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Uniformly Starlike and Uniformly Convexity Properties Pertaining to Certain Special Functions

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Uniformly Starlike and Uniformly Convexity Properties Pertaining to Certain Special Functions

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Abstract - The aim of this paper is to establish sufficient conditions for the function $z\{{}_p\overline{\psi}_q(z)\}$ to be in the classes of uniformly starlike and uniformly convex functions associated with the parabolic region $\text{Re}\{\omega\} > |\omega - 1|$. Further, convolution of the functions which are analytic in the open unit disk with negative coefficients have been investigated. Finally, similar results using an integral operator have also been obtained.

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I. Introduction

Let A denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \qquad \ldots (1.1)$$

that are analytic in the open unit disk $\Delta = \{z : |z| < 1\}$.

Also let S denote the subclass of A consisting of all functions f (z) of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n, \quad a_n \geq 0. \qquad \ldots (1.2)$$

A function $f \in A$ is said to be uniformly convex in Δ if f is a univalent convex function having the property that for every circular arc γ contained in Δ with centre also in Δ ,the image curve $f(\gamma)$ is a convex arc. Denoting the class of all uniformly convex functions by UCV, it was shown in [10, 13] that

$$f \in UCV \Leftrightarrow \left|\frac{z f''(z)}{f'(z)}\right| < \text{Re}\left(1 + \frac{z f''(z)}{f'(z)}\right), (z \in \Delta). \qquad \ldots (1.3)$$

To give the geometric interpretation of (1.3), let

$$\Omega_p = \{\omega : \omega = u + iv, \text{Re}(\omega) > \omega - 1\}$$

which is the interior of the parabola $v^2 = 2u - 1$.

$$\text{Then } f \in UCV \Leftrightarrow 1 + \frac{z f''(z)}{f'(z)} \in \Omega_p \qquad \ldots (1.4)$$

A class closely related to the class UCV is the class of parabolic starlike functions denoted by US_p [15]. This class US_p and the class UC_P of uniformly convex functions have been studied in [1,6,7,11,15,16,17]. A survey of these functions can the works of [12].

Let $f \in S$, $0 \leq \lambda < \infty$, and $0 \leq \mu < 1$, then [2,13]

$$f \in UC_p(\lambda,\mu) \Leftrightarrow \text{Re}\left(1 + \frac{z f''(z)}{f'(z)}\right) \geq \lambda \left|\frac{z f''(z)}{f'(z)}\right| + \mu \qquad \ldots (1.5)$$

and

$$f \in US_p(\lambda,\mu) \Leftrightarrow \text{Re}\left(\frac{z f'(z)}{f(z)}\right) \geq \lambda \left|\frac{z f''(z)}{f'(z)} - 1\right| + \mu \qquad \ldots (1.6)$$

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We recall here that the hadamard product (or convolution) of f (z) of the form (1.1) and $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$, is defined as

$$(f*g)\,z = z - \sum_{n=2}^{\infty} a_n b_n z^n, \quad z \in \Delta. \qquad \ldots(1.7)$$

The generalized Fox-Wright function [8] appearing in the present paper is defined by

$${}_p\overline{\Psi}_q(z) = {}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p};\\ (b_j,\beta_j;B_j)_{1,q};\end{matrix}\; z\right]$$

$$= \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p}\{\Gamma(a_j+\alpha_j n)\}^{A_j}\, z^n}{\prod_{j=1}^{q}\{\Gamma(b_j+\beta_j n)\}^{B_j}\, n!}, \qquad \ldots(1.8)$$

where α_j $(j = 1,\ldots, p)$ and β_j $(j = 1,\ldots, q)$ are real and positive ,$1+\sum_{j=1}^{q}\beta_j > \sum_{j=1}^{p}\alpha_j$, and A_j $(j = 1,\ldots, p)$ and B_j $(j = 1,\ldots, q)$ can take non -integer values.

