Two Summation Formulae Based On Half Argument Associated To Hypergeometric Function

GJSFR-F; A.M.S. Subject Classification (2000): 33C60, 33C70

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***Abstract*-The aim of the present paper is to obtain two summation formulae based on half argument associated to Hypergeometric function.The results derived in this paper are of general character and are believed to be new.**

A.M.S. Subject Classification (2000): 33C60 , 33C70

Keywords and Phrases: Contiguous relation,Recurrence relation, Gauss second summation theorem.

I. Introduction

Generalized Gaussian Hypergeometric Function of one variable is difined by

$$ {}_AF_B\left[\begin{array}{lll} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} $$

or

$$ {}_AF_B\left[\begin{array}{lll} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right] \equiv {}_AF_B\left[\begin{array}{lll} (a_j)_{j=1}^A & ; & \\ & & z \\ (b_j)_{j=1}^B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

Contiguous Relation is defined by

[Andrews p.363(9.16), E. D. p.51(10), H.T. F. I p.103(32)]

$$ (a-b)\ {}_2F_1\left[\begin{array}{lll} a,\ b\ ; & & \\ c\ \ ; & & z \end{array}\right] = a\ {}_2F_1\left[\begin{array}{lll} a+1,\ b & ; & \\ c & ; & z \end{array}\right] - b\ {}_2F_1\left[\begin{array}{lll} a,\ b+1 & ; & \\ c\ ; & & z \end{array}\right] \tag{2} $$

Recurrence relation is defined by

$$ (z+1) = z\ \Gamma(z) \tag{3} $$

Gauss second summation theorm is defined by [Prud., 491(7.3.7.5)]

$$ {}_2F_1\left[\begin{array}{lll} a,\ b & ; & \\ \frac{a+b+1}{2} & ; & \frac{1}{2} \end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{4} $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{(b)\ \Gamma(\frac{a+1}{2})} \tag{5} $$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prud., p.491(7.3.7.3)]

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$$ {}_2F_1\left[\begin{matrix} a, \ b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & ; & \end{matrix}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{6} $$

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$ {}_2F_1\left[\begin{matrix} a, \ b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & ; & \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{7} $$

B. MAIN RESULTS OF SUMMATION FORMULAE

$$ {}_2F_1\left[\begin{matrix} a, \ b & ; & \frac{1}{2} \\ \frac{a+b+21}{2} & ; & \end{matrix}\right] = \frac{2^b\ \Gamma(\frac{a+b+21}{2})}{(a-b)\ \Gamma(b)} \times $$

$$ \times\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\left\{\frac{512a(-34459425+71697105a-53809164a^2+20570444a^3-4574934a^4)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+\right.\right. $$

$$ +\frac{512a(626934a^5-53676a^6+2796a^7-81a^8+a^9+127734435b-74042088ab+158767116a^2b)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+ $$

$$ +\frac{512a(-27708840a^3b+12096426a^4b-856824a^5b+129276a^6b-3192a^7b+171a^8b)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+ $$

$$ +\frac{512a(17316372b^2+224894564ab^2-24458852a^2b^2+50456476a^3b^2-2913460a^4b^2)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+ $$

$$ +\frac{512a(1236444a^5b^2-27132a^6b^2+3876a^7b^2+65285444b^3+10669336ab^3+65808020a^2b^3)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+ $$

$$ +\frac{512a(-2041360a^3b^3+3959980a^4b^3-69768a^5b^3+27132a^6b^3+6670026b^4+27977614ab^4)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+ $$

$$ +\frac{512a(1272620a^2b^4+4954820a^3b^4-41990a^4b^4+75582a^5b^4+3074466b^5+1144712ab^5)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+ $$

$$ +\frac{512a(2479348a^2b^5+33592a^3b^5+92378a^4b^5+150708b^6+453492ab^6+34884a^2b^6)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+ $$

