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The Concept of Heart-Oriented Rhotrix Multiplication

By Ezugwu E. Absalom, Sani B., Junaidu B. Sahalu
Ahmadu Bello University, Zaria-Nigeria

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THE CONCEPT OF HEART-ORIENTED RHOTRIX MULTIPLICATION

Strictly as per the compliance and regulations of:

The Concept of Heart-Oriented Rhotrix Multiplication

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Abstract-This paper introduces the concept of heart-oriented rhotrix multiplication. A generalized representation of *n*-dimensional heart-oriented rhotrices viewed from the direction of single array indices and double array row-wise indices is presented. We provided a simplified compact mathematical expression for the two multiplication process presented in the paper. The given methods were implemented based on the sequential algorithm developed.

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I. Introduction

The fundamental concept of this paper can be found in [1] which tailored down from the idea of matrix-tertion and matrix-noitrites[2]. The extension of this idea was presented by Ajibade [1] and referred to as rhotrices. In his paper Ajibade presented the initial concept of rhotrix algebra in which he established some interesting relationships between a rhotrix and its hearts.A rhotrix is defined as a mathematical array which is in some way between 2 x 2 matrix and 3 x 3 matrix is given as.

$$R = \left\{ \left\langle \begin{matrix} & & a & & \\ & b & c & d & \\ e & f & h(R) & h & i \\ & j & k & l & \\ & & m & & \end{matrix} \right\rangle : a, b, c, \ldots\ldots, m \in \Re \right\}$$

The above rhotrix is of the fifth dimension. h(R) is called the heart of the rhotrix. An extension is possible, thereby increasing the dimension which is always odd. The number of entries in an n-dimensional rhotrix is given by $\frac{1}{2}(n^2+1)$. Where n is the dimension of the rhotrix.

II. High dimensional heart-oriented rhotrices

In this paper we present a general idea on heart-oriented rhotrix multiplication and its formal representation for *n*-dimensional rhotrix. This new concept of the multiplication of rhotrices gave room for the initial conception of heart-oriented sequential computational implementation which is the basis for this paper.

Ajibade [1] , indicated that the dimension of rhotrices can be increased although a rhotrix would always have an odd dimension. He also indicated that a rhotrix R_n of dimension *n* will have $|R_n|$ entries where $|R_n| = \frac{1}{2}(n^2+1)$. Let's consider generalizing any given rhotrix R_n with entries $a_1,\ a_2,\ \ldots\ldots, a_{\frac{1}{2}(n^2+1)}$, and we assume that the following holds:

- If we denote the number of entries in a rhotrix by N, then the middle entry, known as the heart element, can be expressed as $H = \frac{1}{2}(N+1)$ from statistical distribution expression for median [3]. In our case, the value of H indicates the index of the heart entry.

- Similarly if $N = |R_n|$ then $N = \frac{1}{2}(n^2+1)$, hence, $H = \dfrac{\frac{1}{2}(n^2+1)+1}{2} \equiv \dfrac{1}{4}\left[n^2+3\right].$

About : Department of Mathematics Ahmadu Bello University, Zaria-Nigeria
Emails: code_abs@yahoo.com , bsani@yahoo.co.uk, sahalu@abu.edu.ng

III. SINGLE INDICES HEART-ORIENTED RHOTRICES

Some results: we can derive the general representation of high dimensional rhotrix by considering a sequence of 3, 5, 7 and 9 dimensional rhotrices as illustrated below:

For a rhotrix of dimension 3, we have 2 x 1 + 1 = 3

For a rhotrix of dimension 5, we have 2 x 2 + 1 = 5

For a rhotrix of dimension 7, we have 2 x 3 + 1 = 7

For a rhotrix of dimension 9, we have 2 x 4 + 1 = 9

… ….

… ….

For a rhotrix of dimension *n*, we have 2 x *k* + 1 = *n*

Where k is the k^{th} term of the incremental value and ***n*** is the dimension of the rhotrix, and then we have: $k = \frac{1}{2}(n-1)$

- Again, if we denote the direction of the leftmost entry of a rhotrix by *L* and the number of entries by $|R_n| = \frac{1}{2}(n^2+1)$ then the rhotrix entry at *L* is given by

  $$L = \frac{1}{4}\left[n^2+3\right] - \frac{n-1}{2}$$

- Similarly, the rightmost rhotrix entry denoted by *R*, is given by

  $$R = \frac{1}{4}\left[n^2+3\right] + \frac{n-1}{2}$$

- It is important to note that *L* and *R* denote the leftmost and the rightmost indices of *a* and *b* respectively in the rhotrix.

