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Normal Mode Analysis of Micropolar Elastic Medium with Void under Inviscid Fluid

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Abstract - The present investigation is concerned with the two dimensional problem of micropolar elastic medium with void. Normal mode analysis is used to obtain the expression of components of stresses, displacement components and acoustic pressure of the inviscid fluid. Numerically simulated results are obtained and presented graphically to depict the impact of void for a particular model.

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I. INTRODUCTION

The micropolar theory of elasticity constructed by Eringen [1] was intended to be applied to such materials and for such problems where the ordinary classical theory of elasticity fails because of microstructure of the material. Also micropolar theory is more appropriate for geological materials like rocks, soil since this theory takes into account the intrinsic rotation and predicts the behavior of material with inner structure. For engineering problem, it can model composites with rigid chopped fibers, elastic solids with rigid inclusion and other industrial materials such as liquid crystal.

The mechanical behavior of solids with voids; solid containing microscopic components cannot be described by classical theory of elasticity. Hence, the theory for granular materials with interstitial voids was presented by Goodman and Cowin [2]. A theory for the behavior of porous solids, in which the skeletal or matrix material is elastic and the interstices are voids of the material, was established by Nunziato and Cowin [3], Cowin and Nunziato [4]. Various author's [5, 6] discussed different problems in micropolar elastic medium with voids. Othman [7] discussed effect of rotation on plane waves in generalized thermoelasticity by using normal mode analysis. Recently, Ezzat and co-author's [8] discussed two dimensional coupled problems in electro-magneto thermoelasticity by using normal mode analysis. The aim of the present problem is to find the components of displacement, stress components, acoustic pressure and volume fraction field in a homogenous isotropic micropolar elastic solid with voids under inviscid liquid by using normal mode analysis.

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II. BASIC EQUATIONS

Following Eringen [1] and Quintanilla [9], the equation of motion and the constitutive relation in a homogenous isotropic micropolar elastic solid with voids in the absence of body forces, body couples are given as:

$$(\lambda + 2\mu + K)\nabla(\nabla \cdot \vec{u}) - (\mu + K)\nabla \times \nabla \times \vec{u} + K(\nabla \times \vec{\phi}) + \xi \nabla \psi = \rho \frac{\partial^2 \vec{u}}{\partial t^2} \quad (1)$$

$$(\alpha + \beta + \gamma)\nabla(\nabla \cdot \vec{\phi}) - \gamma \nabla \times (\nabla \times \vec{\phi}) + K(\nabla \times \vec{u}) - 2K \vec{\phi} + \zeta \nabla \psi = \rho j \frac{\partial^2 \vec{\phi}}{\partial t^2} \quad (2)$$

$$d \nabla^2 \psi - \xi \nabla \cdot \vec{u} - \zeta \nabla \cdot \vec{\phi} - \omega_1^* \frac{\partial \psi}{\partial t} - a \psi = \rho \chi \frac{\partial^2 \psi}{\partial t^2} \quad (3)$$

$$t_{ij} = \lambda \delta_{ij} e_{rr} + \mu (u_{i,j} + u_{j,i}) + K (u_{j,i} - \epsilon_{ijk} \phi_k) + \xi \psi \delta_{ij} \quad (4)$$

$$m_{ij} = \alpha \phi_{r,r} \delta_{ij} + \beta \phi_{i,j} + \gamma \phi_{j,i} + \zeta \psi \delta_{ij} \quad (5)$$

where λ and μ - Lamé's constants, t_{ij} - components of the stress tensor, m_{ij} - components of couple stress tensor, ρ - density, u_i - displacement components, ψ - change in volume fraction, δ_{ij} - Kronecker delta, ϵ_{ijk} - alternative tensor, ϕ_i - microrotation vector, t - time, j - microrotation inertia, K, α, β, γ - material constant, $d, \xi, \zeta, a, \omega_1^*$ and χ - material constants due to presence of void.

