Some Geometric And Physical Properties Of Generalized Peres Space-Time

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***Abstract*-In this paper we have investigated some geometric and physical properties of generalized Peres space-time P* in the sense of Takeno (1961) and made some discussions.**

***Keywords* - General relativity,plane fronted waves, Ricci tensor, curvature tensor, electromagnetic energy momentum tensor, co-ordinate condition,phase velocity, energy momentum pseudo tensors,parallel vector field vi etc.**

I. INTRODUCTION

The pioneer work of Einstein (1916, 18) and Rosen (1937) forms the corner stone of the investigations of plane gravitational waves in general relativity.Plane and plane fronted gravitational waves in general relativity have been studied by number of authors, Peres (1959), Takeno H (1961), Zakhanov (1973) etc.According to Takeno[4], a plane wave ij g is a non flat solution of $R_{ij}=0$ and in some co-ordinate system all its components are functions of A single variable $Z = Z(x^i)$. In formulating this definition he assumed a cartesian like co-ordinate system and obtained a plane wave metric for both $Z = (z - t)$ and $Z = (t z)$ type plane gravitational waves in 4 V represented by

$ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - (C - E)dz^2 - 2Edzdt + (C + E)dt^2$, and (1)

$ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - Z^2(C - E)dz^2 - 2ZEdzdt + (C + E)dt^2$ (2)

Where *A,B,C, D* and E are functions of *Z*.

A.Peres [2] has found an exact plane-wave like solutions of field equations by considering the line element of the space-time P ,

ds2 = - dx2 - dy2 - dz2 + dt 2 - 2 f (x, y,Z) (dz - dt)2 , (3)

where f = f (x, y,Z) satisfying $\partial_{11}f + \partial_{22}f = 0.$ (4)

He also showed that the curvature tensor of the space-time belongs to thesecond class of petrov's classifications .Furthermore Takeno[5] call the space-time *P* defined in (1.3) in which $f = f(x, y, Z)$ is an arbitrary function of its arguments and accordingly does not satisfy (1.4) in general, a space-time *P* and the coordinate system in which the metric takes the form (1.3), the (0)-system alongwith he investigated physical, geometrical and differential properties of space time (1.1), (1.2) and (1.3) respectively with respect to, phase velocity,the coordinate condition, the energy momentum pseudo tensor, parallel null vector field *Vi* , canonical form of curvature tensor, etc.

Takeno [3] considered the generalized Peres space-time represented by the metric

$ds^2 = - Adx^2 - Bdy^2 - (1- E)dz^2 - {}^2Edzdt + (1+ E)dt^2$ (5)

and we denote it by P^* space-time,

where $A = A(x, Z)$, $B = B(y, Z)$ and $E = E(x, y, Z)$; $Z = Z - t$, and satisfies Einsteins field equations for empty region. *E* is a function of *x*, *y* and Z satisfying,

$$\epsilon = 0 \text{ where } \epsilon = \frac{\alpha}{A} + \frac{\beta}{B}, \tag{6}$$

$$\alpha = \frac{1}{2}\left[\bar{A} - E_{xx} - \frac{1}{2}\left(\frac{A^2 - A_x E_x}{A}\right)\right], \beta = \frac{1}{2}\left[\bar{B} - E_{yy} - \frac{1}{2}\left(\frac{B^2 - B_y E_y}{B}\right)\right].$$

The space-time *P** have been studied by number of authors, Pandey [8] ,Lal and Pandey [7], Patel and Vaidya [6] etc. and obtained the plane wave solutions of the field equations. Lal and Pradhan [9] obtained the plane wave- like solution of weakend field equations. Recently Khapekar and Deshmukh [11] obtained the generalized Peres plane wave-like solution of Einstein Maxwell field equation in thepresence of null fluid and null currents for space- time (1.5).Summarizing above investigations, in this paper we have obtained and discussed some geometric and physical properties of space- time *P** in the sense of Takeno (1961).

