T*-Semi Generalized Continuous Maps In Topological Spaces

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[1]A.Pushpalatha and [2]S.Eswaran

Abstract- **In this paper, we introduce new class of maps called τ*-Semi Generalized Continuous maps in topological spaces and study its relationship with some existing maps. 2000 Mathematics Subject Classification: 54A05.**

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I. Introduction

In 1970, Levine[7] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Using generalized closed sets, Dunham [4] introduced the concept of the closure operator cl^* and a topology τ^* and studied some of its properties. Pushpalatha, Eswaran and Rajarubi [10] introduced and studied τ^*-generalized closed sets and τ^*-generalized open sets. Using τ^*-generalized closed sets, Eswaran and Pushpalatha [5] introduced and studied τ^*-generalized continuous maps. Several authors have introduced and studied various maps in topological spaces.

The purpose of this paper is to introduce and study the concept of a new class of maps, namely τ*-semigeneralized continuous maps.

Throughout this paper X and Y are topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A of a topological space X, cl(A), $cl^*(A)$, scl(A), spcl(A), $cl_\alpha(A)$ and A^c denote the closure, closure*, semi-closure, semi-preclosure, α-closure and complement of A respectively.

II. Preliminaries

We recall the following definitions:

Definition 2.1- For the subset A of a topological X, the generalized closure operator cl*[4] is defined by the intersection of all g-closed sets containing a.

Definition 2.2- For the subset A of a topological X, the topology τ* [4] is defined by $\tau^* = \{G : cl^*(G^C) = G^C\}$

Definition 2.3-A subset A of a topological space X is called τ^*-generalized closed set (briefly τ^*-g-closed) [10] if $cl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is τ^*-open. The complement of τ^*-generalized closed set is called the τ^*-generalized open set (briefly τ^*-g-open).

Definition 2.4-The τ* generalized closure operator cl_τ^* [10] for a subset A of a topological space (X, τ*) is defined by the intersection of all τ*-g-closed set containing A. That is $cl_\tau^*(A) = \cap\{G : A \subseteq G$ and G is τ*-g-closed$\}$

Definition 2.5- A map $f : X \to Y$ from a topological space X into a topological space Y is called :

1) continuous if the inverse image of every closed set (or open set) in Y is closed (or open) in X.
2) generalized continuous[2](g-continuous) if the inverse image of every closed set in Y is g-closed in X.
3) strongly sg-continuous [12] if the inverse image of each sg-open set of Y is open in X.
4) strongly gs-continuous [12] if the inverse image of each gs-open set of Y is open in X.
5) semi continuous [13] if the inverse image of each closed set of Y is semi-closed in X.
6) sg-continuous [11] if the inverse image of each closed set of Y is sg-closed in X.
7) gs-continuous [14] if the inverse image of each closed set of Y is gs-closed in X.
8) gsp-continuous [3] if the inverse image of each closed set of Y is gsp-closed in X.
9) αg- continuous [6] if the inverse image of each closed set of Y is αg-closed in X.
10) pre-continuous [8] if the inverse image of each open set of Y is pre-open in X.
11) α- continuous [9] if the inverse image of each open set of Y is α-open in X.
12) sp-continuous [1] if the inverse image of each open set of Y is semipreopen in X.
13) weakly sg-continuous [12] if the inverse image of each sg-open set of Y is semiopen in X.
14) weakly gs-continuous [12] if the inverse image of each gs-open set of Y is semiopen in X.
15) sg*-continuous [12] if the inverse image of each semiopen set of Y is sg-open set in X
16) gs*-continuous [12] if the inverse image of each semiopen set of Y is gs-open set in X.
17) τ*-g-continuous [5] if the inverse image of each g-closed set in Y is τ*-g-closed in X.

Remark 2.6- In [10] it has been proved in Theorem 3.2 that every closed set is τ*-g-closed.

Remark 2.7-In [10] it has been proved in Theorem 3.4 that every g-closed set is τ*-g-closed.