Following Achenbach [10], the field equations can be expressed in terms of velocity potential for inviscid fluid as

$$p = -\bar{\rho}\dot{\phi}^f$$

$$\left(\nabla^2 - \frac{1}{\alpha^{f^2}} \frac{\partial^2}{\partial t^2}\right) \phi^f = 0 \tag{6}$$

$$\dot{\mathbf{u}} = \nabla \phi^f \tag{7}$$

where $\alpha^{f^2} = \bar{\lambda} / \bar{\rho}$, $\bar{\lambda}$ is the bulk modulus, $\bar{\rho}$ is the density of the liquid, $\dot{\mathbf{u}}$ is the velocity vector and p is the acoustic pressure in the inviscid fluid,.

For two-dimensional problem, we take

$$\bar{\mathbf{u}} = (u_1, 0, u_3), \quad \bar{\phi} = (0, \phi_2, 0) \tag{8}$$

Also, we introduce the non-dimensional quantities defined by the expressions

$$x'_i = \frac{\omega^*}{c_1} x_i, \quad u'_i = \frac{\omega^*}{c_1} u_i, \quad \{\phi'_2, \psi'\} = \left(\frac{\rho c_1^2}{K}\right) \{\phi_2, \psi\},$$

$$t' = \omega^* t, \quad \phi'^f = \frac{\omega^*}{c_1^2} \phi^f, \quad t'_{3i} = \frac{1}{\mu} t_{3i}, \quad m'_{32} = \frac{\omega^*}{\mu c_1} m_{32}$$

$$p' = \bar{\lambda} p, \quad \dot{u}'_i = \frac{\omega^*}{c_1} \dot{u}_i, \quad \omega^{*2} = \frac{K}{\rho j}, \quad c_1 = \frac{\lambda + 2\mu + K}{\rho},$$

$$i = 1, 3 \tag{9}$$

The displacement components u_1 and u_3 are related to the potential functions as,

$$u_1 = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial z}, \quad u_3 = \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi}{\partial x}, \tag{10}$$

Using equations (8), (9) and (10) on equations (1) – (3), (6) (suppressing primes), we get

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) \Phi + a_4 \psi = 0, \tag{11}$$

$$\left(a_2 \nabla^2 - \frac{\partial^2}{\partial t^2}\right) \Psi + a_3 \phi_2 = 0 \tag{12}$$

$$\left(a_5 \nabla^2 - 2a_7 - \frac{\partial^2}{\partial t^2}\right) \phi_2 - a_6 \nabla^2 \Psi = 0, \tag{13}$$

$$\left(a_8 \nabla^2 - a_{10} \frac{\partial}{\partial t} - a_{11} - \frac{\partial^2}{\partial t^2}\right) \psi - a_9 \nabla^2 \Phi = 0 \tag{14}$$

$$\left(\nabla^2 - a_{12} \frac{\partial^2}{\partial t^2}\right) \phi^f = 0 \tag{15}$$

where

$$a_1 = \frac{\lambda + \mu}{\rho c_1^2}, \quad a_2 = \frac{K + \mu}{\rho c_1^2}, \quad a_3 = \frac{K^2}{\rho^2 c_1^4}, \quad a_4 = \frac{\xi K}{\rho^2 c_1^4},$$

$$a_5 = \frac{\gamma}{\rho j c_1^2}, \quad a_6 = \frac{c_1^2}{j \omega^{*2}}, \quad a_7 = \frac{K}{\rho j \omega^{*2}}, \quad a_8 = \frac{d}{\chi \rho c_1^2},$$

$$a_9 = \frac{\xi c_1^2}{\chi k \omega^{*2}}, \quad a_{10} = \frac{\omega_1^*}{\chi \rho \omega^*}, \quad a_{11} = \frac{a}{\chi \rho \omega^{*2}}, \quad a_{12} = \frac{c_1^2}{\alpha^{f^2}}$$

III. NORMAL MODE ANALYSIS

The solution of the considered physical variable can be decomposed in terms of normal modes as following:

$$[\Phi, \psi, \phi_2, \Psi, \phi^f] = [\bar{\Phi}(z), \bar{\psi}(z), \bar{\phi}_2(z), \bar{\Psi}(z), \bar{\phi}^f(z)] e^{i(kx - \omega t)} \tag{16}$$

where ω is the complex time constant and k is the wave number in the x -direction. Using equation (16), equations (11)-(15) takes the form