This is represented as:

$$R_n = \left\langle \begin{array}{ccccccccc}
 & & & & a_1 & & & & \\
 & & & a_2 & a_3 & a_4 & & & \\
 & & a_5 & a_6 & a_7 & a_8 & a_9 & & \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
a_{\frac{1}{4}(n^2+3)-\frac{n-1}{2}} & \cdots & \cdots & \cdots & a_{\frac{1}{4}(n^2+3)} & \cdots & \cdots & \cdots & a_{\frac{1}{4}(n^2+3)+\frac{n-1}{2}} \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
 & & a_{\frac{1}{2}(n^2+1)-8} & a_{\frac{1}{2}(n^2+1)-7} & a_{\frac{1}{2}(n^2+1)-6} & a_{\frac{1}{2}(n^2+1)-5} & a_{\frac{1}{2}(n^2+1)-4} & & \\
 & & & a_{\frac{1}{2}(n^2+1)-3} & a_{\frac{1}{2}(n^2+1)-2} & a_{\frac{1}{2}(n^2+1)-1} & & & \\
 & & & & a_{\frac{1}{2}(n^2+1)} & & & &
\end{array} \right\rangle$$

$$
Q_n = \left\langle \begin{array}{ccccccccc}
 & & & & b_1 & & & & \\
 & & & b_2 & b_3 & b_4 & & & \\
 & & b_5 & b_6 & b_7 & b_8 & b_9 & & \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
b_{\frac{1}{4}(n^2+3)-\frac{n-1}{2}} & \cdots & \cdots & \cdots & b_{\frac{1}{4}(n^2+3)} & \cdots & \cdots & \cdots & b_{\frac{1}{4}(n^2+3)+\frac{n-1}{2}} \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
 & & b_{\frac{1}{2}(n^2+1)-8} & b_{\frac{1}{2}(n^2+1)-7} & b_{\frac{1}{2}(n^2+1)-6} & b_{\frac{1}{2}(n^2+1)-5} & b_{\frac{1}{2}(n^2+1)-4} & & \\
 & & & b_{\frac{1}{2}(n^2+1)-3} & b_{\frac{1}{2}(n^2+1)-2} & b_{\frac{1}{2}(n^2+1)-1} & & & \\
 & & & & b_{\frac{1}{2}(n^2+1)} & & & &
\end{array} \right\rangle \quad (2.1)
$$

We further simplify equation (2.2) as follow:

since $R = \dfrac{\frac{1}{2}(n^2+1)+1}{2} + \dfrac{n-1}{2}$ then $R = \dfrac{\frac{1}{2}(n^2+1)+1+n-1}{2} = \dfrac{\frac{1}{2}n^2+\frac{1}{2}+n}{2} = \dfrac{\frac{1}{2}n^2+\frac{1}{2}+n}{2}$ which implies that

$$R = \frac{1}{4}n^2 + \frac{1}{4} + \frac{n}{2} = \frac{n^2+2n+1}{4}$$

And hence $R = \dfrac{n^2+2n+1}{4}$

We also do the same for L

since $L = \dfrac{\frac{1}{2}(n^2+1)+1}{2} - \dfrac{n-1}{2}$ then $L = \dfrac{\frac{1}{2}(n^2+1)+1-n+1}{2} = \dfrac{\frac{1}{2}n^2+\frac{1}{2}+1-n+1}{2}$

$$L = \frac{1}{4}n^2 + \frac{5}{4} - \frac{2n}{4} = \frac{n^2-2n+5}{4}$$

Thus, $L = \dfrac{n^2-2n+5}{4}$

$$R_n = \left\langle \begin{array}{ccccccccc}
 & & & & a_1 & & & & \\
 & & & a_2 & a_3 & a_4 & & & \\
 & & a_5 & a_6 & a_7 & a_8 & a_9 & & \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
a_{\frac{n^2-2n+5}{4}} & \cdots & \cdots & \cdots & a_{\frac{1}{4}(n^2+3)} & \cdots & \cdots & \cdots & a_{\frac{n^2+2n+1}{4}} \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
 & & a_{\frac{1}{2}(n^2+1)-8} & a_{\frac{1}{2}(n^2+1)-7} & a_{\frac{1}{2}(n^2+1)-6} & a_{\frac{1}{2}(n^2+1)-5} & a_{\frac{1}{2}(n^2+1)-4} & & \\
 & & & a_{\frac{1}{2}(n^2+1)-3} & a_{\frac{1}{2}(n^2+1)-2} & a_{\frac{1}{2}(n^2+1)-1} & & & \\
 & & & & a_{\frac{1}{2}(n^2+1)} & & & &
\end{array} \right\rangle$$