$$+\frac{512a(50388a^3b^6+22116b^7+7752ab^7+11628a^2b^7+399b^8+969ab^8+19b^9)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512b(-34459425+127734435a+17316372a^2+65285444a^3+6670026a^4+3074466a^5)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512b(150708a^6+22116a^7+399a^8+19a^9+71697105b-74042088ab+224894564a^2b)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512b(10669336a^3b+27977614a^4b+1144712a^5b+453492a^6b+7752a^7b+969a^8b)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512b(-53809164b^2+158767116ab^2-24458852a^2b^2+65808020a^3b^2+1272620a^4b^2)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512b(2479348a^5b^2+34884a^6b^2+11628a^7b^2+205704444b^3-27708840ab^3+50456476a^2b^3)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512b(-2041360a^3b^3+4954820a^4b^3+33592a^5b^3+50388a^6b^3-4574934b^4+12096426ab^4)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512b(-2913460a^2b^4+3959980a^3b^4-41990a^4b^4+92378a^5b^4+626934b^5-856824ab^5)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512b(1236444a^2b^5-69768a^3b^5+75582a^4b^5-53676b^6+129276ab^6-27132a^2b^6)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512b(27132a^3b^6+2796b^7-3192ab^7+3876a^2b^7-81b^8+171ab^8+b^9)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{1024(34459425+127734435a-17316372a^2+65285444a^3-6670026a^4)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{1024(3074466a^5-150708a^6+22116a^7-399a^8+19a^9+71697105b+74042088ab)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{1024(224894564a^2b-10669336a^3b+27977614a^4b-1144712a^5b+453492a^6b-7752a^7b)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{1024(969a^8b+53809164b^2+158767116ab^2+24458852a^2b^2+65808020a^3b^2-1272620a^4b^2)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{1024(2479348a^5b^2-34884a^6b^2+11628a^7b^2+205704444b^3+27708840ab^3+50456476a^2b^3)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{1024(2041360a^3b^3+4954820a^4b^3-33592a^5b^3+50388a^6b^3+4574934b^4+12096426ab^4)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{1024(2913460a^2b^4+3959980a^3b^4+41990a^4b^4+92378a^5b^4+626934b^5+856824ab^5)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{1024(1236444a^2b^5+69768a^3b^5+75582a^4b^5+53676b^6+129276ab^6+27132a^2b^6)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{1024(27132a^3b^6+2796b^7+3192ab^7+3876a^2b^7+81b^8+171ab^8+b^9)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{1024(34459425+71697105a+53809164a^2+20570444a^3+4574934a^4+626934a^5+53676a^6)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{1024(2796a^7+81a^8+a^9+1277344335b+74042088ab+158767116a^2b+27708840a^3b)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{1024(12096426a^4b+856824a^5b+129276a^6b+3192a^7b+171a^8b-17316372b^2)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{1024(224894564ab^2+24458852a^2b^2+50456476a^3b^2+2913460a^4b^2+1236444a^5b^2)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{1024(27132a^6b^2+3876a^7b^2+652855444b^3-10669336ab^3+65808020a^2b^3+2041360a^3b^3)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{1024(3959980a^4b^3+69768a^5b^3+27132a^6b^3-6670026b^4+27977614ab^4-1272620a^2b^4)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{1024(4954820a^3b^4+41990a^4b^4+75582a^5b^4+3074466b^5-1144712ab^5+2479348a^2b^5)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{1024(-33592a^3b^5+92378a^4b^5-150708b^6+453492ab^6-34884a^2b^6+50388a^3b^6)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{1024(22116b^7-7752ab^7+11628a^2b^7-399b^8+969ab^8+19b^9)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}\Bigg\}\Bigg] \qquad (8)$$

$${}_2F_1\left[\begin{matrix} a, \quad b & ; & \frac{1}{2} \\ \frac{a+b+22}{2} & ; & \end{matrix}\right]=\frac{2^b\,\Gamma(\frac{a+b+22}{2})}{(a-b)\,\Gamma(b)}\times$$

