hydrogen bond, a part of the proton-lattice coupling, and lattice anharmonicity upto fourth order. The approximation, is based on the cluster expansion of the partition function of an order-disorder type hydrogen bonded ferroelectric crystal. The interactions between the four protons surrounding a given PO4 group are taken into account exactly and the rest is replaced by a molecular field, which is determined self-consistently[6]. An attempt is made to account for the effect of one proton on the tunneling integral of another. The present approximation reduces in the classical limit to the Senko-Uehling modification of Slater theory. For KH_2PO_4 the smallest cluster which is compatible with the crystal structure is a four particle one, which takes into account the correlation in the motion of the four protons surrounding a given PO_4 group. The four-body forces, considering the four particle cluster, which is the smallest cluster appropriate for the short-range effect, has been considered for KDP-system. This model is based on the four-particle approximation which was used successfully to describe the static properties of KDP-system along z-direction[9, 20, 22]. The advantage of this approximation over mean field

approximation is that, in addition to the long range molecular-field two-body forces, the ST short range four-body interactions are also taken into account.

In the present study, the model Hamiltonian proposed by Blinc and Žekš [32] has been modified by considering the lattice anharmonicities for the stochastic motion of $H_2PO_4^-$ groups in a KH_2PO_4, system. The model hamiltonian for this system is a combination of the proton Hamiltonian, the lattice Hamiltonian, the lattice-proton interaction terms and the anharmonic terms upto fourth order. The correlation functions were evaluated using the Green's function technique[41] and Dyson's equation[42]. The higher order Green's functions were evaluated using symmetric decoupling scheme, the cross combinations were not considered because they do not contribute significantly. The expressions for collective proton frequency width, collective phonon mode frequency were obtained. Using parameters values given by[43,44-46],in the theoretical expression for $< S_q^z >$ and $< S_q^x >$, the collective proton wave half width $\Gamma_s(q,\omega)$,the collective phonon mode frequency is obtained as $\tilde{\tilde{\omega}}_{q\pm}^2$ will be calculated. The temperature dependence of $< S^z >$ and $< S^x >$ in KH_2PO_4 for different values of J' in temperature range (0 to 150 K) will be calculated and temperature dependence of the collective proton wave half width, collective phonon mode frequency in PE phase for KH_2PO_4 will be evaluated.

II. Theory

Model Hamiltonian-The model Hamiltonian for KDP-system in four particle cluster approximation can be considered as a combination of following terms:

$$H = H_p + H_L + H_I + H_A \quad \ldots(1)$$

Where H_p the proton self-energy Hamiltonian, H_L lattice Hamiltonian, H_I proton-lattice interaction term, and H_A anharmonic term.

Proton Hamiltonian-The proton self-energy Hamiltonian can be written as:

$$H_p = H_t + H_{SR} + H_{LR}. \quad \ldots(2a)$$

here H_t is a sum of one particle operators, which extends over all PO_4 groups in the crystal as well as overall four hydrogen sites near a given PO_4 groups:

$$H_t = -\frac{1}{2}\Omega\sum_{i=1}^{N}\sum_{j=1}^{4}\left(S_{ij}^{+} + S_{ij}^{-}\right) = -2\Omega\sum_{i} S_i^x. \quad \ldots(2a.1)$$

where Ω is the proton tunneling frequency, $S_{ij}^{+} = b_{ij\uparrow}^{\dagger} b_{ij\downarrow}$ and $S_{ij}^{-} = b_{ij\downarrow}^{\dagger} b_{ij\uparrow}$ with $b_{ijk}^{\dagger}$ and b_{ijk}. ($(k = \uparrow, \downarrow)$ standing for the proton creation and annihilation operators at i, j matrix site with state k, and obey the usual anticommutation rules. The sign $\uparrow$ or $\downarrow$ means that the proton is created or annihilated near the upper or lower oxygen atom respectively, of a given PO_4^- ion. S_i^x is called the tunneling operator, which measures the tunneling power of the proton between the hydrogen double well. The short range energy term:

$$H_{SR} = -\sum_{i=1}^{N} H_{PO_4} = \sum_{i=1}^{N} \sum_{k_1,k_2,k_3,k_4=\uparrow,\downarrow} \varepsilon_{k_1k_2k_3k_4} n_{i1k_1} n_{i2k_2} n_{i3k_{31}} n_{i4k_4}$$

$$= -\frac{1}{2}\sum_{ij} J_{ij} S_i^z S_j^z. \quad \ldots(2a.2)$$

where n_{ijk} are proton number operators, defined as: $n_{ijk} = b_{ijk}^{\dagger} b_{ijk}$ obeying the $n_{ij\uparrow} + n_{ij\downarrow} = 1$, J_{ij} the two-body coupling coefficient, is the same for energy pair of protons in KDP. The short range measures the energies $\varepsilon_{k_1k_2k_3k_4}$ associated with different proton configuration around a given PO_4ion as introduced by Slater and Takagi[20]. The long range energy term:

$$H_{LR} = -\frac{1}{4}\sum_{i,i'=1}^{N}\sum_{j,j'=1}^{4} J_{ii'jj'}\left(n_{ij\uparrow} - n_{ij\downarrow}\right)\left(n_{i'j'\uparrow} - n_{i'j'\downarrow}\right) = -\frac{1}{4}\sum_{ij} J_{ijkl} S_i^z S_j^z S_k^z S_l^z. \quad \ldots(2a.3)$$

describes the interaction between those proton sites which do not belong to the same PO_4^-ion. where J_{ijkl} the four body coupling coefficient refers to the four hydrogen bonds in the PO_4 group in KDP. S_i^z is the half of the difference of occupation probabilities for the proton to be found in the two equilibrium positions of the hydrogen bond.

Lattice Hamiltonian-The Hamiltonian of the lattice vibrations in the absence of proton motion can be written as:

$$H_L = \frac{1}{4}\sum_{k}\omega_k\left(A_k^{+}A_k + B_k^{+}B_k\right) \quad \ldots(2b)$$

where ω_k is the initial phonon frequency and A_k and B_k are displacement and momentum operators and are related with normal coordinates: $Q_k = \hbar^{1/2}(2\omega_k)^{1/2} A_k$ and momenta: $P_k = -i\left(\frac{1}{2}\hbar\omega_k\right)^{1/2} B_k$, with $A_k = a_k + a_{-k}^{\dagger} = A_{-k}^{\dagger}$, $B_k = a_k - a_{-k}^{\dagger} = -B_{-k}^{\dagger}$. Where $a_{-k}^{\dagger}$ and a_k are phonon creation and annihilation operators with wave vector k.

Proton-Lattice Interaction Hamiltonian-The proton-lattice term can be written as:

$$H_I = -\sum_{i}\overline{V_{ik}} S_i^z A_k, \quad \ldots(2c)$$

where $\bar{V}_{ik} = \frac{V_{ik}}{(2\omega_k)^{1/2}}$ is the proton-lattice coupling constant.

Anharmonic Hamiltonian-The Hamiltonian of the anhamonic crystal containing N unit cells with n atoms per unit cell in the second quantized form, inclusive of third and fourth order anharmonicities, is given by [47]:

$$H_A = H_A^{(3)} + H_A^{(4)} = \sum_{\vec{k}_1,\vec{k}_2,\vec{k}_3} V_3(\vec{k}_1\vec{k}_2\vec{k}_3) A_{\vec{k}_1} A_{\vec{k}_2} A_{\vec{k}_3} + \sum_{\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4} V_4(\vec{k}_1\vec{k}_2\vec{k}_3\vec{k}_4) A_{\vec{k}_1} A_{\vec{k}_2} A_{\vec{k}_3} A_{\vec{k}_4}, \quad \ldots(2d)$$

where V_3 and V_4 are Fourier transform of the third- and fourth- order anharmonoc coefficient and are given by Maradudin et al [47]:

$$V_3 = \frac{1}{(6N)^{1/2}} \Delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \times \left(\frac{\hbar^3}{8\omega_{k_1}\omega_{k_2}\omega_{k_3}} \right) \times \Phi(\vec{k}_1 \vec{k}_2 \vec{k}_3),$$

and

$$V_4 = \frac{1}{(24N)} \Delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) \times \left(\frac{\hbar^3}{8\omega_{k_1}\omega_{k_2}\omega_{k_3}\omega_{k_4}} \right) \times \Phi(\vec{k}_1\vec{k}_2\vec{k}_3\vec{k}_4),$$

$\Phi^{'}$s are coulomb coefficient defined by Born an Huang[48] and Semwal and Sharma[49].

Combining equations 2. The resultant Hamiltonian, used in this paper, can be written as:

$$H = -2\Omega \sum_i S_i^x - \frac{1}{2}\sum_{i,j} J_{i,j} S_i^z S_j^z - \frac{1}{4}\sum_{i,j,k,l} J'_{i,j,k,l} S_i^z S_j^z S_k^z S_l^z + \frac{1}{4}\omega_k \left(A_k^+ A_k + B_k^+ B_k \right)$$

$$- \sum_i \overline{V_{ik}} S_i^z A_k + \sum_{\vec{k}_1,\vec{k}_2,\vec{k}_3} V_3(\vec{k}_1\vec{k}_2\vec{k}_3) A_{\vec{k}_1} A_{\vec{k}_2} A_{\vec{k}_3} + \sum_{\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4} V_4(\vec{k}_1\vec{k}_2\vec{k}_3\vec{k}_4) A_{\vec{k}_1} A_{\vec{k}_2} A_{\vec{k}_3} A_{\vec{k}_4}. \quad \ldots (3)$$

Using this modified Hamiltonian for the KDP-system and using the green's function method the dielectric properties of KDP-type ferroelectrics will be discussed.

III . GREEN"S FUNCTION

Following Zubarev[50], the proton Green's function is as follow:

$$G_{qq'}^{zz}(\omega) = << S_q^z(t); S_{q'}^z(t') >>$$

$$= -j\theta(t-t') < [S_q^z(t); S_{q'}^z(t')] > \quad \ldots(4)$$

where the angular brackets denote the average over the large canonical ensemble and $\theta(t)$ is the usual Heaviside step function, having properties:

$\theta(t) = 1$ for $t > 0$ and $\theta(t) = 0$ for $t < 0$, $j = (-1)^{1/2}$.

Differentiating (4) twice with respect to time't' with the help of model Hamiltonian (3) and taking Fourier transformation, one obtains:

$$(\omega^2 - \Omega^2) G_{qq'}^{zz}(\omega) = \frac{\Omega < S_q^x > \delta_{qq'}}{\pi} + \frac{\Omega}{\pi} << F_q(t); S_{q'}^z(t') >>, \quad \ldots(5)$$

and the higher order Green's functions:

$$P_{qq'}(t,t') = << F_q(t); S_{q'}^z(t') >> \quad \ldots(6)$$

with

$$F_q(t) = \pi \Big[-2\bar{V}_q S_q^x A_q - J_q(S_q^z S_{q'}^x + S_q^x S_{q'}^z) - \frac{1}{2} J'_q (S_q^x S_{q'}^z S_{-q}^z S_{-q'}^z + S_{q'}^z S_q^x S_{-q}^z S_{-q'}^z + S_{-q}^z S_q^z S_{q'}^x S_{-q'}^z + S_{-q'}^z S_q^z S_{q'}^z S_{-q}^x) \Big] \quad \ldots(7)$$

To calculate $P_{qq'}(t,t')$, Eq. (6) is differentiating twice with respect to't' from right hand side using the model Hamiltonian Eq. (3), taking Fourier transformation, one obtains:

$$(\omega^2 - \Omega^2) G_{qq'}^{zz}(\omega) = \frac{\Omega j < F_q(t), S_q^y >}{\pi} + \frac{\Omega}{\pi} << F_q(t); F_q(t') >>, \quad \ldots(8)$$

where

$$F_{q'}(t') = \pi \Big[-2\bar{V}_q S_q^x A_q - J_q(S_q^x S_{q'}^z + S_q^x S_{q'}^z) - \frac{1}{2} J'_q (S_q^x S_{q'}^z S_{-q}^z S_{-q'}^z + S_{q'}^z S_q^x S_{-q}^z S_{-q'}^z + S_{-q}^z S_q^z S_{q'}^x S_{-q'}^z + S_{-q'}^z S_q^z S_{q'}^z S_{-q}^x) \Big] \quad \ldots(9)$$

and

$$\neg_{qq'}(t - t') = << F_q(t); F_q(t') >> \quad \ldots (10)$$

Substituting the value of $\neg_{qq'}(t - t')$ from equation (8) to equation (5) and rearranging, and applying the Dyson's equation[51], writing $G_{qq'}^{zz}(\omega)$ in the first approximation. After arranging the terms, the final form of $G_{qq'}^{zz}(\omega)$ becomes:

$$G_{qq'}^{zz}(\omega) = \frac{\Omega < S_q^z > \delta_{qq'}}{\pi \left[\omega - \tilde{\Omega}^2 - \frac{\Omega}{2\pi < S_q^z >} \Gamma(\omega) \right]}, \quad \ldots (11)$$

where the renormalized frequency is

$$\tilde{\Omega}^2 = a^2 + b^2 - bc \quad \ldots (12)$$

with

$$a = J_0 < S_q^z > + J_0' < S_q^z >^3, \quad \ldots(12a)$$

$$b = 2\Omega, \quad \ldots(12b)$$

$$c = J_0' < S_q^z > + 3J_0' < S_q^x >< S_q^z >, \quad \ldots(12c)$$

and

$$J_0 = \sum_q J_q \text{, and } J_0^{'} = \sum_q J_q^{'}$$

…(12d)

The expectation values of the proton collective mode components at site q have been obtained by Blink and Žekš [32] as:

$$< S_q > = \frac{Tr\, S_q\, e^{-\beta|\tilde{\Omega}|MFA}}{Tr\, e^{-\beta|\tilde{\Omega}|MFA}}$$

$$= \frac{d.\log_e|\tilde{\Omega}|}{d.\beta|\tilde{\Omega}|}$$

$$= \frac{1}{2}\frac{|\tilde{\Omega}|}{\tilde{\Omega}}\tanh\left(\frac{\beta\tilde{\Omega}}{2}\right)$$

…(13)

so

$$< S_q^x > = \frac{\Omega}{\tilde{\tilde{\Omega}}}\tanh\left(\frac{\beta\tilde{\Omega}}{2}\right),$$

…(13a)

$$< S_q^y > = 0,$$

…(13b)

$$< S_q^z > = \frac{(J_0 < S_q^z > + J_0^{'} < S_q^z >^3)}{2\tilde{\Omega}}\tanh\left(\frac{\beta\tilde{\Omega}}{2}\right).$$

…(13c)

Equations (13) represent a system of 3N equations for the average values of the collective mode components. The solution of this system will, however, be stable only if they minimize the free energy, i.e., if

$$< S_q^z > = < S_q^y > = 0.$$

and so

$$< S_q^x > = \frac{\Omega}{\left[4\Omega^2 - 2\Omega J_0 < S^x >\right]^{1/2}}\tanh\left(\frac{\left[4\Omega^2 - 2\Omega J_0 < S^x >\right]^{1/2}}{2k_\beta T}\right)$$

…(14)

Equation (14) exists at all temperature $(T > T_c)$.It represents PE phase. In the ferroelectric phase $(T < T_c)$.[45]

$$< S_q^x > = \frac{2\Omega}{J_0 + J_0^{'} < S_q^z >^2},$$

and

$$< S_q^z > = \frac{1}{2}\tanh\left(\frac{J_0 < S_q^z > + J_0^{'} < S_q^z >^3}{2k_\beta T}\right)$$

…(15)

The higher order Green's functions are evaluated using symmetrical decoupling scheme, the cross combinations are not considered because they do not contribute significantly. Putting the evaluated value of higher order Green's functions $<< F_q(t); F_{q'}(t') >>$; in Equation (11), one gets:

$$\lim_{\varepsilon\to 0} G_{qq'}^{zz}(\omega + j\varepsilon) = \frac{\Omega < S_q^x > \delta_{qq'}}{\pi\left[\omega^2 - \tilde{\tilde{\Omega}}^2 + j\Gamma_s(q,\omega)\right]},$$

…(16)

where $\tilde{\tilde{\Omega}}$ is the proton renormalized frequency of the coupled system, which on solving self consistently takes the form:

$$\tilde{\tilde{\Omega}}^2 = \tilde{\Omega}^2 + 2\Omega\Delta_s(q,\omega),$$

…(17)

and

$$\Delta_s(q,\omega) = \frac{1}{2\Omega}\sum_{i=1}^{3} G_{si}^{'}(q,\omega)$$

…(18)

where

$$G_{s1}^{'}(q,\omega) = \frac{2\bar{V}_q^2\omega_q < S_q^x > (\omega^2 - \tilde{\omega}_q^2)}{\Omega\left[(\omega^2 - \tilde{\omega}_q^2)^2 + 4\omega_q^2\Gamma_p^2\right]},$$

…(18a)

$$G_{s2}^{'}(q,\omega) = \frac{bc^2}{(\omega^2 - \Omega^2)},$$

…(18b)

and

$$G_{s3}^{'}(q,\omega) = \frac{a^2\hat{\Omega}^2}{2\Omega(\omega^2 - \hat{\Omega}^2)}$$

…(18c)

with

$$\hat{\Omega}^2 = (a^2 + n_q\bar{V}_q^2)$$

…(19)

and $\Gamma_s(q,\omega)$ is the collective proton wave half width, given by:

$$\Gamma_s(q,\omega) = \pi\sum_{i=1}^{3} G_{si}^{''}(q,\omega)$$

…(20)

where

$$\Gamma_{s1}(q,\omega) = \pi G_{s1}^{''} = \frac{-4\pi\bar{V}_q^2\omega_q^2 < S_q^x > \delta_{qq}^{'}\Gamma_p}{\Omega[(\omega^2 - \tilde{\tilde{\omega}}_q^2)^2 + 4\omega_q^2\Gamma_p^2]},$$

…(20a)

$$\Gamma_{s2}(q,\omega) = \pi G''_{s2} = \frac{\pi b c^2}{2\tilde{\Omega}}\left\{\delta(\omega-\tilde{\Omega})-\delta(\omega+\tilde{\Omega})\right\}, \quad \ldots(20b)$$

and

$$\Gamma_{s3}(q,\omega) = \pi G''_{s3} = \frac{\pi a^2 \hat{\Omega}}{2b}\left\{\delta(\omega-\hat{\Omega})-\delta(\omega+\hat{\Omega})\right\} \quad \ldots(20c)$$

The acoustic phonon width and shift are obtained analogously from the acoustic phonon Green's function $<< A_q; A_q^+ >>$. Using the model Hamiltonian, equation (3), one has

$$G_{qq'}(\omega) = \frac{\omega_q \delta_{qq'}}{\pi(\omega^2 - \tilde{\tilde{\omega}}_q^2 + 2j\omega_q \Gamma_p(q,\omega)}, \quad \ldots(21)$$

where

$$\tilde{\tilde{\omega}}_q^2 = \tilde{\omega}_q^2 + 2\omega_q \Delta_p(q,\omega) \quad \ldots(22)$$

with

$$\tilde{\omega}_q^2 = \omega_q^2 + \omega_q\left\{16\sum_q V_3(q)\coth\left(\frac{\beta\omega_q}{2}\right)+8\sum_{q'} V_4(q')\coth\left(\frac{\beta\omega_{q'}}{2}\right)\right\} \quad \ldots(23)$$

and the collective phonon mode frequency shift

$$\Delta_p(q,\omega) = \sum_{i=1}^{3} G'_{ip}(q,\omega); \quad \ldots(24)$$

where

$$G'_{1p}(q,\omega) = \frac{2\bar{V}_q^2 \Omega < S_q^x > (\omega^2 - \tilde{\tilde{\Omega}}^2)\delta'_{qq}}{\omega_q\left[(\omega^2-\tilde{\tilde{\Omega}}^2)^2 + 4\Omega^2\Gamma_s^2(q,\omega)\right]}, \quad \ldots(24a)$$

$$G'_{2p}(q,\omega) = \frac{24\sum_q V_3^2(qq')\omega_q n_q}{\tilde{\omega}(\omega^2 - 4\tilde{\omega}_q^2)}, \quad \ldots(24b)$$

and

$$G'_{3p}(q,\omega) = \frac{24\sum_q V_4^2(qq')\omega_q\omega_{q'}}{\tilde{\omega}_q^2\tilde{\omega}_{q'}}\left[\frac{(1+2n_{q'}+n_q^2)(2\tilde{\omega}_q+\tilde{\omega}_{q'})}{\omega^2-(2\tilde{\omega}_q+\tilde{\omega}_{q'})^2}+\frac{(n_q^2-1)(2\tilde{\omega}_q-\tilde{\omega}_{q'})}{\omega^2-(2\tilde{\omega}_q-\tilde{\omega}_{q'})^2}+\frac{2\omega_{q'}(n_q^2-1)}{(\omega^2-\tilde{\omega}_q^2)}\right]$$

$$+\frac{72\sum_q V_4^2(qq')\omega_q^2}{\tilde{\omega}_q^2}\left[\frac{(1+3n_q^2)}{(\omega^2-9\tilde{\omega}_q^2)}+\frac{(n_q^2-1)}{(\omega^2-\tilde{\omega}_q^2)}\right] \quad \ldots \quad (24c)$$

Calculating equation (22) self consistently and approximating, the collective phonon mode frequency is obtained as:

$$\tilde{\tilde{\omega}}_{q\pm}^2 = \frac{1}{2}(\tilde{\omega}_q^2+\tilde{\tilde{\Omega}}^2) \pm \frac{1}{2}\left[(\tilde{\omega}_q^2+\tilde{\tilde{\Omega}}^2)^2 + 16\bar{V}_q^2\omega_q\Omega < S^x >\right]^{1/2} \quad \ldots(25)$$

where

$$\tilde{\omega}_q^2 = \omega_q^2 + 8\omega_q(2V_3+V_4)\coth\left(\frac{\beta\omega_q}{2}\right) \quad \ldots \quad (26)$$

