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A Quarter Symmetric Non-metric Connection in a Generalized Co-symmpletic Manifolds

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I. Introduction

In 1975, S. Golab [4] introduced the notion of quarter symmetric non-connections in a Riemannian manifold with affine connection. After that S. C. Rastogi ([5],[6]) continue the systematic study quarter symmetric metric connection. In 1980, R. S. Mishra and S. N. Pandey [3] study a quarter symmetric metric connections in Riemannian, Kaehlarian and Sasakian manifolds. In 1992, S. Mukhopadhya, A. K. Roy and B. Barua [7] studied quarter symmetric metric connection in a Riemannian manifold with almost complex structure. In 1997, U. C. De and S. C. Biswas [8] studied quarter symmetric metric connection on an SP-Ssaskian manifold. Also in 2008, Sular, Ozgur and De [9] quarter symmetric metric connection in Kenmotsu manifold. In 2009, Abul Kalam, Mondal and De [1] studied some properties quarter symmetric metric connection on a Sasakian manifold. In this paper we studied a type of quarter symmetric non-metric connection in a generalized cosyplectic manifold and investigated the properties of this connection in the same manifold.

II. Preliminaries

An $n-$dimensional differentiable manifold M_n is an almost contact manifold if it admits a tensor field of type F, a vector field U and 1-form u satisfying for arbitrary vector field X.

$$\text{(a)} \quad \overline{\overline{X}} + X = A(X)\,T \qquad \text{(b)} \quad \overline{U} = 0 \tag{2.1}$$

where $\overline{X} \stackrel{def}{=} FX$

Again (2.1) (a) and (2.1) (b), gives

$$\text{(a)} \quad u(\overline{X}) = 0 \qquad \text{(b)} \quad u(U) = 1 \tag{2.2}$$

An almost contact manifold M_n in which a Riemannian metric tensor g of type $(0,\,2)$ satisfies

$$\text{(a)} \quad g(\overline{X},\overline{Y}) = g(X,Y) - u(X)u(Y) \qquad \text{(b)} \quad g(X,U) = u(X) \tag{2.3}$$

for arbitrary vector field X and Y, is called an almost contact metric manifold.

Let us put

$$'F(X,Y) = g(\overline{X},Y)$$

Then, we obtain

$$\text{(a)} \quad 'F(\overline{X},\overline{Y}) = {}'F(X,Y) \qquad \text{(b)} \quad 'F(X,Y) = g(\overline{X},Y) = -g(X,\overline{Y}) = -{}'F(Y,X) \tag{2.4}$$

An almost contact metric manifold satisfying

$$(D_X\,'F)(Y,Z) = u(Y)(D_X u)(\overline{Z}) - u(Z)(D_Z u)(\overline{Y}) \tag{2.5}$$

$$\begin{aligned}(D_X\,'F)(Y,Z) + (D_Y\,'F)(Z,X) + (D_Z\,'F)(X,Y) + u(X)\left[(D_Y u)(\overline{Z}) - (D_Z F)(\overline{Y})\right] \\ + u(Y)\left[(D_Z u)(\overline{Z}) - (D_X u)(\overline{Z})\right] + u(Z)\left[(D_X u)(\overline{Y}) - (D_Y u)(\overline{X})\right] = 0\end{aligned} \tag{2.6}$$

for arbitrary vector field X, Y, Z are respectively called generalized co-symplectic and generalized quasi-Sasakian manifolds[11].

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If on any manifold U, satisfies

(a) $(D_X u)(\bar{Y}) = -(D_{\bar{X}} u)(Y) = (D_Y u)(\bar{X})$ (2.7)

(b) $(D_X u)(Y) = -(D_{\bar{X}} u)(\bar{Y}) = -(D_Y u)(X)$ and

(c) $(D_{U_1} F) = 0$

then U_1 is said to be the first class.

If on an almost contact metric manifold U satisfies

(a) $(D_X u)(\bar{Y}) = (D_{\bar{X}} u)(Y) = -(D_Y u)(\bar{X}) \Leftrightarrow$ (2.8)

(b) $(D_X u)(Y) = -(D_{\bar{X}} u)(\bar{Y}) = -(D_Y u)(X)$ and

(c) $(D_{U_2} F) = 0$

then U_2 is said to be the second class.