$$\times\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{1024(-185794560a+262803456a^2-150105600a^3+46315520a^4-8618400a^5+1012368a^6)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}\times\right.$$

$$+\frac{1024(-75600a^7+3480a^8-90a^9+a^{10}+185794560b+802676736ab-241497600a^2b)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{1024(451573760a^3b-62262240a^4b+22911264a^5b-1390800a^6b+182400a^7b-3990a^8b)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{1024(190a^9b+262803456b^2+241497600ab^2+886415360a^2b^2-62739520a^3b^2)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{1024(115995760a^4b^2-5555600a^5b^2+2015520a^6b^2-38760a^7b^2+4845a^8b^2+150105600b^3)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{1024(451573760ab^3+62739520a^2b^3+193593280a^3b^3-4522000a^4b^3+7648640a^5b^3)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{1024(-116280a^6b^3+38760a^7b^3+46315520b^4+62262240ab^4+115995760a^2b^4+4522000a^3b^4)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{1024(11757200a^4b^4-83980a^5b^4+125970a^6b^4+8618400b^5+22911264ab^5+5555600a^2b^5)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{1024(7648640a^3b^5+83980a^4b^5+184756a^5b^5+1012368b^6+1390800ab^6+2015520a^2b^6)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{1024(116280a^3b^6+125970a^4b^6+75600b^7+182400ab^7+38760a^2b^7+38760a^3b^7)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{1024(3480b^8+3990ab^8+4845a^2b^8+90b^9+190ab^9+b^{10})}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{4096b(166035456a+34142208a^2+43209472a^3+4122240a^4+1370064a^5+60672a^6+7368a^7)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}\times$$

$$+\frac{4096b(120a^8+5a^9+166035456b+192064768a^2b+14013184a^3b+14821520a^4b+593408a^5b)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{4096b(172824a^6b+2736a^7b+285a^8b-34142208b^2+192064768ab^2+42987424a^3b^2)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{4096(1240320a^4b^2+1103368a^5b^2+15504a^6b^2+3876a^7b^2+43209472b^3-14013184ab^3)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{4096b(42987424a^2b^3+2648600a^4b^3+25840a^5b^3+19380a^6b^3-4122240b^4+14821520ab^4)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{4096b(-1240320a^2b^4+2648600a^3b^4+41990a^5b^4+1370064b^5-593408ab^5+1103368a^2b^5)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{4096b(-25840a^3b^5+41990a^4b^5-60672b^6+172824ab^6-15504a^2b^6+19380a^3b^6+7368b^7)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\left.\frac{4096b(-2736ab^7+3876a^2b^7-120b^8+285ab^8+5b^9)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}\right\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{4096a(166035456a-34142208a^2+43209472a^3-4122240a^4+1370064a^5-60672a^6)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{4096a(7368a^7-120a^8+5a^9+166035456b+192064768a^2b-14013184a^3b+14821520a^4b)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{4096a(-593408a^5b+172824a^6b-2736a^7b+285a^8b+34142208b^2+192064768ab^2)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{4096a(429874244a^3b^2-1240320a^4b^2+1103368a^5b^2-15504a^6b^2+3876a^7b^2+43209472b^3)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{4096a(14013184ab^3+42987424a^2b^3+2648600a^4b^3-25840a^5b^3+19380a^6b^3+4122240b^4)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{4096a(14821520ab^4+1240320a^2b^4+2648600a^3b^4+41990a^5b^4+1370064b^5+593408ab^5)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{4096a(1103368a^2b^5+25840a^3b^5+41990a^4b^5+60672b^6+172824ab^6+15504a^2b^6)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{4096a(19380a^3b^6+7368b^7+2736ab^7+3876a^2b^7+120b^8+285ab^8+5b^9)}{\left(\prod_{\beta=0}^{9}\{a-b-2\beta\}\right)\left(\prod_{\sigma=1}^{10}\{a-b+2\sigma\}\right)}+$$

