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# Uniformly Starlike And Uniformly Convexity Properties For Certain Special Functions

By V. B. L. Chaurasia, Yaghvendra Kumawat  
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**Abstracts** - The aim of the present paper is to establish the sufficient condition for the function  $yz \left\{ {}_p \overline{\psi}_q(z) \right\}$  to be in the classes of uniformly starlike and uniformly convex functions. Similar types of result using integral operators also obtained. +

**Keywords and Phrases** : Generalized Fox-Wright function, uniformly starlike function, uniformly convex function, univalent.

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# Uniformly Starlike And Uniformly Convexity Properties For Certain Special Functions

V. B. L. Chaurasia<sup>1</sup>, Yaghvendra Kumawat<sup>2</sup>

**Abstract:** The aim of the present paper is to establish the sufficient condition for the function  $yz \left\{ \overline{\psi}_q(z) \right\}$  to be in the classes of uniformly starlike and uniformly convex functions. Similar types of result using integral operators also obtained. +

**Keywords and Phrases :** Generalized Fox-Wright function, uniformly starlike function, uniformly convex function, univalent.

## I. INTRODUCTION AND DEFINITIONS

Let  $C$  denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad (1)$$

where  $a_n \geq 0$  and  $n \in N$ , that are analytic in the open unit disk  $U = \{z : |z| < 1\}$ .

A function  $f \in C$  is said to be starlike univalent of order  $\alpha$ ,  $0 \leq \alpha < 1$ , if and only if  $\operatorname{Re} \left( \frac{zf'(z)}{f(z)} \right) > \alpha$ ,  $z \in U$ . Also  $f(z)$  of the form (1) is uniformly starlike, whenever  $\operatorname{Re} \left( \frac{f(z) - f(\xi)}{(z - \xi)f'(z)} \right) \geq 0$ ,  $(z, \xi) \in U \times U$ . This class of all uniformly starlike functions is denoted by UST ([3]) (see also [11], [14] and [7]).

The function  $f \in C$  of the form (1) is uniformly convex in  $U$  whenever  $\operatorname{Re} \left( 1 + (z - \xi) \frac{f''(z)}{f'(z)} \right) \geq 0$ , where  $(z, \xi) \in U \times U$ . This class of all uniformly convex functions is denoted by UCV ([2]) (also refer [1], [12], [4] and [6]). Further it is said to be in the class  $UCV(\alpha)$ ,  $\alpha \geq 0$ , if  $\operatorname{Re} \left( 1 + \frac{zf''(z)}{f'(z)} \right) \geq \left| \frac{zf''(z)}{f'(z)} \right| + \alpha$ .

H. Silverman [8] introduced a subclass  $B$  of  $C$  consisting of functions of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n, \quad a_n \geq 0. \quad (2)$$

A function  $f$  of the form (2) is said to be in the class  $USTN(\alpha)$ ,  $0 \leq \alpha < 1$ , if

$$\operatorname{Re} \left( \frac{f(z) - f(\xi)}{(z - \xi)f'(z)} \right) \geq \alpha, \quad \text{where } (z, \xi) \in U \times U.$$

In the present paper, we shall use analogues of the lemmas in [8] and [9]. Respectively in the following form:

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**Lemma 1.** A function  $f$  of the form (1) is in the class  $UST(\alpha)$ , if  $\sum_{n=2}^{\infty} [(3-\alpha)n-2]|a_n| \leq (1-\alpha)N$ ,

where  $N > 0$  is a suitable constant. In particular,  $f \in UST$  whenever  $\sum_{n=2}^{\infty} (3n-2)|a_n| \leq N$ .

**Lemma 2.** A sufficient condition for a function  $f$  of the form (1) to be in the class  $UCV(\alpha)$  is that

$\sum_{n=2}^{\infty} n[(\alpha+1)n-\alpha]a_n \leq N$ , where  $N > 0$  is a suitable constant. In particular,  $f \in UCV$  whenever

$$\sum_{n=2}^{\infty} n^2 a_n \leq N.$$

**Lemma 3.** A necessary and sufficient condition for a function  $f$  of the form (2) to be in the class

$$2 \quad UCV(\alpha) \text{ is that } \sum_{n=2}^{\infty} n[(\alpha+1)n-\alpha]a_n \leq N.$$

The generalized Fox-Wright function is defined by ([5], p.271, eqn.(7))

$${}_p\overline{\Psi}_q(z) = {}_p\overline{\Psi}_q \left[ \begin{matrix} (a_j, \alpha_j; A_j)_{1,p} \\ (b_j, \beta_j; B_j)_{1,q} \end{matrix}; z \right] = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j n)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j n)\}^{B_j}} \frac{z^n}{n!} \quad (3)$$

Where  $1 + \sum_{j=1}^q \beta_j > \sum_{j=1}^p \alpha_j$ ,  $\alpha_j$  ( $j = 1, \dots, p$ ) and  $\beta_j$  ( $j = 1, \dots, q$ ) are real and positive and  $A_j$  ( $j = 1, \dots, p$ ) and  $B_j$  ( $j = 1, \dots, q$ ) can take non-integer values.

It is interesting to note that  ${}_p\overline{F}_q$  ([5], p.271, eqn.(9)) is obtained by taking  $\alpha_i = \beta_j = 1$  ( $i = 1, \dots, p$ ;  $j = 1, \dots, q$ ) in eqn. (3) and  ${}_pF_q$  can also be obtained by taking  $A_i = B_j = \alpha_j = \beta_j = 1$  ( $i = 1, \dots, p$ ;  $j = 1, \dots, q$ )s in eqn (1.3).

For the sake of brevity, we use here the following notation:

$$y = \frac{\prod_{j=1}^q \{\Gamma(b_j)\}^{B_j}}{\prod_{j=1}^p \{\Gamma(a_j)\}^{A_j}}.$$

## II. MAIN RESULT

**Theorem 2.1** If  $a_i > 0$  ( $i = 1, \dots, p$ ),  $b_j > 0$  ( $j = 1, \dots, q$ ),  $\sum_{j=1}^q b_j > \sum_{j=1}^p a_j + 1$  and  $1 + \sum_{j=1}^q B_j \beta_j > \sum_{j=1}^p A_j \alpha_j$ , then

a sufficient condition for the function  $yz\{{}_p\overline{\Psi}_q(z)\}$  to be in the class  $UST(\alpha)$ ,  $0 \leq \alpha < 1$ , is that

$$\left( \frac{3-\alpha}{1-\alpha} \right) {}_p\overline{\Psi}_q \left[ \begin{matrix} (a_j + \alpha_j, \alpha_j; A_j)_{1,p} \\ (b_j + \beta_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] + {}_p\overline{\Psi}_q \left[ \begin{matrix} (a_j, \alpha_j; A_j)_{1,p} \\ (b_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] \leq y^{-1}(1+N). \quad (4)$$



**Proof.** Since

$$yz\left\{{}_p\overline{\psi}_q(z)\right\}=z+\sum_{n=2}^{\infty}\frac{\prod_{j=1}^p\left\{\Gamma(a_j+\alpha_j(n-1))\right\}^{A_j}yz^n}{\prod_{j=1}^q\left\{\Gamma(b_j+\beta_j(n-1))\right\}^{B_j}(n-1)!},$$

according to Lemma 1, we need only to show that,

$$\sum_{n=2}^{\infty}[(3-\alpha)n-2]\left|\frac{\prod_{j=1}^p\left\{\Gamma(a_j+\alpha_j(n-1))\right\}^{A_j}y}{\prod_{j=1}^q\left\{\Gamma(b_j+\beta_j(n-1))\right\}^{B_j}(n-1)!}\right|\leq(1-\alpha)N.$$

$$\begin{aligned} \text{Now } & \sum_{n=2}^{\infty}[(3-\alpha)n-2]\left|\frac{\prod_{j=1}^p\left\{\Gamma(a_j+\alpha_j(n-1))\right\}^{A_j}y}{\prod_{j=1}^q\left\{\Gamma(b_j+\beta_j(n-1))\right\}^{B_j}(n-1)!}\right| \\ &= y(3-\alpha)\sum_{n=0}^{\infty}\left|\frac{\prod_{j=1}^p\left\{\Gamma(a_j+\alpha_j(n+1))\right\}^{A_j}}{\prod_{j=1}^q\left\{\Gamma(b_j+\beta_j(n+1))\right\}^{B_j}n!}\right|+y(1-\alpha)\sum_{n=1}^{\infty}\left|\frac{\prod_{j=1}^p\left\{\Gamma(a_j+\alpha_jn)\right\}^{A_j}}{\prod_{j=1}^q\left\{\Gamma(b_j+\beta_jn)\right\}^{B_j}n!}\right| \\ &= y(3-\alpha){}_p\overline{\psi}_q\left[\begin{matrix} (a_j+\alpha_j,\alpha_j;A_j)_{1,p} \\ (b_j+\beta_j,\beta_j;B_j)_{1,q} \end{matrix};1\right]+y(1-\alpha){}_p\overline{\psi}_q\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q} \end{matrix};1\right]-(1-\alpha) \\ &\leq(1-\alpha)N, \end{aligned}$$

this gives the desired result from the Lemma 1.