The Nijenhuise tensor in a generalized co-symplectic manifold is given by

(a) $N(X,Y) = (D_X F)Y - (D_{\bar{Y}} F)(X) - \overline{(D_X F)(Y)} + \overline{(D_Y F)(X)}$ (2.9)

(b) ${}'N(X,Y,Z) = (D_{\bar{X}}\, {}'F)(Y,Z) - (D_{\bar{Y}}\, {}'F)(X,Z) + (D_X\, {}'F)(Y,\bar{Z}) - (D_Y\, {}'F)(X,\bar{Z})$

III. Quarter Symmetric Non-Metric Connection in a Generalized Co-Symplectic Manifold

Let (M_n, g) be a generalized co-symplectic manifold with Riemannian connection D.we define a linear connection B on (M^n, g) by

$$B_X Y = D_X Y + u(Y)X + a(X)FY \tag{3.1}$$

where u and a is 1-form associated with vector field ξ and A on (M^n, g) that is

(a) $g(X,U) = u(X)$ and (3.2)

(b) $g(X,A) = a(X)$

for all vector field $X \in \mathcal{X}(M_n)$, where $\mathcal{X}(M_n)$ is the set of all differentiable vector fields on (M^n, g).

Using (3.1) the torsion tensor T of (M^n, g) with respect to connection B is given by

$$T(X,Y) = u(Y)X - u(X)FY + a(X)FY - a(Y)FX \tag{3.3}$$

A linear connection satisfying (3.3) is called Quarter-symmetric connection and metric tensor g satisfies [10].

$$(B_X g)(Y,Z) = -u(Y)g(FX,Z) - u(Z)g(FX,Y) - 2a(X)g(FY,Z) \tag{3.4}$$

for arbitrary vector field X, Y, Z.

Then a linear connection B defined by (3.1) satisfies (3.3) and (3, 4) is called a quarter - symmetric non-metric connection.

If we put

$$B_X Y = D_X Y + H(X,Y)$$

Where H is a tensor of type (0, 2), then we have

$$\begin{cases} (i) & H(X,Y) = u(Y)X + a(X)FY \\ (ii) & {}'H(X,Y,Z) = u(Y)g(X,Z) + a(X)g(FY,Z) \\ (iii) & {}'T(X,Y,Z) = u(Y)g(FX,Z) - u(X)g(FY,Z) + a(X)g(FY,Z) - a(Y)g(FX,Z) \\ (iv) & (B_X u)(Y) = (D_X u)(Y) + g(X,Y) + {}'F(X,Y) \end{cases} \tag{3.5}$$

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where

$$'H(X,Y,Z) \underline{\underline{def}} g(H(X,Y)Z)$$

$$'T(X,Y,Z) \underline{\underline{def}} g(T(X,Y)Z)$$

we have

$$X('F(Y,Z) = (D_X{}'F)(Y,Z) + 'F(D_XY,Z) + 'F(Y,D_XZ)$$
$$= (B_X{}'F)(Y,Z) + 'F(B_XY,Z) + 'F(Y,B_XZ)$$

Using (3.1) in the above equation, we get

$$X('F(Y,Z) = (B_X{}'F)(Y,Z) + 'F(D_XY + u(Y)X + a(X)FY,Z) + 'F(Y,D_XZ + u(Z)X + a(X)FZ)$$
$$(B_X{}'F)(Y,Z) = (D_X{}'F)(Y,Z) + u(Y)'F(X,Z) + a(X)'F(FY,Z) + u(Z)'F(Y,X) + a(X)'F(Y,FZ) \quad (3.6)$$

The Nijenhuis tensor N in term of quarter symmetric non metric connection B is given by

$$\begin{cases} (i) & N(X,Y) = (B_{\overline{X}}F)(Y) - (B_{\overline{Y}}F)(X) + \overline{(B_XF)(Y)} + \overline{(B_XF)(X)} \\ (ii) & 'N(X,Y,Z) = (B_{\overline{X}}{}'F)(Y,Z) - (B_{\overline{Y}}{}'F)(X,Z) - (B_X{}'F)(Y,\overline{Z}) - (B_Y{}'F)(X,\overline{Z}) \end{cases} \quad (3.7)$$

Theorem 3.1 : A generalized co-symplectic manifold admitting quarter symmetric non−metric connection such that $B_X{}'F = 0$, then $'F$ is locally killing provided the vector fields X,Y,Z are orthogonal to U.

