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Development of a Summation Formula Related To Hypergeometric Functions

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DEVELOPMENT OF A SUMMATION FORMULA RELATED TO HYPERGEOMETRIC FUNCTIONS

Strictly as per the compliance and regulations of:

Development of a Summation Formula Related To Hypergeometric Functions

Salahuddin[α] , M.P.Chaudhary[Ω]

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Keywords : Contiguous relation, Gauss second summation theorem.

I. Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ccc} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} $$

or

$$ {}_AF_B\left[\begin{array}{ccc} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right] \equiv {}_AF_B\left[\begin{array}{ccc} (a_j)_{j=1}^{A} & ; & \\ & & z \\ (b_j)_{j=1}^{B} & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \quad (1) $$

Where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and A, B are non-negative integers.

Contiguous Relation is defined by $[\text{Andrews p.363(9.16), E.D.P.51(10), H.T.F.I p.103(32)}]$

$$ (a-b)\ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ c & ; & z \end{array}\right] = a\ {}_2F_1\left[\begin{array}{ccc} a+1,\ b & ; & \\ c & ; & z \end{array}\right] - b\ {}_2F_1\left[\begin{array}{ccc} a,\ b+1 & ; & \\ c\ ; & & z \end{array}\right] \quad (2) $$

Gauss second summation theorem is defined by $[\text{Prud., } 491(7.3.7.5)]$

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ \frac{a+b+1}{2} & ; & \frac{1}{2} \end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \quad (3) $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \quad (4) $$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prud., p.491(7.3.7.3)]

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ \frac{a+b-1}{2} & ; & \frac{1}{2} \end{array}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \quad (5) $$

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form,

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ \frac{a+b-1}{2} & ; & \frac{1}{2} \end{array}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \quad (6) $$

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II. Main Summation Formulae

$$
{}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b+26}{2} & & ; & \end{matrix}\right] = \frac{2^b\,\Gamma(\frac{a+b+26}{2})}{(a-b)\,\Gamma(b)} \times
$$

$$
\times\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})}\left\{\frac{4096(-81749606400a+123436892160a^2-77270003712a^3+26946067456a^4)}{\left(\prod\limits_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod\limits_{\eta=1}^{12}\{a-b+2\eta\}\right)}+\right.\right.
$$

$$
+\frac{4096(-5887453440a^5+853730240a^6-84401856a^7+5718768a^8-261360a^9+7700a^{10})}{\left(\prod\limits_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod\limits_{\eta=1}^{12}\{a-b+2\eta\}\right)}+
$$

$$
+\frac{4096(-132a^{11}+a^{12}+81749606400b+367298150400ab-118779801600a^2b)}{\left(\prod\limits_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod\limits_{\eta=1}^{12}\{a-b+2\eta\}\right)}+
$$

$$
+\frac{4096(242509385728a^3b-38595539712a^4b+16452876672a^5b-1303280832a^6b)}{\left(\prod\limits_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod\limits_{\eta=1}^{12}\{a-b+2\eta\}\right)}+
$$

$$
+\frac{4096(217911936a^7b-8342928a^8b+621368a^9b-9108a^{10}b+276a^{11}b+123436892160b^2)}{\left(\prod\limits_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod\limits_{\eta=1}^{12}\{a-b+2\eta\}\right)}+
$$

$$
+\frac{4096(118779801600ab^2+464899287040a^2b^2-37227551232a^3b^2+76928096320a^4b^2)}{\left(\prod\limits_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod\limits_{\eta=1}^{12}\{a-b+2\eta\}\right)}+
$$

$$
+\frac{4096(-4716831168a^5b^2+2038636096a^6b^2-67277760a^7b^2+11193732a^8b^2-148764a^9b^2)}{\left(\prod\limits_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod\limits_{\eta=1}^{12}\{a-b+2\eta\}\right)}+
$$

$$
+\frac{4096(10626a^{10}b^2+77270003712b^3+242509385728ab^3+37227551232a^2b^3)}{\left(\prod\limits_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod\limits_{\eta=1}^{12}\{a-b+2\eta\}\right)}+
$$

$$
+\frac{4096(125264532736a^3b^3-3669038016a^4b^3+7098446208a^5b^3-181484352a^6b^3)}{\left(\prod\limits_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod\limits_{\eta=1}^{12}\{a-b+2\eta\}\right)}+
$$