**Theorem 2.2** If  $a_i > 0 (i=1, \dots, p)$ ,  $b_j > 0 (j=1, \dots, q)$ ,  $\sum_{j=1}^q b_j > \sum_{j=1}^p a_j + 1$  and  $1 + \sum_{j=1}^q B_j \beta_j > \sum_{j=1}^p A_j \alpha_j$ ,

then a sufficient condition for the function  $yz\left\{{}_p\overline{\psi}_q(z)\right\}$  to be in the class  $USTN(\alpha)$ ,  $0 \leq \alpha < 1$ , is that

$$\left(\frac{3-\alpha}{1-\alpha}\right){}_p\overline{\psi}_q\left[\begin{matrix} (a_j+\alpha_j,\alpha_j;A_j)_{1,p} \\ (b_j+\beta_j,\beta_j;B_j)_{1,q} \end{matrix};1\right]+{}_p\overline{\psi}_q\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q} \end{matrix};1\right]\leq y^{-1}(1+N). \quad (5)$$

**Proof.** The proof directly follows from the Theorem 1.

**Theorem 2.3** If  $a_i > 0 (i=1, \dots, p)$ ,  $b_j > 0 (j=1, \dots, q)$ ,  $\sum_{j=1}^q b_j > \sum_{j=1}^p a_j + 2$  and  $1 + \sum_{j=1}^q B_j \beta_j > \sum_{j=1}^p A_j \alpha_j$ ,

then a sufficient condition for the function  $yz\left\{{}_p\overline{\psi}_q(z)\right\}$  to be in the class  $UCV(\alpha)$ ,  $0 \leq \alpha < 1$ , is that

$$(1+\alpha){}_p\overline{\psi}_q\left[\begin{matrix} (a_j+2\alpha_j,\alpha_j;A_j)_{1,p} \\ (b_j+2\beta_j,\beta_j;B_j)_{1,q} \end{matrix};1\right]+(3+2\alpha){}_p\overline{\psi}_q\left[\begin{matrix} (a_j+\alpha_j,\alpha_j;A_j)_{1,p} \\ (b_j+\beta_j,\beta_j;B_j)_{1,q} \end{matrix};1\right]$$

$$+ {}_p\overline{\psi}_q \left[ \begin{matrix} (a_j, \alpha_j; A_j)_{1,p} \\ (b_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] \leq y^{-1}(1+N). \quad (6)$$

**Proof.** By Lemma 2, it suffices to show that

$$\sum_{n=2}^{\infty} n[(\alpha+1)n-\alpha] \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j(n-1))\}^{A_j} y}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j(n-1))\}^{B_j} (n-1)!} \leq N.$$

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$$\begin{aligned} \text{Now, } & \sum_{n=2}^{\infty} n[(\alpha+1)n-\alpha] \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j(n-1))\}^{A_j} y}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j(n-1))\}^{B_j} (n-1)!} \\ &= y(1+\alpha) \sum_{n=2}^{\infty} (n-1)^2 \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j(n-1))\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j(n-1))\}^{B_j} (n-1)!} \\ & \quad + y(2+\alpha) \sum_{n=2}^{\infty} (n-1) \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j(n-1))\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j(n-1))\}^{B_j} (n-1)!} + y \sum_{n=2}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j(n-1))\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j(n-1))\}^{B_j} (n-1)!} \\ &= y(1+\alpha) \sum_{n=0}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j + 2\alpha_j + \alpha_j n)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + 2\beta_j + \beta_j n)\}^{B_j} n!} + y(3+2\alpha) \sum_{n=0}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j + \alpha_j n)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j + \beta_j n)\}^{B_j} n!} \\ & \quad + y \sum_{n=0}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j n)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j n)\}^{B_j} n!} - 1 \\ &= y(1+\alpha) {}_p\overline{\psi}_q \left[ \begin{matrix} (a_j + 2\alpha_j, \alpha_j; A_j)_{1,p} \\ (b_j + 2\beta_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] + y(3+2\alpha) {}_p\overline{\psi}_q \left[ \begin{matrix} (a_j + \alpha_j, \alpha_j; A_j)_{1,p} \\ (b_j + \beta_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] \\ & \quad + y {}_p\overline{\psi}_q \left[ \begin{matrix} (a_j, \alpha_j; A_j)_{1,p} \\ (b_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] - 1, \end{aligned}$$

this last expression is bounded above by N if and only if (6) holds. Hence the Theorem 3 is proved.

## III. AN INTEGRAL OPERATORS

In this section we obtain sufficient conditions for the function  $y {}_p\overline{\psi}_q \left[ \begin{matrix} (a_j, \alpha_j; A_j)_{1,p} \\ (b_j, \beta_j; B_j)_{1,q} \end{matrix}; z \right] = y \int_0^z {}_p\overline{\psi}_q(x) dx$  to be in the classes UST and UCV.

**Theorem 3.1** If  $a_i > 0 (i=1, \dots, p)$ ,  $b_j > 0 (j=1, \dots, q)$ ,  $\sum_{j=1}^q b_j > \sum_{j=1}^p a_j$  and  $1 + \sum_{j=1}^q B_j \beta_j > \sum_{j=1}^p A_j \alpha_j$ , then a sufficient condition for the function  $y \{ {}_p\overline{\phi}_q(z) \} = y \int_0^z {}_p\overline{\psi}_q(x) dx$  to be in the class UST, is that

$$3 {}_p\overline{\psi}_q \left[ \begin{matrix} (a_j, \alpha_j; A_j)_{1,p} \\ (b_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] - 2 {}_p\overline{\psi}_q \left[ \begin{matrix} (a_j - \alpha_j, \alpha_j; A_j)_{1,p} \\ (b_j - \beta_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] + 2 \frac{\prod_{j=1}^p \{\Gamma(a_j - \alpha_j)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j - \beta_j)\}^{B_j}} \leq y^{-1}(1 + N). \quad (7)$$

**Proof.** we have,

$$y \{ {}_p\overline{\phi}_q(z) \} = y \int_0^z {}_p\overline{\psi}_q(x) dx = z + \sum_{n=2}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j(n-1))\}^{A_j} y z^n}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j(n-1))\}^{B_j} n!}. \quad (8)$$

Now,

$$\begin{aligned} & \sum_{n=2}^{\infty} (3n-2) \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j(n-1))\}^{A_j} y}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j(n-1))\}^{B_j} n!} \\ &= 3y \sum_{n=0}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j n)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j n)\}^{B_j} n!} - 2y \sum_{n=0}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j - \alpha_j + \alpha_j n)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j - \beta_j + \beta_j n)\}^{B_j} n!} + 2y \frac{\prod_{j=1}^p \{\Gamma(a_j - \alpha_j)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j - \beta_j)\}^{B_j}} - 1 \\ &= 3y {}_p\overline{\psi}_q \left[ \begin{matrix} (a_j, \alpha_j; A_j)_{1,p} \\ (b_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] - 2y {}_p\overline{\psi}_q \left[ \begin{matrix} (a_j - \alpha_j, \alpha_j; A_j)_{1,p} \\ (b_j - \beta_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] \\ &+ 2y \frac{\prod_{j=1}^p \{\Gamma(a_j - \alpha_j)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j - \beta_j)\}^{B_j}} - 1, \end{aligned} \quad (9)$$

which in view of Lemma 1, (9) gives the result (7).



**Theorem 3.2** If  $a_i > 0 (i=1, \dots, p)$ ,  $b_j > 0 (j=1, \dots, q)$ ,  $\sum_{j=1}^q b_j > \sum_{j=1}^p a_j$  and  $1 + \sum_{j=1}^q B_j \beta_j > \sum_{j=1}^p A_j \alpha_j$ , then a

sufficient condition for the function  $y\{\bar{\phi}_q(z)\} = y \int_0^z {}_p\bar{\psi}_q(x) dx$  to be in the class  $UCV$ , is that

$${}_p\bar{\psi}_q \left[ \begin{matrix} (a_j + \alpha_j, \alpha_j; A_j)_{1,p} \\ (b_j + \beta_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] + {}_p\bar{\psi}_q \left[ \begin{matrix} (a_j, \alpha_j; A_j)_{1,p} \\ (b_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] \leq y^{-1}(1+N). \quad (10)$$

**Proof.** we have,

$$\begin{aligned} & \sum_{n=2}^{\infty} n^2 \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j(n-1))\}^{A_j} y}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j(n-1))\}^{B_j} n!} \\ &= y \sum_{n=2}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j(n-1))\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j(n-1))\}^{B_j} (n-2)!} + y \sum_{n=1}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j n)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j n)\}^{B_j} n!} \\ &= y \sum_{n=0}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j(n+1))\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j(n+1))\}^{B_j} n!} + y \sum_{n=0}^{\infty} \frac{\prod_{j=1}^p \{\Gamma(a_j + \alpha_j n)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j + \beta_j n)\}^{B_j} n!} - 1 \\ &= y {}_p\bar{\psi}_q \left[ \begin{matrix} (a_j + \alpha_j, \alpha_j; A_j)_{1,p} \\ (b_j + \beta_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] + y {}_p\bar{\psi}_q \left[ \begin{matrix} (a_j, \alpha_j; A_j)_{1,p} \\ (b_j, \beta_j; B_j)_{1,q} \end{matrix}; 1 \right] - 1, \end{aligned} \quad (11)$$

Theorem 3.2 follows from (10), (11) and Lemma 2.

