Global Journal of Science Frontier Research
Volume 11 Issue 8 Version 1.0 November 2011
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN : 2249-4626 & Print ISSN:0975-5896

Construction of A Summation Formula Allied with Hyper - geometric Function and Involving Recurrence Relation

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GJSFR-A Classification : *FOR Code: 010206, 080604*

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I. Introduction

Generalized Gaussian Hypergeometric function of one variable :

$$ {}_AF_B\left[\begin{array}{ll} a_1, a_2, \cdots, a_A & ; \\ & z \\ b_1, b_2, \cdots, b_B & ; \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \quad (1) $$

or

$$ {}_AF_B\left[\begin{array}{ll} (a_A) & ; \\ & z \\ (b_B) & ; \end{array}\right] \equiv {}_AF_B\left[\begin{array}{ll} (a_j)_{j=1}^A & ; \\ & z \\ (b_j)_{j=1}^B & ; \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \quad (2) $$

where the parameters $b1, b2, \cdots, b_B$ are neither zero nor negative integers and A, B are non-negative integers.

Contiguous Relation :

[Abramowitz p.558(15.2.19)]

$$ (a-b)\,(1-z)\,{}_2F_1\left[\begin{array}{ll} a,\ b\ ; & \\ c\ \ ; & z \end{array}\right] = (c-b)\,{}_2F_1\left[\begin{array}{ll} a,\ b-1\ ; & \\ c\ \ ; & z \end{array}\right] + (a-c)\,{}_2F_1\left[\begin{array}{ll} a-1,\ b\ ; & \\ c\ ; & z \end{array}\right] \quad (3) $$

Recurrence relation :

$$ \Gamma(z+1) = z\,\Gamma(z) \quad (4) $$

Legendre's duplication formula :

$$ \sqrt{\pi}\,\Gamma(2z) = 2^{(2z-1)}\,\Gamma(z)\,\Gamma\left(z+\frac{1}{2}\right) \quad (5) $$

$$ \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)}\,\Gamma(\frac{b}{2})\,\Gamma(\frac{b+1}{2})}{\Gamma(b)} \quad (6) $$

$$ = \frac{2^{(a-1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\Gamma(a)} \quad (7) $$

In the monograph of Prudnikov et al., a summation formula is given in the form [Prudnikov, 491, equation(7.3.7.3)]

$$ {}_2F_1\left[\begin{array}{ll} a,\ \ b\ \ ; & \\ \frac{a+b-1}{2}\ ; & \frac{1}{2} \end{array}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\Gamma(\frac{b+1}{2})} + \frac{2\,\Gamma(\frac{a+b-1}{2})}{\Gamma(a)\Gamma(b)}\right] \quad (8) $$

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Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above formula can be written in the form

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & & ; & \frac{1}{2} \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})}+\frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2}+\frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \quad (9) $$

It is noted that the above formula [Prudnikov, 491, equation (7.3.7.3)], i.e.equation(8) or (9) is not correct.The correct form of equation(8) or (9) is obtained by [Asish et. al(2008), p.337(10)]

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & & ; & \frac{1}{2} \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})}\left\{\frac{(b+a-1)}{(a-1)}\right\}+\frac{2\ \Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\right] \quad (10) $$

Involving the formula obtained by [Asish et. al (2008), p.337 (10)], we establish the main formula.

II. Main Summation Formula

For the main formula $a \neq b$

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-33}{2} & & ; & \frac{1}{2} \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-33}{2})}{(a-b)\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-33}{2})}\left\{\frac{(-63326598707628506 25a)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+\right.\right. $$

$$ +\frac{(151884650291143250 25a^2-143545106916107132 40a^3+752431412791255 1832a^4)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+ $$

$$ +\frac{(-252369860620076319 6a^5+585146416702456764a^6-98283050207112680a^7)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+ $$

$$ +\frac{(12319487399406824a^8-1174199725349222a^9+86014818744998a^{10}-4862169489320a^{11})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+ $$

$$ +\frac{(211577650856a^{12}-7020044668a^{13}+174281212a^{14}-3132760a^{15}+38488a^{16}-289a^{17})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+ $$

$$ +\frac{(a^{18}+6332659870762850625b-21685865075950122360a^2b+27174273987848799888a^3b)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+ $$

$$ +\frac{(-14941382816881136916a^4b+5514641320597784784a^5b-1223922579902059240a^6b)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+ $$

$$ +\frac{(223951372274069328a^7b-25702619937058218a^8b+2705684289022352a^9b)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+ $$

