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# Finite Integrals Pertaining To a Product of Special Functions

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FINITE INTEGRALS PERTAINING TO A PRODUCT OF SPECIAL FUNCTIONS

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# Finite Integrals Pertaining To a Product of Special Functions

V.B.L. Chaurasia[α], Yudhveer Singh[Ω]

*Abstract -* An attempt has been made to establish an integral concerning the product of generalized Lauricella function and two H-function of several complex variables (Srivastava and Panda [4,5]) By giving suitable values to the parameters, the main integral reduces to F function. Mainly we are using the series representation of H-function given by Olkha and Chaurasia [2,3].

*Keywords : Multivariable H-function, H-function in Series form, Lauricella function, Jacobi polynomial, Kampé de Fériet function.*

## I. Introduction

The series representation of the H-function of several complex variable studied by Olkha and Chaurasia [2,3] is given as follows:

$$H[z_1,\dots,z_r]=H^{0,\lambda':(u',v');\dots;(u^{(r)},v^{(r)})}_{A',C':[B',D'];\dots;[B^{(r)},D^{(r)}]}\left[\begin{matrix}[(a):\theta',\dots,\theta^{(r)}]:[b':\phi'];\dots;[b^{(r)}:\phi^{(r)}];\\ [(c):\psi',\dots,\psi^{(r)}]:[d':\delta'];\dots;[d^{(r)}:\delta^{(r)}];\end{matrix}\; z_1,\dots,z_r\right]$$

$$=\sum_{m_i=1}^{u^{(i)}}\sum_{n_i=0}^{\infty}\Phi_1\Phi_2\frac{\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}}{\prod_{i=1}^{r}(\delta^{(i)}_{(m_i)}n_i!)},\qquad\dots(1.1)$$

Where

$$\Phi_1=\frac{\prod_{j=1}^{\lambda'}\Gamma\left(1-a_j+\sum_{i=1}^{r}\theta_j^{(i)}U_i\right)}{\prod_{j=\lambda'+1}^{A'}\Gamma\left(a_j-\sum_{i=1}^{r}\theta_j^{(i)}U_i\right)\prod_{j=\lambda'+1}^{C'}\Gamma\left(1-c_j+\sum_{i=1}^{r}\psi_j^{(i)}U_i\right)}\qquad\dots(1.2)$$

$$\Phi_2=\frac{\prod_{\substack{j=1\\ j\neq m_i}}^{u^{(i)}}\Gamma(d_j^{(i)}-\delta_j^{(i)}U_i)\prod_{j=1}^{v^{(i)}}\Gamma(1-b_j^{(i)}+\phi_j^{(i)}U_i)}{\prod_{j=u^{(i)}+1}^{D^{(i)}}\Gamma(1-d_j^{(i)}+\delta_j^{(i)}U_i)\prod_{j=v^{(i)}+1}^{B^{(i)}}\Gamma(b_j^{(i)}-\phi_j^{(i)}U_i)}\qquad\dots(1.3)$$

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$$U_i = \frac{d_{m_i}^{(i)} + n_i}{\delta_{m_i}^{(i)}}, \quad i = 1, \ldots, r \qquad \ldots(1.4)$$

which is valid under the following condition

$$\delta_{m_i}^{(i)}[d_j^{(i)} + p_i] \neq \delta_j^{(i)}[d_{m_i}^{(i)} + n_i] \qquad \ldots(1.5)$$

for $j \neq m_i, m_i = 1, \ldots, u^{(i)}; p_i, n_i = 0,1,2,\ldots; z_i \neq 0$

$$\nabla_i = \sum_{j=1}^{A'} \theta_j^{(i)} - \sum_{j=1}^{C'} \psi_j^{(i)} + \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} - \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} < 0, \ \forall\, i = 1, \ldots, r \qquad \ldots(1.6)$$