For (1.8), we have

$$z\{{}_p\overline{\Psi}_q(z)\} = \frac{\prod_{j=1}^{p}\{\Gamma a_j\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma b_j\}^{B_j}}\, z +$$

$$\sum_{n=2}^{\infty} \frac{\prod_{j=1}^{p}\{\Gamma[a_j+\alpha_j(n-1)]\}^{A_j}\, z^n}{\prod_{j=1}^{q}\{\Gamma[b_j+\beta_j(n-1)]\}^{B_j}\,(n-1)!}$$

or

$$z\,{}_p\overline{\Omega}_q = z + \sum_{n=2}^{\infty} \frac{\prod_{j=1}^{q}\{\Gamma b_j\}^{B_j}}{\prod_{j=1}^{q}\{\Gamma a_j\}^{A_j}} \qquad \ldots(1.9)$$

$$\frac{\prod_{j=1}^{p}\{\Gamma[a_j+\alpha_j(n-1)]\}^{A_j}\, z^n}{\prod_{j=1}^{q}\{\Gamma[b_j+\beta_j(n-1)]\}^{B_j}\,(n-1)!}$$

where

$${}_p\overline{\Omega}_q = \frac{\prod_{j=1}^{q}\{\Gamma b_j\}^{b_j}}{\prod_{j=1}^{q}\{\Gamma a_j\}^{A_j}}\; {}_p\overline{\Psi}_q. \qquad \ldots(1.10)$$

Now, we define a linear operator $\overline{G}_q^p : S \to S$ as follows:

$$\overline{G}_q^p f(z) = z\,{}_p\overline{\Omega}_q * f(z)$$

$$= z - \sum_{n=2}^{\infty} \frac{\prod_{j=1}^{q} \{\Gamma b_j\}^{B_j} \prod_{j=1}^{p} \{\Gamma[a_j+\alpha_j(n-1)]\}^{A_j} z^n}{\prod_{j=1}^{p} \{\Gamma a_j\}^{A_j} \prod_{j=1}^{q} \{\Gamma[b_j+\beta_j(n-1)]\}^{B_j} (n-1)!} a_n$$

or $$\bar{G}_q^p = z - \sum_{n=2}^{\infty} \bar{B}(a_j, b_j, A_j, B_j, n)\, a_n z^n \qquad \ldots(1.11)$$

Where

$$\bar{B}(a_j, b_j, A_j, B_j, n) = \frac{\prod_{j=1}^{q} \{\Gamma b_j\}^{B_j}}{\prod_{j=1}^{p} \{\Gamma a_j\}^{A_j}} \frac{\prod_{j=1}^{p} \{\Gamma[a_j+\alpha_j(n-1)]\}^{A_j}}{\prod_{j=1}^{q} \{\Gamma[b_j+\beta_j(n-1)]\}^{B_j} (n-1)!} \qquad \ldots(1.12)$$

Corresponding to the operator $\bar{G}_q^p$ defined in (1.11), we let $\bar{S}_q^p$ (λ) for $-1 \le \gamma < 1$, denotes the subclass of functions f ∈ S satisfying the inequality

$$\mathrm{Re}\left\{ \frac{z(\bar{G}_q^p f(z))'}{\bar{G}_q^p f(z)} - \gamma \right\} \ge \left| \frac{z(\bar{G}_q^p f(z))'}{\bar{G}_q^p f(z)} - 1 \right| \qquad \ldots(1.13)$$

In the present paper we shall use the following Lemmas [2, 18] to establish our main results:

Lemma 1.1. A function f (z) of the form (1.1) is in the class $US_p(\lambda,\mu)$ if

$$\sum_{n=2}^{\infty} [n(1+\lambda)-(\lambda+\mu)]\, a_n \le (1-\mu)\, M_1, \qquad \ldots(1.14)$$

where $M_1 > 0$ is a suitable constant.

Lemma 1.2. A function f(z) of the form (1.1) is in the class $UC_p(\lambda,\mu)$ if

$$\sum_{n=2}^{\infty} n[n(1+\lambda)-(\lambda+\mu)]\, a_n \le (1-\mu)\, M_2, \qquad \ldots(1.15)$$

where $M_2 > 0$ is a suitable constant.