$$+\frac{1024(185794560a+262803456a^2+150105600a^3+46315520a^4+8618400a^5+1012368a^6)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{1024(75600a^7+3480a^8+90a^9+a^{10}-185794560b+802676736ab+241497600a^2b)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{1024(451573760a^3b+62262240a^4b+22911264a^5b+1390800a^6b+182400a^7b+3990a^8b)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{1024(190a^9b+262803456b^2-241497600ab^2+886415360a^2b^2+62739520a^3b^2)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{1024(115995760a^4b^2+5555600a^5b^2+2015520a^6b^2+38760a^7b^2+4845a^8b^2-150105600b^3)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{1024(451573760ab^3-62739520a^2b^3+193593280a^3b^3+4522000a^4b^3+7648640a^5b^3}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{1024(116280a^6b^3+38760a^7b^3+46315520b^4-62262240ab^4+115995760a^2b^4-4522000a^3b^4)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{1024(11757200a^4b^4+83980a^5b^4+125970a^6b^4-8618400b^5+22911264ab^5-5555600a^2b^5)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{1024(7648640a^3b^5-83980a^4b^5+184756a^5b^5+1012368b^6-1390800ab^6+2015520a^2b^6)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$+\frac{1024(-116280a^3b^6+125970a^4b^6-75600b^7+182400ab^7-38760a^2b^7+38760a^3b^7+3480b^8)}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}+$$

$$\left.\left.+\frac{1024(-3990ab^8+4845a^2b^8-90b^9+190ab^9+b^{10})}{\left(\prod_{\alpha=0}^{10}\{a-b-2\alpha\}\right)\left(\prod_{\delta=1}^{9}\{a-b+2\delta\}\right)}\right\}\right] \tag{9}$$

C. DERIVATION OF SUMMATION FORMULA (8) :

Substituting $c=\frac{a+b+21}{2}$ and $z=\frac{1}{2}$ in equation (2), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+21}{2} & ; & \end{matrix}\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b & ; & \frac{1}{2} \\ \frac{a+b+21}{2} & ; & \end{matrix}\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1 & ; & \frac{1}{2} \\ \frac{a+b+21}{2} & ; & \end{matrix}\right]$$

Now using Gauss second summation theorem, we get

$$L.H.S = a\,\frac{2^b\ \Gamma(\frac{a+b+21}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\left\{\frac{512(-34459425+71697105a-53809164a^2+20570444a^3)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+\right.\right.$$

$$+\frac{512(-4574934a^4+626934a^5-53676a^6+2796a^7-81a^8+a^9+127734435b-74042088ab)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(158767116a^2b-27708840a^3b+12096426a^4b-856824a^5b+129276a^6b-3192a^7b)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(171a^8b+17316372b^2+224894564ab^2-24458852a^2b^2+50456476a^3b^2-2913460a^4b^2)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(1236444a^5b^2-27132a^6b^2+3876a^7b^2+652854446b^3+10669336ab^3+65808020a^2b^3)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(-2041360a^3b^3+3959980a^4b^3-69768a^5b^3+27132a^6b^3+6670026b^4+27977614ab^4)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(1272620a^2b^4+4954820a^3b^4-41990a^4b^4+75582a^5b^4+3074466b^5+1144712ab^5)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(2479348a^2b^5+33592a^3b^5+92378a^4b^5+150708b^6+453492ab^6+34884a^2b^6)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(50388a^3b^6+22116b^7+7752ab^7+11628a^2b^7+399b^8+969ab^8+19b^9)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+2}{2})}\Bigg\{\frac{512(34459425+127734435a-17316372a^2+65285444a^3-6670026a^4+3074466a^5)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(-150708a^6+22116a^7-399a^8+19a^9+716971056+74042088ab+224894564a^2b)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(-10669336a^3b+27977614a^4b-1144712a^5b+453492a^6b-7752a^7b+969a^8b)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(538091646b^2+158767116ab^2+24458852a^2b^2+65808020a^3b^2-1272620a^4b^2)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(2479348a^5b^2-34884a^6b^2+11628a^7b^2+205704444b^3+27708840ab^3+50456476a^2b^3)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(2041360a^3b^3+4954820a^4b^3-33592a^5b^3+50388a^6b^3+4574934b^4+12096426ab^4)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(2913460a^2b^4+3959980a^3b^4+41990a^4b^4+92378a^5b^4+626934b^5+856824ab^5)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{512(1236444a^2b^5+69768a^3b^5+75582a^4b^5+53676b^6+129276ab^6+27132a^2b^6)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}+$$