[1]*A.Pushpalatha and* [2]*S.Eswaran*
[1]*Department of Mathematics, Government Arts College, Udumelpet, Tamilnadu, India – 642 126*
[2]*Department of Mathematics, KPR Institute of Engineering and Technology, Coimbatore, Tamilnadu, India – 641 407*
[1] *velu_pushpa@yahoo.co.in, 2 rigeeas@yahoo.co.in*

III. SEMI GENERALIZED CONTINUOUS MAPS IN TOPOLOGICAL SPACES

S.Eswaran and A.Pushpalatha [5] introduced and studied a map called τ*-generalized continuous map in topological space. In this section, we introduce a new class of map namely τ*-semi generalized continuous map in topological spaces and study some of its properties and relationship with some existing mappings.

Definition 3.1-A map $f: X \to Y$ from a topological space X into a topological space Y is called τ*-semi generalized continuous map (briefly τ*-sg-continuous) if the inverse image of every sg-closed set in Y is τ*-g-closed in X.

Theore 3.2.-Let $f: X \to Y$ be a map from a topological space (X, τ*) into a topological space (Y, σ*).

i. The following statements are equivalent:
   a. f is τ*-sg-continuous.
   b. The inverse image of each sg-open set in Y is τ*-g-open in X.

ii. If $f: X \to Y$ is τ*-sg-continuous, then $f(cl_\tau^*(A)) \subseteq cl(f(A))$ for every subset A of X.

Proof -1- Assume that $f: X \to Y$ is τ*-sg-continuous. Let G be sg-open in Y. Then G^C is sg-closed in Y. Since f is τ*-sg-continuous, $f^{-1}(G^C)$ is τ*-g-closed in X. But $f^{-1}(G^C) = X - f^{-1}(G)$. Thus $X - f^{-1}(G)$ is τ*-g-closed in X and so $f^{-1}(G)$ is τ*-g-open in X. Therefore (a) ⇒ (b).
Conversely assume that the inverse image of each sg-open set in Y is τ*-g-open in X. Let F be any sg-closed set in Y. Then F^C is sg-open in Y. By assumption, $f^{-1}(F^C)$ is τ*-g-open in X. But $f^{-1}(F^C) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is τ*-g-open in X and so $f^{-1}(F)$ is τ*-g-closed in X. Therefore f is τ*-sg-continuous. Hence (b) ⇒ (c). Thus (a) and (b) are equivalent.
2-Assume that f is τ*-sg-continuous. Let A be any subset of X. Then cl(f(A)) is sg-closed set in Y. Since f is τ*-sg-continuous, $f^{-1}(cl(f(A)))$ is τ*-g-closed in X and it contains A. But $cl_\tau^*(A)$ is the intersection of all τ*-g-closed set containing A. Therefore $cl_\tau^*(A) \subseteq f^{-1}(cl(f(A)))$ and so $f(cl_\tau^*(A)) \subseteq cl((f(A))$.

Theorem 3.3-If a map $f: X \to Y$ from a topological space X into a topological space Y is continuous then it is τ^*-sg-continuous but not conversely.

Proof -Let $f: X \to Y$ be continuous. Let V be a closed set in Y. Since *f* is continuous, $f^{-1}(V)$ is closed in X. Since every closed set is sg-closed , Y is sg-closed. Also by Remark 2.6, $f^{-1}(V)$ is τ^*-g-closed. Thus, f is τ^*-sg-continuous.
The converse of the theorem need not be true as seen from the following example.

Example 3.4-Let X=Y= {a, b, c}, τ = {X, ϕ, {a,}, {b}, {a, b}, {a, c}}and σ = {Y, ϕ, {a}, {b, c}}. Let $f: X \to Y$ be an identity map. Then f is τ*-sg-continuous. But f is not continuous. Since for the closed set F = {c} in Y, $f^{-1}(F)$ = {c} is not closed in X.

Theorem 3.5.-If a map $f: X \to Y$ from a topological space X into a topological space Y is g-continuous then it is τ^*-sg-continuous but not conversely.