Srivastava and Panda [5] have introduced the multivariable H-function

$$H[y_1, \ldots, y_R] = H^{0,\lambda:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}_{A,C:[M',N'];\ldots;[M^R,N^R]}$$

$$\left[\begin{matrix} [(g):\gamma',\ldots,\gamma^{(R)}]:[q':n'];\ldots;[q^{(R)}:n^{(R)}]; \\ [(f):\xi',\ldots,\xi^{(R)}]:[p':\varepsilon'];\ldots;[p^{(R)}:\varepsilon^{(R)}]; \end{matrix} \; y_1, \ldots, y_R \right] \qquad \ldots(1.7)$$

$$T_i = \sum_{j=1}^{A} \gamma_j^{(i)} - \sum_{j=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \varepsilon_j^{(i)} \leq 0, \qquad \ldots(1.8)$$

$$\Omega_i = -\sum_{j=\lambda+1}^{A} \gamma_j^{(i)} - \sum_{j=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{\beta^{(i)}} \eta_j^{(i)} - \sum_{j=\beta^{(i)}+1}^{M^{(i)}} \eta_j^{(i)} + \sum_{j=1}^{\alpha^{(i)}} \varepsilon_j^{(i)} - \sum_{j=\alpha^{(i)}+1}^{N^{(i)}} \varepsilon_j^{(i)} > 0, \qquad \ldots(1.9)$$

$$(1.10)\quad |\arg(y_i)| < \frac{T_i \pi}{2}, \ \forall\, i = 1, \ldots, R.$$

## II. The Main Integral Transformation

We obtained the following integral transformation for H-function of several complex variables defined by Srivastava and Panda [4] (see also [3])

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma} H[y_1 x^{h_1}(1-x)^{k_1}; \ldots; y_R\, x^{h_R}(1-x)^{k_R}]$$

$$. \, H\left[z_1 x^{h_1'}(1-x)^{k_1'}; \ldots; z_r x^{h_r'}(1-x)^{k_r'}\right] F^{v:P';\ldots;P^S;0;0}_{\sigma:Q';\ldots;Q^{(S)};1;1}$$

$$\left[\begin{matrix}[\alpha_v]:[a';\ldots;a^{(s)}\gamma,\gamma]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]:[\ldots];[\ldots];z_1',\ldots,z_s',-xt,(1-x)t\\ [\beta_\sigma]:[b';\ldots;b^{(s)},\mu,\mu]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1;1];\end{matrix}\right]dx$$

$$=\sum_{m_i=1}^{u^{(i)}}\sum_{k,n,n_i=0}^{\infty}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}t^n(-n)_k(\alpha+\beta+n+1)_k}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}\left(\delta_{m_i}^{(i)}n_i!\right)(\alpha+1)_n(\beta+1)_n k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$.F^{v:P';\ldots;P^{(s)}}_{\sigma:Q';\ldots;Q^{(s)}}\left[\begin{matrix}[\alpha_v+n\gamma_v]:[a';\ldots;a^{(s)}]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}];\\ [\beta_\sigma+n\mu_\sigma):[b';\ldots;b^{(s)}]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',\ldots,z_s'\right]$$

$$.H^{0,\lambda+2\quad:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}_{A+2,C+1:[M',N'];\ldots;[M^{(R)},N^{(R)}]}\left[\begin{matrix}\left[1-\rho-\sum_{i=1}^{r}h_i'U_i-k;h_1;\ldots;h_R\right];\left[-\sigma-\sum_{i=1}^{r}k_i'U_i;k_1,\ldots,k_R\right];\\ [(f):\xi',\ldots,\xi^{(R)}];\end{matrix}\right.$$

$$\left.\begin{matrix}[(g):\gamma',\ldots,\gamma^{(R)}];[q':\eta'];\ldots;[q^{(R)}:\eta^{(R)}];\\ \left[-\rho-\sigma-k-\sum_{i=1}^{r}(k_i'+h_i')U_i;(h_1+k_1),\ldots,(h_R+k_R)\right];[p':\varepsilon'];\ldots;[p^{(R)}:\varepsilon^{(R)}];\end{matrix}\; y_1,\ldots,y_R\right],\qquad\ldots(2.1)$$

Where

$$\mathrm{Re}\left[\rho+\sum_{i=1}^{R}h_i\frac{p_j^{(i)}}{\varepsilon_j^{(i)}}+\sum_{\ell=1}^{r}h_\ell'\frac{d_j^{\ell}}{\delta_j^{(\ell)}}\right]>0,$$

$$\mathrm{Re}\left[\sigma+\sum_{i=1}^{R}\frac{k_ip_j^i}{\varepsilon_j^{(i)}}+\sum_{\ell=1}^{r}k_\ell'\frac{d_j^{\ell}}{\delta_j^{(\ell)}}\right]>-1,$$

$$j=1,\ldots,\alpha^{(i)},k_i>0,h_i>0,h_\ell',k_\ell'>0,T_i>0,|\arg(y_i)|<\frac{T_i\pi}{2},i=1,\ldots,R,\ell=1,\ldots,r,|t|<1,$$

Where the series on the right hand side is convergent and is given by (1.8).