$$+\left.\left.\frac{512(27132a^3b^6+2796b^7+3192ab^7+3876a^2b^7+81b^8+171ab^8+b^9)}{\left(\prod_{\zeta=1}^{9}\{a-b-(2\zeta-1)\}\right)\left(\prod_{\eta=1}^{10}\{a-b+(2\eta-1)\}\right)}\right\}\right]-$$

$$-b\,\frac{2^{b+1}\,\Gamma(\frac{a+b+21}{2})}{\Gamma(b+1)}\left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\left\{\frac{512(34459425+71697105a+53809164a^2+20570444a^3)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+\right.\right.$$

$$+\frac{512(4574934a^4+626934a^5+53676a^6+2796a^7+81a^8+a^9+127734435b+74042088ab)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(158767116a^2b+27708840a^3b+12096426a^4b+856824a^5b+129276a^6b+3192a^7b)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(171a^8b-17316372b^2+224894564ab^2+24458852a^2b^2+50456476a^3b^2+2913460a^4b^2)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(1236444a^5b^2+27132a^6b^2+3876a^7b^2+652854444b^3-10669336ab^3+65808020a^2b^3)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(2041360a^3b^3+3959980a^4b^3+69768a^5b^3+27132a^6b^3-6670026b^4+27977614ab^4)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(-1272620a^2b^4+4954820a^3b^4+41990a^4b^4+75582a^5b^4+3074466b^5-1144712ab^5)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(2479348a^2b^5-33592a^3b^5+92378a^4b^5-150708b^6+453492ab^6-34884a^2b^6)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(+50388a^3b^6+22116b^7-7752ab^7+11628a^2b^7-399b^8+969ab^8+19b^9)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{512(-34459425+127734435a+17316372a^2+65285444a^3+6670026a^4)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(3074466a^5+150708a^6+22116a^7+399a^8+19a^9+71697105b-74042088ab)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(224894564a^2b+10669336a^3b+27977614a^4b+1144712a^5b+453492a^6b+7752a^7b)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(969a^8b-53809164b^2+158767116ab^2-24458852a^2b^2+65808020a^3b^2+1272620a^4b^2)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(2479348a^5b^2+34884a^6b^2+11628a^7b^2+205704444b^3-27708840ab^3+50456476a^2b^3)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(-2041360a^3b^3+4954820a^4b^3+33592a^5b^3+50388a^6b^3-4574934b^4+12096426ab^4)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(-2913460a^2b^4+3959980a^3b^4-41990a^4b^4+92378a^5b^4+626934b^5-856824ab^5)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(1236444a^2b^5-69768a^3b^5+75582a^4b^5-53676b^6+129276ab^6-27132a^2b^6)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}+$$

$$+\frac{512(27132a^3b^6+2796b^7-3192ab^7+3876a^2b^7-81b^8+171ab^8+b^9)}{\left(\prod_{=1}^{10}\{a-b-(2\psi-1)\}\right)\left(\prod_{\xi=1}^{9}\{a-b+(2\xi-1)\}\right)}\Bigg\}\Bigg]$$

On simplification ,we prove the result (8).
Similarly, we can prove the result (9).

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