Proof : From (3.6), we have

$$(B_X{}'F)(Y,Z) = (D_X{}'F)(Y,Z) + u(Y)'F(X,Z) + a(X)'F(\overline{Y},Z) + u(Z)'F(Y,X) + a(X)'F(Y,\overline{Z})$$

Since $B_X{}'F = 0$, we get

$$(D_X{}'F)(Y,Z) = -u(Y)'F(X,Z) - a(X)'F(\overline{Y},Z) - u(Z)'F(Y,Z) - a(X)'F(Y,\overline{Z}) \quad (3.8)$$

Similarly

$$(D_Y{}'F)(X,Z) = -u(X)'F(Y,Z) - a(Y)'F(\overline{X},Z) - u(Z)'F(X,Z) - a(Y)'F(X,\overline{Z}) \quad (3.9)$$

By virtue of equation (3.8) and (3.9), we get

$$(D_X{}'F)(Y,Z) + (DF)(X,Z) = [u(X) + u(Z)]'F(Y,Z) - [u(Y) + u(Z)]'F(X,Z) \quad (3.10)$$

Taking the vector field X,Y,Z orthogonal to U, we get

we get the required result.

Theorem 3.2 : A generalized co-symplectic manifolds admitting quarter symmetric non-metric connection is locally closed with respect to this connection D if and only if it is locally closed with respect to Riemannian connection provided the vector fields X,Y,Z orthogonal to ξ.

Proof : We have

$$X('F(Y,Z) = (D_X{}'F)(Y,Z) + 'F(D_XY,Z) + 'F(Y,D_XZ)$$
$$= (B_X{}'F)(Y,Z) + 'F(B_XY,Z) + 'F(Y,B_XZ)$$

Using (3.1), we get

$$(B_X{}'F)(Y,Z) = (D_X{}'F)(Y,Z) + u(Y)'F(X,Z) + a(X)'F(FY,Z) + u(Z)'F(Y,X) + a(X)'F(Y,FZ) \quad (3.11)$$

from (3.11), we obtained

$$\begin{aligned}(B_X{}'F)(Y,Z) + (B_Y{}'F)(Z,X) + (B_Z{}'F)(X,Y) &= (D_X{}'F)(Y,Z) + (D_Y{}'F)(Z,X) + (D_Z{}'F)(X,Y) \\ &+ 2u(Y)'F(X,Z) + a(X)\left['F(\overline{Y},Z) + 'F(Y,\overline{Z})\right] \\ &+ 2u(Z)'F(Y,X) + a(Y)\left['F(\overline{Z},X) + 'F(Z,\overline{X})\right] \\ &+ 2u(X)\left['F(Z,Y) + F(Z,Y)\right] + a(Z)\left['F(\overline{X},Y) + 'F(X,\overline{Y})\right]\end{aligned}$$

Using (2.4)(b) in above, we have

$$\begin{aligned}(B_X{}'F)(Y,Z) + (B_Y{}'F)(Z,X) + (B_Z{}'F)(X,Y) &= (D_X{}'F)(Y,Z) + (D_Y{}'F)(Z,X) + (D_Z{}'F)(X,Y) \\ &+ 2\left[u(X)'F(Z,Y) + u(Y)'F(X,Z) + u(Z)'F(Y,X)\right]\end{aligned}$$