#### IV. SPECIAL CASES

If we take  $\alpha_j = \beta_k = 1 (j=1, \dots, p; k=1, \dots, q)$  in Theorems (2.1), (2.2), (2.3), (3.1) and (3.2) respectively, the  ${}_p\bar{\psi}_q$  function will reduce to the  ${}_p\bar{F}_q$  function; we get the following Theorems:

**Theorem 4.1** If  $a_i > 0 (i=1, \dots, p)$ ,  $b_j > 0 (j=1, \dots, q)$ ,  $\sum_{j=1}^q b_j > \sum_{j=1}^p a_j + 1$ , then a sufficient condition for

the function  $yz\{{}_p\bar{F}_q(z)\}$  to be in the class  $UST(\alpha)$ ,  $0 \leq \alpha < 1$ , is that

$$\left( \frac{3-\alpha}{1-\alpha} \right) {}_p\bar{F}_q \left[ \begin{matrix} (a_j + 1, 1; A_j)_{1,p} \\ (b_j + 1, 1; B_j)_{1,q} \end{matrix}; 1 \right] + {}_p\bar{F}_q \left[ \begin{matrix} (a_j, 1; A_j)_{1,p} \\ (b_j, 1; B_j)_{1,q} \end{matrix}; 1 \right] \leq y^{-1}(1+N). \quad (12)$$

**Theorem 4.2** If  $a_i > 0 (i=1, \dots, p)$ ,  $b_j > 0 (j=1, \dots, q)$ ,  $\sum_{j=1}^q b_j > \sum_{j=1}^p a_j + 1$ , then a sufficient condition for

the function  $yz\{{}_p\bar{F}_q(z)\}$  to be in the class  $USTN(\alpha)$ ,  $0 \leq \alpha < 1$ , is that

$$\left( \frac{3-\alpha}{1-\alpha} \right) {}_p\bar{F}_q \left[ \begin{matrix} (a_j + 1, 1; A_j)_{1,p} \\ (b_j + 1, 1; B_j)_{1,q} \end{matrix}; 1 \right] + {}_p\bar{F}_q \left[ \begin{matrix} (a_j, 1; A_j)_{1,p} \\ (b_j, 1; B_j)_{1,q} \end{matrix}; 1 \right] \leq y^{-1}(1+N). \quad (13)$$

**Theorem 4.3** If  $a_i > 0 (i=1, \dots, p)$ ,  $b_j > 0 (j=1, \dots, q)$ ,  $\sum_{j=1}^q b_j > \sum_{j=1}^p a_j + 2$ , then a sufficient condition for the function  $y z \left\{ {}_p \overline{F}_q(z) \right\}$  to be in the class  $UCV(\alpha)$ ,  $0 \leq \alpha < 1$ , is that

$$\begin{aligned} (1+\alpha) {}_p \overline{F}_q \left[ \begin{matrix} (a_j+2, 1; A_j)_{1,p} \\ (b_j+2, 1; B_j)_{1,q} \end{matrix}; 1 \right] + (3+2\alpha) {}_p \overline{F}_q \left[ \begin{matrix} (a_j+1, 1; A_j)_{1,p} \\ (b_j+1, 1; B_j)_{1,q} \end{matrix}; 1 \right] \\ + {}_p \overline{F}_q \left[ \begin{matrix} (a_j, 1; A_j)_{1,p} \\ (b_j, 1; B_j)_{1,q} \end{matrix}; 1 \right] \leq y^{-1}(1+N). \end{aligned} \quad (14)$$

**Theorem 4.4** If  $a_i > 0 (i=1, \dots, p)$ ,  $b_j > 0 (j=1, \dots, q)$ ,  $\sum_{j=1}^q b_j > \sum_{j=1}^p a_j$ , then a sufficient condition for the function  $y \left\{ {}_p \overline{G}_q(z) \right\} = y \int_0^z {}_p \overline{F}_q(x) dx$  to be in the class  $UST$ , is that

$$\begin{aligned} 3 {}_p \overline{F}_q \left[ \begin{matrix} (a_j, 1; A_j)_{1,p} \\ (b_j, 1; B_j)_{1,q} \end{matrix}; 1 \right] - 2 {}_p \overline{F}_q \left[ \begin{matrix} (a_j-1, 1; A_j)_{1,p} \\ (b_j-1, 1; B_j)_{1,q} \end{matrix}; 1 \right] \\ + 2 \frac{\prod_{j=1}^p \{\Gamma(a_j-1)\}^{A_j}}{\prod_{j=1}^q \{\Gamma(b_j-1)\}^{B_j}} \leq y^{-1}(1+N). \end{aligned} \quad (15)$$

**Theorem 4.4** If  $a_i > 0 (i=1, \dots, p)$ ,  $b_j > 0 (j=1, \dots, q)$ ,  $\sum_{j=1}^q b_j > \sum_{j=1}^p a_j$ , then a sufficient condition for the function  $y \left\{ {}_p \overline{G}_q(z) \right\} = y \int_0^z {}_p \overline{F}_q(x) dx$  to be in the class  $UCV$ , is that

$${}_p \overline{F}_q \left[ \begin{matrix} (a_j+1, 1; A_j)_{1,p} \\ (b_j+1, 1; B_j)_{1,q} \end{matrix}; 1 \right] + {}_p \overline{F}_q \left[ \begin{matrix} (a_j, 1; A_j)_{1,p} \\ (b_j, 1; B_j)_{1,q} \end{matrix}; 1 \right] \leq y^{-1}(1+N).$$

If we set  $A_i = B_j = 1 (i=1, \dots, p; j=1, \dots, q)$ ,  $N \rightarrow My$ , Theorems (2.1), (2.2), (2.3), (3.1) and (3.2) reduce to the results recently obtained by Chaurasia and Srivastava ([16]).

Further on taking  $\alpha_k = \beta_l = 1 (k=1, \dots, p; l=1, \dots, q)$  and  $N = \frac{\prod_{j=1}^p \Gamma(a_j)}{\prod_{j=1}^q \Gamma(b_j)}$ , we arrive at the results of

Shanmugam, Ramachandran, Sivasubramanian and Gangadharan ([12]).

By specifying the parameters suitably, the results of this paper readily yield the results due to Dixit and Verma ([1]).

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## On Ramanujan's Generating Relation For Tau Function

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**Abstracts** - Present paper concerns mainly with verification and extension of the table for  $\tau(1), \tau(2), \tau(3), \dots, \tau(30)$  of Ramanujan. Our extended table for  $\tau(31), \tau(32), \tau(33), \dots, \tau(211)$  is obtained without using certain arithmetical functions defined by Ramanujan and also the theory of elliptic functions.

**Keywords** : Ramanujan's tau function; Generating relation and function; Ordinary finite difference table.

**Classification**: GJSFR-F Classification: 2010 AMS Subject Classifications: 33A30.



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# On Ramanujan's Generating Relation For Tau Function

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**Abstracts** - Present paper concerns mainly with verification and extension of the table for  $\tau(1)$ ,  $\tau(2)$ ,  $\tau(3)$ ,  $\dots$ ,  $\tau(30)$  of Ramanujan. Our extended table for  $\tau(31)$ ,  $\tau(32)$ ,  $\tau(33)$ ,  $\dots$ ,  $\tau(211)$  is obtained without using certain arithmetical functions defined by Ramanujan and also the theory of elliptic functions.

**Keywords** : Ramanujan's tau function; Generating relation and function; Ordinary finite difference table.

## I. INTRODUCTION

In this paper, we obtain the values of  $\tau(1)$ ,  $\tau(2)$ ,  $\dots$ ,  $\tau(211)$ , where  $\tau(n)$  is Tau function of Ramanujan, defined as follows:

$$\sum_{n=1}^{\infty} \tau(n) x^n = x \left\{ \prod_{n=1}^{\infty} (1 - x^n) \right\}^{24} \quad (1.1)$$

The right hand side of (1.1) is called generating function for  $\tau(n)$ . Ramanujan[3,p.196, Table(V); see also 1;2] calculated the values of  $\tau(1)$ ,  $\tau(2)$ ,  $\dots$ ,  $\tau(30)$ , by means of the theory of elliptic functions and certain arithmetical functions such as  $F_{r,s}(x)$ ,  $\Phi_{r,s}(x)$ ,  $E_{r,s}(n)$ ,  $\sigma_s(n)$ , Riemann's Zeta function  $\zeta(n)$ , greatest integer function  $[x]$ , theory of symbols  $o, O$ , continued fraction, asymptotic expansion, some trigonometrical identities, inequalities, Gamma function, theory of order of error terms, number theory, convergence and divergence of infinite series.

