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- Validating Regression Models
- Solar Power Distillation
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On Validating Regression Models with Bootstraps and Data Splitting Techniques

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Abstract - Model validity is the stability and reasonableness of the regression coefficients, the plausibility and usability of the regression function and ability to generalize inference drawn from the regression analysis. Model validation is an important step in the modeling process and helps in assessing the reliability of models before they can be used in decision making. This research work therefore seeks to study regression model validation process by bootstrapping approach and data splitting techniques. We review regression model validation by comparing predictive index accuracy of data splitting techniques and residual resampling bootstraps. Various validation statistic such as the mean square error (MSE), Mallow's cp and R2 were used as criteria for selecting the best model and the best selection procedure for each data set. The study shows that bootstrap provides the most precise estimate of R2 which reduce the risk over fitted models than in data splitting techniques..

Keywords : Validation, bootstrap, Data splitting techniques, coefficient of determination, and stepwise regression .

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On Validating Regression Models with Bootstraps and Data Splitting Techniques

A.I Oredein[α], T.O Olatayo[Ω], A.C Loyinmi[β]

Abstract - Model validity is the stability and reasonableness of the regression coefficients, the plausibility and usability of the regression function and ability to generalize inference drawn from the regression analysis. Model validation is an important step in the modeling process and helps in assessing the reliability of models before they can be used in decision making. This research work therefore seeks to study regression model validation process by bootstrapping approach and data splitting techniques.

We review regression model validation by comparing predictive index accuracy of data splitting techniques and residual resampling bootstraps. Various validation statistic such as the mean square error (MSE), Mallow's cp and R^2 were used as criteria for selecting the best model and the best selection procedure for each data set. The study shows that bootstrap provides the most precise estimate of R^2 which reduce the risk over fitted models than in data splitting techniques.

Keywords : Validation, bootstrap, Data splitting techniques, coefficient of determination, and stepwise regression.

I. Introduction

Model selection and validation are critical in predicting a dependent variable given the independent variable. The correct selection of variables minimizes the model mismatch error while the selection of suitable model reduces the model estimation error. Models are validated to minimize the model prediction error. A more flexible model can better represent the data may also more easily lead the user astray by noise in the data. Determining the right form of the model in order to reduce model mismatch error is accomplished during model construction phase, whereas determining the correct model parameter can be achieved at the model selection and validation.

Once a regression model has been constructed, it is important to confirm the goodness of fit of the model and the statistic significance of the estimated parameters, commonly used are check of goodness of fit include analysis of the pattern of residuals and hypothesis testing, statistically significance checked by an f-test of the overall fit , followed by t-test of individual parameters interpretation of these diagnostic tests.

Validation is an essential part of model building, its application and levels of confidence in usage are highly important. It entails checking the R^2 statistic from the regression fit, carrying out a diagnostic of the residual either through exploratory statistic, checking the mean confirmatory statistics, checking the mean square error and also the mallow C_p statistic.

Model validity refers to stability and reasonableness of the regression coefficients, the plausibility and usability of the regression function and ability to generalize inferences drawn from the regression analysis. Validation is a useful and necessary part of the model building process. A good fit of a model to the data set is not an only goal of model validation but also to get a perfect fit by n-1 parameter to a data set with n cases. i.e. its predictive accuracy of the model is how the model validates a new dataset.

Model validation requires checking the model against independent data to see how well is predicts. Several researchers have work extensively on model validation using Jackniffing, Data splitting techniques, data resampling bootstraps regression without assuring fixed X or identically distributed errors. A drawn back of cross validation is the choice of the number of observation to hold out each fit. Also cross validation may not fully represent the variability of variable selection. The major disadvantages of data splitting techniques in model validation is that different investigators using the same data could split the data differently generate different models, hence obtain different validating result. Snee (1997) researched extensively on method of validation, Neumann et al (1977) and Shapiro (1984) have employed Monte Carlo testing to estimate artificial predictability in tropical Cyclane prediction models. Renduer and Run (1980) and Lanzante (1984) carried out set of Monte Carlo test to examine false predictability and the inflation of R^2 as a function of sample size, size of the predictor and number of predictors selected.

Model validation is an important step in the modeling process and helps in assessing the reliability of models before they can be used in decision making (Jannath and Tsuchido 1988). Hall and Wilson(1991),Davison and Hinkely(1997),Efron(1998)

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considered the application of bootstrap method to regression models from model based resampling approach.

These research works examine the validation of regression by comparing the predictive accuracy of data splitting techniques and the newly introduced bootstrapping approach by Efron (1993), to check the significance of each method in regression model validation. This work proposes a procedure for construction, selection and validation of regression models.

However, in regression model validation analysis, fewer reports have shown how bootstrap can be used in estimating the distribution of any validation statistic in random simulation with replicated runs. Unfortunately, this simplicity and versatile techniques of bootstrapping approach in validation seems not to be well known among simulation users and researchers. A fewer recent publication on bootstrapping in simulations are Cheng (2004), Deflandre and Kleijnen etal (2001) and Willemain etal (2003. This research work will extensively shows how bootstrap technique can be applied in checking the validity of a regression models using residual resampling. This work will rely less on theoretical sampling distribution like the normal, X^2, t and F, whose appropriateness for any given always rest on untestable assumptions. Instead we will construct appropriate sampling distribution empirically through bootstrap method using the data at hand.

II. Material and Methodology

Validating regression model was implemented in this work by bootstrapping approach and using the technique of data splitting. In data splitting, three different regression procedures were used to fit regression model to two different data sets. The data sets have many variables predicting the response variable. In data splitting, we split the data sets into two separate samples using one part for modeling and the other for testing the model. We also hope to see if the peculiarities of the original set will be seen in the split modeling set.

The first data set is a stock exchange data using Number of deals; 'Quality traded' and 'values of shares' as the independent variables predicting the 'All share index' per week. The observations were selected over 50 weeks.

The second data set pertains to different hourly readings of bytes received in telecommunication industry. 'Bytes transmitted', link utilization received, link utilization transmitted, 'Real time' and 'Best effort' were used as the independent variables predicting, 'Bytes received'. The observations were recorded over 130 hours. In data splitting techniques we employed the approach of stepwise regression procedures in selecting variables into a regression model. These include Forward Selection, Backwards Elimination and Best subset Regression. They add or remove variables one at a time until some stopping rule is satisfied.

The forward selection regression procedure sequentially adds variables to the model one at a time. It starts with an empty model and adds the variable that has the smallest p value usually less than 0.05 or 0.1 to the model.

Aside the p-value criterion, at any stage in the selection process, forward selection adds the variable that has the highest partial correlation, increases R^2 the most, and gives the largest absolute t of F statistic to the model. This procedure is a model reduction method. The Backward Elimination regression procedure starts with all the predictors in the model and sequentially deletes variables from the model. At any stage, in the selection process, it deletes the variables with the smallest absolute t or F-statistic, largest p-value and smallest R^2 ,backward Elimination procedure gives an adequate model since the procedure involves starting the model building with all the variables and deleting the variables that add nothing to the model.

Best subset regression examines all possible models and chooses the one with the most favorable value of some summary measure such as large adjusted, R^2 smallest Mallow's C_p and smallest standard error. All possible regression has a large advantage over stepwise procedures in that it can let the analyst see competing models, models that are almost as good as the best.

Data splitting has the advantage of allowing hypothesis tests to be confirmed in the test sample, however, the major disadvantages it has is that different investigators using the same data could split the data differently and generate different models, hence obtaining different validating results.

a) Bootstrap Estimate Of Standard Error

The bootstrap was introduced in 1979 as a computer based method for estimating the standard error of $\hat{\theta}$ .The bootstrap estimate of standard error requires no theoretical calculations, and is available no matter how mathematically complicated the estimator $\hat{\theta} = s(x)$. Bootstrap methods depend on the notion of a bootstrap sample . A bootstrap sample is defined to be a random sample of size n drawn from F , X^* is defined as

$$X^* = (x_1^*, x_2^* \ldots\ldots\ldots\ldots, x_n^*)$$

And

$$\hat{F} \rightarrow (x_1^*, x_2^* \ldots\ldots\ldots, x_n^*)$$

The star notation indicates that is not the actual data set *x*, but rather a randomized or resample version of X, in other word. Bootstrap sample can be defined as bootstrap data points x_1^*, x_2^* ...., x_n^* that are random sample of size n drawn with replacement from the population of n objects$(x_1, x_2 \ldots\ldots\ldots, x_n)$The bootstrap data set$(x_1^*, x_2^*, \ldots\ldots\ldots\ldots x_n^*)$ consists of the original data set $(x_1, x_2 \ldots\ldots\ldots, x_n)$ some appearing zero times, once, twice etc.

Corresponding to a bootstrap data set $X^*$is a bootstrap replication of $\hat{\theta}$,

$$\hat{\theta}^* = s(x^*)$$

s(x^*) is the mean of the bootstrap data set.

$$\bar{x}^* = \frac{\sum_{i=1}^{n} x_i^*}{n}$$

The bootstrap estimate of $S_{ef}(\hat{\theta})$i.e.the standard error of a statistical $\hat{\theta}$, is a plug-in estimate that uses the empirical distribution function $\hat{F}$ in place of the unknown distribution *F*. Specifically the bootstrap estimate of $S_{ef}(\hat{\theta}^*)$. is defined by

$$S_{ef}(\hat{\theta}^*).$$

In other words, the bootstrap estimate of $S_{ef}(\hat{\theta})$ is the standard error of $\hat{\theta}$ for the data sets of size n randomly sampled from F.

b) The Bootstrap Algorithm For Estimating Standard Errors

1. Select *B* independent bootstrap samples (X^{*1}, X^{*2}. ....,X^{*B}) each consisting of n data values drawn with replacement from $x = (x_1, x_2 \ldots, x_n)$. (*B* is the number of bootstrap samples used).
2. Evaluate the bootstrap replication corresponding to each bootstrap sample:

   $$\hat{\theta}(b) = S(X^{*b}) \text{ where b = 1, 2..., } B.$$

3. Estimate the standard error $S_{ef}(\hat{\theta})$ by the sample standard deviation of the *B* replicatication where

$$\widehat{se}_B = \frac{[\sum \theta^*(b) - \theta^*(.)^2]}{(B-1)^{1/2}}$$

where

$$\hat{\theta}^*(.) = \sum_{b=1}^{B} \frac{\theta^*(b)}{B}$$

$$\widehat{bias}_B = \hat{\theta}(.) - t(\widehat{F})$$

The bootstrap algorithm above works by drawing many independent bootstrap samples, evaluating the corresponding bootstrap replications and estimating the standard error of $\hat{\theta}$ by the empirical standard deviation of the replications.

c) Bootstrap Estimate of Bias

F is an unknown probability distribution, given data $x = (x_1, x_2 \ldots, x_n)$ by random sampling

$$F \rightarrow x$$

To estimate a real value parameter

$$\theta = (F)$$

Let statistic $\hat{\theta}$ = s(x) to be an estimator, using plug-in-estimate

$$\hat{\theta} = t(\hat{F})$$

The bias of $\hat{\theta} = s(x)$ as an estimate of θ is defined to be the difference between the expectation of $\hat{\theta}$ and the value of the parameter $\hat{\theta}$.

$$bias_f = bias_f(\hat{\theta}, \theta) = E_F[s(x)] - t(F)$$

The bootstrap estimate of bias is defined to be the estimate $bias_F$

$$bias_{\hat{F}} = E_{\hat{F}}[s(X^*)] - t(\hat{F})$$

d) Validation Using Bootstrap

Efron and Gong, Efron and Tibshirani,(1993) describe several bootstrapping procedures for obtaining nearly unbiased estimates of future model performance without holding back data when making the final state of model parameters. With the "simple bootstrap", one repeatedly fits the model in a bootstrap sample and evaluates the performance of the model on the original data.

A simple regression bootstrap called residual resampling was achieved through the following algorithm with the aid of computer

(i) Perform regression with the original sample; calculate predicted values ($\hat{Y}$) and residuals(e)
(ii) Randomly resample the residuals, but leave X and ($\hat{Y}$) unchanged.
(iii) Construct new Y^* values by adding the original predicted values to the bootstrap residuals i.e $Y^* = \hat{Y} + e^*$
(iv) Regress Y* on the original X variable(s).
(v) Repeat step (ii) – (iv) several times.
(vi) Estimate parameter of interest in validation of regression models such as R^2 and MSE.

The ability to study the arbitrariness of how a stepwise variable selection algorithm selects "important" factors is a major benefit of bootstrapping.

III. Results

Summary of result obtained in data splitting techniques using validating set of stock exchange data

	R^2	ADJ R^2	MSE
LSE	0.617	0.625	0.036
FORWARD	0.3955	0.3952	0.008
BACKWARD	0.3955	0.3922	0.008
BEST SUBSET	0.624	0.630	0.007

We went further in comparing the MSE obtained from the modeling set and validation of stock exchange data.

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The table below shows the summary of the MSE obtained

Set	No. of Obs.	LSQ.	FWD	BKWD	Best Subset
Modeling	30	0.036	0.036	0.0362	0.0356
Validating	20	0.007	0.008	0.008	0.007

Hence the validated model is

$$Y \quad -1.417 - 0.023x_1 - 0.001x_2 + 0.176x_3$$

$$R^2 \quad 0.4985$$

$$R^2\,Adj \quad 0.4960$$

$$MSE \quad 0.007$$

$$SE \quad 0.08026$$

a) Analysis of Telecommunication Data

Summary of the result obtained in data splitting techniques using validating set of telecommunication data

	R^2	ADJ R^2	MSE
LSE	0.3124	0.2846	24.42
FORWARD	0.403	0.486	29.28
BACKWARD	0.402	0.482	29.28
BEST SUBSET	0.426	0.415	24.4

Summary of the MSE for Telecommunication Data

Set	No. of Obs.	LSQ.	FWD	BKWD	Best Subset
Modeling	110	163.18	157.5	157.52	163.07
Validating	20	24.42	29.28	29.28	24.4

MSE's from the validating set are smaller than those from the modeling set. This is not far from our expectation as this can be attributed to the distance of the observation of the validating set from the modeling Hence the validated model of telecommunication data is

$$Y \quad 3.67 + 0.87x_1 + 0.002x_4$$

$$R^2 \quad 0.426$$

$$Adj\ R^2 \quad 0.415$$

$$Cp \quad 4.78$$

$$S\,E. \quad 4.93$$

$$MSE \quad 29.816$$

b) Statistical Analysis of Bootsrap Approach In Validating Regression Models

The validating model obtained for stock exchange data using bootstrap residual resampling is

$Y = -34.4188 - 0.0793X_1 + 0{,}2684X_2 + 0.6504X_3$

$R^2 = 0.9854$, Adj $R^2 = 0.9848$, S.E = 0.6424, MSE =0.8015, N=50

Also s

$Y = 4.5094 + 0.6077X_1 - 0.0380X_2 + 0.0169X_3 + 0.0829X_4 - 0.0168X_5$ with $R^2 = 0.9899$, Adj $R^2 = 0.9895$, S.E =0.8826, MSE =0.7790, N=130 was obtained as the validating model for telecommunication data set using bootstrap residual resampling procedures. The above models were chosen because they generated highest and lowest value of R^2 and MSE respectively, as a criterion of model validation. Summary of validating statistics in validated models using data splitting and bootstraps

VALIDATING TECHNIQUES		R^2	Adj R^2	MSE
Data Splitting	Stock	0.4985	0.4960	0.007
	Telecommu-nication	0.426	0.4150	29.816
Bootstra-pping				
	Stock	0.9854	0.9848	0.8015
	Telecommu-nication	0.9899	0.9895	0.7790

IV. Discussion of Results

The bootstrap models were obtained from 100 bootstrap replication. The bootstrap Y values were computed by adding resample residuals onto the ordinary least squares regression fit. The B=100 bootstrap samples were generated randomly to reflect the exact behaviour of bootstrap estimations.

Residual resampling assumes fixed X values and independently and identically distributed errors (but not necessarily normal) that i.e. it assumes that the residual found for the i^{th} case could equally well have occurred with the j^{th} case instead, residual resampling randomly reassigns the original-sample residuals to new case. The n sets of X values from the original sample remain unchanged in each bootstrap sample. Using a Monte Carlo algorithm, B bootstrap sample are generated by drawing from the empirical distribution with replacement. For each boot sample, the statistics of interest such as R^2, MSE, and standard error of estimate was calculated.

Two set of data samples were considered to check how bootstrap approach and data splitting techniques work on small and larger data set in validating regression models. The number of bootstrap replications B depends on the application and size of sample and computer availability.

From above results, it was discover that the larger the bootstrap replicate, the higher and stable R^2 is i.e. it gives a better validity model. It was also observed that in validating regression model bootstrap approach gives a better and higher R^2 in each replicates than the value of R^2 in the validated models of data splitting techniques.

advantages of allowing hypothesis tests to be confirmed in the test samples, but the major disadvantages of this method compared to bootstrap approach in model validation is that different investigators using the same data could split the data differently and senate difference models, hence obtain different validating results. It was observed from the above, comparing the data set in Telecommunication and stock exchange data, bootstrap give a better R^2 both in small and large sample data sets compared to stepwise regression i.e. the risk of over fitting was reduced. Also R^2 varies inversely with SSE, and it also increases if and only if MSE decreases, R^2 does not take account of number of parameters in the regression model.

This research work has demonstrated the use of validating regression models by data splitting techniques and bootstrap. In data splitting techniques, the data were split according to time into two samples with a view of using the second samples to validate the predictions made by the first sample.

Three regression procedures were used to build models on each data set for comparison purposes and to test their predictive abilities on each unique data set. Our criterion for test was the validation MSE(s) which was obtained by predicting each dependent variable value in the validation sample and averting the squared errors.

In bootstrapping approach, validation of the regression models was achieved by adding the resample residuals unto the least square regression fit, holding the regression design fixed. The least squares estimate from each bootstrap samples was obtained and the validation statistic of interest such as R^2 than stepwise regression in data splitting techniques.

Bootstrapping seems to work better than stepwise regression in validating regression models. In the simplest form of bootstrapping, instead of repeatedly analyzing subset of the data, analyst repeatedly analyse subsamples of the data and each subsamples is a random sample with replacement from the full sample. Bootstrapping allows us to gather many alternative version of the single statistic that would ordinarily be calculated from one sample and compute the statistical interest for each of the data sets.

V. Conclusion

Bootstrapping the model fitting process is a much better way to get unbiased estimates of model performance without sacrificing sample size, here we are validating the full n- subject model.

The most important advantage of bootstrapping in validating regression models over data splitting techniques are to need smaller sample than data splitting techniques and its practical performance is frequently much better in the sense that the risk of over fitted models are reduced as it gives a better and stable value of R^2.

Bootstrap method in regression model validation accomplish the goal of constructing appropriate sampling distributions empirically using the data at hand instead of statistician relying on theoretical sampling distributions like the normal, t and f where appropriateness for any given problem always rest on untestable assumptions.

In a nutshell in validating regression models, bootstrapping procedures are useful than data splitting in the following situation:

(i) When the theoretical distribution of a statistic is complicated or unknown.
(ii) When the sample size is insufficient for straightforward statistical inference.
(iii) When power calculations have to be performed and a small pilot sample is available.

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Mathematical Morphology and Fractal Geometry

By Ramkumar P.B , K.V Pramod

Abstract - Mathematical morphology examines the geometrical structure of an image by probing it with small patterns, called 'structuring elements', of varying size and shape. This procedure results in nonlinear image operators which are suitable for exploring geometrical and topological structures. A series of such operators is applied to an image in order to make certain features more clear. Scale-space is an accepted and often used formalism in image processing and computer vision. Today, this formalism is so important because it makes the choice at what scale visual observations are to be made explicit. Fractal Geometry is a very new branch in Mathematics. An attempt to link Morphological operators and Fractals is made in this paper.

Keywords : *Dilation , Erosion, Morphological Space, Fractal .*

GJSFR Classification: *MSC: 47A20*

MATHEMATICAL MORPHOLOGY AND FRACTAL GEOMETRY

Strictly as per the compliance and regulations of:

Mathematical Morphology and Fractal Geometry

Ramkumar P.B[α], K.V Pramod[Ω]

Abstract - Mathematical morphology examines the geometrical structure of an image by probing it with small patterns, called 'structuring elements', of varying size and shape. This procedure results in nonlinear image operators which are suitable for exploring geometrical and topological structures. A series of such operators is applied to an image in order to make certain features more clear. Scale-space is an accepted and often used formalism in image processing and computer vision. Today, this formalism is so important because it makes the choice at what scale visual observations are to be made explicit. Fractal Geometry is a very new branch in Mathematics. An attempt to link Morphological operators and Fractals is made in this paper.

Keywords : Dilation , Erosion, Morphological Space, Fractal.

I. Introduction

Mathematical Morphology is a tool for extracting image components that are useful for representation and description. It provides a quantitative description of geometrical structures. Morphology is useful to provide boundaries of objects, their skeletons, and their convex hulls. It is also useful for many pre- and post-processing techniques, especially in edge thinning and pruning.

Most morphological operations are based on simple expanding and shrinking operations. Morphological operations preserve the main geometric structures of the object. Only features `smaller than' the structuring element are affected by transformations. All other features at `larger scales' are not degraded. (This is not the case with linear transformations, such as convolution).

The primary application of morphology occurs in binary images, though it is also used on grey level images. It can also be useful on range images. (A range image is one where grey levels represent the distance from the sensor to the objects in the scene rather than the intensity of light reflected from them).

a) Preliminaries

i. *Notation and Image Definitions*

Types of Images

An image is a mapping denoted as I, from a set, N_P, of pixel coordinates to a set, M, of values such that for every coordinate vector, $\mathbf{p} = (p_1, p_2)$ in N_P, there is a value I(**p**) drawn from M. N_P is also called the image plane.[1]

Under the above defined mapping a real image maps an n-dimensional Euclidean vector space into the real numbers. Pixel coordinates and pixel values are real.

A discrete image maps an n-dimensional grid of points into the set of real numbers. Coordinates are n-tuples of integers, pixel values are real. A digital image maps an n-dimensional grid into a finite set of integers. Pixel coordinates and pixel values are integers. A binary image has only 2 values. That is, M= $\{m_{fg}, m_{bg}\}$, where m_{fg}, is called the foreground value and m_{bg} is called the background value.

The foreground value is $m_{fg} = 0$, and the background is $m_{bg} = -\infty$. Other possibilities are $\{m_{fg}, m_{bg}\} = \{0,\infty\}, \{0,1\}, \{1,0\}, \{0,255\}$, and $\{255,0\}$.

b) Dilation and Erosion

Morphology uses 'Set Theory' as the foundation for many functions [1]. The simplest functions to implement are 'Dilation' and 'Erosion'

i. *Definition :* Dilation of the object *A* by the structuring element *B* is given by

$$A \oplus B = \{x : \hat{B}_x \cap A \neq \emptyset\}.$$

Usually A will be the signal or image being operated on A and B will be the Structuring Element'

ii. *Definition Erosion*

The opposite of dilation is known as erosion. Erosion of the object *A* by a structuring element *B* is given by

$$A \ominus B = \{x : B_x \subseteq A\}.$$

Erosion of A by B is the set of points x such that B translated by x is contained in A.

iii. *Definition Opening*

The opening of *A* by *B*, denoted by $A \circ B$, is given by the erosion by *B*, followed by the dilation by *B*, that is

iv. Closing

The opposite of opening is 'Closing' defined by $A \bullet B = (A \oplus B) \ominus B$.