$$Q_n = \left\langle \begin{array}{ccccccccc}
 & & & & b_1 & & & & \\
 & & & b_2 & b_3 & b_4 & & & \\
 & & b_5 & b_6 & b_7 & b_8 & b_9 & & \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
b_{\frac{n^2-2n+5}{4}} & \cdots & \cdots & \cdots & b_{\frac{1}{4}(n^2+3)} & \cdots & \cdots & \cdots & b_{\frac{n^2+2n+1}{4}} \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
 & & b_{\frac{1}{2}(n^2+1)-8} & b_{\frac{1}{2}(n^2+1)-7} & b_{\frac{1}{2}(n^2+1)-6} & b_{\frac{1}{2}(n^2+1)-5} & b_{\frac{1}{2}(n^2+1)-4} & & \\
 & & & b_{\frac{1}{2}(n^2+1)-3} & b_{\frac{1}{2}(n^2+1)-2} & b_{\frac{1}{2}(n^2+1)-1} & & & \\
 & & & & b_{\frac{1}{2}(n^2+1)} & & & &
\end{array} \right\rangle \qquad (2.2)$$

Substituting the values $n = 3$, $n = 5$ and $n = 7$ in equation (2.2) we have the following rhotrices:

$$R_3 = \left\langle \begin{array}{ccc} & a_1 & \\ a_2 & a_3 & a_4 \\ & a_5 & \end{array} \right\rangle, \; R_5 = \left\langle \begin{array}{ccccc} & & a_1 & & \\ & a_2 & a_3 & a_4 & \\ a_5 & a_6 & a_7 & a_8 & a_9 \\ & a_{10} & a_{11} & a_{12} & \\ & & a_{13} & & \end{array} \right\rangle \text{ and } R_7 = \left\langle \begin{array}{ccccccc} & & & a_1 & & & \\ & & a_2 & a_3 & a_4 & & \\ & a_5 & a_6 & a_7 & a_8 & a_9 & \\ a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ & a_{17} & a_{18} & a_{19} & a_{20} & a_{21} & \\ & & a_{22} & a_{23} & a_{24} & & \\ & & & a_{25} & & & \end{array} \right\rangle$$

Similarly, from (2.2), we can define the multiplication of any two heart-oriented rhotrices in the following way.

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$$R_n \circ Q_n = \left\langle \begin{array}{ccccccccc}
 & & & & a_1 & & & & \\
 & & & a_2 & a_3 & a_4 & & & \\
 & & a_5 & a_6 & a_7 & c_8 & a_9 & & \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
a_{\frac{n^2-2n+5}{4}} & \cdots & \cdots & \cdots & a_{\frac{1}{4}(n^2+3)} & \cdots & \cdots & \cdots & a_{\frac{n^2+2n+1}{4}} \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
 & & a_{\frac{1}{2}(n^2+1)-8} & a_{\frac{1}{2}(n^2+1)-7} & a_{\frac{1}{2}(n^2+1)-6} & a_{\frac{1}{2}(n^2+1)-5} & a_{\frac{1}{2}(n^2+1)-4} & & \\
 & & & a_{\frac{1}{2}(n^2+1)-3} & a_{\frac{1}{2}(n^2+1)-2} & a_{\frac{1}{2}(n^2+1)-1} & & & \\
 & & & & a_{\frac{1}{2}(n^2+1)} & & & &
\end{array} \right\rangle \circ$$

$$\left\langle \begin{array}{ccccccccc}
 & & & & b_1 & & & & \\
 & & & b_2 & b_3 & b_4 & & & \\
 & & b_5 & b_6 & b_7 & b_8 & b_9 & & \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
b_{\frac{n^2-2n+5}{4}} & \cdots & \cdots & \cdots & b_{\frac{1}{4}(n^2+3)} & \cdots & \cdots & \cdots & b_{\frac{n^2+2n+1}{4}} \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
 & & b_{\frac{1}{2}(n^2+1)-8} & b_{\frac{1}{2}(n^2+1)-7} & b_{\frac{1}{2}(n^2+1)-6} & b_{\frac{1}{2}(n^2+1)-5} & b_{\frac{1}{2}(n^2+1)-4} & & \\
 & & & b_{\frac{1}{2}(n^2+1)-3} & b_{\frac{1}{2}(n^2+1)-2} & b_{\frac{1}{2}(n^2+1)-1} & & & \\
 & & & & b_{\frac{1}{2}(n^2+1)} & & & &
\end{array} \right\rangle$$

