Subclass Of K−Uniformly Starlike Functions Associated With Wright Generalized Hypergeometric Functions

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***Abstract*- In this paper we consider the class of functions of the form**

$$f(z) = a_1 z - \sum_{n=2}^{\infty} a_n z^n, \ (a_n \geq 0),$$

that are satisfying the condition

$$(1-\mu)\frac{f(z_0)}{z_0} + \mu f'(z_0) = 1, \ (0 \leq \mu \leq 1; \ -1 < z_0 < 1 \text{ and } z_0 \neq 0).$$

We obtain coefficient bounds, distortion theorem and extreme points of the subclass of starlike functions defined by linear operator. Furthermore, we discuss radius of convexity and closure properties.

Keywords-Univalent, convex, starlike, uniformly convex, uniformly starlike, Linear operator.

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I. Introduction

Let S be the class of functions $f(z)$ that are analytic in the unit disc $U = \{z : |z| < 1\}$ with f(0) = 0. Denote by T, the subclass of *S* consists of functions of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \quad (a_n \geq 0) \tag{1}$$

and also denote T_1 ,the subclass of *S* consisting of functions of the form

$$f(z) = a_1 z - \sum_{n=2}^{\infty} a_n z^n, \ (a_n \geq 0, \ a_1 > 0).$$

$$\text{where, either } f(z_0) = z_0 \left(-1 < z_0 < 1; z_0 \neq 0\right) \text{ or, } f'(z_0) = 1 \ (-1 < z_0 < 1). \tag{2}$$

Let T_μ be the subclass of T_1 satisfying

$$(1-\mu)\frac{f(z_0)}{z_0} + \mu f'(z_0) = 1 \text{ where } (-1 < z_0 < 1), \ 0 \leq \mu \leq 1. \tag{3}$$

Following Goodman [9, 10] , Rønning[15] defined two subclasses of S,

(i)for the functions f in S is said to be k− uniformly starlike functions of order if γ if

$$\text{Re}\left\{\frac{zf'(z)}{f(z)} - \gamma\right\} > k\left|\frac{zf'(z)}{f(z)} - 1\right|, \ z \in U, \ -1 < \gamma \leq 1, \text{ and } k \geq 0$$

and (ii)for the functions f in S is said to be k− uniformly convex functions of order γ if

$$\text{Re}\left\{1 + \frac{zf''(z)}{f'(z)} - \gamma\right\} > k\left|\frac{zf''(z)}{f'(z)}\right|, \ z \in U, \ -1 < \gamma \leq 1, \text{ and } k \geq 0.$$

For positive real parameters $\alpha_1, A_1 \ldots, \alpha_p, A_p$ and $\beta_1, B_1 \ldots, \beta_q, B_q$ $(p, q \in N = 1, 2, 3, .$

that

$$1 + \sum_{n=1}^{q} B_n - \sum_{n=1}^{p} A_n \geq 0. \ \ z \in U. \tag{4}$$

The Wright generalized hypergeometric function[20]

$${}_p\Psi_q[(\alpha_1, A_1), \ldots, (\alpha_p, A_p); (\beta_1, B_1), \ldots, (\beta_q, B_q); z] = {}_p\Psi_q[(\alpha_n, A_n)_{1,p}(\beta_n, B_n)_{1,q}; z]$$

is defined by

$${}_p\Psi_q[(\alpha_t, A_t)_{1,p}(\beta_t, B_t)_{1,q}; z] = \sum_{n=0}^{\infty}\{\prod_{t=0}^{p}\Gamma(\alpha_t + nA_t)\}\{\prod_{t=0}^{q}\Gamma(\beta_t + nB_t)\}^{-1}\frac{z^n}{n!}, z \in U.$$

If $A_t = 1(t = 1, 2, ..., p)$ and $B_t = 1(t = 1, 2, ..., q)$we have the relationship:

$$\Omega_p\Psi_q[(\alpha_t, 1)_{1,p}(\beta_t, 1)_{1,q}; z] \equiv {}_pF_q(\alpha_1, \ldots \alpha_p; \beta_1, \ldots, \beta_q; z) = \sum_{n=0}^{\infty}\frac{(\alpha_1)_n \ldots (\alpha_p)_n}{(\beta_1)_n \ldots (\beta_q)_n}\frac{z^n}{n!} \tag{5}$$