The frequencies $\tilde{\tilde{\omega}}_{\pm}$ are the normal modes of the system and are the frequencies which be used for comparison with other measured responses of the system[52]. Furthermore, $\tilde{\tilde{\omega}}_{\pm}$ are approximately the same frequencies that obtained by fitting each part of spectrum independently[53]. The $\tilde{\tilde{\omega}}_{-}$ mode frequency approaches zero at the T_c. The $\tilde{\tilde{\omega}}_{+}$ mode, on the other hand, has no critical temperature dependence. The $\tilde{\tilde{\omega}}_{-}$ mode corresponds to the longitudinal soft $B_2(z)$ mode[54],which softens when temperature approaches Tc and $\tilde{\tilde{\omega}}_{+}$ mode corresponds to the transverse E(x,y) mode[54],which has by far less temperature dependence than the $\tilde{\tilde{\omega}}_{-}$ mode. $\tilde{\tilde{\omega}}_{-}$ and $\tilde{\tilde{\omega}}_{+}$ modes originate from a zone center phonon (q = 0) corresponding to a collective proton motion in the a-b plane. Equation (15a) is obtained using the model Hamiltonian equation (3) and equation (24b) and (24c). The higher order correlations in the phonon response function are obtained without any decoupling and using the renormalized Hamiltonian:

$$H_{ren.} = -2\Omega\sum_q S_q^x - \frac{1}{2}\sum_{q,q'} J_q S_q^z S_{q'}^z - \frac{1}{4}\sum_{q,q',-q,-q'} J'_q S_q^z S_{q'}^z S_{-q}^z S_{-q'}^z + \frac{1}{4}\sum_q \frac{\tilde{\omega}_q^2}{\omega_q}\left(A_q^+ A_q + B_q^+ B_q\right) \quad \ldots(27)$$

The collective phonon half width is obtained as:

$$\Gamma_P(q,\omega) = \Gamma_{P1} + \Gamma_{P2} + \Gamma_{P3} = \pi\sum_{i=1}^{3} G_i''(q,\omega), \quad \ldots \quad (28)$$

$$\Gamma_{P1}(q,\omega)=\pi G_1^{''}(q,\omega)=\frac{-4\bar{V}_q^{\,2}\Omega^2<S_q^z>\Gamma_s(q,\omega)}{\omega_q\left[(\omega^2-\tilde{\tilde{\Omega}}^2)^2\right]+4\Omega^2\Gamma_s^2(q,\omega)}; \quad \ldots(28a)$$

$$\Gamma_{P2}(q,\omega)=\pi G_2^{''}(q,\omega)=6\pi\sum_q V_3^2(qq')\frac{\omega_q n_q}{\omega_q}\left[\delta(\omega-2\tilde{\omega}_q)-\delta(\omega-2\tilde{\omega}_q)\right]; \quad \ldots(28b)$$

$$\begin{aligned}\Gamma_{P3}(q,\omega)=\pi G_3^{'}(q,\omega)=12\pi\sum_q&\frac{V_3^2(qq')\omega_q\omega_q'}{\tilde{\omega}_q^2\tilde{\omega}_q'}\left[(1+2n_q n_q'+n_q^2)\left\{\delta(\omega-2\tilde{\omega}_q-\tilde{\omega}_q')\right.\right.\\&\left.-\delta(\omega+2\tilde{\omega}_q+\tilde{\omega}_q')\right\}+(n_q^2-1)\left\{\delta(\omega-2\tilde{\omega}_q+\tilde{\omega}_q')-\delta(\omega+2\tilde{\omega}_q-\tilde{\omega}_q')\right\}\\&+2(n_q^2-1)\left\{\delta(\omega-\tilde{\omega}_q')-\delta(\omega+\tilde{\omega}_q')\right\}\\&+36\sum_q V_4^2(qq')\frac{\omega_q^2}{\tilde{\omega}_q^2}\left[\frac{(1+3n_q^2)}{3\tilde{\omega}_q}\left\{\delta(\omega-3\tilde{\omega}_q)-\delta(\omega+3\tilde{\omega}_q)\right\}\right.\\&\left.+(n_q^2-1)\left\{\delta(\omega-\tilde{\omega}_q')-\delta(\omega+\tilde{\omega}_q')\right\}\right]\end{aligned} \quad (28c)$$

where $n_q=\frac{\omega_q}{\tilde{\omega}_q}\coth\left(\frac{\beta\tilde{\omega}_q}{2}\right)$ is the phonon occupation number and $\beta=(k_BT)^{-1}$, k_B is Boltzman constant and T the absolute temperature, In the PE phase $<S^z>=0$, and the stability limit of the PE phase is determined by the temperature where $\tilde{\tilde{\omega}}_{q^-}$ approaches zero,

i.e., $\tilde{\tilde{\omega}}_{q^-}\to 0$ as $T\to T_C$. In the vicinity of transition temperature in the PE phase one may expand $\tilde{\tilde{\omega}}_{q^-}^2$ in the power of $(T-T_C)$ around its value at T_C getting immediately

$$\tilde{\tilde{\omega}}_{q^-}^2=\left(\frac{\partial\tilde{\tilde{\omega}}_-^2}{\partial T}\right)_{T=T_C}(T-T_C), \quad \ldots(23a)$$

$$\tilde{\tilde{\omega}}_{q^-}^2=\gamma(T-T_C). \quad \ldots(23b)$$

With

$$\gamma=\frac{\Omega^2\hat{J}}{k_BT_C^2\cosh^2\left(\frac{\Omega}{k_BT_C}\right)}$$

When $T\to T_C$, $\tilde{\tilde{\omega}}_{q^-}\to 0$, equation (17) gives

$$T_C=\frac{\Omega}{k_B\tanh^{-1}\left(\frac{4\Omega}{\hat{J}}\right)} \quad (24)$$

where

$$\hat{J}=J_0+\left.\frac{2\bar{V}_q^{\,2}\omega_q}{\tilde{\omega}_q}\right|_{T=T_C} \quad (25)$$

Now following Kuo[55] and Zubarev[56] the real part of dielectric constant (ε) of KDP crystal can be expressed with the help of Green function (4) as:

$$\varepsilon-1=-\frac{8\pi N\mu^2\tilde{\omega}\left(\omega^2-\tilde{\tilde{\omega}}^2\right)}{\left(\omega^2-\tilde{\tilde{\omega}}^2\right)^2+4\omega^2\Gamma_P^2(\omega)}, \quad \ldots(27)$$

Numerical calculations of width, shift and collective phonon mode frequency in PE phase

By using the parameter values from the literature [40,43-46], the temperature dependence for $<S_q^z>$ and $<S_q^x>$ in KH_2PO_4 for different values of J'. (a) $J'=J/3$, (b) $J'=4J/3$, (c) $J'=7J/3$, the collective proton frequency width $\Gamma_s(q,\omega)$ in PE phase for KH_2PO_4, and the collective phonon mode frequency $\tilde{\tilde{\omega}}_{q\pm}^2$ in PE phase for

KH_2PO_4 are calculated using respective equations and are presented in Table 1 and 2.

Temperature dependence of $< S_q^z >$ **and** $< S_q^x >$.

Using calculated values of $< S_q^z >$ and $< S_q^x >$ from Table 1.The $< S_q^z >$ and $< S_q^x >$ versus temperature plot for KH_2PO_4 is shown in Fig. 1. The theoretical results are good agreement with theoretical result of others[37-40]. In Fig. 1, curve 'a' is the case of $J^{'} = J/3 < 4J/3,$ curve 'b' $J^{'} = 4J/3 < 7J/3,$ curve 'c' $J^{'} = 7J/3 > 4J/3.$ In curve 'a' and 'b', value of $< S^z >$ increases to the saturated value 0.5 from zero, when temperature decreases from transition temperature. That is the case of second order phase transition. But in curve 'c' the change of $< S^z >$ with temperature starts from a non zero value $< S^z >$ at point A that is to say, when temperature decreases $< S^z >$ increases to the saturation value from the finite value of $< S^z >$. This is the case of first order phase transition. The temperature at point 'A' is transition temperature (T_c), and the value of $< S^z >$ at 'A' is discontinuity of $< S^z >$. The value of $< S^x >$ decreases when temperature decreases in the ferroelectric phase. On the other hand, in PE phase the value of $< S^x >$ decreases when temperature increases from transition temperature (T_c). Our theoretical results fairly agree with experimentally reported [55-77] result within experimental errors.

Table 1 Calculated values for KDP crystal.

Temperature (K)	$< S_q^z >$			$< S_q^x >$		
	$J^{'} = J/3$	$J^{'} = 4J/3$	$J^{'} = 7J/3$	$J^{'} = J/3$	$J^{'} = 4J/3$	$J^{'} = 7J/3$
20	0.5	0.499	0.499	0.454	0.369	0.311
40	0.498	0.499	0.499	0.455	0.369	0.311
60	0.459	0.481	0.498	0.460	0.376	0.312
80	0.257	0.278	0.487	0.482	0.446	0.317
100	0.137	0.143	0.454	0.489	0.479	0.332
120	0.102	0.104	0.409	0.490	0.485	0.354
122	0.100	0.100	0.403	0.491	0.486	0.357
123	0.00	0.00	0.400	0.492	0.492	0.358
125	0.00	0.00	0.398	0.484	0.484	0.449
130	0.00	0.00	0.389	0.468	0.468	0.457
135	0.00	0.00	0.373	0.446	0.446	0.443
140	0.00	0.00	0.352	0.426	0.426	0.436
145	0.00	0.00	0.320	0.408	0.408	0.410
150	0.00	0.00	0.30	0.396	0.396	0.392

Table 2 Calculated values KDP crystal.

Temperature (K)	125	130	135	140	145
$\Gamma\left(cm^{-1}\right)\times 10^{-4}$	2.87	2.31	1.76	1.88	1.90
$\tilde{\tilde{\omega}}_{-}\left(cm^{-1}\right)$	45.65	57.04	58.69	63.04	64.91

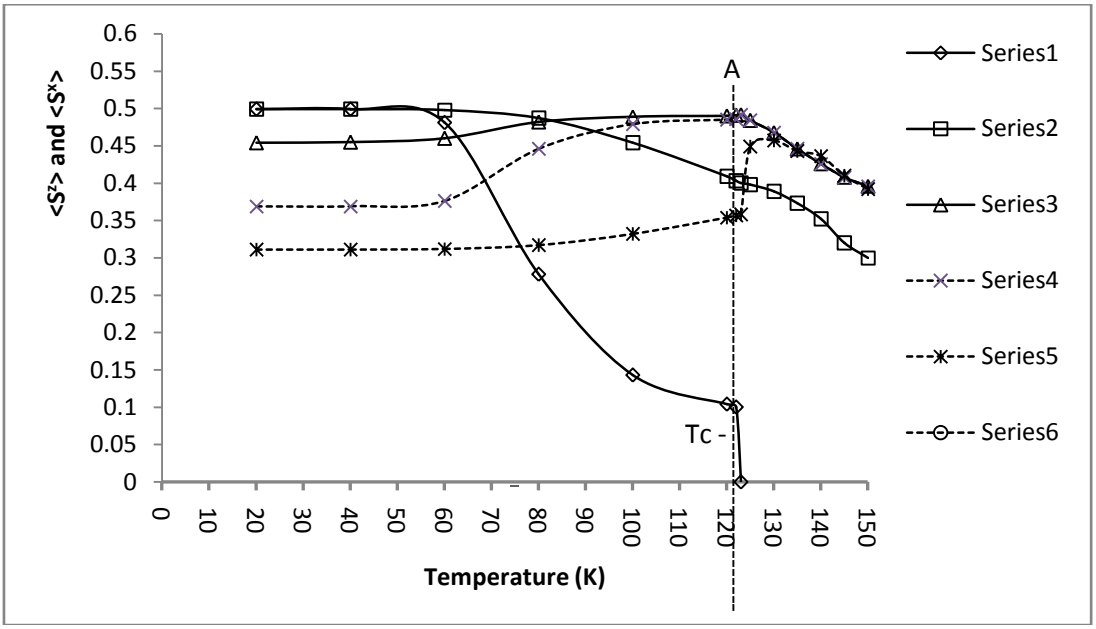

Fig. 1 Temperature dependence of $< S^z >$ for different values of $J^{'}$. (Series1) $J^{'} = J/3$, (Series2) $J^{'} = 4J/3$, (Series3) $J^{'} = 7J/3$ (present study), and $< S^x >$ for different values of $J^{'}$. (Series4) $J^{'} = J/3$, (Series5) $J^{'} = 4J/3$, (Series6) $J^{'} = 7J/3$ (present study), in KH_2PO_4

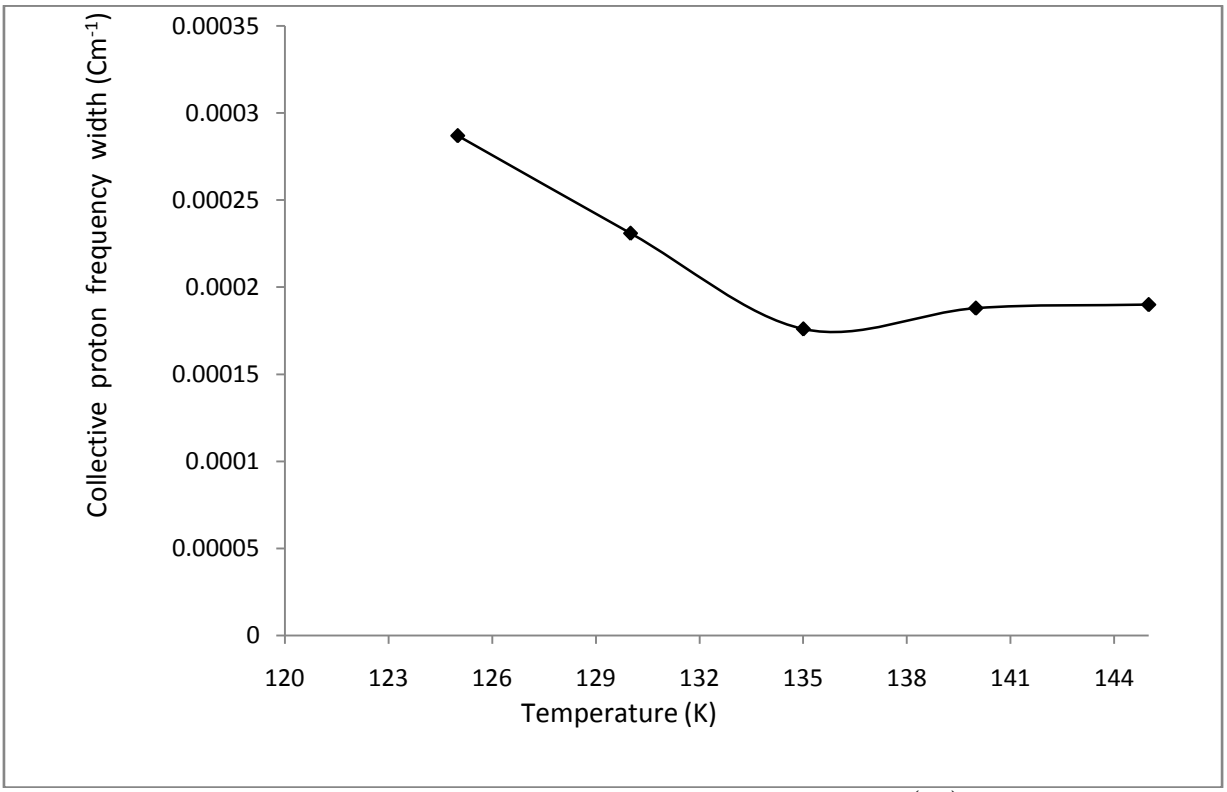

Fig 2 Temperature dependence of collective proton frequency width (Γ) in PE phase for KH_2PO_4 (present study).

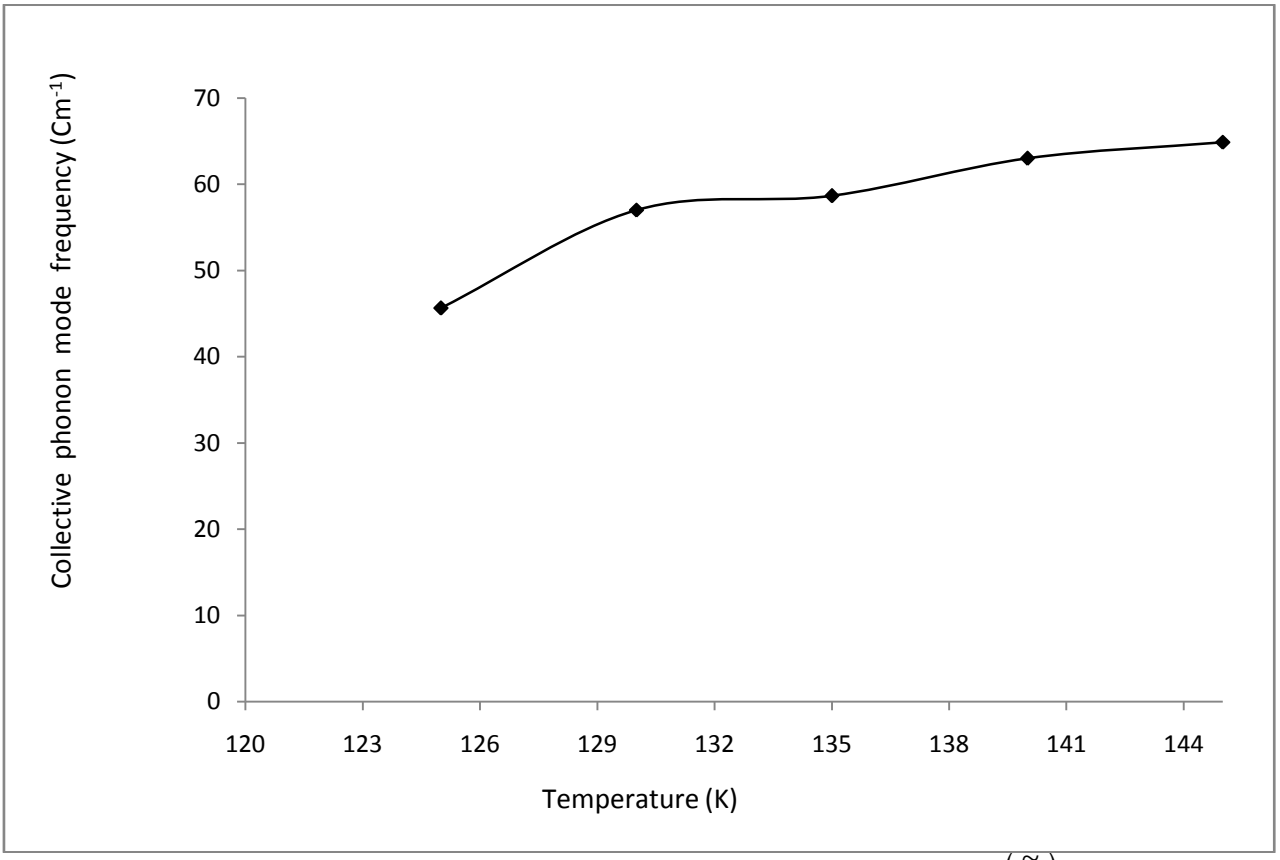

Fig. 3 Temperature dependence of collective phonon mode frequency $\left(\tilde{\tilde{\omega}}\right)$ in PE phase for KH_2PO_4 (present study).

C.. Temperature dependence of collective proton frequency width and collective phonon mode frequency in PE phase

Using calculated values of collective proton frequency width (Γ) and collective phonon mode frequency ($\tilde{\tilde{\omega}}_-$) in PE phase for KH_2PO_4 from Table 2. The temperature variations are shown in graphs as shown in Fig. 2 and 3.

IV. Conclusion

From the present study, it can be concluded that the consideration of four - particle cluster Hamiltonian alongwith the third - and fourth - order anharmonicities for the KDP- type ferroelectrics lead to the renormalization and stabilization of the relaxational soft mode and renormalization of the pseudo-spin exchange interaction constant. The decoupling of the correlation appearing in the dynamical equation results in shift in frequency and facilitates the calculation of damping parameter, which is related to the relaxation time. In the present study the modified four-particle cluster model with phonon anharmonic interaction terms up to fourth order have been used to obtain expressions for the collective proton wave half width, collective phonon mode frequency and shift. The method of double time thermal Green's function technique and Dyson's equation has been used for the evaluation. Many worker 9,19-24 used four proton cluster model but could not explain most of the features of KDP-system except the difference between the Curie and Curie-Weiss temperature. Vaks and Zinenko 22 Yoshimitsu and Matsubara 23and Havlin and Sompolinsky 24performed extensive calculations for the static thermodynamics behaviour in the four-particle cluster approximation and found satisfactory agreement with the experimental data, but they could not explain the observed relaxational behavour of dielectric properties and ultrasonic attenuation explicitly. Ganguli et al 6 modified Ramakrishnan and Tanaka theory by considering anharmonic interaction. Their treatment explains many features of order-disorer ferroelectrics. However, due to insufficient treatment of anharmonic interactions, they could not obtain quantitatively good results and could not describe some intersting properties, like dielectric, ultrasonic attenuation, etc.

Blinc and Zeks 32 obtain, the expectation value of proton collective mode component at site q: $< S_q^z >$ and $< S_q^z >$, in present study, we use different values of J'. In lower value of J', $< S^z >$ shows a second order phase transition. While higher value J', $< S^z >$ is the case of first order phase transition. The value of $< S^x >$ decreases

when temperature decreases in the ferroelectric phase, in PE (PE) phase the value of $< S^x >$ decreases when temperature increases from transition temperature (Tc). Silsbee, Uehling and Schmidt (SUS)57 showed that the Slater-Takagi model can predict either first or second order transitions. Later it was found experimentally 58 that the KDP transition is of first order.

The present results reduce to the results of others6,34 if the width and shift are neglected. The method of double time thermal Green's function and Dyson's equation formalism have been found convenient and systematic to give the static and dynamical properties of a single framework of KDP-type system using four-cluster Hamiltonian alongwith phonon anharmonicities. Our theoretical calculation fairly agree with experimentally59-61 reported results within experimental errors.