$$
+\frac{4096(74296992a^7b^3-865260a^8b^3+134596a^9b^3+26946067456b^4+38595539712ab^4)}{\left(\prod\limits_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod\limits_{\eta=1}^{12}\{a-b+2\eta\}\right)}+
$$

$$
+\frac{4096(76928096320a^2b^4+3669038016a^3b^4+10621924768a^4b^4-124807200a^5b^4)}{\left(\prod\limits_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod\limits_{\eta=1}^{12}\{a-b+2\eta\}\right)}+
$$

$$+\frac{4096(219244648a^6b^4-1961256a^7b^4+735471a^8b^4+5887453440b^5+16452876672ab^5)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(4716831168a^2b^5+7098446208a^3b^5+124807200a^4b^5+312018000a^5b^5-1248072a^6b^5)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(1961256a^7b^5+853730240b^6+1303280832ab^6+2038636096a^2b^6+181484352a^3b^6)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(219244648a^4b^6+1248072a^5b^6+2704156a^6b^6+84401856b^7+217911936ab^7)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(67277760a^2b^7+74296992a^3b^7+1961256a^4b^7+1961256a^5b^7+5718768b^8)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(8342928ab^8+11193732a^2b^8+865260a^3b^8+735471a^4b^8+261360b^9+621368ab^9)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(148764a^2b^9+134596a^3b^9+7700b^{10}+9108ab^{10}+10626a^2b^{10}+132b^{11})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(+276ab^{11}+b^{12})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768b(38385745920a+8277098496a^2+11938545664a^3+1293154816a^4+518927616a^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(29747424a^6+4842288a^7+138864a^8+9944a^9+110a^{10}+3a^{11}+38385745920b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(50799591424a^2b+4130196480a^3b+5067819264a^4b+257208448a^5b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(92965264a^6b+2517856a^7b+370392a^8b+4048a^9b+253a^{10}b-8277098496b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(50799591424ab^2+14021624320a^3b^2+502850656a^4b^2+529633328a^5b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(12452384a^6b^2+3991328a^7b^2+40986a^8b^2+5313a^9b^2+119385456664b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(-4130196480ab^3+14021624320a^2b^3+1206518544a^4b^3+19283936a^5b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(17521952a^6b^3+153824a^7b^3+43263a^8b^3-1293154816b^4+5067819264ab^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(-502850656a^2b^4+1206518544a^3b^4+35778064a^5b^4+208012a^6b^4+163438a^7b^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(518927616b^5-257208448ab^5+529633328a^2b^5-19283936a^3b^5+35778064a^4b^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(312018a^6b^5-297474224b^6+92965264ab^6-12452384a^2b^6+17521952a^3b^6)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(-208012a^4b^6+312018a^5b^6+4842288b^7-2517856ab^7+3991328a^2b^7)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(-153824a^3b^7+163438a^4b^7-138864b^8+370392ab^8-40986a^2b^8+43263a^3b^8)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\left.\frac{32768b(9944b^9-4048ab^9+5313a^2b^9-110b^{10}+253ab^{10}+3b^{11})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}\right\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\left\{\frac{32768a(38385745920a-8277098496a^2+11938545664a^3-1293154816a^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+\right.$$