$(p \leq q+1; \ p, q \in N_0 = N \cup \{0\}; z \in U)$ is the generalized hypergeometric function(see for details[8]) where N denotes the set of all positive integers and $(\alpha)_n$ is the Pochhammer symbol and

$$\Omega = \left(\prod_{t=0}^{p}\Gamma(\alpha_t)\right)^{-1}\left(\prod_{t=0}^{q}\Gamma(\beta_t)\right). \tag{6}$$

By using the generalized hypergeometric function Dziok and Srivastava [8] introduced the linear operator. In[4] Dziok and Raina extended the linear operator by using

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Wright generalized hypergeometric function . First we define a function

$$_p\phi_q[(\alpha_t, A_t)_{1,p}; (\beta_t, B_t)_{1,q}; z] = \Omega z_p\Psi_q[(\alpha_t, A_t)_{1,p}(\beta_t, B_t)_{1,q}; z].$$

Let $W[(\alpha_n, A_n)_{1,p}; (\beta_n, B_n)_{1,q}] : S \to S$ be a linear operator defined by

$$W[(\alpha_t, A_t)_{1,p}; (\beta_t, B_t)_{1,q}]f(z) := z\ _p\phi_q m[(\alpha_t, A_t)_{1,p}; (\beta_t, B_t)_{1,q}; z] * f(z)$$

We observe that , for f(z) of the form(1.1),we have

$$W[(\alpha_t, A_t)_{1,p}; (\beta_t, B_t)_{1,q}]f(z) = z + \sum_{n=2}^{\infty} \Omega\sigma_n(\alpha_1)\ a_n z^n \tag{7}$$

where Ω is given by (1.6)and $\sigma_n(\alpha_1)$is defined by

$$\sigma_n(\alpha_1) = \frac{\Gamma(\alpha_1 + A_1(n-1))\ldots\Gamma(\alpha_p + A_p(n-1))}{(n-1)!\Gamma(\beta_1 + B_1(n-1))\ldots\Gamma(\beta_q + B_q(n-1))}. \tag{8}$$

If, for convenience, we write

$$W[\alpha_1]f(z) = W[(\alpha_1, A_1), \ldots, (\alpha_l, A_p); (\beta_1, B_1), \ldots, (\beta_q, B_q)]f(z) \tag{9}$$

introduced by Dziok and Raina[4]. In view of the relationship (1.5) the linear operator(1.7) and by setting $A_t = 1(t = 1, ..., q)$ and $B_t = 1(t = 1, ..., s)$ we are led immediately to the aforementioned Dziok- Srivastava operator which contains, as its further special cases, such other linear operators of Geometric Function Theory as the Hohlov operator, the Carlson-Shaffer operator[3], the Ruscheweyh derivative operator[16], the generalized Bernardi-Libera-Livingston operator[2], the fractional derivative operator[7], and so on (see, for the precise relationships,Dziok and Srivastava [4, 5, 6, 8]).

For $-1 \le \gamma < 1,$ we let $W_a^p([\alpha_1], \gamma, k, z_0)$ denote the subclass of starlike functions corresponding to the family UCV for functions f(z) of the form (??) such that

$$\text{Re}\left\{\frac{z(W_q^p[\alpha_1]f(z))'}{W_q^p[\alpha_1]f(z)} - \gamma\right\} \ge k\left|\frac{z(W_q^p[\alpha_1]f(z))'}{W_q^p[\alpha_1]f(z)} - 1\right| \tag{10}$$

For $k \ge 0$ and $-1 \le \gamma < 1,$ we let

$$TW_q^p([\alpha_1], \gamma, k, z_0) = W_q^p([\alpha_1], \gamma, k, z_0) \cap T_\mu$$

the subclass of Tµ consisting of functions of the form (1.2) and satisfying the analytic criterion(1.3).