The anomalous behaviour of order-disorder KDP type ferroelectrics finds explanation by the consideration of collective proton-phonon interaction and third-and fourth-order phonon anharmonicities in the four-particle cluster Hamiltonian. The dielectric properties and ultrasonic attenuation strongly depend on the relaxational behaviour of the stochastic motion of $H_2PO_4^-$ group in KDP type ferroelectrics

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VI. References

1) Schrieffer J R *Theory of superconductivity*,{WA Benjamin Inc., New York, 1964}
2) Guyer R *In Solid state physics* {Academic Press Inc, New York 1969} 23 499
3) Van Hove L Hagenholtz and Howalnd *Quantum theory of many particle system* {W A Benjamin Inc., New York, 1961}
4) Lifshitz J M Nuovo *Climento (Suppl.)* 3, (1956) 716
5) Kobayashi K K *J Phys Soc Jpn*, 24 (1968) 497
6) Ganguli S Nath D and Chaudhari B K *Phys Rev*, B21 (1980) 2937
7) Rawat M S Semwal B S *Ind J Pure and appl phys* 22, (1984) 528
8) Panwar NS and Semwal B *S bull Matter Sci*, 15 (1992) 237
9) Tokunaga M and Matsubara T *prog theor phys*, 35 (1996) 581
10) Tokunaga M *prog theor phy* ,36 (1996) 587
11) West J Z *Krist* ,74 (1930) 306
12) De Quervain M *Helv phys acta* ,17 (1944) 509
13) Frazer B C & Pepinsky *Acta Cryst* ,6 (1953) 273
14) Bacon G E *in neutron diffraction* {clarendon press, oxford,1950}
15) Bacon G E & Pease R S *Proc roy soc london* ,A 220 (1953) 397
16) Bacon G E & Pease R S *Proc roy soc london*, A 230 (1955) 359
17) Peterson S W Levy A H & Simonsen *J chem phys*, 21 (1953) 2084
18) Levy H a Peterson S W & Simonsen S H *Phys Rev*, 93 (1954) 1120
19) Matsubara T & Yoshimitsu K *prog theor phys jpn*, 37 (1967) 634
20) Blinc R & Svetina S *phys rev*, 147 (1966) 423
21) Vaks V G Zein N E & Strukov B a *phys stat so*, A30 (1975) 801
22) Vaks V G & Zinenko *sov phys JEPT*, 37 (1974) 330
23) Yoshimitsu K & Matsubara T, *Prog. Theor. Phys (Supp)*, 109(1968) 36.
24) Havlin S H & Sompolinsky, *J Phys*, C12 (1982) 3135.
25) Cowley R A *rep progr phys*, 31 (1968) 123
26) Brout R Mullar KA & Thomas H *solid state commun* ,4 (1966) 507
27) Konwant H, *Phys Stat Sol*, 28(1968) 39
28) Villain J & Stamenkovic *phys state so,l* 15 (1966) 585
29) Novakovic L *j phys chem solids*, 31 (1970) 431
30) Samara G A *phys lett* ,27 (1971) 103
31) Houston G D & Bolton H C *phys* ,C4 (1971) 2894
32) Blinc R Zeks B *adv phys* ,21C (1972) 293
33) Pak K N *phy stat sol* ,B60 (1973) 233
34) Ramakrishana V & Tanaka T, *Phys Rev*, B16 (1977) 422.
35) Chaudhuri B *K ind j pure & applied phys*, 16 (1978) 831
36) Jhang Z Qin J & Wang C *phys re,c* B101 (1990) 159
37) Upadhyay T C Panwar NS & Semwal B S *int j mod phys*, B9 (1995) 45
38) Upadhyay T C & Semwal B S procedding iind cof. Disordered system allahabad december 2000
39) Upadhay T C & Kandpal B *In J of pure and applied phys*, 47 (2009) 134
40) Upadhay T C & Semwal BS *pramana* ,60 (2003) 525
41) Dovner B & Comes R In dynamics of solids and liquids by neutron scattering ed S W
42) Gairola R P& Semwal B S, *J Phys Sco. Jpn*, 42(1977) 975.
43) SamaraA *Ferroelectrics*, 20 (1978) 91
44) Chaudhuri B K Ganguli S & Nath D *phys rev* ,B23 (1981) 2368
45) Zikai Q Jinbo Z & Chunlei W *Ferroelectri,c* 101 (1988) 164
46) Lyddane R H Sachs R G & Teller *Phys rev* ,59 (1941) 673
47) Maradudin A a Fein A E *phys rev* ,128 (1962) 2589
48) Born M & Huang K *Dynamical theory of crystal lattice oxford university press New York 1964*

49) Semwal B S & Sharma P K *phys rev*, B5 3905 1972
50) Zubarev D N *sov phys usp*, 3 (1960) 320
51) Jaynes E T in ferroelectricity princeton university press princeton 1953
52) Peercy P S & Samara G A, *Phys Rev*, B8 (1973)2033.
53) Peercy P S, *Phy Rev*, B9(1974) 4868.
54) Takagi Y & Shigenari T, *J Phys Soc Jpn*, 39(1975) 440.
55) Kubo R,*J Phys Soc Jpn*, 12(1957) 570.
56) Zubarev D N, *Uspekhi Fiz Nauk*, 71(1960) 71.
57) Silsbee H B Uehling E A Schmidt V H *phys rev* ,133A (1964) 165
58) Reese W *phys rev* ,181 (1969) 905
59) Kaminow I P & Harding G O, *Phys Rev*, 129(1963) 1562.
60) Kaminow I P *Phy rev* ,A 138 (1965) 1539
61) Gervais F Simon P *ferroelectrics*, 72 77 (1987) 379

Certain Summation Formulae Associated To Gauss Second Summation Theorem

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GJSFR Classification - F (FOR)
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***Abstract*-The main object of present paper is to obtain some summation formuale involving Contiguous relation,Recurrence relation, Gauss second summation theorem ,and Legendre duplication formula**

A.M.S. Subject Classification-33 Special Functions.

Keywords and Phrases-Contiguous relation,Recurrence relation, Gauss second summation theorem ,and Legendre duplication formula.

I. INTRODUCTION

The Pochhammer's symbol is defined by

$$(\alpha,\ k) = (\alpha)_k = \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)} = \begin{cases} \alpha(\alpha+1)(\alpha+2)\cdots(\alpha+k-1); & \text{if } k = 1,2,3,\cdots \\ 1 \quad ; & \text{if } k = 0 \\ k! \quad ; & \text{if } \alpha = 1 \end{cases} \tag{1}$$

Generalized Gaussian Hypergeometric function of one variable is defined by

$${}_AF_B\left[\begin{array}{c} a_1, a_2, \cdots, a_A \ ; \\ b_1, b_2, \cdots, b_B \ ; \end{array} z\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!}$$

or

$${}_AF_B\left[\begin{array}{c} (a_A) \ ; \\ (b_B) \ ; \end{array} z\right] \equiv {}_AF_B\left[\begin{array}{c} (a_j)_{j=1}^{A} \ ; \\ (b_j)_{j=1}^{B} \ ; \end{array} z\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{2}$$

Where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and A, B are non-negative integers.
Contiguous Relation is defined by
[Andrews p.367(8), E. D. p.52(19), H.T. F. I p.103(38)]

$$c\,(1-z)\ {}_2F_1\left[\begin{array}{c} a,\ b\ ; \\ c\ ; \end{array} z\right] = c\ {}_2F_1\left[\begin{array}{c} a-1,\ b\ ; \\ c\ ; \end{array} z\right] - (c-b)\ z\ {}_2F_1\left[\begin{array}{c} a,\ b\ ; \\ c+1\ ; \end{array} z\right] \tag{3}$$

Recurrence relation

$$\Gamma(z+1) = z\,\Gamma(z) \tag{4}$$

Legendre's duplication formula

$$\sqrt{\pi}\,\Gamma(2z) = 2^{(2z-1)}\,\Gamma(z)\,\Gamma\left(z+\frac{1}{2}\right) \tag{5}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)}\,\Gamma(\frac{b}{2})\,\Gamma(\frac{b+1}{2})}{\Gamma(b)} \tag{6}$$

$$= \frac{2^{(a-1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\Gamma(a)} \tag{7}$$

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Gauss second summation theorem [Prud., 491(7.3.7.5)]

$$ {}_2F_1\left[\begin{matrix} a,\ b \\ \frac{a+b+1}{2} \end{matrix};\ \frac{1}{2}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{8} $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{9} $$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prud.,p.491(7.3.7.3)]

$$ {}_2F_1\left[\begin{matrix} a,\ b \\ \frac{a+b-1}{2} \end{matrix};\ \frac{1}{2}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{10} $$

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$ {}_2F_1\left[\begin{matrix} a,\ b \\ \frac{a+b-1}{2} \end{matrix};\ \frac{1}{2}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{11} $$

II. Main Results Of Summation Formulae For a < 1 and a > 12

$$
{}_2F_1\left[\begin{matrix} a, & b \\ \frac{a+b-12}{2}; & \end{matrix}\ \frac{1}{2}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-12}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-12}{2})}\left\{\frac{(1024a+3072b-12288)}{(a-12)} - \right.\right.
$$

$$
-\frac{(a-b-12)(2816a+7424b-28160)}{(a-12)(a-10)}+\frac{(a-b-12)(a-b-10)(2816a+6400b-22528)}{(a-12)(a-10)(a-8)}-
$$

$$
-\frac{(a-b-12)(a-b-10)(a-b-8)(1232a+2352b-7392)}{(a-12)(a-10)(a-8)(a-6)}+
$$

$$
+\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(220a+340b-880)}{(a-12)(a-10)(a-8)(a-6)(a-4)}-
$$

$$
\left.-\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(a-b-4)(11a+13b-22)}{(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)}\right\}+
$$

$$
+\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-11}{2})}\left\{\frac{(2048a+2048b-20480)}{(a-11)}-\frac{(a-b-12)(4864a+4352b-39424)}{(a-11)(a-9)}+\right.
$$

$$
+\frac{(a-b-12)(a-b-10)(4032a+3136b-25088)}{(a-11)(a-9)(a-7)}-
$$

$$
-\frac{(a-b-12)(a-b-10)(a-b-8)(1360a+880b-5920)}{(a-11)(a-9)(a-7)(a-5)}+
$$

$$
+\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(160a+80b-400)}{(a-11)(a-9)(a-7)(a-5)(a-3)}-
$$

$$
\left.\left.-\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(a-b-4)(3a+b-2)}{(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}\right\}\right] \qquad (12)
$$

For $a < 1$ and $a > 13$

$$
{}_2F_1\left[\begin{matrix} a, & b \\ \frac{a+b-13}{2}; & \end{matrix}\ \frac{1}{2}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-13}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-13}{2})}\left\{\frac{(2048a+6144b-26624)}{(a-13)} - \right.\right.
$$

$$
-\frac{(a-b-13)(6144a+16384b-67584)}{(a-13)(a-11)}+\frac{(a-b-13)(a-b-11)(6912a+16128b-62208)}{(a-13)(a-11)(a-9)}-
$$

$$
-\frac{(a-b-13)(a-b-11)(a-b-9)(3584a+7168b-25088)}{(a-13)(a-11)(a-9)(a-7)}+
$$

$$
+\frac{(a-b-13)(a-b-11)(a-b-9)(a-b-7)(840a+1400b-4200)}{(a-13)(a-11)(a-9)(a-7)(a-5)}-
$$

$$
-\frac{(a-b-13)(a-b-11)(a-b-9)(a-b-7)(a-b-5)(72a+96b-216)}{(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)}+
$$

$$+\frac{(a-b-13)(a-b-11)(a-b-9)(a-b-7)(a-b-5)(a-b-3)(a+b-1)}{(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}\Bigg\}+$$
$$+\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-12}{2})}\Bigg\{\frac{(4096a+4096b-45056)}{(a-12)}-\frac{(a-b-13)(10752a+9728b-97792)}{(a-12)(a-10)}+$$
$$+\frac{(a-b-13)(a-b-11)(10240a+8192b-73728)}{(a-12)(a-10)(a-8)}-$$
$$-\frac{(a-b-13)(a-b-11)(a-b-9)(4256a+2912b-22624)}{(a-12)(a-10)(a-8)(a-6)}+$$
$$+\frac{(a-b-13)(a-b-11)(a-b-9)(a-b-7)(720a+400b-2480)}{(a-12)(a-10)(a-8)(a-6)(a-4)}-$$
$$-\frac{(a-b-13)(a-b-11)(a-b-9)(a-b-7)(a-b-5)(34a+14b-54)}{(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)}\Bigg\}\Bigg] \quad (13)$$

The above summation theorems can be easily verified by using computer algebra system programming languages, like Maple, Matlab, Scilab, Octave or Mathematica.

III. DERIVATIONS OF SUMMATION FORMULAE (12) TO (13)

Derivation of (12): Substituting c = a+b−12 2 and z = 12 in equation (3), we get

$$\left(\frac{a+b-12}{4}\right)\,{}_2F_1\left[\begin{matrix}a,\ b\\ \frac{a+b-12}{2}\end{matrix};\ \frac{1}{2}\right]=\left(\frac{a+b-12}{2}\right)\,{}_2F_1\left[\begin{matrix}a-1,\ b\\ \frac{a+b-12}{2}\end{matrix};\ \frac{1}{2}\right]-$$
$$-\left(\frac{a-b-12}{4}\right)\,{}_2F_1\left[\begin{matrix}a,\ b\\ \frac{a+b-10}{2}\end{matrix};\ \frac{1}{2}\right]$$
$${}_2F_1\left[\begin{matrix}a,\ b\\ \frac{a+b-12}{2}\end{matrix};\ \frac{1}{2}\right]=2.\,{}_2F_1\left[\begin{matrix}a-1,\ b\\ \frac{a+b-12}{2}\end{matrix};\ \frac{1}{2}\right]-\left(\frac{a-b-12}{a+b-12}\right)\,{}_2F_1\left[\begin{matrix}a,\ b\\ \frac{a+b-10}{2}\end{matrix};\ \frac{1}{2}\right]$$

Now using (11), we get

$${}_2F_1\left[\begin{matrix}a,\ b\\ \frac{a+b-12}{2}\end{matrix};\ \frac{1}{2}\right]=\frac{2^{(b-1)}\,\Gamma(\frac{a+b-12}{2})}{\Gamma(b)}\Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-12}{2})}\Bigg\{\frac{(1024a+3072b-12288)}{(a-12)}-$$
$$-\frac{(a-b-12)(2560a+6656b-25600)}{(a-12)(a-10)}+\frac{(a-b-12)(a-b-10)(2240a+4928b-17920)}{(a-12)(a-10)(a-8)}-$$
$$-\frac{(a-b-12)(a-b-10)(a-b-8)(800a+1440b-4800)}{(a-12)(a-10)(a-8)(a-6)}+$$
$$+\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(100a+140b-400)}{(a-12)(a-10)(a-8)(a-6)(a-4)}-$$

$$-\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(a-b-4)(2a+2b-4)}{(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)}\Bigg\}+$$
$$+\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-11}{2})}\Bigg\{\frac{(2048a+2048b-20480)}{(a-11)}-\frac{(a-b-12)(4352a+3840b-35328)}{(a-11)(a-9)}+$$
$$+\frac{(a-b-12)(a-b-10)(3072a+2304b-19200)}{(a-11)(a-9)(a-7)}-$$
$$-\frac{(a-b-12)(a-b-10)(a-b-8)(800a+480b-3520)}{(a-11)(a-9)(a-7)(a-5)}+$$
$$+\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(56a+24b-144)}{(a-11)(a-9)(a-7)(a-5)(a-3)}\Bigg\}\Bigg]-$$
$$-(\frac{a-b-12}{a+b-12})\frac{2^{(b-1)}\,\Gamma(\frac{a+b-10}{2})}{\Gamma(b)}\Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-10}{2})}\Bigg\{\frac{(256a+768b-2560)}{(a-10)}-$$
$$-\frac{(a-b-10)(576a+1472b-4608)}{(a-10)(a-8)}+\frac{(a-b-10)(a-b-8)(432a+912b-2592)}{(a-10)(a-8)(a-6)}-$$
$$-\frac{(a-b-10)(a-b-8)(a-b-6)(120a+200b-480)}{(a-10)(a-8)(a-6)(a-4)}+$$
$$+\frac{(a-b-10)(a-b-8)(a-b-6)(a-b-4)(9a+11b-18)}{(a-10)(a-8)(a-6)(a-4)(a-2)}\Bigg\}+$$
$$+\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-9}{2})}\Bigg\{\frac{(512a+512b-4096)}{(a-9)}-\frac{(a-b-10)(960a+832b-5888)}{(a-9)(a-7)}+$$
$$+\frac{(a-b-10)(a-b-8)(560a+400b-2400)}{(a-9)(a-7)(a-5)}-$$
$$-\frac{(a-b-10)(a-b-8)(a-b-6)(104a+56b-256)}{(a-9)(a-7)(a-5)(a-3)}+$$
$$+\frac{(a-b-10)(a-b-8)(a-b-6)(a-b-4)(3a+b-2)}{(a-9)(a-7)(a-5)(a-3)(a-1)}\Bigg\}\Bigg]$$
$$=\frac{2^{(b-1)}\,\Gamma(\frac{a+b-12}{2})}{\Gamma(b)}\Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-12}{2})}\Bigg\{\frac{(1024a+3072b-12288)}{(a-12)}-$$
$$-\frac{(a-b-12)(2560a+6656b-25600)}{(a-12)(a-10)}+\frac{(a-b-12)(a-b-10)(2240a+4928b-17920)}{(a-12)(a-10)(a-8)}-$$
$$-\frac{(a-b-12)(a-b-10)(a-b-8)(800a+1440b-4800)}{(a-12)(a-10)(a-8)(a-6)}+$$
$$+\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(100a+140b-400)}{(a-12)(a-10)(a-8)(a-6)(a-4)}-$$

$$+\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(220a+340b-880)}{(a-12)(a-10)(a-8)(a-6)(a-4)}-$$
$$-\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(a-b-4)(11a+13b-22)}{(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)}\Big\}+$$
$$+\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-11}{2})}\Big\{\frac{(2048a+2048b-20480)}{(a-11)}-\frac{(a-b-12)(4864a+4352b-39424)}{(a-11)(a-9)}+$$
$$+\frac{(a-b-12)(a-b-10)(4032a+3136b-25088)}{(a-11)(a-9)(a-7)}-$$
$$-\frac{(a-b-12)(a-b-10)(a-b-8)(1360a+880b-5920)}{(a-11)(a-9)(a-7)(a-5)}+$$
$$+\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(160a+80b-400)}{(a-11)(a-9)(a-7)(a-5)(a-3)}-$$
$$-\frac{(a-b-12)(a-b-10)(a-b-8)(a-b-6)(a-b-4)(3a+b-2)}{(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}\Big\}\Big]$$

Thus , we prove the result (12)
Similarly, we can prove the result(13).

IV. References

1) Arora, Asish, Singh, Rahul , Salahuddin. ;Development of a family of summationformulae of half argument using Gauss and Bailey theorems Journal of RajasthanAcademy of Physical Sciences., 7(2008), 335-342.
2) Choi, J., Harsh, H. and Rathie, A. K.; Some summation formulae for the Apple'sfunction F1, East Asian Math. Journal, 17(2001), 233-237.
3) Erd´elyi, A., Magnus, W., Okerhettinger, F. and Tricomi, F. G.; Higher transcendentalfunctions Vol.1 (Bateman Manuscript Project) McGraw-Hill book P. Inc. NewYork, Toronto and London, 1953.
4) Krupnikov, E. D., K¨olbig, K. S.; Some special cases of the generalized hypergeometricfunction q+1Fq, Journal of computational and Applied Math., 78(1997), 79-95.
5) Lavoie, J. L.; Notes on a paper by J. B. Miller, J. Austral. Math. Soc. Ser. B,29(1987), 216-220.
6) Lavoie, J. L.; Some summation formulae for the series 3F2, Math. Comput., 49(1987),269-274.
7) Lavoie, J. L., Grondin, F. and Rathie, A.K.; Generalizations of Watson's theorem on the sum of a 3F2, Indian J. Math., 34(1992), 23-32.
8) Lavoie, J. L., Grondin, F. and Rathie, A.K.; Generalizations of Whipple's theoremon the sum of a 3F2, J. Comput. Appl. Math., 72(1996), 293-300.
9) Lavoie, J. L., Grondin, F. Rathie, A. K. and Arora, K.; Generalizations of Dixon'stheorem on the sum of a 3F2, Math. Comput., 62, 267-276.
10) Mitra, C. S.; J. Indian Math. Soc. (N.S.), 7(1943), 102-109.Prudnikov, A. P., Brychkov, Yu. A. and Marichev, O.I.; Integrals and Series Vol.More Special Functions. Nauka, Moscow, 1986. Translated from the Russian by G.G. Gould, Gordon and Breach Science Publishers, New York, Philadelphia, London, Paris, Montreux, Tokyo, Melbourne, 1990.
11) Rainville, E. D.; The contiguous function relations for pFq with applications to Bateman's Ju,v n and Rice's Hn (_, p, _), Bull. Amer. Math. Soc., 51(1945),714-723.
12) Shashikant, Sharma, S. and Rathie, A. K.; Some summation formulae for the Apple'sfunction F1, Proc. of the fourth Int. Conf. SSFA, 4(2003), 81-84.

Fuzzy Anti-bounded Linear Functionals

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010301.010206.010108

***Abstract*-Fuzzy anti-bounded linear functional and fuzzy anti-dual spaces are *defined*. Hahn-Banach theorem and some of its consequences on fuzzy anti-normed linear space are studied. Two fundamental theorems; namely, open mapping theorem and closed graph theorem are established.**

Keywords-Fuzzy anti-norm, ***α-norm***, Fuzzy anti-complete, Fuzzy anti-bounded linear functional, Fuzzy anti-dual space.

I. INTRODUCTION

During the last few years, there is a growing interest and much work has been done in extension of fuzzy set theory which is a useful tool to describe situations in which data are imprecise, vague or uncertain. Fuzzy set theory handle this situation by attributing a degree of membership to which a certain object belongs to a set. Fuzzy set theory was first introduce by Zadeh [17] in 1965 and thereafter, the concept of fuzzy set theory applied on different branches of pure and applied mathematics in different ways by different authors.