Closing is the dual operation of opening and is denoted by $A \bullet B$. It is produced by the dilation of A by B, followed by the erosion by B :

c) Morphological Operators defined on a Lattice

i. *Definition Dilation*

Let $(L, \leq)$ be a complete lattice, with infimum and minimum symbolized by $\wedge$ and $\vee$, respectively. [1],[2].[11]

A dilation is any operator $\delta : L \rightarrow L$ that distributes over the supremum and preserves the least element. $\bigvee_i \delta(X_i) = \delta\left(\bigvee_i X_i\right)$, $\delta(\emptyset) = \emptyset$.

ii. *Definition Erosion*

An erosion is any operator $\varepsilon : L \rightarrow L$ that distributes over the infimum $\bigwedge_i \varepsilon(X_i) = \varepsilon\left(\bigwedge_i X_i\right)$, $\varepsilon(U) = U$.

iii. *Galois connections*

Dilations and erosions form Galois connections. That is, for all dilation δ there is one and only one erosion ε that satisfies $X \leq \varepsilon(Y) \Leftrightarrow \delta(X) \leq Y$ for all $X, Y \in L$.

Similarly, for all erosion there is one and only one dilation satisfying the above connection.
Furthermore, if two operators satisfy the connection, then δ must be a dilation, and ε an erosion.

iv. *Definition Adjunctions*

Pairs of erosions and dilations satisfying the above connection are called "adjunctions", and the erosion is said to be the adjoint erosion of the dilation, and vice-versa.

v. *Opening and Closing*

For all adjunction (ε, δ), the morphological opening $\gamma : L \rightarrow L$ and morphological closing $\phi : L \rightarrow L$ are defined as follows:[2]

$\gamma = \delta\varepsilon$, and $\phi = \varepsilon\delta$.

The morphological opening and closing are particular cases of algebraic opening (or simply opening) and algebraic closing (or simply closing). Algebraic openings are operators in *L* that are idempotent, increasing, and anti-extensive. Algebraic closings are operators in *L* that are idempotent, increasing, and extensive.

II. MORPHOLOGICAL OPERATORS DEFINED AS AN ALGEBRAIC STRUCTURE

a) Definition : Morphogenetic field

Let X $\neq \phi$ and W $\subseteq P(X)$ such that i) ϕ, X ∈ W , ii) If B ∈ W then its complement $\overline{B}$ ∈ W , iii) If$_i$B ∈ W is a sequence of signals defined in X, then $\bigcup_{n\ 1}^{\infty} Bi$ ∈ W.

Let A= $\{\phi : W \rightarrow U \,/\, \phi(\cup A_i) = \vee\phi(A_i) \,\&\, \phi(\wedge A_i) = \wedge\phi(A_i)\}$. Then W_U is called Morphogenetic field [16] where the family W_u is the set of all image signals defined on the continuous or discrete image Plane[11],[12] X and taking values in a set U .The pair (W_u, A) is called an operator space where A is the collection of operators defined on X.

i. *Definition : Morphological space*

The triplet (X, W_u, A) consisting of a set X, a morphogenetic field W_u and an operator A(or collection of operators) defined on X is called a Morphological space [16].

Note : If X = Z^2 then it is called Discrete Morphological space

ii. *Definition : Concave morphological space*

Let (X, W_u, A) be a morphological space and (W_U, A) be an operator space in (X, W_u, A).

If X is a class of concave functions [14],[15] then (X, W_u, A) is called concave morphological space. If X is a class of convex functions then (X, W_u, A) is called convex morphological space.[16].

iii. *Definition : Sensitive operator*

Let (X, W_u, A) be a Morphological space[16]. Let B_1 be the neighbourhood of x∈ X i.e., N(x)=B_1⊆ X.Then ∀$x\epsilon X$, x∈B_1, y∈B_1, ∃ B_2 such that B_1⊆ B_2⊆X and α^n(x) ∈B2 andα^n(y) ∉ $\overline{B2}$, nϵZ^+. Then $\alpha \in A$ is called a sensitive operator and the operator space [16] (W_u, A) is called a sensitive space.

Example : Dilation is sensitive. Constant signals f(x)= c are not sensitive.

iv. *Proposition*

Let N: X→P(X) be defined such that $N(x)=\{y\in X / x\rho y\}$ where ρ is the relation, dilation defined between x and y $\forall x, y \in X$. i.e., $x\rho y \Rightarrow y=\delta(x)$ where δ is the dilation [8],[9],[10],[11] and for $\alpha \in A$, α $=\delta$ $\Rightarrow$ δ^n(x) ∈ B_2 =N(x), $\delta^n(y) \notin \overline{B}_2$ $n\in Z^+$, $x, y \in B_1$ 1. Thus δ is sensitive.

v. *Definition : Perfect Set*

Let F⊆ X . Define $S(F)=\{\alpha / \alpha \in (W_u, A)$ *besensitive* $[6] by \alpha \in F\}$. If S(F) ≠ ϕ and (X, W_u, A) is a convex morphological space [16] then F is called Perfect.

vi. *Definition : Stirring Operator*

(Let (X, W_u, A) be a Morphological space and let U, V⊆ X be two sets. Let α ∈A. Then α is called stirring [6] if given any neighbourhoods N_1 and N_2,of U and V, $\forall x \in U$, y∈V in X, $\exists k \in Z^+$ such that

$$\alpha^k(N_1)\cap\alpha^k(N_2) \neq \phi .$$

α is strongly stirring if $\exists k \in Z^+$ and a set G in X such that $G \subseteq \alpha^k(N_1)\cap\alpha^k(N_2)$.

vii. *Definition : Partial Similarity*

Let (X, Wu, A) be a Morphological space.

Let K ⊆ X. K is called Partial self similar or α *similar* if $\exists K_1, K_2, ... K_t$ such that $K=\bigcup_{i=1}^{t} K_i$ and for each K_i,∃ contraction maps $\varphi(i,j,k)$,for i=1….. t,r=1 t ,j=1,….t and k=1. $w(i,j)$ with $w(i,j)$>0 such that K_i.

viii. *Definition : Scale space*

Let S a scaling on an image space L. The family {T(t)},t> 0 of operators on L is called an (S,+) scale – space [2],[5] if T(t).T(s)=T(t+ s) ,s ,t > 0 and T(t).S(t)=S(t).T(1) ,t > 0

ix. *Proposition*

The erosion $\varepsilon(f) = f \ominus b$ with a convex structuring element b induces an ($S^{1/2}$, + ½) scale space and f is ½ similar.

x. *Definition : Anamorphic Scaling*

A family $S=\{S(t)/t>0\}$ of operators on L is called a scaling if S(1) = identity element.

S(t)S(s)=S(ts) for s,t > 0.Two scalings S and $\hat{S}$ are said to be anamorphic [2].[5]if ∃ an increasing bijection γ on T such that $S(\gamma(t))=S'(t)$ $\forall t \in T$ Also .

xi. *Proposition*

Anamorphic scaling are α – similar

xii. *Proposition*

The erosion $\varepsilon(f) = f \ominus b$ with b∈SP(k) for K>1 induces a $(S^\alpha ,+ v)$ scale space if $v=1-\alpha+K^*(2\alpha-1)$ which implies that f is α - similar , b is called the structuring function.

xiii. *Proposition*

Let (X, Wu, A) be a Morphological space. Let f be α similar. Then $\exists \psi \in A$ such that $\psi^\alpha(f)=\alpha\psi(f)$.

xiv. *Definition*

The cross – section $X_t(f)$ [1],[2],[5] of f at level t is the set obtained by thresholding f at level t. $X_t(f)=\{x/f(x)\geq t\}$, where $-\infty<t<\infty$

xv. *Proposition*

If f is a fractal then $\exists i\in I$ such that $\forall$ i, $X_{t_i}(f)$ are self similar and $X=\bigcup_{\forall i} X_t(f)$.

III. Morphological Fractals

a) Surface area of a compact set

Morphological operators extracts the impact of a particular shape on images [13] using structuring elements. It encodes the primitive shape information. The transformed image is obtained by using a structuring element .Therefore it can be treated as a function of the structuring element.

Dilation of a set X [5],[17] with a structuring element Y is given by the expression $X\oplus Y=\{x/ Y^x\cup X\neq\phi\}$, Y^x denotes the translation of a set Y with x.

Dilation operation can be used to define the surface area of a compact set.

Surface area [19]of a compact set X with respect to a compact convex structuring element Y which is symmetrical with respect to the origin is given by

$$S(X,Y)\quad \lim_{\rho\to 0}\frac{V(\partial X\oplus\rho Y)}{2\rho}$$

Where ∂X is the boundary of set X and $\oplus$ denotes the dilation of the boundary of X by the structuring element Y and ρ is a scaling factor. Volume of a set X is denoted by $V(X)$.

b) Particular Case – Fractals

If the object is regular, the surface area will not change with ρ_i .For a fractal object, S is increases exponentially with decreasing ρ .

c) Definition : Fractal Identification

An image is segmented into the regions $R_1,R_2,........R_n$ if $\exists$ a relation ρ on Regions such that $R_i\rho R_j$ if $R_i\cap R_j=\phi$ and $\bigcup R_i=X$.

Also Image Property of $R_i\cap R_j=\phi$,if $i\neq j$ .If *Image Property of* R_i = *Property of* R_j then each R_i is a fractal.

Note : Converse is not always true. For every Fractal, it is not necessary that Image *Property of* R_i = *Property of* R_j

d) Definition : Class of Fractal Regions- X(k, t)

Let $F(p)=\prod_{i\ 1}^{m} f_i(p_i)$ ....(1)where ($p=(p_1,p_2,...p_m)$) and f_i, $i=1,2,....m$ is a set of completely defined functions and F is uniquely defined on R.

Define G(F) as $p\in G(F)$ iff F(p)=1.i.e F is a characteristic function of $G(F)$.The set of graphs which can be generated from (1) by allowing each f_i to vary over all possible logic functions is defined as Class of Fractal Graphs, [18] denoted by $G(k,t)$ where the vectors ($k=(k_1,k_2,...k_m)$) and t= $(t_1,t_2,...t_m)$).

e) Definition : Compression

Let (X, Wu, A) be a Morphological space. Let R =X be a rectangular plane and is divided into $2^{n_1}\times 2^{n_2}$ grids represented by R($2^{n_1}\times 2^{n_2}$).X1 is a region on R and χ:R→{0,1} is its characteristic function.

Given two integers r1 and r2,0< r1< n1 ,0< r2<n2,construct a rectangular plane R' regarding its left lower corner as an origin. A function χ' :R' →{0,1} is defined as follows.

∀p'=(x',y')ε R' ,if ∃ integers α_1 & α_2 where0<α1< 2^{r1}, 0<α2< 2^{r2} such that χ (p)=1where p=(x,y) εR and then χ'(p')=1,otherwise χ'(p')=0.

Region X' with χ' as its characteristic function is called a compressed region [18]of X based on (r1,r2) and a compressed region of X is denoted by X' = Comp(r1,r2)(X) and χ'(p') ∀p', p'εR' is given below. χ'(p')=1 ,if there exist 'a' in R such that χ (p)=1, χ'(p') =0 otherwise.

f) Definition : Similarity

Let (X, Wu, A) be a Morphological space. Assume that X1 and X2are two regions on a plane. If ∃ a compression transformation in A, Comp(r1,r2) such that X1is compressed into X3=Comp(r1,r2) (X1) and X2 can coincide with X3 through translating X2,then X1 and X2 are similar, denoted by X1≅X2.

Note: Compression is nonreversible. Therefore Similarity is an asymmetric relation.

g) Definition : Self Similarity

Let (X, Wu, A) be a Morphological space. If ∃ a partition X1,X2,…..X r of X and Xi is a proper sub region of X such that X and each non empty sub region Xi of X are similar.

For any two non empty sub regions Xi and X j ,X j and Xi are similar or Xi and X j are similar,[1],[4]
Then X is said to be a self similar region.

h) The Order of Self Similarity

Let (X, Wu, A) be a Morphological space. X is a self similar region, if any proper sub region among all partitions of X which satisfy the definition of self similarity [18]is not a self similar region, then the order of similarity of X is 1.

If the maximal order of sub regions among all partitions of X which satisfy the definition of self similarity is m, then the order of self –similarity of X is m+1.

i) Definition: Mutual Similarity

Let (X, Wu, A) be a Morphological space. Let X be partitioned into X1,X2,…. of X and Xi is a proper sub region of X such that for any two non empty sub regions Xi and X j,Xi and Xj are similar or Xj and Xi are similar, then X is a mutually similar region[18].

j) Definition: Fractal Regions X(k,t)

Let (X, Wu, A) be a Morphological space. If set X can be partitioned into several sub regions and the sub regions are mutually similar and each sub region can further be partitioned into mutually similar sub regions etc, then G is said to be a mutual- similar region.

Note: If X can be identified as a representation in terms of graphs then X(k, t) is a mutual similar graph.

IV. Conclusion

Morphological operators[3],[4],[9] are very useful for gathering informations from images. Most of the operators can also be applied in Medical Imaging [7]. Some results are given in generalized structure [16].The regions can also be taken as graph points. So we can also apply the results from the already developed Graph Theory. We can also reconstruct a fractal image using Dilation and a fractal structuring element. Morphological fractals are useful in Medical imaging and other areas. It is possible to construct soft wares for this particular job.

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Integral Formulae for Certain Product of Special Function Generalized Fractional Calculus

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Abstract - Fractional calculus and special functions have contributed a lot to the science and engineering. In view of great importance and usefulness of fractional calculus operators in different directions, we derive the images of the product of certain special function under the multiple Erdélyi - Kober operator due to Galué et al. The result obtained are general in character and includes, as special cases, the result for Riemann -Liouville operator, Erdélyi - Kober operator and Saigo operator etc. involving the product of certain special function of general argument.

Keywords : *Multivariable H-function, Erdélyi-Kober operator, Saigo operator, Fractional calclulus, Jacobi polynomials, Series representation of the H-function.*

GJSFR-F Classification: *MSC: 26A33, 33C05, 33C40*

INTEGRAL FORMULAE FOR CERTAIN PRODUCT OF SPECIAL FUNCTION GENERALIZED FRACTIONAL CALCULUS

Strictly as per the compliance and regulations of:

Integral Formulae for Certain Product of Special Function Generalized Fractional Calculus

V.B.L. Chaurasia[α] , Vinod Gill [Ω]

Abstract - Fractional calculus and special functions have contributed a lot to the science and engineering. In view of great importance and usefulness of fractional calculus operators in different directions, we derive the images of the product of certain special function under the multiple Erdélyi - Kober operator due to Galué et al. The result obtained are general in character and includes, as special cases, the result for Riemann -Liouville operator, Erdélyi - Kober operator and Saigo operator etc. involving the product of certain special function of general argument.

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I. Introduction

Fractional calculus is a field of applied mathematics that deals with derivatives and integrals of arbitrary orders. During the last three decades, fractional calculus has been applied to almost every field of science and engineering.

The multiple Erdélyi-Kober operator of Weyl type, introduced by Galué et al. [3] is defined as

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w,(\lambda_w),r} f(x)=\begin{cases}\int_1^\infty H^{r,0}_{r,r}\left[\frac{1}{y}\left|\begin{matrix}(\eta_w+\zeta_w+1/\tau_w.1/\tau_w)_1^r\\(\eta_w+1/\lambda_w,1/\lambda_w)_1^r\end{matrix}\right.\right]f(x,y)\,dy\,,\ \text{if}\ \sum_1^r \zeta_w>0\\ f(x),\ \text{If}\ \zeta_w=0,\lambda_w=\tau_w,\ w=1,2,...,r,\end{cases} \quad ...(1.1)$$

Where $\sum_{w=1}^{r}\frac{1}{\lambda_w}\geq\sum_{w=1}^{r}\frac{1}{\tau_w}$ and $f(x)\ \in C^*_\beta$.

The class C^*_β is defined in the form [3, p.56]

$$C^*_\beta=\{f(x)=x^q\ \breve{f}(x);\ q<\beta^*,\breve{f}\in C\ (0,\infty),|\breve{f}(x)|<A_{\breve{f}}\} \quad ...(1.2)$$

And $\beta^*\leq\max\ (\lambda_w,\eta_w)$.

Galué et al. [3, p.56] represented that

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w),(\lambda_w),r}x^\rho=\prod_{w=1}^{r}\frac{\Gamma(\eta_w-\rho/\lambda_w)}{\Gamma(\eta_w+\zeta_w-\rho/\lambda_w)}x^\rho. \quad ...(1.3)$$

In the form of Pochhammer symbol $(a)_n$, defined as

$$(a)_n=\frac{\Gamma(a+n)}{\Gamma(a)}=\begin{cases}1 & \text{if}\ n=0\\ a\,(a+1)...(a+n-1),\forall n\in N\end{cases}. \quad ...(1.4)$$

We can write

$$(1-x)^{-\alpha}=\sum_{n=0}^{\infty}\frac{(\alpha)_n}{n!}x^n. \quad (1.5)$$

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A general class of multivariable polynomials of Srivastava and Grag [14] is defined and represented in the following form

$$S_n^{w_1,\dots,w_s}[x_1,\dots,x_s]=\sum_{k_1,\dots,k_s=0}^{w_1k_1+\dots+w_sk_s\le n}(-n)_{w_1k_1+\dots+w_sk_s}\,A(n:k_1,\dots,k_s)\frac{x_1^{k_1}}{k_1!},\dots,\frac{x^{k_s}}{k_s!}, \quad (1.6)$$

$n, w_1,\dots,w_s \in N_0 \quad \{0,1,2,\dots\}$ and the coefficients A(n : $k_1,\dots,k_s$), ($k_j \in N_0$; j = 1,…,s) are arbitrary constants, real or complex. For s = 1, the polynomial (1.6) reduces to a general class of polynomials due to Srivastava [12].

$$S_n^w[x]=\sum_{k=0}^{[n/w]}\frac{(-n)_{wk}}{k!}A_{n,k}\,x^k,\ n=0,1,2,\dots \quad (1.7)$$

where w is an arbitrary positive integer, the coefficients $A_{n,k}(n,k\in N_0)$ are arbitrary constants, real are arbitrary constants, real or complex. The following are the interesting special cases of this polynomials [15].

(i) Since

$$H_n(x)=\sum_{k=0}^{[n/2]}\frac{(-1)^k\ n!}{k!\,(n-2k)!}(2x)^{n-2k} \quad (1.8)$$

define Hermite polynomials therefore in this case, if we take

$$w=2,\ A_{n,k}=(-1)^k,\ S_n^2(x)\to x^{n/2}H_n(1/2\sqrt{x}). \quad (1.9)$$

(ii) On setting $w=1,\ A_{n,k}=\binom{n+\alpha}{n}\dfrac{(\alpha+\beta+n+1)_k}{(\alpha+1)_k},\ S_n^1$ reduces to the Jacobi polynomials $P_n^{(\alpha,\beta)}$ $(1-2x),$ defined by Szegö [16, p.68, eqn. (4.3.2)].

$$P_n^{(\alpha,\beta)}(x)=\sum_{k=0}^{n}\binom{n+\alpha}{n-k}\binom{n+\beta}{k}\left(\frac{x-1}{2}\right)^k\left(\frac{x+1}{2}\right)^{n-k} \quad (1.10)$$

$$.\binom{n+\alpha}{n}{}_2F_1\left[-n,\alpha+\beta+n+1;\alpha+1;\frac{1-x}{2}\right].$$

The following series representation of the H-function of several complex variables has been recently studied and given by Olkha and Chaurasia [6, p.39], as follows:

$$H[z_1,\dots,z_r]=\sum_{m_i=1}^{u^{(i)}}\sum_{n_i=0}^{\infty}\frac{\prod_{j=1}^{\lambda}\Gamma(1-a_j+\sum_{i=1}^{r}\theta_j^{(i)}U_i)}{\prod_{j=\lambda+1}^{A}\Gamma(a_j-\sum_{i=1}^{r}\theta_j^{(i)}U_i)\prod_{j=1}^{C}\Gamma(1-c_j+\sum_{i=1}^{r}\psi_j^{(i)}U_i)}$$

$$.\frac{\prod_{j=1}^{u^{(i)}}\Gamma(d_j^{(i)}-\delta_j^{(i)}U_i)\prod_{j=1}^{v^{(i)}}\Gamma(1-b_j^{(i)}+\phi_j^{(i)}U_i)}{\prod_{j=u^{(i)}+1}^{D^{(i)}}\Gamma(1-d_j^{(i)}+\delta_j^{(i)}U_i)\prod_{j=v^{(i)}+1}^{B^{(i)}}\Gamma(b_j^{(i)}-\phi_j^{(i)}U_i)}\frac{\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}}{\prod_{i=1}^{r}(\delta_{m_i}^{(i)}n_i!)}, \quad \dots(1.11)$$

where $U_i = \frac{d^{(i)}_{m_i} + n_i}{\delta^{(i)}_{m_i}}$, $\sum_{m_i=1}^{u^{(i)}}$ and $\sum_{n_i=0}^{\infty}$ denote the multiple sums $\sum_{m_1=1}^{u'} \dots \sum_{m_r=1}^{u^{(r)}}$ $\sum_{n_1=0}^{\infty} \dots \sum_{n_r=0}^{\infty}$ respectively and ∀ i ∊ (1,…,r).

The multivariable H-function due to Srivastava and Panda [13] will be required in the proof.