## II. VERIFICATION AND EXTENSION

Consider the expanded form of (1.1), we have

$$\sum_{n=1}^{\infty} \tau(n) x^n = x \{ (1-x)(1-x^2)(1-x^3)(1-x^4) \cdots (1-x^{210}) \cdots \}^{24} \quad (2.1)$$

$$= x \{ (1-x)^3 (1-x^2)^3 (1-x^3)^3 (1-x^4)^3 \cdots (1-x^{210})^3 \cdots \}^8 = x T^8 = x \{ (T^2)^2 \}^2 \quad (2.2)$$

where

$$T = (1-x)^3 (1-x^2)^3 (1-x^3)^3 (1-x^4)^3 \cdots (1-x^{210})^3 \cdots \quad (2.3)$$

Now consider the product of first two hundred ten polynomials in (2.3) and collecting the terms upto  $x^{210}$ , we get

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$$T = +1 - 3x + 5x^3 - 7x^6 + 9x^{10} - 11x^{15} + 13x^{21} - 15x^{28} + 17x^{36} - 19x^{45} + 21x^{55} - \\ - 23x^{66} + 25x^{78} - 27x^{91} + 29x^{105} - 31x^{120} + 33x^{136} - 35x^{153} + 37x^{171} - 39x^{190} + 41x^{210} + \dots \quad (2.4)$$

It is to be noted that the coefficients in (2.4) are alternatively positive and negative such that the sequence 1, 3, 5, 7, 9, ... form arithmetic progression. Suppose the powers of  $x$  (*i.e.* the sequence 0, 1, 3, 6, 10, 15, ...) are generated by the function  $F(k)$ , therefore

$$T = \sum_{k=1}^{\infty} (-1)^{k-1} (2k-1) x^{F(k)} \quad (2.5)$$

Now we shall find the function  $F(k)$  using the following ordinary finite difference table:

$k$	$F(k)$	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	...
1	0															
		1														
2	1		1													
		2		0												
3	3		1		0											
		3		0		0										
4	6		1		0		0									
		4		0		0		0								
5	10		1		0		0		0							
		5		0		0		0		0						
6	15		1		0		0		0		0					
		6		0		0		0		0		0				
7	21		1		0		0		0		0		0			
		7		0		0		0		0		0		0		
8	28		1		0		0		0		0		0		0	
		8		0		0		0		0		0		0		
9	36		1		0		0		0		0		0			
		9		0		0		0		0		0				
10	45		1		0		0		0		0					
		10		0		0		0		0						
11	55		1		0		0		0							
		11		0		0		0								
12	66		1		0		0									
		12		0		0										
13	78		1		0											
		13		0												
14	91		1													
		14														
15	105															
⋮																

ORDINARY FINITE DIFFERENCE TABLE



Since second order ordinary differences are equal, therefore third and higher order differences will be zero and so  $F(k)$  will be a polynomial of second degree (by means of fundamental theorem of finite difference calculus). Thus:

$$F(k) = A + Bk + Ck^2 \quad (2.6)$$

where the unknowns  $A, B$  and  $C$  are to be calculated.

Now selecting any three values of  $k$  and also their corresponding values of  $F(k)$  from above table and putting them in (2.6), we get a system of three linear equations which on simplification gives  $A = 0$ ,  $B = -\frac{1}{2}$  and  $C = \frac{1}{2}$ .

Therefore suitable  $F(k)$  is given by

$$F(k) = -\frac{1}{2}k + \frac{1}{2}k^2 = \frac{k(k-1)}{2}$$

Consequently (2.5) reduces to:

$$T = \sum_{k=1}^{\infty} (-1)^{k-1} (2k-1) x^{\frac{k(k-1)}{2}} \quad (2.7)$$

Now squaring the expansion in (2.4) and collecting the terms upto  $x^{210}$ , we have

$$\begin{aligned} T^2 = & +1 - 6x + 9x^2 + 10x^3 - 30x^4 + 11x^6 + 42x^7 - 70x^9 + 18x^{10} - 54x^{11} + 49x^{12} + 90x^{13} - 22x^{15} - \\ & - 60x^{16} - 110x^{18} + 81x^{20} + 180x^{21} - 78x^{22} + 130x^{24} - 198x^{25} - 182x^{27} - 30x^{28} + 90x^{29} + 121x^{30} + \\ & + 84x^{31} + 210x^{34} - 252x^{36} - 102x^{37} - 270x^{38} + 170x^{39} - 69x^{42} + 330x^{43} - 38x^{45} + 420x^{46} - 190x^{48} - \\ & - 390x^{49} - 108x^{51} - 300x^{55} + 99x^{56} + 442x^{57} + 210x^{58} + 418x^{60} - 294x^{61} - 510x^{64} + 378x^{65} - \\ & - 540x^{66} + 138x^{67} - 230x^{69} - 462x^{70} + 611x^{72} + 570x^{73} + 132x^{76} + 50x^{78} - 150x^{79} + 110x^{81} - \\ & - 630x^{83} - 350x^{84} - 598x^{87} + 450x^{88} + 361x^{90} + 660x^{91} + 162x^{92} - 550x^{93} + 420x^{94} + 378x^{97} + \\ & + 650x^{99} - 798x^{100} - 486x^{101} - 782x^{102} + 58x^{105} - 330x^{106} + 290x^{108} + 441x^{110} + 468x^{111} - 702x^{112} + \\ & + 850x^{114} + 522x^{115} + 810x^{119} - 700x^{120} - 780x^{121} - 1260x^{123} + 1188x^{126} - 918x^{127} - 558x^{130} + \\ & + 529x^{132} + 180x^{133} + 682x^{135} + 1092x^{136} - 198x^{137} + 330x^{139} + 180x^{141} - 462x^{142} - 1150x^{144} - \\ & - 540x^{146} + 930x^{148} - 1102x^{150} - 726x^{151} - 70x^{153} + 210x^{154} - 779x^{156} + 2100x^{157} + 490x^{159} + \\ & + 1218x^{160} - 630x^{163} - 990x^{164} + 1178x^{165} + 770x^{168} - 1350x^{169} - 1260x^{171} + 900x^{172} - 540x^{174} - \\ & - 1302x^{175} - 518x^{177} + 462x^{181} + 729x^{182} + 1450x^{183} + 612x^{186} - 1190x^{189} - 78x^{190} + 1620x^{191} + \\ & + 962x^{192} - 390x^{193} - 1020x^{196} - 220x^{198} - 1110x^{199} - 702x^{200} - 1518x^{202} + 858x^{205} + 1258x^{207} - \\ & - 1470x^{208} + 923x^{210} + \dots \end{aligned} \quad (2.8)$$

Further repeating the same process for  $(T^2)^2$ , we get

$$T^4 = +1 - 12x + 54x^2 - 88x^3 - 99x^4 + 540x^5 - 418x^6 - 648x^7 + 594x^8 + 836x^9 + 1056x^{10} - 4104x^{11} -$$