Also this is used to develop the concept of norm in fuzzy environment and the concept of fuzzy norm was introduced by Katsaras [12] in 1984. In 1992, Felbin[9] introduced the idea of fuzzy norm on a linear space. Cheng-Moderson [5] introduced another idea of fuzzy norm on a linear space whose associatedmetric is same as the associated metric of Kramosil-Michalek [13]. Latter on Bag and Samanta [2] modified the definition of fuzzy norm of Cheng-Moderson[5] and established the concept of continuities and boundedness of a functionwith respect to their fuzzy norm in[2].

Later on Jebril and Samanta [11] introduced the concept of fuzzy anti-norm on a linear space. The motivation of introducing fuzzy anti-norm is to study fuzzy set theory with respect to the non-membership function. It is useful in the process of decision making. Following the definition of Jebril and Samanta[11], we have modified the definition of fuzzy anti-norm in [7]. Also the results on finite dimensional fuzzy anti-normed linear space, fuzzy anti-α-convergence,various types of fuzzy anti-continuities and fuzzy anti-boundedness and their

relations were studied in [7] and [8]. In this paper, after defining Fuzzy anti-bounded linear functional and fuzzy anti-dual spaces, Hahn-Banach theorem is established and some of it consequences are studied. Thereafter open mapping theorem and closed graph theorem are proved.

II. PRELIMINARIES

This section contain some basic **definition** and preliminary results which will be needed in the sequel.

Definition 2.1 ([15]). A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is a t-conorm if $\diamond$ satisfies the following conditions :

(i) $\diamond$ is commutative and associative ,

(ii) $a \diamond 0 = a \quad \forall \; a \in [0,1]$,

(iii) $a \diamond b \leq c \diamond d$ whenever $a \leq c$, $b \leq d$ and $a, b, c, d \in [0,1]$. A few examples of continuous t-conorm are $a \diamond b = a + b - ab$, $a \diamond b = \max\{a, b\}$, $a \diamond b = \min\{a + b, 1\}$.

Remark 2.3 ([14]). (a) For any $r_1, r_2 \in (0,1)$ with $r_1 > r_2$, there exists $r_3 \in (0,1)$ such that $r_1 > r_2 \diamond r_3$.
(b) For any $r_4 \in (0,1)$ there exists $r_5 \in (0,1)$ such that $r_5 \diamond r_5 \leq r_4$.

Definition 2.2 ([7]). Let $\diamond$ be linear space over the field F (= R or C). A fuzzy subset ν of $V \times R$ is called a **fuzzy antinorm** on V if and only if for all $x, y \in V$ and $c \in F$

(i) For all $t \in R$ with $t \leq 0$, $\nu(x,t) = 1$;

(ii) For all $t \in R$ with $t > 0$, $\nu(x,t) = 0$ if and only if $x = \theta$;

(iii) For all $t \in R$ with $t > 0$, $\nu(cx,t) = \nu\left(x, \frac{t}{|c|}\right)$ if $c \neq 0, c \in F$;

(iv) For all $s, t \in R$ with $\nu(x + y, s + t) \leq \nu(x,s) \diamond \nu(y,t)$;

(v) $\lim_{t \to \infty} \nu(x,t) = 0$.

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We further assume that for any fuzzy anti-normed linear space (V, A^*),

(vi) $\nu(x,t) < 1, \forall t > 0 \Rightarrow x = \theta$.

(vii) $\nu(x,\cdot)$ is a continuous function of **R** and strictly decreasing on the subset $\{t: 0 < \nu(x,t) < 1\}$ of **R**.

Theorem 2.3 [7] Let (V, A^*) be a fuzzy antinormed linear space satisfying (vi) and (vii). Let

$$\| x \|_\alpha^* = \wedge \{t: \nu(x, t) \leq 1 - \alpha\}, \alpha \in (0,1).$$

Also, let $\nu': V \times R \to [0,1]$ be defined by

$$\nu'(x,t) = \wedge \{1 - \alpha : \| x \|_\alpha^* \leq t\}, \quad \text{if } (x,t) \neq (\theta, 0)$$

$$= 1 \text{ , if } (x,t) = (\theta, 0)$$

Then $\nu' = \nu$.

Definition 2.4 [11] A subset A of a fuzzy antinormed linear space (U, N^*) is said to be **bounded** if and only if there exist $t > 0$, $r \in (0,1)$ such that

$$N^*(x,t) < r \quad \forall x \in A.$$

Definition 2.5 [11] Let, (U, N^*) be a fuzzy antinormed linear space. A subset B of U is said to be **closed** if for any sequence $\{x_n\}_n$ in B converges to $x \in B$, that is,

$$\lim_{n \to \infty} N^*(x_n - x, t) = 0, \quad \forall t > 0 \Rightarrow x \in B.$$

Definition 2.6 [8] A linear operator $T: (U, A^*) \to (V, B^*)$ is said to be **strongly fuzzy anti-bounded** on U if and only if there exist a positive real number M such that for all $x \in U$ and for all $t \in R^+$,

$$\nu_V(T(x), t) \leq \nu_U\left(x, \frac{t}{M}\right).$$

Definition 2.7 [8] A linear operator $T: (U, A^*) \to (V, B^*)$ is said to be **weakly fuzzy anti-bounded** on U if and only if for any $\alpha \in (0,1)$ there exist $M_\alpha (> 0)$ such that for all $x \in U$ and for all $t \in R^+$,

$$\nu_U\left(x, \frac{t}{M_\alpha}\right) \leq 1 - \alpha \Rightarrow \nu_V(T(x), t) \leq 1 - \alpha.$$

Definition 2.8 [8] A linear operator $T: (U, A^*) \to (V, B^*)$ is said to be **uniformly fuzzy anti-bounded** if and only if there exist $M > 0$ such that

$$\| T(x) \|_\alpha^* \geq M \| x \|_\alpha^*, \alpha \in (0,1)$$

where $\{\| \cdot \|_\alpha^* : \alpha \in (0,1)\}$ is ascending family of norms on U.

III. HAHN-BANACH THEOREM AND ITS CONSEQUENCES

In this section the real line **R** or the complex plane **C** is denoted by $\mathcal{K}$.

Definition 3.1 A strongly fuzzy anti-bounded linear operator defined from a fuzzy anti-normed linear space (U, ν) to $\mathcal{K}$ is called a strongly fuzzy anti-bounded linear functional.

The set of all strongly fuzzy anti-bounded linear functionals over (U, ν) is denoted by U^*.

Definition 3.2 A weakly fuzzy anti-bounded linear operator defined from a fuzzy anti-normed linear space (U, ν) to $\mathcal{K}$ is called a weakly fuzzy anti-bounded linear functional.

The set of all weakly fuzzy anti-bounded linear functionals over (U, ν) is denoted by $U^\#$.

Definition 3.3 Let (U, ν) be a fuzzy anti-normed linear space satisfying (vi) and (vii). Let $T \in U^*$ and $\{\| \cdot \|_\alpha^* : \alpha \in (0,1)\}$ be the family of norms on U. We define

$$\| T \|_\alpha^{*^*} = \wedge \frac{|T(x)|}{\| x \|_{1-\alpha}^*} \qquad \forall x \in U, \forall x \neq \theta, \forall \alpha \in (0,1)$$

Then $\{ \| \cdot \|_\alpha^{*^*} : \alpha \in (0,1) \}$ is an ascending family of norms on U^*. Again we define

$$\nu^*(T,s) = \wedge \{ 1-\alpha \in (0,1) : \| T \|_\alpha^{*^*} \leq s \} \quad \text{for } (T,s) \neq (\theta,0)$$
$$= 1 \quad \text{for } (T,s) = (\theta,0)$$

Then it can be shown that ν^* is a fuzzy anti-norm on U^* and hence (U^*, ν^*) is a fuzzy anti-normed linear space. The first fuzzy strong anti-dual space of U is denoted by U^*.

Definition 3.4 Let (U, ν) be a fuzzy anti-normed linear space satisfying (vi) and (vii). Let $T \in U^\#$ and $\{ \| \cdot \|_\alpha^* : \alpha \in (0,1) \}$ be the family of norms on U. We define

$$\| T \|_\alpha^{*^\#} = \wedge \frac{|T(x)|}{\| x \|_{1-\alpha}^*} \qquad \forall x \in U, \forall x \neq \theta, \forall \alpha \in (0,1)$$

Then $\{ \| \cdot \|_\alpha^{*^\#} : \alpha \in (0,1) \}$ is an ascending family of norms on $U^\#$. Again we define

$$\nu^\#(T,s) = \wedge \{ 1-\alpha \in (0,1) : \| T \|_\alpha^{*^\#} \leq s \} \quad \text{for } (T,s) \neq (\theta,0)$$
$$= 0 \quad \text{for } (T,s) = (\theta,0)$$

Then it can be shown that $\nu^\#$ is a fuzzy anti-norm on $U^\#$ and hence $(U^\#, \nu^\#)$ is a fuzzy anti-normed linear space. The first fuzzy weak anti-dual space of U is denoted by $U^\#$.

Definition 3.5 We define U_α^* be the set of all linear functional defined from (U, ν) to $\mathcal{K}$ which are fuzzy anti-bounded with respect to $\| \|_\alpha^*$ where $\| \|_\alpha^*$ denotes norm on U for $\alpha \in (0,1)$.

Theorem 3.6 (Hahn-Banach Theorem): Let (U, ν) be a fuzzy anti-normed linear space satisfying (vi) and (vii) and W be a subspace of U. Let f be a strongly fuzzy anti-bounded linear functional defined on (W, ν). Then for each $\alpha \in (0,1)$ there exists $f_\alpha \in U_{1-\alpha}^*$ such that $f_\alpha(x) = f(x)$ for all $x \in W$ (i.e., f_α is an extension of f) and if $f \neq \theta$ then $\nu'(f, \| f_\alpha \|_{1-\alpha}^*) \leq 1-\alpha$, where ν' is a fuzzy anti-norm on W^*.

Proof. Since $f : (W, \nu) \to \mathcal{K}$ is strongly fuzzy anti-bounded linear functional, therefore

$$\| f \|_\alpha^{*^*} = \wedge \frac{|f(x)|}{\| x \|_{1-\alpha}^*} \qquad \forall x \in W, \forall x \neq \theta, \forall \alpha \in (0,1)$$

is finite and the function ν' is defined by

$$\nu'(f,s) = \wedge \{ 1-\beta \in (0,1) : \| f \|_\beta^{*^*} \leq s \} \quad \text{for } (f,s) \neq (\theta,0)$$
$$= 1 \quad \text{for } (f,s) = (\theta,0)$$

is a fuzzy anti-norm on W^*. Also, $f : (W, \| \cdot \|_{1-\alpha}^*) \to \mathcal{K}$ is uniformly fuzzy anti-bounded for all $\alpha \in (0,1)$, where $\| \cdot \|_{1-\alpha}^*$ is fuzzy $(1-\alpha)$-anti-norms of ν Now by Hahn-Banach theorem over the normed linear space $(U, \|\cdot\|_{1-\alpha})$, it follows that for each $\alpha \in (0,1)$, there exists a fuzzy anti-bounded linear functional say $f_\alpha \in U_{1-\alpha}^*$ which is an extension of f such that

$$\| f_\alpha \|_{1-\alpha}^* = \| f \|_\alpha^{*^*} \qquad (1)$$

Now,

$$\nu'(f, \| f_\alpha \|_{1-\alpha}^*) = \wedge \{ 1-\beta \in (0,1) : \| f \|_\beta^{*^*} \leq \| f_\alpha \|_{1-\alpha}^* \} \text{ for } f \neq \theta.$$

$$\Rightarrow \nu'(f, \| f_\alpha \|_{1-\alpha}^*) \leq 1-\alpha \quad \big(\text{by } (1)\big).$$

Corollary 3.7 If f is weakly fuzzy anti-bounded linear functional then the Theorem 3.6 also holds. Since, f is weakly fuzzy anti-bounded implies that f is bounded with respect to the fuzzy α-anti-norms of ν for each $\alpha \in (0,1)$.

Theorem 3.8 Let (U, ν) be a fuzzy anti-normed linear space satisfying (vi) and (vii) and $x_0 (\neq \theta) \in U$. Then for each $\alpha \in (0,1)$, there exists $f_\alpha \in U^*_{1-\alpha}$ such that $\| f_\alpha \|^*_{1-\alpha} = 1$ and $f_\alpha(x_0) = \| x_0 \|^*_{1-\alpha}$.

Proof. Since (U, ν) be a fuzzy anti-normed linear space satisfying (vi) then $(U, \| \cdot \|^*_{1-\alpha})$ is a fuzzy anti-normed linear space for each $\alpha \in (0,1)$. For each $\alpha \in (0,1)$, by Hahn-Banach theorem over the normed linear space $(U, \| \cdot \|^*_{1-\alpha})$ there exists $f_\alpha \in U^*_{1-\alpha}$ such that $\| f_\alpha \|^*_{1-\alpha} = 1$ and $f_\alpha(x_0) = \| x_0 \|^*_{1-\alpha}$.

Theorem 3.9 Let (U, ν) be a fuzzy anti-normed linear space satisfying (vi) and (vii) then for $x_0 (\neq \theta) \in U$

$$N\left(x, \wedge \frac{|f(x)|}{\| f(x) \|^*_{1-\alpha}} \right) \leq \alpha \qquad \alpha \in (0,1), f \in U^*_{1-\alpha}, f \neq \theta.$$

Proof. Since (U, ν) be a fuzzy anti-normed linear space satisfying (vi) then $(U, \| \cdot \|^*_{1-\alpha})$ is a fuzzy anti-normed linear space for each $\alpha \in (0,1)$. Now by applying the Hahn-Banach theorem over the normed linear space $(U, \| \cdot \|^*_{1-\alpha})$ we have

$$\| x \|^*_{1-\alpha} = \wedge \frac{|f(x)|}{\| f(x) \|^*_{1-\alpha}} = s_\alpha \text{ (say)} \quad \forall f \in U^*_{1-\alpha}, \forall f \neq \theta, \forall \alpha \in (0,1) \qquad (2)$$

Hence for $x \neq \theta$ we have

$$\nu(x, s_\alpha) = \wedge \{ 1-\beta \in (0,1) : \| x \|^*_\beta \leq s_\alpha \} \Rightarrow \nu(x, t_\alpha) \leq \alpha \text{ ,(by (1))}$$

$$\text{i.e., } \nu\left(x, \wedge \frac{|f(x)|}{\| f(x) \|^*_{1-\alpha}} \right) \leq \alpha \qquad \alpha \in (0,1), \quad f \in U^*_{1-\alpha}, \quad f \neq \theta.$$

IV. Open mapping theorem

Open mapping theorem is one of the fundamental theorem for the theory of fuzzy anti-normed linear space.

Definition 4.1 A mapping $T : (U, \nu') \to (V, \nu'')$, where (U, ν') and (V, ν'') are fuzzy anti-normed linear spaces, is said to be an open map if it maps an open set to an open set.

Definition 4.2 A fuzzy anti-normed linear space (V, ν) is said to be complete if every fuzzy anti-cauchy sequence in V converges to an element of V.

Definition 4.3 A complete fuzzy anti-normed linear space is called fuzzy anti-Banach space.

Definition 4.4 Let, $0 < r < 1$, $t \in \mathbf{R}^+$ and $x \in V$. Then the set

$$B(x, r, t) = \{ y \in V : \nu(x-y, t) < r \}$$

is called an **open ball** in (V, A^*) with x as its center and r as its radious with respect to t.

Theorem 4.5 (Open mapping theorem): If T is a continuous linear operator from the fuzzy anti-Banach space (U, ν') onto the fuzzy anti-Banach space (V, ν'') then T is an open mapping.

Proof. Let, E be a neighborhood of θ in X. Then it can be shown that $\theta \in (\overline{T(E)})^\circ$. Since $\theta \in E$ and E is an open set, then there exist $0 < \alpha < 1$ and $t_0 \in (0, \infty)$ such that $B(\theta, \alpha, t_0) \subset E$ and for $0 < \alpha < 1$ a sequence

$\{\varepsilon_n\}_n$ can be found such that $\varepsilon_n \to 0$ and $\lim_{n\to\infty} \varepsilon_1 \diamond \varepsilon_2 \diamond \cdots \diamond \varepsilon_n < \alpha$. Now, $\theta \in T(\overline{B(\theta, \varepsilon_n, t_n')})$ where $t_n' = \frac{1}{2^n} t_0$. Therefore, there exist $\sigma_n \in (0,1)$ and $t_n > 0$ such that $B(\theta, \sigma_n, t_n) \subset T(\overline{B(\theta, \varepsilon_n, t_n')})$. Since the set $\{B(0, \frac{1}{n}, \frac{1}{n})\}$ has a countable local base at zero and $t_n' \to 0$ as $n \to \infty$, so t_n and σ_n can be chosen such that $t_n \to 0$ and $\sigma_n \to 0$ as $n \to \infty$.

It is shown that $B(\theta, \sigma_1, t_1) \subset (T(E))^\circ$. Suppose y_0 in $B(\theta, \sigma_1, t_1)$ then y_0 in $T(B(\theta, \varepsilon_1, t'))$ and so for $\sigma_2 > 0$ and $t_2 > 0$, the ball $B(\theta, \sigma_2, t_2)$ intersects $T(B(\theta, \varepsilon_1, t')$. Therefore there exists $x_1 \in B(\theta, \varepsilon_1, t')$ such that

$$T(x_1) \in B(\theta, \sigma_2, t_2) \qquad \text{i.e., } \nu''(y_0 - T(x_1), t_2) < \sigma_2$$

or equivalently $y_0 - T(x_1) \in B(\theta, \sigma_2, t_2) \subset T(B(\theta, \varepsilon_1, t'))$ and by the similar argument there exists x_2 in $B(\theta, \varepsilon_2, t')$ such that

$$\nu''(y_0 - (T(x_1) + T(x_2))), t_3) = \nu''((y_0 - T(x_1)) - T(x_2), t_3) < \sigma_3$$

If this process is continued, it leads to a sequence $\{x_n\}_n$ such that $x_n \in B(\theta, \varepsilon_n, t_n')$ and

$$\nu''(y_0 - \textstyle\sum_{j=1}^{n-1} T(x_j), t_n) < \sigma_n$$

Now if $n \in \mathbf{N}$ and $\{p_n\}_n$ is a positive increasing sequence then

$$\nu'(\textstyle\sum_{j=1}^{n} x_j - \sum_{j=1}^{n+p_n} x_j, t) = \nu'(\sum_{j=n+1}^{n+p_n} x_j, t)$$
$$\leq \nu'(x_{n+1}, t_1) \diamond \cdots \nu'(x_{n+p_n}, t_{p_n})$$

where $t_1 + \cdots + t_{p_n} = t$.

By putting $t_0 = \min\{t_1, \cdots, t_{p_n}\}$, since $t_n' \to 0$ therefore there exists n_0 such that $0 < t_n' \leq t_0$ for $n > n_0$. Thus,

$$\nu'(x_{n+1}, t_0) \diamond \cdots \nu'(x_{n+p}, t_{p_n}) \leq \nu'(x_{n+1}, t_{n+1}') \diamond \cdots \nu'(x_{n+p_n}, t_{n+p_n}')$$
$$\leq \varepsilon_{n+1} \diamond \cdots \diamond \varepsilon_{n+p_n}.$$

Therefore, $\lim_{n\to\infty} \nu'(\sum_{j=n+1}^{n+p_n} x_j, t) \leq \lim_{n\to\infty} [\varepsilon_{n+1} \diamond \cdots \diamond \varepsilon_{n+p_n}] = 0.$

That is, $\nu'(\sum_{j=n+1}^{n+p_n} x_j, t) \to 0$ for all $t > 0$. So, the sequence $\{\sum_{j=1}^{n} x_j\}_n$ is a cauchy sequence and consequently the series $\{\sum_{j=1}^{\infty} x_j\}_n$ converges to some point $x_0 \in U$ because U is complete. By fixing $t > 0$, there exist n_0 such that $t > t_n$ for $n > n_0$ because $t_n \to 0$. Therefore it follows that

$$\nu''(y_0 - T(\textstyle\sum_{j=1}^{n-1} x_j), t) \leq \nu''(y_0 - T(\sum_{j=1}^{n-1} x_j), t_n) \leq \sigma_n$$

and thus $\nu''(y_0 - T(\sum_{j=1}^{n-1} x_j), t) \to 0$. Hence $y_0 = \lim_{n\to\infty} T(\sum_{j=1}^{n-1} x_j) = T(x_0)$. But, $\nu'(x_0, t_0) \leq \lim_{n\to\infty} \inf \nu'(\sum_{j=1}^{n-1} x_j, t_0) \leq \lim_{n\to\infty} \inf [\nu'(x_1, t_1') \diamond \cdots \diamond \nu'(x_n, t_n')]$

$$\leq \lim_{n\to\infty} \inf [\varepsilon_1 \diamond \cdots \diamond \varepsilon_n] = \alpha$$

Hence $x_0 \in B(\theta, \alpha, t_0)$. Let G be an open subset of U and $x_0 \in G$. Then

$$T(G) = T(x_0 + (-x_0) + G) = T(x_0) + T(-x_0 + G) \supset T(x_0) + (T(-x_0 + G))^{\circ}$$

Since x_0 is arbitrary, $T(G)$ is open because it includes each of its points.

Hence the proof.

V. Closed Graph Theorem

Although closed graph theorem is derived from open mapping theorem, it some time more useful than open mapping theorem.

Definition 5.1 Let (U, ν') and (V, ν'') be two fuzzy anti-normed linear space over the same field F and $T : (U, \nu') \to (V, \nu'')$. If the graph of T, $G_T = \{(x, T(x)) : x \in U\}$ is closed, then T is called a closed graph.

Theorem 5.2 (Closed graph theorem): Let (P_1, ν') and (P_2, ν'') be two fuzzy anti-Banach spaces. If T is a linear transformation from P_1 to P_2 then T is fuzzy anti-continuous if and only if the graph of T is closed.
To prove the theorem we use the following lemmas.

Lemma 5.3 A strongly fuzzy anti-bounded linear operator from the fuzzy anti-normed linear space (U, ν') to the fuzzy anti-normed linear space (V, ν'') is fuzzy anti-continuous.