$$ +\frac{(-173934158896520a^{10}b+11136166030000a^{11}b-402445286516a^{12}b+15725518704a^{13}b)}{\prod_{\clubsuit=1}^{17}a-\{(2\clubsuit-1)\}}+ $$

$$+\frac{(-296201880a^{14}b+6703984a^{15}b-50575a^{16}b+560a^{17}b-15188465029114325025b^{2})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(21685865075950122360ab^{2}-13682691432034310808a^{3}b^{2}+11066683826498381100a^{4}b^{2})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(-3872556582071104200a^{5}b^{2}+1077310489531409840a^{6}b^{2}-161824805437466776a^{7}b^{2})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(24012617457411330a^{8}b^{2}-1869004163521880a^{9}b^{2}+166427533185760a^{10}b^{2})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(-6955392355464a^{11}b^{2}+386494342780a^{12}b^{2}-8156169560a^{13}b^{2}+280185840a^{14}b^{2})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(-2314312a^{15}b^{2}+45815a^{16}b^{2}+14354510691610713240b^{3}-27174273987848799888ab^{3})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(13682691432034310808a^{2}b^{3}-2823412677568587720a^{4}b^{3}+1630202936508633872a^{5}b^{3})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(-374886600752864200a^{6}b^{3}+82069972989712224a^{7}b^{3}-8266545280520120a^{8}b^{3})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(1016286884813200a^{9}b^{3}-51348408517560a^{10}b^{3}+3905028076096a^{11}b^{3}-95843023640a^{12}b^{3})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(4626006000a^{13}b^{3}-43361560a^{14}b^{3}+1298528a^{15}b^{3}-7524314127912551832b^{4})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(14941382816881136916ab^{4}-11066683826498381100a^{2}b^{4}+2823412677568587720a^{3}b^{4})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(-239701353260432756a^{5}b^{4}+105012931049313980a^{6}b^{4}-15843532755817360a^{7}b^{4})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(2814256196843080a^{8}b^{4}-182217872348820a^{9}b^{4}+18878387187660a^{10}b^{4})}{\prod_{\clubsuit=1}^{17}\left\{a-(2\clubsuit-1)\right\}}+$$

$$+\frac{(-558624827320a^{11}b^4+36604618960a^{12}b^4-400108940a^{13}b^4+16811300a^{14}b^4)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(2523698606200763196b^5-5514641320597784784ab^5+3872556582071104200a^2b^5)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-1630202936508633872a^3b^5+239701353260432756a^4b^5-9306573013861200a^6b^5)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(3224351601798624a^7b^5-307619347906332a^8b^5+45369021123888a^9b^5)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-1706864738040a^{10}b^5+151079424048a^{11}b^5-1985995284a^{12}b^5+112971936a^{13}b^5)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-5851464167024567764b^6+1223922579902059240ab^6-1077310489531409840a^2b^6)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(374886600752864200a^3b^6-105012931049313980a^4b^6+9306573013861200a^5b^6)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-170407715551920a^7b^6+48117783663180a^8b^6-2634364332600a^9b^6+328465903920a^{10}b^6)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-5456031000a^{11}b^6+417225900a^{12}b^6+98283050207112680b^7-223951372274069328ab^7)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(161824805437466776a^2b^7-82069972989712224a^3b^7+15843532755817360a^4b^7)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-3224351601798624a^5b^7+170407715551920a^6b^7-1398935518200a^8b^7+329434234800a^9b^7)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-7887861960a^{10}b^7+843621600a^{11}b^7-12319487399406824b^8+25702619937058218ab^8)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-24012617457411330a^2b^8+8266545280520120a^3b^8-2814256196843080a^4b^8)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(307619347906332a^5b^8-48117783663180a^6b^8+1398935518200a^7b^8-4059928950a^9b^8)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(811985790a^{10}b^8+1174199725349222b^9-2705684289022352ab^9+1869004163521880a^2b^9)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-1016286884813200a^3b^9+182217872348820a^4b^9-45369021123888a^5b^9)}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(2634364332600a^6b^9-329434234800a^7b^9+4059928950a^8b^9-86014818744998b^{10})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(173934158896520ab^{10}-166427533185760a^2b^{10}+51348408517560a^3b^{10})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-18878387187660a^4b^{10}+1706864738040a^5b^{10}-328465903920a^6b^{10}+7887861960a^7b^{10})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-811985790a^8b^{10}+4862169489320b^{11}-11136166030000ab^{11}+6955392355464a^2b^{11})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-3905028076096a^3b^{11}+558624827320a^4b^{11}-151079424048a^5b^{11}+5456031000a^6b^{11})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-843621600a^7b^{11}-211577650856b^{12}+402445286516ab^{12}-386494342780a^2b^{12})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(95843023640a^3b^{12}-36604618960a^4b^{12}+1985995284a^5b^{12}-417225900a^6b^{12})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(7020044668b^{13}-15725518704ab^{13}+8156169560a^2b^{13}-4626006000a^3b^{13}+400108940a^4b^{13})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-112971936a^5b^{13}-174281212b^{14}+296201880ab^{14}-280185840a^2b^{14}+43361560a^3b^{14})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(-16811300a^4b^{14}+3132760b^{15}-6703984ab^{15}+2314312a^2b^{15}-1298528a^3b^{15}-38488b^{16})}{\prod_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}+$$