II. The Main Results

Images under Multiple Erdélyi-Kober operator Letting

$$f(x) = x^{\rho}(x^{\mu}+c^{\mu})^{-\sigma} S_m^{w_1,\dots,w_s}[x^{\ell_1}(x^{\mu}+c^{\mu})^{-v_1},\dots,x^{\ell_s}(x^{\mu}+c^{\mu})^{-v_s}]$$

$$.\ H[z_1 x^{-h_1}(x^{\mu}+c^{\mu})^{-\rho_1},\dots,z_N x^{-h_N}(x^{\mu}+c^{\mu})^{-\rho_N}]$$

$$.\ H[y_1 x^{t_1}(x^{\mu}+c^{\mu})^{-\eta_1},\dots,y_r x^{t_r}(x^{\mu}+c^{\mu})^{-\eta_r}] \qquad (2.1)$$

With

$$\operatorname{Re}[-\alpha^{*} + \min_{1\le k\le r}(\lambda_k \gamma_k)] > 0, \sum_{i=1}^{r}\frac{1}{\lambda_i} \ge \sum_{j=1}^{r}\frac{1}{\tau_i} \text{ and}$$

$\rho,\sigma,h_i,\rho_i\ (i\quad 1,\dots,N),\ell_i,v_i\ (i\quad 1,\dots,s),\eta_i\ (i\quad 1,\dots,r) > 0$ then there holds the following formula

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w),(\lambda_w),r}[f(x)] = x^{\rho} c^{-\mu\sigma} \sum_{k_1\dots k_s=0}^{w_1k_1+\dots+w_sk_s\le m} (-m)_{w_1k_1+\dots+w_sk_s} A(m:k_1\dots k_s)$$

$$.\ \frac{c^{-\mu\sum_{i=1}^{s} v_i k_i}}{k_1!\dots k_s!} x^{\sum_{i=1}^{s}\ell_i k_i} \sum_{n=0}^{\infty}\frac{(-1)^n x^{\mu n}}{n!\, c^{\mu n}} H^{0,\lambda_1:[m_1,n_1];\dots;[m_{(r)},n_{(r)}]}_{p,q:[p_1,q_1];\dots;[p_{(r)},q_{(r)}]}$$

$$.\left[\begin{matrix}[(e):\alpha^{(1)},\dots,\alpha^{(r)}]:[(E^{(1)}):r^{(1)}];\dots;[(E^{(r)}):r^{(r)}];\\ [(f):\beta^{(1)},\dots,\beta^{(r)}]:[(F^{(1)}):W^{(1)}];\dots;[(F^{(r)}):W^{(r)}];\end{matrix}\ \frac{y_1 x^{t_1}}{c^{\mu\eta_1}},\dots,\frac{y_r x^{t_r}}{c^{\mu\eta_r}}\right]$$

$$H^{0,\lambda+r+1\ :(u^{(1)},v^{(1)});\dots;(u^{(r)},v^{(r)})}_{A+r+1,C+r+1:[B^{(1)},D^{(1)}];\dots;[B^{(N)},D^{(N)}]}\left[\begin{matrix}\frac{z_1}{x^{h_1}c^{\mu\rho_1}}\\ \vdots\\ \frac{z_n}{x^{h_N}c^{\mu\rho_N}}\end{matrix}\ \middle|\ \begin{matrix}[1-\Delta-n:\rho_1,\dots,\rho_N]\left[1-\eta_w+E:\frac{h_1}{\lambda_w},\dots,\frac{h_N}{\lambda_w}\right]_1^r,\\ [(c):\psi_1^{(1)},\dots,\psi^{(N)}][(d^{(1)}):\delta^{(1)}];\dots;[(d^{(N)}):\delta^{(N)}],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta^{(1)},...,\theta^{(N)}]:[(b^{(1)}):\phi^{(1)}];...;[(b^{(N)}):\phi^{(N)}] \\ [1-\Delta:\rho_1,...,\rho_N],\left[1-\eta_w-\zeta_w+E:\frac{h_1}{\lambda_w},...,\frac{h_N}{\lambda_w}\right]_1^r\end{matrix}\right], \qquad ...(2.2)$$

Where

$$\Delta=\sigma+\sum_{i=1}^{r} v_i k_i+\sum_{i=1}^{r}\eta_i U_i,$$

$$E=\frac{\left[\rho+\sum_{i=1}^{r}\ell_i k_i+\sum_{i=1}^{r} t_i U_i+\mu n\right]}{\lambda_w}.$$

and the series (2.2) is convergent.

Proof of 2.2

To establish (2.2), we express the general class of polynomial, on multivariable H-function in series form by using (1.6) and (1.11) and another multivariable H-function in terms of Mellin-Barnes contour integrals [13]. Then changing the order of integration and summations which is permissible under the conditions surrounding (2.2) and appealing to the result (1.3), we arrive at the desired result.

III. Applications

As an application of the result (2.2), we derive some interesting special cases. More special cases associated with various orthogonal polynomials and special functions can be derived by using the special cases of the polynomial $S_m^w[x]$ and the H-function of several variables.

(I) Taking s = 1 in (2.2), the polynomial (1.6) will reduce to and consequently, we obtain the following result

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w),(\lambda_w),r}[f_1(x)]=x^{\rho}c^{-\mu\sigma}\sum_{k=0}^{[m/w]}\frac{(-m)_{wk}}{k!}A_{m,k}\,x^{\ell k}c^{-\mu v k}$$

$$.\sum_{n=0}^{\infty}\frac{(-1)^n x^{\mu n}}{n!\,c^{\mu n}}H^{0,\lambda_1:[m_1,n_1];...;[m_{(r)},n_{(r)}]}_{p,q:[p_1,q_1];...;[p_{(r)},q_{(r)}]}$$

$$.\left[\begin{matrix}[(e):\alpha^{(1)},...,\alpha^{(r)}]:[(E^{(1)}):r^{(1)}];...;[(E^{(r)}):r^{(r)}]; \\ [(f):\beta^{(1)},...,\beta^{(r)}]:[(F^{(1)}):W^{(1)}];...;[(F^{(r)}):W^{(r)}];\end{matrix}\ \frac{y_1x^{t_1}}{c^{\mu\eta_1}}...\frac{y_rx^{t_r}}{c^{\mu\eta_r}}\right]$$

$$.H^{0,\lambda+r+1\ :(u^{(1)},v^{(1)});...;(u^{(r)},v^{(r)})}_{A+r+1,C+r+1:[B^{(1)},D^{(1)}];...;[B^{(N)},D^{(N)}]}\left[\begin{matrix}\frac{z_1}{x^{h_1}c^{\mu\rho_1}} \\ \vdots \\ \frac{z_n}{x^{h_N}c^{\mu\rho_N}}\end{matrix}\left|\begin{matrix}[1-\Delta^*-n:\rho_1,...,\rho_N]\left[1-\eta_w+E^*:\frac{h_1}{\lambda_w},...,\frac{h_N}{\lambda_w}\right]_1^r, \\ [(c):\psi_1^{(1)},...,\psi^{(N)}][(d^{(1)}):\delta^{(1)}];...;[(d^{(N)}):\delta^{(N)}],\end{matrix}\right.\right.$$

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$$\left.\begin{matrix}[(a):\theta^{(1)},...,\theta^{(N)}]:[(b^{(1)}):\phi^{(1)}];...;[(b^{(N)}):\phi^{(N)}] \\ [1-\Delta^*:\rho_1,...,\rho_N],\left[1-\eta_w-\zeta_w+E^*:\frac{h_1}{\lambda_w},...,\frac{h_N}{\lambda_w}\right]_1^r\end{matrix}\right], \quad ...(3.1)$$

$$\Delta^* = \sigma + vk + \sum_{i=1}^{r} \eta_i U_i\,,\; E^* = \frac{\left[\rho + \ell k + \sum_{i=1}^{r} t_i U_i + \mu n\right]}{\lambda_w},$$

and $f_1(x) = x^{\rho}(x^{\mu}+c^{\mu})^{-\sigma} S_m^w[x^{\ell}(x^{\mu}+c^{\mu})^{-v}]$

$$.\ H[z_1 x^{-h_1}(x^{\mu}+c^{\mu})^{-\rho_1} ... z_N x^{-h_N}(x^{\mu}+e^{\mu})^{-\rho_N}$$

$$.\ H[y_1 x^{t_1}(x^{\mu}+c^{\mu})^{-\eta_1} ... y_r x^{t_r}(x^{\mu}+c^{\mu})^{-\eta_r}].$$

(II) Setting s = 1, w = 2 and $A_{m,k}$ = (- 1)^k in (2.2), the by virtue of the result (1.9), we find that

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w),(\lambda_w),r}[f_2(x)] = x^{\rho} c^{-\mu\sigma} \sum_{k=0}^{[m/2]} (-1)^k (-m)_{2k} \frac{c^{-\mu v k} x^{\ell k}}{k!}$$

$$.\ \sum_{n=0}^{\infty} \frac{(-1)^n x^{\mu n}}{n!\, c^{\mu n}} H^{0,\lambda_1:[m_1,n_1];...;[m_{(r)},n_{(r)}]}_{p,q:[p_1,q_1];...;[p_{(r)},q_{(r)}]}$$

$$.\left[\begin{matrix}[(e):\alpha^{(1)},...,\alpha^{(r)}]:[(E^{(1)}):r^{(1)}];...;[(E^{(r)}):r^{(r)}]; \\ [(f):\beta^{(1)},...,\beta^{(r)}]:[(F^{(1)}):W^{(1)}];...;[(F^{(r)}):W^{(r)}];\end{matrix}\ \frac{y_1 x^{t_1}}{c^{\mu\eta_1}},...,\frac{y_r x^{t_r}}{c^{\mu\eta_r}}\right]$$

$$.\,H^{0,\lambda+r+1\ :(u^{(1)},v^{(1)});...;(u^{(r)},v^{(r)})}_{A+r+1,C+r+1:[B^{(1)},D^{(1)}];...;[B^{(N)},D^{(N)}]}\left[\begin{matrix}\frac{z_1}{x^{h_1}c^{\mu\rho_1}} \\ \vdots \\ \frac{z_n}{x^{h_N}c^{\mu\rho_N}}\end{matrix}\ \middle|\ \begin{matrix}[1-\Delta^*-n:\rho_1,...,\rho_N]\left[1-\eta_w+E^*:\frac{h_1}{\lambda_1},...,\frac{h_N}{\lambda_w}\right]_1^r, \\ [(c):\psi_1^{(1)},...,\psi^{(N)}][(d^{(1)}):\delta^{(1)}];...;[(d^{(N)}):\delta^{(N)}],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta^{(1)},...,\theta^{(N)}]:[(b^{(1)}):\phi^{(1)}];...;[(b^{(N)}):\phi^{(N)}] \\ [1-\Delta^*:\rho_1,...,\rho_N],\left[1-\eta_w-\zeta_w+E^*:\frac{h_1}{\lambda_w},...,\frac{h_N}{\lambda_w}\right]_1^r\end{matrix}\right], \quad ...(3.2)$$

where E^* and Δ^* are defined in equation (3.1), the series in (3.2) is convergent and the conditions given with (2.2) are satisfied for s = 1 and

$$f_2(x) = x^{\rho+\frac{n\ell}{2}} (x^\mu + c^\mu)^{-\sigma-\frac{nv}{2}}]$$

$$.\ H[z_1 x^{-h_1} (x^\mu + c^\mu)^{-\rho_1} \ldots z_N x^{-h_N} (x^\mu + e^\mu)^{-\rho_N}$$

$$.\ H[y_1 x^{t_1} (x^\mu + c^\mu)^{-\eta_1} \ldots y_r x^{t_r} (x^\mu + c^\mu)^{-\eta_r}] H_n\left[\frac{(x^\mu + c^\mu)^{v/2}}{2x^{\ell/2}}\right].$$

(III) Next, if we set s = 1, w = 1 and

$$A_{m,k} = \binom{m+\alpha}{m} \frac{(\alpha+\beta+m+1)_k}{(\alpha+1)_k}$$

then by virtue of (1.10), S_m^1 (x) reduces to the Jacobi polynomials and consequently, it yields

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w),(\lambda_w),r}[f_3(x)] = x^\rho c^{-\mu\sigma} \sum_{k=0}^{m} (-m)_k \binom{m+\alpha}{m} \frac{(\alpha+\beta+m+1)_k}{(\alpha+1)_k} \frac{x^{\ell k} c^{-\mu v k}}{k!}$$

$$.\ \sum_{n=0}^{\infty} \frac{(-1)^n x^{\mu n}}{n!\, c^{\mu n}} H^{0,\lambda_1:[m_1,n_1];\ldots;[m_{(r)},n_{(r)}]}_{p,q:[p_1,q_1];\ldots;[p_{(r)},q_{(r)}]}$$

$$.\left[\begin{matrix}[(e):\alpha^{(1)},\ldots,\alpha^{(r)}]:[(E^{(1)}):r^{(1)}];\ldots;[(E^{(r)}):r^{(r)}]; \\ [(f):\beta^{(1)},\ldots,\beta^{(r)}]:[(F^{(1)}):W^{(1)}];\ldots;[(F^{(r)}):W^{(r)}];\end{matrix}\ \frac{y_1 x^{t_1}}{c^{\mu\eta_1}} \ldots \frac{y_r x^{t_r}}{c^{\mu\eta_r}}\right]$$

$$.\ H^{0,\lambda+r+1\ :(u^{(1)},v^{(1)});\ldots;(u^{(r)},v^{(r)})}_{A+r+1,C+r+1:[B^{(1)},D^{(1)}];\ldots;[B^{(N)},D^{(N)}]}\left[\begin{matrix}\frac{z_1}{x^{h_1} c^{\mu\rho_1}} \\ \vdots \\ \frac{z_N}{x^{h_N} c^{\mu\rho_N}}\end{matrix}\ \middle|\ \begin{matrix}[1-\Delta^*-n:\rho_1,\ldots,\rho_N]\left[1-\eta_w+E^*:\frac{h_1}{\lambda_w},\ldots,\frac{h_N}{\lambda_w}\right]_1^r, \\ [(c):\psi_1^{(1)},\ldots,\psi^{(N)}][(d^{(1)}):\delta^{(1)}];\ldots;[(d^{(N)}):\delta^{(N)}],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta^{(1)},\ldots,\theta^{(N)}]:[(b^{(1)}):\phi^{(1)}];\ldots;[(b^{(N)}):\phi^{(N)}] \\ [1-\Delta^*:\rho_1,\ldots,\rho_N],\left[1-\eta_w-\zeta_w+E^*:\frac{h_1}{\lambda_w},\ldots,\frac{h_N}{\lambda_w}\right]_1^r\end{matrix}\right], \qquad \ldots(3.3)$$

where E^* and Δ^* are defined in equation (3.1), the series in (3.3) is convergent and the conditions given with (2.2) are satisfied for s = 1.

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A New Summation Formula Allied With Hypergeometric Function

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Abstract - The main objective of the present paper is to derive a summation formula of half argument associated to Bailey theorem .The result presented here is presumably new.

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GJSFR Classification: *2000 MSC No: 33C60 , 33C70.*

A NEW SUMMATION FORMULA ALLIED WITH HYPERGEOMETRIC FUNCTION

Strictly as per the compliance and regulations of:

A New Summation Formula Allied With Hypergeometric Function

Salahuddin

Abstract - The main objective of the present paper is to derive a summation formula of half argument associated to Bailey theorem .The result presented here is presumably new.

Keywords and Phrases : Gaussian Hypergeometric function , Contiguous function, Re-currence relation, Bailey summation theorem and Legendre duplication formula.

I. Introduction

Generalized Gaussian hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ll} a_1, a_2, \cdots, a_A & ; \\ & & z \\ b_1, b_2, \cdots, b_B & ; \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} $$

or

$$ {}_AF_B\left[\begin{array}{ll} (a_A) & ; \\ & & z \\ (b_B) & ; \end{array}\right] \equiv {}_AF_B\left[\begin{array}{ll} (a_j)_{j=1}^A & ; \\ & & z \\ (b_j)_{j=1}^B & ; \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and A, B are non-negative integers.

Contiguous Relation is defined as follows [E. D. p.51(10), Andrews p.363(9.16), H.T. F. I p.103(32)]

$$ (a-b)\ {}_2F_1\left[\begin{array}{ll} a,\ b\ ; & \\ c\ ; & z \end{array}\right] = a\ {}_2F_1\left[\begin{array}{ll} a+1,\ b\ ; & \\ c\ ; & z \end{array}\right] - b\ {}_2F_1\left[\begin{array}{ll} a,\ b+1\ ; & \\ c\ ; & z \end{array}\right] \tag{2} $$

Recurrence relation of gamma function is defined as follows

$$ \Gamma(z+1) = z\ \Gamma(z) \tag{3} $$

Legendre duplication formula is defined as follows

$$ \sqrt{\pi}\ \Gamma(2z) = 2^{(2z-1)}\ \Gamma(z)\ \Gamma\left(z+\frac{1}{2}\right) \tag{4} $$

$$ \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{b+1}{2})}{\Gamma(b)} \tag{5} $$

$$ = \frac{2^{(a-1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\Gamma(a)} \tag{6} $$

Bailey summation theorem [Prud, p.491(7.3.7.8)]is defined as follows

$$ {}_2F_1\left[\begin{array}{ll} a, 1-a & ; \\ c & ; \end{array} \frac{1}{2}\right] = \frac{\Gamma(\frac{c}{2})\ \Gamma(\frac{c+1}{2})}{\Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})} = \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-1}\ \Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})} \tag{7} $$

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II. Main Result Of Summation Formula

$$ {}_2F_1\left[\begin{matrix} a \;\; , & -a-35 & ; & \frac{1}{2} \\ c & & ; & 2 \end{matrix}\right] = \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+35}} \times \Bigg[\frac{4(415017197290314178560000)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(-7011347247419528369664 00a + 346312606003048571708160a^2 - 630449057748142770681 60a^3)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(25201620309558040233 12a^4 + 347909921081488886400a^5 - 17050603760335860120a^6)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(-1215731406744910440a^7 + 22493679752250729a^8 + 2293529179788000a^9 + 21722140394460a^{10})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(-1179007608240a^{11} - 30913573194a^{12} - 149940000a^{13} + 3136500a^{14} + 42840a^{15} + 153a^{16})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(995391244148036404838400c - 1188738087797910855260160ac + 433443627220853784109824a^2c)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(-54690068587015435432320a^3c + 2224941907438523118 24a^4c + 308870212868652946560a^5c)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(-26051932559773463 28a^6c - 81139091839306116 0a^7c - 7344331962050919a^8c)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(809731932055200a^9c + 20301736826844a^{10}c - 49464547440a^{11}c - 6878340714a^{12}c - 76792800a^{13}c)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(-92940a^{14}c + 2520a^{15}c + 9a^{16}c + 940737212685399702896640c^2 - 842944939817358014300160ac^2)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(229804932215451704039424a^2c^2 - 19087576455710013350400a^3c^2 - 598038150920943774720a^4c^2)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(97496924379205317120a^5c^2 + 1795534443248012544a^6c^2 - 172218004133868000a^7c^2)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(-4900104145807200a^8c^2 + 63835461597600a^9c^2 + 3660595210272a^{10}c^2 + 32261090400a^{11}c^2)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(-301193760a^{12}c^2 - 5997600a^{13}c^2 - 24480a^{14}c^2 + 49311345068687699686 1952c^3)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(-340008605858413227294720ac^3 + 69119287604777891371008a^2c^3 - 3360401286427327654400a^3c^3)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(-25929527467416654592 0a^4c^3 + 14355397477659293440a^5c^3 + 651193244570304128a^6c^3)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$ + \frac{4(-13393051014173600a^7c^3 - 809327950990240a^8c^3 - 4254392973600a^9c^3 + 232222999008a^{10}c^3)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})} + $$

$$+\frac{4(3623373600a^{11}c^3+8336160a^{12}c^3-117600a^{13}c^3-480a^{14}c^3+165393111855973216813056c^4)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-88619859281749602508800ac^4+13234031762431394360320a^2c^4-267732463853898489600a^3c^4)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-50852045281263121280a^4c^4+885721501247352000a^5c^4+92742286806954400a^6c^4)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(188394608001600a^7c^4-58996252798560a^8c^4-853038648000a^9c^4+3592105440a^{10}c^4)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(131947200a^11c^4+628320a^12c^4+38348155565012206485504c^5-15980556939785111511040ac^5)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(1703747269799832083456a^2c^5+7094137991114845440a^3c^5-5780224308733390208a^4c^5)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-22987619341431360a^5c^5+6958079968315168a^6c^5+100932053893440a^7c^5-1873167815904a^8c^5)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-45925387200a^9c^5-162823584a^{10}c^5+1552320a^{11}c^5+7392a^{12}c^5+6441077978373336596480c^6)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-2075899486553471631360ac^6+150541710347476783104a^2c^6+4035388382214604800a^3c^6)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-406469269661199360a^4c^6-7894718282626560a^5c^6+279461650288128a^6c^6+7133593420800a^7c^6)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-4463585280a^8c^6-1055577600a^9c^6-6031872a^{10}c^6+807369926207525617664c^7)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-198998299950923038720ac^7+9013154655402692608a^2c^7+464897900224665600a^3c^7)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-17366159133649920a^4c^7-603327725468160a^5c^7+4491951518208a^6c^7+237685324800a^7c^7)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(1232056320a^8c^7-8870400a^9c^7-50688a^{10}c^7+76952353200486612992c^8)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-14260711716681830400ac^8+337711544275522560a^2c^8+30600618278630400a^3c^8)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-376535905025280a^4c^8-24535845734400a^5c^8-73578785280a^6c^8+3920716800a^7c^8)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(28005120a^8c^8+5637067161272188928c^9-766928822089830400ac^9+5176262833730560a^2c^9)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(1300268697728000a^3c^9+1160910150400a^4c^9-575089715200a^5c^9-4430666240a^6c^9)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(25625600a^7c^9+183040a^8c^9+318647139652075520c^{10}-307977594993868880ac^{10})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-191736922144768a^2c^{10}+36337203302400a^3c^{10}+305641656320a^4c^{10}-7318671360a^5c^{10})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-69701632a^6c^{10}+13865952926498816c^{11}-909358510161920ac^{11}-14383991078912a^2c^{11})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(646743552000a^3c^{11}+8097689600a^4c^{11}-39137280a^5c^{11}-372736a^6c^{11}+460065647362048c^{12})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-19154958950400ac^{12}-430851133440a^2c^{12}+6653337600a^3c^{12}+95047680a^4c^{12})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(11421693509632c^{13}-272285081600ac^{13}-7252725760a^2c^{13}+30105600a^3c^{13}+430080a^4c^{13})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(205308559360c^{14}-2339635200ac^{14}-66846720a^2c^{14}+2522349568c^{15}-9175040ac^{15})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4(-262144a^2c^{15}+18939904c^{16}+65536c^{17})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-6110663484726998765568000a+5999292753779459361024000a^2-1835414095881644284688640a^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{173306115070278868568448a^4+68473630878394926592 80a^5-12771252398874487140 64a^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-2940310184799111516 0a^7+347196302738172585 6a^8+126277195151524725a^9}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-2216897933318673a^{10}-171425296918260a^{11}-2149251178156a^{12}+31660673310a^{13}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{1068483738a^{14}+8985060a^{15}+852a^{16}-315a^{17}-a^{18}+6110663485082686193664000c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-1650826675511851316613 1200ac+9924029599401067550100480a^2c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-2045268821850893695203840a^3c+970694618865608546570 88a^4c+11370086277363321580800a^5c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-633824951895804745440a^6c-41696927468297529120a^7c+867582829527332292a^8c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{81786749980320000a^9c+738130579599600a^{10}c-42829190268480a^{11}c-1109263754088a^{12}c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-5325868800a^{13}c+113207760a^{14}c+1542240a^{15}c+5508a^{16}c+1050897400256245939568 6400c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-1591233842767430061656064 0ac^2+662772637016643906011289 6a^2c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-9237207550831336935129 60a^3c^2+8432672530427463216672a^4c^2+5300412627789403850880a^5c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-583235592255586297 44a^6c^2-1435976664087879 1440a^7c^2-117708974158778046a^8c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{14649826634150400a^9c^2+361244419738488a^{10}c^2-955517381280a^{11}c^2-123959873268a^{12}c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-1380153600a^{13}c^2-1664280a^{14}c^2+45360a^{15}c^2+162a^{16}c^2+7823722925976479795773440c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-8155904534327159641866240ac^3+2452519572753235312705536a^2c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-222301377892875933388800a^3c^3-59899639895783296358 40a^4c^3+11480547565033055308 80a^5c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{1934410939063508505 6a^6c^3-2070540243026640000a^7c^3-57385728514224000a^8c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{786409798204800a^9c^3+43840425315456a^{10}c^3+383966352000a^{11}c^3-3629404800a^{12}c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-71971200a^{13}c^3-293760a^{14}c^3+3400140872524649384116224c^4-2609068981199287139205120ac^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{572280147968391303917568a^2c^4-303895387006060985856 00a^3c^4-2162804762047330894080a^4c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{129800158011581904000a^5c^4+5652662058893572800a^6c^4-123530509599789600a^7c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-7227575616428640a^8c^4-36913079095200a^9c^4+2094884247456a^{10}c^4+32563792800a^{11}c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{74803680a^{12}c^4-1058400a^{13}c^4-4320a^{14}c^4+976353339464085078540288c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-566616560808405897216000ac^5+89945371865302855372800a^2c^5-2065831359960639237120a^3c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-35159107288624739481 6a^4c^5+6655650171513235200a^5c^5+65766669544344384 0a^6c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{1100553967054080a^7c^5-424605744275328a^8c^5-6103877472000a^9c^5+26080306560a^{10}c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{950019840a^{11}c^5+4523904a^{12}c^5+198112157533999839838208c^6-87792282223271875952640ac^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