Using the techniques of Silverman [17] and motivated by the earlier works [12, 11, 14] and [19], in this paper we obtain the coefficient bounds, distortion bounds, extreme points, radius of starlikeness and closure theorems for the functions belong to the class $TW_q^p([\alpha_1], \gamma, k, z_0).$

II. Main Results

Theorem 1. A function f(z) of the form (1.2) is in the class $W_q^p([\alpha_1], \gamma, k, z_0)$ if and only if

$$\sum_{n=2}^{\infty}[n(1+\beta) - (\alpha+\beta)]\Omega\sigma_n(\alpha_1)a_n \le a_1(1-\alpha), \tag{1}$$

$-1 \le \gamma < 1, k \ge 0.$

The proof of the Theorem 1 is similar to that of Theorem 2.2, in [1], hence we omit the details.

Theorem 2-Let f(z) be defined by (1.2). Then $f(z) \in TS(\mu, \alpha, \beta, z_0)$ if and only if

$$\sum_{n=2}^{\infty}\left\{\frac{[n(1+k)-(k+\gamma)]}{1-\alpha}\Omega\sigma_n(\alpha_1) - [(1-\mu)+n\mu]z_0^{n-1}\right\}a_n \le 1, \tag{2}$$

$-1 \le \gamma < 1, k \ge 0.$

Proof- Since

$(1-\mu)\frac{f(z_0)}{z_0} + \mu f'(z_0) = 1,\ (0 \le \mu \le 1;\ -1 < z_0 < 1$ and $z_0 \ne 0)$, we get

$$a_1 = 1 + \sum_{n=2}^{\infty}[(1-\mu)+n\mu]a_n z_0^{n-1}.$$

Substituting for α_1 in (2.1) we get (2.2). (a)

Corollary 1-Let the function f(z) defined by (1.2) belongs $TW_q^p([\alpha_1], \gamma, k, z_0)$. *Then*

$$a_n \le \left\{\frac{[n(1+k)-(k+\gamma)]}{1-\alpha}\Omega\sigma_n(\alpha_1) - [(1-\mu)+n\mu]z_0^{n-1}\right\}^{-1}$$

$n \ge 2,\ -1 \le \alpha < 1,\ \beta \ge 0$ with equality for

$$f(z) = \frac{[n(1+k)-(k+\gamma)]\Omega\sigma_n(\alpha_1)z - (1-\gamma)z^n}{[n(1+k)-(k+\gamma)]\Omega\sigma_n(\alpha_1) - (1-\gamma)[(1-\mu)+n\mu]z_0^{n-1}} \tag{3}$$

Theorem 3-Let

$$f_1(z) = z \text{ and } f_n(z) = \frac{[n(1+k)-(k+\gamma)]\Omega\sigma_n(\alpha_1)z - (1-\gamma)z^n}{[n(1+k)-(k+\gamma)]\Omega\sigma_n(\alpha_1) - (1-\gamma)[(1-\mu)+n\mu]z_0^{n-1}}\ (n \ge 2). \tag{4}$$

Then $f(z) \in TW_q^p([\alpha_1], \gamma, k, z_0)$ if and only if it can be expressed in the form

$$f(z) = \sum_{n=1}^{\infty}\lambda_n f_n(z), \text{ where } \lambda_n \ge 0 \text{ and } \sum_{n=1}^{\infty}\lambda_n = 1.$$

Proof-The proof of the Theorem 3, follows on line similar to the proof of the theorem on extreme points given in Silverman [17] (b)

III. A Distortion Theorem

Theorem 4-Let the function f(z) defined by (1.2) belong to $TW_q^p([\alpha_1], \gamma, k, z_0).$

$$|f(z)| \ge a_1|z|\left\{1 - \frac{(1-\alpha)}{\Omega\sigma_2(\alpha_1)(2+k+\gamma)}|z|\right\} \tag{1}$$

$$|f(z)| \le a_1|z|\left\{1 + \frac{(1-\alpha)}{\Omega\sigma_2(\alpha_1)(2-k-\gamma)}|z|\right\} \tag{2}$$

for $z \in U$.