Proof -Let $f: X \to Y$ be g-continuous. Let V be a closed set in Y. Since f is g-continuous, $f^{-1}(V)$ is g-closed in X. Since every closed set is sg-closed, V is sg-closed. Also by Remark 2.7, $f^{-1}(V)$ is τ^*-g-closed. Thus, f *is* τ^*-sg-continuous.
Converse of the theorem need not be true as seen from the following example.

Example 3.6-Let X=Y= {a, b, c}, τ = {X, ϕ, {a,}, {b}, {a, b}, {b, c}}and σ = {Y, ϕ, {c}}. Let $f: X \to Y$ be an identity map. Then f is τ*-sg-continuous. But f is not g-continuous. Since for the closed set F = {a, b} in Y, $f^{-1}(F)$ = {a, b} is not g-closed in X.

Theorem 3.7-If a map $f: X \to Y$ from a topological space X into a topological space Y is strongly sg-continuous then it is τ^*-sg-continuous but not conversely.

Proof -Let $f: X \to Y$ be strongly sg-continuous. Let F be a sg-closed set in Y. Then F^C is sg-open in Y. Since f is strongly sg-continuous, $f^{-1}(F^C)$ is open in X. But $f^{-1}(F^C) = X - f^{-1}(F)$. Therefore $f^{-1}(F)$ is closed in X. By Remark 2.6, $f^{-1}(F)$ is τ*-g-closed in X. Thus, f is τ*-sg-continuous.
Converse of the theorem need not be true as seen from the following example.

Example 3.8.-Let X=Y= {a, b, c}, τ = {X, ϕ, {c}}, σ = {Y, ϕ, {a,}, {b}, {a, b}, {a, c}}. Let $f: X \to Y$ be an identity map. Then f is τ*-sg-continuous. But f is not strongly sg-continuous. Since for the sg-open set V = {a, b} in Y, $f^{-1}(V)$ = {a, b} is not open in X.

Theorem 3.9.-If a map $f: X \to Y$ from a topological space X into a topological space Y is strongly gs-continuous then it is τ^*-sg-continuous but not conversely.

Proof - Let $f: X \to Y$ be strongly gs-continuous. Let V be a sg-closed set in Y. Since every sg-closed set is gs-closed, we have V is gs-closed set in Y. Since f is strongly gs-continuous, $f^{-1}(V^C)$ is open in X. But $f^{-1}(V^C) = X - f^{-1}(V)$. Therefore $f^{-1}(V)$ is closed in X. By Remark 2.6, $f^{-1}(V)$ is τ*-g-closed in X. Thus, f is τ*-sg-continuous.
Converse of the theorem need not be true as seen from the following example.

Example 3.10-Let X=Y= {a, b, c}, τ = {X, ϕ, {a}, {b, c}} σ = {Y, ϕ, {a,}, {a, b}, {a, c}}. Let $f: X \to Y$ be an identity map. Then f is τ*-sg-continuous. But f is not strongly gs-continuous. Since for the gs-open set V = {a, b} in Y, $f^{-1}(V)$ = {a, b} is not open in X.