$$\begin{aligned}
& -209x^{12} + 4104x^{13} - 594x^{14} + 4256x^{15} - 6480x^{16} - 4752x^{17} - 298x^{18} + 5016x^{19} + 17226x^{20} - \\
& -12100x^{21} - 5346x^{22} - 1296x^{23} - 9063x^{24} - 7128x^{25} + 19494x^{26} + 29160x^{27} - 10032x^{28} - \\
& -7668x^{29} - 34738x^{30} + 8712x^{31} - 22572x^{32} + 21812x^{33} + 49248x^{34} - 46872x^{35} + 67562x^{36} + 2508x^{37} - \\
& -47520x^{38} - 76912x^{39} - 25191x^{40} + 67716x^{41} + 32076x^{42} + 7128x^{43} + 29754x^{44} + 36784x^{45} - \\
& -51072x^{46} + 45144x^{47} - 122398x^{48} - 53460x^{49} + 11286x^{50} - 27256x^{51} + 57024x^{52} + 122364x^{53} + \\
& + 99902x^{54} + 3576x^{55} - 29646x^{56} - 221616x^{57} + 41382x^{58} - 52272x^{59} + 130549x^{60} - 206712x^{61} - \\
& -180036x^{62} + 336512x^{63} + 145200x^{64} + 100980x^{65} - 73568x^{66} + 221616x^{67} - 317142x^{68} - 148324x^{69} + \\
& + 15552x^{70} - 225720x^{71} - 32076x^{72} + 108756x^{73} + 196614x^{74} + 74360x^{75} - 58806x^{76} + 229824x^{77} + \\
& + 120878x^{78} - 233928x^{79} + 361152x^{80} - 111340x^{81} - 349920x^{82} - 491832x^{83} - 196569x^{84} - \\
& -82764x^{85} + 707454x^{86} + 18392x^{87} + 92016x^{88} + 493668x^{89} - 559450x^{90} + 416856x^{91} - 16092x^{92} + \\
& + 320760x^{93} - 361152x^{94} - 724032x^{95} + 7106x^{96} + 270864x^{97} - 530442x^{98} + 56168x^{99} - 261744x^{100} + \\
& + 52272x^{101} + 930204x^{102} + 406296x^{103} + 451440x^{104} - 339196x^{105} + 562464x^{106} - 653400x^{107} - \\
& -374528x^{108} - 810744x^{109} - 248292x^{110} + 779360x^{111} + 20691x^{112} - 744876x^{113} - 272746x^{114} + \\
& + 570240x^{115} - 153846x^{116} - 69984x^{117} + 922944x^{118} + 1154736x^{119} + 657074x^{120} - 694980x^{121} - \\
& -489402x^{122} - 349448x^{123} - 812592x^{124} + 1341900x^{125} - 2216160x^{126} - 384912x^{127} + 132354x^{128} + \\
& + 26224x^{129} + 58806x^{130} + 943272x^{131} + 1052676x^{132} - 357048x^{133} + 967518x^{134} - 518320x^{135} - \\
& -441408x^{136} - 112860x^{137} + 2222726x^{138} - 421344x^{139} - 196614x^{140} - 1552276x^{141} - 541728x^{142} - \\
& -1515888x^{143} - 1067021x^{144} + 1468776x^{145} - 1072170x^{146} - 414072x^{147} + 2216160x^{148} + \\
& + 1715472x^{149} + 1064800x^{150} - 135432x^{151} - 1875852x^{152} + 1585892x^{153} + 327072x^{154} - 730728x^{155} + \\
& + 584858x^{156} + 470448x^{157} - 2482866x^{158} - 320760x^{159} - 1468368x^{160} + 496584x^{161} + 87362x^{162} - \\
& -1198824x^{163} + 114048x^{164} + 377948x^{165} + 29502x^{166} + 1177848x^{167} + 639122x^{168} + 355752x^{169} + \\
& + 2298240x^{170} + 2276560x^{171} + 2659392x^{172} - 2904660x^{173} - 3991570x^{174} - 1715472x^{175} + \\
& + 1429218x^{176} - 2531088x^{177} + 627264x^{178} + 1161864x^{179} - 1777203x^{180} - 1566588x^{181} + \\
& + 3648348x^{182} - 1089232x^{183} - 1705374x^{184} - 1715472x^{185} + 3505766x^{186} + 2160432x^{187} + \\
& + 248292x^{188} + 4043852x^{189} - 4038144x^{190} + 5187456x^{191} - 2566080x^{192} + 1197900x^{193} - \\
& -950346x^{194} - 2437776x^{195} - 1211760x^{196} - 4153248x^{197} - 520738x^{198} + 882816x^{199} + 764370x^{200} - \\
& -1779008x^{201} - 1360314x^{202} - 160920x^{203} + 2640506x^{204} + 3805704x^{205} + 674784x^{206} + \\
& + 3656664x^{207} + 1779888x^{208} - 4980204x^{209} - 237994x^{210} + \dots \quad (2.9)
\end{aligned}$$

Finally adopting the same procedure for  $(T^4)^2$  and multiplying  $(T^4)^2$  by  $x$  and comparing the coefficients of  $x, x^2, x^3, x^4, \dots, x^{210}, x^{211}$  with the coefficients of left hand side of (2.2), we get the values of  $\tau(1), \tau(2), \tau(3), \tau(4), \dots, \tau(210), \tau(211)$  and are given in tabular form as follows:

III. EXTENDED TABLE FOR  $\tau(n)$  ;  $n \in \{1, 2, 3, 4, 5, \dots, 211\}$ 

$n$	$\tau(n)$	$n$	$\tau(n)$	$n$	$\tau(n)$
1	+1	37	-182213314	73	+1463791322
2	-24	38	-255874080	74	+4373119536
3	+252	39	-145589976	75	-6425804700
4	-1472	40	+408038400	76	-15693610240
5	+4830	41	+308120442	77	-8951543328
6	-6048	42	+101267712	78	+3494159424
7	-16744	43	-17125708	79	+38116845680
8	+84480	44	-786948864	80	+4767866880
9	-113643	45	-548895690	81	+1665188361
10	-115920	46	-447438528	82	-7394890608
11	+534612	47	+2687348496	83	-29335099668
12	-370944	48	+248758272	84	+6211086336
13	-577738	49	-1696965207	85	-33355661220
14	+401856	50	+611981400	86	+411016992
15	+1217160	51	-1740295368	87	+32358470760
16	+987136	52	+850430336	88	+45164021760
17	-6905934	53	-1596055698	89	-24992917110
18	+2727432	54	+1758697920	90	+13173496560
19	+10661420	55	+2582175960	91	+9673645072
20	-7109760	56	-1414533120	92	-27442896384
21	-4219488	57	+2686677840	93	-13316478336
22	-12830688	58	-3081759120	94	-64496363904
23	+18643272	59	-5189203740	95	+51494658600
24	+21288960	60	-1791659520	96	-49569988608
25	-25499225	61	+6956478662	97	+75013568546
26	+13865712	62	+1268236032	98	+40727164968
27	-73279080	63	+1902838392	99	-60754911516
28	+24647168	64	+2699296768	100	+37534859200
29	+128406630	65	-2790474540	101	+81742959102
30	-29211840	66	-3233333376	102	+41767088832
31	-52843168	67	-15481826884	103	-225755128648
32	-196706304	68	+10165534848	104	-48807306240
33	+134722224	69	+4698104544	105	-20380127040
34	+165742416	70	+1940964480	106	+38305336752
35	-80873520	71	+9791485272	107	+90241258356
36	+167282496	72	-9600560640	108	+107866805760

$n$	$\tau(n)$	$n$	$\tau(n)$	$n$	$\tau(n)$
109	+73482676310	144	-112181096448	178	+599830010640
110	-61972223040	145	+620204022900	179	+1681384224780
111	-45917755128	146	-35130991728	180	+807974455680
112	-16528605184	147	-427635232164	181	-996774496018
113	-85146862638	148	+268217998208	182	-232167481728
114	-64480268160	149	-1115433620850	183	+1753032622824
115	+90047003760	150	+154219312800	184	+1574983618560
116	-189014559360	151	-824447297848	185	-880090306620
117	+65655879534	152	+900676761600	186	+319595480064
118	+124540889760	153	+784811057562	187	-3691995187608
119	+115632958896	154	+214837039872	188	-3955776986112
120	+102825676800	155	-255232501440	189	+1226984915520
121	+498319933	156	+214308444672	190	-1235871806400
122	-166955487888	157	+1315116754406	191	+2762403350592
123	+77646351384	158	-914804296320	192	+680222785536
124	+77785143296	159	-402206035896	193	+5442387685442
125	-359001100500	160	-950091448320	194	-1800325645104
126	-45668121408	161	-312162946368	195	-703199584080
127	-262717201024	162	-39964520664	196	+2497932784704
128	+338071388160	163	-357832759588	197	-2876091504354
129	-4315678416	164	-453553290624	198	+1458117876384
130	+66971388960	165	+650708341920	199	+728391402200
131	+631528759932	166	+704042392032	200	-2154174528000
132	-198311113728	167	+2754833892216	201	-3901420374768
133	-178514816480	168	-356462346240	202	-1961831018448
134	+371563845216	169	-1458379197393	203	-2150040612720
135	-353937956400	170	+800535869280	204	+2561714781696
136	-583413304320	171	-1211595753060	205	+1488221734860
137	-297198746214	172	+25209042176	206	+5418123087552
138	-112754509056	173	-950387449578	207	-2118677359896
139	+596793577940	174	-776603298240	208	-570305978368
140	+119045821440	175	+426959023400	209	+5699723069040
141	+677211820992	176	+527734751232	210	+489123048960
142	-234995646528	177	-1307679342480	211	-6793168439188
143	-308865667656				

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## Screening of Indian Medicinal Plants and their potentials as Antimicrobial Agents

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**Abstracts** - Ethanol extracts of certain Indian Medicinal Plants *Curculigo orchioides*, *Symplocos racemosa*, *Puerariatuberosa*, *Scindapsus officinarum*, *Luffa acutangula* and *Acacia nilotica* were examined for their anti-microbial potentials against selected bacteria and fungi. The purpose of screening is to justify, authenticate and validate the use of Indian Medicinal Plants in ethno-medicinal or folklore as traditional treasure to cure various ailments. In present investigations attempts were made to screen the Indian Medicinal Plants as antibiotics. The extracts were tested against selected test bacteria and fungi as antimicrobial assay through disc diffusion assay where standard tetracycline is used and solvent ethanol as control. Indian Medicinal Plants have a traditional background that they have potentials to use as antimicrobial agents. The results showed that all the extracts possess good antimicrobial activity against selected test bacteria and intermediate against fungus. The present results therefore offer a scientific basis for traditional use of ethanolic extracts *Curculigo orchioides*, *Symplocos racemosa*, *Pueraria tuberosa*, *Scindapsus officinarum*, *Luffa acutangula* and *Acacia nilotica*. These results explain that certain plants showed potential antimicrobial activity against *S. aureus* negative can be used as a very good treatment for acne if added to daily diet. Further, almost all the selected plants have also possessed antimicrobial potentials against all test bacteria and fungi which explains that their use in daily life will generate a resistant or immunity to fight against microorganisms.