Proof. Let T be strongly fuzzy anti-bounded. Then there exists $s \in (0, \infty)$ such that

$$\nu''(T(x), t) \leq \nu'(x, \tfrac{t}{s}) \quad \text{for every } x \in U.$$

Given $t > 0$, $\varepsilon \in (0,1)$, choose $s_x \in (0, \varepsilon)$ and $t_x = \frac{t}{s_x}$.

Whenever $\nu'\left(x - y, \frac{t}{s_x}\right) < s_x$ we have $\nu''(T(x-y), t) \leq \nu'\left(x - y, \frac{t}{s_x}\right) < s_x < \varepsilon$.

i.e., $\nu'(x - y, t_x) < s_x \Rightarrow \nu''(T(x) - T(y), t) < \varepsilon$

$\Rightarrow T$ is fuzzy anti-continuous at $x \in U$. Since $x \in U$ is arbitrary T is fuzzy anti-continuous on U.

Lemma 5.4 If T is a fuzzy anti-continuous linear transformation from a fuzzy anti-normed linear space (U, ν') to a fuzzy anti-normed linear space (V, ν'') and $\{x_n\}_n$ is a sequence in U converging to x in U then $\{T(x_n)\}_n$ converging to $T(x)$ in V.

Proof. Since T is fuzzy anti-continuous we have, for any given $\varepsilon > 0$, $t > 0$ there exist $\delta > 0$, $t' > 0$ such that

$$\nu'(x - y, t') < \delta \Rightarrow \nu''(T(x) - T(y), t) < \varepsilon \quad \forall\, x \in U.$$

Since $x_n \to x$ there exists a positive integer n_0 such that $\nu'(x_n - x, t') < \delta \quad \forall\, n \geq n_0$.

By fuzzy anti-continuity of T, we have

$$\nu'(x_n - x, t') < \delta \Rightarrow \nu''(T(x_n) - T(x), t) < \varepsilon \quad \forall\, n \geq n_0$$
$$\Rightarrow T(x_n) \to T(x).$$

Proof of the main theorem:

$\Rightarrow$ **part:** Suppose T is fuzzy anti-continuous. We have to show that the graph of T (denoted by G_T) is closed. It is enough to show that $\overline{G_T} \subset G_T$.

Let $(x, y) \in \overline{G_T}$. This means that there exist a sequence $\{(x_n, T(x_n))\}_n$ in G_T converging to (x, y). This implies $x_n \to x$ and $T(x_n) \to y$. But since, T is fuzzy anti-continuous by lemma 5.3 we have $x_n \to x \Rightarrow T(x_n) \to T(x)$. From this we get $y = T(x)$. Thus,

$$(x,y) = (x,T(x)) \in G_T \quad \text{i.e., } \overline{G_T} \subset G_T$$

⇐ **part:** Suppose that G_T is closed. We have to show that T is fuzzy anti-continuous.
Let (P,ν) denotes another fuzzy anti-normed linear space with ν defined by $\nu(x,2t) = \nu'(x,t) \diamond \nu''(T(x),t)$. We can easily verified that ν is a fuzzy anti-norm and with respect to this fuzzy anti-norm P is a fuzzy anti-Banach space.
Now, we prove that the linear transformation $T : (P,\nu) \to (P_2,\nu'')$ is strongly fuzzy anti-bounded.

$$\begin{aligned}\nu''(T(x),t) &= 0 \diamond \nu''(T(x),t)\\ &\leq \nu'(x,t) \diamond \nu''(T(x),t)\\ &= \nu(x,2t) = \nu(x,\tfrac{t}{\frac{1}{2}})\end{aligned}$$

Hence T is strongly fuzzy anti-bounded.
Therefore, T is fuzzy anti-continuous (by lemma 5.3).
To prove $T : (P_1,\nu') \to (P_2,\nu'')$ is fuzzy anti-continuous it is enough to show that (P_1,ν') and (P,ν) are homeomorphic to each other.
Consider the identity map $I : (P,\nu) \to (P_1,\nu')$ defined by $I(x) = x$.

$$\begin{aligned}\nu'(I(x),t) &= 0 \diamond \nu'(I(x),t)\\ &\leq \nu''(x,t) \diamond \nu'(I(x),t)\\ &= \nu(x,2t) = \nu(x,\tfrac{t}{\frac{1}{2}})\end{aligned}$$

Hence T is strongly fuzzy anti-bounded.
Therefore, $I : (P,\nu) \to (P_1,\nu')$ is fuzzy anti-continuous. Hence by open mapping theorem I is an open mapping. This implies that $I : P \to P_1$ is a homeomorphism i.e., P_1 and P are homeomorphic to each other. Therefore $T : (P_1,\nu') \to (P_2,\nu'')$ is fuzzy anti-continuous.
Hence the proof.

VI. References

1) Bag T, Samanta SK. *Finite dimensional fuzzy normed linear space*, The J.Fuzzy Mathematics (2003); 11(3): 687 - 705.
2) Bag T, Samanta SK. *Fuzzy bounded linear operators* , Fuzzy Sets and Systems (2005); 151: 513 - 547.
3) Barro S, Martin R. *Fuzzy logic in medicine*, Heidelberg: Physica-Verlag:2002.
4) Barros LC, Bassanezi RC, Tonelli PA. *Fuzzy modelling in population dynamics*, Ecol Model (2000); 128: 27 - 33.
5) Cheng SC, Mordeson JN. *Fuzzy Linear Operators and Fuzzy Normed Linear Spaces*, Bull. Cal. Math. Soc. (1994); 86: 429 - 436.
6) Dinda B, Samanta TK. *Intuitionistic Fuzzy Continuity and Uniform Convergence* , Int. J. Open Problems Compt.Math.,(2010);3(1): 8-26.
7) Dinda B, Samanta TK, Jebril IH. *Fuzzy Anti-norm and Fuzzy α-anti-convergence* (Comunicated).
8) Dinda B, Samanta TK, Jebril IH. *Fuzzy Anti-bounded Linear Operators* (Comunicated).
9) Felbin C. *The completion of fuzzy normed linear space*, Journal of mathmatical analysis and application (1993); 174(2): 428-440.
10) Felbin C. *Finite dimentional fuzzy normed linear space*, Journal of analysis (1999); 7: 117-131.
11) Jebril IH, Samanta TK. *Fuzzy anti-normed linear space*, Journal of mathematics and Technology, February,(2010) 66 - 77.
12) Katsaras AK. *Fuzzy topological vector space*, Fuzzy Sets and Systems(1984); 12: 143 - 154.
13) [13] Kramosil O, Michalek J. *Fuzzy metric and statisticalmetric spaces*, Kybernetica (1975); 11: 326 - 334.
14) Samanta TK, Jebril IH. *Finite dimentional intuitionistic fuzzy normed linear space*, Int. J. Open Problems Compt. Math., (2009); 2(4): 574-591.
15) Schweizer B, Sklar A. *Statistical metric space*, Pacific journal of mathhematics (1960); 10: 314-334.
16) Vijayabalaji S, Thillaigovindan N, Jun YB. *Intuitionistic Fuzzy n-normed linear space* , Bull. Korean Math. Soc. (2007); 44: 291 - 308.
17) Zadeh LA. *Fuzzy sets*, Information and control (1965); 8: 338-353.

The Lists of Plant Synonyms in De materia medica of Dioscorides

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***Abstract*-The purpose of this research is to formulate viable answers to the most important questions surrounding the lists of plant synonyms appearing in some editions of Dioscorides' De materia medica: Who included these synonyms into Dioscorides' work, when and why did this happen, and which is the most probable source for these lists?**

I. Introduction: About Dioscorides and His De Materia Medica

Dioscorides was a Cilician Greek born in Anazarbos (or Anazarba) and therefore known as Dioscorides Anazarbeus. When Greeks worked for Romans, they adopted the name of a Roman gens: Dioscorides took the name of the gens Pedania, thus calling himself Dioscorides Pedanios Anazarbeus.

It is most likely that he lived somewhere around the middle of the first century A.D., during the reigns of the emperors Claudius and Nero. Dioscorides was a learned physician, who probably studied in Tarsos and Alexandria, the capitals of science in those times. It is believed that in Tarsos he was taught by Areios , to whom he dedicated his De materia medica.

Dioscorides practiced medicine as a physician of the Roman army, serving under Claudius and Nero, or under Nero and Vespasian . As such, he had the opportunity to accumulate extensive knowledge regarding the flora of foreign countries and it is generally believed that his interest in plants started to develop at a young age .

In De materia medica, a work consisting of five volumes, Dioscorides shortly describes the aspect, mentions the occurrence and points out the properties and effects of about 700 plants and vegetal products, more than anyone before him . It is for the first time in botanical history that the succession of remedies follows practical and pharmacological criteria.

Dioscorides' work remained the standard medical botany and pharmacology of the West until Renaissance. Starting with the 5th century, De materia medica was translated into Latin, Syrian, and Arabic. The Greek version was printed in 1499 in Venice by Aldus Manutius and afterwards five more times in the 16th century. This famous herbal has been translated into many languages during Renaissance, and it has exercised its influence on drug therapy as late as the beginning of the 19th century .

II. The Presence of Synonyms in Some De Materia Medica Manuscripts

Difficulties in identifying the plants of Dioscorides' herbal have led to the preparation of copies of his work, provided with pictures of plants (e.g. the Juliana Anicia MS. of 515 at Vienna, the earliest surviving complete Greek herbal) . Another change that was made to the original version was the alphabetic rearrangement of the succession of remedies for educational purposes and for easier consultation . The alphabetic

It is interesting though that Dioscorides is scarcely familiar with the flora growing in regions where the Roman army was mainly stationed, i.e. along the Rhine and Danube, in Spain or Northern Africa. Vivian Nutton (2004, 175) believes Dioscorides may have served in Syria, Egypt or Armenia

Dioscorides himself reveals this aspect of his life in the preface to De materia medica (see also Singer, 1927, 19).

Berendes (1902), 1-12; Morton (1981), 67-68; Greene (1983), 218-223.

Touwaide (2000), 464.

The Oxford Classical Dictionary, s.v. Dioscorides Pedanius and s.v. Botany, pgf. 6-8.

In the Preface to his work, Dioscorides tells us that some of his peers have used the alphabetical arrangement, which he does not find appropriate: "[…] Moreover, they have offended in the classification of medicines: some

versions that were abbreviated or completed as needed, contain some addings of exceptional value, i.e. the Latin and often also Gallic, Etruscan, Sicilian, Dacian, Dardanian, Egyptian, Spanish, African, Armenian, and Syrian synonyms of the plants .

There are no such detailed lists of synonyms in the best manuscripts of the so-called "genuine" Dioscorides (Parisinus 2179, Laurentianus LXXIV, 23). In the Vaticano-Palatinus 77, the oldest pages contain synonyms written on their margins. Wellmann believes that the source of these lists was most likely the work of Pamphilos (end of the 1st century A.D.), and that at least some of the 12 Latin synonyms (with Greek endings) found in this "genuine" Dioscorides are original .

III. Opinions on the Origin of the Synonyms

There are three different opinions regarding the provenience of the synonyms:

1.All synonyms are original, having been added to the text by Dioscorides himself.

2.Only some of the synonyms are original, the others were added later on.

3.All synonyms have been added to the text by peers of the author or later copyists.

About-Eduard Nebelthau Gymnasium in Bremen-Lesum, Germany

The first assumption is found in just a few scholars, of which the most representative appears to be Kurt Sprengel. In the preface to his Dioscorides-edition as well as in his History

couple together those of quite contrary faculties, others follow an alphabetical arrangement in their writing, and have separated both the kinds and the operations of things that are closely related, so that thereby they come to be harder to remember." (Translation of J. Goodyer, in R. T. Gunther, The Greek Herbal of Dioscorides, 1933 and 1959).

Wellmann (1898), 363-364.

Wissowa (1905), V, cols. 1138-1139; Wellmann (1898), 364-365.

Sprengel (1829-1830), XVI. See also comments on this preface in Meyer (1855), II, 102-107; Berendes (1902), 5-7; Tomaschek (1975), 152.

of Botany , Sprengel claims that Dioscorides himself would have gathered the various synonyms by means of the good relationship of the Romans to other peoples. He also believed that the many Celtic and Old-Thracian phytonymes should have been familiar to Dioscorides, because given the fact that Cilicia was his country of birth, his first language consisted of rests of Thracian mixed up with Celtic language. Therefore, the pure Attic dialect should have been foreign to him.

Most researchers believe that only the Greek and some of the Latin synonyms are original, and that the rest of synonyms were taken either from the work of Apuleius Platonicus (Pseudo-Apuleius) or from Xenokrates and/or Pamphilos, and have been introduced into De materia medica either during the 3rd century or in the period between the 5th and the 7th centuries A.D. In the following, I will review the most relevant evidence sustaining these theses .

According to scientific tradition, Lambeck (or Lambecius) was the first scholar, who believed that the synonyms were added to Dioscorides' work later on, and that they were most probably copied from the work of Pamphilos.

As previously mentioned, there were also other possible sources for the synonyms. Ackermann thinks they were borrowed from Apuleius Platonicus, whereas Berendes considers beside Pamphilos also the work of Xenokrates as a potential source for the synonyms.

In his Geschichte der Botanik , Meyer excludes the hypothesis according to which Dioscorides alone would have gathered all the synonyms, and he concludes that their provenience should be the registers of phytonymes. But, in his opinion, it is impossible to decide whether the

Sprengel (1817), I, 135-136.

For the complete and, at the same time, chronological review see Meyer (1855), II, 102-107 and Váczy (1969), 116-118.

Tomaschek (1975), 152.

Fabricii Bibliotheca graeca (1795), IV, 681.

(1902), Einleitung zu De materia medica, 5-7.

(1855), II, 106-107.

synonyms were introduced from the start by Dioscorides himself or later on, by the copyists of his work.

The most thorough examination of the synonym-issue was undertaken by Wellmann in 1898. After analyzing the manuscripts, he came to the conclusion that the alphabetical rearrangement of the succession of plants in De materia medica (the so-called "alphabetic Dioscorides") must have taken place during the 3rd century A.D. Wellmann also tried to prove that the synonyms had been taken from the work of Pamphilos, and that the only "genuine" ones are those appearing in the "non-alphabetic" Dioscorides (considered to be the closest to the original manuscript, as shown above), i.e. the 12 Latin synonyms with Greek endings. Of these 12 Latin synonyms, Wellmann found that just 1-7 of them might have been introduced by Dioscorides himself.

In 1927, Charles Singer, while commenting on the large variety of Greek manuscripts of Dioscorides, referred also to the issue of the synonyms. He believes they could have been added to the text before the end of the 3rd century, and perhaps before the end of the 2nd . Singer too mentions the Alexandrian lexicographer Pamphilos as a possible source for the synonyms, of which he gives the full list: "African, 'Andreae medici', Armenian, 'Bessicum', Boeotian, Cappadocian, Dacian, 'Dardana', 'Democriti', Egyptian, Ethiopian, Gaulish, Spanish, 'Istrici', 'Lucanica', 'Marsum', Osthanis, 'Prophetae', Pythagoreans, Romans, Tuscans, and Zoroastrians".

Due to the "static character of mediaeval thought", these synonyms have been copied in Greek and Latin herbals as late as the 16th century, although the languages, to which they belonged, had been extinct for more than a thousand years. Singer also believes that those, who

[1] *363-375.*

[1] *Singer (1927), 22-24*

added the synonyms, wanted to show their knowledge of the market. "In the vast and polyglot Roman Empire", there were no scientific botanists, who would have been able to preserve a standard, so the presence of the synonyms was therefore justified .

Wellmann's opinion seems to have prevailed, because it is found even in more recent works. Riddle considers that the Latin synonyms have been given by Dioscorides himself, but recommends caution concerning the other ones: He believes that they were added to the text by someone else "sometime before the 5th century", that they increased the work's usefulness and certainly did not distract from Dioscorides' authority. The original name, the description, and the picture of the plant identified it well enough, Dioscorides' work having above all a practical purpose.

In 2004, Vivian Nutton clearly shows that due to the fact that the organization of De materia medica was difficult to follow, "some copyists reverted to familiar practice and rewrote the whole book with the substances in alphabetical order within the larger divisions. Others assembled lists of synonyms or added the names for the plants in a variety of languages, such as Dacian. All this helped Dioscorides and his herbal to become the bible of medical botany and to

exercise an enormous influence on pharmacology and botany well into the 17th century" .
After all, why would Dioscorides have used any other language but Greek? Even if, according to Sprengel, he might not have been familiar with the pure Attic dialect due to the fact that he grew up in Cilicia, in Rome he certainly had the opportunity to improve his knowledge of such, for in his time, the language of Roman medicine was Greek beyond any doubt. .

Singer (1927), 33.
(1985), 28.
Nutton (2004), 174-177.
Jackson (1988), 56-57; Scarborough (1969), 109-121.

Dioscorides could have been easily read by his peers as well as by educated Romans, who enjoyed bilingual education. Under these circumstances, any other synonyms, except for the Latin ones, would have been of little importance. Beside this, a list of synonyms for every plant name would have "imbalanced" the description, putting too much weight on the name. Since *De materia medica* was indeed exhaustive for those times, Dioscorides' successors could probably not have had much more to add to it beside these lists of names and the pictures of the plants, for easier identification

IV. The Preface to De Materia Medica

One main idea pervades the whole preface, i.e. the fact that the treatise was written by a specialist for his peers. Dioscorides dedicated *De materia medica* to his teacher, Areios, who, as shown above, is believed to be a famous pharmacologist of Tarsos.
There are three distinct parts in the preface to *De materia medica*:

1. A review of the positive and negative features of the works of Dioscorides' predecessors.
2. The merits of *De materia medica*, presented in contrast to the negative aspects of the previously mentioned works.
3. The most important principles to be followed in the gathering and storing of plants.

If the author himself had added any synonyms to his work, it is most likely that he would have mentioned this fact in part two of this preface. Such synonyms would have been a quite important feature of the treatise, and as such it would be hard to believe that Dioscorides could have forgotten to mention it as a useful particularity of his work. Therefore, even the originality of the 1-7 Latin synonyms, which Wellmann believes they were added by Dioscorides himself,

becomes questionable. Why would Dioscorides have added only these Latin synonyms, and not all of them?
On the other hand, the detailed description accompanied only by the Greek name of the plant could not have led to its doubtless identification. In his article on the semantics of Greek names for plants, R. M. Dawkins (1936) clearly demonstrates the fact that in Ancient Greece, the same name was often used for different plants. Thus, it is understandable that posterity considered it necessary to add drawings and/or synonyms to the text. And indeed, in the polyglot Roman Empire, the "internationalization" of such a valuable treatise like De materia medica by means of synonyms in various languages must have been a necessity.

IV. Conclusions

Dioscorides is the author of De materia medica, the standard medical botany and pharmacology of the West until Renaissance. Among the changes that posterity has performed on this famous work, is also the adding of the lists of plant synonyms in various languages. Their provenience and the period, in which they were added to De materia medica, have been subjected to debates especially during the 19th and 20th centuries.
Modern scholars believe that all synonyms were added to the text by Dioscorides' successors, and this research tried to focus on the evidence in favor of this opinion. The origin of the synonyms and the time frame, in which they became part of De materia medica, remain uncertain. Nevertheless, there is no doubt that in the polyglot Roman Empire, the use of synonyms became a necessity.
See bibliography

V. References

1) Berendes, J. (Ed). (1902). Des Pedanios Dioskurides aus Anazarbos Arzneimittellehre in fünf Büchern. Stuttgart: Enke.
2) Cohen, M. R., Drabkin, I. E. (1948). A Source Book in Greek Science. New York, Toronto, London: McGraw-Hill Book Co.
3) Dawkins, R. M. (1936). The Semantics of Greek Names for Plants. Journal of Hellenic Studies, 56/1, 1-11.
4) Iohannis Alberti Fabricii (1795). Bibliotheca Graeca, vol. IV. Hamburgi: Apud Carolum Ernestum Bohn.
5) Greene, E. L. (1983). Landmarks of Botanical History, part I. Stanford, CA: Stanford University Press.
6) Gunther, R. T. (Ed., 1933) (reprinted 1959). The Greek Herbal of Dioscorides. New York: Hafner Publishing Co.
7) Jackson, R. (1988). Doctors and Diseases in the Roman Empire. London: British Museum Publications.
8) Meyer, E. H. F. (1855). Geschichte der Botanik, vol. II. Königsberg: Bornträger.
9) Morton, A. G. (1981). History of Botanical Science: An Account of the Development of Botany from Ancient Times to the Present Day. London & New York: Academic Press.
10) Neuburger, M. (1906). Geschichte der Medizin, vol. I. Stuttgart: Enke.
11) Nutton, V. (2004). Ancient Medicine. London & New York: Routledge.

12) Riddle, J. M. (1985). Dioscorides on Pharmacy and Medicine, History of Science Series 3. Austin: University of Texas Press.
13) Scarborough, J. (1969). Roman Medicine. London & Southampton: Thames and Hudson.
14) Singer, Ch. (1927). The Herbal in Antiquity and its Transmission to Later Ages. Journal of Hellenic Studies, 47/1, 1-52
15) Sprengel, K. (Ed) (1829-1830). Ped. Dioskoridis Anazarbei de materia medica libri V. Lipsiae: Teubner.
16) Sprengel, K. (1817). Geschichte der Botanik, vol. 1. Altenburg: Brockhaus.
17) Tomaschek, W. (1893-1894 & 1975). Die alten Thraker. Eine ethnologische Untersuchung. Osnabrück: Zeller.
18) Touwaide, A. (2000). Pedanios Dioskurides. In H. Cancik & H. Schneider (Eds). Der Neue Pauly (Enzyklopädie der Antike), vol. IX, pp. 462-465. Stuttgart & Weimar: J.B. Metzler.
19) Váczy, C. (1969). Nomenclatura dacică a plantelor la Dioscorides şi Pseudo-Apuleius, part II. In Acta Musei Napocensis VI, 115-129.
20) Wellmann, M. (1898). Die Pflanzennamen des Dioskurides. Hermes 33, 360-422.
21) Wissowa, G. (Ed). (1905). Paulys Real-Encyclopädie der classischen Altertumswissenschaft, vol. V. Stuttgart: De Gruyter Saur

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Multidimensional Fractional Integral Operators Involving General Class Of Polynomial And $\overline{H}$-Function

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***Abstract*-In the present paper, we first define a pair of multidimensional fractional integral operators whose kernels involve the product of multivariable polynomial $S_V^{U_1,\ldots,U_k}(x_1,\ldots,x_k)$ and $\overline{H}$-function. First we obtain images of two useful functions in our operator of study. Next, we establish two theorems giving the multidimensional generalized Stieltjes transform of fractional integral operators and conversely, the fractional integrals of multidimensional generalized Stieltjes transform. Finally, we present results concerning Mellin transform, Mellin convolutions and inversion formulae for these operators. The fractional integral operators studied by us are quite general in nature and may be considered as extensions and unifications of a number of (known or new) results for simpler fractional integral operators.**

Keywords-Fractional integral operator, $\overline{H}$ - function, Mellin transform, Stieltjes transform, General class of multivariable polynomials.