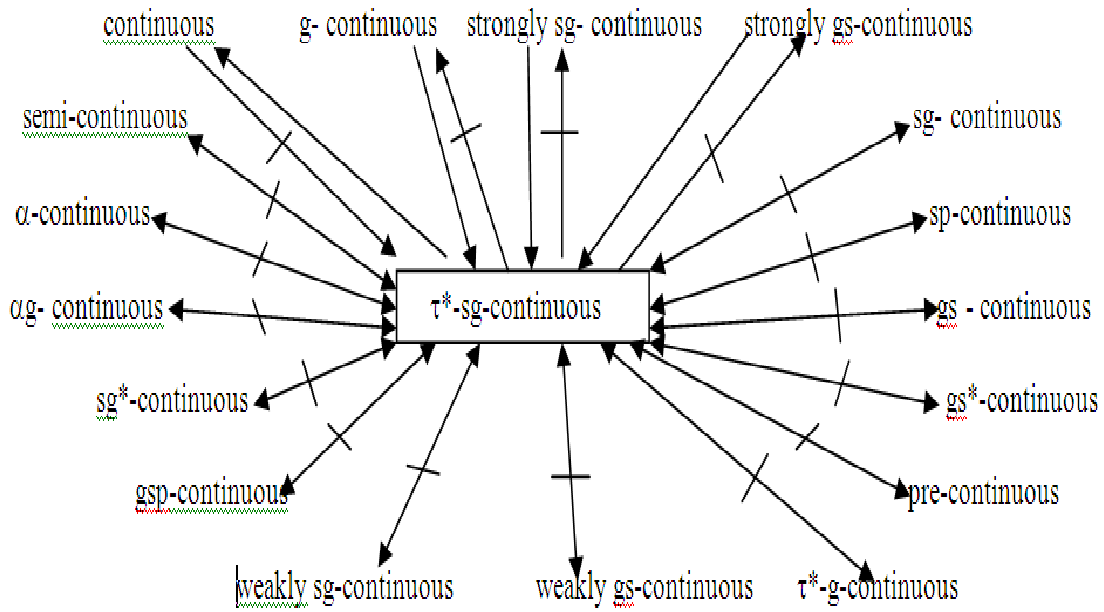
Remark 3.11-Following example shows that τ*-sg-continuous map is independent from the following mappings.

Let X = Y = {a, b, c}. Let $f: X \to Y$ be an identity map.