Proof-In the view of (2.1) and the fact that $\Omega\sigma_n(\alpha_1)$ is non-decreasing for $n \geq 2$, we have

$$\Omega\sigma_2(\alpha_1)(2-\alpha+\beta)\sum_{n=2}^{\infty} a_n \leq \sum_{n=2}^{\infty}[n(1+k)-(k+\gamma)]\Omega\sigma_n(\alpha_1)a_n \leq a_1(1-\gamma) \quad (3)$$

which is equivalent to,

$$\sum_{n=2}^{\infty} a_n \leq \frac{a_1(1-\gamma)}{\Omega\sigma_2(\alpha_1)(2+k-\gamma)}. \quad (4)$$

Using (1.2) and (3.4), we obtain

$$\begin{aligned}|f(z)| &\geq a_1|z| - |z|^2\sum_{n=2}^{\infty} a_n \\ &\geq a_1|z| - |z|^2\frac{a_1(1-\gamma)}{\Omega\sigma_2(\alpha_1)(2+k-\gamma)} \\ &\geq a_1|z|\left\{1-\frac{(1-\gamma)}{\Omega\sigma_2(\alpha_1)(2+k-\gamma)}|z|\right\}\end{aligned}$$

and

$$|f(z)| \leq a_1|z|\left\{1+\frac{(1-\gamma)}{\Omega\sigma_2(\alpha_1)(2+k-\gamma)}|z|\right\}. \quad (5)$$

IV. CLOSURE THEOREMS

Let the functions fj(z) be defined for j = 1, 2, . . .m by

$$f_j(z) = a_{1,j}\, z - \sum_{n=2}^{\infty} a_{n,j}\, z^n \quad (1)$$

$a_{1,j} > 0,\ a_{n,j} \geq 0,\ z \in U.$

Theorem 5. Let fj(z) defined by (4.1) be in the class $TW_q^p([\alpha_1],\gamma,k,z_0)$. Then the function h(z) defined by

$$h(z) = \sum_{j=1}^{m} d_j f_j(z), \quad d_j \geq 0 \quad (2)$$

also in the same class $TW_q^p([\alpha_1],\gamma,k,z_0)$, where

$$\sum_{j=1}^{m} d_j = 1. \quad (3)$$

Proof-From (4.2) we have

$$h(z) = b_1 z + \sum_{n=2}^{\infty} b_n z^n \quad (4)$$

where

$b_1 = \sum_{j=1}^{m} d_j a_{1,j}$ and $b_n = \sum_{j=1}^{m} d_j a_{n,j}\ (n = 2,3,\dots)$.

Since $f_j(z) \in TW_q^p([\alpha_1],\gamma,k,z_0)\ (j = 1,2,\dots m)$ and by applying Theorem 2, we get

$$\sum_{n=2}^{\infty}\left\{\frac{[n(1+k)-(k+\gamma)]}{1-\alpha}\Omega\sigma_n(\alpha_1) - [(1-\mu)+n\mu]z_0^{n-1}\right\}a_{n,j} \leq 1,\ (j = 1,2,\dots m).$$

Therefore, we have

$$\begin{aligned}&\sum_{n=2}^{\infty}\left\{\frac{[n(1+k)-(k+\gamma)]}{1-\alpha}\Omega\sigma_n(\alpha_1) - [(1-\mu)+n\mu]z_0^{n-1}\right\}\left(\sum_{j=1}^{m} d_j a_{n,j}\right) \\ &= \sum_{j=1}^{m} d_j\left[\sum_{n=2}^{\infty}\left\{\frac{[n(1+k)-(k+\gamma)]}{1-\alpha}\Omega\sigma_n(\alpha_1) - [(1-\mu)+n\mu]z_0^{n-1}\right\}a_{n,j}\right] \\ &\leq \sum_{j=1}^{m} d_j = 1 \quad (\text{by Theorem 2 and by } (4.3)).\end{aligned}$$

Which implies that h $h(z) \in TW_q^p([\alpha_1],\gamma,k,z_0)$ and so the proof is complete.

V. RADIUS OF CONVEXITY AND STARLIKENESS

In this section we obtain the radius of starlikeness of order $\delta\ (0 \leq \delta < 1)$, radius of convexity of order $\delta\ (0 \leq \delta < 1)$, for the class $TW_q^p([\alpha_1],\gamma,k,z_0)$.