$$(D^4 + AD^2 + B)(\bar{\Phi}, \bar{\psi}) = 0 \tag{17}$$

$$(D^4 + LD^2 + M)(\bar{\phi}_2, \bar{\Psi}) = 0 \tag{18}$$

$$(D^2 + N)\bar{\phi}^f = 0 \tag{19}$$

where

$$D = \frac{d}{dz}, \quad N = k^2 + a_{12} \omega^2$$

$$A = \frac{a_5(\omega^2 - 2a_2 k^2 - 2a_3 a_6 k^2) + (\omega^2 - 2a_7)(a_2 + a_3 a_6)}{a_5(a_2 + a_3 a_6)},$$

$$B = \frac{(\omega^2 - 2a_7)(\omega^2 - a_2k^2 - a_3a_6k^2) - a_5k^2(\omega^2 - a_2k^2 - a_3a_6k^2)}{a_5(a_2 + a_3a_6)}$$

$$L = \frac{a_8(\omega^2 - k^2) - (a_8k^2 + \omega^2 + i\omega a_{10} + a_{11} + a_4a_9)}{a_8}$$

$$M = \frac{(\omega^2 - k^2)(a_8k^2 + \omega^2 + i\omega a_{10} + a_{11} - a_4a_9k^2)}{a_8}$$

The solution of equations (17) and (18) satisfying radiation conditions that $\bar{\Phi}, \bar{\psi}, \bar{\Psi}, \bar{\phi}_2, \phi^f \rightarrow 0$ as $x_3 \rightarrow \infty$ are:

$$\{\bar{\Phi}, \bar{\psi}\} = \sum (1, d_i) A_i e^{-m_i x_3}$$

$$\{\bar{\phi}_2, \bar{\Psi}\} = \sum (1, d_j) B_j e^{-m_j x_3},$$

$$\phi^f = E e^{-m_5 x_3} \quad i = 1, 2 \text{ and } j = 1, 2 \quad (20)$$

IV. BOUNDARY CONDITIONS

The boundary conditions in this case are:

$$t_{33} - p = -F e^{i(kx - \omega t)},$$

$$t_{31} = 0,$$

$$m_{32} = 0,$$

$$\frac{d\psi}{dz} = 0,$$

$$\dot{u}_3 = u_3^f \text{ at } x_3 = 0 \quad (21)$$

where F is well defined function.

Making use of the equations (4)-(5), (7) and (8) and applying normal mode analysis defined by (16) and substitute the values of $\bar{\Phi}, \bar{\psi}, \bar{\phi}_2, \bar{\Psi}, \phi^f$ from equation (20) in the resulting equations, we obtain the expression for components of displacement, stresses, volume fraction and acoustic pressure as

$$u_3 = F_1 [-(m_1 \Delta_1 e^{-m_1 x_3} - m_2 \Delta_2 e^{-m_2 x_3}) + ik(\Delta_3 e^{-m_3 x_3} + \Delta_4 e^{-m_4 x_3})], \quad (22)$$

$$u_3^f = F_1 m_5 \Delta_5 e^{-m_5 x_3}, \quad (23)$$

$$t_{33} = F_1 [\Delta_1 s_1 e^{-m_1 x_3} - \Delta_2 s_2 e^{-m_2 x_3} + \Delta_3 s_3 e^{-m_3 x_3} + \Delta_4 s_4 e^{-m_4 x_3}] \quad (24)$$

$$t_{31} = F_1 [\Delta_1 s_6 e^{-m_1 x_3} - \Delta_2 s_7 e^{-m_2 x_3} + \Delta_3 s_8 e^{-m_3 x_3} + \Delta_4 s_9 e^{-m_4 x_3}] \quad (25)$$

$$m_{32} = F_1 [m_3 \Delta_3 s_3 e^{-m_3 x_3} + m_4 \Delta_4 s_4 e^{-m_4 x_3}], \quad (26)$$

$$p = F_1 s_5 e^{-m_5 x_3}, \quad (27)$$

$$\psi = F_1 [\Delta_1 d_1 e^{-m_1 x_3} - \Delta_2 d_2 e^{-m_2 x_3}], \quad (28)$$

where

$$\Delta = \begin{vmatrix} s_1 & s_2 & s_3 & s_4 & s_5 \\ s_6 & s_7 & s_8 & s_9 & 0 \\ 0 & 0 & m_3 & m_4 & 0 \\ m_1 d_1 & m_2 d_2 & 0 & 0 & 0 \\ i\omega m_1 & i\omega m_2 & \omega k & \omega k & m_5 \end{vmatrix}$$