II. THE CURVATURE TENSOR AND RICCI TENSOR

We confine to the space-time (1.5) for which it follows that

$$(g_{ij}) = \begin{bmatrix} -A & 0 & 0 & 0 \\ 0 & -B & 0 & 0 \\ 0 & 0 & -(1-E) & -E \\ 0 & 0 & -E & 1+E \end{bmatrix} \tag{1}$$

and

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$$(g^{ij}) = \begin{bmatrix} -1/A & 0 & 0 & 0 \\ 0 & -1/B & 0 & 0 \\ 0 & 0 & -(1+E) & -E \\ 0 & 0 & -E & 1-E \end{bmatrix} \tag{2}$$

The non-vanishing components of christoffel symbol, curvature tensor $_{ijkl}R$ and Ricci Tensor $_{ij}R$ are respectively,

$$\left.\begin{array}{l} \left\{\begin{matrix}1\\11\end{matrix}\right\} = \frac{A_x}{2A}, \left\{\begin{matrix}1\\13\end{matrix}\right\} = -\left\{\begin{matrix}1\\14\end{matrix}\right\} = \frac{\bar{A}}{2A}, \left\{\begin{matrix}1\\33\end{matrix}\right\} = -\left\{\begin{matrix}1\\34\end{matrix}\right\} = \left\{\begin{matrix}1\\44\end{matrix}\right\} = \frac{E_x}{2A}, \\ \left\{\begin{matrix}2\\22\end{matrix}\right\} = \frac{B_y}{2B}, \left\{\begin{matrix}2\\23\end{matrix}\right\} = -\left\{\begin{matrix}2\\24\end{matrix}\right\} = \frac{\bar{B}}{2B}, \left\{\begin{matrix}2\\33\end{matrix}\right\} = -\left\{\begin{matrix}2\\34\end{matrix}\right\} = \left\{\begin{matrix}2\\44\end{matrix}\right\} = \frac{E_y}{2B}, \\ \left\{\begin{matrix}3\\11\end{matrix}\right\} = \left\{\begin{matrix}4\\11\end{matrix}\right\} = -\frac{\bar{A}}{2}, \left\{\begin{matrix}3\\22\end{matrix}\right\} = \left\{\begin{matrix}4\\22\end{matrix}\right\} = -\frac{\bar{B}}{2}, \left\{\begin{matrix}a\\13\end{matrix}\right\} = -\left\{\begin{matrix}a\\14\end{matrix}\right\} = -\frac{E_x}{2} \\ \left\{\begin{matrix}a\\23\end{matrix}\right\} = -\left\{\begin{matrix}a\\24\end{matrix}\right\} = -\frac{E_y}{2}, \left\{\begin{matrix}a\\33\end{matrix}\right\} = -\left\{\begin{matrix}a\\34\end{matrix}\right\} = \left\{\begin{matrix}a\\44\end{matrix}\right\} = -\frac{\bar{E}}{2}, \text{where a =3,4.} \end{array}\right\} \tag{3}$$

Here bar **(-)** over a letter denotes the derivative with respect to Z.The non-vanishing components of the curvature tensor R_{ijkl} and Ricci tensor R_{ij} are

$$\left.\begin{array}{l} R_{1313} = -R_{1314} = R_{1414} = \alpha, \\ R_{1323} = -R_{1324} = -R_{1423} = R_{1424} = \frac{1}{2}E_{xy}, \end{array}\right\} \tag{4}$$

$$R_{2323} = -R_{2324} = R_{2424} = \beta.$$

$$R_{33} = -R_{34} = R_{44} = \epsilon \quad . \tag{5}$$

From (2.1) and (2.5)

$$R = g^{ij}R_{ij} = 0.$$

Hence from (2.3) we have,

Theorem - A condition that a space-time (1.5) be the Minkoswkian is

$$\alpha = \beta = E_{xy} = 0, \tag{6}$$

in the *(0)*- system

III. Exact solution of field equations in the case when an electromagnetic field is present

We shall consider the following field equations for a system in which an electromagnetic field $_{ij}F$ co-exists with gravitational field.