II. Main Results

Theorem 2.1. If $\sum_{j=1}^{q} b_j > \sum_{j=1}^{p} a_j + 1$, and $1 + \sum_{j=1}^{q} B_j\beta_j > \sum_{j=1}^{p} A_j\alpha_j$,

then a sufficient condition for the function $z\{{}_p\bar{\Psi}_q(z)\}$ to be in the class $US_p(\lambda, \mu)$, $0 \le \lambda < \infty$, and $0 \le \mu < 1$, is

$$\left(\frac{1+\lambda}{1-\mu}\right) {}_p\bar{\Psi}_q\left[\begin{matrix}(a_j+\alpha_j, \alpha_j; A_j)_{1,p};\\ (b_j+\beta_j, \beta_j; B_j)_{1,q};\end{matrix}\; 1\right] + {}_p\bar{\Psi}_q\left[\begin{matrix}(a_j, \alpha_j; A_j)_{1,p};\\ (b_j, \beta_j; B_j)_{1,q};\end{matrix}\; 1\right] \le M_1 + \frac{\prod_{j=1}^{p} \{\Gamma a_j\}^{A_j}}{\prod_{j=1}^{q} \{\Gamma b_j\}^{B_j}}. \qquad \ldots(2.1)$$

Proof. Since

$$z\{{}_p\overline{\Psi}_q(z)\} = \frac{\prod_{j=1}^{p}\{\Gamma a_j\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma b_j\}^{B_j}}\, z + $$

$$\sum_{n=2}^{\infty} \frac{\prod_{j=1}^{p}\{\Gamma[a_j+\alpha_j(n-1)]\}^{A_j}\, z^n}{\prod_{j=1}^{q}\{\Gamma[b_j+\beta_j(n-1)]\}^{B_j}\,(n-1)!}$$

so by virtue of Lemma 1.1, we need only to show that

$$\sum_{n=2}^{\infty}[(1+\lambda)n-(\lambda+\mu)]\left[\frac{\prod_{j=1}^{p}\{\Gamma[a_j+\alpha_j(n-1)]\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma[b_j+\beta_j(n-1)]\}^{B_j}\,(n-1)!}\right]$$

$$\leq (1-\mu)M_1 \qquad \ldots (2.2)$$

Now, we have

$$\sum_{n=2}^{\infty}[(1+\lambda)n-(\lambda+\mu)]\left[\frac{\prod_{j=1}^{p}\{\Gamma[a_j+\alpha_j(n-1)]\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma[b_j+\beta_j(n-1)]\}^{B_j}\,(n-1)!}\right]$$

$$= \sum_{n=0}^{\infty}[(1+\lambda)(n+2)-(\lambda+\mu)] \times \left[\frac{\prod_{j=1}^{p}\{\Gamma[a_j+\alpha_j(n+1)]\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma[b_j+\beta_j(n+1)]\}^{B_j}\,(n+1)!}\right]$$

$$= (1+\lambda)\sum_{n=0}^{\infty}\left[\frac{\prod_{j=1}^{p}\{\Gamma[(a_j+\alpha_j)+n\alpha_j]\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma[(b_j+\beta_j)+n\beta_j]\}^{B_j}\, n!}\right] + (1-\mu)\left[\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}\{\Gamma(a_j+\alpha_j n)\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma(b_j+\beta_j n)\}^{B_j}}\frac{1}{n!} - \frac{\prod_{j=1}^{p}\{\Gamma a_j\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma b_j\}^{B_j}}\right]$$

$$= (1+\lambda)\ {}_p\overline{\Psi}_q\left[\begin{matrix}(|a_j+\alpha_j|,\alpha_j;A_j)_{1\,p}\,; \\ (|b_j+\beta_j|,\beta_j;B_j)_{1\,q}\,;\end{matrix}\ 1\right]$$

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$$+ (1-\mu)\ {}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\ ; \\ (b_j,\beta_j;B_j)_{1,q}\ ;\end{matrix}\ 1\right] - (1-\mu)\ \frac{\prod_{j=1}^{p}\{\Gamma a_j\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma b_j\}^{B_j}}$$

$$\le (1-\mu)M_1$$

which in view of Lemma 1.1 gives the desired result .