$$+\frac{32768a(518927616a^5-297474224a^6+4842288a^7-138864a^8+9944a^9-110a^{10}+3a^{11})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(38385745920b+50799591424a^2b-4130196480a^3b+5067819264a^4b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(-257208448a^5b+92965264a^6b-2517856a^7b+370392a^8b-4048a^9b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(253a^{10}b+8277098496b^2+50799591424ab^2+14021624320a^3b^2-502850656a^4b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(529633328a^5b^2-12452384a^6b^2+3991328a^7b^2-40986a^8b^2+5313a^9b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(119385456664b^3+4130196480ab^3+14021624320a^2b^3+1206518544a^4b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(-19283936a^5b^3+17521952a^6b^3-153824a^7b^3+43263a^8b^3+1293154816b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(5067819264ab^4+502850656a^2b^4+1206518544a^3b^4+35778064a^5b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(-208012a^6b^4+163438a^7b^4+5189276166b^5+257208448ab^5+529633328a^2b^5)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(19283936a^3b^5+35778064a^4b^5+312018a^6b^5+29747424b^6+92965264ab^6)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(12452384a^2b^6+17521952a^3b^6+208012a^4b^6+312018a^5b^6+4842288b^7)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(2517856ab^7+3991328a^2b^7+153824a^3b^7+163438a^4b^7+138864b^8+370392ab^8)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(40986a^2b^8+43263a^3b^8+9944b^9+4048ab^9+5313a^2b^9+110b^{10}+253ab^{10}+3b^{11})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(81749606400a+123436892160a^2+77270003712a^3+26946067456a^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(5887453440a^5+853730240a^6+84401856a^7+5718768a^8+261360a^9+7700a^{10})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(132a^{11}+a^{12}-81749606400b+367298150400ab+118779801600a^2b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(242509385728a^3b+38595539712a^4b+16452876672a^5b+1303280832a^6b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(217911936a^7b+8342928a^8b+621368a^9b+9108a^{10}b+276a^{11}b+123436892160b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(-118779801600ab^2+464899287040a^2b^2+37227551232a^3b^2+76928096320a^4b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(4716831168a^5b^2+2038636096a^6b^2+67277760a^7b^2+11193732a^8b^2+148764a^9b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(10626a^10b^2-77270003712b^3+242509385728ab^3-37227551232a^2b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(125264532736a^3b^3+3669038016a^4b^3+7098446208a^5b^3+181484352a^6b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(74296992a^7b^3+865260a^8b^3+134596a^9b^3+26946067456b^4-38595539712ab^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(76928096320a^2b^4-3669038016a^3b^4+10621924768a^4b^4+124807200a^5b^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(219244648a^6b^4+1961256a^7b^4+735471a^8b^4-5887453440b^5+16452876672ab^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(-4716831168a^2b^5+7098446208a^3b^5-124807200a^4b^5+312018000a^5b^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(1248072a^6b^5+1961256a^7b^5+853730240b^6-1303280832ab^6+2038636096a^2b^6)}{\Big(\prod_{\nu=0}^{12}\{a-b-2\nu\}\Big)\Big(\prod_{\mu=1}^{11}\{a-b+2\mu\}\Big)}+$$

$$+\frac{4096(-181484352a^3b^6+219244648a^4b^6-1248072a^5b^6+2704156a^6b^6-84401856b^7)}{\Big(\prod_{\nu=0}^{12}\{a-b-2\nu\}\Big)\Big(\prod_{\mu=1}^{11}\{a-b+2\mu\}\Big)}+$$

$$+\frac{4096(217911936ab^7-67277760a^2b^7+74296992a^3b^7-1961256a^4b^7+1961256a^5b^7)}{\Big(\prod_{\nu=0}^{12}\{a-b-2\nu\}\Big)\Big(\prod_{\mu=1}^{11}\{a-b+2\mu\}\Big)}+$$

$$+\frac{4096(5718768b^8-8342928ab^8+11193732a^2b^8-865260a^3b^8+735471a^4b^8-261360b^9)}{\Big(\prod_{\nu=0}^{12}\{a-b-2\nu\}\Big)\Big(\prod_{\mu=1}^{11}\{a-b+2\mu\}\Big)}+$$

$$+\frac{4096(621368ab^9-148764a^2b^9+134596a^3b^9+7700b^{10}-9108ab^{10}+10626a^2b^{10})}{\Big(\prod_{\nu=0}^{12}\{a-b-2\nu\}\Big)\Big(\prod_{\mu=1}^{11}\{a-b+2\mu\}\Big)}+$$

$$+\frac{4096(-132b^{11}+276ab^{11}+b^{12})}{\Big(\prod_{\nu=0}^{12}\{a-b-2\nu\}\Big)\Big(\prod_{\mu=1}^{11}\{a-b+2\mu\}\Big)}\Bigg\}\Bigg] \tag{7}$$

III. Derivation of Summation Formulae (7)

Substituting $c=\frac{a+b+26}{2}$ and $z=\frac{1}{2}$ in equation (2), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+26}{2} & ; & \end{matrix}\right]=a\ {}_2F_1\left[\begin{matrix} a+1,\ b\ ; & \frac{1}{2} \\ \frac{a+b+26}{2} & ; \end{matrix}\right]-b\ {}_2F_1\left[\begin{matrix} a,\ b+1 & ; & \frac{1}{2} \\ \frac{a+b+26}{2} & ; & \end{matrix}\right]$$