$$R_{ij} = -8\pi E_{ij}, \quad (i,j,k = 1,\ldots,4), \tag{1}$$

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0, \tag{2}$$

$$F^{ij}_{;j} = 0. \tag{3}$$

Where E_{ij} is the electromagnetic energy tensor. We consider in (1.5)
a transverse electromagnetic field whose components in the (0)-system are,

$$F_{31} = F_{14} = \sigma, \quad -F_{23} = F_{24} = -\rho \qquad F_{12} = F_{34} = 0, \tag{4}$$

where r and s are functions of x, y and Z .
The non-vanishing components of E $_{ij}$ are,

$$E_{33} = -E_{34} = E_{44} = \frac{\sigma^2}{A} + \frac{\rho^2}{B}, \tag{5}$$

and the field equations (8.3.1) becomes,

$$\epsilon = -8\pi\left(\frac{\sigma^2}{A} + \frac{\rho^2}{B}\right). \tag{6}$$

The generalized Maxwell equations (3.2) and (3.3) are equivalent to the following Cauchy-Riemann type equations:

$$\partial_1\rho = -\partial_2\sigma \quad , \quad \partial_2\rho = \partial_1\sigma. \tag{7}$$

Hence we have
Theorem : A necessary and sufficient conditions that ij g given by (2.1) and ij F given by (3.4) where E, r, s are functions of x, y and Z satisfy the field equations (3.1), (3.2) and (3.3) is that E, r, s satisfy (3.6) and (3.7).

IV. Some properties of P* and F ij

Some tensor properties of P^* and F_{ij} can be obtained as direct consequences of formulas in section §2, which we shall enumerate in terms of simple theorems

Theorem 1-g ^{ij} given by (1.5) cannot satisfy the co-ordinate condition

$$\frac{\partial}{\partial x^j}\left(\sqrt{-g}\,g^{ij}\right) = 0.$$

Theorem 2 -The phase velocity of the waves is not the fundamental velocity general in the sense that the equiphase hypersurface E = constant are not null in general. The velocity is the fundamental only when the (1.5) is Minkowskian.

Proof -We can consider that $_{ij}g$ given by (1.5) is of the form $g_{ij}\,g_{ij}(E)$ =,where the phase function E is the$E = E(x, y, Z)$ i.e the phase velocity propogating in the negative direction of z-axis.
Consider the relation

$$g^{ij}(\partial_i E)(\partial_j E) = 0,$$

$$g^{11}(\partial_1 E)(\partial_1 E) + g^{22}(\partial_2 E)(\partial_2 E) + g^{33}(\partial_3 E)^2 + 2g^{34}(\partial_3 E)(\partial_4 E) + g^{44}(\partial_4 E)^2 = 0,$$

$$-\frac{E_x^2}{A} - \frac{E_y^2}{B} - \bar{E}^2 - E\bar{E}^2 + 2E\bar{E}^2 + \bar{E}^2 - E\bar{E}^2 = 0,$$

$$\frac{E_x^2}{A} + \frac{E_y^2}{B} = 0,$$

$$\frac{E_x^2}{A} = 0 \quad \text{or} \quad \frac{E_y^2}{B} = 0,$$

$$E_x = 0 \quad \text{or} \quad E_y = 0 \quad , \quad \text{since A, B} \neq 0,$$

$$\therefore \partial_1 E = \partial_2 E = 0.$$

Hence E is independent of x and y or E = constant.
Hence the result.

Theorem 3-The g_{ij} of (1.5) is null in the sense of Ehlers and Sachs,

$$i.e \quad R_{ijkl}R^{kl}_{pq} = 0.$$

Proof-

$$R_{ijkl}R^{kl}_{pq} = R_{ij13}R^{13}_{pq} + R_{ij14}R^{14}_{pq} + R_{ij23}R^{23}_{pq} + R_{ij24}R^{24}_{pq}$$

$$= (R_{1313} + R_{1413} + R_{2313} + R_{2413})(g^{11}g^{33}R_{1313} + g^{11}g^{33}R_{1413} + g^{11}g^{33}R_{2313} + g^{11}g^{33}R_{2413} + g^{11}g^{34}R_{1314} + g^{11}g^{34}R_{1414} + g^{11}g^{34}R_{2314} + g^{11}g^{34}R_{2414})$$