$$R_5 \circ Q_5 = \left\langle \begin{array}{ccccccccc}
 & & & & c_1 & & & & \\
 & & & c_2 & c_3 & c_4 & & & \\
 & & c_5 & c_6 & c_7 & c_8 & c_9 & & \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
c_{\frac{n^2-2n+5}{4}} & \cdots & \cdots & \cdots & c_{\frac{1}{4}(n^2+3)} & \cdots & \cdots & \cdots & c_{\frac{n^2+2n+1}{4}} \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\
 & & c_{\frac{1}{2}(n^2+1)-8} & c_{\frac{1}{2}(n^2+1)-7} & c_{\frac{1}{2}(n^2+1)-6} & c_{\frac{1}{2}(n^2+1)-5} & c_{\frac{1}{2}(n^2+1)-4} & & \\
 & & & c_{\frac{1}{2}(n^2+1)-3} & c_{\frac{1}{2}(n^2+1)-2} & c_{\frac{1}{2}(n^2+1)-1} & & & \\
 & & & & c_{\frac{1}{2}(n^2+1)} & & & &
\end{array} \right\rangle \quad (2.3)$$

Definition (heart-oriented multiplication)

Let $R_5 = \left\langle \begin{array}{ccccc} & & a_1 & & \\ & a_2 & a_3 & a_4 & \\ a_5 & a_6 & a_7 & a_8 & a_9 \\ & a_{10} & a_{11} & a_{12} & \\ & & a_{13} & & \end{array} \right\rangle$ and $Q_5 = \left\langle \begin{array}{ccccc} & & b_1 & & \\ & b_2 & b_3 & b_4 & \\ b_5 & b_6 & b_7 & b_8 & b_9 \\ & b_{10} & b_{11} & b_{12} & \\ & & b_{13} & & \end{array} \right\rangle$ be two rhotrices of dimension 5 with entries from $\mathbb{R}$, the set of real numbers. We follow the multiplication (called *heart-oriented multiplication* in this paper) which was first defined in [1] on rhotrices of third dimension as follows:

$$R_5 \circ Q_5 = \left\langle \begin{array}{ccccc} & & a_1 & & \\ & a_2 & a_3 & a_4 & \\ a_5 & a_6 & a_7 & a_8 & a_9 \\ & a_{10} & a_{11} & a_{12} & \\ & & a_{13} & & \end{array} \right\rangle \circ \left\langle \begin{array}{ccccc} & & b_1 & & \\ & b_2 & b_3 & b_4 & \\ b_5 & b_6 & b_7 & b_8 & b_9 \\ & b_{10} & b_{11} & b_{12} & \\ & & b_{13} & & \end{array} \right\rangle =$$

$$b_7 \circ \left\langle \begin{array}{ccccc} & & a_1 & & \\ & a_2 & a_3 & a_4 & \\ a_5 & a_6 & a_7 & a_8 & a_9 \\ & a_{10} & a_{11} & a_{12} & \\ & & a_{13} & & \end{array} \right\rangle + a_7 \circ \left\langle \begin{array}{ccccc} & & b_1 & & \\ & b_2 & b_3 & b_4 & \\ b_5 & b_6 & b_7 & b_8 & b_9 \\ & b_{10} & b_{11} & b_{12} & \\ & & b_{13} & & \end{array} \right\rangle$$

$$= \left\langle \begin{array}{ccccc} & & a_1b_7 + a_7b_1 & & \\ & a_2b_7 + a_7b_2 & a_3b_7 + a_7b_3 & a_4b_7 + a_7b_4 & \\ a_5b_7 + a_7b_5 & a_6b_7 + a_7b_6 & a_7b_7 & a_8b_7 + a_7b_8 & a_9b_7 + a_7b_9 \\ & a_{10}b_7 + a_7b_{10} & a_{11}b_7 + a_7b_{11} & a_{12}b_7 + a_7b_{12} & \\ & & a_{13}b_7 + a_7b_{13} & & \end{array} \right\rangle \qquad (2.4)$$

This can be expressed in the following way:

$$c_1 = a_1b_7 + a_7b_1 \qquad c_2 = a_2b_7 + a_7b_2 \qquad c_3 = a_3b_7 + a_7b_3 \qquad c_4 = a_4b_7 + a_7b_4$$

$$c_5 = a_5b_7 + a_7b_5 \qquad c_6 = a_6b_7 + a_7b_6 \qquad c_7 = a_7b_7 \qquad c_8 = a_8b_7 + a_7b_8$$

$$c_9 = a_9b_7 + a_7b_9 \qquad c_{10} = a_{10}b_7 + a_7b_{10} \qquad c_{11} = a_{11}b_7 + a_7b_{11} \qquad c_{12} = a_{12}b_7 + a_7b_{12}$$

$$c_{13} = a_{13}b_7 + a_7b_{13} \qquad (2.5)$$

where a_7 and b_7 denote the hearts indices defined by $\frac{1}{4}[n^2+3]$ of the two rhotrices, the resulting value of c_7 is the product of the two hearts from the two rhotrices. We can now extend this to accommodate rhotrices of arbitrary dimensions. Going by (2.5), we can represent the rhotrix's heart entry as $a_{\frac{1}{4}[n^2+3]}$ and $b_{\frac{1}{4}[n^2+3]}$ respectively.