Taking the vector field X, Y, Z orthogonal to U, we get

$$(B_X{}'F)(Y,Z)+(B_Y{}'F)(Z,X)+(B_Z{}'F)(X,Y) = (D_X{}'F)(Y,Z)+(D_Y{}'F)(Z,X)+(D_Z{}'F)(X,Y)=0 \tag{3.12}$$

Theorem 3.3 : A generalized co-symplectic manifolds admitting quarter symmetric non-metric connection satisfies the following relations

i. $(B_{\overline{X}}{}'F)(Y)=(D_{\overline{X}}{}'F)(Y)$

ii. $(B_{\overline{X}}{}'F)(\overline{Y})=(D_{\overline{X}}{}'F)(\overline{Y})$

iii. $N(X,Y)=0$ (Complete integrable) if $D_X{}'F=0$

iv. ${}'H(\overline{X},\overline{Y},\overline{Z})={}'T(\overline{X},\overline{Y},\overline{Z})$

Proof : From (3.1), we have

$$B_X Y = D_X Y + H(X,Y) \tag{3.13}$$

where

$$H(X,Y)=u(Y)X+a(X)FY$$

for any vector field for $\overline{Y}$, equation (3.13) can be written as

$$(B_X F)(Y)=(D_X F)(Y)-\overline{B_X Y}+(\overline{D_X Y}-a(X)Y+A(Y)a(X)T \tag{3.14}$$

Operating both side equation (3.13) by

$$\overline{B_X Y}-\overline{D_X Y}=u(Y)\overline{X}-a(X)Y+a(X)A(Y)T \tag{3.15}$$

Using (3.15) in (3.14), we get

$$(B_X{}'F)(Y)=(D_X{}'F)(Y)-u(Y)\overline{X} \tag{3.16}$$

Barring X, Y and respectively in (3.16) and using (2.2) (a), we get the required the result (i, ii).

Since $(D_X{}'F)(Y)=0$,

Then from (3.16), we get

$$(B_X{}'F)(Y)=-u(Y)\overline{X} \tag{3.17}$$

Barring X and using (2.1)(a), we get

$$(B_{\overline{X}}{}'F)(Y)=u(Y)X-u(Y)A(X)T \tag{3.18}$$

Interchanging X and Y, we get

$$(B_{\overline{Y}}{}'F)(X)=u(X)Y-u(X)A(Y)T \tag{3.19}$$

Operating both side equation (3.17) by F, we get

$$\overline{(B_X F)(Y)}=u(Y)X-u(Y)A(X)T \tag{3.20}$$

Interchanging X and Y we get

$$\overline{(B_Y F)(X)}=u(X)Y-u(X)A(Y)T \tag{3.21}$$

Using (3.18),(3.19),(3.20)and(3.21)in (3.7)(i), we get the result (iii).

Finally barring X, Y, Z in (3.5) (ii, iii), we get the result (iv).

Theorem 3.4 : If U is a killing on generalized co-symplectic manifolds with quarter symmetric non-metric connection then

$$(B_X{}'F)(Y,\overline{Z})+(B_Y{}'F)(\overline{Z},X)+(B_{\overline{Z}}{}'F)(X,Y)={}'N(X,Y,Z)-\{u(Y)+u(Z)\}{}'F(\overline{X},Z)$$

Proof : From (3.11) and (2.9) (b), we have

$$\begin{aligned}{}'N(X,Y,Z)-(B_X{}'F)(Y,\overline{Z})-(B_Y{}'F)(X,\overline{Z})-(B_{\overline{Z}}{}'F)(X,Y)&=(D_{\overline{X}}{}'F)(Y,Z)-(D_{\overline{Y}}{}'F)(X,Z)\\&-(D_{\overline{Z}}{}'F)(X,Y)+u(Y){}'F(\overline{X},Z)+u(Z){}'F(Y,\overline{X})\\&-u(X){}'F(\overline{Y},Z)-u(Z){}'F(X,\overline{Y})-u(Z){}'F(\overline{Z},X)\end{aligned}$$

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$$-u(X)'F(Y,\bar{Z})$$

Again using (2.5) in above equation, we get

$$'N(X,Y,Z)-(B_X{}'F)(Y,\bar{Z})-(B_Y{}'F)(X,\bar{Z})-(B_{\bar{Z}}{}'F)(X,Y)=-u(X)\left[(D_{\bar{Y}}u)(\bar{Z})+(D_{\bar{Z}}u)(Y)\right]$$
$$+u(Y)\left[(D_{\bar{X}}u)(\bar{Z})+(D_{\bar{Z}}u)(\bar{X})\right]+u(Z)\left[(D_{\bar{Y}}u)(\bar{X})-(D_{\bar{X}}u)(\bar{Y})\right]$$
$$+(u(Y)+u(Z))'F(\bar{X},Z)$$