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$$+\frac{984581569427670410649 6a^2c^6+234952064274888576 00a^3c^6-3395159070731336832 0a^4c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-1127886830806982 40a^5c^6+415582562323125 12a^6c^6+5956185804307 20a^7c^6-11290688684352a^8c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-275179766400a^9c^6-974812608a^{10}c^6+9313920a^{11}c^6+44352a^{12}c^6+2958922858433630699 5200c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-1000692834729017278464 0ac^7+757910734442793271296a^2c^7+1930286356655247360 0a^3c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-2072186286738923520a^4c^7-39427376325765120a^5c^7+1440727996409856a^6c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{36498857472000a^7c^7-24299827200a^8c^7-5428684800a^9c^7-31021056a^{10}c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{3338242567056453009408c^8-854705277266778685440ac^8+40269792261059260416a^2c^8}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{2022273891437184000a^3c^8-78204798453196800a^4c^8-2683947042120960a^5c^8+20452779502848a^6c^8}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{1068200179200a^7c^8+5534369280a^8c^8-39916800a^9c^8-228096a^{10}c^8+289112052804832198656c^9}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-55212638992422912000ac^9+1361953471309516800a^2c^9+120228388600320000a^3c^9}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-1524394295424000a^4c^9-97704262656000a^5c^9-290133043200a^6c^9+15682867200a^7c^9}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{112020480a^8c^9+19391941584781049856c^{10}-2701252922655006720ac^{10}+19575184610961408a^2c^{10}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{4638318400235520a^3c^{10}+3645345639936a^4c^{10}-2067739914240a^5c^{10}-15925797888a^6c^{10}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{92252160a^7c^{10}+658944a^8c^{10}+1010033600684359680c^{11}-99417087242403840ac^{11}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-596583567949824a^2c^{11}+118442715955200a^3c^{11}+993438351360a^4c^{11}-23952015360a^5c^{11}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-228114432a^6c^{11}+40711783303872512c^{12}-2706854827622400ac^{12}-42586761584640a^2c^{12}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{1937882419200a^3c^{12}+24259522560a^4c^{12}-117411840a^5c^{12}-1118208a^6c^{12}+1256826574209024c^{13}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{-52847972352000ac^{13}-1187511091200a^2c^{13}+18424627200a^3c^{13}+263208960a^4c^{13}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{29143244734464c^{14}-699335884800ac^{14}-18626273280a^2c^{14}+77414400a^3c^{14}+1105920a^4c^{14}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$+\frac{490922311680c^{15}-5615124480ac^{15}-160432128a^2c^{15}+5668601856c^{16}-20643840ac^{16}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}+$$

$$\left.+\frac{-589824a^2c^{16}+40108032c^{17}+131072c^{18}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+36}{2})}\right] \quad (8)$$

Derivation of result (8):

putting b = −a − 35, z = $\frac{1}{2}$ in known result (2),we get

$$(2a+35)\ {}_2F_1\left[\begin{matrix} a\ , & -a-35 & ; & \frac{1}{2} \\ c & & ; & 2\end{matrix}\right]$$

$$= a\ {}_2F_1\left[\begin{matrix} a+1\ , & -a-35 & ; & \frac{1}{2} \\ c & & ; & 2\end{matrix}\right] + (a+35)\ {}_2F_1\left[\begin{matrix} a\ , & -a-34 & ; & \frac{1}{2} \\ c & & ; & 2\end{matrix}\right]$$

Now involving Bailey theorem, we get

$$\text{L.H.S} = a\ \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+34}} \times \left[\frac{-5210554925457720115200 0+3228732334584959892480 0a}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+\right.$$

$$\frac{4665314080497377305728 0a^2-3223336755530305033459 2a^3+5488847144083484177904a^4}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-542141883466318548 64a^5-366521420029415994 00a^6+150767267386517376a^7}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{1078972008786061 83a^8+1994945981790927a^9-88817701723740a^{10}-3580089931276a^{11}}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-26449725078a^{12}+660294138a^{13}+13846860a^{14}+81492a^{15}-9a^{16}-a^{17}}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$\frac{-69462745690104564940800c-70551909979177708154880ac+13007348902410890135500 8a^2c}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$\frac{-4465816818046341643699 2a^3c+4094833228285214588448a^4c+1679239961967933388 80a^5c}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-23094422267494862256a^6c-7247594634082149 60a^7c+37308814013226162a^8c}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{1755542571140160a^9c+29707423698 48a^{10}c-923367713184a^{11}c-16387628628a^{12}c}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-44372160a^{13}c+1185000a^{14}c+9936a^{15}c+18a^{16}c+1138852367664206708736 0c^2}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1559219037566482626232 32ac^2+11231887442991231682252 8a^2c^2-23342463632776772242176a^3c^2}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{948734345567285867520a^4c^2+1153724231323117681 92a^5c^2-40227226421810440 32a^6c^2}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-3243874365818122 08a^7c^2+834141495369600a^8c^2+336406309039680a^9c^2+5563247526144a^{10}c^2}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-34990935840a^{11}c^2-1631387520a^{12}c^2-12667200a^{13}c^2-8640a^{14}c^2+144a^{15}c^2}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{554424650163115933040 64c^3-111068382282816419364864ac^3+48393399324811963375616a^2c^3}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-6287216055695980984320a^3c^3+1155517658973076224 0a^4c^3+289503331695615797 76a^5c^3}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{43930305906512256a^6c^3-5110137044427360 0a^7c^3-876496444556480a^8c^3+20374584310080a^9c^3}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{670195665216a^{10}c^3+3981539520a^{11}c^3-39305280a^{12}c^3-463680a^{13}c^3-960a^{14}c^3}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{37527463344249570066432c^4-42645015165231037800448ac^4+12397193443150168611840a^2c^4}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-965457019813117879040a^3c^4-326093993713326854 40a^4c^4+35932830138900365 44a^5c^4}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{91343821462370400a^6c^4-3395230689331360a^7c^4-121523975385120a^8c^4-212408787360a^9c^4}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{28154760480a^{10}c^4+301328160a^{11}c^4+433440a^{12}c^4-3360a^{13}c^4+13510118305163970347008c^5}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-10323758302057734266880ac^5+2056491071757425938432a^2c^5-82878606240028752384a^3c^5}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-67349590966064028 16a^4c^5+219850789996252800a^5c^5+11976686903483456a^6c^5}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-415576069413 12a^7c^5-6696675023808a^8c^5-68070119040a^9c^5+307699392a^{10}c^5+6120576a^{11}c^5}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{14784a^{12}c^5+3130056044504000102400c^6-1700270404273278939136ac^6}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{230786652805037564928a^2c^6-268935657192180070 4a^3c^6-686916413160848640a^4c^6}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{29202959959317 76a^5c^6+740064574884096a^6c^6+7029696406656a^7c^6-157484194560a^8c^6}{\Gamma(\frac{c+a+35}{2})\;\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-2561623680a^9c^6-5854464a^{10}c^6+29568a^{11}c^6+504999310792104869888c^7}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1987851308493920010024ac^7+178192072078270300016a^2c^7+2064475552641638400a^3c^7}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-41382372982579200a^4c^7-488097174039552a^5c^7+23698855842816a^6c^7+422140815360a^7c^7}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-551823360a^8c^7-34974720a^9c^7-101376a^{10}c^7+59161852372726185984c^8}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-169003860702583808800ac^8+937163970349009920a^2c^8+30707029480765440a^3c^8}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1489251061267200a^4c^8-35818350720768a^5c^8+316518935040a^6c^8+9989168640a^7c^8}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{33834240a^8c^8-126720a^9c^8+5152744512277446656c^9-1056982167398277120ac^9}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{31698064973752320a^2c^9+1844571268700160a^3c^9-28075061414400a^4c^9-1155058544640a^5c^9}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1891169280a^6c^9+101038080a^7c^9+366080a^8c^9+337670204648325120c^{10}-48683180589801472ac^{10}}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{546819979689984a^2c^{10}+65208351143936a^3c^{10}-65191526400a^4c^{10}-19405754368a^5c^{10}}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-98402304a^6c^{10}+292864a^7c^{10}+16698720466501632c^{11}-1635615902269440ac^{11}}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-3551832653824a^2c^{11}+1420850995200a^3c^{11}+8211374080a^4c^{11}-154312704a^5c^{11}-745472a^6c^{11}}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{619584954433536c^{12}-39175823138816ac^{12}-436475719680a^2c^{12}+18239836160a^3c^{12}}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{150958080a^4c^{12}-372736a^5c^{12}+16973437272064c^{13}-640631685120ac^{13}-10411089920a^2c^{13}}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{118702080a^3c^{13}+860160a^4c^{13}+332921733120c^{14}-6598082560ac^{14}-116490240a^2c^{14}}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{245760a^3c^{14}+4420796416c^{15}-36175872ac^{15}-524288a^2c^{15}+35586048c^{16}-65536ac^{16}+131072c^{17}}{\Gamma(\frac{c+a+35}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{12449059983360000-3835978717115265477120000a+3498473724759821441088800a^2}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-91897064286234680718720a^3+5349183000720094955952a^4+624669804151667381584a^5}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-35315751928499458920a^6-2851177925475811680a^7+30576386460688299a^8}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{5649027499407633a^9+106277649831540a^{10}-1707700340180a^{11}-85722805854a^{12}}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-1008568218a^{13}+167580a^{14}+89580a^{15}+603a^16+a^{17}+383597914530722217984000c}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-984494237718394284134400ac+533683276801989101360640a^2c-88673539520059738539264a^3c}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{937411648798849628448a^4c+602576665340697774720a^5c-3095068771370511600a^6c}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-1813741731860501520a^7c-32446407661384398a^8c+1545568399393920a^9c+60904486614840a^{10}c}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{428069571552a^{11}c-11769254868a^{12}c-240186240a^{13}c-1381080a^{14}c+432a^{15}c+18a^{16}c}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{634646928998497701888000c^2-891544960641224822353920ac^2+323497946519474767570944a^2c^2}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-33039865708098194060544a^3c^2-1069107709209191124480a^4c^2+191538372418841669760a^5c^2}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{5047298563885611264a^6c^2-336320453706868752a^7c^2-14028936710025600a^8c^2}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{13044949325760a^9c^2+8229644260992a^{10}c^2+133856866080a^{11}c^2+220993920a^{12}c^2-11282880a^{13}c^2}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-86400a^{14}c^2-144a^{15}c^2+449758803541459552174080c^3-424361084473430996287488ac^3}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{106699621776076144541696a^2c^3-5943566750811274905600a^3c^3-511600651806731988480a^4c^3}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{26180828185511231232a^5c^3+1593174370131501696a^6c^3-15025123090190400a^7c^3}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-1913536321611200a^8c^3-25716298386240a^9c^3+304613660736a^{10}c^3+9607590720a^{11}c^3}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{64478400a^{12}c^3-20160a^{13}c^3-960a^{14}c^3+184187526298792576843776c^4}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-124561329541509091192832ac^4+21709061172519601090560a^2c^4-431132806280069582080a^3c^4}{\frac{c+a+36}{2}\qquad\frac{c-a-1}{2}}+$$

$$+\frac{-1019756642927247456OOa^4c^4+1054283755518504896a^5c^4+206173676796780000a^6c^4}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{2226847463869600a^7c^4-108572981600160a^8c^4-2646472345440a^9c^4-9261917280a^{10}c^4}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{225573600a^{11}c^4+2005920a^{12}c^4+3360a^{13}c^4+49339504936023346905088c^5}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-24507367998079394856960ac^5+2888874316719236392960a^2c^5+28254033881806182912a^3c^5}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-11337146664891072256a^4c^5-141887361281339520a^5c^5+13500845824262720a^6c^5}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{327039780203136a^7c^5-1523026642368a^8c^5-105685050240a^9c^5-851484480a^{10}c^5+266112a^{11}c^5}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{14784a^{12}c^5+9243724230164377927680c^6-3392714026389984235520ac^6}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{256000715426159176704a^2c^6+9560726388764655104a^3c^6-745346714565277440a^4c^6}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-22673639805013120a^5c^6+402088325683968a^6c^6+17963712945024a^7c^6+103427976960a^8c^6}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-1653590400a^9c^6-17563392a^{10}c^6-29568a^{11}c^6+1260981912550139494400c^7}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-340145536546911879168ac^7+14669030470350012416a^2c^7+1031175960548659200a^3c^7}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-26915981666365440a^4c^7-1466661042975744a^5c^7-1392908504064a^6c^7+483138754560a^7c^7}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{4867737600a^8c^7-1520640a^9c^7-101376a^{10}c^7+128421640911359803392c^8}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-25086382139421306880ac^8+461793624582343680a^2c^8+64904964334172160a^3c^8}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-257288995434240a^4c^8-52460877756672a^5c^8-476345802240a^6c^8+5667340800a^7c^8}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{74891520a^8c^8+126720a^9c^8+9909451880787869696c^9-1367832022323486720ac^9}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-554339253811200a^2c^9+2617395227166720a^3c^9+21121170470400a^4c^9-1046816010240a^5c^9}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-14068454400a^6c^9+4392960a^7c^9+366080a^8c^9+583327157087600640c^{10}-54833650369372160ac^{10}}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-799709448265728a^2c^{10}+67865020647424a^3c^{10}+1036668272640a^4c^{10}-9819729920a^5c^{10}}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-172204032a^6c^{10}-292864a^7c^{10}+26186417921064960c^{11}-1586469855559680ac^{11}}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-39937560592384a^2c^{11}+1065864683520a^3c^{11}+21495685120a^4c^{11}-6709248a^5c^{11}-745472a^6c^{11}}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{889333785919488c^{12}-31864244199424ac^{12}-1058624286720a^2c^{12}+8328785920a^3c^{12}}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{218050560a^4c^{12}+372736a^5c^{12}+22449979654144c^{13}-409953976320ac^{13}-16542310400a^2c^{13}}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{5160960a^3c^{13}+860160a^4c^{13}+408015175680c^{14}-2744729600ac^{14}-143032320a^2c^{14}-245760a^3c^{14}}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\left.\frac{5043650560c^{15}-1572864ac^{15}-524288a^2c^{15}+37945344c^{16}+65536ac^{16}+131072c^{17}}{\Gamma(\frac{c+a+36}{2})\ \Gamma(\frac{c-a-1}{2})}\right]+$$

$$+(a+35)\ \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+34}}\times\left[\frac{-1797253965991562158080000a+1762784654780725986816000a^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+\right.$$

$$+\ \frac{-539984429150605283520000a^3+52121430584431286987520a^4+1630946460604286072160a^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\ \frac{-348709899597669411200a^6-6310935555192674000a^7+86522703948738904a^8}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\ \frac{2692481789166367a^9-51357622524560a^{10}-3216119281100a^{11}-33810573664a^{12}+474997418a^{13}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\ \frac{12624080a^{14}+81500a^{15}+8a^{16}-a^{17}+1797253966200790056960000c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\ \frac{-4800798949938789159936000ac+2866075382186120788761600a^2c-589236816007670925980160a^3c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\ \frac{29365548336133402524480a^4c+2894013235460045932800a^5c-171228894179788956000a^6c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\ \frac{-9606787445266519200a^7c+217869186779308020a^8c+1678621152574080a^9c+12060050654520a^{10}c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\ \frac{-730595054112a^{11}c-15707443668a^{12}c-59928960a^{13}c+1038120a^{14}c+9648a^{15}c+18a^{16}c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\ \frac{3038014295865405997056000c^2-4537832568590869291008000ac^2+1868314299643574342000640a^2c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\ \frac{-259145950179612438466560a^3c^2+32290213229361236352000a^4c^2+1326908812599632908800a^5c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-1755729792096314496a^6c^2-3196278077787725248a^7c^2-1938186903906000a^8c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{279199198531440a^9c^2+5844089982192a^{10}c^2-16398985680a^{11}c^2-1467565680a^{12}c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-12531120a^{13}c^2-10800a^{14}c^2+144a^{15}c^2+22117416165852762341376\,0c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-22661891224278283203379\,2ac^3+67036582057662381981696a^2c^3-60465498689350017536\,00a^3c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-1308127692446157900800a^4c^3+276651065340095649\,28a^5c^3+37552539957994969\,6a^6c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-434349930870416000a^7c^3-1030384714673600a^8c^3+138999298838\,40a^9c^3+623936233536a^{10}c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{4417385280a^{11}c^3-33364800a^{12}c^3-450240a^{13}c^3-960a^{14}c^3+9349902090782711060889\,6c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-7018793995795766152396\,8ac^4+15063413339614862888960a^2c^4-80102551752092577792\,0a^3c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-490952981462121024000a^4c^4+29806918636205751\,04a^5c^4+111731381007400000a^6c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-2433965084610400a^7c^4-118394832346560a^8c^4-477481102560a^9c^4+24869718720a^{10}c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{295864800a^{11}c^4+477120a^{12}c^4-3360a^{13}c^4+25966303486948763762688c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-14657409272023175741440ac^5+2262698965968343705600a^2c^5-539808809479591787\,52a^3c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-7653568354604736256a^4c^5+147484321071669120a^5c^5+120858629442320\,00a^6c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{9530353117824a^7c^5-6071200056768a^8c^5-70813733760a^9c^5+241348800a^{10}c^5+5943168a^{11}c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{14784a^{12}c^5+5063113354325031649280c^6-2167153902409193062400ac^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{234714970109273206784a^2c^6+7296539320338329\,6a^3c^6-690673666259709440a^4c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-1363969989536000a^5c^6+686661075881088a^6c^6+8198063803776a^7c^6-134697911040a^8c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-2501452800a^9c^6-6179712a^{10}c^6+29568a^{11}c^6+721356330294449274880c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-233641252827241971712ac^7+16956797879563452416\,a^2c^7+366636901242163200\,a^3c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-38601213445386240a^4c^7-621398831133696a^5c^7+20731335668736a^6c^7+425308477440a^7c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-241612800a^8c^7-33960960a^9c^7-101376a^{10}c^7+76967242257705992192c^8}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-18676816839483883520ac^8+836470098009722880a^2c^8+36299867555466240a^3c^8}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-1305758760176640a^4c^8-37509602473728a^5c^8+247552757760a^6c^8+9713932800a^7c^8}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{34974720a^8c^8-126720a^9c^8+6239553251386064896c^9-1114738046535393280ac^9}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{26007420905523200a^2c^9+1945362268129280a^3c^9-22331646937600a^4c^9-1141610229760a^5c^9}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-2588185600a^6c^9+98109440a^7c^9+366080a^8c^9+386834950982205440c^10-49581031165952000ac^{10}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{350996352458752a^2c^{10}+65277038002176a^3c^{10}+30350960640a^4c^{10}-18809190400a^5c^{10}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-100452352a^6c^{10}+292864a^7c^{10}+18329372050063360c^{11}-1624283296563200ac^{11}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{-7763585449984a^2c^{11}+1386477281280a^3c^{11}+8971755520a^4c^{11}-149839872a^5c^{11}-745472a^6c^{11}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{658306213347328c^{12}-38248757886976ac^{12}-490285752320a^2c^{12}+17632276480a^3c^{12}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{152821760a^4c^{12}-372736a^5c^{12}+17603540025344c^{13}-619456839680ac^{13}-10762035200a^2c^{13}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{115261440a^3c^{13}+860160a^4c^{13}+339403079680c^{14}-6364364800ac^{14}-117227520a^2c^{14}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{245760a^3c^{14}+4456448000c^{15}-35127296ac^{15}-524288a^2c^{15}+35651584c^{16}-65536ac^{16}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{131072c^{17}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+34}{2})}+$$

$$+\frac{830034394580628357120000-1377065451088614086553600a+650917761399667233123840a^2}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-106441825212269888974848a^3+1797340338149203219104a^4+775661574779850021264a^5}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-1495285615230779512 0a^6-2905674724521079336a^7-152417310625 00822a^8}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4510465479373073a^9+119693002445720a^{10}-757636334884a^{11}-72635836868a^{12}-1001843738a^{13}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-1104440a^{14}+80068a^{15}+586a^{16}+a^{17}+19907824882960728096768 00c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-23186120260533594998169 60ac+7992932997228759621319 68a^2c-863970594558471487802 88a^3c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-206087009599090041955 2a^4c+585055100899622895360a^5c+8575377898850894544a^6c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-1505688369793705680a^7c-43691973648968238a^8c+962487629468640a^9c+55486260156888a^{10}c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{551069327424a^{11}c-8772675828a^{12}c-220815840a^{13}c-1385400a^{14}c+144a^{15}c+18a^{16}c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{1881474425370799405793280c^2-1632458853872025818923008ac^2+414369892955453180203008a^2c^2}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-26959982583521703297024a^3c^2-1940300862028789774080a^4c^2+15497710340920288588 8a^5c^2}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{7009302342933356928a^6c^2-224562920489457792a^7c^2-1379796999059160 0a^8c^2}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-619458793192 80a^9c^2+6774945182544a^{10}c^2+130356127440a^{11}c^2+359874480a^{12}c^2}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-10088400a^{13}c^2-84240a^{14}c^2-144a^{15}c^2+986226901373753993723904c^3}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-653423370678042386006016ac^3+121223070338904522633216a^2c^3-3667640265557653038080a^3c^3}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-61821194433278099456 0a^4c^3+16410099785215273984a^5c^3+1646990973593760256a^6c^3}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-676053491748800a^7c^3-1669935333999680a^8c^3-28248215244480a^9c^3+203190542016a^{10}c^3}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{8832626880a^{11}c^3+64653120a^{12}c^3-6720a^{13}c^3-960a^{14}c^3+33078622371194643362611 2c^4}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-168860896838707866058752ac^4+22383105664203945687040a^2c^4-1672698649718374656 0a^3c^4}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-10423968625124888064 0a^4c^4-13024544043303654 4a^5c^4+187765957371977600a^6c^4}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{3001342598673760a^7c^4-85207747806720a^8c^4-2541885524640a^9c^4-11611797120a^{10}c^4}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{201764640a^{11}c^4+1962240a^{12}c^4+3360a^{13}c^4+766963111300244129710080c^5}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-301557940845742103756800ac^5+273770381467084984115200a^2c^5+71925252918545478144a^3c^5}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-10436737037868418816a^4c^5-215952412981683840a^5c^5+111776212400504960a^6c^5}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{335521585779072a^7c^5-610214582208a^8c^5-97158821760a^9c^5-853435968a^{10}c^5+88704a^{11}c^5}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{14784a^{12}c^5+128821559567466731929600c^6-38731675470428668887040ac^6}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{22307884941216097484800a^2c^6+123079578836494551040a^3c^6-626568475975805440a^4c^6}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-24714927694057216a^5c^6+279373545365376a^6c^6+1707885772454 4a^7c^6+117524816640a^8c^6}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-1479582720a^9c^6-17238144a^{10}c^6-29568a^{11}c^6+16147398524150512353280c^7}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-3662897272840476098560ac^7+1142864240594102681600a^2c^7+11242177721019801600a^3c^7}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-19620139023513600a^4c^7-1448430437465088a^5c^7-4638476688384a^6c^7+444154275840a^7c^7}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{4876861440a^8c^7-506880a^9c^7-101376a^{10}c^7+1539047064009732259840c^8}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-258144847468423168000ac^8+26605234427599872000a^2c^8+654192326280345600a^3c^8}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-2322924172800a^4c^8-49487966744832a^5c^8-513930869760a^6c^8+5072770560a^7c^8+73751040a^8c^8}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{126720a^9c^8+112741343225443778560c^9-13589607924577484800ac^9-826954086120448000a^2c^9}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{2522723887523840a^3c^9+261440955776000a^4c^9-962333532160a^5c^9-14088954880a^6c^9}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{1464320a^7c^9+366080a^8c^9+6372942793041510400c^{10}-530348311514644480ac^{10}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-9969888801832960a^2c^{10}+636235840880640a^3c^{10}+1083194112000a^4c^{10}-8792655872a^5c^{10}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-170153984a^6c^{10}-292864a^7c^{10}+27731905852997632c^{11}-1503483152138240ac^{11}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{-43006124621824a^2c^{11}+979829760000a^3c^{11}+21518049280a^4c^{11}-2236416a^5c^{11}-745472a^6c^{11}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{920131294724096c^{12}-29722879606784ac^{12}-1082306068480a^2c^{12}+7460311040a^3c^{12}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{216186880a^4c^{12}+372736a^5c^{12}+22843387019264c^{13}-376857313280ac^{13}-16552632320a^2c^{13}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$+\frac{1720320a^3c^{13}+860160a^4c^{13}+410617118720c^{14}-2459402240ac^{14}-142295040a^2c^{14}-245760a^3c^{14}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}+$$

$$\left.+\frac{5044699136c^{15}-524288ac^{15}-524288a^2c^{15}+37879808c^{16}+65536ac^{16}+131072c^{17}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+35}{2})}\right]$$

After simplification , the result (8) is proved.