$\Delta_i, i = 1, 5$ are obtained by replacing i^{th} column of Δ with $[-F \ 0 \ 0 \ 0 \ 0]$

where

$$s_i = k^2 b_1 + b_2 m_i + b_3 d_i, \quad s_j = ik m_j (b_1 - b_2),$$

$$s_5 = -b_6 i\omega, \quad \{s_6, s_7\} = (b_4 - 1) ik m_i,$$

$$\{s_8, s_9\} = -(m_j^2 b_4 + b_5 d_j + k^2), \quad b_1 = \frac{\lambda}{\mu}, \quad b_3 = \frac{\xi K}{\mu \rho c_1^2}$$

$$b_2 = \frac{\lambda + 2\mu + K}{\mu}, \quad b_4 = \frac{\mu + K}{\mu}, \quad b_5 = \frac{K^2}{\mu \rho c_1^2}, \quad b_6 = \frac{\bar{\rho} c_1^2}{\lambda^f},$$

$$F_1 = \frac{F e^{i(kx - \omega t)}}{\Delta} \quad i = 1, 2 \ \& \ j = 3, 4$$

V. PARTICULAR CASE

1. Micropolar Elastic Solid: Neglecting void effects in equations (22)-(28), we obtain the corresponding expression for components of displacement, stresses, and acoustic pressure in micropolar elastic media under inviscid fluid.

2. In absence of inviscid liquid: if $\bar{\rho} \rightarrow 0$, then we obtain corresponding expression for micropolar elasticity with void

VI. NUMERICAL DISCUSSION

In order to study, the problem considered in greater details numerically simulated results are computed for a particular model and are presented graphically. For this purpose, we have taken the case of magnesium crystal like material. Following Eringen [11], the physical constants are:

$$\lambda = 9.4 \times 10^{11} \text{ dyn cm}^{-2}, \quad \mu = 4 \times 10^{11} \text{ dyn cm}^{-2},$$

$$K = 1 \times 10^{11} \text{ dyn cm}^{-2}, \quad \rho = 1.7 \text{ gm cm}^{-3},$$

$$\gamma = 0.779 \times 10^{-4} \text{ dyn}, \quad j = 0.2 \times 10^{-15} \text{ cm}^2,$$

$$\bar{\lambda} = 2.1904 \times 10^{10} \text{ dyn cm}^{-2}, \quad \bar{\rho} = 1.0 \times 10^3 \text{ gm cm}^{-3}$$

and the void parameters are

$$d = 3.688 \times 10^{-4} \text{ dyn}, \quad a = 1.475 \times 10^{11} \text{ dyn cm}^{-2},$$

$$\xi = 1.13849 \times 10^{11} \text{ dyn cm}^{-2}, \quad \omega_1^* = 0.0787 \text{ dyn cm}^{-2}$$

The computations were carried out for small values of time $t = 0.1$. The numerical results for the stress components (t_{33}, t_{31}, m_{32}) , volume fraction field ψ normal velocity u_3^f and acoustic pressure p of inviscid fluid are shown graphically in figures (1)-(6) for different ω i.e. for $\omega = 0.1$ and $\omega = 0.5$ with distance $0 \leq x \leq 10$. The solid line and dashed line corresponds to Micropolar elastic with void (MEV) for $\omega = 0.1$ and $\omega = 0.5$ respectively, whereas solid line with center symbol 'triangle' and dashed line with center symbol 'circle' corresponds to Micropolar elasticity (ME) for $\omega = 0.1$ and $\omega = 0.5$ respectively.

Figure 1 depicts the variations of t_{33} with distance x . It is noticed that the values of t_{33} for ME at $\omega = 0.1$ and $\omega = 0.5$ are similar in nature in entire range, whereas values of t_{33} for MEV at $\omega = 0.1$ and $\omega = 0.5$ are opposite in nature, which is accounted as void effect.