I. Introduction

The multivariable polynomial $S_V^{U_1,\ldots,U_k}(x_1,\ldots,x_k)$ introduced by Srivastava and Garg (1987) [6, p. 686, eq. (1.4)] is defined in the following manner:

$$S_V^{U_1,\ldots,U_k}[x_1,\ldots,x_k] = \sum_{R_1,\ldots,R_k=0}^{\sum_{i=1}^{k}U_iR_i\le V} (-V)_{\sum_{i=1}^{k}U_iR_i}\, A(V,R_1,\ldots,R_k)\frac{x_i^{R_i}}{R_i!}$$

$$V=0,1,2,\ldots \qquad (1.1)$$

Where $U_1,\ldots,U_k$ are arbitrary positive integers and the coefficients $A(V,R_1,\ldots,R_k)$ are arbitrary constants (real or complex).

The $\overline{H}$-function will be defined and represented in the following manner

$$\overline{H}_{P,Q}^{M,N}[z] = \overline{H}_{P,Q}^{M,N}\left(z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right)$$

$$=\frac{1}{2\pi i}\int_L \bar{\phi}(\xi)z^{\xi}d\xi \qquad (z\neq 0) \qquad (1.2)$$

where

$$\bar{\phi}(\xi)=\frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \qquad (1.3)$$

The following sufficient conditions for the absolute convergence of the defining integral for $\overline{H}$ - Function given by (1.2) have been recently given by Gupta, Jain and Agrawal (2007) [3].

(i) $|\arg(z)|<1/2\Omega\pi \text{ and } \Omega>0$

(ii) $|\arg(z)|=1/2\Omega\pi \text{ and } \Omega\ge 0$ (1.4)

and (a) $\mu\neq 0$ and the contour L is so chosen that $(c\mu+\lambda+1)<0$

(b) $\mu=0 \text{ and } (\lambda+1)<0$

where

$$\Omega=\sum_1^M\beta_j+\sum_1^N\alpha_jA_j-\sum_{M+1}^Q\beta_jB_j-\sum_{N+1}^P\alpha_j \qquad (1.5)$$

$$\mu=\sum_1^N\alpha_jA_j+\sum_{N+1}^P\alpha_j-\sum_1^M\beta_j-\sum_{M+1}^Q\beta_jB_j \qquad (1.6)$$

$$\lambda=\mathrm{Re}\left(\sum_1^Mb_j+\sum_{M+1}^Qb_jB_j-\sum_1^Na_jA_j-\sum_{N+1}^Pa_j\right)+\frac{1}{2}\left(-M-\sum_{M+1}^QB_j+\sum_1^NA_j+P-N\right) \qquad (1.7)$$

It may be noted that the conditions of validity given above are more general than those given earlier by Buschman (1990)[1].

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The following series representation of the $\overline{H}$ -function was given by Rathie (1997) [4].

$$\overline{H}_{P,Q}^{M,N}\left(z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right)$$

$$=\sum_{\upsilon=1}^{M}\sum_{P=0}^{\infty}\overline{\theta}\left(S_{P,\upsilon}\right)z^{S_{P,\upsilon}} \qquad (1.8)$$

where

$$\overline{\theta}(S_{P,\upsilon})=\frac{\prod_{j=1,j\neq\upsilon}^{M}\Gamma(b_j-\beta_jS_{P,\upsilon})\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_jS_{P,\upsilon})\}^{A_j}(-1)^P}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_jS_{P,\upsilon})\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_jS_{P,\upsilon})P!\beta_\upsilon},\quad S_{P,\upsilon}=\frac{b_\upsilon+P}{\beta_\upsilon} \qquad (1.9)$$

To the sequel, we shall also, make use of the following behavior of the $\overline{H}$ -function for small and large value of z as recorded by Saxena (2002) [5, p.112, eqs. (2.3) and (2.4)].

$$\overline{H}_{P,Q}^{M,N}[z]=O\left[|z|^{\alpha}\right] \text{ for small z, where}$$

$$\alpha=\min_{1\le j\le M}\left[\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)\right] \qquad (1.10)$$

$$\overline{H}_{P,Q}^{M,N}[z]=O\left[|z|^{\beta}\right] \text{ for large z, where}$$

$$\beta=\max_{1\le j\le N}\left[\mathrm{Re}\left(\frac{a_j-1}{\alpha_j}\right)\right] \qquad (1.11)$$

and the conditions (1.4) are satisfied.

II. Multidimensional Fractional Integral Operators.

In the present paper we study the following fractional integral operators.

$$I_x\left[f(t_1,...,t_s)\right]=I_{x;U,V;Z}^{\rho,\sigma;e,f;\eta,\lambda}\left[f(t_1,...,t_s);x_1,...,x_s\right]$$

$$=\left(\prod_{j=1}^{s}x_j^{-\rho_j-\sigma_j}\right)\int_0^{x_1}...\int_0^{x_s}\left[\prod_{j=1}^{s}t_j^{\rho_j}(x_j-t_j)^{\sigma_j-1}\right]S_V^{U_1,...,U_s}\left[E_1\left(\frac{t_1}{x_1}\right)^{e_1}\left(1-\frac{t_1}{x_1}\right)^{f_1},...,E_s\left(\frac{t_s}{x_s}\right)^{e_s}\left(1-\frac{t_s}{x_s}\right)^{f_s}\right]$$

$$\times\overline{H}_{P,Q}^{M,N}\left[Z\prod_{j=1}^{s}\left(\frac{t_j}{x_j}\right)^{\eta_j}\left(1-\frac{t_j}{x_j}\right)^{\lambda_j}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]f(t_1,...,t_s)dt_1...dt_s \qquad (1.12)$$

$$J_x\left[f(t_1,...,t_s)\right]=J_{x;U,V;Z}^{\rho,\sigma;e,f;\eta,\lambda}\left[f(t_1,...,t_s);x_1,...,x_s\right]$$

$$=\left(\prod_{j=1}^{s}x_j^{\rho_j}\right)\int_{x_1}^{\infty}...\int_{x_s}^{\infty}\left[\prod_{j=1}^{s}t_j^{-\rho_j-\sigma_j}(t_j-x_j)^{\sigma_j-1}\right]S_V^{U_1,...,U_s}\left[E_1\left(\frac{x_1}{t_1}\right)^{e_1}\left(1-\frac{x_1}{t_1}\right)^{f_1},...,E_s\left(\frac{x_s}{t_s}\right)^{e_s}\left(1-\frac{x_s}{t_s}\right)^{f_s}\right]$$

$$\overline{H}_{P,Q}^{M,N}\left[Z\prod_{j=1}^{s}\left(\frac{x_j}{t_j}\right)^{\eta_j}\left(1-\frac{x_j}{t_j}\right)^{\lambda_j}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]f(t_1,...,t_s)dt_1...dt_s \qquad (1.13)$$

Throughout the paper we assume that

$$f(t_1,...,t_s)=\begin{cases}O\prod_{j=1}^{s}\left(|t_j|^{U_j}\right) & \max\{|t_j|\}\to 0\\ O\prod_{j=1}^{s}\left(|t_j|^{-V_j}e^{-W_j|t_j|}\right) & \min\{|t_j|\}\to\infty\end{cases}\quad j=1,...,s$$

$$f(t_1,...,t_s)=\begin{cases}O\prod_{j=1}^{s}\left(|t_j|^{U_j}\right) & \max\{|t_j|\}\to 0\\ O\prod_{j=1}^{s}\left(|t_j|^{-V_j}e^{-W_j|t_j|}\right) & \min\{|t_j|\}\to\infty\end{cases}\quad j=1,...,s \qquad (1.14)$$

Such a class of function will be represented symbolically as $f(t_1,...,t_s)\in A$.

We also assume that $\int...\int_{\Omega_s}|f(t_1,...,t_s)|dt_1...dt_s<\infty$ for every bounded s-dimensional region Ω_s excluding the origin. The operators defined by (1.12) and (1.13) exists if $\min\mathrm{Re}(e_j,f_j,\eta_j,\lambda_j)\ge 0\quad(j=1,...,s)$ not all zero simultaneously.

$$\left.\begin{aligned}&\min_{1\le k\le M}\mathrm{Re}\left[1+\rho_j+U_j+\eta_j\frac{b_k}{\beta_k}\right]>0,\\&\min_{1\le k\le M}\mathrm{Re}\left[\sigma_j+\lambda_j\frac{b_k}{\beta_k}\right]>0\\&\mathrm{Re}(W_j)=0,\min_{1\le k\le M}\mathrm{Re}\left[\rho_j+V_j+\eta_j\frac{b_k}{\beta_k}\right]>0,\\&\min_{1\le k\le M}\mathrm{Re}\left[\sigma_j+\lambda_j\frac{b_k}{\beta_k}\right]>0\ \text{or}\\&\mathrm{Re}(W_j)>0,\min_{1\le k\le M}\mathrm{Re}\left[\sigma_j+\lambda_j\frac{b_k}{\beta_k}\right]>0\end{aligned}\right\} \qquad (1.15)$$

II *Some Useful Images*

Now we shall obtain the images of some useful functions in our operators of study.

$$I_x\left[\prod_{j=1}^{s}t_j^{\gamma_j}(h_j+t_j)^{-\delta_j}\right]=\left(\prod_{j=1}^{s}x_j^{\gamma_j}(x_j+h_j)^{-\delta_j}\right)$$

$$\sum_{j=1}^{s}U_jR_j\le V\ \sum_{R_1,...,R_s=0}^{\infty}(-V)_{\sum_{j=1}^{s}U_jR_j}A(V,R_1,...,R_s)\frac{E_1^{R_1}}{R_1!}...\frac{E_s^{R_s}}{R_s!}$$

$$\sum_{l=0}^{\infty}\left[\prod_{j=1}^{s}x_j^l(x_j+h_j)^{-l}(\delta_j)_l\right]$$

$$\overline{H}_{P+2s,Q+s}^{M,N+2s}\left[Z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\prod_{j=1}^{s}\{(-\rho_j-\gamma_j-e_jR_j;\eta_j;1)(1-\sigma_j-l-f_jR_j;\lambda_j;1)\}\\\prod_{j=1}^{s}\{1-\rho_j-\gamma_j-\sigma_j-l-(e_j+f_j)R_j;(\eta_j+\lambda_j);1\}\end{matrix}\right] \qquad (2.1)$$

where $\min\mathrm{Re}(e_j,f_j,\eta_j,\lambda_j)\ge 0\quad(j=1,...,s)$ not all zero simultaneously,

$$\min_{1\le k\le M}\operatorname{Re}\left[1+\rho_j+\gamma_j+\eta_j\frac{b_k}{\beta_k}\right]>0;$$

$$\min_{1\le k\le M}\operatorname{Re}\left[\sigma_j+\lambda_j\frac{b_k}{\beta_k}\right]>0$$

.

$$J_x\left[\prod_{j=1}^{s}t_j^{\gamma_j}\left(h_j+t_j\right)^{-\delta_j}\right]=\left(\prod_{j=1}^{s}x_j^{\gamma_j}\left(x_j+h_j\right)^{-\delta_j}\right)$$

$$\sum_{R_1,\ldots,R_s=0}^{\sum_{j=1}^{s}U_jR_j\le V}(-V)_{\sum_{j=1}^{s}U_jR_j}\,A(V,R_1,\ldots,R_s)\frac{E_1^{R_1}}{R_1!}\ldots\frac{E_s^{R_s}}{R_s!}$$

$$\sum_{l=0}^{\infty}\left[\prod_{j=1}^{s}x_j^l h_j^l\left(x_j+h_j\right)^{-l}\left(\delta_j\right)_l\right]$$

$$\overline{H}^{M,N+2s}_{P+2s,Q+s}\left[Z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right.$$

$$\left.\left.\begin{matrix}\prod_{j=1}^{s}\left\{\left(1-\rho_j+\gamma_j-\delta_j-e_jR_j;\eta_j;1\right)\left(1-\sigma_j-l-f_jR_j;\lambda_j;1\right)\right\}\\\prod_{j=1}^{s}\left\{1-\rho_j-\delta_j-\sigma_j-l+\gamma_j-\left(e_j+f_j\right)R_j;\left(\eta_j+\lambda_j\right);1\right\}\end{matrix}\right.\right]$$

(2.2)

$$\left.\left.\begin{matrix}\prod_{j=1}^{s}\left\{\left(1-\rho_j+\gamma_j-\delta_j-e_jR_j;\eta_j;1\right)\left(1-\sigma_j-l-f_jR_j;\lambda_j;1\right)\right\}\\\prod_{j=1}^{s}\left\{1-\rho_j-\delta_j-\sigma_j-l+\gamma_j-\left(e_j+f_j\right)R_j;\left(\eta_j+\lambda_j\right);1\right\}\end{matrix}\right.\right]$$

where $\min\operatorname{Re}(e_j,f_j,\eta_j,\lambda_j)\ge 0\quad(j=1,\ldots,s)$ not all zero simultaneously,

$$\min_{1\le k\le M}\operatorname{Re}\left[\rho_j-\gamma_j+\delta_j+\eta_j\frac{b_k}{\beta_k}\right]>0;\ \min_{1\le k\le M}\operatorname{Re}\left[\sigma_j+\lambda_j\frac{b_k}{\beta_k}\right]>0$$

$$I_x\left[\prod_{j=1}^{s}t_j^{\gamma_j}H\left(\begin{matrix}z_1\prod_{j=1}^{s}t_j^{u_j^{(1)}}\left[1+\left(h_jt_j\right)^{K_j}\right]^{-v_j^{(1)}}\\\vdots\\z_r\prod_{j=1}^{s}t_j^{u_j^{(r)}}\left[1+\left(h_jt_j\right)^{K_j}\right]^{-v_j^{(r)}}\end{matrix}\right)\right]$$

$$=\left(\prod_{j=1}^{s}x_j^{\gamma_j}\right)\sum_{R_1,\ldots,R_s=0}^{\sum_{j=1}^{s}U_jR_j\le V}(-V)_{\sum_{j=1}^{s}U_jR_j}\,A(V,R_1,\ldots,R_s)$$

$$\frac{E_1^{R_1}}{R_1!}\ldots\frac{E_s^{R_s}}{R_s!}\sum_{\upsilon=1}^{M}\sum_{P=0}^{\infty}\bar{\theta}\left(S_{P,\upsilon}\right)z^{S_{P,\upsilon}}\Gamma\left(\sigma_j+\lambda_jS_{P,\upsilon}+\sum_{j=1}^{s}f_jR_j\right)$$

$$H^{0,n+2s;m_1,n_1;\ldots;m_r,n_r;1,0;\ldots;1,0}_{p+2s,q+2s;p_1,q_1;\ldots;p_r,q_r;0,1;\ldots;0,1}\left[\left.\begin{matrix}z_1\prod_{j=1}^{s}x_j^{u_j^{(1)}}\\\vdots\\z_r\prod_{j=1}^{s}x_j^{u_j^{(r)}}\\\left(h_1x_1\right)^{K_1}\\\vdots\\\left(h_sx_s\right)^{K_s}\end{matrix}\right|\begin{matrix}A^*:C^*\\B^*:D^*\end{matrix}\right]$$

(2.3)

where

$$A^*=\left(a_k;\alpha_k^{(1)},\ldots,\alpha_k^{(r)},\underbrace{0,\ldots,0}_{2s}\right)_{1,p}\left(1;v_1^{(1)},\ldots,v_1^{(r)},1,\underbrace{0,\ldots,0}_{s-1}\right)$$

$$,\ldots,\left(1;v_s^{(1)},\ldots,v_s^{(r)},\underbrace{0,\ldots,0}_{s-1},1\right)\left(-\rho_1-\gamma_1-\eta_1S_{P,\upsilon}-\sum_{j=1}^{s}e_jR_{j,};\right.$$

$$\left.u_1^{(1)},\ldots,u_1^{(r)},k_1,\underbrace{0,\ldots,0}_{s-1}\right),\ldots,\left(-\rho_s-\gamma_s-\eta_sS_{P,\upsilon}\right.$$

$$\left.-\sum_{j=1}^{s}e_jR_{j,};u_s^{(1)},\ldots,u_s^{(r)},\underbrace{0,\ldots,0}_{s-1},k_s\right)$$

(2.4)

$$B^*=\left(b_k;\underbrace{0,\ldots,0}_{2s},\beta_k^{(1)},\ldots,\beta_k^{(r)},\right)_{1,q}\left(1;\underbrace{0,\ldots,0}_{s-1},v_1^{(1)},\ldots,v_1^{(r)}\right)$$

$$,\ldots,\left(1;\underbrace{0,\ldots,0}_{s-1}v_s^{(1)},\ldots,v_s^{(r)}\right)\left(-\rho_1-\gamma_1-\sigma_1-\left(\lambda_1+\eta_1\right)S_{P,\upsilon}\right.$$

$$\left.-\sum_{j=1}^{s}\left(e_j+f_j\right)R_{j,};u_1^{(1)},\ldots,u_1^{(r)},k_1,\underbrace{0,\ldots,0}_{s-1}\right),\ldots,\left(-\rho_s-\gamma_s-\sigma_s\right.$$

$$\left.-\left(\lambda_s+\eta_s\right)S_{P,\upsilon}-\sum_{j=1}^{s}\left(e_j+f_j\right)R_{j,};u_s^{(1)},\ldots,u_s^{(r)},\underbrace{0,\ldots,0}_{s-1},k_s\right)$$

(2.5)

$$C^*=\left(c_k^{(1)},\gamma_k^{(1)}\right)_{1,p_1};\ldots;\left(c_k^{(r)},\gamma_k^{(r)}\right)_{1,p_r}$$

$$D^*=(0,1);\ldots;(0,1);\left(d_k^{(1)},\delta_k^{(1)}\right)_{1,q_1};\ldots;\left(d_k^{(r)},\delta_k^{(r)}\right)_{1,q_r}$$

where $\bar{\theta}(S_{P,\upsilon}), S_{P,\upsilon}$ shall be obtained from (1.9) and $\underbrace{0,...,0}_{s}$ occurring on the right hand side of equation (2.4) and (2.5) would means s zeros and so on. Provided that $\min \operatorname{Re}(e_j, f_j, \eta_j, \lambda_j, u_j^{(i)}, v_j^{(i)}) \geq 0 (i=1,...,r,\ j=1,...,s)$ not all zero simultaneously,

$$\min_{1\leq k\leq M} \operatorname{Re}\left[1+\rho_j+\gamma_j+\eta_j\frac{b_k}{\beta_k}+\sum_{i=1}^{r}u_j^{(i)}\frac{d_k^{(i)}}{\delta_k^{(i)}}\right]>0$$

Proof: To prove (2.1), first of all we express the I-operator involved in its left hand side in the integral form with the help of equation (1.12). Next, we express $S_V^{U_1,...,U_k}(x_1,...,x_k)$ polynomials occurring therein in the series form using (1.1). Then, we change the order of the series and t_j -integrals and express the $\overline{H}$ -function in terms of Mellin Barnes type contour integrals with the help of (1.2). Now we change the order of ξ and t_j -integrals $(j=1,...,s)$ (which is permissible under the conditions stated). Finally, evaluating the t_j -integrals with the help of known result (Gradshteyn and Ryzhik (1980)) [2, p. 287, eq. 3.197(8)] we get

$$I_x\left[\prod_{j=1}^{s}t_j^{\gamma_j}\left(h_j+t_j\right)^{-\delta_j}\right]$$

$$=\sum_{R_1,...,R_s=0}^{\sum_{j=1}^{s}U_jR_j\leq V}(-V)_{\sum_{j=1}^{s}U_jR_j}A(V,R_1,...,R_s)\frac{E_1^{R_1}}{R_1!}...\frac{E_s^{R_s}}{R_s!}$$

$$\frac{1}{2\pi i}\int_L\bar{\theta}(\xi)Z^{\xi}\prod_{j=1}^{s}x_j^{\gamma_j}h_j^{-\delta_j}$$

$$B\left(\sigma_j+f_jR_j+\lambda_j\xi,\rho_j+\gamma_j+e_jR_j+\eta_j\xi+1\right)$$

$${}_2F_1\left[\begin{matrix}\delta_j,\rho_j+\gamma_j+e_jR_j+\eta_j\xi+1\\ \rho_j+\gamma_j+\sigma_j+\left(e_j+f_j\right)R_j+\left(\eta_j+\lambda_j\right)\xi+1\end{matrix};\frac{-x_j}{h_j}\right]$$

Where

$$\left|\arg\left(\frac{x_j}{h_j}\right)\right|<\pi,\ \operatorname{Re}\left(\sigma_j+f_jR_j+\lambda_j\xi\right)>0,$$

$$\operatorname{Re}\left(\rho_j+\gamma_j+e_jR_j+\eta_j\xi+1\right)$$

Now reinterpreting the result thus obtained in terms of the $\overline{H}$ -function, we easily arrive at the desired result after a little simplification.

Again the proof of result (2.2) can be developed similarly using the formula [Gradshteyn and Ryzhik (1980) 2, p.286, 3.197(2)]. Result (2.6) can be similarly established on expressing the $\overline{H}$ -function involved in the operator in its series form using (1.8).