Equation (2.5) can be represented in rhotrix for as:

$$R_5 \circ Q_5 = C = \left\langle \begin{array}{ccccc} & & c_1 & & \\ & c_2 & c_3 & c_4 & \\ c_5 & c_6 & c_7 & c_8 & c_9 \\ & c_{10} & c_{11} & c_{12} & \\ & & c_{13} & & \end{array} \right\rangle \qquad (2.6)$$

In general, given two rhotrices R_n and Q_n of dimension n, the entries of the heart-oriented product C_n of R_n and Q_n can be expressed as follows:

$$C_i = b_{\frac{1}{4}[n^2+3]} a_i + a_{\frac{1}{4}[n^2+3]} b_i + (1-\lambda)(a_{\frac{1}{4}[n^2+3]} b_{\frac{1}{4}[n^2+3]}), \text{ for } i = 1,2,\ldots,\tfrac{1}{2}(n^2+1) \qquad (2.7)$$

where $\lambda = 0$ when the index value corresponds to that of the heart and $\lambda = 1$ when otherwise. This condition is further illustrated in table I and II.

Alternatively, suppose that we now represent the n-dimensional rhotrix in equation (2.1) by $R_n = \langle a_i, \quad a_h \rangle$ where of course a_i and a_h represents the a_i entries and its heart respectively, with $i = 1, 2, 3, \ldots |R_n|$ and $h = 3, 7, 13, \ldots \frac{1}{4}[n^2+3]$.

Consider an i dimensional rhotrix, having number of entries $|R_i| = \frac{1}{2}(i^2+1)$. Let $\bar{i}$ be the mean of these entries computed as shown in table I:

Table I Setting conditions for λ over a three dimensional rhotrix entries

i	$\bar{i}$	$i-\bar{i}$	$\lvert i-\bar{i} \rvert$	
1	3	-2	2	
2	3	-1	1	$\lambda = 0$
3	3	0	0	$\lambda = 1$
4	3	1	1	
5	3	2	2	$\lambda = 0$

Table II Setting conditions for λ over a five dimensional rhotrix entries

i	$\bar{i}$	$i-\bar{i}$	$\lvert i-\bar{i} \rvert$	
1	7	-6	6	
2	7	-5	5	
3	7	-4	4	
4	7	-3	3	$\lambda = 0$
5	7	-2	2	
6	7	-1	1	
7	7	0	0	$\lambda = 1$
8	7	1	1	
9	7	2	2	
10	7	3	3	$\lambda = 0$
11	7	4	4	
12	7	5	5	
13	7	6	6	

Let the heart entry of rhotrix Q_n denoted by b_h multiply all the entries of rhotrix R_n denoted by a_i and vice versa for the heart entry of the rhotrix R_n denoted by a_h. Subsequently, the two corresponding hearts of R_n and Q_n multiply each other as a single resulting product. Based on the derived value of λ, we can then define a function on λ as:

$$\lambda = \begin{cases} 0, & |i - \bar{i}| > 0 \\ 1, & |i - \bar{i}| = 0 \end{cases}$$

We define multiplication thus, of any two heart-oriented rhotrices of the same dimension as follow:

$$R_n \circ Q_n = b_h \circ \langle a_i \rangle + a_h \langle b_i \rangle \circ (1 - \lambda)$$

It follows that,

$$C(i) = \begin{cases} b_h \langle a_i \rangle & for \;\; i = h, \;\lambda = 1 \\ b_h \langle a_i \rangle + a_h \langle b_i \rangle & for \;\; i \neq h, \;\lambda = 0 \end{cases}$$

Hence, we can thus generalize this as.

$$C(i) = b_h \langle a_i \rangle + a_h \langle b_i \rangle (1 - \lambda) \qquad (2.8)$$

Where $\lambda = 1$ for $i = h$ and $\lambda = 0$ for $i \neq h$

It can also be verified further, though not discussed in this work that multiplication of any two rhotrices is commutative, associative, and distributive over addition. Hence we say that the set R of all rhotrices is a commutative algebra [4].