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## III. Proof

To prove that (2.1), we start with the following result [6] :

$$F^{v:P';\ldots;P^{s};0;0}_{\sigma:Q';\ldots;Q^{(s)};1;1}\left[\begin{matrix}[\alpha_v]:[a';\ldots;a^{(s)},\gamma,\gamma]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]:[\ldots];[\ldots];z_1',\ldots,z_s',-xt,(1-x)t\\ [\beta_\sigma]:[b';\ldots;b^{(s)},\mu,\mu]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1;1]\end{matrix}\right]$$

$$=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\,t^n\,P_n^{(\alpha,\beta)}(1-2x)}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\,(\alpha+1)_n\,(\beta+1)_n}$$

$$F^{v:P';\ldots;P^{(s)}}_{\sigma:Q';\ldots;Q^{(s)}}\left[\begin{matrix}[(\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}];\\ [\beta_\sigma+n\mu_\sigma):b';\ldots;b^{(s)}]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];\end{matrix}\;z_1',\ldots,z_s'\right], \qquad \ldots(3.1)$$

where $\gamma_i\ a_j^{(i)}$; j = 1,…,v and i = 1,…,s $\mu_i\ \beta_j^{(i)}$; j = 1,…, σ and i = 1,…,s; $\rho_i\ \tau_k^{(j)}$; i = 1,…,$P^{(i)}$ ; j = 1,…,s and k = 1,…,$Q^{(s)}$ are all real and positive, $(\alpha_v)$ stand for the sequence of parameter $\alpha_1,\ldots,\alpha_v$ ; $(\ell^{(j)})$ stand for the sequence of $P^{(j)}$ parameters $\ell_1^{(j)},\ldots,\ell_{P^{(j)}}^{(j)}$, j = 1,…,s, and also similar interpretation for $(\beta_\sigma)$ and $(m^{(J)})$, j = 1,…,s and F denote the generalized Lauricella function of s-complex variables $z_1',\ldots,z_s'$ given by Srivastava and Daoust [8]. Now the proof of main integral transform (2.1) is obtained after multiplying both side (3.1) by

$$x^{\rho-1}(1-x)^{\sigma}\,H[y_1x^{h_1}(1-x)^{k_1},\ldots,y_Rx^{h_R}(1-x)^{k_R}]$$

$$.\ H[z_1x^{h_1'}(1-x)^{k_1'},\ldots,z_r\,x^{h_r'}(1-x)^{k_r'}]$$

And integrate with respect to x from 0 to 1.

Now we represent the $H[z_1x^{h_1'}(1-x)^{k_1'},\ldots,z_r\,x^{h_r'}(1-x)^{k_r'}]$ in series form [3], given by (1.1) and interchange the order of integrations and summations. The required formula is obtained by evaluating the innermost integral with the help of result [5].

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma}\,H\left[y_1x^{h_1}(1-x)^{k_1},\ldots,y_Rx^{h_R}(1-x)^{k_R}\right]P_n^{\alpha,\beta}(1-2x)\,dx$$

$$=\sum_{k=0}^{\infty}\frac{(-n)_k(\alpha+\beta+n+1)_k}{k!(\alpha+1)_k}H^{0,\lambda+2\ :(\alpha',\beta');\ldots;(\alpha^R,\beta^R)}_{A+2,C+1:[M',N'];\ldots;[M^R,N^R]}$$

$$\left[\begin{matrix}[1-\rho-k;h_1,...,h_R],[-\sigma;k_1,...,k_R],[(g):\gamma',...,\gamma^{(R)}];[(q'):\eta'];...;[q^{(R)}:\eta^{(R)}];\\ [(f);\xi',...,\xi^{(R)}],[1-\rho-\sigma-k;(h_1+k_1),...,(h_R+k_R)],[p':\varepsilon'];...;[(p^{(R)}):\varepsilon^{(R)}];\end{matrix}\; y_1,...,y_R\right], \qquad ...(3.2)$$