Now using Gauss second summation theorem, we get

$$L.H.S=a\,\frac{2^b\ \Gamma(\frac{a+b+26}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+2}{2})}\left\{\frac{2048(-81749606400a+123436892160a^2)}{\Big(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\Big)\Big(\prod_{\eta=1}^{12}\{a-b+2\eta\}\Big)}+\right.\right.$$

$$+\frac{2048(-77270003712a^3+26946067456a^4-5887453440a^5+853730240a^6-84401856a^7)}{\Big(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\Big)\Big(\prod_{\eta=1}^{12}\{a-b+2\eta\}\Big)}+$$

$$+\frac{2048(5718768a^8-261360a^9+7700a^{10}-132a^{11}+a^12+817496064000b}{\Big(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\Big)\Big(\prod_{\eta=1}^{12}\{a-b+2\eta\}\Big)}+$$

$$+\frac{2048(367298150400ab-118779801600a^2b)+242509385728a^3b-38595539712a^4b)}{\Big(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\Big)\Big(\prod_{\eta=1}^{12}\{a-b+2\eta\}\Big)}+$$

$$+\frac{2048(16452876672a^5b-1303280832a^6b+217911936a^7b-8342928a^8b+621368a^9b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(-9108a^{1}0b+276a^{1}1b+123436892160b^2+118779801600ab^2+464899287040a^2b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(-37227551232a^3b^2+76928096320a^4b^2-4716831168a^5b^2+2038636096a^6b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(-67277760a^7b^2+11193732a^8b^2-148764a^9b^2+10626a^{10}b^2+77270003712b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(242509385728ab^3+37227551232a^2b^3+125264532736a^3b^3-3669038016a^4b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(7098446208a^5b^3-181484352a^6b^3+74296992a^7b^3-865260a^8b^3+134596a^9b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(26946067456b^4+38595539712ab^4+76928096320a^2b^4+3669038016a^3b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(10621924768a^4b^4-124807200a^5b^4+219244648a^6b^4-1961256a^7b^4+735471a^8b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(5887453440b^5+16452876672ab^5+4716831168a^2b^5+7098446208a^3b^5)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(124807200a^4b^5+312018000a^5b^5-1248072a^6b^5+1961256a^7b^5+853730240b^6)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(1303280832ab^6+2038636096a^2b^6+181484352a^3b^6+219244648a^4b^6+1248072a^5b^6)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(2704156a^6b^6+844018560b^7+217911936ab^7+67277760a^2b^7+74296992a^3b^7)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(1961256a^4b^7+1961256a^5b^7+5718768b^8+8342928ab^8+11193732a^2b^8)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(865260a^3b^8+735471a^4b^8+261360b^9+621368ab^9+148764a^2b^9+134596a^3b^9)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(7700b^{10}+9108ab^{10}+10626a^2b^{10}+132b^{11}+276ab^{11}+b^{12})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{32768(38385745920a-8277098496a^2+11938545664a^3-1293154816a^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(518927616a^5-29747424a^6+4842288a^7-138864a^8+9944a^9-110a^{10}+3a^{11})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(38385745920b+50799591424a^2b-4130196480a^3b+5067819264a^4b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(-257208448a^5b+92965264a^6b-2517856a^7b+370392a^8b-4048a^9b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(253a^{10}b+8277098496b^2+50799591424ab^2+14021624320a^3b^2-502850656a^4b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(529633328a^5b^2-12452384a^6b^2+3991328a^7b^2-40986a^8b^2+5313a^9b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(11938545664b^3+4130196480ab^3+14021624320a^2b^3+1206518544a^4b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(-19283936a^5b^3+17521952a^6b^3-153824a^7b^3+43263a^8b^3+1293154816b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(5067819264ab^4+502850656a^2b^4+1206518544a^3b^4+35778064a^5b^4-208012a^6b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(163438a^7b^4+518927616b^5+257208448ab^5+529633328a^2b^5+19283936a^3b^5)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(35778064a^4b^5+312018a^6b^5+29747424b^6+92965264ab^6+12452384a^2b^6)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(17521952a^3b^6+208012a^4b^6+312018a^5b^6+4842288b^7+2517856ab^7)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(3991328a^2b^7+153824a^3b^7+163438a^4b^7+138864b^8+370392ab^8+40986a^2b^8)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$\left.\left.+\frac{32768(43263a^3b^8+9944b^9+4048ab^9+5313a^2b^9+110b^{10}+253ab^{10}+3b^{11})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}\right\}\right]-$$