Algorithm 1: Algorithm for single indexed Multiplication of Rhotrices

Input:

$$h = \frac{1}{4}\left[n^2 + 3\right]$$

$a[0..k]$

$b[0..k]$

Output:

$c[0..k]$

for i =: 1 to k

if $(i == \frac{1}{4}[n^2 + 3])$

$c[i] \leftarrow a[i] * b[i]$;

else

$c[i] \leftarrow b[h] \times a[i] + a[h] \times b[i]$;

endif

endfor

IV. Row-Wise double Indices Heart-Oriented Rhotrix Multiplication

The row-wise rhotrix multiplication tends to put into consideration the position and direction of each entry in the cause of the multiplicative operations. The operation is performed in a row-wise direction indicated by the entry indices *i* and *j*. The index *i* indicate the row entry position while the index *j* indicates the row direction. The general representation of the row-wise rhotrices is as depicted in (3.1). It is important to note that division in this case are all integer division, since we are less concerned with the resulting decimal values.

Definition: The row of any given rhotrix is an array of entries running diagonally from the top-most left to the bottom rightmost direction of the rhotrix.

$$R_n = \left\langle \begin{array}{ccccccccc} & & & & a_{11} & & & & \\ & & & a_{31} & a_{21} & a_{12} & & & \\ & & a_{5,1} & a_{4,1} & a_{32} & a_{22} & a_{13} & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ a_{n,1} & \cdots & \cdots & \cdots & a_{\frac{n+1}{2},\left\lfloor\frac{n+3}{4}\right\rfloor} & \cdots & \cdots & \cdots & a_{1,\frac{n+1}{2}} \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & a_{n,\frac{n+1}{2}-2} & a_{n-1,\frac{n+1}{2}-2} & a_{n-2,\frac{n+1}{2}-1} & a_{n-3,\frac{n+1}{2}-1} & a_{n-4,\frac{n+1}{2}} & & \\ & & & a_{n,\frac{n+1}{2}-1} & a_{n-1,\frac{n+1}{2}-1} & a_{n-2,\frac{n+1}{2}} & & & \\ & & & & a_{n,\frac{n+1}{2}} & & & & \end{array} \right\rangle \qquad (3.1)$$

We use commas in (3.1) to avoid any ambiguity with respect to the array indexes separation.
Further simplification of equation 3.1 gives:

$$R_n = \left\langle \begin{array}{ccccccccc} & & & & a_{11} & & & & \\ & & & a_{31} & a_{21} & a_{12} & & & \\ & & a_{51} & a_{41} & a_{32} & a_{22} & a_{13} & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ a_{n,1} & \cdots & \cdots & \cdots & a_{\frac{n+1}{2},\left\lfloor\frac{n+3}{4}\right\rfloor} & \cdots & \cdots & \cdots & a_{1,\frac{n+1}{2}} \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & a_{n,\frac{n-3}{2}} & a_{n-1,\frac{n-3}{2}} & a_{n-2,\frac{n-1}{2}} & a_{n-3,\frac{n-1}{2}} & a_{n-4,\frac{n+1}{2}} & & \\ & & & a_{n,\frac{n-1}{2}} & a_{n-1,\frac{n-1}{2}} & a_{n-2,\frac{n+1}{2}} & & & \\ & & & & a_{n,\frac{n+1}{2}} & & & & \end{array} \right\rangle \qquad (3.2)$$

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$$R_n \circ Q_n = \left\langle \begin{array}{ccccccccc} & & & & a_{11} & & & & \\ & & & a_{31} & a_{21} & a_{12} & & & \\ & & a_{51} & a_{41} & a_{32} & a_{22} & a_{13} & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ a_{n1} & \cdots & \cdots & \cdots & a_{\frac{n+1}{2},\left\lfloor\frac{n+3}{4}\right\rfloor} & \cdots & \cdots & \cdots & a_{1,\frac{n+1}{2}} \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & a_{n,\frac{n-3}{2}} & a_{n-1,\frac{n-3}{2}} & a_{n-2,\frac{n-1}{2}} & a_{n-3,\frac{n-1}{2}} & a_{n-4,\frac{n+1}{2}} & & \\ & & & a_{n,\frac{n-1}{2}} & a_{n-1,\frac{n-1}{2}} & a_{n-2,\frac{n+1}{2}} & & & \\ & & & & a_{n,\frac{n+1}{2}} & & & & \end{array} \right\rangle \circ \left\langle \begin{array}{ccccccccc} & & & & b_{11} & & & & \\ & & & b_{31} & b_{21} & b_{12} & & & \\ & & b_{51} & b_{41} & b_{32} & b_{22} & b_{13} & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ b_{n,1} & \cdots & \cdots & \cdots & b_{\frac{n+1}{2},\left\lfloor\frac{n+3}{4}\right\rfloor} & \cdots & \cdots & \cdots & b_{1,\frac{n+1}{2}} \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & b_{n,\frac{n-3}{2}} & b_{n-1,\frac{n-3}{2}} & b_{n-2,\frac{n-1}{2}} & b_{n-3,\frac{n-1}{2}} & b_{n-4,\frac{n+1}{2}} & & \\ & & & b_{n,\frac{n-1}{2}} & b_{n-1,\frac{n-1}{2}} & b_{n-2,\frac{n+1}{2}} & & & \\ & & & & b_{n,\frac{n+1}{2}} & & & & \end{array} \right\rangle \quad (3.3)$$