Theorem 6-Let $f \in TS(\mu,\alpha,\beta,z_0)$. Then

1) f is starlike of order $\delta\ (0 \leq \delta < 1)$, in the disc $|z| < r_1$; that is, $Re\left\{\frac{zf'(z)}{f(z)}\right\} > \delta,\ (|z| < r_1\ ;\ 0 \leq \delta < 1)$, where

$$r_1 = \inf_{n \leq 2}\left\{\Omega\sigma_n(\alpha_1)\frac{1-\delta}{n-\delta}\frac{[n(1+k)-(k+\gamma)]}{1-\alpha}\right\}^{\frac{1}{n-1}}.$$

2) f is convex of order $\delta\ (0 \leq \delta < 1)$, in the unit disc $|z| < r_2$, that is Re $\left\{1+\frac{zf''(z)}{f'(z)}\right\} > \delta$, $(|z| < r_2; 0 \leq \delta < 1)$, where

$$r_2 = \inf_{n \leq 2}\left\{\Omega\sigma_n(\alpha_1)\frac{(1-\delta)[n(1+k)-(k+\gamma)]}{n(n-\delta)(1-\gamma)}\right\}^{\frac{1}{n-1}}.$$

Each of these results are sharp for the extremal function f(z) given by (2.4).

Proof-Given $f \in T_1$, and f is starlike of order δ, we have

$$\left|\frac{zf'(z)}{f(z)} - 1\right| < 1-\delta. \quad (1)$$

For the left hand side of (5.1) we have

$$\left|\frac{zf'(z)}{f(z)} - 1\right| \leq \frac{\sum_{n=2}^{\infty}(n-1)a_n\,|z|^{n-1}}{a_1 - \sum_{n=2}^{\infty} a_n\,|z|^{n-1}}.$$

The last expression is less than $1-\delta$ if

$$\sum_{n=2}^{\infty}\frac{n-\delta}{1-\delta}a_n\,|z|^{n-1} < a_1. \quad (2)$$

Substituting $a_1 = 1 + \sum_{n=2}^{\infty}[(1-\mu)+n\mu]a_n z_0^{n-1}$ in (5.2), we have

$$\sum_{n=2}^{\infty}\left\{\frac{n-\delta}{1-\delta}|z|^{n-1}-[(1-\mu)+n\mu]z_0^{n-1}\right\}a_n \le 1 \tag{3}$$

Using the fact, that $f \in TW_q^p([\alpha_1],\gamma,k,z_0)$ if and only if

$$\sum_{n=2}^{\infty}\left\{\frac{[n(1+k)-(k+\gamma)]}{1-\alpha}\Omega\sigma_n(\alpha_1)-[(1-\mu)+n\mu]z_0^{n-1}\right\}a_n < 1.$$

We can say (5.1) is true if

$$\frac{n-\delta}{1-\delta}|z|^{n-1}-[(1-\mu)+n\mu]z_0^{n-1} \le \frac{[n(1+k)-(k+\gamma)]}{1-\alpha}\Omega\sigma_n(\alpha_1)-[(1-\mu)+n\mu]z_0^{n-1}.$$

Or, equivalently,

$$|z|^{n-1} < \frac{(1-\delta)[n(1+k)-(k+\gamma)]}{(n-\delta)(1-\gamma)}\Omega\sigma_n(\alpha_1)$$

which yields the starlikeness of the family. (2) Using the fact that f is convex if and only if zf0 is starlike, we can prove (2), on lines similar the proof of (1). (a)

Remark-We note that the radius of starlikeness and convexity are independent of the fixed point z0.

VI. Convex Families

Suppose B is nonempty subset of the real interval (0, 1),we define $TW_q^p([\alpha_1],\gamma,k,B)$ by

$$TW_q^p([\alpha_1],\gamma,k,B)=\bigcup_{z_i\in B} TW_q^p([\alpha_1],\gamma,k,z_i).$$

If B consists of a single element say z0 then $TW_q^p([\alpha_1],\gamma,k,z_0)$ is a convex family. Because if f1(z) and f2(z) are in $TW_q^p([\alpha_1],\gamma,k,z_0)$, then it can be seen that for $0\le\lambda\le 1$, $\lambda f_1(z)+(1-\lambda)f_2(z)$ is in $TW_q^p([\alpha_1],\gamma,k,z_0)$. To examine this class for other subsets of B, we prove the following lemma

Lemma 1.- If $f(z)\in TW_q^p([\alpha_1],\gamma,k,z_0)\cap TW_q^p([\alpha_1],\gamma,k,z_1)$, where z0 and z1 are distinct positive numbers then f(z) = z.