Since U is a killing then putting $(D_X u)(Y)+(D_Y u)(x)=0$ in above equation, we obtained

$$(B_X{}'F)(Y,\bar{Z})+(B_Y{}'F)(\bar{Z},X)+(B_{\bar{Z}}{}'F)(X,Y)={}'N(X,Y,Z)-u(Z)\left[(D_{\bar{Y}}u)(\bar{X})-(D_{\bar{X}}u)(\bar{Y})\right]$$
$$-(u(Y)+u(Z))'F(\bar{X},Z) \tag{3.22}$$

By virtue of equation (2.7) (b), equation (3.22) will be reduces

$$(B_X{}'F)(Y,\bar{Z})+(B_Y{}'F)(\bar{Z},X)+(B_Z{}'F)(X,Y)={}'N(X,Y,Z)-\{u(Y)+u(Z)\}'F(\bar{X},Z)$$

we get the required result.

Theorem 3.5 : A generalized co-symplectic manifold of first class with quarter symmetric non-metric connection satisfy

$$(B_{\bar{\bar{X}}}{}'F)(\bar{Y},\bar{Z})+(B_{\bar{\bar{Y}}}{}'F)(\bar{Z},\bar{X})+(B_{\bar{\bar{Z}}}{}'F)(\bar{X},\bar{Y})=0$$

Proof : From equation (2.5) and (3.11), we get

$$(B_{\bar{X}}{}'F)(Y,Z)=u(Y)\left[(D_{\bar{X}}u)(\bar{Z})+{}'F(\bar{X},Z)\right]-u(Z)(D_{\bar{X}}u)(\bar{Y})+a(\bar{X})'F(\bar{Y},Z) \tag{3.23}$$

Taking cyclic sum of equation (3.23) in X,Y,and Z we get

$$(B_{\bar{X}}{}'F)(Y,Z)+(B_{\bar{Y}}{}'F)(Z,X)+(B_{\bar{Z}}{}'F)(X,Y)=u(X)\left[(D_{\bar{Z}}u)(\bar{Y})-(D_{\bar{Y}}u)(\bar{Z})\right]$$
$$+u(Y)\left[(D_{\bar{X}}u)(\bar{Z})-(D_{\bar{Z}}u)(X)\right]$$
$$+u(Z)\left[(D_{\bar{Y}}u)(\bar{X})-(D_{\bar{X}}u)(Y)\right] \tag{3.24}$$
$$+u(Y)'F(\bar{X},Z)+u(Z)'F(\bar{Y},X)$$
$$+u(X)'F(\bar{Z},Y)$$

Barring X,Y and Z in equation (3.24) and using (2.2)(a), we get

$$(B_{\bar{\bar{X}}}{}'F)(\bar{Y},\bar{Z})+(B_{\bar{\bar{Y}}}{}'F)(\bar{Z},\bar{X})+(B_{\bar{\bar{Z}}}{}'F)(\bar{X},\bar{Y})=0$$

we get the required result.

Theorem 3.6 : An almost contact metric manifold admitting quarter symmetric non-metric connection B is a generalized co-symplectic if

$$(B_X{}'F)(Y,Z)=u(Y)\left[\ (B_X u)(\bar{Z})+2'F(X,Z)-{}'F(X,\bar{Z})\ \right]+u(Z)\left[2'F(X,Y)-(B_X u)(\bar{Y})+{}'F(X,\bar{Y})\right]$$

Proof : From equation (2.5) and (3.11), we have

$$(B_X{}'F)(Y,Z)=u(Y)\left[\ (D_X u)(\bar{Z})+{}'F(X,Z)\ \right]+u(Z)\left[{}'F(Y,X)-(D_X u)(\bar{Y})\right] \tag{3.25}$$

Using (2.4)(b), (3.5)(iv) in (3.25), we obtain

$$(B_X{}'F)(Y,Z)=u(Y)\left[\ (B_X u)(\bar{Z})+2'F(X,Z)-{}'F(X,\bar{Z})\ \right]+u(Z)\left[2'F(X,Y)-(B_X u)(\bar{Y})+{}'F(X,\bar{Y})\right] \tag{3.26}$$

we get the required result.