Where

$$\mathrm{Re}\left(\rho+\sum_{i=1}^{R}\frac{h_i\,p_j^{(i)}}{\varepsilon_j^{(i)}}\right)>0,\ \mathrm{Re}\left(\sigma+\sum_{i=1}^{R}\frac{k_i\,p_j^{(i)}}{\varepsilon_j^{(i)}}\right)>-1,$$

$j=1,...,\alpha^{(i)},k_i>0,h_i>0,T_i>0,|\arg(y_i)|<\frac{T_i\pi}{2},i=1,...,R$ and $T_i$ is given by $(1.8)$.

## IV. Special Cases

Taking $h_i\to 0(i=1\text{ to }R),h_i'\to 0(i=1\text{ to }r)$ in (2.1), we obtain the following result

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma}H\left[y_1(1-x)^{k_1},...,y_R(1-x)^{k_R}\right]$$

$$.\ H\left[z_1(1-x)^{k_1'},...,z_r(1-x)^{k_r'}\right]$$

$$F^{v:P';...;P^{(s)};0;0}_{\sigma:Q';...;Q^{(s)};1;1}\left[\begin{matrix}[\alpha_v]:[a';...;a^{(s)},\gamma,\gamma]:[(\ell'):\rho'];...;[(\ell^{(s)}):\rho^{(s)}]:[...];[...];z_1',...,z_s',-xt,(1-x)t\\ [\beta_\sigma]:[b';...;b^{(s)},\mu,\mu]:[(m'):\tau'];...;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1;1];\end{matrix}\right]dx$$

$$=\sum_{m_i=1}^{u^{(i)}}\sum_{k,n,n_i=0}^{\infty}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}t^n(-n)_k(\alpha+\beta+n+1)_k\Gamma(\rho+k)}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}\left(\delta_{m_i}^{(i)}n_i!\right)(\alpha+1)_n(\beta+1)_n k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$.\ F^{v:P';...;P^{(s)}}_{\sigma:Q';...;Q^{(s)}}\left[\begin{matrix}[\alpha_v+n\gamma_v):a';...;a^{s}]:[(\ell'):\rho'];...;[(\ell^{(s)}):\rho^{(s)}];\\ [\beta_\sigma+n\mu_\sigma):b';...;b^{(s)}]:[(m'):\tau'];...;[(m^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',...,z_s'\right]$$

$$.\ H^{0,\lambda+1\ :(\alpha',\beta');...;(\alpha^{(R)},\beta^{(R)})}_{A+1,C+1:[M',N'];...;[M^{(R)},N^{(R)}]}\left[\begin{matrix}\left[-\sigma-\sum_{i=1}^{r}k_i'U_i;k_1,...,k_R\right];\\ [(f):\xi',...,\xi^{(R)}];\end{matrix}\right.$$

$$\left.\begin{matrix} [(g):\gamma',\ldots,\gamma^{R}];[q':\eta'];\ldots;[q^{R}:\eta^{R}]; \\ \left[-\rho-\sigma-k-\sum_{i=1}^{R} k_i^{'}\,U_i;k_1,\ldots,k_R\right];[p':\varepsilon'];\ldots;[p^{(R)}:\varepsilon^{(R)}]; \end{matrix}\quad y_1,\ldots,y_R \right], \qquad \ldots(4.1)$$

Where

$$\operatorname{Re}(\rho)>0,\operatorname{Re}\left(\sigma+\sum_{i=1}^{R}\frac{k_i p_j^{(i)}}{\varepsilon_j^{i}}+\sum_{\ell=1}^{r} k_{\ell}^{'}\frac{d_j^{(\ell)}}{\delta_j^{(\ell)}}\right)>-1,$$

$$j=1,\ldots,\alpha^{(i)},k_i>0,k_{\ell}^{'}>0,T_i>0,|\arg(y_i)|<\frac{T_i\pi}{2},i=1,\ldots,R,\ell=1,\ldots,r,|t|<1.$$