**Classification:** *GJSFR-F Classification: FOR Code: 060599*



*Strictly as per the compliance and regulations of:*



# Screening of Indian Medicinal Plants and their potentials as Antimicrobial Agents

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**Abstract**—Ethanol extracts of certain Indian Medicinal Plants *Curculigo orchioides*, *Symplocos racemosa*, *Pueraria tuberosa*, *Scindapsus officinarum*, *Luffa acutangula* and *Acacia nilotica* were examined for their anti-microbial potentials against selected bacteria and fungi. The purpose of screening is to justify, authenticate and validate the use of Indian Medicinal Plants in ethno-medicinal or folklore as traditional treasure to cure various ailments. In present investigations attempts were made to screen the Indian Medicinal Plants as antibiotics. The extracts were tested against selected test bacteria and fungi as antimicrobial assay through disc diffusion assay where standard tetracycline is used and solvent ethanol as control. Indian Medicinal Plants have a traditional background that they have potentials to use as antimicrobial agents. The results showed that all the extracts possess good antimicrobial activity against selected test bacteria and intermediate against fungus. The present results therefore offer a scientific basis for traditional use of ethanolic extracts *Curculigo orchioides*, *Symplocos racemosa*, *Pueraria tuberosa*, *Scindapsus officinarum*, *Luffa acutangula* and *Acacia nilotica*. These results explain that certain plants showed potential antimicrobial activity against *S. aureus* negative can be used as a very good treatment for acne if added to daily diet. Further, almost all the selected plants have also possessed antimicrobial potentials against all test bacteria and fungi which explains that their use in daily life will generate a resistant or immunity to fight against microorganisms.

## I. INTRODUCTION

Nature has been a source of medicinal agents since times immemorial. The importance of herbs in the management of human ailments cannot be over emphasized. It is clear that the plant kingdom harbors an inexhaustible source of active ingredients invaluable in the management of many intractable diseases. Ayurveda is ancient health care system and is practiced widely in India, Srilanka and other countries (Chopra and Doiphode, 2002). Ayurveda system of medicine use plants to cure the ailments and diseases. Despite the availability of different approaches

for the discovery of therapeutically, natural products still remain as one of the best reservoir of new structural types. They are used directly as therapeutic agents, as well as starting material for the synthesis of drugs or as models for pharmacologically active compounds (Cowan, 1999). In modern time plants have been sources of analgesics, anti-inflammatory, anti-neoplastic drugs, medicine for asthma, anti arrhythmic agents and anti hypertensive.

Furthermore, the active components of herbal remedies have the advantage of being combined with many other substances that appear to be inactive. However, these complementary components give the plant as a whole a safety and efficiency much superior to that of its isolated and pure active components (Shariff, 2001).

In last three decades numbers of new antibiotics have produced, but clinical efficacy of these existing antibiotics is being threatened by the emergence of multi drug resistant pathogens (Bandow et al., 2003). In general, bacteria have the genetic ability to transmit and acquire resistance to drugs (Cohen, 1992). According to World Health Organization (WHO) medicinal plants would be the best source to obtain a variety of drugs (Santos et al., 1995).

Antibiotic resistance has become a global concern (Westh et al., 2004). There has been an increasing incidence of multiple resistances in human pathogenic microorganisms in recent years, largely due to indiscriminate use of commercial antimicrobial drugs commonly employed in the treatment of infectious diseases. This has forced scientist to search for new antimicrobial substances from various sources like the medicinal plants. Search for new antibacterial agents should be continued by screening many plant families. Recent work revealed the potential of several herbs as sources of drugs (Iwu, 2002). The screening of plant extracts and plant products for antimicrobial activity has shown that higher plants represent a potential source of novel antibiotic prototypes (Afolayan, 2003).

Numerous studies have identified compounds within herbal plants that are effective antibiotics. Traditional healing systems around the world that utilize herbal remedies are an important source for the

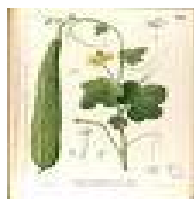
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discovery of new antibiotics. Some traditional remedies have already produced compounds that are effective against antibiotic resistant strains of bacteria (Kone et al., 2004). The results of this indicate the need for further research into traditional health system. It also facilitates pharmacological studies leading to synthesis of a more potent drug with reduced toxicity. The need of the hour is to screen a number of medicinal plants for promising biological activity.

Therefore, in present project attempts have been made to six medicinal plants *Curculigo orchioides*, *Symplocos racemosa*, *Pueraria tuberosa*, *Scindapsus officinarum*, *Luffa acutangula* and *Acacia nilotica* each belonging to different families were evaluated for antibacterial potentials. Further, all the selected medicinal plants were used to justify and authenticate on scientific basis where antimicrobial characters will be aid as a markers to characterize these drugs from their adulterants. These biomarkers can be used further for formation of Indian Pharmacopoeia.



*Luffa acutangula*      *Picrorhizza kurroa*      *Pueraria tuberosa*  
*Orchioides*              *Acacia nilotica*              *Curculigo*

## II. MATERIALS AND METHODS

**Collection:** Plant samples (*Curculigo orchioides*, *Symplocos racemosa*, *Pueraria tuberosa*, *Scindapsus officinarum*, *Luffa acutangula* and *Acacia nilotica*) were collected from various tribes living in tribal pockets of Mt. Abu, arid zone of Rajasthan. These plants were used by these tribes in their daily lives to cure various ailments and few from Chunnilal Attar Ayurvedic Store, Ghat Gate, Jaipur in the month of May, 2009.

**Identification:** All the samples were authenticated and were given identification number *Curculigo orchioides*, *Symplocos racemosa*, *Pueraria tuberosa*, *Scindapsus*

*officinarum*, *Luffa acutangula* and *Acacia nilotica*. These samples were authenticated and submitted in Ethnomedicinal Herbarium, Centre of Excellence funded by DST, MGias, Jaipur (Rajasthan).

**Sources of test organisms:** Bacteria-Pure culture of all test organisms, namely *Pseudomonas aeruginosa*, *Staphylococcus aureus* positive, *Escherichia coli*, *Staphylococcus aureus* negative and fungi *Candida albicans*, were obtained through the courtesy of Mahatma Gandhi Institute of applied Sciences(MGias), Jaipur, which were maintained on Nutrient broth media.

**Culture of test microbes:** For the cultivation of bacteria, Nutrient Agar Medium (NAM) was prepared by using 20 g Agar, 5 g Peptone, 3 g beef extract and 3 g NaCl in 1 L distilled water and sterilized at 15 lbs pressure and 121°C temperature for 25-30 min. Agar test plates were prepared by pouring approximately 15 mL of NAM into the Petri dishes (10 mm) under aseptic conditions. A saline solution was prepared (by mixing 0.8% NaCl) in distilled water, followed by autoclaving and the bacterial cultures were maintained on this medium by regular sub-culturing and incubation at 37°C for 24-48 h.

To prepare the test plates, in bacteria, 10-15 mL of the respective medium was poured into the Petri plates and used for screening. For assessing the bactericidal efficacy, a fresh suspension of the test bacteria was prepared in saline solution from a freshly grown Agar slant.

**Preparation of test extracts:** Crushed powder (50 g) of all the species were successively soxhlet extracted with ethanol. Later, each of the homogenates was filtered and the residue was re-extracted twice for complete exhaustion, the extracts were pooled individually. Each filtrate was concentrated to dryness *in vitro* and re dissolved in respective solvents, out of which 80 mg/10 disc i.e. 8 mg disc<sup>-1</sup> concentration were stored at 4°C in a refrigerator, until screened for antibacterial activity.

**Bactericidal assay:** For both, bactericidal *in vitro* Disc diffusion method was adopted (Gould and Bowie, 1952), because of reproducibility and precision. The different test organisms were proceeded separately using a sterile swab over previously sterilized culture medium plates and the zone of inhibition were measured around sterilized dried discs of Whatman No. 1 paper (6 mm in diameter), which were containing 4 mg of the test extracts, its control (of the respective solvent) and tetracycline as reference drugs (standard disk) separately. Such treated discs were air-dried at room temperature to remove any residual solvent, which might interfere with the determination, sterilized and inoculated. These plates were initially placed at low temperature for 1 h so as to allow the maximum diffusion of the compounds from the test disc into the agar plate and later, incubated at 37°C for 24 h in case of bacteria, after which the zones of inhibition could be easily observed. Five replicates of each test extract were examined and the mean values were then referred.

The Inhibition Zone (IZ) in each case were recorded and the Activity Index (AI) was calculated as compared with those of their respective standard reference drugs (AI = Inhibition Zone of test sample/Inhibition zone of standard).