Theorem 3.7 : A quasi-Sasakian manifold is normal if and only if

$$(B_X{}'F)(Y,Z)=u(Y)\left[\ (B_Z u)(\bar{X})-{}'F(Z,\bar{X})\ \right]+u(Z)\left[(B_{\bar{X}}u)(Y)-2'F(X,Y)-{}'F(\bar{X},Y)\right]$$

Proof : The necessary and sufficient conditions for a quasi-Sasakin manifold to be normal [11] is

$$(D_X{}'F)(Y,Z)=u(Y)(D_Z u)(\bar{X})+u(Z)(D_{\bar{X}}u)(Y) \tag{3.27}$$

Using (3.11) in (3.27), we get

$$(B_X{}'F)(Y,Z)=u(Y)\left[(D_{\bar{Z}}u)(\bar{X})+{}'F(X,Z)\right]+u(Z)\left[(D_{\bar{X}}u)(Y)+{}'F(Y,X)\right] \tag{3.28}$$

By virtue of equation (2.4)(b), (3.5)(iv) and (3.28), we obtain

$$(B_X{}'F)(Y,Z)=u(Y)[\,(B_Z u)(\overline{X})-{}'F(Z,\overline{X})\,]+u(Z)\left[(B_{\overline{X}}u)(Y)-2{}'F(X,Y)-{}'F(\overline{X},Y)\right]$$

we get the required result.

Theorem 3.8 : A generalized co-symplectic manifold is quasi-Sasakian manifold if

$$(B_X u)(\overline{Z})=(B_Z u)(\overline{X})+2{}'F(X,Y)$$

where B is the quarter symmetric non-metric connection.

Proof : From (3.11), we have

$$\begin{aligned}(D_X{}'F)(Y,Z)+(D_Y{}'F)(Z,X)+(D_Z{}'F)(X,Y)&=(B_X{}'F)(Y,Z)+(B_Y{}'F)(Z,X)+(B_Z{}'F)(X,Y)\\&-2u(Y){}'F(X,Z)-a(X)\left[{}'F(\overline{Y},Z)+{}'F(Y,\overline{Z})\right]\\&-2u(Z){}'F(Y,X)-a(Y)\left[{}'F(\overline{Z},X)+{}'F(Z,\overline{X})\right]\\&-2u(X)\left[{}'F(Z,Y)+F(Z,Y)\right]-a(Z)\left[{}'F(\overline{X},Y)+{}'F(X,\overline{Y})\right]\end{aligned} \tag{3.29}$$

By virtue of equation (3.26) and (3.29), we get

$$\begin{aligned}(D_X{}'F)(Y,Z)+(D_Y{}'F)(Z,X)+(D_Z{}'F)(X,Y)&=u(Y)\left[(B_X u)(\overline{Z})-2{}'F(X,Z)-(B_Z u)(\overline{X})\right]\\&+u(Z)\left[(B_Y u)(\overline{X})-(B_X u)(\overline{Y})-2{}'F(Y,X)\right]\\&+2u(X)\left[(B_Z u)(\overline{Y})-(B_Y u)(\overline{Z})-2{}'F(Z,Y)\right]\end{aligned} \tag{3.30}$$

Since the manifold is quasi-Sasakian , therefore

$$(D_X{}'F)(Y,Z)+(D_Y{}'F)(Z,X)+(D_Z{}'F)(X,Y)=0 \tag{3.31}$$

From equation (3.30) and (3.31) ,we get

$$(B_X u)(\overline{Z})=(B_Z u)(\overline{X})+2{}'F(X,Y)$$

we get the required result.