$$-b\,\frac{2^{b+1}\,\Gamma(\frac{a+b+26}{2})}{\Gamma(b+1)}\left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\left\{\frac{2048(81749606400a+123436892160a^2+77270003712a^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+\right.\right.$$

$$+\frac{2048(26946067456a^4+5887453440a^5+853730240a^6+84401856a^7+5718768a^8)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(261360a^9+7700a^{10}+132a^{11}+a^{12}-81749606400b+367298150400ab)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(118779801600a^2b+242509385728a^3b+38595539712a^4b+16452876672a^5b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(1303280832a^6b+217911936a^7b+8342928a^8b+621368a^9b+9108a^{10}b+276a^{11}b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(123436892160b^2-118779801600ab^2+464899287040a^2b^2+37227551232a^3b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(76928096320a^4b^2+4716831168a^5b^2+2038636096a^6b^2+67277760a^7b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

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$$+\frac{2048(11193732a^8b^2+148764a^9b^2+10626a^{10}b^2-772700037121b^3+242509385728ab^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(-37227551232a^2b^3+125264532736a^3b^3+3669038016a^4b^3+7098446208a^5b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(181484352a^6b^3+74296992a^7b^3+865260a^8b^3+134596a^9b^3+26946067456b^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(-38595539712ab^4+76928096320a^2b^4-3669038016a^3b^4+10621924768a^4b^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(124807200a^5b^4+219244648a^6b^4+1961256a^7b^4+735471a^8b^4-5887453440b^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(16452876672ab^5-4716831168a^2b^5+7098446208a^3b^5-124807200a^4b^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(312018000a^5b^5+1248072a^6b^5+1961256a^7b^5+853730240b^6-1303280832ab^6)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(2038636096a^2b^6-181484352a^3b^6+219244648a^4b^6-1248072a^5b^6+2704156a^6b^6)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(-844018856b^7+217911936ab^7-67277760a^2b^7+74296992a^3b^7-1961256a^4b^7)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(1961256a^5b^7+5718768b^8-8342928ab^8+11193732a^2b^8-865260a^3b^8+735471a^4b^8)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(-261360b^9+621368ab^9-148764a^2b^9+134596a^3b^9+7700b^{10}-9108ab^{10})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$\left.+\frac{2048(10626a^2b^{10}-132b^{11}+276ab^{11}+b^{12})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}\right\}-$$

$$-\frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{32768(38385745920a+8277098496a^2+11938545664a^3+1293154816a^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(518927616a^5+29747424a^6+4842288a^7+138864a^8+9944a^9+110a^{10}+3a^{11})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(38385745920b+50799591424a^2b+4130196480a^3b+5067819264a^4b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(257208448a^5b+92965264a^6b+2517856a^7b+370392a^8b+4048a^9b+253a^{10}b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(-8277098496b^2+50799591424ab^2+14021624320a^3b^2+502850656a^4b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(529633328a^5b^2+12452384a^6b^2+3991328a^7b^2+40986a^8b^2+5313a^9b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(11938545664b^3-4130196480ab^3+14021624320a^2b^3+1206518544a^4b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(19283936a^5b^3+17521952a^6b^3+153824a^7b^3+43263a^8b^3-1293154816b^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(35778064a^4b^5+312018a^6b^5-29747424b^6+92965264ab^6-12452384a^2b^6)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(+17521952a^3b^6-208012a^4b^6+312018a^5b^6+4842288b^7-2517856ab^7)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(3991328a^2b^7-153824a^3b^7+163438a^4b^7-138864b^8+370392ab^8-40986a^2b^8)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(43263a^3b^8+9944b^9-4048ab^9+5313a^2b^9-110b^{10}+253ab^{10}+3b^{11})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}\Bigg\}\Bigg]$$

On simplification we get the result (7).

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