It is noticed from figure (2), which is plot for t_{31} with distance x that value of t_{31} at $\omega = 0.1$ for MEV increases in range $3 \leq x \leq 6$ and $9 \leq x \leq 10$, decreases in remaining range while for ME values of t_{31} at $\omega = 0.1$ decreases in range $0 \leq x \leq 2$, $5 \leq x \leq 8$ and vice-versa trends are noticed in remaining range.

Whereas values of t_{31} at $\omega = 0.5$ for MEV and ME show similar oscillatory behavior in entire range, magnitude of values for ME are greater as compared to MEV, which reveals the impact of void effect.

The variations of m_{32} with x are noticed in figure 3. It is noticed that values of m_{32} for MEV and ME at $\omega = 0.1$ shows similar behavior in entire range i.e. their values increases and decreases alternately with x , while values of MEV for $\omega = 0.5$ oscillates with greater magnitude as compared to those noticed for MEV and ME for different ω , which clearly shows the impact of complex time constant.

Figure (4) shows the variations of volume fraction ψ with x . It is noticed that the trends of ψ for MEV at $\omega = 0.1$ and $\omega = 0.5$ are opposite in nature in entire range.

The variations of u_3^f are shown in figure (5). It is noticed that values of u_3^f at different ω for MEV and ME increases in range $2 \leq x \leq 5$, $8 \leq x \leq 10$ and vice-versa trends are noticed in remaining range with significant difference in their magnitude.

Figure (6) depicts the variations of acoustic pressure p with x at $\omega = 0.1$ and $\omega = 0.5$. It is noticed that trends for MEV are opposite in nature as compared to ME for both ω , which is accounted as absence of void effect.

VII. CONCLUSION

Normal mode technique is employed to solve the problem of micropolar elasticity solid given by Eringen [1] and Quintanilla [9]. From the above discussion, we noticed that presence of void effect shows significant impact on the components of stresses, normal velocity and volume fraction field. Also different values of parameter ω show relevant impact on different calculated parameters in micropolar elasticity and in inviscid fluid.

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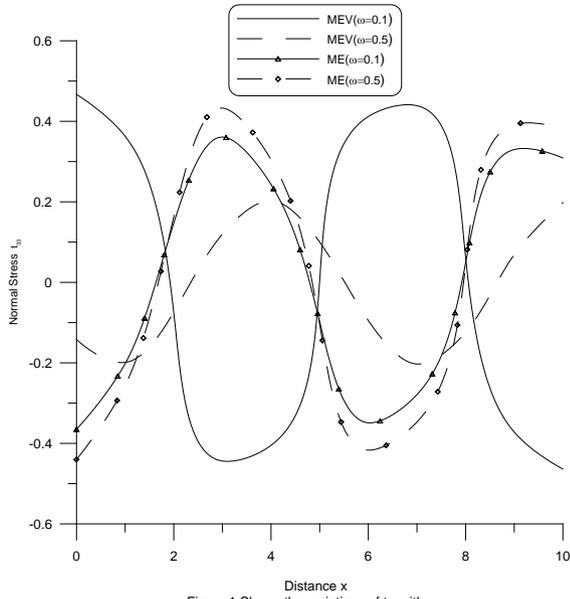