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Household Consumption of Cassava Products in Oyo State

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Abstract - This paper analyses household consumption of cassava products in Oyo State using Almost Ideal Demand System. Information on different type's cassava products consumed by the household was obtained using a multistage random technique. The result showed that demand for gari and fufu are elastic than demands for lafun meaning that lafun is a price inelastic cassava products. Expenditure elasticities of all the cassava products were examined and were found to be less than one. The highest expenditure elasticity is found for fufu suggesting that its demand will grow faster than the demand for other products as the economy develops and income increases.

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Strictly as per the compliance and regulations of:

Household Consumption of Cassava Products in Oyo State

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I. Introduction

Cassava was probably first cultivated by the Maya from which its use as food was introduced to many parts of the tropical world. The starchy root was considered of low food value hence its use as slave food. This was prejudicial to the emergence of cassava as an essential food crop with commercial potential (Mroso, 2003).The large population of the inhabitants in the tropics depends on tuber crops for the supply of carbohydrate in their diet. This is more so in the rain forest zone of the tropics where the growing of cereal is difficult (Akanbi *et al.*, 2004).

Cassava is a very versatile commodity with numerous uses and by-products. Each components of the plant can be valuable to its cultivator. The leaves may be consumed as a vegetable, or cooked as a soup ingredient or dried and fed to livestock as a protein feed supplement. The stem is used for planting propagation and grafting. The roots are typically processed for human and industrial consumption. Various products can be gotten from cassava which includes gari, lafun, wet pulp, starch, smoked cassava balls ("Kumkum"), dried cassava among others. (Truman *et al.*, 2004).

According to Tonukari (2004), cassava ranks very high among crops that convert the greatest amount of solar energy into soluble carbohydrates per unit of area. Nigeria is the largest cassava producing country in the world, Nigeria's production is 19% of world output, 34% of Africa's output and 46% of West African countries output (West Africa's countries accounts for 75% of Africa's output).Among the starchy staples, cassava gives a carbohydrate production which is about 40% higher than rice and *25%* more than maize, with the result that cassava is the cheapest source of calories for both human nutrition and animal feeding. A typical composition of the cassava root is moisture (70%), starch (24%), fiber (2%), protein (1%) and other substances including minerals (3%). A recent study on cassava shows that it accounts for about 70% of the total calories intake of more than half of the population (Nneoyi *et. al.,* 2008).

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Household consumption of cassava products is not possible without processing of cassava parts to finished products. Fresh cassava roots cannot be stored for long because they rot within 3-4days of harvest. They are bulky with about 70% moisture content and roots and leaves which contain varying amounts of cyanide which is toxic to humans and animals, while the raw cassava roots and uncooked leaves are not palatable. Reasons for processing cassava are to increase the shelf life of the products, facilitate transportation and marketing, reduce cyanide content and improve palatability, to also improve the nutritional status through fortification with other protein rich crops, to reduce food losses and stabilize seasonal fluctuations in the supply of the crop. (Hahn,1989).

World's processed products commonly known are Gari, Lafun, Fufu a dry granular meal made from moist and fermented cassava commonly used in West Africa (FAO, 1999) and are produced for human consumption(Kormawa and Akoroda,2003). It is produced mainly by small farmers especially in South and Central Nigeria and cultivated as a food and cash crop (TARCA, 2005).

Cassava therefore performs 5 major roles according to Nweke *et al.*, (2002) namely; Famine reserve crops, rural food staple, cash crop for urban consumption, industrial raw materials and earner of foreign exchange.

There exists a well-developed literature on the relationship between consumer theory and demand functions, and on empirical specification of demand functions. Deaton and Muellbauer (1980b) provide excellent reviews of both the consumer theory's implications on demand and empirical specifications. Other works on the same subjects, with somewhat different focuses, are Barten and Böhm (1982), Deaton (1986), Blundell (1988), Pollak and Wales (1992) and Barten (1993). Pollak and Wales (1992) also give a thorough treatment of functional forms used in analyses of demand systems. In this analysis, I employ the linear approximate almost ideal demand system's (LA-AIDS) technique (Deaton and Muellbauer, 1980a) via a cross-sectional model. The AIDS technique was preferred to alternative functional forms (such as the Rotterdam and translog systems) because it has the advantage that it is linear and formulated in levels. It may accordingly be encountered as more intuitive and easier to use.

In view of the above, this study therefore examined the household consumption of cassava product in Oyo State, Nigeria.

II. Methodology

The data for this study were obtained from Oyo state of Nigeria. Multistage random sampling technique was employed in the selection of respondents for the study. In the first stage the study area was stratified into four strata based on agricultural zone. The second stage involved purposive selection of two zones due to the high number of cassava product consumers. In the third stage, two local government areas (LGAs) were randomly selected from each of the zone making a total of four LGAs. The fourth stage involved random selection of two villages from each LGA making a total of eight villages. The last stage involved random selection of fifteen households in each village making a total of one hundred and twenty (120) respondents.

The study employed the Almost Ideal Demand System (AIDS) developed by Deaton and Muellbauer (1980a). The model is flexible enough to allow the assumptions of homogeneity and symmetry to be tested or successfully imposed during empirical analysis. It is easy to estimate, gives arbitrary first order estimation to any demand system, and satisfies the axioms of choice. Many of these good attributes have contributed tremendously to the application of the model to demand equation estimation in many parts of the world.

The (Almost Ideal Demand System) AIDS model of Deaton and Muelbauer (1980b) has enjoyed great popularity in applied demand analysis. Starting from a specific cost function, the AIDS model gives the share equations in an n-good system as

$$w_i \quad \alpha_i + \sum_{j}^{n} \gamma_{ij} \ln P_j + \beta_i \ln(\frac{X}{P*}) + e_i \qquad (1)$$

where w_i = budget share of ith commodity defined by

$$P_j Q_i / X$$

P_j = price of jth commodity within the group

γ_{ij} =estimated coefficient of prices

β_i =estimated expenditure coefficient

X= total expenditure on the group of goods being analyzed

P=price index for the group

The price index can be further defined as:

$$\ln P \quad \alpha_0 + \sum_{k} \alpha_k \ln P_k + \frac{1}{2} \sum_{j} \sum_{k} \gamma_{jk} \ln P_k \ln P_j \qquad (2)$$

The price index makes equation 1 to be non-linear. In order to linearize it, the Stone's index has been incorporated.

$$\ln P* \quad \sum w_j \ln p_j \qquad (3)$$

Homogeneity, symmetry, and adding up are respectively imposed on the system through the following parameter restrictions:

$$\sum_{j} \gamma_{ij} \quad 0; \gamma_{ij} \quad \gamma_{ji}; \sum_{i} a_i \quad 1; \sum a_{is} \quad 0; \sum \beta_i \quad 0; \sum \gamma_{ij} \quad 0. \qquad (4)$$

Following Chalfant (1984) and Ahmed and Shams (1994), the Marshalian and Hicksian elasticities are computed from the estimated parameters of the Linear Approximation AIDS model (LA-AIDS) in equation 4 as follows;

Marshalian (Uncompensated)

$$\varepsilon_{ij} \quad -1 + (\gamma_{ij} / w_i) - \beta_i \text{ (Own-price)} \qquad (5)$$

$$\varepsilon_{ij} \quad (\gamma_{ij} / w_i) - \beta_i (w_j / w_i) \text{ (Cross-price)} \qquad (6)$$

The expenditure elasticity is derived as

$$E_i \quad -1 + \beta_i / w_i \qquad (7)$$

III. Results and Discussions

Consumption of Cassava Products

Table 1 shows that majority of the respondents consume gari, lafun and fufu with a percentage of 79.2%, 92.5% and 71.7% respectively. From this finding, one can say that cassava products are highly consumed in the study area. The Table also shows that 20.8% of the respondents did not consume Gari, 28.3% of the respondents did not consume fufu while 7.5% did not consume Lafun. This implies that Lafun is the major staple food of the respondents.

Table 1 : Distribution of Respondents by Gari Consumption

Cassava products	Yes	No
Gari	95(79.2)	25(20.8)
Lafun	111(92.5)	9(7.5)
Fufu	86(71.7)	34(28.3)
Starch	47(39.2)	73(60.8)
Tapioca	59(49.2)	61(50.8)
Total	120	100.00

Source : Field Survey; 2010

Note : The figures in parentheses are percentages Almost Ideal Demand System.

The almost ideal Demand Systems (AIDS) was derived by Dealton and Muelbauer (1980a) from expenditure or cost function. Using this model i.e. AIDs, the demand equation for cassava products were estimated without imposition of any restrictions. From the table, the test for homogeneity was carried out. The results of the tests showed that in the consumption of cassava products, there is a significant violation of the homogeneity conditions. This result is in line with the findings of Deaton and Muellbauer (1980a), Ahmed and Shams (1994), Tsegai and Kormawa(2002) and Awoyemi *et al.,* (2006).The result of the analysis present in Table 2 shows the unconstraint parameter estimates All the Durbin –Watson statistics were shown to be within the plausible region in Table 2. The dependent and the independent variable can therefore be said to have performed their roles.

Using the budget share of Gari as the dependent variable, three variables were found to be significant which are soup at 5%, rice at 5% and expenditure at 1% level of significance. It simply implies that as the household expenditure increases, there is a decrease in the budget share of Gari. Also, there is an indirect relationship between the price of Gari and its budget share. A 1% increase in the price of soup will lead to a 0.012% increase in their budget share. Also a 1% increase in the price of rice will lead to a 0.025% increase in their budget share.

Using the budget share of Lafun as the dependent variable, five variables were found to be statistically significant at the 10%,5% and 1% level of significance. The significant variables are the prices of Lafun, Fufu, Soup, yam and total food expenditure. There is a direct relationship between the prices of Lafun, Fufu, Soup, yam and the budget share of Lafun while there is an indirect relationship between total food expenditure and budget share of Lafun. This implies that as household expenditure increase, there is a decrease in the budget share of Lafun while as the price of Fufu and soup increases the budget share of Lafun increases. In other words, the budget share of Lafun increases with increase in the price of Fufu, soup and yam. There is also a direct relationship between the price of Lafun and its budget. A 1% increase in the prices of Fufu and Soup will lead to 0.005% and 0.04a% increase in their budget share respectively.

Also using the budget share of Fufu, three variables were found to be significant which are stew at 1%, rice at 5% and expenditure at 1% level of significance. This shows that as the household expenditure increases, there is a decrease in the budget share of Fufu. There is also an indirect relationship between the price of Fufu and it's budget share. A 1% increase in the price of Stew will lead to a 0.013% increase in their budget share and a 1% increase in the price of Rice will lead to a 0.034% decrease in their budget share.

Table 2 : Unconstraint parameter estimates and test of homogeneity

Comm-odities	Const	Gari	Lafun	Fufu	Veg	Soup	Fish	Meat	Rice	Yam	Exp	R2	DW
Gari	0.075 (2.131)	-0.002 (-1.094)	0.002 (0.737)	0.002 (1.002)	0.012 (0.870)	0.012** (2.009)	0.015 (0.183)	0.030 (0.758)	0.025** (2.070)	-0.045 (-0.652)	-0.016*** (-3.031)	0.53	1.987
Lafun	0.529 (0.000)	0.002 (0.760)	0.012*** (3.245)	0.005** (2.020)	0.007 (0.029)	0.049*** (6.075)	-0.013 (-0.922)	-0.010 (-1.494)	0.028 (1.317)	0.002* (1.911)	-0.103*** (-32.663)	0.937	1.959
Fufu	0.174 (0.000)	0.001 (1.025)	0.002 (1.006)	-0.003 (-1.591)	0.012 (0.682)	0.013** (2.133)	0.020 (0.217)	-0.526 (-1.184)	-0.034** (-2.516)	0.011 (1.521)	-0.028*** (-5.151)	0.323	2.176

Source : Field survey, 2010.

NOTE : Value in parenthesis represents t-value

*Represents significant at 10% level of significance

** Represents significant at 5% level of significance

***Represents significant at 1% level of significance

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Own Price and Cross Price Elasticities

Table 3 presents the full matrices of the uncompensated (Marshallian) own price and cross price elasticities. The own-price elasticities of all cassava products under study show a negative sign (as expected), which is consistent with the law of demand. This is in line with earlier findings by Tsegai and Kormawa(2002) and Jumah *et al.*, (2008).

The estimates of own price elasticities of lafun is less than one while that of Fufu and Gari are greater than one. The own-price elasticity of lafun is equal to -0.834, meaning that an increase in the price of lafun by 10percent would decrease lafun consumption by 8.34 percent. To compensate for such consumption, household would increase gari and fufu consumption by 0.2% and 0.36% respectively. So lafun is a price inelastic cassava products and the indication of this is that households in Oyo State are insensitive to changes in the price of lafun because it serves as their major staple food. Gari and fufu however have elastic own price elasticities which means that households in Oyo State are sensitive to changes in the price of gari and fufu as they are secondary to Lafun as a staple food. The implication of this is that if the price of lafun comes down, or there is an increase in the per capita income, household consumption will not be so much affected.

The cross price elasticities are recorded as non-diagonal elements in Table 3. Yam, fufu, lafun and rice have a positive sign with respect to Gari which shows that, they bear a substitute relationship with gari. Also, fish has a negative sign implying its complementary relationship with Lafun other commodities have a substitute relationship with lafun owing to their positive signs.

Rice and meat both have negative signs indicative of their complementary relationship with fufu while other commodities have positive signs which implies that they are substitute to fufu

The gari-to lafun, the gari-to-fufu, the lafun-to-fufu etc cross-price elasticities are positive showing that they are strong substitute goods. Since all the products are substitute, higher prices for gari will lead to an increase in demand for fufu and lafun. Given that all the products are produced from cassava, an increase in the price of cassava leads to an increase in the prices of all products simultaneously, and a subsequent rise in the consumption of all products. This study is not consistent with earlier findings by Jumah *et al.*, (2008).

Table 3 : Own Price and Cross Price Elasticities

Commodities	Gari	Lafun	Fufu	Veg	Soup	Fish	Meat	Rice	Yam
Gari	-1.092	0.272	0.123	0.684	0.727	0.916	1.708	1.420	2.351
Lafun	0.021	-0.834	0.036	0.059	0.308	-0.004	0.107	0.192	0.167
Fufu	0.089	0.429	-1.148	0.772	0.913	1.374	-30.776	-1.868	0.868

Source: Field survey, 2010

Expenditure Elasticity

As shown in Table 4, the expenditure elasticities are all positive suggesting that all the cassava products are normal goods whose consumption will increase with increasing total expenditure on cassava products (see also, Abdulai and Jain, 1999: Jumah *et al.*,2008). The expenditure elasticity of all the cassava products is less than one. Cassava products are expenditure inelastic. The consumption of each of these products will decline as per capita income increases. According to the AIDS setting, the sign of the coefficient for the expenditure variable establishes whether a product group is a luxury good or a necessity. These revelations suggest that all the cassava products are necessities. This implies that as total expenditure on cassava food products increases, consumers tend to spend proportionately less on gari, lafun and fufu. The expenditure elasticity obtained for this study were not similar to that obtained by Tsegai and Kormawa (2002) for the Kaduna area and Jumah *et al.*,2008 for Lagos.

The mean budget share is considered. The highest percentage of budget for cassava product went for Lafun (84%) followed by Gari (8.2%) while Fufu had 7.8%.

Table 4 : Elasticities of Cassava Product

Commodities	Mean Budget Share (%)	Expenditure Elasticity
Gari	8.2	0.135
Lafun	84.0	0.455
Fufu	7.8	0.647

Source: Field Survey; 2010.

IV. Conclusion

The study conclude that all the estimates of own price elasticities conform to the law of demand with negative signs. Using the estimated coefficients, uncompensated price and expenditure elasticities are evaluated at the sample means. Own-price elasticity for gari is -1.092 and that of fufu is 1.148, which is relatively more elastic than that for lafun. This result indicates that demand for gari and fufu are elastic than demands for lafun in this study. All the cassava products are expenditure inelastic with fufu having the highest expenditure followed by lafun and gari respectively. Based on findings, it can be concluded that cassava products are well established staples among the inhabitants of Oyo State.

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Hypergeometric Forms of Well Known Partial Fraction Expansions of Some Meromorphic Functions

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Abstract - In this paper, we obtain hypergeometric forms of some meromorphic functions $\frac{1}{e^z-1}$, $\sec^2 z$, $\operatorname{cosec}^2 z$, $\tan z$, $\cot z$, $\operatorname{cosec} z$, $\sec z$, $\operatorname{sech}^2(z)$, $\operatorname{cosech}^2(z)$, $\tanh(z)$, $\coth(z)$, $\operatorname{cosech}(z)$, $\operatorname{sech}(z)$, $\frac{\pi}{8z^3}\frac{\sinh(2\pi z)+\sin(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)}$, $\frac{\pi}{4z}\frac{\sinh(2\pi z)-\sin(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)}$ and $\frac{\pi}{4z^2}\frac{\sinh(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)}$, from corresponding partial fraction expansions .

Keywords : Maclaurin expansion, Mittag-Leffler theorem, Bernoulli and Euler numbers, Ramanujan's transcendental functions.

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HYPERGEOMETRIC FORMS OF WELL KNOWN PARTIAL FRACTION EXPANSIONS OF SOME MEROMORPHIC FUNCTIONS

Strictly as per the compliance and regulations of:

Hypergeometric Forms of Well Known Partial Fraction Expansions of Some Meromorphic Functions

M.I. Qureshi[α], Izharul H. Khan[Ω], M.P.Chaudhary[β]

Abstract - In this paper, we obtain hypergeometric forms of some meromorphic functions $\frac{1}{e^z-1}$, $\sec^2 z$, $\operatorname{cosec}^2 z$, $\tan z$, $\cot z$, $\operatorname{cosec} z$, $\sec z$, $\operatorname{sech}^2(z)$, $\operatorname{cosech}^2(z)$, $\tanh(z)$, $\coth(z)$, $\operatorname{cosech}(z)$, $\operatorname{sech}(z)$, $\frac{\pi}{8z^3}\frac{\sinh(2\pi z)+\sin(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)}$, $\frac{\pi}{4z}\frac{\sinh(2\pi z)-\sin(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)}$ and $\frac{\pi}{4z^2}\frac{\sinh(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)}$, from corresponding partial fraction expansions.

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I. Introduction

In the monumental work of Prudnikov et al.[8,Chapter 7] and other literature of Special functions, hypergeometric forms of following functions $\sin z$, $\cos z$, $\sin^2 z$, $\cos^2 z$, $\sinh(z)$, $\cosh(z)$, $\sinh^2 z$, $\cosh^2 z$, $\sin^{-1} z$, $(\sin^{-1} z)^2$, $\cos^{-1} z$, $\sec^{-1} z$, $\operatorname{cosec}^{-1} z$, $\tan^{-1} z$, $\cot^{-1} z$, $\frac{\sin^{-1} z}{\sqrt{1-z^2}}$, $\sinh^{-1} z$, $(\sinh^{-1} z)^2$, $\cosh^{-1} z$, $\operatorname{sech}^{-1} z$, $\operatorname{cosech}^{-1} z$, $\tanh^{-1} z$, $\coth^{-1} z$, $\frac{\sinh^{-1} z}{\sqrt{1+z^2}}$, $\log_a(1\pm z)$, $\ln(1\pm z)$, $e^{\pm z}$, $a^{\pm z}$, $(1\pm z)^{\pm a}$, $\sin(a\sin^{-1} z)$, $\cos(a\sin^{-1} z)$, $\frac{\cos(a\sin^{-1} z)}{\sqrt{1-z^2}}$, $\frac{\sin(a\sin^{-1} z)}{\sqrt{1-z^2}}$, associated composite functions and transcendental functions, are available.

In the Maclaurin's expansions of $\tan z$, $\cot z$, $\operatorname{cosec} z$, $\tanh(z)$, $\coth(z)$, $\operatorname{cosech}(z)$ and $\sec z$, $\operatorname{sech}(z)$, the coefficients of z^n are associated with Bernoulli numbers and Euler numbers [15] respectively.From Maclaurin's expansions, we are unable to obtain their corresponding hypergeometric forms.

Now we shall find the hypergeometric forms of $\tan z$, $\operatorname{cosec} z$, $\cot z$, $\sec z$ and other associated composite functions by means of corresponding partial fraction expansions obtained by Mittag-Leffler theorem or Fourier series method [5; pp.602-603 .

The Pochhammer's symbol or Shifted factorial (h_r) is defined by

$$(h)_r = \frac{\Gamma(h+r)}{\Gamma(h)} = \begin{cases} 1 & ;\ \text{if } r=0 \\ h(h+1)\cdots(h+r-1) & ;\ \text{if } r=1,2,3,\cdots \end{cases} . \tag{1}$$

where h $0,-1,-2,\cdots$ and the notation (Γ) stands for Gamma function.

Lemma: If a, p and n are suitably adjusted real or complex numbers such that associated Pochhammer's symbols are well-defined, then we have

$$(a+pn) = \frac{a\left(\frac{a+p}{p}\right)_n}{\left(\frac{a}{p}\right)_n} \tag{2}$$

Mittag - Leffler's expansion theorem[4;7;14;15;16]

(i) Suppose that the only singularities of f( ), except at infinity, in the finite plane are the simple poles at the points $z=a_1, z=a_2, z=a_3,\cdots$ arranged in order of increasing absolute value, that is: $\cdots\cdots \geq |a_3| \geq |a_2| \geq |a_1| > 0$

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(ii) Let the residues of $f(z)$ at $a_1, a_2, a_3, \cdots$ be $b_1, b_2, b_3, \cdots$ respectively.

(iii) Let C_N be circles of radius R_N which do not pass through any poles and on which $|f(z)| < M$, where M is independent of N and $R_N \to \infty$ as $N \to \infty$.

