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On The Solutions of Generalized Fractional Kinetic Equations Involving the Functions for the Fractional Calculus

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Abstract - The paper is devoted to the study of the solution of generalized fractional kinetic equations. Results are obtained in a compact form in terms of $K4$ - Function introduced by sharma [9]. The results obtained in this paper are the extensions of the results given earlier by Chaurasia and Pandey[19-20] believed to be new.

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On the Solutions of Generalized Fractional Kinetic Equations Involving the Functions for the Fractional Calculus

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Abstract - The paper is devoted to the study of the solution of generalized fractional kinetic equations. Results are obtained in a compact form in terms of K_4 - Function introduced by sharma [9]. The results obtained in this paper are the extensions of the results given earlier by Chaurasia and Pandey[19-20] believed to be new.

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I. Introduction

Fractional Calculus and special functions have contributed a lot to mathematical physics and its various branches. The great use of mathematical physics in distinguished astrophysical problems has attracted astronomers and physicists to pay more attention to available mathematical tools that can be widely used in solving several problems of astrophysics/physics. The fractional kinetic equations discussed here can be used to investigate a wide class of known fractional kinetic equations. Fractional kinetic equations have gained importance during the last decade due to their occurrence in certain problems in science and engineering. A spherically symmetric non-rotating , self-gravitating model of star like the sun is assumed to be in thermal equilibrium and hydrostatic equilibrium. The star is characterized by its mass, luminosity effective surface temperature, radius central density and central temperature. The stellar structures and their mathematical models are investigated on the basis of above characters and some additional information related to the equation of nuclear energy generation rate and the opacity.

Consider an arbitrary reaction characterized by a time dependent quantity $N = N(t)$.

It is possible to calculate rate of change $dN/dt = -d + p.$

In general, through feedback or other interaction mechanism, destruction and production depend on the quantity N itself: $d = d(N)$ or $p = p(N)$. This dependence is complicated since the destruction or production at time t depends not only on $N(t)$ but also on the past history $N(\tau), \tau < t$, of the variable N .This may be represented by Haubold and Mathai[7]

$$dN/dt = -d(N_t) + p(N_t), \tag{1.1}$$

where N_t denotes the function defined by $N_t(t^*) = N(t - t^*), t^* > 0.$

Haubold and Mathai[7] studied a special case of this equation , when spatial fluctuation or inhomogenities in quantities $N(t)$ are neglected, is given by the equation

$$dN_i/dt = -c_i N_i(t) \tag{1.2}$$

with the initial condition that $N_i(t = 0) = N_0$ is the number density of species i at time $t = 0$;constant $c_i > 0$,known as standard kinetic equation. A detailed discussion of the above equation is given in Kourganoff[21]. The solution of (1.2) is given by

$$N_i(t) = N_0 e^{-c_i t} \tag{1.3}$$

An alternative form of this equation can be obtained on integration:

$$N(t) - N_0 = c\, {}_0D_t^{-1} N(t) , \tag{1.4}$$

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where ${}_0D_t^{-1}$ is the standard integral operator.Haubold and Mathai[7] have given the fractional generalization of the standard kinetic equation(1.2) as

$$N(t) - N_0 = c^{\nu}\, {}_0D_t^{-1} N(t), \tag{1.5}$$

where ${}_0D_t^{-1}$ is well known Riemann-Liouville fractional integral operator (Oldham and Spanier[8]; Samko, Kilbas and Marichev[16]; Miller and Ross[10]) defined by

$${}_0D_t^{-\nu} N(t) = \frac{1}{\Gamma(\nu)} \int_0^t (t-u)^{\nu-1} f(u)du, \mathrm{Re}(\nu) > 0. \tag{1.6}$$

The solution of the fractional kinetic equation(1.5) is given by (see Haubold and Mathai[7])

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(\nu k+1)} (ct)^{\nu k}. \tag{1.7}$$

Fractional kinetic equations are studied by many authors notably Hille and Tamarkin[5], Glockle and Nonnenmacher[22], Saichev and Zaslavsky[1], Saxena et al.[11-13], Zaslavsky[6], Saxena and Kalla[15], Chaurasia and Pandey[18-19], Chaurasia and Kumar[17] etc. for their importance in the solution of certain physical problems. Recently, Saxena et al. [14] investigated the solutions of the fractional reaction equation and the fractional diffusion equation. Laplace transform technique is used.