(2) Taking $k_i\to 0(i=1\text{ to }R),k_i^{'}\to 0(i=1\text{ to }r)$ in (2.1), we obtain the following result

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma}\,H\left[y_1x^{h_1},\ldots,y_Rx^{h_R}\right]\,H\left[z_1x^{h_1^{'}},\ldots,z_rx^{h_r^{'}}\right]$$

$$.\,F^{\,v:P';\ldots;P^{(s)};0;0}_{\,\sigma:Q';\ldots;Q^{S};1;1}\left[\begin{matrix}[\alpha_v]:[a';\ldots;a^{(s)}\gamma,\gamma]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]:[\ldots];[\ldots];z_1^{'},\ldots,z_s^{'},-xt,(1-x)t\\ [\beta_\sigma]:[b';\ldots;b^{(s)},\mu,\mu]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1;1];\end{matrix}\right]dx$$

$$=\sum_{m_i=1}^{u^{(i)}}\sum_{k,n,n_i=0}^{\infty}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}\,t^{n}(-n)_k(\alpha+\beta+n+1)_k\,\Gamma(\sigma+1)}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}\left(\delta_{m_i}^{(i)}\,n_i!\right)(\alpha+1)_n(\beta+1)_n\,k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$.\,F^{\,v:P';\ldots;P^{(s)}}_{\,\sigma:Q';\ldots;Q^{(s)}}\left[\begin{matrix}[\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}];\\ [\beta_\sigma+n\mu_\sigma):b';\ldots;b^{(s)}]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];\end{matrix}\quad z_1^{'},\ldots,z_s^{'}\right]$$

$$.\,H^{\,0,\lambda+1\quad:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}_{\,A+1,C+1:[M',N'];\ldots;[M^{(R)},N^{(R)}]}\left[\begin{matrix}\left[1-\rho-k-\sum_{i=1}^{r}h_i^{'}U_i;h_1,\ldots,h_R\right];\\ [(f):\xi',\ldots,\xi^{(R)}];\end{matrix}\right.$$

$$\left.\begin{matrix}[(g):\gamma',\ldots,\gamma^{R}];[q':\eta'];\ldots;[q^{(R)}:\eta^{(R)}];\\ \left[-\rho-\sigma-k-\sum_{i=1}^{R}h_i^{'}\,U_i;h_1,\ldots,h_R\right];[p':\varepsilon'];\ldots;[p^{(R)},\varepsilon^{(R)}];\end{matrix}\quad y_1,\ldots,y_R\right], \qquad \ldots(4.2)$$

Where
$$\mathrm{Re}\left(\rho+\sum_{i=1}^{R}\frac{h_i p_j^{(i)}}{\varepsilon_j^i}+\sum_{\ell=1}^{r} h'_\ell \frac{d_j^{(\ell)}}{\delta_j^{(\ell)}}\right)>0,$$

$$\mathrm{Re}(\sigma)>-1,\ j=1,\ldots,\alpha^{(i)}, h_i>0, h'_\ell>0, T_i>0, |\arg(y_i)|<\frac{T_i\pi}{2}, i=1,\ldots,R, \ell=1,\ldots,r, |t|<1.$$

(3) Reducing the H-function of several complex variables to the generalized Lauricella function [8] by putting $\lambda=A, \alpha^i=1, \beta^{(i)}=M^i, N^{(i)}=N^{(i)}+1,$ $\forall$ i = 1,…,R in (2.1), we get the following result:

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma} H\left[z_1 x^{h'_1}(1-x)^{k'_1};\ldots;z_r x^{h'_r}(1-x)^{k'_r}\right]$$

$$.F^{v:P';\ldots;P^{(s)};0;0}_{\sigma:Q';\ldots;Q^{(s)};1;1}\left[\begin{matrix}[\alpha_v]:[a';\ldots;a^{(s)}\gamma,\gamma]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]:[\ldots];[\ldots];z'_1,\ldots,z'_s-xt,(1-x)t\\ [\beta_\sigma]:[b';\ldots;b^{(s)},\mu,\mu]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1;1];\end{matrix}\right]$$