When these conditions are satisfied then Mittag-Leffler's expansion theorem states that

$$f(z) = f(0) + \sum_{n=1}^{\infty} b_n \left\{ \frac{1}{z - a_n} + \frac{1}{a_n} \right\} \tag{3}$$

for all values of z except the poles.

In the literature of calculus of residues [2 to 7; 12 to 16], following partial fraction expansions are available.

[2, pp.296-297; 6, p.240(Q.No.3); 16, p.113]

$$\frac{1}{e^z - 1} = \frac{1}{z} - \frac{1}{2} + 2z \sum_{n=1}^{\infty} \frac{1}{4n^2\pi^2 + z^2} \qquad ; z \neq 0, \pm 2i\pi, \pm 4i\pi, \qquad \cdots (4)$$

[15, p.187]

$$\sec^2 z = 4\left\{ \frac{1}{(\pi - 2z)^2} + \frac{1}{(\pi + 2z)^2} + \frac{1}{(3\pi - 2z)^2} + \frac{1}{(3\pi + 2z)^2} + \cdots \right\}$$

$$= 4 \sum_{n=-\infty}^{+\infty} \frac{1}{[(2n+1)\pi + 2z]^2} \qquad ; z \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \qquad \cdots (5)$$

[7, p.135; 16, p.113]

$$\operatorname{cosec}^2 z = \frac{1}{z^2} + \frac{1}{(z - \pi)^2} + \frac{1}{(z + \pi)^2} + \frac{1}{(z - 2\pi)^2} + \frac{1}{(z + 2\pi)^2} + \cdots$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{(z - n\pi)^2} \qquad ; z \neq 0, \pm\pi, \pm 2\pi, \pm 3\pi, \qquad \cdots (6)$$

[2, p.296; 4, p.157(Q.No.36); 16, p.113]

$$\tan z = 8z\left\{ \frac{1}{\pi^2 - 4z^2} + \frac{1}{9\pi^2 - 4z^2} + \frac{1}{25\pi^2 - 4z^2} + \cdots \right\} \qquad ; z \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \cdots$$

$$= 2z \sum_{n=0}^{\infty} \frac{1}{(n + \frac{1}{2})^2\pi^2 - z^2} = -\sum_{n=-\infty}^{+\infty} \left\{ \frac{1}{z - (n + \frac{1}{2})\pi} + \frac{1}{(n + \frac{1}{2})\pi} \right\} \tag{7}$$

[3, p.122(Q.No.8); 5, p.602; 12, p.310(Q.No.14)]

$$\cot z = \frac{1}{z} + 2z\left\{ \frac{1}{z^2 - \pi^2} + \frac{1}{z^2 - 4\pi^2} + \frac{1}{z^2 - 9\pi^2} + \cdots \right\}$$

$$= \frac{1}{z} + 2z \sum_{n=1}^{\infty} \frac{1}{z^2 - n^2\pi^2} \qquad ; z \neq 0, \pm\pi, \pm 2\pi, \pm 3\pi, \qquad \cdots (8)$$

[3, p.122(Q.No.9); 4, p.147; 7, pp.132-133]

$$\operatorname{cosec} z = \frac{1}{z} - 2z\left\{\frac{1}{z^2-\pi^2} - \frac{1}{z^2-4\pi^2} + \frac{1}{z^2-9\pi^2} - \cdots\right\}$$

$$= \frac{1}{z_n} + 2z\sum_{=1}^{\infty}\frac{(-1)^{n-1}}{n^2\pi^2-z^2} \qquad ; z \neq 0, \pm\pi, \pm 2\pi, \pm 3\pi, \qquad \cdots (9)$$

[4, p.156(Q.No.34); 5, p.603; 7, p.137(Q.No.18)]

$$\sec z = 4\pi\left\{\frac{1}{\pi^2-4z^2} - \frac{3}{9\pi^2-4z^2} + \frac{5}{25\pi^2-4z^2} - \cdots\right\}$$

$$= 2\pi\sum_{n=0}^{\infty}\frac{(-1)^n\,(n+\frac{1}{2})}{(n+\frac{1}{2})^2\pi^2-z^2} \qquad ; z \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \qquad \cdots (10)$$

[2, p.296; 15, p.187]

$$\tanh(z) = 8z\left\{\frac{1}{\pi^2+4z^2} + \frac{1}{9\pi^2+4z^2} + \frac{1}{25\pi^2+4z^2} + \cdots\right\}$$

$$= 2z\sum_{n=0}^{\infty}\frac{1}{(n+\frac{1}{2})^2\pi^2+z^2} \qquad ; z \neq \pm\frac{i\pi}{2}, \pm\frac{3i\pi}{2}, \pm\frac{5i\pi}{2}, \qquad \cdots (11)$$

[2, p.296; 7, p.134]

$$\coth(z) = \frac{1}{z} + 2z\left\{\frac{1}{z^2+\pi^2} + \frac{1}{z^2+4\pi^2} + \frac{1}{z^2+9\pi^2} + \cdots\right\}$$

$$= \frac{1}{z} + 2z\sum_{n=1}^{\infty}\frac{1}{z^2+n^2\pi^2} \qquad ; z \neq 0, \pm i\pi, \pm 2i\pi, \pm 3i\pi, \qquad \cdots (12)$$

[2, p.296; 7, p.135]

$$\operatorname{cosech}(z) = \frac{1}{z} - 2z\left\{\frac{1}{z^2+\pi^2} - \frac{1}{z^2+4\pi^2} + \frac{1}{z^2+9\pi^2} - \cdots\right\}$$

$$= \frac{1}{z} + 2z\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2\pi^2+z^2} \qquad ; z \neq 0, \pm i\pi, \pm 2i\pi, \pm 3i\pi, \qquad \cdots (13)$$

[14, p.175; 15, p.187]

$$\operatorname{sech}(z) = 4\pi\left\{\frac{1}{\pi^2+4z^2} - \frac{3}{9\pi^2+4z^2} + \frac{5}{25\pi^2+4z^2} - \cdots\right\}$$

$$= 2\pi\sum_{n=0}^{\infty}\frac{(-1)^n\,(n+\frac{1}{2})}{(n+\frac{1}{2})^2\pi^2+z^2} \qquad ; z \neq \pm\frac{i\pi}{2}, \pm\frac{3i\pi}{2}, \pm\frac{5i\pi}{2}, \qquad \cdots (14)$$

Ramanujan's partial fraction expansions [1, Part -IV, pp.380-381]

Ramanujan's systematic work on ordinary hypergeometric series is contained primarily in Chapters XII, XIII, XV of first notebook [9] and Chapters III, X and XI of second notebook [10]. Ramanujan evidently had an affinity for partial fraction expansions, which can be found in several places in his notebooks. The heaviest concentrations lie in Chapters 14 and 18 and in the unorganized pages at the end of the second notebook. See Berndt's books [part-II] and [part-III] for accounts of the material in Chapters 14 and 18, respectively. In this paper, we obtain the hypergeometric forms of three partial fraction decompositions in the unorganized pages of second notebook.

When $z \neq \frac{m}{2}(1 \pm i) \quad ; m = 0, \pm 1, \pm 2, \pm 3, \cdots$

then [1(Part-IV), pp.380-381, Entry 13; see also 7, p.137 (Q.No.20 i)]

$$\frac{\pi}{8z^3}\,\frac{\sinh(2\pi z)+\sin(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)} = \frac{1}{8z^4} + \sum_{n=1}^{\infty}\frac{1}{4z^4+n^4} \tag{15}$$

[1(Part-IV), pp.380-381, Entry 14]

$$\frac{\pi}{4z}\,\frac{\sinh(2\pi z)-\sin(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)} = \sum_{n=1}^{\infty}\frac{n^2}{4z^4+n^4} \tag{16}$$

[1(Part-IV), pp.380-381, Entry 15]

$$\frac{\pi}{4z^2}\,\frac{\sinh(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)} = \frac{1}{8z^3} + \sum_{n=1}^{\infty}\frac{n}{4z^4+n^4} + \frac{1}{2z}\sum_{n=1}^{\infty}\frac{1}{z^2+(z+n)^2} \tag{17}$$

II. Hypergeometric Forms of Some Partial Fraction Expansions

If we apply the Lemma (2) in real or complex linear factors of quadratic and biquadratic polynomials in n, associated with the denominators of partial fraction expansions (4) to (17), we get the following hypergeometric forms:

$$\frac{1}{e^z-1} = \frac{1}{z} - \frac{1}{2} + \frac{2z}{(z^2+4\pi^2)}\,{}_3F_2\left[\begin{matrix}1\,, & \frac{2\pi+iz}{2\pi}, & \frac{2\pi-iz}{2\pi}\,; \\ & \frac{4\pi+iz}{2\pi}\,, & \frac{4\pi-iz}{2\pi}\,;\end{matrix}\;1\right] \;; z \neq 0, \pm 2i\pi, \qquad \cdots(18)$$

$$\sec^2 z = \frac{4}{(2z+\pi)^2}\,{}_2H_2\left[\begin{matrix}\frac{\pi+2z}{2\pi}, & \frac{\pi+2z}{2\pi}\,; \\ \frac{3\pi+2z}{2\pi}\,, & \frac{3\pi+2z}{2\pi}\,;\end{matrix}\;1\right] \qquad ; z \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \qquad \cdots(19)$$

$$\sec^2 z = \frac{4}{(2z-\pi)^2}\,{}_3F_2\left[\begin{matrix}1\,, & \frac{\pi-2z}{2\pi}, & \frac{\pi-2z}{2\pi}\,; \\ & \frac{3\pi-2z}{2\pi}\,, & \frac{3\pi-2z}{2\pi}\,;\end{matrix}\;1\right] + \frac{4}{(2z+\pi)^2}\,{}_3F_2\left[\begin{matrix}1\,, & \frac{\pi+2z}{2\pi}, & \frac{\pi+2z}{2\pi}\,; \\ & \frac{3\pi+2z}{2\pi}\,, & \frac{3\pi+2z}{2\pi}\,;\end{matrix}\;1\right] \tag{20}$$

$$\mathrm{cosec}^2 z = \frac{1}{z^2}\,{}_2H_2\left[\begin{matrix}-\frac{z}{\pi}, & -\frac{z}{\pi}\,; \\ \frac{\pi-z}{\pi}\,, & \frac{\pi-z}{\pi}\,;\end{matrix}\;1\right] \qquad ; z \neq 0, \pm\pi, \pm 2\pi, \pm 3\pi, \qquad \cdots(21)$$

$$\mathrm{cosec}^2 z = \frac{1}{(z+\pi)^2}\,{}_3F_2\left[\begin{matrix}1\,, & \frac{\pi+z}{\pi}, & \frac{\pi+z}{\pi}\,; \\ & \frac{2\pi+z}{\pi}\,, & \frac{2\pi+z}{\pi}\,;\end{matrix}\;1\right] + \frac{1}{z^2}\,{}_3F_2\left[\begin{matrix}1\,, & -\frac{z}{\pi}, & -\frac{z}{\pi}\,; \\ & \frac{\pi-z}{\pi}\,, & \frac{\pi-z}{\pi}\,;\end{matrix}\;1\right] \tag{22}$$

$$\tan z = \frac{8z}{(\pi^2-4z^2)}\, {}_3F_2\left[\begin{array}{ccc} 1\,, & \frac{\pi+2z}{2\pi}, & \frac{\pi-2z}{2\pi}\,; \\ & \frac{3\pi+2z}{2\pi}\,, & \frac{3\pi-2z}{2\pi}\,; \end{array} 1\right] \quad ; z \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \qquad \cdots(23)$$

$$\cot z = \frac{1}{z} + \frac{2z}{(z^2-\pi^2)}\, {}_3F_2\left[\begin{array}{ccc} 1\,, & \frac{\pi+z}{\pi}, & \frac{\pi-z}{\pi}\,; \\ & \frac{2\pi+z}{\pi}\,, & \frac{2\pi-z}{\pi}\,; \end{array} 1\right] \quad ; z \neq 0, \pm\pi, \pm 2\pi, \qquad \cdots(24)$$

$$\text{cosec}\, z = \frac{1}{z} + \frac{2z}{(\pi^2-z^2)}\, {}_3F_2\left[\begin{array}{ccc} 1\,, & \frac{\pi+z}{\pi}, & \frac{\pi-z}{\pi}\,; \\ & \frac{2\pi+z}{\pi}\,, & \frac{2\pi-z}{\pi}\,; \end{array} -1\right] \quad ; z \neq 0, \pm\pi, \pm 2\pi, \qquad \cdots(25)$$

$$\sec z = \frac{4\pi}{(\pi^2-4z^2)}\, {}_4F_3\left[\begin{array}{cccc} 1\,, & \frac{3}{2}\,, & \frac{\pi+2z}{2\pi}\,, & \frac{\pi-2z}{2\pi}\,; \\ & \frac{1}{2}\,, & \frac{3\pi+2z}{2\pi}\,, & \frac{3\pi-2z}{2\pi}\,; \end{array} -1\right] \quad ; z \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \qquad \cdots(26)$$

By replacing by in (19) to (26) and using the identities $\sec(iz) = \text{sech}(z)$, $\text{cosec}\,(iz) = -i\,\text{cosech}(z)$,$\tan(iz) = i\tanh(z)$,$\cot(iz) = -i\coth(z)$, we get the hypergeometric forms of corresponding hyperbolic functions.

$$\text{sech}^2(z) = \frac{4}{(2iz+\pi)^2}\, {}_2H_2\left[\begin{array}{cc} \frac{\pi+2iz}{2\pi}, & \frac{\pi+2iz}{2\pi}\,; \\ \frac{3\pi+2iz}{2\pi}\,, & \frac{3\pi+2iz}{2\pi}\,; \end{array} 1\right] \quad ; z \neq \pm\frac{i\pi}{2}, \pm\frac{3i\pi}{2}, \qquad \cdots(27)$$

$$\text{sech}^2(z) = \frac{4}{(\pi-2iz)^2}\, {}_3F_2\left[\begin{array}{ccc} 1\,, & \frac{\pi-2iz}{2\pi}, & \frac{\pi-2iz}{2\pi}\,; \\ & \frac{3\pi-2iz}{2\pi}\,, & \frac{3\pi-2iz}{2\pi}\,; \end{array} 1\right] + \frac{4}{(\pi+2iz)^2}\, {}_3F_2\left[\begin{array}{ccc} 1\,, & \frac{\pi+2iz}{2\pi}, & \frac{\pi+2iz}{2\pi}\,; \\ & \frac{3\pi+2iz}{2\pi}\,, & \frac{3\pi+2iz}{2\pi}\,; \end{array} 1\right] \qquad (28)$$

$$\text{cosech}^2(z) = \frac{1}{z^2}\, {}_2H_2\left[\begin{array}{cc} -\frac{iz}{\pi}, & -\frac{iz}{\pi}\,; \\ \frac{\pi-iz}{\pi}\,, & \frac{\pi-iz}{\pi}\,; \end{array} 1\right] \quad ; z \neq 0, \pm i\pi, \pm 2i\pi, \pm 3i\pi, \qquad \cdots(29)$$

$$\text{cosech}^2(z) = \frac{1}{z^2}\, {}_3F_2\left[\begin{array}{ccc} 1\,, & -\frac{iz}{\pi}, & -\frac{iz}{\pi}\,; \\ & \frac{\pi-iz}{\pi}\,, & \frac{\pi-iz}{\pi}\,; \end{array} 1\right] - \frac{1}{(\pi+iz)^2}\, {}_3F_2\left[\begin{array}{ccc} 1\,, & \frac{\pi+iz}{\pi}, & \frac{\pi+iz}{\pi}\,; \\ & \frac{2\pi+iz}{\pi}\,, & \frac{2\pi+iz}{\pi}\,; \end{array} 1\right] \qquad (30)$$

$$\tanh(z) = \frac{8z}{(\pi^2+4z^2)}\, {}_3F_2\left[\begin{array}{ccc} 1\,, & \frac{\pi+2iz}{2\pi}, & \frac{\pi-2iz}{2\pi}\,; \\ & \frac{3\pi+2iz}{2\pi}\,, & \frac{3\pi-2iz}{2\pi}\,; \end{array} 1\right] \quad ; z \neq \pm\frac{i\pi}{2}, \pm\frac{3i\pi}{2}, \qquad \cdots(31)$$

$$\coth(z) = \frac{1}{z} + \frac{2z}{(z^2+\pi^2)}\, {}_3F_2\left[\begin{array}{ccc} 1\,, & \frac{\pi+iz}{\pi}, & \frac{\pi-iz}{\pi}\,; \\ & \frac{2\pi+iz}{\pi}\,, & \frac{2\pi-iz}{\pi}\,; \end{array} 1\right] \quad ; z \neq 0, \pm i\pi, \pm 2i\pi, \qquad \cdots(32)$$

$$\operatorname{cosech}(z)=\frac{1}{z}-\frac{2z}{(\pi^2+z^2)}\,{}_3F_2\left[\begin{matrix}1\,,\ \frac{\pi+iz}{\pi},\ \frac{\pi-iz}{\pi}\ ;\\ \frac{2\pi+iz}{\pi}\,,\ \frac{2\pi-iz}{\pi}\ ;\end{matrix}\ -1\right]\ ;z\neq 0,\pm i\pi,\pm 2i\pi,\qquad\cdots(33)$$

$$\operatorname{sech}(z)=\frac{4\pi}{(\pi^2+4z^2)}\,{}_4F_3\left[\begin{matrix}1\,,\ \frac{3}{2}\,,\ \frac{\pi+2iz}{2\pi}\,,\ \frac{\pi-2iz}{2\pi}\ ;\\ \frac{1}{2}\,,\ \frac{3\pi+2iz}{2\pi}\,,\ \frac{3\pi-2iz}{2\pi}\ ;\end{matrix}\ -1\right]\ ;z\neq\pm\frac{i\pi}{2},\pm\frac{3i\pi}{2},\qquad\cdots(34)$$

When $z\neq\frac{m}{2}(1\pm i)\quad;m=0,\pm1,\pm2,\pm3,\cdots$,then

$$\frac{\pi}{8z^3}\,\frac{\sinh(2\pi z)+\sin(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)}=\frac{1}{8z^4}+\frac{1}{(4z^4+1)}\times$$

$$\times\,{}_5F_4\left[\begin{matrix}1\,,\ -z+1+iz\,,\ -z+1-iz\,,\ z+1+iz\,,\ z+1-iz\ ;\\ -z+2+iz\,,\ -z+2-iz\,,\ z+2+iz\,,\ z+2-iz\ ;\end{matrix}\ 1\right]\qquad(35)$$

$$\frac{\pi}{4z}\,\frac{\sinh(2\pi z)-\sin(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)}=\frac{1}{(4z^4+1)}\times$$

$$\times\,{}_6F_5\left[\begin{matrix}2\,,\ 2\,,\ -z+1+iz\,,\ -z+1-iz\,,\ z+1+iz\,,\ z+1-iz\ ;\\ 1\,,\ -z+2+iz\,,\ -z+2-iz\,,\ z+2+iz\,,\ z+2-iz\ ;\end{matrix}\ 1\right]\qquad(36)$$

$$\frac{\pi}{4z^2}\,\frac{\sinh(2\pi z)}{\cosh(2\pi z)-\cos(2\pi z)}=\frac{1}{8z^3}+$$

$$+\frac{1}{(4z^4+1)}\,{}_5F_4\left[\begin{matrix}2\,,\ -z+1+iz\,,\ -z+1-iz\,,\ z+1+iz\,,\ z+1-iz\ ;\\ -z+2+iz\,,\ -z+2-iz\,,\ z+2+iz\,,\ z+2-iz\ ;\end{matrix}\ 1\right]+$$

$$+\frac{1}{(4z^3+4z^2+2z)}\,{}_3F_2\left[\begin{matrix}1\,,\ z+1+iz\,,\ z+1-iz\ ;\\ z+2+iz\,,\ z+2-iz\ ;\end{matrix}\ 1\right]\qquad(37)$$

Above hypergeometric forms are not available in the literature. It is to be noted that the hypergeometric series ${}_3F_2$, ${}_4F_3$, ${}_5F_4$ and ${}_6F_5$ are convergent.

III. Proofs

To derive (18), consider the following partial fraction expansion

$$\frac{1}{e^z-1}=\frac{1}{z}-\frac{1}{2}+2z\sum_{n=1}^{\infty}\frac{1}{z^2+4n^2\pi^2}\qquad;z\neq 0,\pm2i\pi,\pm4i\pi,\pm6i\pi,\cdots$$

$$=\frac{1}{z}-\frac{1}{2}+2z\sum_{n=0}^{\infty}\frac{1}{[z+2i(n+1)\pi]\ [z-2i(n+1)\pi]}$$

$$=\frac{1}{z}-\frac{1}{2}+2z\sum_{n=0}^{\infty}\frac{1}{[(z+2i\pi)+(2i\pi)n]\ [(z-2i\pi)+(-2i\pi)n]}$$

Now using the beautiful Lemma (2), we get

$$\frac{1}{e^z-1} = \frac{1}{z}-\frac{1}{2}+2z\sum_{n=0}^{\infty}\frac{\left(\frac{z+2i\pi}{2i\pi}\right)_n\left(\frac{z-2i\pi}{-2i\pi}\right)_n}{(z+2i\pi)\left(\frac{z+4i\pi}{2i\pi}\right)_n(z-2i\pi)\left(\frac{z-4i\pi}{-2i\pi}\right)_n}$$

$$= \frac{1}{z}-\frac{1}{2}+\frac{2z}{(z^2+4\pi^2)}\,{}_3F_2\left[\begin{matrix}1\,, & \frac{2i\pi+z}{2i\pi}, & \frac{2i\pi-z}{2i\pi}\,; \\ & & & 1 \\ & \frac{4i\pi+z}{2i\pi}\,, & \frac{4i\pi-z}{2i\pi}\,;\end{matrix}\right]$$

$$\frac{1}{e^z-1} = \frac{1}{z}-\frac{1}{2}+\frac{2z}{(z^2+4\pi^2)}\,{}_3F_2\left[\begin{matrix}1\,, & \frac{2\pi+iz}{2\pi}, & \frac{2\pi-iz}{2\pi}\,; \\ & & & 1 \\ & \frac{4\pi+iz}{2\pi}\,, & \frac{4\pi-iz}{2\pi}\,;\end{matrix}\right]$$

To derive (21) and (22), consider the following expansion

$$\operatorname{cosec}^2 z = \sum_{n=-\infty}^{+\infty}\frac{1}{(z-n\pi)^2} \qquad ; z\neq 0,\pm\pi,\pm2\pi,\pm3\pi,\cdots\cdots$$

$$= \sum_{n=-\infty}^{+\infty}\frac{1}{[z+(-\pi)n]\,[z+(-\pi)n]}$$

Now using the beautiful Lemma (2), we get

$$\operatorname{cosec}^2 z = \sum_{n=-\infty}^{+\infty}\frac{\left(\frac{z}{-\pi}\right)_n\left(\frac{z}{-\pi}\right)_n}{z\left(\frac{z-\pi}{-\pi}\right)_n z\left(\frac{z-\pi}{-\pi}\right)_n} = \frac{1}{z^2}\,{}_2H_2\left[\begin{matrix}-\frac{z}{\pi}, & -\frac{z}{\pi}\,; \\ & & 1 \\ \frac{\pi-z}{\pi}\,, & \frac{\pi-z}{\pi}\,;\end{matrix}\right]$$

which is the hypergeometric form (21).