$$+\frac{(50575ab^{16}-45815a^2b^{16}+289b^{17}-560ab^{17}-b^{18})}{\prod\limits_{\clubsuit=1}^{17}\{a-(2\clubsuit-1)\}}\Bigg\}+\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-32}{2})}\Bigg\{\frac{(10133413135603654050a)}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-16516534255341284160a^2+12913410201153578352a^3-5134255758481893696a^4)}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(1587404582027587896a^5-283859488061889600a^6+46858165261036304a^7)}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-4462194675588672a^8+439713443065292a^9-23589705501120a^{10}+1446665758736a^{11})}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-43308178368a^{12}+1644188728a^{13}-25149120a^{14}+554608a^{15}-3264a^{16}+34a^{17})}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-10133413135603654050b+16931174426365770288a^2b-14906340904637522304a^3b)}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(7884962414212007016a^4b-2042954269149525760a^5b+485108793688889104a^6b)}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-59536895575144064a^7b+8016646272897108a^8b-523874269585920a^9b)}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(43542297741008a^{10}b-1537398584960a^{11}b+80953284776a^{12}b-1435006720a^{13}b)}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(46940400a^{14}b-319872a^{15}b+5950a^{16}b+16516534255341284160b^2)}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-16931174426365770288ab^2+6725033931916644528a^3b^2-3600430749436861760a^4b^2)}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(1397408542187076048a^5b^2-241437978982668160a^6b^2+45693039943591216a^7b^2)}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-3794731512686400a^8b^2+425328212299760a^9b^2-18176612312320a^{10}b^2)}{\prod\limits_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

Global Journal of Science Frontier Research Volume XI Issue VIII Version I November 2011