Theorem 2.2. Let f (z) be given by (1.1) and -1 ≤ γ < 1, then $f(z) \in \overline{S}_q^p(\gamma)$ if

$$\sum_{n=2}^{\infty}(2n-1-\gamma)\overline{B}(a_j,b_j,A_j,B_j,n)\,|a_n| \le 1-\gamma \qquad \ldots (2.3)$$

where $\overline{B}\,(a_j,b_j,A_j,B_j,n)$ is given by (1.12).

Proof. By virtue of (1.13), it is sufficient to show that

$$\mathrm{Re}\left\{\frac{z(\overline{G}_q^p f(z))'}{\overline{G}_q^p f(z)}-\gamma\right\} \ge \left|\frac{z(\overline{G}_q^p f(z))'}{\overline{G}_q^p f(z)}-1\right|$$

or $$2\left|\frac{z(\overline{G}_q^p f(z))'}{\overline{G}_q^p f(z)}-1\right| \le 1-\gamma$$

or $$2\left|\frac{z\left\{1-\sum_{n=2}^{\infty}\overline{B}(a_j,b_j,A_j,B_j,n)\,a_n\,n\,z^{n-1}\right\}}{z-\sum_{n=2}^{\infty}\overline{B}(a_j,b_j,A_j,B_j,n)\,a_n\,z^n}-1\right| \le 1-\gamma$$

or

$$2\sum_{n=2}^{\infty}(n-1)\overline{B}(a_j,b_j,A_j,B_j,n)\,|a_n|$$

$$\le (1-\gamma)\left\{1-\sum_{n=2}^{\infty}\overline{B}(a_j,b_j,A_j,B_j,n)|a_n|\right\}$$

or $$\sum_{n=2}^{\infty}(2n-1-\gamma)\overline{B}(a_j,b_j,A_j,B_j,n)\,|a_n| \le 1-\gamma$$

Hence the theorem.

III. An Integral Operator

In this section we obtain sufficient conditions for the function ${}_p\overline{\varphi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\ ; \\ (b_j,\beta_j;B_j)_{1,q}\ ;\end{matrix}\ z\right] = \int_0^z {}_p\overline{\Psi}_q(x)\,dx$ to be in the classes $US_p(\lambda,\mu)$ and $UC_p(\lambda,\mu)$.

Theorem 3.1. If $\sum_{j=1}^{q} b_j > \sum_{j=1}^{p} a_j + 1$, and $1+\sum_{j=1}^{q} B_j\beta_j > \sum_{j=1}^{p} A_j\alpha_j$, then a sufficient condition for the function ${}_p\overline{\varphi}_q(z) = \int_0^z {}_p\overline{\Psi}_q(x)\,dx$ to be in the class $US_p(\lambda,\mu)$, $0 \le \lambda < \infty$ and $0 \le \mu < 1$, is

$$\frac{(1+\lambda)}{(1-\mu)}\ {}_p\overline{\Psi}_q(1)-\frac{(\lambda+\mu)}{(1-\mu)}\ {}_p\overline{\Psi}_q\left[\begin{matrix}(a_j-\alpha_j,\alpha_j;A_j)_{1,p}\,; \\ (b_j-\beta_j,\beta_j;B_j)_{1,q}\ ;\end{matrix}\ 1\right]$$

$$+\frac{(\lambda+\mu)}{(1-\mu)}\frac{\prod_{j=1}^{p}\{\Gamma(a_j-\alpha_j)\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma(b_j-\beta_j)\}^{B_j}}\le M_1+\frac{\prod_{j=1}^{p}\{\Gamma a_j\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma b_j\}^{B_j}}. \qquad \ldots (3.1)$$