Now replacing z by iz and using suitable circular-hyperbolic identity, we get (29).

Now again consider,

$$\operatorname{cosec}^2 z = \sum_{n=-\infty}^{+\infty}\frac{1}{(z-n\pi)^2} \qquad ; z\neq 0,\pm\pi,\pm2\pi,\pm3\pi,\cdots\cdots$$

$$= \sum_{n=-\infty}^{-1}\frac{1}{(z-n\pi)^2}+\sum_{n=0}^{\infty}\frac{1}{(z-n\pi)^2} = \sum_{n=1}^{\infty}\frac{1}{(z+n\pi)^2}+\sum_{n=0}^{\infty}\frac{1}{(z-n\pi)^2}$$

$$= \sum_{n=0}^{\infty}\frac{1}{[(z+\pi)+\pi n]^2}+\sum_{n=0}^{\infty}\frac{1}{[z+(-\pi)n]^2}$$

Now using the Lemma (2), we get

$$\operatorname{cosec}^2 z = \sum_{n=0}^{\infty}\frac{(1)_n\left(\frac{z+\pi}{\pi}\right)_n^2}{(z+\pi)^2\left(\frac{z+2\pi}{\pi}\right)_n^2 n!}+\sum_{n=0}^{\infty}\frac{(1)_n\left(\frac{z}{-\pi}\right)_n^2}{z^2\left(\frac{z-\pi}{-\pi}\right)_n^2 n!}$$

$$= \frac{1}{(z+\pi)^2}\,{}_3F_2\left[\begin{matrix}1\,, & \frac{\pi+z}{\pi}, & \frac{\pi+z}{\pi}\,; \\ & & & 1 \\ & \frac{2\pi+z}{\pi}\,, & \frac{2\pi+z}{\pi}\,;\end{matrix}\right] + \frac{1}{z^2}\,{}_3F_2\left[\begin{matrix}1\,, & -\frac{z}{\pi}, & -\frac{z}{\pi}\,; \\ & & & 1 \\ & \frac{\pi-z}{\pi}\,, & \frac{\pi-z}{\pi}\,;\end{matrix}\right]$$

which is the hypergeometric form (22).

Now replacing z by iz, we get (30).

Similarly, we can obtain the hypergeometric forms (19), (20), (23) to (28), (31) to (37) of remaining partial fraction expansions.

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Solar Powered Distillation of Lagos Bar Beach Water

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Abstract - In this study Lagos Bar-beach water was distilled using a rectangular solar still. The solar still is a single slope type inclined 22o to the horizontal in the north-south direction. The various chemical and biological parameters of the seawater were analysed before and after distillation. The results showed a total dissolved solid value of 4,014 mg/l and total coliform count of 380 cfu/ml. Thus, Lagos bar-beach water is not suitable for human consumption due to basically high total coliform count far above acceptable maximum limit of 10 cfu/ml. However after the solar distillation zero coliform count and no dissolved solids were recorded. Data generated from the performance evaluation of the solar still over an ambient day time temperature range of between 23–31 °C, and a daily global irradiation range of between 7.5–17.1 MJ m2 that involved physical monitoring of the temperatures of different components of the solar still to determine its efficiency, also validated this result.

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GJSFR Classification: *FOR Code: 090605*

SOLAR POWERED DISTILLATION OF LAGOS BAR BEACH WATER

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J. I. Eze, Onyekwere[α], Ojikeand Ejilah, I.R[Ω]

Abstract - In this study Lagos Bar-beach water was distilled using a rectangular solar still. The solar still is a single slope type inclined 22° to the horizontal in the north-south direction. The various chemical and biological parameters of the seawater were analysed before and after distillation. The results showed a total dissolved solid value of 4,014 mg/l and total coliform count of 380 cfu/ml. Thus, Lagos bar-beach water is not suitable for human consumption due to basically high total coliform count far above acceptable maximum limit of 10 cfu/ml. However after the solar distillation zero coliform count and no dissolved solids were recorded. Data generated from the performance evaluation of the solar still over an ambient day time temperature range of between 23–31 °C, and a daily global irradiation range of between 7.5–17.1 MJ m^2 that involved physical monitoring of the temperatures of different components of the solar still to determine its efficiency, also validated this result.

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I. Introduction

Safe drinking water remains inaccessible for about 1.1 billion people in the world, and the hourly toll from biological contamination of drinking water is 400 deaths of children below age 5 (Gadgil, 1998). Availability of plentiful and safe water for domestic use has long been known to be fundamental to the development process, with benefits, such as labour productivity, spread across all sectors. Most recently, the UN General Assembly declared the period from 2005 to 2015 as the International Decade for Action, "Water for Life" (WHO, 2008). According to Kalogirou (2005) "Water is one of the most abundant resources on earth, covering three-fourths of the planet's surface. About 97% of the earth's water is salt water in the oceans and 3% (about 36 million km^3) is fresh water contained in the poles, ground water, lakes and rivers, which supply most of human and animal needs. Nearly, 70% from this tiny 3% of the world's fresh water is frozen in glaciers, permanent snow cover, ice and permafrost. Thirty percent of all fresh water is underground, most of it in deep, hard-to-reach aquifers. Lakes and rivers together contain just a little more than 0.25% of all fresh water". The only nearly inexhaustible sources of water are the oceans. Their main demerit is their high salinity. Thus, desalination is a major way of making ocean water (seawater) accessible to mankind. Desalination refers to the removal of salts and minerals, as in soil desalination. Water is desalinated in order to convert salt water to fresh water to make it suitable for human consumption or irrigation. Sometimes the process produces table salt as a by-product.

Large-scale desalination typically uses extremely large amounts of energy as well as specialized, expensive infrastructure, making it very costly compared to the use of fresh water from rivers or groundwater. Generally, the energy requirements of desalination processes are high (Fischetti, 2007) making it difficult in developing countries or isolated areas where electricity is erratic, unreliable, and a high percentage of the population is not on the electricity grid (UNDP, 2002). According to (García-Rodríguez, 2003), since most arid regions have high renewable energy resources, the use of renewable energies in seawater desalination exhibits an interesting chance, or even the only way to offer a secure source of fresh water.

Despite the continual technological progress in desalination methods, the conventional solar still continues to be a choice that can be made, mainly for remote areas, due to the known advantages it has, such as use of free energy without harming the environment, autonomous operation independent of conventional energy sources and need for simple technological and construction solutions that can be implemented locally (Mathioulakis and Belessiotis, 2003). Solar still uses the principle of distillation in its operation.

Solar distillation is a technique to distillate water using solar energy. Distillation is the oldest and most commonly used method of desalination. It is a phase separation method whereby saline water is heated to produce water vapour, which is then condensed to produce freshwater. Distillation units routinely use designs that conserve as much thermal energy as possible by interchanging the heat of condensation and heat of vaporization within the units. The major energy requirement in the distillation process thus becomes providing the heat for vaporization to the feed water.

In this study, Lagos bar beach water (seawater) was distilled using a locally made solar still to obtain a distilled water and the result compared to the WHO standards for drinking water.

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II. Theory of the Passive Solar Still

The solar still operation is governed by various heat and mass transfer modes occurring in the system. Within the solar still the convection heat and mass transfer from water surface to the inner glass cover surface can be observed (Fig. 1). The trapped long wave radiation from incident solar radiation heats up the water by way of "green house effect". The hot water evaporates and condenses on the transparent cover. Energy considerations in a passive solar still yield the following equations (Kurmar et al., 2000; Azi and Iyoha, 2007):

Glass cover;

$$\alpha_g I(t)A_g + h_{wg}(T_w - T_g)A_w = h_{ga}(T_g - T_a)A_g \qquad (1)$$

Water;

$$\alpha_w(1-\alpha_g)I(t)A_w + h_w(T_b - T_w)A_b = (m_w C_w)\frac{dT_w}{dt} + h_{wg}(T_w - T_g)A_w \qquad (2)$$

Basin;

$$\alpha_w(1-\alpha_g)(1-\alpha_w)I(t)A_b = [h_{bw}(T_b - T_w) + h_{ba}(T_b - T_a)]A_b \qquad (3)$$

Given T_g and T_w, the hourly production of the still is calculated as

$$\dot{m}_{ew} = \frac{h_{ew}(T_w - T_g)\times 3600}{L_w}\,Kg/m^2 - h \qquad (4)$$

Where T_o, T_g, T_w and T_a are outer surface cover, inside surface cover, basin saline water and ambient temperatures respectively while h_{ew} and L_w are evaporative heat transfer coefficient from the water surface to the glass cover and latent heat of vapourisation of water respectively. The above equations were derived based on the assumptions that; the unit is in the quasi-steady state, airtight and perfectly insulated. Absorption coefficients and heat capacities of the transparent cover and water are deemed negligible.

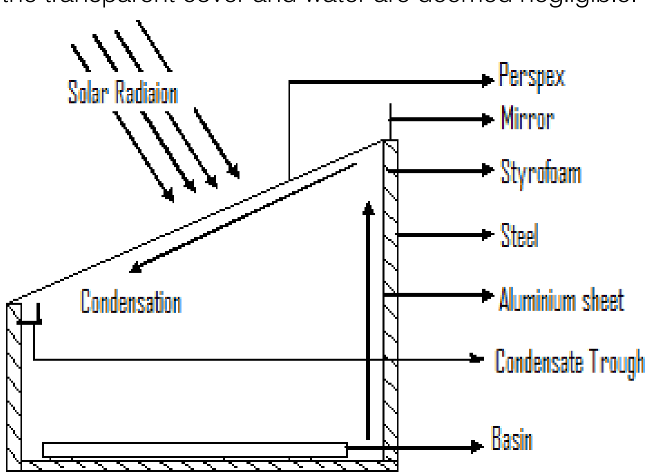

Fig. 1 : Schematic diagram of a solar still

Still Efficiency Determination: the experimental steady state efficiency (η) of the solar still is given as (Hamdan et al., 1999)

$$\eta = \frac{mL_w}{GA_g\Delta t} \qquad (5)$$

where m, L_w, G, A_g and Δt are the mass condensate collected in a time interval, water latent heat of evaporation, hourly solar radiation flux, the glass collecting area and the time interval, respectively. Also, the daily efficiency (η_d) of the solar still is given as (Swelam, 2005):

$$\eta_d = \frac{\sum mL_w}{\sum AGt} \qquad (6)$$

Equation was used to determine the still daily efficiency, as it is summing up the hourly condensate production (m) multiplied by the latent heat of evaporation (L_w), divided by the summation of the average daily solar radiation (G), the whole still area (A) and time of (t).

III. Materials and Methods

The solar still used is shown in Fig. 2 and developed at the National Centre for energy research and development, University of Nigeria Nsukka. It is a rectangular box with a transparent slanted Perspex cover. The perspex cover is inclined at an angle of 22^0 to the horizontal. The solar still is oriented facing south as recommended by Duffie and Beckman (1991).

Fig. 2: The Solar Still

The effective total absorber area is $0.6m^2$. The body of the box is made of prefabricated fibre reinforce plastic 0.005m thick. The interior of the rectangular basin is painted black. Inside the solar still is a rectangular black steel basin of area 0.5m2 and height 0.06m. The basin serves as the container for the seawater. A mirror of $0.18m^2$ was fitted on the still side walls. During the study, four litres of Lagos Bar-beach water was poured into the solar still basin for distillation. The water evaporates only to condense on the underside of the transparent cover, leaving other constituents of the seawater behind. The gentle slope of the glass directs the condensate to a condensate trough from where the water runs out to a storage vessel. Periodically, the temperatures of the seawater, the base of solar still, transparent cover and the ambient were measured using I-Bk thermocouples. At sunset each day the volume of water distilled in the container is measured. Some samples of the sea water and distilled water were analysed to determine their chemical and biological components. Water pH, Turbidity and Total dissolved solid, Total viable cell count and Total coliform count were determined using the method of Franson (1976), Total hardness, calcium, magnesium, sulphate and chloride were determined by AOAC (1990) method. The evaluation was done between 24th May and 4th June, 2011.

IV. Results and Discussion

Results presented in figures 3 to 5 are typical hourly averages as usual in a work of this nature where Amb, Basin, Perspex, and water represent ambient, still interior base, transparent cover and saline water temperature respectively while Solar Rad represents solar radiation.

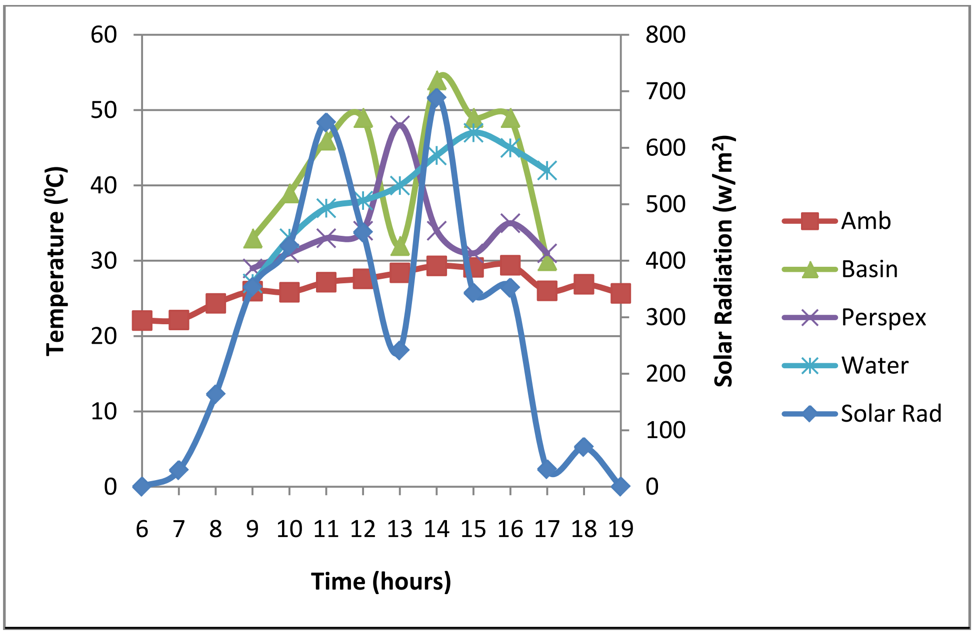

Fig. 3 : Temperature and Solar radiation of 24th May, 2011 Vs Time

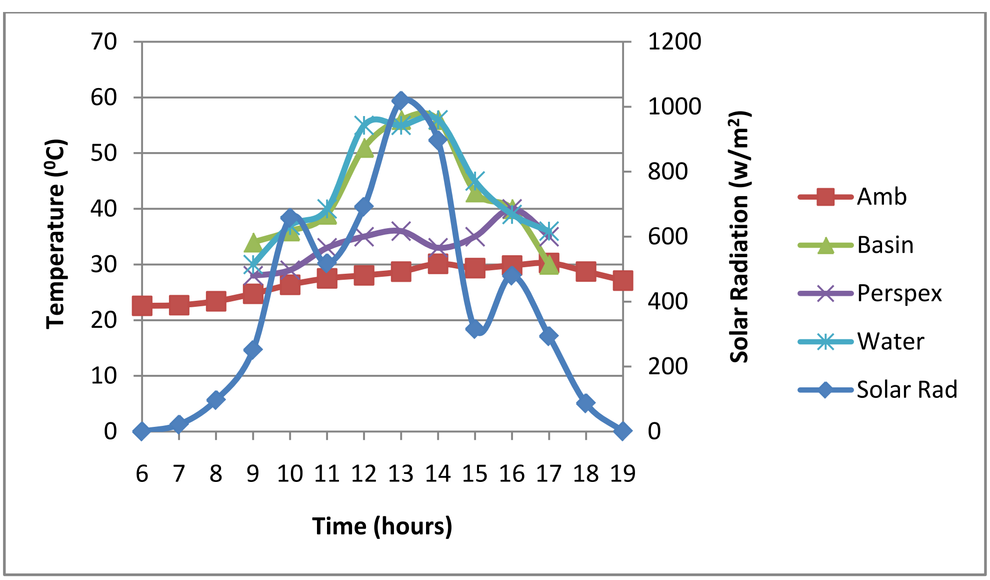

Fig. 4 : Temperature and Solar radiation of 25th May, 2011 Vs Time

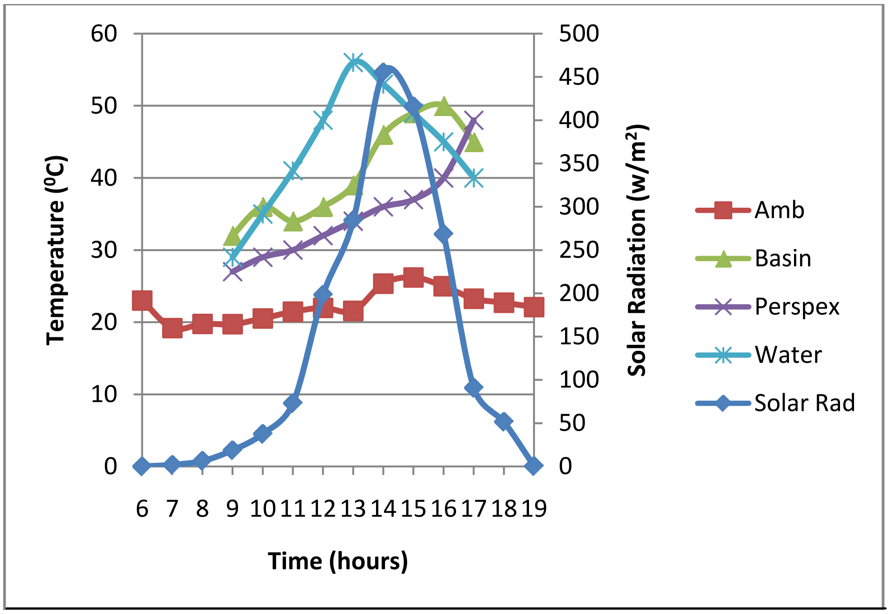

Fig. 5 : Temperature and Solar radiation of 26th May, 2011 vs Time

It is observed that as sun begin to rise from 06:00 hours, solar radiation value started increasing till it reached its peak period between the hours of 13:00 and 14:00 when the sun is vertically overhead (Duffie and Bechman, 1991). After this, radiation value started decreasing till sunset at about 18:00 hour. Among the three days selected, May 26th recorded lowest radiation values. This low values were as a result of the cloudiness of the weather on that day. It was also noticed that all the temperatures measure varied in sympathy with the solar radiation. Also, it is observed that, still interior components temperatures were higher than ambient temperature due to existence of the transparent cover which traps the solar energy inside the solar still (greenhouse effect). The transparent cover which is opaque to the infrared rays from the absorber interior (Badran, and Al-Hayek, 2004) resulted in a higher temperatures inside solar still (Radwan et al., 2009).

Table 1 represents the total and average daily still efficiency using equation 6. The average efficiency of 36.8% is in sympathy with the average 35% efficiency for flat plate passive solar systems (Duffie and Beckman, 1991; Okonkwo 1993

Table 1: The total and average daily still efficiency.

Days	Efficiency (%)
24th May	43.5
25th May	24.7
26th May	42.4
Average	36.8

The results of the laboratory analysis of seawater before and after distillation in comparison with acceptable WHO standards for drinking water are shown in tables 2 and 3.

Table 2 : Parameters and Maximum Allowable Limits for Drinking Water

Parameter	Maximum Permitted Level	Health Impact
pH	6.5-8.5	None
Chloride (CL)	250 mg/L	None
Calcium	75 mg/L	None
Magnesium	0.20 mg/L	Consumer acceptability
Total hardness	500 mg/L	None
Sulphate	100 mg/L	None
Total dissolved solids	500 mg/L	None
Colour	15 TCU	None
Taste	Unobjectionable	None
Odour	Unobjectionable	None
Turbidity	5 NTU	None
Total Viable cell count	0 cfu/100ml	Urinary track infections, bacteraemia ,meningities, diarrhea, acute renal failure and haemoltic
Total coliform count	10 cfu/ml	Indication of faecal contamination

Source : NIS, 2007

Table 3 : Lagos Bar beach water Analysis

Parameter	Before Distillation	After Distillation
pH	7.94	6.86
Chloride (CL)	2.84 mg/L	NIL
Calcium	53.2 mg/L	3.04 mg/L
Magnesium	2720.80 mg/L	0.76 mg/L
Total hardness	2774 mg/L	3.80 mg/L
Sulphate	19.2 mg/L	1.46 mg/L
Total dissolved solids	4014 mg/L	0.07 mg/L
Colour	Colourless	Colourless
Taste	Very salty	Tasteless
Odour	Odourless	Odourless
Turbidity	Nil	Nil
Total Viable cell count	1640 cfu/100ml	Nil
Total coliform count	380 cfu/ml	Nil

Comparing tables 2and 3, it can easily be seen that apart from the pH, chloride and calcium all other components of the bar beach water are beyond the acceptable limit for drinking water. Thus there is the need for distillation of the water especially because of the health implications of the very high values of the total viable cell count and coliform count.

It is observed that the pH of the distilled water decreased but within the acceptable range. As the solar radiation heats up the water, Co_2 reacts with water to form carbonic acid thereby reducing water pH. Equally, all the salt components of the water reduced significantly. This is as a result of the evaporation of water vapour leaving behind these components in the solar still.

Furthermore, the solar still successfully reduced the Total viable cell and coliform counts to zero which is the acceptable level for drinking water. This is as a result of the destruction of these microbes by ultraviolet rays of the solar radiation (Gadgil, 1998).

V. Conclusion

Lagos Bar beach water was distilled using a solar still developed at the National Centre for Energy Research and Development, University of Nigeria, Nsukka. Analysis of the water shows that there are very high values of the Total viable cell and coliform counts in the water which make it unsuitable for human consumption. However, after distillation of the seawater using the solar still both the total viable cell and coliform counts were reduced to zero which is the required condition for among other parameters necessary for its consumption. The solar still with an average efficiency of 36.8% performed within the acceptable range for passive solar systems. Hence the system is recommended for use to especially people living in coastal areas.

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Sumudu Homotopy Perturbation Technique

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Abstract - In this paper, a combinatory method of the sumudu transform and the homotopy perturbation method is proposed for solving one dimensional non-homogeneous partial differential equations with a variable coefficient. This method presents an accurate methodology to solve nonhomogeneous partial differential equations with a variable coefficient. The obtained approximate solutions are compared with exact solutions and those obtained by other analytical methods, showing reliability of the present method. The comparison shows a precise agreement between the results, and introduces this new method as an applicable one which it needs fewer computations and is much easier and more convenient than others, so it can be widely used in science and engineering.

Keywords and Phrases : Sumudu transform, Homotopy perturbation method, Non-homogenous partial differential equations.

Sumudu Homotopy Perturbation Technique

Devendra Kumar[α], Jagdev Singh[Ω], Sushila[β]

Abstract - In this paper, a combinatory method of the sumudu transform and the homotopy perturbation method is proposed for solving one dimensional non-homogeneous partial differential equations with a variable coefficient. This method presents an accurate methodology to solve non-homogeneous partial differential equations with a variable coefficient. The obtained approximate solutions are compared with exact solutions and those obtained by other analytical methods, showing reliability of the present method. The comparison shows a precise agreement between the results, and introduces this new method as an applicable one which it needs fewer computations and is much easier and more convenient than others, so it can be widely used in science and engineering.

Keywords and Phrases : Sumudu transform, Homotopy perturbation method, Non-homogenous partial differential equations.