III. Theorems connecting the multidimensional generalized Stieltjes transform and fractional integral operators

The multidimensional generalized Stieltjes transform of a function $\phi(t_1,...,t_s)$ is defined as

$$S_{w_1,...,w_s}(\phi)(h_1,...,h_s)$$

$$=\int_0^{\infty}...\int_0^{\infty}\phi(t_1,...,t_s)\prod_{j=1}^{s}\left\{\left(t_j+h_j\right)^{-w_j}\right\}dt_1,...,dt_s$$

provided that the integral exist.

The following theorem gives the multidimensional generalized Stieltjes transform of the generalized fractional operators given by (1.12) and (1.13).

Theorem 1. Let $\phi(t_1,...,t_s)\in A$,

$\min(e_j, f_j, \eta_j, \lambda_j)\geq 0 \quad (j=1,...,s)$ not all zero simultaneously and $\min_{1\leq k\leq M}\operatorname{Re}\left[\sigma_j+\lambda_j\frac{b_k}{\beta_k}\right]>0$,

Then for $\min_{1\leq k\leq M}\operatorname{Re}\left[\rho_j+\delta_j+\eta_j\frac{b_k}{\beta_k}\right]>0$

$$\min_{1\leq k\leq M}\operatorname{Re}\left[1+\rho_j+U_j+\eta_j\frac{b_k}{\beta_k}\right]>0,(j=1,...,s)$$

$$S_{w_1,...,w_s}(I_t\phi)(h_1,...,h_s)$$

$$=\int_0^{\infty}...\int_0^{\infty}\phi(x_1,...,x_s)\psi_1(x_1,...,x_s;h_1,...,h_s)\,dx_1,...,dx_s$$

and for

$$\min_{1\leq k\leq M}\operatorname{Re}\left[1+\rho_j+\eta_j\frac{b_k}{\beta_k}\right]>0,\ \operatorname{Re}(w_j)>0\ ,$$

$$\operatorname{Re}(w_j)=0\ \min_{1\leq k\leq M}\operatorname{Re}\left[\rho_j+V_j+\eta_j\frac{b_k}{\beta_k}\right]>0,(j=1,...,s)$$

$$S_{w_1,...,w_s}(J_t\phi)(h_1,...,h_s)$$

$$=\int_0^{\infty}...\int_0^{\infty}\phi(x_1,...,x_s)\psi_2(x_1,...,x_s;h_1,...,h_s)dx_1,...,dx_s$$

where

$$\psi_1(x_1,...,x_s;h_1,...,h_s)=J_x\left[\prod_{j=1}^{s}t_j^{\gamma_j}\left(h_j+t_j\right)^{-w_j}\right]$$

$$=\left(\prod_{j=1}^{s}\left(x_j+h_j\right)^{-w_j}\right)\sum_{R_1,...,R_s=0}^{\sum_{j=1}^{s}U_jR_j\leq V}(-V)_{\sum_{j=1}^{s}U_jR_j}A(V,R_1,...,R_s)$$

$$\frac{E_1^{R_1}}{R_1!}...\frac{E_s^{R_s}}{R_s!}\sum_{l=0}^{\infty}\left[\prod_{j=1}^{s}h_j^l\left(x_j+h_j\right)^{-l}\left(w_j\right)_l\right]$$

$$\overline{H}_{P+2s,Q+s}^{M,N+2s}\left[Z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\prod_{j=1}^{s}\left\{\left(1-\rho_j-w_j-e_jR_j;\eta_j;1\right)\left(1-\sigma_j-l-f_jR_j;\lambda_j;1\right)\right\}\\ \prod_{j=1}^{s}\left\{\left(1-\rho_j-w_j-\sigma_j-l-\left(e_j+f_j\right)R_j;\left(\eta_j+\lambda_j\right);1\right)\right\}\end{matrix}\right]$$

$$\psi_2(x_1,\ldots,x_s;h_1,\ldots,h_s)=I_x\left[\prod_{j=1}^{s}\left(h_j+t_j\right)^{-w_j}\right]$$

$$=\left(\prod_{j=1}^{s}\left(x_j+h_j\right)^{-w_j}\right)\sum_{R_1,\ldots,R_s=0}^{\sum_{j=1}^{s}U_jR_j\le V}(-V)_{\sum_{j=1}^{s}U_jR_j}A(V,R_1,\ldots,R_s)$$

$$\frac{E_1^{R_1}}{R_1!}\ldots\frac{E_s^{R_s}}{R_s!}\sum_{l=0}^{\infty}\left[\prod_{j=1}^{s}x_j^l\left(x_j+h_j\right)^{-l}\left(w_j\right)_l\right]$$

$$.\overline{H}^{M,N+2s}_{P+2s,Q+s}\left[Z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

$$\left.\begin{matrix}\prod_{j=1}^{s}\left\{\left(-\rho_j-e_jR_j;\eta_j;1\right)\left(1-\sigma_j-l-f_jR_j;\lambda_j;1\right)\right\}\\ \prod_{j=1}^{s}\left\{\left(-\rho_j-\sigma_j-l-\left(e_j+f_j\right)R_j;\left(\eta_j+\lambda_j\right);1\right)\right\}\end{matrix}\right] \quad (3.5)$$

It is assumed that the integrals on the right hand side of equations (3.2) and (3.3) exist.

Proof: To prove first part of theorem 1, we express the left hand side of (3.2) with the help of (1.12) and (3.1), then we interchange the order of t_j- and x_j integrals (which is permissible under the conditions stated with the theorem). Finally evaluating the inner t_j-integrals with the help of result (2.1) (taking $\gamma_j=0$ therein), we easily arrive at desired result after a little simplification.

Similarly the second result (3.3) of theorem1 can be established on using (2.2).

The following theorem gives the fractional integrals of generalized Stieltjes transform given by (3.1)

Theorem 2. Let $\phi(t_1,\ldots,t_s)\in A$,

$\min(e_j,\ f_j,\eta_j,\lambda_j)\ge 0\,(j=1,\ldots,s)$ not all zero

simultaneously $\mathrm{Re}(w_j)>0;\ \min \mathrm{Re}\left[\sigma_j+\lambda_j\frac{b_k}{\beta_k}\right]>0;$

$\mathrm{Re}(w_j)=0;\mathrm{Re}(V_j+w_j)>0$

then for $\min_{1\le k\le M}\mathrm{Re}\left[1+\rho_j+\eta_j\frac{b_k}{\beta_k}\right]>0\ ,\ (j=1,\ldots,s)$

$I_y(S_{w_1,\ldots,w_s}(\phi)(x_1,\ldots,x_s)$

$=\int_0^{\infty}\ldots\int_0^{\infty}\phi(t_1,\ldots,t_s)\psi_2(t_1,\ldots,t_s;x_1,\ldots,x_s)dt_1,\ldots,dt_s$

and for $\min_{1\le k\le M}\mathrm{Re}\left[\rho_j+w_j+\eta_j\frac{b_k}{\beta_k}\right]>0\ ,(j=1,\ldots,s)$

$J_y(S_{w_1,\ldots,w_s}\phi)(x_1,\ldots,x_s)$

$=\int_0^{\infty}\ldots\int_0^{\infty}\phi(t_1,\ldots,t_s)\psi_1(t_1,\ldots,t_s;x_1,\ldots,x_s)dt_1,\ldots,dt_s$

where $\psi_1(t_1,\ldots,t_s;x_1,\ldots,x_s)$ and $\psi_2(t_1,\ldots,t_s;x_1,\ldots,x_s)$ are as given in (3.4) and (3.5) respectively, provided that the integrals in the right hand side of equations (3.6) and (3.7) exists.

Proof: Results (3.6) and (3.7) of Theorem 2 can be obtained on the similar lines to the proof of Theorem 1.

We can easily obtain the one dimensional analogues of the theorem1 and 2, however, we omit the details here.

IV Mellin Transforms, Inversion Formulas and convolutions

The multidimensional Mellin Transform of the function $f(t_1,\ldots,t_s)\in \mathrm{A}$ is defined by the following equation [Saxena and Panda (1978) 7, part I, p. 125, eq. (3.5)].

$$M\left[f(t_1,\ldots,t_s);\gamma_1,\ldots,\gamma_s\right]=\int_0^{\infty}\ldots\int_0^{\infty}\left(\prod_{j=1}^{s}t_j^{\gamma_j-1}\right)f(t_1,\ldots,t_s)dt_1\ldots dt_s \quad (4.1)$$

where $\mathrm{Re}(\gamma_j+U_j)>0,\mathrm{Re}(W_j)>0$ or $\mathrm{Re}(W_j)=0,\mathrm{Re}(V_j-\gamma_j)>0\quad(j=1,\ldots,s)$.

Now we shall establish the following results

Result 1

If $M\left[I_x\left\{f(t_1,\ldots,t_s);\gamma_1,\ldots,\gamma_s\right\}\right]$ exists, then

$$M\left[I_x\left\{f(t_1,\ldots,t_s);\gamma_1,\ldots,\gamma_s\right\}\right]=M\left[f(t_1,\ldots,t_s);\gamma_1,\ldots,\gamma_s\right]\chi(\gamma_1,\ldots,\gamma_s) \quad (4.2)$$

Result 2

If $M\left[J_x\left\{f(t_1,\ldots,t_s);\gamma_1,\ldots,\gamma_s\right\}\right]$ exists, then

$$M\left[J_x\left\{f(t_1,\ldots,t_s);\gamma_1,\ldots,\gamma_s\right\}\right]=M\left[f(t_1,\ldots,t_s);\gamma_1,\ldots,\gamma_s\right]\chi(1-\gamma_1,\ldots,1-\gamma_s) \quad (4.3)$$

where

$$\chi(\gamma_1,\ldots,\gamma_s)=\sum_{R_1,\ldots,R_s=0}^{\sum_{j=1}^{s}U_jR_j\le V}(-V)_{\sum_{j=1}^{s}U_jR_j}A(V,R_1,\ldots,R_s)$$

$$\frac{E_1^{R_1}}{R_1!}\ldots\frac{E_s^{R_s}}{R_s!}\overline{H}^{M,N+2s}_{P+2s,Q+s}\left[Z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right.$$
$$\left.\left\{\prod_{j=1}^{s}\left(1-\rho_j-\gamma_j-\sum_{j=1}^{s}e_jR_{j,}\eta_j;1\right)\left(1-\sigma_j-\sum_{j=1}^{s}f_jR_{j,}\lambda_j;1\right)\right\}\right.$$
$$\left.\left\{\prod_{j=1}^{s}\left(1-\rho_j-\gamma_j-\sigma_j-\sum_{j=1}^{s}(e_j+f_j)R_{j,}(\eta_j+\lambda_j),1\right)\right\}\right] \quad (4.4)$$

The conditions under which the above results are valid can easily be obtained from (4.1) and (1.15).

Proof: To prove result 1, first of all we write the multidimensional Mellin Transform of the I-operator with the help of equation (4.1), then we change the order of t_j and x_j-integrals. Next, with the help of (2.1) and (4.1) we easily arrive at the desired result (4.2) after a little simplification.

The proof of result 2 can be developed by proceeding on the lines similar to those indicated above.

Inversion Formulas

On making use of the inversion theorems for the multidimensional Mellin Transform (4.1), given by Srivastava and Panda (1978) [7, part I,p.125, Lemma 2], we easily get from (4.2) and (4.3) the following inversion formula for the fractional integral operators defined by (1.12) and (1.13).

Result 3.

$$f(t_1,\ldots,t_s)=\frac{1}{(2\pi i)^s}\int_{c_1-i\infty}^{c_1+i\infty}\ldots\int_{c_s-i\infty}^{c_s+i\infty}\frac{\prod_{j=1}^{s}t_j^{-\gamma_j}}{\chi(\gamma_1,\ldots,\gamma_s)}$$
$$M\left[I_x\left\{f(t_1,\ldots,t_s);\gamma_1,\ldots,\gamma_s\right\}\right]d\gamma_1\ldots d\gamma_s \quad (4.5)$$

Result 4.

$$f(t_1,\ldots,t_s)=\frac{1}{(2\pi i)^s}\int_{c_1-i\infty}^{c_1+i\infty}\ldots\int_{c_s-i\infty}^{c_s+i\infty}\frac{\prod_{j=1}^{s}t_1^{-\gamma_j}}{\chi(1-\gamma_1,\ldots,1-\gamma_s)}$$
$$M\left[J_x\left\{f(t_1,\ldots,t_s);\gamma_1,\ldots,\gamma_s\right\}\right]d\gamma_1\ldots d\gamma_s \quad (4.6)$$

The precise conditions under which the inversion formulas (4.5) and (4.6) are valid can be deduced from the existence condition of the various fractional integral operators and their multidimensional Mellin transforms stated earlier.

IV. Mellin Convolutions

The multidimensional Mellin convolutions of two functions $f(t_1,\ldots,t_s)$ and $g(t_1,\ldots,t_s)$ will be defined by

$$(f*g)(t_1,\ldots,t_s)=(g*f)(t_1,\ldots,t_s)$$
$$=\int_0^\infty\ldots\int_0^\infty\left(\prod_{j=1}^{s}x_j^{-1}\right)f\left(\frac{t_1}{x_1},\ldots,\frac{t_s}{x_s}\right)g(x_1,\ldots,x_s)\,dx_1\ldots dx_s \quad (4.7)$$

provided the multiple integral exists

If $f(t_1,\ldots,t_s)\in A$, then the fractional integral operators defined by (1.12) and (1.13) can readily be expressed as multidimensional Mellin convolutions in the following form.

Result 5.

$$I^{\rho,\sigma;e,f;\eta,\lambda}_{x;U,V;Z}f(t_1,\ldots,t_s)$$
$$=\left(I_{\rho,\sigma;e,f;\eta,\lambda;x;U,V;Z}*f\right)(x_1,\ldots,x_s) \quad (4.8)$$

where

$$I_{\rho,\sigma;e,f;\eta,\lambda;x;U,V;Z}$$
$$=\left(\prod_{j=1}^{s}x_j^{-\rho_j-\sigma_j}(x_j-1)^{\sigma_j-1}U(x_j-1)\right)$$
$$S_V^{U_1,\ldots,U_s}\left[E_1(x_1)^{-e_1-f_1}(x_1-1)^{f_1},\ldots,E_s(x_s)^{-e_s-f_s}(x_s-1)^{f_s}\right]$$
$$\overline{H}^{M,N}_{P,Q}\left[Z\prod_{j=1}^{s}(x_j)^{-\eta_j-\lambda_j}(x_j-1)^{\lambda_j}\right]$$

Result 6.

$$J^{\rho,\sigma;e,f;\eta,\lambda}_{x;U,V;Z}f(t_1,\ldots,t_s)$$
$$=\left(J_{\rho,\sigma;e,f;\eta,\lambda;x;U,V;Z}*f\right)(x_1,\ldots,x_s) \quad (4.10)$$

where

$$J_{\rho,\sigma;e,f;\eta,\lambda;x;U,V;Z}$$
$$=\left(\prod_{j=1}^{s}x_j^{\rho_j}(1-x_j)^{\sigma_j-1}U(1-x_j)\right)$$
$$S_V^{U_1,\ldots,U_s}\left[E_1(x_1)^{e_1}(1-x_1)^{f_1},\ldots,E_s(x_s)^{e_s}(1-x_s)^{f_s}\right]$$
$$\overline{H}^{M,N}_{P,Q}\left[Z\prod_{j=1}^{s}(x_j)^{\eta_j}(1-x_j)^{\lambda_j}\right] \quad (4.11)$$

U(x) being the Heaviside's unit function.

Proof: To prove result 5, first we write the I-operator defined by (1.12) in the following form with the help of the Heaviside unit function.

$$I_{x;U,V;Z}^{\rho,\sigma;e,f;\eta,\lambda} f(t_1,...,t_s)$$

$$= \int_0^\infty ... \int_0^\infty \left(\prod_{j=1}^{s} t_j^{-1} \right) \left\{ \prod_{j=1}^{s} \left[\left(\frac{x_j}{t_j} \right)^{-\rho_j-\sigma_j} \left(\frac{x_j}{t_j} - 1 \right)^{\sigma_j - 1} U\left(\frac{x_j}{t_j} - 1 \right) \right] \right\}$$

$$S_V^{U_1,...,U_s} \left[E_1 \left(\frac{x_1}{t_1} \right)^{-e_1-f_1} \left(\frac{x_1}{t_1} - 1 \right)^{f_1}, ..., E_s \left(\frac{x_s}{t_s} \right)^{-e_s-f_s} \left(\frac{x_s}{t_s} - 1 \right)^{f_s} \right]$$

$$\overline{H}_{P,Q}^{M,N} \left[Z \prod_{j=1}^{s} \left(\frac{x_j}{t_j} \right)^{-\eta_j-\lambda_j} \left(\frac{x_j}{t_j} - 1 \right)^{\lambda_j} \right] f(t_1,...,t_s)\, dt_1 ... dt_s \tag{4.12}$$

Now making use of the equation (4.9) and the definition of the Mellin convolutions given by (4.7) in the above equation, we easily arrive at the desired result. The proof of the result 6 can be developed on the same lines.

V. References

1. Buschman R.G. and Srivastava H.M. (1990), The $\overline{H}$ -function associated with a certain class of Feynman integrals, *J. Phys. A: Math. Gen.*, **23,** 4707-4710.
2. Gradshteyn, I. S. and Ryzhik, I. M.: Table of Integrals, Series and Products. Academic Press Inc., NewYork (1980)
3. Gupta K.C., Jain R. and Agrawal R. (2007), On existence conditions for generalized Mellin-Barnes type integral, *Nat. Acad. Sci. Lett.* **30**(5&6), 169-172.
4. Rathie, A.K. (1997), A new generalization of generalized hypergeometric functions, *Le Mathematiche Fasc. II*, 52,297-310.
5. Saxena ,R.K. ;Chena Ram and Kalla, S.L. (2002), Application of generalized H-function in Bivariate distribution, *Rev. Acad. Can. Ciene., XIV (Nums 1-2)*, 111-120.
6. Srivastava H.M. and Garg M. (1987) Some integral involving a general class of polynomials and the multivariable H-Function, *Rev. Romaine Phys.*, **32** 685-692.
7. Srivastava H.M. and Panda R. (1978), Certain multidimensional integral transformations, I and II, *Nederl. Akad. Wetensch.Proc. Ser. A 81= Indag. Math.*, 40, 118-131 and 132-144.

Fourth-Order Method For One-Dimensional Heat Equation With A Nonlocal Boundary Condition

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Abstract **In this paper a fourth-order numerical technique is developed and im- plemented for the solution of the homogeneous heat equation ut = ®uxx: The results obtained show that the numerical method based on the proposed technique is fourth-order accurate. Also the e±ciency and the accuracy of the new scheme is in good agreement with the exact solution as compared to the alternative techniques.**

Keywords-Heat equation, non-local boundary condition, fourth- order numerical methods, method of lines, parallel splitting. American Mathematical Subject Classi¯cation 2000: 65M06, 65N40

I. Introduction

In the last two decades, the development of the numerical techniques for the solution of non-local boundary value problems has been an important research topic in many branches of science and engineering. Particularly thermo-elasticity has been the subject of some resent works [3, 4]. Many physical phenomena are modelled by nonclassical boundary value problems with non-local boundary conditions. These can be classi¯ed into two types: (i) boundary value problems with non-local initial conditionsor (ii) boundary value problems with non-local boundary conditions. The present work focuses on the ¯rst group of these non-local boundary value problems. This paper considers the problem of obtaining numerical approximation to the concentration $u = u(x; t)$ which satis¯es the partial di®erential equa-tion

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad 0 < t \leq T, \tag{1}$$

subject to the initial condition

$$u(x, 0) = f(x), \quad 0 \leq x \leq 1, \tag{2}$$

the boundary condition

$$u(0, t) = g(t), \quad 0 < t \leq T \tag{3}$$

and the non-local boundary condition

$$\int_0^1 u(x, t)dx = M(t), \quad 0 < t \leq T, \quad 0 < x < 1 \tag{4}$$

where f; g;M and s are known functions and are assumed to be su±ciently smooth to produce a smooth classical solution of u. ® being a constant and T is given positive constant. Much attention has been paid in the literature for the development, anal- ysis and implementation of accurate methods for the numerical solution of time dependent partial di®erential equation with non-local boundary condi-tion. A number of sequential numerical procedures (other than new scheme) have been suggested in the literature: see, for instance [1, 2, 6, 8, 5]. In the present paper the method of lines, semi discretization approach,
will be used to transform the model partial di®erential equation (PDE) into a system of ¯rst-order, linear, ordinary di®erential equations (ODEs), the solution of which satis¯es a recurrence relation involving matrix exponen- tial terms. A suitable rational approximation will be used to approximate such exponential functions leading to an L0-stable algorithm which may be parallelized through the partial fraction splitting technique.