Theorem 3.9 : If the generalized co-symplectic manifold is of first class with respect to Riemannian connection D then it is also first class with respect to the quarter symmetric non-metric connection B and satisfies .

$$(B_X{}'F)(Y,Z)=u(Y)\left[\,(B_Z u)(\overline{Z})+2{}'F(X,Z)-{}'F(X,\overline{Y})\right]-u(Z)\left[(B_{\overline{X}}u)(\overline{Y})+2{}'F(X,Y)-{}'F(X,\overline{Y})\right]$$

Proof : Barring X and Y in (3.5)(iv) respectively and using(2.3)(a), we have

$$(D_{\overline{X}}u)(Y)=(B_{\overline{Z}}u)(Y)-g(\overline{X},Y)-{}'F(\overline{X},Y) \tag{3.32}$$

and

$$(D_X u)(\overline{Y})=(B_X u)(\overline{Y})-g(X,\overline{Y})-{}'F(X,\overline{Y}) \tag{3.33}$$

Adding (3.32), (3.33) and using (2.4)(b), we get

$$(D_{\overline{X}}u)(Y)+(D_X u)(\overline{Y})=(B_{\overline{X}}u)(Y)+(B_X u)(\overline{Y}) \tag{3.34}$$

By virtue of (2.7)(a) and (3.34), we have

$$(B_{\overline{X}}u)(Y)=-(B_X u)(\overline{Y}) \tag{3.35}$$

Again in similar way, we have

$$(B_X u)(\overline{Y})=(B_Y u)(\overline{X}) \tag{3.36}$$

From (3.35) and (3.36), we get

$$(B_X u)(\overline{Y})=-\,(B_{\overline{X}}u)(Y)=(B_Y u)(\overline{X}) \tag{3.37}$$

Taking covariant derivative of $FY=\overline{Y}$ with respect to B and using (2.1)(a),(2.2)(a) and (3.11), we get

$$(B_X F)(Y)=(D_X F)(Y)-u(Y)\overline{X} \tag{3.38}$$

Replacing X by U in (3.38) and using (2.1)(a), (2.8)(c), we obtain

$$(B_U F)(Y)=0 \tag{3.39}$$

By virtue of equation (2.5), (3.5) (iv) and (3.11), we get

$$(B_X{}^{/}F)(Y,Z) = u(Y)\left[(B_Z u)(\overline{Z}) + 2^{/}F(X,Z) - {}^{/}F(X,\overline{Y})\right] - u(Z)\left[(B_{\overline{X}} u)(\overline{Y}) + 2^{/}F(X,Y) - {}^{/}F(X,\overline{Y})\right]$$

We get the required result.

References Références Referencias

1. A.Kalam. Mondal, and U.C.De, Some properties of quarter-symmetric metric connection on a Sasakian manifold. Bulletin of mathematical analysis and app.vol.1, Issue3 (2009), 99-108.
2. I.Mihai, and, K. Yano, Quarter-symmetric metric connection and there curvature tensor. Tensor, N.S., 38(1982), 13-18.
3. R.S. Mishra and S.N. Pandey, On quarter symmetric metric f-connections, Tensor, N.S.34 (1980), 1-7.
4. S. Golab, On semi-symmetric and quarter-symmetric linear connection, Tensor.N.S.29 (1975), 249-254.
5. S.C. Rastogi, On quarter-symmetric metric connection, Tensor, 44, 2(1987), 133-141.
6. S. C. Rastogi, On quarter-symmetric metric connection, CR. Acad. Sci. Bulgar, 31(1978),811-814.
7. S. Mukhopadhyay, A.K. Roy and B. Barua, Some properties of quarter symmetric metric connection\ Riemannian manifold. Soochaw J. of math.17, 2 (1991), 205-211.
8. S. C. Biswas, and U. C. De, Quarter-symmetric metric connection in an SP-Sasakian manifold, commn. Fac. sci. Univ. ank. Servies, 46(1997), 49-56.
9. S. Sular, C. Ozugar and U.C. De, Quarter-symmetric metric connection in a Kenmotsu manifold, SUT Journal of mathematics Vol.44, No.2(2008), 297-306.
10. S.K. Yadav and D. L .Suthar, On a quarter symmetric non-metric connection on a Lorentzian Para-cosympletic manifold(communicated for publication).
11. R. S. Mishra, Olmost contact metric manifold, Monograph (I) Tensor Society of India, Lucknow, (1991).

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