$$+ (R_{1314} + R_{1414} + R_{2314} + R_{2414})(g^{11}g^{43}R_{1313} + g^{11}g^{43}R_{1413} + g^{11}g^{43}R_{2313} + g^{11}g^{43}R_{2413} + g^{11}g^{44}R_{1314} + g^{11}g^{44}R_{1414} + g^{11}g^{44}R_{2314} + g^{11}g^{44}R_{2414})$$

$$+ (R_{1323} + R_{1423} + R_{2323} + R_{2423})(g^{22}g^{33}R_{1323} + g^{22}g^{33}R_{1423} + g^{22}g^{33}R_{2323} + g^{22}g^{33}R_{2423} + g^{22}g^{33}R_{1324} + g^{22}g^{33}R_{1424} + g^{22}g^{33}R_{2324} + g^{22}g^{33}R_{2424})$$

$$+ (R_{1324} + R_{1424} + R_{2324} + R_{2424})(g^{22}g^{43}R_{1323} + g^{22}g^{43}R_{1423} + g^{22}g^{43}R_{2323} + g^{22}g^{43}R_{2423} + g^{22}g^{44}R_{1324} + g^{22}g^{44}R_{1424} + g^{22}g^{44}R_{2324} + g^{22}g^{44}R_{2424})$$

$$= \left(\alpha - \alpha + \frac{1}{2}E_{xy} - \frac{1}{2}E_{xy}\right)(\kappa_1) + \left(-\alpha + \alpha - \frac{1}{2}E_{xy} + \frac{1}{2}E_{xy}\right)(\kappa_2)$$

$$+ \left(\frac{1}{2}E_{xy} - \frac{1}{2}E_{xy} + \beta - \beta\right)(\kappa_3) + \left(-\frac{1}{2}E_{xy} + \frac{1}{2}E_{xy} - \beta + \beta\right)(\kappa_4)$$

$= 0,$ [From(2.5)].

Where

$$\kappa_1 = (g^{11}g^{33}R_{1313} + g^{11}g^{33}R_{1413} + g^{11}g^{33}R_{2313} + g^{11}g^{33}R_{2413} + g^{11}g^{34}R_{1314} + g^{11}g^{34}R_{1414} + g^{11}g^{34}R_{2314} + g^{11}g^{34}R_{2414}),$$

$$\kappa_2 = (g^{11}g^{43}R_{1313} + g^{11}g^{43}R_{1413} + g^{11}g^{43}R_{2313} + g^{11}g^{43}R_{2413} + g^{11}g^{44}R_{1314} + g^{11}g^{44}R_{1414} + g^{11}g^{44}R_{2314} + g^{11}g^{44}R_{2414}),$$

$$\kappa_3 = (g^{22}g^{33}R_{1323} + g^{22}g^{33}R_{1423} + g^{22}g^{33}R_{2323} + g^{22}g^{33}R_{2423} + g^{22}g^{33}R_{1324} + g^{22}g^{33}R_{1424} + g^{22}g^{33}R_{2324} + g^{22}g^{33}R_{2424}),$$

$$\kappa_4 = (g^{22}g^{43}R_{1323} + g^{22}g^{43}R_{1423} + g^{22}g^{43}R_{2323} + g^{22}g^{43}R_{2423} + g^{22}g^{44}R_{1324} + g^{22}g^{44}R_{1424} + g^{22}g^{44}R_{2324} + g^{22}g^{44}R_{2424}).$$

Theorem 4-Four eigen values of Ricci tensor are all zero.

Proof -Consider, $\det(R_{ij} - \lambda g_{ij}) = 0,$

$$\begin{vmatrix} \lambda A & 0 & 0 & 0 \\ 0 & \lambda B & 0 & 0 \\ 0 & 0 & \epsilon + \lambda(1-E) & -\epsilon + \lambda E \\ 0 & 0 & -\epsilon + \lambda E & \epsilon - \lambda(1+E) \end{vmatrix} = 0,$$

$$\lambda^4 = 0.$$

Theorem 5:-Four eigen values of Fij are all zero.