1) Let τ = {X, ϕ, {a}, {a, c}} and σ = {Y, ϕ, {a}, {a, b}, {a, c}}. Then f is semi-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {c} in Y, $f^{-1}(V)$ = {c} is not τ*-g-closed in X.
2) Let τ = {X, ϕ, {c}} and σ = {Y, ϕ, {b}, {c}, {b, c}}. Then f is τ*-sg-continuous But it is not semi-continuous. Since for the closed set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not semi-closed in X.
3) Let τ = {X, ϕ, {a}, {b}, {a, b}, {a, c}} and σ = {Y, ϕ, {a}, {a, c}}. Then f is sg-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a, b} in Y, $f^{-1}(V)$ = {a, b} is not τ*-g-closed in X.
4) Let τ = {X, ϕ, {c}} and σ = {Y, ϕ, {b}, {c}, {b, c}}. Then f is τ*-sg-continuous But it is not sg-continuous. Since for the closed set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not sg-closed in X.
5) Let τ = {X, ϕ, {a, b}} and σ ={Y, ϕ, {a}, {b}, {a, b}}. Then f is gs-continuous. But it is not τ*-sg-continuous. Since for the gs-closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not τ*-g-closed in X.
6) Let τ = {X, ϕ, {a}}, σ ={Y, ϕ, {b}, {c}, {b, c}, {a, c}} Then f is τ*-sg-continuous But it is not gs-continuous. Since for the closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not gs-closed in X.
7) Let τ = {X, ϕ, {a, c}} and σ ={Y, ϕ, {a}, {c}, {a, c}, {a, b}}. Then f is gsp-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {c} in Y, $f^{-1}(V)$ = {c} is not τ*-g-closed in X.
8) Let τ = {X, ϕ, {c}}, σ ={Y, ϕ, {a}, {b}, {a, b}, {b, c}}. Then f is τ*-sg-continuous But it is not gsp-continuous. Since for the closed set V = {c} in Y, $f^{-1}(V)$ = {c} is not gsp-closed in X.
9) Let τ = {X, ϕ, {a, b}} and σ ={Y, ϕ, {b}, {a, b}}. Then f is αg-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not τ*-g-closed in X.
10) Let τ = {X, ϕ, {b}}, σ ={Y, ϕ, {c}, {a, c}}. Then f is τ*-sg-continuous But it is not αg -continuous. Since for the closed set V = {b} in Y, $f^{-1}(V)$ = {b} is not αg-closed in X.
11) Let τ = {X, ϕ, {a, b}} and σ ={Y, ϕ, {a}, {b}, {a, b}}. Then f is pre-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not τ*-g-closed in X.
12) Let τ = {X, ϕ, {a}}, σ ={Y, ϕ, {c}, {a, c}, {b, c}}. Then f is τ*-sg-continuous But it is not pre-continuous. Since for the open set V = {b, c} in Y, $f^{-1}(V)$ = {b, c} is not pre-open in X.
13) Let τ = {X, ϕ, {c}, {b, c}} and σ ={Y, ϕ, {c}, {a, c}, {b, c}}. Then f is α-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {b} in Y, $f^{-1}(V)$ = {b} is not τ*-g-closed in X.
14) Let τ = {X, ϕ, {a}, {b, c}}, σ ={Y, ϕ, {a}, {c}, {a, c}, {b, c}}.. Then f is τ*-sg-continuous But it is not α-continuous. Since for the open set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not α-open in X.
15) Let τ = {X, ϕ, {a}, {a, c}} and σ ={Y, ϕ, {a}, {a, b}, {a, c}}. Then f is sp-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {c} in Y, $f^{-1}(V)$ = {c} is not τ*-g-closed in X.
16) Let τ = {X, ϕ, {c}}, σ ={Y, ϕ, {b}, {c}, {b, c}, {a, c}}. Then f is τ*-sg-continuous. But it is not sp-continuous. Since for the open set V = {b} in Y, $f^{-1}(V)$ = {b} is not sp-open in X.
17) Let τ = {X, ϕ, {c}, {b, c}} and σ ={Y, ϕ, {c}, {a, c}, {b, c}}. Then f is weakly sg-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {b} in Y, $f^{-1}(V)$ = {b} is not τ*-g-closed in X.
18) Let τ = {X, ϕ, {a}, {b, c}} and σ ={Y, ϕ, {c}, {b, c}}. Then f is τ*-sg-continuous. But it is not weakly sg-continuous. Since for the sg-open set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not semiopen in X.
19) Let τ = {X, ϕ, {c}, {a, c}} and σ ={Y, ϕ, {c}, {a, c}, {b, c}}.Then f is weakly gs-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not τ*-g-closed in X.
20) Let τ = {X, ϕ, {a}, {b, c}} and σ ={Y, ϕ, {c}, {a, c}, {b, c}}. Then f is τ*-sg-continuous. But it is not weakly gs-continuous. Since for the gs-open set V = {c} in Y, $f^{-1}(V)$ = {c} is not semiopen in X.
21) Let τ = {X, ϕ, {c}, {a, c}} and σ ={Y, ϕ, {c}, {a, c}, {b, c}}. Then f is sg*-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not τ*-g-closed in X.
22) Let τ = {X, ϕ, {b}} and σ ={Y, ϕ, {a}, {a, b}, {a, c}}. Then f is τ*-sg-continuous. But it is not sg*-continuous. Since for the semiopen set V = {a} in Y, $f^{-1}(V)$ = {a} is not sg-open in X.
23) Let τ = {X, ϕ, {a}, {a, b}, {a, c}} and σ ={Y, ϕ,{a, b}}. Then f is gs*-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not τ*-g-closed in X.
24) Let τ = {X, ϕ, {b}, {a, b}} and σ ={Y, ϕ, {a}}. Then f is τ*-sg-continuous. But it is not gs*-continuous. Since for the semiopen set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not gs-open in X.
25) Let τ = {X, ϕ, {a}, {a, b}} and σ ={Y, ϕ, {a}, {c}, {a, c}}. Let $f: X \to Y$ be defined by f(a) = c, f(b) = a and f(c) = b . Then f is τ*-g-continuous. But it is not τ*-sg-continuous. Since for the sg-closed set V = {a} in Y, $f^{-1}(V)$ = {b} is not τ*-g-closed in X.

26) Let τ = {X, ϕ, {b}, {c}, {b, c}, {a, b}} and σ ={Y, ϕ, {b}, {a, b}}. Then f is τ*-sg-continuous. But it is not τ*-g-continuous. Since for the g-closed set V = {b, c} in Y, $f^{-1}(V)$ = {b, c} is not τ*g-closed in X.

Remark 3.12-From the above discussion, we obtain the following implications.



A ⟶ B means A implies B, A ⟶ B means A does not imply B and A ⟷ B means A and B are independent

IV. References

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