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$$C_n = \left\langle \begin{array}{ccccccccc} & & & & c_{11} & & & & \\ & & & c_{31} & c_{21} & c_{12} & & & \\ & & c_{51} & c_{41} & c_{32} & c_{22} & c_{13} & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ c_{n,1} & \cdots & \cdots & \cdots & c_{\frac{n+1}{2},\left\lfloor\frac{n+3}{4}\right\rfloor} & \cdots & \cdots & \cdots & c_{1,\frac{n+1}{2}} \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & c_{n,\frac{n+1}{2}-2} & c_{n-1,\frac{n+1}{2}-2} & c_{n-2,\frac{n+1}{2}-1} & c_{n-3,\frac{n+1}{2}-1} & c_{n-4,\frac{n+1}{2}} & & \\ & & & c_{n,\frac{n+1}{2}-1} & c_{n-1,\frac{n+1}{2}-1} & c_{n-2,\frac{n+1}{2}} & & & \\ & & & & c_{n,\frac{n+1}{2}} & & & & \end{array} \right\rangle \quad (3.4)$$

Definition: (Row-wise heart-oriented rhotrix multiplication)

Let $a, b, c, \ldots\ldots, c \in \Re$, $R_n \circ Q_n = a_{11} \times b_{\frac{n+1}{2},\left\lfloor\frac{n+3}{4}\right\rfloor} + b_{11} \times a_{\frac{n+1}{2},\left\lfloor\frac{n+3}{4}\right\rfloor} \;...\; a_{n,\frac{n+1}{2}} \times b_{\frac{n+1}{2},\left\lfloor\frac{n+3}{4}\right\rfloor} + b_{n,\frac{n+1}{2}} \times a_{\frac{n+1}{2},\left\lfloor\frac{n+3}{4}\right\rfloor}$

.

Where $a_{\frac{n+1}{2},\left\lfloor\frac{n+3}{4}\right\rfloor}$ and $b_{\frac{n+1}{2},\left\lfloor\frac{n+3}{4}\right\rfloor}$ are the hearts of the two rhotrices. This is illustrated as in (3.5) and (3.6).

$$R_5 \circ Q_5 = \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{31} & a_{21} & a_{12} & \\ a_{51} & a_{41} & a_{32} & a_{22} & a_{13} \\ & a_{52} & a_{42} & a_{33} & \\ & & a_{53} & & \end{array} \right\rangle \circ \left\langle \begin{array}{ccccc} & & b_{11} & & \\ & b_{31} & b_{21} & b_{12} & \\ b_{51} & b_{41} & b_{32} & b_{22} & b_{13} \\ & b_{52} & b_{42} & b_{33} & \\ & & b_{53} & & \end{array} \right\rangle \quad (3.5)$$

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$$R_5 \circ Q_5 = C_5 = \left\langle \begin{array}{ccccc} & & c_{11} & & \\ & c_{31} & c_{21} & c_{12} & \\ c_{51} & c_{41} & c_{32} & c_{22} & c_{13} \\ & c_{52} & c_{42} & c_{33} & \\ & & c_{53} & & \end{array} \right\rangle \qquad (3.6)$$