Proof-For the functions of the form (1.2), we have

$$a_1=1+\sum_{n=2}^{\infty}a_n[(1-\mu)+n\mu]z_0^{n-1}$$

and

$$a_1=1+\sum_{n=2}^{\infty}a_n[(1-\mu)+n\mu]z_1^{n-1}.$$

That is,

$$a_n[(1-\mu)+n\mu][z_1^{n-1}-z_0^{n-1}]=0.$$

Hence $a_n\equiv 0$ for $n\ge 2$, and so the results follows. (a)

Theorem 7.- If B is contained in the interval (0, 1) and $0\le\alpha<1,\beta\ge 0$, then $TW_q^p([\alpha_1],\gamma,k,B)$ is a convex family if and only if B is connected.

Proof-Let B be connected. Suppose of the form (1.2) is in $z_0,z_1\in B$ with $z_0\le z_1$. If $f(z)\in TW_q^p([\alpha_1],\gamma,k,z_0)$ and $g(z)=b_1z-\sum_{n}^{\infty}b_nz^n$ is in $TW_q^p([\alpha_1],\gamma,k,z_1)$ then for $0\le\lambda\le 1$, we shall prove that there exists a z_2 $(z_0\le z_2\le z_1)$ such that $h(z)=\lambda f(z)+(1-\lambda)g(z)$ is in $TW_q^p([\alpha_1],\gamma,k,z_2)$. Set

$$\begin{aligned} t(z) &= [(1-\mu)\frac{h(z}{z}+\mu h'(z)] \\ &= \lambda\left[a_1-\sum_{n=2}^{\infty}a_nz^{n-1}((1-\mu)+n\mu)\right]+(1-\lambda)\left[b_1-\sum_{n=2}^{\infty}b_nz^{n-1}((1-\mu)+n\mu)\right] \\ t(z) &= 1+\lambda\sum_{n=2}^{\infty}[a_n\{(1-\mu)+n\mu\}(z_0^{n-1}-z^{n-1})] \\ &\quad +(1-\lambda)\sum_{n=2}^{\infty}[b_n\{(1-\mu)+n\mu\}(z_1^{n-1}-z^{n-1})] \end{aligned} \tag{6}$$

and we observe that f(z) is real when z is real with $t(z_0)\ge 1$ and $t(z_1)\le 1$.

Hence for some z_1, $z_0\le z_2\le z_1$, we have $t(z_2) = 1$. Since z_1, z_2 and γ are arbitrary, the family $TS(\mu,\alpha,\beta,B)$ Conversely, suppose B is not connected. Then we can take z0, z1 z0, z1 2 B, z_2 /2 B such that $z_0 < z_2 < z_1$. Let us assume f(z) and g(z) are not both identity function. Then using (6.1)

fixing z = z_2 and allow γ to vary,

$$\begin{aligned} t(z) &= t(z_2,\lambda) \\ &= 1+\lambda\sum_{n=2}^{\infty}[a_n\{(1-\mu)+n\mu\}(z_0^{n-1}-z_2^{n-1})] \\ &\quad +(1-\lambda)\sum_{n=2}^{\infty}[b_n\{(1-\mu)+n\mu\}(z_1^{n-1}-z_2^{n-1})]. \end{aligned}$$

Since $t(z_2,0)>1$ and $t(z_2,1)<1$, there must exists λ_0, $0<\lambda_0<1$, for which $t(z_2,\lambda_0)=1$. Hence $h(z)\in TW_q^p([\alpha_1],\gamma,k,z_2)$ for $\lambda=\lambda_0$. Since $z_2\notin B$

Since $z_2\notin B$ from the Lemma 1, it follows that $h(z)\in TW_q^p([\alpha_1],\gamma,k,B)$.

Therefore $\in TW_q^p([\alpha_1],\gamma,k,B)$. is not a convex family. (b)

Concluding Remarks

Observe that, if A_t = 1(t = 1, 2, ..., p) and B_t = 1(t = 1, 2, ..., q) specializing the parameters p, q, α_1, α2, . . . , αp, and $\beta_1,\beta_2,\ldots,\beta_q,\gamma$, in the class $TW_q^p([\alpha_1],\gamma,k,z_0)$ we obtain various classes introduced and studied in the literature (see [12, 14, 17, 19]).

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