$$+\frac{(1288646146704a^{11}b^2 - 26912849600a^{12}b^2 + 1222187120a^{13}b^2 - 9694080a^{14}b^2 + 272272a^{15}b^2)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-1291341020115357835 2b^3 + 14906340904637522304ab^3 - 6725033931916644528a^2b^3)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(978138257795363280a^4b^3 - 334206391921947776a^5b^3 + 10022468751912232 0a^6b^3)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-11475840301911040a^7b^3 + 1778313651358640a^8b^3 - 95547195004800a^9b^3)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(9056404988400a^{10}b^3 - 227804308480a^{11}b^3 + 13899856880a^{12}b^3 - 129852800a^{13}b^3)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(5101360a^{14}b^3 + 5134255758481893696b^4 - 7884962414212007016ab^4)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(3600430749436861760a^2b^4 - 978138257795363280a^3b^4 + 62527488580332776a^5b^4)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-13739063876827520a^6b^4 + 3300339203735200a^7b^4 - 240722395488960a^8b^4)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(31178568063720a^9b^4 - 977960392640a^{10}b^4 + 79379902000a^{11}b^4 - 890081920a^{12}b^4)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(47071640a^{13}b^4 - 1587404582027587896b^5 + 2042954269149525760ab^5)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-1397408542187076048a^2b^5 + 334206391921947776a^3b^5 - 62527488580332776a^4b^5)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(1910209218847776a^6b^5 - 261663896938752a^7b^5 + 51709067642232a^8b^5 - 2176238211840a^9b^5)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(239444018352a^{10}b^5 - 3327537024a^{11}b^5 + 233646504a^{12}b^5 + 283859488061889600b^6)}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-485108793688889104ab^{6}+2414379789826681 60a^{2}b^{6}-100224687519122320a^{3}b^{6})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(13739063876827520a^{4}b^{6}-1910209218847776a^{5}b^{6}+28409218881120a^{7}b^{6})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-2209930430400a^{8}b^{6}+366743454000a^{9}b^{6}-6778316160a^{10}b^{6}+641886000a^{11}b^{6})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-468581652610363304b^{7}+595368955751440 64ab^{7}-456930399435912 16a^{2}b^{7})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(11475840301911040a^{3}b^{7}-3300339203735200a^{4}b^{7}+261663896938752a^{5}b^{7})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-28409218881120a^{6}b^{7}+194057780400a^{8}b^{7}-6550473600a^{9}b^{7}+927983760a^{10}b^{7})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(4462194675588672b^{8}-8016646272897108ab^{8}+3794731512686400a^{2}b^{8})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-1778313651358640a^{3}b^{8}+240722395488960a^{4}b^{8}-51709067642232a^{5}b^{8})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(2209930430400a^{6}b^{8}-194057780400a^{7}b^{8}+477638700a^{9}b^{8}-4397134430652 92b^{9})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(523874269585920ab^{9}-425328212299760a^{2}b^{9}+95547195004800a^{3}b^{9}-3117856806 3720a^{4}b^{9})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(2176238211840a^{5}b^{9}-366743454000a^{6}b^{9}+6550473600a^{7}b^{9}-477638700a^{8}b^{9})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(235897055011 20b^{10}-43542297741008ab^{10}+18176612312320a^{2}b^{10}-9056404988400a^{3}b^{10})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(977960392640a^{4}b^{10}-239444018352a^{5}b^{10}+6778316160a^{6}b^{10}-927983760a^{7}b^{10})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-1446665758736b^{11}+1537398584960ab^{11}-1288646146704a^{2}b^{11}+227804308480a^{3}b^{11})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-79379902000a^{4}b^{11}+3327537024a^{5}b^{11}-641886000a^{6}b^{11}+433081783680b^{12})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-80953284776ab^{12}+26912849600a^{2}b^{12}-13899856880a^{3}b^{12}+890081920a^{4}b^{12})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-233646504a^{5}b^{12}-16441887280b^{13}+1435006720ab^{13}-1222187120a^{2}b^{13}+129852800a^{3}b^{13})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(-47071640a^{4}b^{13}+251491200b^{14}-46940400ab^{14}+9694080a^{2}b^{14}-5101360a^{3}b^{14}-554608b^{15})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}+$$

$$+\frac{(319872ab^{15}-272272a^{2}b^{15}+3264b^{16}-5950ab^{16}-34b^{17})}{\prod_{\spadesuit=1}^{16}\{a-2\spadesuit\}}\Bigg\}\Bigg] \qquad (11)$$

III. Derivation of Main Summation Formula :

Substituting $c=\frac{a+b-33}{2}$ and $z=\frac{1}{2}$ in equation (3), we get

$$(a-b)\,{}_2F_1\left[\begin{matrix}a,\ b & ; & \frac{1}{2}\\ \frac{a+b-33}{2} & ; & \end{matrix}\right]=(a-b-33)\,{}_2F_1\left[\begin{matrix}a,\ b-1 & ; & \frac{1}{2}\\ \frac{a+b-33}{2} & ; & \end{matrix}\right]+(a-b+33)\,{}_2F_1\left[\begin{matrix}a-1,\ b & ; & \frac{1}{2}\\ \frac{a+b-33}{2} & ; & \end{matrix}\right]$$

Now proceeding the same way of Ref [5] the main result is proved.

References Références Referencias

1. Abramowitz, Milton., A and Stegun, Irene; Handbook of *Mathematical Functions with Formulas , Graphs , and Mathematical Tables*.National Bureau of Standards, 1970.
2. Arora, Asish, Singh, Rahul , Salahuddin. ; Development of a family of summation formulae of half argument using Gauss and Bailey theorems , *Journal of Rajasthan Academy of Physical Sciences.*, 7(2008),335-342.
3. Prudnikov, A. P., Brychkov, Yu. A. and Marichev, O.I.; *Integrals and Series Vol. 3: More Special Functions*. Nauka, Moscow, 1986. Translated from the Russian by G.G. Gould, Gordon and Breach Science Publishers, New York, Philadelphia, London, Paris, Montreux, Tokyo,Melbourne, 1990.
4. Salahuddin. ; On Certain Summation Formulae based on half argument associated to hypergeometric function , *International Journal of Mathematical Archieve.*, 2(2011), 255-257.
5. Salahuddin. ; Three Summation Formulae using Contiguous Relation and Related to Hypergeometric function , *International Journal of Mathematical Archieve.*, 2(2011), 518-520.