### III. RESULTS

The profile of six medicinal plants used in present investigation. The results of antimicrobial activity of the crude extracts of Selected Indian Medicinal Plants

Table 1: Antimicrobial Efficacy in terms of inhibition zone and activity index of certain Indian Medicinal Plants against selected test bacteria and fungi where tetracycline is used as standard

Ethanollic extract	Measures	<i>Pseudomonas aeruginosa</i>	<i>Escherichia coli</i>	<i>S. aureus</i> positive	<i>S. aureus</i> negative	<i>Candida albicans</i>
	Standard IZ	27	28	29	26	30
<i>Curculigo archioides</i>	IZ (mm)	12	11	10	8	11
	AI	0.63	0.647	0.45	0.347	0.5
<i>Symplocos racemosa</i>	IZ (mm)	10	11	10	10	9
	AI	0.526	0.647	0.45	0.43	0.33
<i>Puraria tuberosa</i>	IZ (mm)	7	10	13	12	9
	AI	0.368	0.35	0.59	0.521	0.33
<i>Scindapsus officinarum</i>	IZ (mm)	10	12	8	9	10
	AI	0.526	0.705	0.363	0.391	0.45
<i>Luffa acutangula</i>	IZ (mm)	10	9	8	8	8
	AI	0.526	0.34	0.363	0.347	0.266
<i>Acacia nilotica</i>	IZ (mm)	7	19	11	11	8
	AI	0.368	0.67	0.37	0.42	0.266

IZ = Inhibition zone, AI = Activity index Standard = tetracycline

While screening of ethanollic extract of *Curculigo orchoides* the antibacterial activity against selected test bacteria showing good inhibition zone. The ethanollic extract have the potential to make inhibition zone against *Pseudomonas aeruginosa* (IZ=12mm), *GPB* (IZ=10mm), *Staphylococcus aureus* positive strain (IZ=10mm), *E.coli* (IZ=11mm), *Staphylococcus aureus* negative strain (IZ=8mm) & in antifungal activity the inhibition zone against *Candida albicans* is 11 mm. These results showed that the given test extracts have maximum activity against *Pseudomonas aeruginosa* & minimum against *Staphylococcus aureus* negative strain. While screening of ethanollic extract of *Symplocos racemosa* the antibacterial activity against selected test bacteria showing good inhibition zone. The ethanollic extract have the potential to make inhibition zone against *Pseudomonas aeruginosa* (IZ=10mm), *GPB* (IZ=10mm), *Staphylococcus aureus* positive strain

(IZ=10mm), *E. coli* (IZ=11mm), *Staphylococcus aureus* negative strain (IZ=10mm) & in antifungal activity the inhibition zone against *Candida albicans* is 9mm. These results showed that the given test extracts have maximum activity against *E.coli* & minimum against *Candida albicans*.

While screening of ethanollic extract of *Pueraria tuberosa* the antibacterial activity against selected test bacteria showing very good inhibition zone. The ethanollic extract have the potential to make inhibition zone against *Pseudomonas aeruginosa* (IZ=7mm), *GPB* (IZ=8mm), *Staphylococcus aureus* positive strain (IZ=13mm), *E. coli* (IZ=1mm), *Staphylococcus aureus* negative strain (IZ=12mm) & in antifungal activity the inhibition zone against *Candida albicans* is 9mm. These results showed that the given test extracts have maximum activity against *Staphylococcus aureus* positive strain & minimum against *E. coli*.

While screening of ethanolic extract of *Scindapsus officinarum* the antibacterial activity against selected test bacteria showing good inhibition zone. The ethanolic extract have the potential to make inhibition zone against *Pseudomonas aeruginosa* (IZ=10mm),

GPB (IZ=9mm), *Staphylococcus aureus* positive strain (IZ=8mm), *E. coli* (IZ=12mm), *Staphylococcus aureus* negative strain (IZ=9mm) & in antifungal activity the inhibition zone against *Candida albicans* is 10mm.

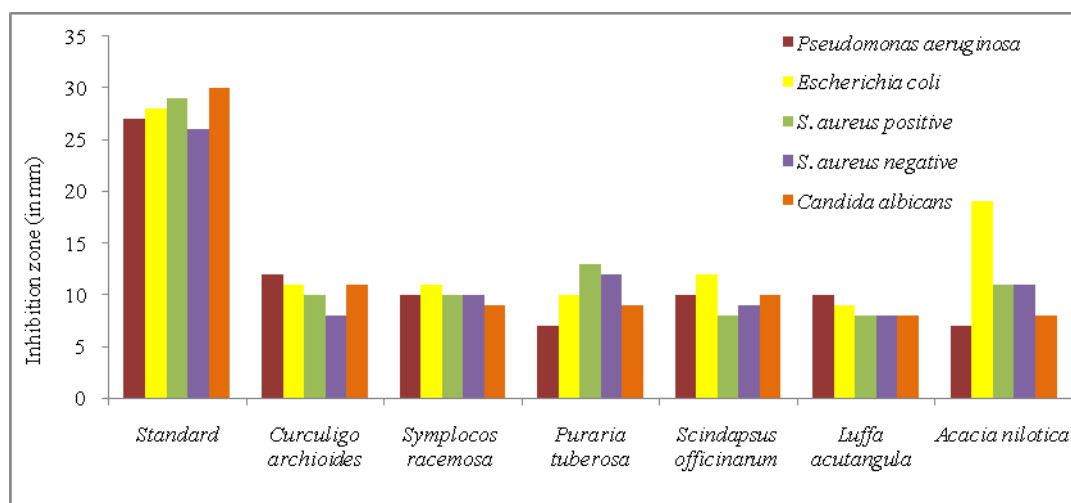


Fig.1. Antimicrobial potentials of certain Indian Medicinal Plants against selected microorganisms in terms of inhibition zone (mm)

These results showed that the given test extracts have maximum activity against *E. coli* & minimum against *Staphylococcus aureus* positive strain. While screening of ethanolic extract of *Luffa acutangula* the antibacterial activity against selected test bacteria showing good inhibition zone. The methanolic extract have the potential to make inhibition zone against *Pseudomonas aeruginosa* (IZ=10mm), GPB (IZ=8mm), *Staphylococcus aureus* positive strain (IZ=8mm), *E. coli* (IZ=9mm), *Staphylococcus aureus* negative strain (IZ=8mm) & in antifungal activity the inhibition zone against *Candida albicans* is 8mm. These results showed that the given test extracts have maximum activity against *Pseudomonas aeruginosa* & minimum against), GPB, *Staphylococcus aureus* positive strain, *Staphylococcus aureus* negative strain *Candida albicans* While screening of ethanolic extract of *Acacia nilotica* the antibacterial activity against selected test bacteria showing very good inhibition zone. The ethanolic extract have the potential to make inhibition zone against *Pseudomonas aeruginosa* (IZ=7mm), GPB (IZ=1mm), *Staphylococcus aureus* positive strain (IZ=11mm), *E. coli* (IZ=19mm), *Staphylococcus aureus* negative strain (IZ=11mm) & in antifungal activity the inhibition zone against *Candida albicans* is 8mm. These results showed that the given test extracts have maximum activity against *E. coli* & minimum against GPB

#### IV. DISCUSSION

Use of ethno-pharmacological knowledge is one attractive way to reduce empiricism and enhance the probability of success in new drug-finding efforts (Patwardhan, 2005). Validation and selection of primary screening assays are pivotal to guarantee sound selection of extracts or molecules with relevant pharmacological action and worthy following up (Cos et al., 2006). The number of multi-drug resistant microbial strains and the appearance of strains with reduced susceptibility to antibiotics are continuously increasing. This increase has been attributed to indiscriminate use of broad-spectrum antibiotics, immunosuppressive agent, intravenous catheters, organ transplantation and ongoing epidemics of HIV infection (Graybill, 1988). In addition, in developing countries, synthetic drugs are not only expensive and inadequate for the treatment of diseases but also often with adulterations and side effects. Therefore, there is need to search new infection-fighting strategies to control microbial infections.

The present results therefore offer a scientific basis for traditional use of ethanolic extracts *Curculigo archioides*, *Symplocos racemosa*, *Pueraria tuberosa*, *Scindapsus officinarum*, *Luffa acutangula* and *Acacia nilotica*. These results explain that Indian Medicinal Plants have potential as antimicrobials against *S. aureus* negative can be used as a very good treatment for acne if added to daily diet and some other showed potentials

against *S. aureus* positive. Further, more or less all the selected Indian Medicinal Plants have also possessed antimicrobial potentials against all test bacteria and fungi which explains that their use in daily life will generate a resistant or immunity to fight against microorganisms.

Ethanollic extracts of certain Indian Medicinal Plants showed promising antimicrobial potentials against selected test bacteria and fungi. The main aim of these studies is to validate and authenticate the antimicrobial potentials of certain plants and simultaneously, justify their use in the daily diet to cure mankind from certain ailments.

## V. ACKNOWLEDGEMENT

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# Growth and Productivity Analysis of non Metallic Minerals Products Industry of Punjab

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**Abstracts** - In the fast changing liberalized global environment where growth and productivity have emerged as the important agents of growth and development, present study is an effort to investigate growth pattern and productivity trends of small scale non metallic mineral products industry in Punjab. The growth of industry has been measured in terms of four variables namely: number of units, fixed investment, direct employment and production. Yearly growth rates have been computed to mirror year to-year fluctuations in growth and compound annual growth rates (CAGRs) have been worked out to find the impact of the policies of liberalized regime on growth of this industry. Productivity trends have been sketched in terms of capital intensity, capital output ratio and partial factor productivities. The study observed that the significant growth rate was observed in the variables namely number of units, fixed investment and production. But the policies of liberalized regime have resulted in qualitative rather than quantitative growth in the small scale non-metallic minerals products industry in Punjab. Highly significant growth rate was recorded in fixed investment and production, a slow growth was noticed in number of units but insignificant growth was gauged in employment during the liberalization period. However, in the overall period of the study, significant growth rate was registered in the case of all the four variables. Thus, it could safely be inferred from the analysis that the liberalization has resulted in jobless growth as the rate of growth of employment has gone down miserably. The profile of labour and capital productivity reflects that in absolute terms the labour and capital productivity and the capital intensity exhibited significant growth rate capital output ratio recorded negative growth during the liberalization period. The comparative profile of pre-liberalization and liberalization period indicates that during liberalization period, productivities of labour and capital accompanied by capital intensity have improved significantly whereas capital output ratio decelerated.