I. Introduction

Partial differential equations are obtained in modeling of real-life science and engineering phenomena that are inherently nonlinear with variable coefficients. Most of these types of equations do not have an analytical solution. Therefore, these problems should be solved by using numerical or semi-analytical techniques. In numeric methods, computer codes and more powerful processors are required to achieve accurate results. Acceptable results are obtained via semi-analytical methods which are more convenient than numerical methods. The main advantage of semi-analytical methods, compared with other methods, is based on the fact that they can be conveniently applied to solve various complicated problems. In the semi-analytical methods such as the homotopy perturbation method, the variational iteration method, and the Adomian method, we can always obtain conveniently acceptable results in analytical forms instead of numerical ones for partial differential equations. These methods have simple solution procedures to solve various complicated problems [1-3]. The non-homogeneous partial differential equations with variable coefficients can be solved by the above said methods, however, with less accurate approximations [4-6] which might not satisfy initial/boundary conditions. To overcome this deficiency, this paper suggests a new method which is a combination of sumudu transform and homotopy perturbation method (SHPM), so that the obtained solutions satisfy the initial/boundary conditions. In early 90's, Watugala [7] introduced a new integral transform, named the sumudu transform and applied it to the solution of ordinary differential equation in control engineering problems. The sumudu transform is defined over the set of functions.

$$A = \{ f(t) \mid \exists M, \tau_1, \tau_2 > 0, |f(t)| < M e^{|t|/\tau_j}, \text{if } t \in (-1)^j \times [0, \infty) \}$$

by the following formula

$$\bar{f}(u) = S[f(t)] = \int_0^\infty f(ut)\, e^{-t}\, dt, \; u \in (-\tau_1, \tau_2) \qquad (1)$$

For further detail and properties of this transform, see [8-10].

II. Sumudu Homotopy Perturbation Method (SHPM)

To illustrate the basic idea of this method, we consider a general nonlinear form of one-dimension non-homogenous partial differential equation with a variable coefficient of the form:

$$\frac{\partial y}{\partial t} = \mu(x) \frac{\partial^2 y}{\partial x^2} + \phi(x,t), \qquad (2)$$

with subject to the boundary conditions

$$y(0,t) = g_0(t), \; y(1,t) = g_1(t). \qquad (3)$$

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And the initial condition

$$y(x,0)=f(x). \tag{4}$$

Taking the sumudu transform on equations (2) and (3), we get

$$\frac{d^2\bar{y}}{dx^2}-\frac{\bar{y}(x,u)}{u\,\mu(x)}+\frac{\bar{\phi}(x,u)+f(x)/u}{\mu(x)}=0, \tag{5}$$

$$\bar{y}(0,u)=\overline{g_0}(u),\ \ \bar{y}(1,u)=\overline{g_1}(u), \tag{6}$$

which is second order boundary value problem. According to HPM, we construct a homotopy in the form

$$H(\nu,p)=(1-p)\left[\frac{d^2\nu}{dx^2}-\frac{d^2\overline{y_0}}{dx^2}\right]+p\left[\frac{d^2\nu}{dx^2}-\frac{\nu}{u\mu(x)}+\frac{\bar{\phi}(x,u)+f(x)/u}{\mu(x)}\right]=0, \tag{7}$$

where $\overline{y_0}$ is the arbitrary function that satisfies boundary conditions (6), therefore

$$\nu(x,u)=\sum_{i=0}^{\infty}p^i\nu_i(x,u)=\nu_0(x,u)+p^1\nu_1(x,u)+p^2\nu_2(x,u)+\cdots \qquad \cdots(8)$$

Taking the inverse sumudu transform from both sides of (10), one obtains

$$\nu(x,t)=\sum_{i=0}^{\infty}p^i\nu_i(x,t)=\nu_0(x,t)+p^1\nu_1(x,t)+p^2\nu_2(x,t)+\cdots \qquad \cdots(9)$$

Setting $p=1$ results in the approximate solutions of eq. (2)

$$y(x,t)=y_0(x,t)+y_1(x,t)+y_2(x,t)+\cdots. \tag{10}$$

In this section, we use sumudu homotopy perturbation method (SHPM) in solving the one-dimension non-homogenous partial differential equations.

Example 4.1 : Consider the problem

$$\frac{\partial y}{\partial t}=\frac{\partial^2 y}{\partial x^2}+e^{-x}(\cos(t)-\sin(t)), \tag{11}$$

subject to the initial condition

$$y(x,0)=x. \tag{12}$$

And the boundary conditions

$$y(0,t)=\sin(t),\ \ y(1,t)=\frac{1+\sin(t)}{e}. \tag{13}$$

This problem has an exact solution that is

$$y(x,t)=x+e^{-x}\sin(t). \tag{14}$$

Taking the sumudu transform of eq. (11) and its boundary conditions with respect to t, and considering the initial condition, we have

$$\frac{d^2\bar{y}}{dx^2}-\frac{\bar{y}}{u}+\frac{x}{u}+e^{-x}\left(\frac{1-u}{1+u^2}\right)=0, \tag{15}$$

$$\bar{y}(0,u)=\frac{u}{1+u^2},\ \ \bar{y}(1,u)=1+\frac{u}{e(1+u^2)}. \tag{16}$$

To solve eq. (15) by means of HPM, a homotopy equation can be readily constructed as follows

$$H(\nu,p)=(1-p)\left[\frac{d^2\nu}{dx^2}-\frac{d^2\overline{y_0}}{dx^2}\right]+p\left[\frac{d^2\nu}{dx^2}-\frac{\nu}{u}+\frac{x}{u}+\frac{e^{-x}(1-u)}{1+u^2}\right]=0,\ p\in[0,1] \tag{17}$$

Now, we obtain a solution of eq. (17) in the form $\nu(x,u)=\sum_{i\ 0}^{\infty}p^i\nu_i(x,u).$ After substituting it into eq. (17) and rearranging the resultant equation based on powers of p-terms, following sets of linear differential equations can be obtained:

$$p^0:\frac{d^2\nu_0}{dx^2}-\frac{d^2\overline{y_0}}{dx^2}=0,\quad \nu_0(0,u)=0,\ \nu_0(1,u)=0 \tag{18.a}$$

$$p^1:\frac{d^2\nu_1}{dx^2}-\frac{\nu_0}{u}+\frac{x}{u}-e^{-x}\left(\frac{1-u}{1+u^2}\right)=0,\quad \nu_1(0,u)=0,\ \nu_{1x}(1,u)=0 \tag{18.b}$$

$$\vdots\quad p^i:\frac{d^2\nu_i}{dx^2}-\frac{\nu_{i-1}}{u}=0,\quad \nu_i(0,u)=0,\ \nu_i(1,u)=0,\ i=2,3,4,\ldots \tag{18.c}$$

The initial approximation $\nu_0(x,u)$ can be freely chosen, Here we set

$$\nu_0(x,u)=\frac{u(1-x)}{1+u^2}+x+\frac{ux}{e(1+u^2)},$$

which satisfies boundary conditions (16).

Using some mathematical software to solve eq. (18b-18c), and taking inverse sumudu transform, we get the following result

$$y_1(x,t)=x+e^{-x}\sin(t)+\frac{1}{6}(6+3x^2-6e^{-x}+x^3(e^{-1}-1)+x(4e^{-1}-8)\cos(t)). \tag{19}$$

Comparison of the obtained result with those obtained by other methods is shown in Table 1. As can be seen from Table 1, SHPM leads to more accurate solution.

Table 1 : Comparison between the results and those in open literature

X=0.1 t	u(x,t) exact	u(x,t) SHPM (one iteration)	u(x,t)LHPM[11] (one iteration)	u (x,t) HPM [5] (five iteration)	u(x,t)VIM[6] (five iteration)
0.1	0.190333011	0.187726613	0.187726613	0.19033301	0.19033301
0.3	0.367397741	0.364895251	0.364895251	0.367396826	0.367396826
0.5	0.533802166	0.531503352	0.531503352	0.533782618	0.533782618
0.9	0.808783498	0.807155201	0.807155201	0.8081252	0.8081252
1.5	1.002570788	1.002385493	1.002385493	0.988816989	0.988816989
3	0.227690664	0.230283934	0.230283934	-0.554986914	-0.554986914
4.5	-0.784505828	-0.783953651	-0.783953651	-8.178595887	-8.178595887
7	0.694466058	0.692491222	0.692491222	-67.88113901	-67.88113901
X=0.9					
0.1	0.940589238	0.938138815	0.938138815	0.940589238	0.940589238
0.3	1.02014955	1.017796817	1.017796817	1.020149139	1.020149139
0.5	1.094919878	1.092758632	1.092758632	1.094911094	1.094911094
0.9	1.218476955	1.2169461	1.2169461	1.218181163	1.218181163
1.5	1.305551197	1.305376991	1.305376991	1.299371217	1.299371217
3	0.957375114	0.959813195	0.959813195	0.605695409	0.605695409
4.5	0.502565913	0.503085045	-0.503085045	-2.819812914	2.819812914
7	1.167110818	1.165254163	1.165254163	-29.64589477	-29.64589477

Example 4.2 : Now, consider the problem

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} + e^x(\cosh(t) - \sinh(t)), \quad (20)$$

subject to the initial condition

$$y(x,0) = \frac{x^3}{6}. \quad (21)$$

And the boundary conditions

$$y(0,t) = \sinh(t), \;\; y(1,t) = \sinh(t) + t + \frac{1}{6}. \quad (22)$$

This problem has an exact solution that is

$$y(x,t) = e^x \sinh(t) + \frac{x^3}{6} + xt. \quad (23)$$

Taking the sumudu transform of eq. (20) and its boundary conditions with respect to t, and considering the initial condition, we have

$$\frac{d^2\bar{y}}{dx^2} - \frac{\bar{y}}{u} + \frac{x^3}{6u} + e^x\left(\frac{1}{1+u}\right) = 0, \quad (24)$$

$$\bar{y}(0,u) = \frac{u}{1-u^2}, \;\; \bar{y}(1,u) = \frac{u}{1-u^2} + u + \frac{1}{6}. \quad (25)$$

To solve eq. (24) by means of HPM, a homotopy equation can be readily constructed as follow

$$H(v,p) = (1-p)\left[\frac{d^2 v}{dx^2} - \frac{d^2 \bar{y}_0}{dx^2}\right] + p\left[\frac{d^2 v}{dx^2} - \frac{\bar{y}}{u} + \frac{x^3}{6u} + e^x\left(\frac{1}{1+u}\right)\right] = 0, \; p \in [0,1] \quad (26)$$

Now, following the same procedure as example 4.1, we assume the solution of equation (26) has a form $v(x,u) = \sum_{i\ 0}^{\infty} p^i v_i(x,u),$ and choose an initial solution in the form

$$v_0(x,u) = \frac{u(1-x)}{1-u^2} + x\left(\frac{1}{6} + u + \frac{eu}{1-u^2}\right),$$

which satisfies boundary conditions (26). Finally solving sets of linear differential equations that obtained from substituting $v(x,u)$ in eq. (26) and taking inverse sumudu transform, we get the following result

$$y_1(x,t) = e^x \sin(t) + xt + \frac{1}{6}(6 + 3x^2 + x^3 - 6e^{-x} + x^3(e-1) + x(5e-8)\cosh(t)). \quad (27)$$

Comparison of the obtained result with those obtained by other methods is shown in Table 2. As it can seen it is so close to the exact solution.

Table 2 : Comparison between the results and those in open literature

X=0.1 t	u(x,t) exact	u(x,t) SHPM (one iteration)	u(x,t)LHPM[11] (one iteration)	u(x,t) HPM[5] (five iteration)	u(x,t)VIM[6] (five iteration)
0.1	0.120868046	0.114140161	0.114140161	0.131708767	0.120868044
0.3	0.366713639	0.35971574	0.35971574	0.458331767	0.366712518
0.5	0.626066044	0.618517281	0.618517281	0.870229167	0.626041953
0.9	1.224643099	1.215049466	1.215049466	1.951824767	1.223815452
1.5	2.503384397	2.48763646	2.48763646	4.222104167	2.485179894
3	11.3716307	11.30423389	11.30423389	13.25116667	10.05353534
4.5	50.18618582	49.88484395	49.88484395	26.71272917	31.61579702
7	606.6832	603.0125545	603.0125545	56.44116667	139.4441579

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X=0.9					
0.1	0.45787045	0.329380987	0.329380987	0.4992421	0.457870447
0.3	1.140499061	1.011722397	1.011722397	1.3590651	1.140496567
0.5	1.853187635	1.723825043	1.723825043	2.3665625	1.853134019
0.9	3.456323732	3.324786111	3.324786111	4.8649581	3.454481771
1.5	6.708682372	6.570598712	6.570598712	9.9684375	6.668167506
3	27.46149634	27.26847624	27.26847624	31.372500	24.52802118
4.5	114.8610462	114.41919	114.41919	68.0090625	73.53188592
7	1355.061543	1351.035919	1351.035919	171.36250	315.201931

IV. Conclusions

In this paper, a new modified HPM, namely the sumudu homotopy perturbation method (SHPM) is introduced and the obtained results are compared with those obtained by LHPM, HPM, VIM and exact solutions for non-homogeneous partial differential equations with a variable coefficient. The results reveal that SHPM is an efficient and has good agreement with the exact solutions. In conclusion, the SHPM may be considered as a nice refinement in existing numerical techniques and might find the wide applications.

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On The Solutions of Generalized Fractional Kinetic Equations Involving the Functions for the Fractional Calculus

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Abstract - The paper is devoted to the study of the solution of generalized fractional kinetic equations. Results are obtained in a compact form in terms of $K4$ - Function introduced by sharma [9]. The results obtained in this paper are the extensions of the results given earlier by Chaurasia and Pandey[19-20] believed to be new.

Keywords and Phrases : Fractional kinetic equations, Fractional calculus, Special functions, Mittag-Leffler function, K4- Function, Lalace transform.

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Kishan Sharma

Abstract - The paper is devoted to the study of the solution of generalized fractional kinetic equations. Results are obtained in a compact form in terms of K_4 - Function introduced by sharma [9]. The results obtained in this paper are the extensions of the results given earlier by Chaurasia and Pandey[19-20] believed to be new.

Keywords : Fractional kinetic equations, Fractional calculus, Special functions, Mittag-Leffler function, K_4 *- Function, Lalace transform.*

I. Introduction

Fractional Calculus and special functions have contributed a lot to mathematical physics and its various branches. The great use of mathematical physics in distinguished astrophysical problems has attracted astronomers and physicists to pay more attention to available mathematical tools that can be widely used in solving several problems of astrophysics/physics. The fractional kinetic equations discussed here can be used to investigate a wide class of known fractional kinetic equations. Fractional kinetic equations have gained importance during the last decade due to their occurrence in certain problems in science and engineering. A spherically symmetric non-rotating , self-gravitating model of star like the sun is assumed to be in thermal equilibrium and hydrostatic equilibrium. The star is characterized by its mass, luminosity effective surface temperature, radius central density and central temperature. The stellar structures and their mathematical models are investigated on the basis of above characters and some additional information related to the equation of nuclear energy generation rate and the opacity.

Consider an arbitrary reaction characterized by a time dependent quantity $N = N(t)$.

It is possible to calculate rate of change $dN/dt = -d + p.$

In general, through feedback or other interaction mechanism, destruction and production depend on the quantity N itself: $d = d(N)$ or $p = p(N)$. This dependence is complicated since the destruction or production at time t depends not only on $N(t)$ but also on the past history $N(\tau), \tau < t$, of the variable N .This may be represented by Haubold and Mathai[7]

$$dN/dt = -d(N_t) + p(N_t), \tag{1.1}$$

where N_t denotes the function defined by $N_t(t^*) = N(t - t^*), t^* > 0.$

Haubold and Mathai[7] studied a special case of this equation , when spatial fluctuation or inhomogenities in quantities $N(t)$ are neglected, is given by the equation

$$dN_i/dt = -c_i N_i(t) \tag{1.2}$$

with the initial condition that $N_i(t = 0) = N_0$ is the number density of species i at time $t = 0$;constant $c_i > 0$,known as standard kinetic equation. A detailed discussion of the above equation is given in Kourganoff[21]. The solution of (1.2) is given by

$$N_i(t) = N_0 e^{-c_i t} \tag{1.3}$$

An alternative form of this equation can be obtained on integration:

$$N(t) - N_0 = c\,{}_0D_t^{-1} N(t), \tag{1.4}$$

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where ${}_0D_t^{-1}$ is the standard integral operator.Haubold and Mathai[7] have given the fractional generalization of the standard kinetic equation(1.2) as

$$N(t) - N_0 = c^{\nu}\, {}_0D_t^{-1} N(t), \tag{1.5}$$

where ${}_0D_t^{-1}$ is well known Riemann-Liouville fractional integral operator (Oldham and Spanier[8]; Samko, Kilbas and Marichev[16]; Miller and Ross[10]) defined by

$${}_0D_t^{-\nu} N(t) = \frac{1}{\Gamma(\nu)} \int_0^t (t-u)^{\nu-1} f(u)du, \mathrm{Re}(\nu) > 0. \tag{1.6}$$

The solution of the fractional kinetic equation(1.5) is given by (see Haubold and Mathai[7])

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(\nu k+1)} (ct)^{\nu k}. \tag{1.7}$$

Fractional kinetic equations are studied by many authors notably Hille and Tamarkin[5], Glockle and Nonnenmacher[22], Saichev and Zaslavsky[1], Saxena et al.[11-13], Zaslavsky[6], Saxena and Kalla[15], Chaurasia and Pandey[18-19], Chaurasia and Kumar[17] etc. for their importance in the solution of certain physical problems. Recently, Saxena et al. [14] investigated the solutions of the fractional reaction equation and the fractional diffusion equation. Laplace transform technique is used.

The K_4-function[9] is defined as

$$K_4^{(\alpha,\beta,\gamma),(a,c):(p;q)}(a_1,\ldots,a_p;b_1,\ldots,b_q;x) = K_4^{(\alpha,\beta,\gamma),(a,c):(p;q)}(x)$$

$$= \sum_{n=0}^{\infty} \frac{(a_1)_n \ldots (a_p)_n}{(b_1)_n \ldots (b_q)_n} \frac{(\gamma)_n\, a^n (x-c)^{(n+\gamma)\alpha-\beta-1}}{n!\,\Gamma((n+\gamma)\alpha-\beta)} \tag{1.8}$$

where $R(\alpha\gamma-\beta) > 0$ and $(a_i)_n (i=1,2,\ldots,p)$ and $(b_j)_n\ (j=1,2,\ldots,q)$ are the Pochhammer symbols and none of the parameters $b_j s$ is a negative integer or zero.

We now proceed to solve the generalized fractional kinetic equation in the next section.

II. Generalized Fractional Kinetic Equation

"'In this section we investigate the solution of generalized fractional kinetic equations'.. The results are obtained in a compact form in terms of K_4- Function and are suitable for computation. The result is presented in the form of a theorem as follows:

Theorem 2.1 If $c>0, b\geq 0, \delta>0, \nu>0, \mu>0$ and $(\delta\nu-\mu)>0$ then there exists the solution of the integral equation

$$N(t) - N_0\, K_4^{(\nu,\mu,\delta),(-c^{\nu},b):(p;q)}(t) = -\sum_{r=0}^{n} \binom{n}{r} c^{r\nu}\, {}_0D_t^{-r\nu} N(t), \tag{2.1}$$

given by

$$N(t) = N_0\, K_4^{(\nu,\mu+\nu n,\delta+n),(-c^{\nu},b):(p;q)}(t). \tag{2.2}$$

Proof: Taking the Laplace transform of both sides of (2.1), we have

$$L\{N(t)\} - L\{N_0\, K_4^{(\nu,\mu,\delta),(-c^{\nu},b):(p;q)}(t)\} = L\{-\sum_{r=0}^{n} \binom{n}{r} c^{r\nu}\, {}_0D_t^{-r\nu} N(t)\} \tag{2.2}$$

or

$$\overline{N(p)} = \frac{N_0\, p^{\mu-\delta\nu} c^{-bp}}{(1+c^{\nu}p^{-\nu})^{\mu+\delta}} \sum_{k=0}^{\infty} \frac{(a_1)_k \ldots (a_p)_k}{(b_1)_k \ldots (b_q)_k} \tag{2.4}$$

Finally, taking the inverse Laplace transform, we have

$$L^{-1}\{\overline{N(p)}\} = N\{t\} = L^{-1}\{\frac{N_0\, p^{\mu-(\delta+n)\nu+pn}\, c^{-bp}}{(1+c^{\nu} p^{-\nu})^{\mu+\delta}} \sum_{k=0}^{\infty} \frac{(a_1)_k \ldots (a_p)_k}{(b_1)_k \ldots (b_q)_k}$$

Or

$$N(t) = N_0\, K_4^{(\nu,\mu+\nu n,\delta+n),(-c^{\nu},b):(p;q)}(t) \tag{2.5}$$

This completes the proof of the theorem(2.1).

If we put $r = s = 0$ in theorem 2.1, we get[18]

Cor.1.1 If $c > 0, b \geq 0, \delta > 0, \nu > 0, \mu > 0$ and $(\delta\nu - \mu) > 0$ then there exists the solution of the integral equation

$$N(t) - N_0 G_{\nu,\mu,\delta}(c^{-\nu}, b, t) = -\sum_{r=0}^{n} \binom{n}{r} c^{r\nu}\, {}_0D_t^{-r\nu} N(t), \tag{2.6}$$

is given by

$$N(t) = N_0 G_{\nu,\mu+\nu n,\delta+n}(c^{-\nu}, b, t). \tag{2.7}$$

If we take $b = 0$ in Corollary.(1.1), we get[19]

Cor.1.2 If $c > 0, \delta > 0, \nu > 0, \mu > 0$ and $(\delta\nu - \mu) > 0$ then there exists the solution of the integral equation

$$N(t) - N_0 G_{\nu,\mu,\delta}(c^{-\nu}, 0, t) = -\sum_{r=0}^{n} \binom{n}{r} c^{r\nu}\, {}_0D_t^{-r\nu} N(t), \tag{2.8}$$

is given by

$$N(t) = N_0 G_{\nu,\mu+\nu n,\delta+n}(c^{-\nu}, 0, t). \tag{2.9}$$

If we take $b = 0$ in Corollary.(1.1), we get[20]

Cor.1.3 Let $c > 0, b \geq 0, \delta > 0, \nu > 0, \mu > 0$ and $(\delta\nu - \mu) > 0$ then the equation

$$N(t) - N_0 G_{\nu,\mu,\delta}(c^{-\nu}, b, t) = -c^{\nu}\, {}_0D_t^{-\nu} N(t),$$

is solvable and its solution is given by (2.10)

$$N(t) = N_0 G_{\nu,\mu+\nu,\delta+1}(c^{-\nu}, b, t). \tag{2.11}$$

where $G_{\nu,\mu,\delta}(a,c,t)$ is the G-function(but not the Meijer's G-function) given by [2].

III. Acknowledgement

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IV. Conclusion

In the present paper, we have derived a solution of generalized fractional kinetic equation in terms of the K_4- Function in a compact and elegant form with the help of Laplace transform. Most of the results obtained are suitable for numerical computation. Fractional kinetic equation can be used to calculate the particle reaction rate and describes the statistical mechanics associated with the particle distribution function.

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the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: As you are doing research in the field of Computer Science, then this point is quite obvious.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

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10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

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15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

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18. **Pick a good study spot:** To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. **Think technically:** Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page

- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

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The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.
- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
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Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

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- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
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- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
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Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
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Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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