II. Discretization and Recurrence Relation

Choosing a positive integer N ¸ 6 and dividing the interval [0; 1] into N +1 subintervals each of width h, so that h = =(N + 1), and the time variable t into time steps each of length l gives a rectangular mesh of points with co-ordinates (xm; tn) = (mh; nl) where (m = 0; 1; 2; :::;N +1 and n = 0; 1; 2; :::) covering the region R = [0 < x < 1]£[t > 0] and its boundary @R consisting of lines x = 0; x = 1 and t = 0. Assuming that u(x; t) is six times continuously di®erentiable with respect to variable x and that these derivatives are uniformly bounded, the space derivative in

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(1) may be approximated to the fourth-order accuracy at approximation It is worth noting that the equation (5) is valid only for (x; t) = (xm; tn) with m = 2; 3; :::;N ¡ 1: To attain the same accuracy at the points (xi; tn) for i = 1 and i = N, special formulae must be developed which approximate @2u(x;t) @x2 not only to fourth-order but also with dominant error term h4 90 @6u(x;t) @x6 for x = x1; xN and t = tn: Such approximations will be It is worth noting that some general point (x; t) of the mesh by using the ¯ve point the equation (5) is valid only for (x; t) = (xm; tn) with m = 2; 3; :::;N ¡ 1: To attain the same accuracy at the points (xi; tn) for i = 1 and i = N, special formulae must be developed which approximate @2u(x;t) @x2 not only to fourth-order but also with dominant error term h4 90@6u(x;t) @x6 for x = x1; xN and t = tn: Such approximations will be

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{12h^2}\{-u(x-2h,t)+16u(x-h,t)-30u(x,t)+16u(x+h,t)$$

$$-\ u(x+2h,t)\}+\frac{h^4}{90}\frac{\partial^6 u(x,t)}{\partial x^6}+O(h^5)\ \ as\ \ h\to 0 \qquad (5)$$

It is worth noting that the equation (5) is valid only for $(x,t)=(x_m,t_n)$ with $m=2,3,...,N-1$. To attain the same accuracy at the points (x_i,t_n) for $i=1$ and $i=N$, special formulae must be developed which approximate $\frac{\partial^2 u(x,t)}{\partial x^2}$ not only to fourth-order but also with dominant error term $\frac{h^4}{90}\frac{\partial^6 u(x,t)}{\partial x^6}$ for $x=x_1,\ \ x_N$ and $t=t_n$. Such approximations will be

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{12h^2}\{9u(x-h,t)-9u(x,t)-19u(x+h,t)+34u(x+2h,t)$$
$$-\ 21u(x+3h,t)+7u(x+4h,t)-u(x+5h,t)\}+\frac{h^4}{90}\frac{\partial^6 u(x,t)}{\partial x^6}$$
$$+\ O(h^5)\ \ as\ \ h\to 0 \qquad (6)$$

and

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{12h^2}\{-u(x-5h,t)+7u(x-4h,t)-21u(x-3h,t)+34u(x-2h,t)$$
$$-\ 19u(x-h,t)-9u(x,t)+9u(x+h,t)\}+\frac{h^4}{90}\frac{\partial^6 u(x,t)}{\partial x^6}$$
$$+\ O(h^5)\ \ as\ \ h\to 0 \qquad (7)$$

at the mesh points (x_1,t_n) and (x_N,t_n) respectively.

III. Treatment of the non-local bound-ary condition

The integral in (4) is approximated by using Simpson's quadrature rule as

where $\alpha_N = 20, \alpha_{N-1} = -28, \alpha_{N-2} = 20, \alpha_{N-3} = 1$

and $\alpha_i = \begin{cases} 4 & \text{for i=1(2)N-4} \\ 2 & \text{for i=2(2)N-5} \end{cases}$

also $\beta_N = -45, \beta_{N-1} = -37, \beta_{N-2} = -2, \beta_{N-3} = -39, \beta_{N-4} = -29, \beta_{N-5} = -19$

and $\beta_i = \begin{cases} -36 & \text{for i=1(2)N-6} \\ -18 & \text{for i=2(2)N-7} \end{cases}$

and the column vector

$$v(t) = \frac{\alpha}{12h^2}[9g(t), -g(t), 0, 0, ..., g(t) - \frac{3}{h}M(t), -9g(t) + \frac{27}{h}M(t)]^T$$

where T stands for transpose of a matrix.

The solution of the system (9) and (10) becomes

$$\mathbf{U}(t) = exp(tA)f + \int_0^t exp[(t-s)A]\mathbf{v(s)}ds, \quad t \geq 0 \tag{11}$$

where $\mathbf{v(s)}$ is due to boundary condition and the source term which satisfies the recurrence relation

$$\mathbf{U}(t+l) = exp(lA)\mathbf{U}(t) + \int_t^{t+l} exp[(t+l-s)A]\mathbf{v(s)}ds, \quad t = 0, l, 2l, ... \tag{12}$$

Eigenvalues of the matrix A are calculated using MATLAB for $N = 9, 19, 39, 79$ and it is observed that they are distinct negative real ones or with negative real parts.

To approximate the matrix exponential in (12) we use a rational approximation consisting of three parameters a_1, a_2, a_3 and given for a real scalar θ it is given by

$$E_4(\theta) = \frac{1 + (1-a_1)\theta + (\frac{1}{2} - a_1 + a_2)\theta^2 + (\frac{1}{6} - \frac{a_1}{2} + a_2 - a_3)\theta^3}{1 - a_1\theta + a_2\theta^2 - a_3\theta^3 + (-\frac{1}{24} + \frac{a_1}{6} - \frac{a_2}{2} + a_3)\theta^4} = \frac{p(\theta)}{q(\theta)} \tag{13}$$

with error constant $C_5 = \frac{1}{30} - \frac{1}{8}a_1 + \frac{1}{3}a_2 - \frac{1}{2}a_3$ [9].

Let λ be an eigenvalue of the matrix A then the amplification symbol of the numerical method arising from (13) is

$$R(-z) = \frac{1-(1-a_1)z+(\frac{1}{2}-a_1+a_2)z^2-(\frac{1}{6}-\frac{a_1}{2}+a_2-a_3)z^3}{1+a_1z+a_2z^2+a_3z^3+(-\frac{1}{24}+\frac{a_1}{6}-\frac{a_2}{2}+a_3)z^4}, \quad (14)$$

where $z = -lRe(\lambda) > 0$. The resulting method is L-stable provided that

$$\left.\begin{array}{l} a_1 > \frac{1}{2}, \\ a_2 > \frac{a_1}{2} - \frac{1}{6}, \\ a_3 > \frac{a_2}{2} - \frac{a_1}{6} + \frac{1}{24}. \end{array}\right\} \quad (15)$$

[9]. So we have

$$exp(lA) = G^{-1}(I+(1-a_1)lA+(\frac{1}{2}-a_1+a_2)l^2A^2+(\frac{1}{6}-\frac{a_1}{a_2}+a_2-a_3)l^3A^3) \quad (16)$$

where

$$G = I - a_1lA + a_2l^2A^2 - a_3l^3A^3 + (-\frac{1}{24}+\frac{a_1}{6}-\frac{a_2}{2}+a_3)l^4A^4 \quad (17)$$

The quadrature term in (12) is approximated by

$$\int_t^{t+l} exp[(t+l-s)A]v(s)ds = W_1v(s_1)+W_2v(s_2)+W_3v(s_3)+W_4v(s_4) \quad (18)$$

where $s_1 = t, s_2 = t + l/3, s_3 = t + 2l/3, s_4 = t + l$ and W_1, W_2, W_3 and W_4 are matrices given by

$$W_1 = \frac{l}{24}\{3I-(19-78a_1+216a_2-324a_3)lA+(3-8a_1+12a_2)l^2A^2\}G^{-1} \quad (19)$$

$$W_2 = \frac{3l}{16}\{2I+(16-56a_1+144a_2-216a_3)lA+(1-4a_1+12a_2-24a_3)l^2A^2\}G^{-1} \quad (20)$$

$$W_3 = \frac{3l}{8}\{I-(7-26a_1+72a_2-108a_3)lA-(1-4a_1+12a_2-24a_3)l^2A^2\}G^{-1} \quad (21)$$

$$W_4 = \frac{l}{48}\{6I + (44 - 168a_1 + 432a_2 - 648a_3)lA + (11 - 44a_1 + 132a_2 - 216a_3)l^2A^2 + (2 - 8a_1 + 24a_2 - 48a_3)l^3A^3\}G^{-1} \quad (22)$$

Hence

$$\mathbf{U}(t+l) = exp(lA)\mathbf{U}(t) + W_1v(t) + W_2v(t+l/3) + W_3v(t+2l/3) + W_4v(t+l) \quad (23)$$

IV. Development of Algorithm

Assuming that a_1, a_2 and a_3 are chosen in such a way that the condition (15) is satisfied and $q(\theta)$ given by (13) possesses real and distinct zeros r_i $(i = 1, 2, 3, 4)$ with $r_i \neq 0$. Then

$$G = (I - \frac{l}{r_1}A)(I - \frac{l}{r_2}A)(I - \frac{l}{r_3}A)(I - \frac{l}{r_4}A) \quad (24)$$

and

$$exp(lA) = \sum_{j=1}^{4} p_j(I - \frac{l}{r_j}A)^{-1} \quad (25)$$

where $p_j(j = 1, 2, 3, 4)$, the partial-fraction coefficients of $E_4(\theta)$, are defined by

$$p_j = \frac{1 + (1 - a_1)r_j + (\frac{1}{2} - a_1 + a_2)r_j^2 + (\frac{1}{6} - \frac{a_1}{2} + a_2 - a_3)r_j^3}{\prod_{\substack{i=1 \\ i \neq j}}^{4} (1 - \frac{r_j}{r_i})}, j = 1, 2, 3, 4 \quad (26)$$

and

$$W_k = \frac{m_k l}{48}\sum_{j=1}^{4} p_{4k+j}(I - \frac{l}{r_j}A)^{-1}, k = 1, 2, 3, 4 \quad (27)$$

in which $m_1 = 2, m_2 = 9, m_3 = 18, m_4 = 1$ and for $j = 1, 2, 3, 4$

$$p_{4+j} = \frac{3 + (-19 + 78a_1 - 216a_2 + 324a_3)r_j + (3 - 8a_1 + 12a_2)r_j^2}{\prod^4_{\substack{i=1 \\ i \neq j}} (1 - \frac{r_i}{r_i})},$$

$$p_{8+j} = \frac{2 + (16 - 56a_1 + 144a_2 - 216a_3)r_j + (1 - 4a_1 + 12a_2 - 24a_3)r_j^2}{\prod^4_{\substack{i=1 \\ i \neq j}} (1 - \frac{r_i}{r_i})},$$

$$p_{12+j} = \frac{1 + (-7 + 26a_1 - 72a_2 + 108a_3)r_j - (1 - 4a_1 + 12a_2 - 24a_3)r_j^2}{\prod^4_{\substack{i=1 \\ i \neq j}} (1 - \frac{r_i}{r_i})},$$

$$\begin{aligned} p_{16+j} &= \frac{1}{\prod^4_{\substack{i=1 \\ i \neq j}} (1 - \frac{r_i}{r_i})} \{6 + (44 - 168a_1 + 432a_2 - 648a_3)r_j \\ &+ (11 - 44a_1 + 132a_2 - 216a_3)r_j^2 + (2 - 8a_1 + 24a_2 - 48a_3)r_j^3\} \end{aligned}$$

Hence

$$\begin{aligned} exp(lA)\mathbf{U}(t) &= [p_1(I - \frac{l}{r_1}A)^{-1} + p_2(I - \frac{l}{r_2}A)^{-1} \\ &+ p_3(I - \frac{l}{r_3}A)^{-1} + p_4(I - \frac{l}{r_4}A)^{-1}]\mathbf{U}(t) \end{aligned} \quad (28)$$

So

$$\mathbf{U}(t + l) = y_1(t) + y_2(t) + y_3(t) + y_4(t) \quad (29)$$

in which y_1, y_2, y_3 and y_4 are solutions of the systems

$$A_1 y_1 = p_1\mathbf{U}(t) + \frac{l}{48}[2p_5 v(t) + 9p_9 v(t + l/3) + 18p_{13} v(t + 2l/3) + p_{17} v(t + l)] \quad (30)$$

$$A_2 y_2 = p_2 \mathbf{U}(t) + \frac{l}{48}[2p_6 v(t) + 9p_{10} v(t+l/3) + 18p_{14} v(t+2l/3) + p_{18} v(t+l)] \quad (31)$$

$$A_3 y_3 = p_3 \mathbf{U}(t) + \frac{l}{48}[2p_7 v(t) + 9p_{11} v(t+l/3) + 18p_{15} v(t+2l/3) + p_{19} v(t+l)] \quad (32)$$

$$A_4 y_4 = p_4 \mathbf{U}(t) + \frac{l}{48}[2p_8 v(t) + 9p_{12} v(t+l/3) + 18p_{16} v(t+2l/3) + p_{20} v(t+l)] \quad (33)$$

respectively. Here $A_i = I - \frac{l}{r_i} A,\ i = 1, 2, 3, 4.$

V. The Parallel Algorithm

Equations (30)-(33) have great importance in the parallel environment sinceb they can be used to solve the corresponding linear algebraic systems on pro-
cessors working simultaneously. The solution vector U($t + l$) in (29) may be bobtained using four di®erent processors or a serial computer by the algorithm given in Table 1.

VI. Numerical Experiments

In this section the numerical method described in this paper will be applied to one problem from the literature and results obtained will be compared with exact solutions as well as with the results existing in the literature. Following [9] we have chosen here a1 = 64=25; a2 = 7=3 and a3 = 547=600.
It can be shown that using these values L-stability is guaranteed ..
EXAMPLE (1)
Consider (1),(2),(3) and (4) with
f(x) = cos(¼ 2 x); 0 < x < 1;

Table 1: Algorithm

Step 0 Input	$h, l, a_1, a_2, a_3, \mathbf{U}_0, A$			
Step 1 Compute	$r_1, r_2, r_3, r_4, p_1, p_2 ... p_{20}$			
	processor 1	processor 2	processor 3	processor 4
Step 2 Distribute the data	r_1 $p_1, p_5, ..., p_{17}$	r_2 $p_2, p_6, ..., p_{18}$	r_3 $p_3, p_7, ..., p_{19}$	r_4 $p_4, p_8, ..., p_{20}$
Step 3 Decompose the data	$A_1 = I - \frac{l}{r_1} A$	$A_2 = I - \frac{l}{r_2} A$	$A_3 = I - \frac{l}{r_3} A$	$A_4 = I - \frac{l}{r_4} A$
Step 4 Evaluate	$v(t), v(t+l/3),$ $v(t+2l/3),$ $v(t+l)$	$v(t), v(t+l/3),$ $v(t+2l/3),$ $v(t+l)$	$v(t), v(t+l/3),$ $v(t+2l/3),$ $v(t+l)$	$v(t), v(t+l/3),$ $v(t+2l/3),$ $v(t+l)$
Step 5 Compute	$W_1(t) =$ $2p_5 v(t)$ $+9p_9 v(t+l/3)$ $+18p_{13} v(t+2l/3)$ $+p_{17} v(t+l)$	$W_2(t) =$ $2p_6 v(t)$ $+9p_{10} v(t+l/3)$ $+18p_{14} v(t+2l/3)$ $+p_{18} v(t+l)$	$W_3(t) =$ $2p_7 v(t)$ $+9p_{11} v(t+l/3)$ $+18p_{15} v(t+2l/3)$ $+p_{19} v(t+l)$	$W_4(t) =$ $2p_8 v(t)$ $+9p_{12} v(t+l/3)$ $+18p_{16} v(t+2l/3)$ $+p_{20} v(t+l)$
Step 6 Solve the system	$L_1 U_1 y_1(t)$ $= p_1 \mathbf{U}(t)$ $+\frac{l}{48} W_1(t)$	$L_2 U_2 y_2(t)$ $= p_2 \mathbf{U}(t)$ $+\frac{l}{48} W_2(t)$	$L_3 U_3 y_3(t)$ $= p_3 \mathbf{U}(t)$ $+\frac{l}{48} W_3(t)$	$L_4 U_4 y_4(t)$ $= p_4 \mathbf{U}(t)$ $+\frac{l}{48} W_4(t)$
Step 7 compute	$\mathbf{U}(t+l) = y_1(t) + y_2(t) + y_3(t) + y_4(t)$			
Step 8	GOTO Step 4 for next time level			
Step 9	STOP the process when solution at given time level is obtained			

Table 2: Relative errors for various spatial lengths

h	Implicit	Galerkin	Keller-Box	RKC	Saulyev I	New scheme
0.0500	9.1×10^{-03}	9.9×10^{-02}	9.4×10^{-02}	9.8×10^{-02}	9.6×10^{-03}	2.1×10^{-06}
0.0250	2.3×10^{-03}	3.0×10^{-02}	2.4×10^{-02}	3.7×10^{-02}	2.5×10^{-03}	2.6×10^{-07}
0.0100	3.8×10^{-04}	4.9×10^{-03}	4.1×10^{-03}	6.1×10^{-03}	3.9×10^{-04}	1.1×10^{-08}
0.0050	9.4×10^{-05}	1.2×10^{-03}	1.0×10^{-03}	1.5×10^{-03}	9.6×10^{-05}	9.0×10^{-10}
0.0025	2.3×10^{-05}	3.1×10^{-04}	2.5×10^{-04}	3.5×10^{-04}	2.5×10^{-05}	1.1×10^{-11}
0.0010	4.1×10^{-06}	5.0×10^{-05}	4.0×10^{-05}	6.0×10^{-05}	4.3×10^{-06}	1.8×10^{-10}

$$g(t) = exp(-\frac{\pi^2}{4}t), \quad 0 < t < 1,$$

$$M(t) = \frac{2}{\pi}exp(-\frac{\pi^2}{4}t), \quad 0 < t \leq 1$$

and with theoretical solution

$$u(x,t) = exp(-\frac{\pi^2}{4}t)cos(\frac{\pi}{2}x) \qquad [5]$$

The relative errors for the results of $u(0.5, 0.1)$ at $h = l = 0.01, 0.005, 0.0025, 0.001$, using the new scheme discussed in this paper as well as by using the implicit method [1], Galerkin technique of [2], Keller-Box method of [6], the Rung Kutta Chebyshev (RKC) scheme [8] and the Saulyev's explicit scheme of [5] are shown in Table 2.

VII . Conclusion

The results obtained using the new scheme developed in this paper are highly accurate as compared to those of [1], [2], [6], [8] and [5]. It is also noted that the method developed fourth-order accurate but it deviates slightly only for very small values of l and h. It is therefore clear that the new scheme is the best candidate for the model problem. This technique can be coded easily on the serial or parallel computers. It is worth mentioning that the use of real arithmetic and multiprocessor architecture especially in multi-space dimensional problems can save remark-able CPU time rather than using complex arithmetic based methods.

VII. References

1) Cannon, J R. The one dimension heat equation. Encyclopedia of math- ematics and its applications. Men10 park(CA)1984;23:347-355.
2) Cannon J R. and Matheson A. A numerical procedure for di®usion subject to the speci¯cation of mass. Int.J.Eng.Sci. 1993;131:347-355.
3) Day, W A. Existence of a proprty of solutions of the heat equationsubject to linear thermo-elasticity and other theories. Quart. Appl. Math. 1982;40:319-330.
4) Day, W A. Parabolic equations and thermodynamics. Quart. Appl. Math. 1992;50:523-533. 13
5) Dehghan, M. The one dimensional heat equation subject to a boundary integral speci¯cation. Chaos Solitons and Fractals. 2007;32:661-675.
6) Ewing, R.E and Lin, TA. Class of parameter estimation techniques of °uid °ow in porous media. Adv. Water Resour. 1991;14:89-97.
7) Gumel, A B. On the numerical solution of the di®usion equation subject to the speci¯cation of mass. J. Austral. Math. Soc. Ser. B 1999;40:475-483.
8) Makarov, V L. and Kulyev, D T. Solution of a boundary value prob- lemfor a quasi linear parabolic equation with nonclassical boundary conditions. Di®erent equt. 1985;21:296-305.
9) Taj, M S A. and Twizell, E H. A family of fourth-order parallel split- ting methods for prabolic partial di®erential equations. Inc. Numer. Methods Partial Di®erential Eq. 1997;13:357-373.

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- Numbering of First Main Headings (Heading 1) must be in Roman Letters, Capital Letter, and Font Size of 10.
- Numbering of Second Main Headings (Heading 2) must be in Alphabets, Italic, and Font Size of 10.

You can use your own standard format also.

Author Guidelines:

1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript's Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

1. GENERAL

Before submitting your research paper, one is advised to go through the details as mentioned in following heads. It will be beneficial, while peer reviewer justify your paper for publication.

Scope

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3. SUBMISSION OF MANUSCRIPTS

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Complete support for both authors and co-author is provided.

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Based on potential and nature, the manuscript can be categorized under the following heads: Original research paper: Such papers are reports of high-level significant original research work.

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Research articles: These are handled with small investigation and applications

Research letters: The letters are small and concise comments on previously published matters.

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The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also. Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

Papers*:* These are reports of significant research (typically less than 7000 words equivalent, including tables, figures, references), and comprise:

(a)*Title* should be relevant and commensurate with the theme of the paper.

(b) A brief Summary, *"Abstract"* (less than 150 words) containing the major results and conclusions.

(c) Up to *ten keywords*, that precisely identifies the paper's subject, purpose, and focus.

(d) An *Introduction*, giving necessary background excluding subheadings; objectives must be clearly declared.

(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; *conclusions* should be summarizing.

(h) Brief Acknowledgements.

(i) References in the proper form.

Authors should very cautiously consider the preparation of papers to ensure that they communicate efficiently. Papers are much more likely to be accepted, if they are cautiously designed and laid out, contain few or no errors, are summarizing, and be conventional to the approach and instructions. They will in addition, be published with much less delays than those that require much technical and editorial correction.

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Abbreviations supposed to be used carefully. The abbreviated name or expression is supposed to be cited in full at first usage, followed by the conventional abbreviation in parentheses.

Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than 1.4 × 10-3 m3, or 4 mm somewhat than 4 × 10-3 m. Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

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Abstract, used in Original Papers and Reviews:

Optimizing Abstract for Search Engines

Many researchers searching for information online will use search engines such as Google, Yahoo or similar. By optimizing your paper for search engines, you will amplify the chance of someone finding it. This in turn will make it more likely to be viewed and/or cited in a further work. Global Journals have compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Key Words

A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy and planning a list of possible keywords and phrases to try.

Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

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- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods*:* Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: *Please make these as concise as possible.*

References

References follow the *Harvard scheme* of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals homepage at the judgment of the Editorial Board.

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Acrobat Reader will be required in order to read this file. This software can be downloaded

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Informal Tips for writing a Science Frontier Research Paper to increase readability and citation

Before start writing a good quality Science Frontier Research Paper, let us first understand what is Science Frontier Research Paper? So, Frontier Research Paper is the paper which is written by professionals or scientists who are associated to Physics, Mathematics, Chemistry, Zoology, Botany, Bio-tech, Geology, Military Science, Environment and all Interdisciplinary & Frontier Subjects etc., or doing research study in these areas. If you are novel to this field then you can consult about this field from your supervisor or guide.

Techniques for writing a good quality Applied Science Research Paper:

1. Choosing the topic- In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Frontier Science. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

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3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

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5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

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12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

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16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. **Pick a good study spot:** To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. **Think technically:** Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical

remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

INFORMAL GUIDELINES OF RESEARCH PAPER WRITING

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

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A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

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· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

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Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

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An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

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- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

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- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
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The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

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- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
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Approach:

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- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

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This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

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- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

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- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.

You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle

- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described.
Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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