Similarly, multiplication of the rhotrices in (3.5) is commutative. The product operation is similar to the method given earlier for the single indices multiplication approach. The multiplication process for the two rhotrices in (3.5) can thus be expressed as:

$$b_{32} \circ \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{31} & a_{21} & a_{12} & \\ a_{51} & a_{41} & a_{32} & a_{22} & a_{13} \\ & a_{52} & a_{42} & a_{33} & \\ & & a_{53} & & \end{array} \right\rangle + a_{32} \circ \left\langle \begin{array}{ccccc} & & b_{11} & & \\ & b_{31} & b_{21} & b_{12} & \\ b_{51} & b_{41} & b_{32} & b_{22} & b_{13} \\ & b_{52} & b_{42} & b_{33} & \\ & & b_{53} & & \end{array} \right\rangle =$$

$$\left\langle \begin{array}{ccccc} & & a_{11}b_{32}+b_{11}a_{32} & & \\ & a_{31}b_{32}+b_{31}a_{32} & a_{21}b_{32}+b_{21}a_{32} & a_{12}b_{32}+b_{12}a_{32} & \\ a_{51}b_{32}+b_{51}a_{32} & a_{41}b_{32}+b_{41}a_{32} & a_{32}b_{32} & a_{22}b_{32}+b_{22}a_{32} & a_{13}b_{32}+b_{13}a_{32} \\ & a_{52}b_{32}+b_{52}a_{32} & a_{42}b_{32}+b_{42}a_{32} & a_{33}b_{32}+b_{33}a_{32} & \\ & & a_{53}b_{32}+b_{53}a_{32} & & \end{array} \right\rangle \qquad (3.8)$$

$c_{11} = a_{11}b_{32} + b_{11}a_{32}$ $c_{12} = a_{12}b_{32} + b_{12}a_{32}$ $c_{13} = a_{13}b_{32} + b_{13}a_{32}$ $c_{21} = a_{21}b_{32} + b_{21}a_{32}$

$c_{22} = a_{22}b_{32} + b_{22}a_{32}$ $c_{31} = a_{31}b_{32} + b_{31}a_{32}$ $c_{32} = a_{32}b_{32}$ $c_{33} = a_{33}b_{32} + b_{33}a_{32}$ $c_{41} = a_{41}b_{32} + b_{41}a_{32}$

$c_{42} = a_{42}b_{32} + b_{42}a_{32}$ $c_{51} = a_{51}b_{32} + b_{51}a_{32}$ $c_{52} = a_{52}b_{32} + b_{52}a_{32}$ $c_{53} = a_{53}b_{32} + b_{53}a_{32}$

Where a_{32} and b_{32} denote the hearts of the two rhotrices whose indices are defined by $\frac{n+1}{2}$ *and* $\left\lfloor \frac{n+3}{4} \right\rfloor$ respectively. The resulting value of c_{32} is the product of the two hearts from the two rhotrices. We can equally extend this to accommodate for rhotrix of n-dimension. Then going by equation (3.8), we can represent the row-wise rhotrix hearts as $a_{\frac{n+1}{2},\left\lfloor \frac{n+3}{4} \right\rfloor}$ and $b_{\frac{n+1}{2},\left\lfloor \frac{n+3}{4} \right\rfloor}$, such that:

$$c_{i,j} = a_{\frac{n+1}{2},\left\lfloor \frac{n+3}{4} \right\rfloor} * a_{i,j} + a_{\frac{n+1}{2},\left\lfloor \frac{n+3}{4} \right\rfloor} * b_{i,j}(1-\lambda) \qquad (3.9)$$

Where λ denotes a constant value that lies between 1 and 0, such that if the indices of a_{ij} and b_{ij} takes the positions of the heart entries, then $\lambda = 0$ and $\lambda = 1$, otherwise.

Algorithm 2: Heart-oriented Row-wise rhotrix multiplication

Input:

$p \leftarrow (n+1)/2$

$q \leftarrow (n+3)/4$

$a[0...row\_upperbound\ ,\ 0...column\_upperboun\ ]$

$b[0...row\_upperbound\ ,\ 0...column\_upperboun\ ]$

Output:

$c[0...row\_upperbound\ ,\ 0...column\_upperboun\ ]$

```
For i ←0 to row_upperbound
For j ←0 to column_upperbound
{
        if (i == p && j == q){
        c[i,j] ←a[i,j] * b[i,j];
}
        else {
    c[i,j] ←a[i][j] * b[p][q] + b[i,j] * a[p,q];
}
endfor
```

V. CONCLUSION AND FUTURE WORK

In this paper, we have presented some fundamental concept relating to the general ideas and methods from an already predefined multiplicative process for rhotrix multiplication. Our major contributions in this area could be found in the two generalization cases presented and the mathematical expression given, that led to the formulation of our sequential algorithm. The implementation of the algorithms showed that extension of rhotrices to higher dimension is possible as indicated in [1]. What is probably left is to find a suitable application area for rhotrices. We actually believe that in some ways, this paper might serve as a catalyst for future researchers who might develop interest in finding such possible application areas.

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