Proof -Consider $\det(F_{ij} - \lambda g_{ij}) = 0$

$$\begin{vmatrix} \lambda A & 0 & -\sigma & \sigma \\ 0 & \lambda B & \rho & -\rho \\ \sigma & -\rho & \lambda(1-E) & \lambda E \\ -\sigma & \rho & \lambda E & -\lambda(1+E) \end{vmatrix} = 0,$$

$$\lambda^4 = 0.$$

$$R_{ij}R^{jl} = 0$$

Theorem 6-

Proof-

$$R_{ij}R^{jl} = g^{jr}g^{ls}R_{ij}R_{rs}$$

$$= g^{jr}g^{ls}R_{1j}R_{rs} + g^{jr}g^{ls}R_{2j}R_{rs} + g^{jr}g^{ls}R_{3j}R_{rs} + g^{jr}g^{ls}R_{4j}R_{rs}$$

$$= g^{jr}(R_{1j} + R_{2j} + R_{3j} + R_{4j})R_{rs}g^{ls}$$

$$= [g^{1r}(R_{11} + R_{21} + R_{31} + R_{41}) + g^{2r}(R_{12} + R_{22} + R_{32} + R_{42}) + g^{3r}(R_{13} + R_{23} + R_{33} + R_{43}) + g^{4r}(R_{14} + R_{24} + R_{34} + R_{44})]R_{rs}g^{ls}$$

$$= (R_{33} + R_{34})(g^{3r} + g^{4r})R_{rs}g^{ls}$$

$= 0,$ [using(2.5)].

$$F_{ij}F^{jk}F_{kl} = 0$$

Theorem 7-

Proof-

$$F_{ij}F^{jk}F_{kl} = F_{ij}\{F^{j1}F_{1l} + F^{j2}F_{2l} + F^{j3}F_{3l} + F^{j4}F_{4l}\}$$

$$= F_{1l}(F_{ij}F^{j1}) + F_{2l}(F_{ij}F^{j2}) + F_{3l}(F_{ij}F^{j3}) + F_{4l}(F_{ij}F^{j4})$$

$$= F_{1l}(F_{i3}F^{31} + F_{i4}F^{41}) + F_{2l}(F_{i3}F^{32} + F_{i4}F^{42}) + F_{3l}(F_{i1}F^{13} + F_{i2}F^{23}) + F_{4l}(F_{i1}F^{14} + F_{i2}F^{24})$$

$$= F_{1l}(F_{i3} + F_{i4})F^{41} + F_{2l}(F_{i3} + F_{i4})F^{23} + (F_{3l} + F_{4l})(F_{i1}F^{13} + F_{i2}F^{24})$$

$= 0,$ where $(F^{i3} = F^{i4}, F_{i3} = -F_{i4})$, [using(3.4)].

Theorem *8*- $R_{ijkl}F^{kl} = 0$.

***Proof*-**

$$\begin{aligned}R_{ijkl}F^{kl} &= R_{ij1l}F^{1l} + R_{ij2l}F^{2l} + R_{ij3l}F^{3l} + R_{ij4l}F^{4l}\\ &= R_{ij11}F^{11} + R_{ij12}F^{12} + R_{ij13}F^{13} + R_{ij14}F^{14} + R_{ij21}F^{21}\\ &+ R_{ij22}F^{22} + R_{ij23}F^{23} + R_{ij24}F^{24} + R_{ij31}F^{31} + R_{ij32}F^{32}\\ &+ R_{ij33}F^{33} + R_{ij34}F^{34} + R_{ij41}F^{41} + R_{ij42}F^{42} + R_{ij43}F^{43}\\ &+ R_{ij44}F^{44}\\ &= 2\left[R_{ij13}F^{13} + R_{ij14}F^{14} + R_{ij23}F^{23} + R_{ij24}F^{24}\right]\\ &= 2\left[\left(R_{ij13} + R_{ij14}\right)F^{14} + \left(R_{ij23} + R_{ij24}\right)F^{24}\right]\\ &= 0. \qquad [\text{using}(2.4)]\end{aligned}$$