The K_4-function[9] is defined as

$$K_4^{(\alpha,\beta,\gamma),(a,c):(p;q)}(a_1,\ldots,a_p; b_1,\ldots,b_q; x) = K_4^{(\alpha,\beta,\gamma),(a,c):(p;q)}(x)$$

$$= \sum_{n=0}^{\infty} \frac{(a_1)_n \ldots (a_p)_n}{(b_1)_n \ldots (b_q)_n} \frac{(\gamma)_n\, a^n (x-c)^{(n+\gamma)\alpha-\beta-1}}{n!\Gamma((n+\gamma)\alpha-\beta)} \tag{1.8}$$

where $R(\alpha\gamma-\beta) > 0$ and $(a_i)_n (i=1,2,\ldots,p)$ and $(b_j)_n$ $(j=1,2,\ldots,q)$ are the Pochhammer symbols and none of the parameters $b_j s$ is a negative integer or zero.

We now proceed to solve the generalized fractional kinetic equation in the next section.

II. GENERALIZED FRACTIONAL KINETIC EQUATION

"'In this section we investigate the solution of generalized fractional kinetic equations'.. The results are obtained in a compact form in terms of K_4- Function and are suitable for computation. The result is presented in the form of a theorem as follows:

Theorem 2.1 If $c > 0, b \geq 0, \delta > 0, \nu > 0, \mu > 0$ and $(\delta\nu - \mu) > 0$ then there exists the solution of the integral equation

$$N(t) - N_0 K_4^{(\nu,\mu,\delta),(-c^{\nu},b):(p;q)}(t) = -\sum_{r=0}^{n} \binom{n}{r} c^{r\nu}\, {}_0D_t^{-r\nu} N(t), \tag{2.1}$$

given by

$$N(t) = N_0 K_4^{(\nu,\mu+\nu n,\delta+n),(-c^{\nu},b):(p;q)}(t). \tag{2.2}$$

Proof: Taking the Laplace transform of both sides of (2.1), we have

$$L\{N(t)\} - L\{N_0 K_4^{(\nu,\mu,\delta),(-c^{\nu},b):(p;q)}(t)\} = L\{-\sum_{r=0}^{n} \binom{n}{r} c^{r\nu}\, {}_0D_t^{-r\nu} N(t)\} \tag{2.2}$$

or

$$\overline{N(p)} = \frac{N_0\, p^{\mu-\delta\nu} c^{-bp}}{(1+c^{\nu} p^{-\nu})^{\mu+\delta}} \sum_{k=0}^{\infty} \frac{(a_1)_k \ldots (a_p)_k}{(b_1)_k \ldots (b_q)_k} \tag{2.4}$$

Finally, taking the inverse Laplace transform, we have

$$L^{-1}\{\overline{N(p)}\} = N\{t\} = L^{-1}\{\frac{N_0\, p^{\mu-(\delta+n)\nu+pn}\, c^{-bp}}{(1+c^{\nu} p^{-\nu})^{\mu+\delta}} \sum_{k=0}^{\infty} \frac{(a_1)_k \ldots (a_p)_k}{(b_1)_k \ldots (b_q)_k}$$

Or

$$N(t) = N_0\, {}^{(\nu,\mu+\nu n,\delta+n),(-c^{\nu},b):(p;q)}K_4\,(t) \tag{2.5}$$

This completes the proof of the theorem(2.1).

If we put $r = s = 0$ in theorem 2.1, we get[18]

Cor.1.1 If $c > 0, b \geq 0, \delta > 0, \nu > 0, \mu > 0$ and $(\delta\nu - \mu) > 0$ then there exists the solution of the integral equation

$$N(t) - N_0 G_{\nu,\mu,\delta}(c^{-\nu}, b, t) = -\sum_{r=0}^{n} \binom{n}{r} c^{r\nu}\, {}_0D_t^{-r\nu} N(t), \tag{2.6}$$

is given by

$$N(t) = N_0 G_{\nu,\mu+\nu n,\delta+n}(c^{-\nu}, b, t). \tag{2.7}$$

If we take $b = 0$ in Corollary.(1.1), we get[19]

Cor.1.2 If $c > 0, \delta > 0, \nu > 0, \mu > 0$ and $(\delta\nu - \mu) > 0$ then there exists the solution of the integral equation

$$N(t) - N_0 G_{\nu,\mu,\delta}(c^{-\nu}, 0, t) = -\sum_{r=0}^{n} \binom{n}{r} c^{r\nu}\, {}_0D_t^{-r\nu} N(t), \tag{2.8}$$

is given by

$$N(t) = N_0 G_{\nu,\mu+\nu n,\delta+n}(c^{-\nu}, 0, t). \tag{2.9}$$

If we take $b = 0$ in Corollary.(1.1), we get[20]

Cor.1.3 Let $c > 0, b \geq 0, \delta > 0, \nu > 0, \mu > 0$ and $(\delta\nu - \mu) > 0$ then the equation

$$N(t) - N_0 G_{\nu,\mu,\delta}(c^{-\nu}, b, t) = -c^{\nu}\, {}_0D_t^{-\nu} N(t), \tag{2.10}$$

is solvable and its solution is given by

$$N(t) = N_0 G_{\nu,\mu+\nu,\delta+1}(c^{-\nu}, b, t). \tag{2.11}$$

where $G_{\nu,\mu,\delta}(a, c, t)$ is the G-function(but not the Meijer's G-function) given by [2].