Figure 1 Shows the variations of t_{33} with x

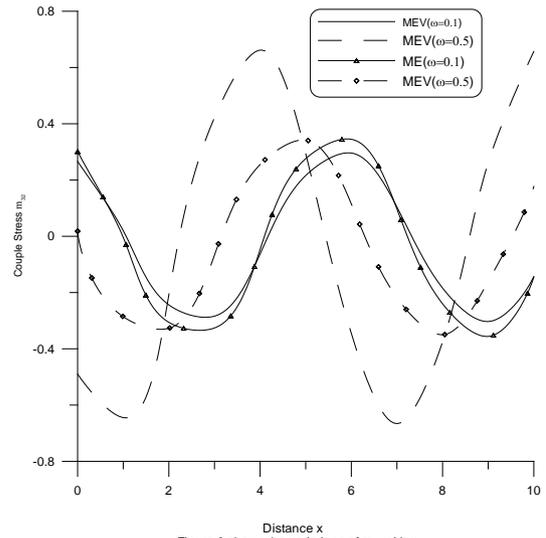


Figure 3 shows the variations of m_{32} with x

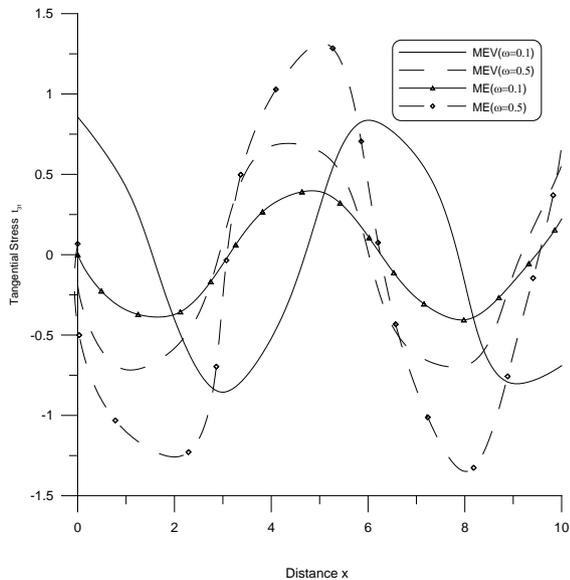


Figure 2 shows the variations of t_{31} with x

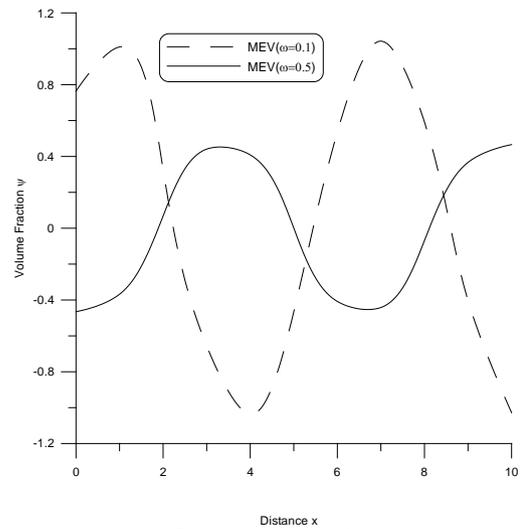
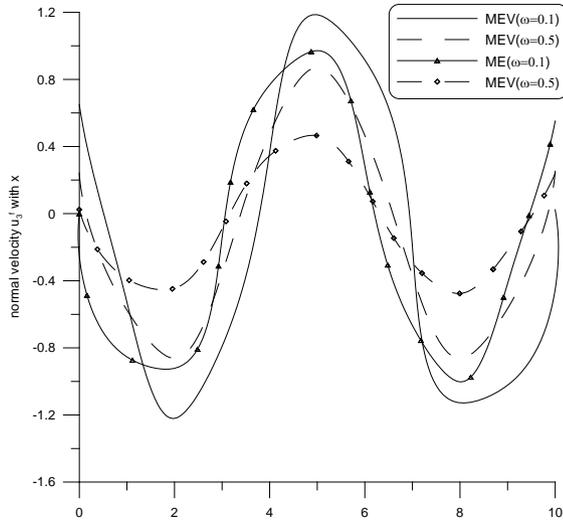
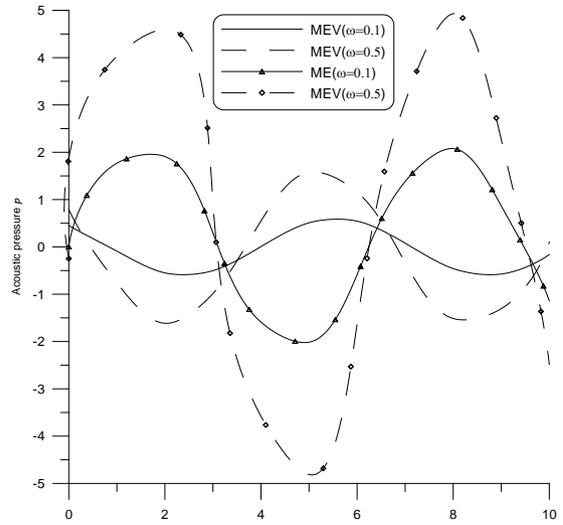


Figure 4 shows the variations of Ψ with x



Distance x
figure 5 shows the variations of u_x' with x



Distance x
Figure 6 shows the variations of p with x



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