Theorem*9*- $R_{ij}R^{jl} = 0$

***Proof*-**

$$\begin{aligned}R_{ij}R^{jl} &= g^{jr}g^{ls}R_{ij}R_{rs}\\ &= g^{jr}\left(g^{l3}R_{r3} + g^{l4}R_{r4}\right)R_{ij}\\ &= R_{ij}\left[g^{j3}g^{l3}R_{33} + g^{j3}g^{l4}R_{34} + g^{j4}g^{l3}R_{43} + g^{j4}g^{l4}R_{44}\right]\\ &= \epsilon\ R_{ij}\left(g^{l3} - g^{l4}\right)\left(g^{j3} - g^{j4}\right)\\ &= \epsilon\ R_{ij}\left(g^{l3} - g^{l4}\right)\left[R_{i3}\left(g^{33} - g^{34}\right) + R_{i4}\left(g^{43} - g^{44}\right)\right]\\ &= -\epsilon\left(g^{l3} - g^{l4}\right)\left[R_{i3} + R_{i4}\right],\\ &= 0, \qquad (\because R_{i3} + R_{i4} = 0), \qquad [\text{using}(2.5)].\end{aligned}$$

Theorem *10*-The curvature tensor satisfies the identity

$$R_{ijkl}R^{jk} = 0.$$

***Proof*-**

$$\begin{aligned}R_{ijkl}R^{jk} &= R_{ijkl}g^{jr}g^{ks}R_{rs}\\ &= R_{ijkl}\left[g^{j3}g^{k3}R_{33} + g^{j3}g^{k4}R_{34} + g^{j4}g^{k3}R_{43} + g^{j4}g^{k4}R_{44}\right]\\ &= R_{ijkl}\,\epsilon\left(g^{j3} - g^{j4}\right)\left(g^{k3} - g^{k4}\right)\\ &= \epsilon\left[\left(g^{33} - g^{34}\right)R_{i33l}\left(g^{33} - g^{34}\right) + \left(g^{33} - g^{34}\right)\left(g^{43} - g^{44}\right)R_{i43l}\right.\\ &\quad \left. + \left(g^{43} - g^{44}\right)\left(g^{33} - g^{34}\right)R_{i34l} + \left(g^{43} - g^{44}\right)\left(g^{43} - g^{44}\right)R_{i44l}\right]\\ &= \epsilon\left[R_{i33l} + R_{i43l} + R_{i34l} + R_{i44l}\right]\\ &= 0, \qquad [\text{using}(2.4)\text{and }(2.5)].\end{aligned}$$

Theorem 11- The ij F is null in the sense of Synge,

$$\text{i.e}\quad F_{ij}F^{ij} = 0.$$

***Proof*-**

$$\begin{aligned}F_{ij}F^{ij} &= F_{1j}F^{1j} + F_{2j}F^{2j} + F_{3j}F^{3j} + F_{4j}F^{4j}\\ &= 2\left[F_{13}F^{13} + F_{14}F^{14} + F_{23}F^{23} + F_{24}F^{24}\right]\\ &= 2\left[F_{13}F^{13} - F_{13}F^{13} + F_{23}F^{23} - F_{23}F^{23}\right]\\ &= 0, \quad \text{where} \quad \left(F_{i3} = -F_{i4}, \quad F^{i3} = F^{i4}\right), [\text{using}(3.4)].\end{aligned}$$

Theorem *12*- $F^{*}_{ij}F^{ij} = 0$, *where* F^{*}_{ij} *is the dual of* F^{ij}.

$$F^{*}_{ij} = \frac{1}{2}\eta_{ijkl}F^{kl}$$

***Proof*-**We have,

Where η_{ijkl} is a tensor whose components changes the sign under interchange of any pair of indices and the components are zero in which pair of indices are identical. The only non-vanishing components are those for which all four indices are different.