$$.F^{A:M';\ldots;M^{(R)}}_{C:N';\ldots;N^{(R)}}\left[\begin{matrix}[1-(g):\gamma',\ldots,\gamma^{(R)}]:[1-(q'):\eta'];\ldots;[1-(q^{(R)}):\eta^{(R)}];\\ [1-(f):\xi',\ldots,\xi^{(R)}]:[1-(p'):\varepsilon'];\ldots;[1-(p^{(R)}):\varepsilon^{(R)}];\end{matrix}\right.$$

$$\left.-y_1 x^{h_1}(1-x)^{k_1},\ldots,-y_R x^{h_R}(1-x)^{k_R}\right]dx$$

$$=\sum_{k,n,n_i=0}^{\infty}\sum_{m_i=1}^{u^{(i)}}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}t^n(-n)_k(\alpha+\beta+n+1)_k}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}\left(\delta_{m_i}^{(i)}n_i!\right)(\alpha+1)_n(\beta+1)_n k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$.F^{v:P';\ldots;P^s}_{\sigma:Q';\ldots;Q^s}\left[\begin{matrix}[\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(\ell'):P'];\ldots;[(\ell^{(s)}):P^{(s)}];\\ [\beta_\sigma+n\mu_\sigma):b';\ldots;b^s]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];\end{matrix}\ z'_1,\ldots,z'_s\right]$$

$$.\frac{\Gamma(1+\sigma+\sum_{i=1}^{r}k'_iU_i)\Gamma(\rho+\sum_{i=1}^{r}h'_iU_i+k)}{\Gamma(1+\rho+\sigma+k+\sum_{i=1}^{r}(h'_i+k'_i)U_i)}$$

$$. F^{A+2:M';\ldots;M^{(R)}}_{C+1:N';\ldots;N^{(R)}}\left[\begin{matrix}\left[1+\sigma+\sum_{i=1}^{r} k_i'U_i;k_1,\ldots,k_R\right];\left[\rho+\sum_{i=1}^{r} h_i'U_i+K;h_1,\ldots,h_R\right], \\ [1-(f):\xi';\ldots;\xi^R];\end{matrix}\right.$$

$$\left.\begin{matrix}[1-(g):\gamma',\ldots,\gamma^{(R)}];[1-(q'):\eta'];\ldots;[1\text{-}(q^{(R)}):\eta^{(R)}]; \\ \left[1+\rho+\sigma+k+\sum_{i=1}^{r}(h_i'+k_i')U_i;(h_1+k_1);\ldots;(h_R+k_R)\right];[1-(p'):\varepsilon'];\ldots;[1-(p^{(R)}):\varepsilon^{(R)}];\end{matrix} -y_1,\ldots,-y_R\right] \quad \ldots(4.3)$$

Which is valid under the same condition as given in (2.1).

(4) Reducing the Lauricella function to the Kampé de Fériet function [7] by putting i = 1,2 in (4.3) and we get the following result :

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma} H\left[z_1 x^{h_1'}(1-x)^{k_1'};\ldots;z_r x^{h_r'}(1-x)^{k_r'}\right]$$

$$. F^{v:P';\ldots;P^S;0;0}_{\sigma:Q';\ldots;Q^{(s)};1;1}\left[\begin{matrix}[\alpha_v][:a';\ldots;a^{(s)},\gamma,\gamma]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]:[\ldots];[\ldots];z_1',\ldots,z_s',-xt,(1-x)t \\ [\beta_\sigma]:[b';\ldots;b^{(s)},\mu,\mu]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1:1]\end{matrix}\right]$$

$$. S^{A:M':M''}_{C:N':N''}\left[\begin{matrix}[(g):\gamma',\gamma'']:[(q'):\eta'];[(q''):\eta''];\\ [(f):\xi',\xi'']:[(p'):\varepsilon'];[(p''):\varepsilon''];\end{matrix}\; y_1 x^{h_1}(1-x)^{k_1}, y_2 x^{h_2}(1-x)^{k_2}\right]dx$$