Proof. Since

$$ {}_p\overline{\varphi}_q(z)=\int_0^z {}_p\overline{\Psi}_q(x)\,dx$$

$$=\frac{\prod_{j=1}^{p}\{\Gamma a_j\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma b_j\}^{B_j}}z+\sum_{n=2}^{\infty}\frac{\prod_{j=1}^{p}\{\Gamma[(a_j-\alpha_j)+\alpha_j n]\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma[(b_j-\beta_j)+\beta_j n]\}^{B_j}}\frac{z^n}{n!}$$

Now, we have $\displaystyle\sum_{n=2}^{\infty}[n(1+\lambda)-(\lambda+\mu)]\frac{\prod_{j=1}^{p}\{\Gamma[(a_j-\alpha_j)+\alpha_j n]\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma[(b_j-\beta_j)+\beta_j n]\}^{B_j}\,n\,!}$

$$=(1+\lambda)\left[\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}\{\Gamma(a_j+\alpha_j n)\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma(b_j+\beta_j n)\}^{B_j}\,n\,!}-\right.$$

$$\left.-\frac{\prod_{j=1}^{p}\{\Gamma(a_j)\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma(b_j)\}^{B_j}\,n\,!}\right]-$$

$$(\lambda+\mu)\left[\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}\Gamma[(a_j-\alpha_j)+\alpha_j n]\}^{A_j}}{\prod_{j=1}^{q}\Gamma[(b_j-\beta_j)+\beta_j n]\}^{B_j}\,n\,!}\right.$$

$$\left.-\frac{\prod_{j=1}^{p}\{\Gamma(a_j-\alpha_j)\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma(b_j-\beta_j)\}^{B_j}}-\frac{\prod_{j=1}^{p}\{\Gamma a_j\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma b_j\}^{B_j}}\right]$$

$$= (1+\lambda)\ {}_p\bar{\psi}_q(1) - (\lambda+\mu)\ {}_p\bar{\psi}_q\left[\begin{matrix}(a_j-\alpha_j,\alpha_j;A_j)_{1,p}\,; \\ (b_j-\beta_j,\beta_j;B_j)_{1,q}\ \ ;\end{matrix}\ 1\right]$$

$$+ (\lambda+\mu)\frac{\prod_{j=1}^{p}\{\Gamma(a_j-\alpha_j)\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma(b_j-\beta_j)\}^{B_j}} - (1-\mu)\frac{\prod_{j=1}^{p}\{\Gamma a_j\}^{A_j}}{\prod_{j=1}^{q}\{\Gamma b_j\}^{B_j}}$$

which in view of Lemma 1.1, leads to the result (3.1).

Theorem 3.2. If $\sum_{j=1}^{q} b_j > \sum_{j=1}^{p} a_j + 1$, and $1 + \sum_{j=1}^{q} B_j\beta_j > \sum_{j=1}^{p} A_j\alpha_j$,

then a sufficient condition for the function ${}_p\bar{\varphi}_q(z) = \int_0^z {}_p\bar{\psi}_q(x)\,dx$ to be in the class $UC_p(\lambda,\mu)$, $0 \le \lambda < \infty$ and $0 \le \mu < 1$, is

Proof. The result follows as direct consequence of the Theorem 3.1, keeping Lemma 1.2 in view.

IV. Particular Cases

4.1 For $\lambda=2$ and $\mu=0$, Theorem 2.1 and Theorem 3.1 corresponds to the results recently obtained by Chaurasia and Kumawat [3] for $\alpha=0$.

4.2 For $A_j = 1 (j = 0,1,\ldots,p)$; $B_j = 1 (j = 0,1,\ldots,q)$, the Theorems established in the present paper readily yield the results due to Bapna and Jain [2].

4.3 The results due to Chaurasia and Srivastava [4], Dixit and Verma [5] and Shanmugam et.al. [14] also follow as particular cases of our main results.

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