$$\begin{aligned}F^{*}_{ij}F^{ij} &= \frac{1}{2}\eta_{ijkl}F^{kl}F^{ij}\\ &= F^{kl}\left[\eta_{13kl}F^{13} + \eta_{14kl}F^{14} + \eta_{23kl}F^{23} + \eta_{24kl}F^{24}\right]\\ &= F^{13}\left[\eta_{1313}F^{13} + \eta_{1413}F^{14} + \eta_{2313}F^{23} + \eta_{2413}F^{24}\right]\\ &\quad + F^{14}\left[\eta_{1314}F^{13} + \eta_{1414}F^{14} + \eta_{2314}F^{23} + \eta_{2414}F^{24}\right]\\ &\quad + F^{23}\left[\eta_{1323}F^{13} + \eta_{1423}F^{14} + \eta_{2323}F^{23} + \eta_{2423}F^{24}\right]\\ &\quad + F^{24}\left[\eta_{1324}F^{13} + \eta_{1424}F^{14} + \eta_{2324}F^{23} + \eta_{2424}F^{24}\right],\\ &= F^{13}F^{14}\left[\eta_{1413} + \eta_{1314}\right] + F^{23}F^{13}\left[\eta_{1323} + \eta_{2313}\right] + F^{24}F^{13}\left[\eta_{1324} + \eta_{2413}\right]\\ &\quad + F^{14}F^{23}\left[\eta_{1423} + \eta_{2314}\right] + F^{24}F^{14}\left[\eta_{1424} + \eta_{2414}\right] + F^{23}F^{24}\left[\eta_{2324} + \eta_{2423}\right],\\ &= F^{13}F^{14}\left[\eta_{1413} - \eta_{1413}\right] + F^{23}F^{13}\left[\eta_{1323} - \eta_{1323}\right] + F^{24}F^{13}\left[\eta_{1324} - \eta_{1324}\right]\\ &\quad + F^{14}F^{23}\left[\eta_{1423} - \eta_{1423}\right] + F^{24}F^{14}\left[\eta_{1424} - \eta_{1424}\right] + F^{23}F^{24}\left[\eta_{2324} - \eta_{2324}\right],\\ &= 0, \quad \text{where} \quad \left(F^{i3} = F^{i4}\right), \qquad [\text{using}(3.4)].\end{aligned}$$

V. ENERGY- MOMENTUM PSEUDO-TENSORS IN P*

With a view to some physical applications, in this section we shall first gives energy-momentum peudo-tensors of Einstein and of Landau-Lifshitz

Theorem-In P* , both the peudo-tensors of Einstein j i t and that of Landau-Lifshitz t*ij does not vanish in general in an O - system.

Proof-*PSEUDO-TENSOR OF EINSTEIN*

The formula which gives the energy-momentum pseudo-tensor introduced by Einstein as

$$\begin{aligned}16\pi\sqrt{-g}\,t_i^{\,j} &= \left\{\begin{matrix} j \\ mn \end{matrix}\right\}\left(\sqrt{-g}\,g^{mn}\right)_{,i} - \left(\log\sqrt{-g}\right)_{,m}\left(\sqrt{-g}\,g^{jm}\right)_{,i}\\ &\quad + \delta_i^{\,j}\left[\left\{\begin{matrix} h \\ mk \end{matrix}\right\}\left\{\begin{matrix} k \\ nh \end{matrix}\right\}g^{mn}\sqrt{-g} - g^{mn}\left\{\begin{matrix} h \\ mn \end{matrix}\right\}\left(\sqrt{-g}\right)_{,h}\right]\end{aligned} \qquad (1)$$

If we substitute (2.2) & (2.3) into (5.1), we can easily obtain components of energy-momentum tensor

$$t_3^{\,3} = t_3^{\,4} = -t_4^{\,3} = -t_4^{\,4} = \chi, \quad \text{other} \quad t_i^{\,j} = 0,$$

$$\text{where } \chi = \frac{\overline{A}\,\overline{B}}{16\pi\, m} \quad \text{and} \quad m = AB.$$

It means some components of Einstein pseudo-tensor survives , not vanish in general.