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IV. Conclusion

In the present paper, we have derived a solution of generalized fractional kinetic equation in terms of the K_4- Function in a compact and elegant form with the help of Laplace transform. Most of the results obtained are suitable for numerical computation. Fractional kinetic equation can be used to calculate the particle reaction rate and describes the statistical mechanics associated with the particle distribution function.

References

1. A.J.Saichev and G.M. Zaslavsky, Fractional kinetic wquations: solutions and applications, Chaos, 7(1997)753-784.
2. Carl F. Lorenzo and Tom T. Hartley, Generalized Functions for the Fractional Calculus, NASA/TP-1999-209424/REV1, (1999), 17 p.; Available electronically at http://gltrs.grc.nasa.gov/reports/1999/TP-1999-209424-REV1.pdf.
3. Carl F. Lorenzo and Tom T. Hartley, R-Function Relationships for Application in the Fractional Calculus, NASA/TM 2000 210361, (2000),22p.; Available electronically at http://gltrs.grc. nasa. gov/reports/2000/TM-2000-210361.pdf
4. Tom T. Hartley and Carl F. Lorenzo, A solution to the Fundamental Linear Fractional Order Differential Equations, NASA/TP-1998-208693 (1998),16 p.; Available electronically at http://gltrs.grc.nasa.gov/reports/1998/TP-1998-208693.pdf
5. E. Hille and J.D. Tamarkin, On the theory of linear integral equations, Annals of Mathematics, 31(1930), 479-528.
6. G.M.Zaslavsky, Fractional kinetic equation for Hamiltonian Chaos, Physica D, 76(1994), 110-122.
7. H.J. Haubold and A.M.Mathai, The fractional reaction equation and thermonuclear functions, Astrophysics and Space Science, 273(2000), 53-63.
8. Keith B. Oldham and Jerome Spanier, The Fractional Calculus; Theory and Applications of Differentiation and integration to Arbitrary Order, Academic Press, New York and London(1974). ISBN 0-12-525550-0.
9. K. Sharma, On Application of Fractional Differintegral Operator to the K_4-Function, Bol. Soc. Paran. Math.Vol.**30** 1, 2012, 91-97.
10. K. S. Miller and B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley and Sons, New York etc (1993).
11. R.K.Saxena, A.M.Mathai and H.J.Haubold, On fractional kinetic equations, Astrophysics and Space Science, 282(2002), 281-287.
12. R.K.Saxena, A.M.Mathai and H.J.Haubold, On generalized fractional kinetic equations, Physica A, 344(2004), 657-664.
13. R.K.Saxena, A.M.Mathai and H.J.Haubold, Unified fractional kinetic equation and a fractional diffusion equation, Astrophysics and Space Science, 290(2002), 299-310.
14. R.K.Saxena, A.M.Mathai and H.J.Haubold, solutions of the fractional reaction equation and the fractional diffusion equation, ArXiv:1001.2293v1,(2010).
15. R. K.Saxena and S.L.Kalla, On the solutions of certain fractional kinetic equations, Appl. Math. Comput., 199(2008), 504-511.
16. S.G. Samko, A. Kilbas and O. Marichev, Fractional Integrals and Derivatives. Theory and Applications. Gordon and Breach Sci. Publ., New York et alibi (1990).
17. V.B.L.Chaurasia and D. Kumar, On the solutions of Generalized Fractional Kinetic Equations, Adv. Studies Theor. Phys., (2010), 773-780.
18. V.B.L.Chaurasia and S.C. Pandey, On the new computable solutions of the generalized fractional kinetic equations involving the generalized function for the fractional calculus and related functions, Astrophysics and Space Science, 317(2008), 213-219.
19. V.B.L.Chaurasia and S.C. Pandey, Computable extensions of generalized fractional kinetic equations in astrophysics, Research in Astron. Astrophys. 2010, Vol. 10,No. 1, 22-32.
20. V.B.L.Chaurasia and S.C. Pandey, 2008, Ap & SS, 317, 213.
21. V.Kourganoff, Introduction to the physics of Stellar interiors, D. Reidel Publishing Company, Dordrecht,(1973).
22. W.G. Glockle and T.F.Nonnenmacher, Fractional integral operators and Fox function in the theory of viscoelasticity, Macromolecules, 24(1991), 6426-6434.

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