$$= \sum_{k,n,n_i=0}^{\infty}\sum_{m_i=1}^{u^{(i)}} \frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)} t^n(-n)_k(\alpha+\beta+n+1)_k}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_\sigma}\prod_{i=1}^{r}\left(\delta^{(i)}_{m_i}\, n_i!\right)(\alpha+1)_n(\beta+1)_n\, k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$. F^{v:P';\ldots;P^{(s)}}_{\sigma:Q';\ldots;Q^{(s)}}\left[\begin{matrix}[\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]; \\ [\beta_\sigma+n\mu_\sigma):b';\ldots;b^{(s)}]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',\ldots,z_s'\right]$$

$$. S^{A+2:M';M''}_{C+1:N';N''}\left[\begin{matrix}\left[1-\rho-\sum_{i=1}^{r} h_i'U_i-k;h_1,h_2\right];\left[-\sigma-\sum_{i=1}^{r} k_i'U_i;k_1,.k_2\right], \\ [(f):\xi';\xi''];\end{matrix}\right.$$

$$\left.\begin{matrix} [(g):\gamma',\gamma''];[(q'):\eta'];\ [(q''):\eta''];\\ [-\rho-\sigma-k-\sum_{i=1}^{r}(h_i'+k_i')U_i;(h_1+k_1),(h_2+k_2)];[p'):\varepsilon'];[(p''):\varepsilon''];\end{matrix}\ y_1,y_2\right] \qquad \ldots(4.4)$$

Which is valid under the same condition as surrounding (2.1).

(5) Reducing the H-function of several complex variables to the product of R mutually independent H- functions by taking $\lambda = A = C = 0$ in (2.1), we get the following result :

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma} H\left[z_1 x^{h_1'}(1-x)^{k_1'};\ldots;z_r x^{k_r'}(1-x)^{h_r'}\right]$$

$$.\ F^{v:P';\ldots;P^{s};0;0}_{\sigma:Q';\ldots;Q^{(s)};1;1}\left[\begin{matrix}[\alpha_v]:[a';\ldots;a^{(s)},\gamma,\gamma]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]:[\ldots];[\ldots];z_1',\ldots,z_s',-xt,(1-x)t\\ [\beta_\sigma]:[b';\ldots;b^{(s)},\mu,\mu]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1:1]\end{matrix}\right]$$

$$.\ \prod_{i=1}^{R}\left\{H^{\alpha^{(i)},\beta^{(i)}}_{M^{(i)},N^{(i)}}\left[y_i\ x^{h_i}(1-x)^{k_i}\left|\begin{matrix}[(q^i):\eta^{(i)}]\\ [(p^i):\varepsilon^{(i)}]\end{matrix}\right.\right]\right\}dx$$

$$=\sum_{k,n,n_i=0}^{\infty}\sum_{m_i=1}^{u^{(i)}}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}t^n(-n)_k(\alpha+\beta+n+1)_k}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_\sigma}\prod_{i=1}^{r}\left(\delta^{(i)}_{m_i}\ n_i!\right)(\alpha+1)_n(\beta+1)_n\ k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$.\ F^{v:P';\ldots;P^{(s)}}_{\sigma:Q';\ldots;Q^{(s)}}\left[\begin{matrix}[\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}];\\ [\beta_\sigma+n\mu_\sigma):b';\ldots;b^{(s)}]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];\end{matrix}\ z_1',\ldots,z_s'\right]$$

$$.\ H^{0,2:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}_{2,1:[M',N'];\ldots;[M^{R},N^{(R)}]}\left[\begin{matrix}\left[1-\rho-\sum_{i=1}^{r}h_i'U_i-k;h_1;\ldots;h_R\right],\\ \left[[(f):\xi',\ldots,\xi^{(R)}];[-\rho-\sigma-K-\sum_{i=1}^{r}(k_i'+h_i')U_i;(h_1+k_1),\ldots,(h_R+k_R)\right]\end{matrix}\right.$$

$$\left.\begin{matrix}\left[-\sigma-\sum_{i=1}^{r}k_i'U_i;k_1,\ldots,k_R\right];[(q'):\eta'];\ldots;[q^{(R)}:\eta^{(R)}];\\ [(p'):\varepsilon'];\ldots;[p^{(R)}:\varepsilon^{(R)}];\end{matrix}\ y_1,\ldots,y_R\right] \qquad \ldots(4.5)$$

which holds under the same condition as for (2.1).

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