PSEUDO-TENSOR OF LANDAU AND LIFSHITZ

To calculate components of t^{*ij} , we use following formula proposed by Landau and Lifshitz

$$16\pi\, t^{*ij} = \left(g^{ik}g^{jl} - g^{ij}g^{kl}\right)\left[2\left\{\begin{matrix} h \\ kl \end{matrix}\right\}\left\{\begin{matrix} m \\ hm \end{matrix}\right\} - \left\{\begin{matrix} m \\ kh \end{matrix}\right\}\left\{\begin{matrix} h \\ lm \end{matrix}\right\} - \left\{\begin{matrix} h \\ kh \end{matrix}\right\}\left\{\begin{matrix} m \\ lm \end{matrix}\right\}\right]$$
$$+ g^{ik}g^{mn}\left[\left\{\begin{matrix} j \\ kh \end{matrix}\right\}\left\{\begin{matrix} h \\ mn \end{matrix}\right\} + \left\{\begin{matrix} j \\ mn \end{matrix}\right\}\left\{\begin{matrix} h \\ kh \end{matrix}\right\} - \left\{\begin{matrix} j \\ nh \end{matrix}\right\}\left\{\begin{matrix} h \\ km \end{matrix}\right\} - \left\{\begin{matrix} j \\ km \end{matrix}\right\}\left\{\begin{matrix} h \\ nh \end{matrix}\right\}\right]$$
$$+ g^{jk}g^{mn}\left[\left\{\begin{matrix} i \\ kh \end{matrix}\right\}\left\{\begin{matrix} h \\ mn \end{matrix}\right\} + \left\{\begin{matrix} i \\ mn \end{matrix}\right\}\left\{\begin{matrix} h \\ kh \end{matrix}\right\} - \left\{\begin{matrix} i \\ nh \end{matrix}\right\}\left\{\begin{matrix} h \\ km \end{matrix}\right\} - \left\{\begin{matrix} i \\ km \end{matrix}\right\}\left\{\begin{matrix} h \\ nh \end{matrix}\right\}\right]$$
$$- g^{hk}g^{mn}\left[\left\{\begin{matrix} i \\ hm \end{matrix}\right\}\left\{\begin{matrix} j \\ kn \end{matrix}\right\} - \left\{\begin{matrix} i \\ hk \end{matrix}\right\}\left\{\begin{matrix} j \\ mn \end{matrix}\right\}\right] \qquad (2)$$

Using (2.2) , (2.3) and (5.2) we obtained non-vanishing t^{*ij} are

$$t^{*33} = t^{*34} = t^{*43} = t^{*44} = \chi^{*}, \quad \text{other} \quad t^{*ij} = 0,$$

where
$$\chi^{*} = -\frac{\left[\left(\overline{A}\overline{B} + \overline{m}^{2}/2m\right)\right]}{m},$$
$$m = AB,$$
and
$$\overline{m} = A\overline{B} + B\overline{A}.$$

Again χ^{*} is not zero in general.

VI. Conclusions

We have investigated some geometrical and physical properties for Generalized Peres space-time *P** in the sense of Takeno (1961). We made following discussions:

i. If we assume that g^{ij} and F^{ij} satisfy (3.1), from theorem 2 we conclude that when the phase velocity is fundamental, *P** becomes Minkowskian,i.e the gravitational wave is apparent and the electromagnetic field F^{ij} must vanish.

ii. The result that both energy momentum pseudo *tensors(5.1)and (5.2)* survives in *P** and carries some energy or momentum in the direction of wave propogation which is striking contrast to the corresponding one for *P* where both tensor vanishes identically and does not carry energy or momentum in the direction of wave propogation but it is analogous to corresponding one for H given by Takeno(1961).

iii. When $A = B = 1$ and $E = 2f$ and when $A=A(Z)$, $B=B(Z)$ and $E = E(x,y,z,t)$ then the results for space-time P and space-time P1 consider by Lal and Pandey (1975) are the special cases of our investigation.

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