**Keywords :** DOM, Productivity, Compound annual growth rate, capital Intensity .

**Classification:** GJSFR-I Classification: FOR Code: 120405, 120103



*Strictly as per the compliance and regulations of:*





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**Abstract :** In the fast changing liberalized global environment where growth and productivity have emerged as the important agents of growth and development, present study is an effort to investigate growth pattern and productivity trends of small scale non metallic mineral products industry in Punjab. The growth of industry has been measured in terms of four variables namely: number of units, fixed investment, direct employment and production. Yearly growth rates have been computed to mirror year to-year fluctuations in growth and compound annual growth rates (CAGRs) have been worked out to find the impact of the policies of liberalized regime on growth of this industry. Productivity trends have been sketched in terms of capital intensity, capital output ratio and partial factor productivities. The study observed that the significant growth rate was observed in the variables namely number of units, fixed investment and production. But the policies of liberalized regime have resulted in qualitative rather than quantitative growth in the small scale non-metallic minerals products industry in Punjab. Highly significant growth rate was recorded in fixed investment and production, a slow growth was noticed in number of units but insignificant growth was gauged in employment during the liberalization period. However, in the overall period of the study, significant growth rate was registered in the case of all the four variables. Thus, it could safely be inferred from the analysis that the liberalization has resulted in jobless growth as the rate of growth of employment has gone down miserably. The profile of labour and capital productivity reflects that in absolute terms the labour and capital productivity and the capital intensity exhibited significant growth rate capital output ratio recorded negative growth during the liberalization period. The comparative profile of pre-liberalization and liberalization period indicates that during liberalization period, productivities of labour and capital accompanied by capital intensity have improved significantly whereas capital output ratio decelerated.

**Keywords :** *DOM, Productivity, Compound annual growth rate, capital Intensity.*

## I. INTRODUCTION

In Punjab, agricultural growth has saturated and the state government is making every effort to develop secondary and tertiary sectors in order to augment the income of its people. But the efforts of the state government faced numerous challenges which are

factors like militancy and global factors like WTO etc. Still, the small scale industry of the state is able to confined not only to the adverse geo-political situation of the state but also relate to various socio-economic withstand all the challenges. The small scale non-metallic minerals products industry which constitutes-manufacture of glass and glass products, cement, lime and plaster, articles of concrete, cutting, shaping and finishing of stones, manufacture of structural non-refractory clay and ceramic products etc., has demonstrated remarkable resilience and succeeded in strengthening its footholds despite the economic challenges unleashed by the policies of the liberalized regime. The policies of the liberalized regime aim to dismantle all the growth retarding structures to trade, investment and productivity. Removal of quantitative and non-quantitative restrictions, rationalization of subsidies, toning up tax administration, easing regulatory controls etc are some of the hallmarks of the liberalized regime. And as a consequence of this, competition has increased manifold, compelling the businesses to enhance productivities in order to survive in the market. Despite the challenges during pre-liberalization and liberalization period, small scale non-metallic minerals products industry of Punjab has made significant growth.. The small scale non-metallic minerals products producing units in the small scale sector were only 756 in the year 1980-81 which soared to 1980 units in 1991-92 and further grew to the level of 2682 in 2004-05. As regards employment the industry provided employment to 8212 persons in the year 1980-81 which surged to 23862 persons in the year 1991-92 and further climbed to the level of 33758 persons in the year 2004-05. In the sphere of fixed capital investment, it was only 6.28 crores of rupees in the year 1980-81 which jumped to Rs. 33.81 crores in 1991-92 and further advanced the level of Rs. 109.36 crores in 2004-05. Similarly the value of production of small scale chemical products industry of Punjab was a only worth Rs. 25.47 crores in the year 1980-81, entailed to the level of Rs. 114.03 crores in the year 1991-92 and further enhanced to the level of Rs. 489.75 crores in the year 2004-05 (Directorate of Industries, Punjab ,2005) .

**About:** Presently working as a senior lecturer in department of Economics at D.A.V. College Hoshiarpur

## II. OBJECTIVES OF THE STUDY

Analysis of growth and productivity of an industry plays an instrumental role in framing a pragmatic and result oriented industrial development strategy. In this study, an attempt has been made to dig the facts about small scale non-metallic minerals products industry in Punjab which can be treated as a catalytic agent for the cause of appropriate policy formulation. The specific objectives of the study were:

- 1 To compute partial productivity of labour (AOLR) and partial productivity of capital (AOCR).
- 2 To analyse the comparative picture of growth of number of units, fixed investment, direct employment and production during pre-liberalization and liberalization periods.
- 3 To calculate average capital output ratio and capital intensity of labour in small scale non-metallic minerals products industry of Punjab.

## III. DATA BASE AND METHODOLOGY

Present study is based on secondary data for the period of 25 years i.e. 1980-81 to 2004-05. (Due to change in classification of industries, it is not possible to get the data as per requirement for the next four years because that data will not be in uniformity with that for the last 25 years, still study finds data for the 25 years sufficient to draw various conclusions). The data relating to number of units, direct employment, fixed capital and production of small scale non-metallic minerals products industry at aggregate level for the above said period were culled from Directorate of Industries, Punjab. Since the figures of fixed capital and production were given at current prices, these have been converted into constant prices by deflating them with index number of the wholesale prices of manufactured products total, taking 1993-94 as the base year. Yearly growth rates for all the four variables were computed to capture year-to-year fluctuations in growth. Partial productivities of labour and capital were obtained as O/L and O/K. For making an assessment of the extent of amount of units of capital that are needed to produce a certain level of output as well as the capital intensity, K/O and K/L ratios were computed. Compound Annual Growth Rates (CAGRs) for overall period (1980-81 to 2004-05) and two sub-periods: pre-liberalization (1980-81 to 1991-92) and liberalization periods (1991-92 to 2004-05) for all the eight variables were estimated by fitting an exponential function of the following form

$$Y_t = \beta_0 \beta_1^t e^{U_t} \quad (1)$$

Where  $Y_t$  is dependent variable,  $\beta_0$  and  $\beta_1$  are the unknown parameters, and  $U_t$  is the disturbance term. The equation (1) could be written in the logarithmic form as follows:

$$\log Y_t = \log \beta_0 + t \log \beta_1 + U_t \quad (2)$$

Above equation was estimated by applying Ordinary Least Square Method and compound rate of growth ( $gr_c$ ) was obtained by taking antilog of estimated regression coefficient, subtracting 1 from it and multiplying the difference by 100, as under:

$$gr_c = (\hat{\beta}_1 - 1) \times 100 \quad (3)$$

Where  $\hat{\beta}_1$  is an estimate for  $\beta_1$ . The significance of growth rates was tested by applying t – test, given as follows:

$$t = \frac{\hat{\beta}_1}{s(\hat{\beta}_1)} \sim t(n-2) \text{ d.f.} \quad (4)$$

Where  $\hat{\beta}_1$  is the regression estimate,  $s(\hat{\beta}_1)$  is the respective standard error. All statistically insignificant growth rates are treated as almost zero growth rates. (Gupta and Kumar, 2006).

## IV. RESULTS AND DISCUSSION

This section presents the results and discussion of the study. The first subsection is devoted to the analysis of compound annual growth rates of number of units, employment, fixed capital and production. The second subsection is devoted to the profile of capital intensity, capital-output ratio and partial productivities of labour and capital in small scale non-metallic minerals products industry.

### V. SECTION – I

#### a. Growth Performance

Measurement of growth has been one of the most extensively researched areas. The growth rate analysis provides the detailed vision of growth. The year to year growth rates and compound annual growth rates (CAGRs) of number of units, fixed investment, employment and production of small scale non-metallic minerals products industry are shown in table I. The results have been discussed in brief under the following four sub heads:-

Insert Table-I

*b. Number of units*

The year-to-year growth rates of number of units as demonstrated in column II of table I suggests a growth trend but with a downward bias. Commencing from the level of 12.17 percent in 1981-82 touched the level of 15.67 percent in 1984-85. Then started the decline in growth rate which continued till 1986-87, however fluctuations in growth rates were observed at odd intervals till 1991-92 and the most noticeable one was in the year 1992-93 when it jumped to the level of 48.94 percent. But in the next year, a sharp fall is found to the tune of -26.01 percent. Improved in next year and observing great fluctuations, finally settled at 0.64 percent in 2004-05. Further perusal of the column reveals that the compound annual growth rates for the pre-liberalization period was found to be 9.1 percent which declined to 1.41 percent in the liberalization period. However, a significant CAGR of 5.06 percent was observed during overall period of the study.

*c. Fixed Investment*

The profile of annual growth rates of fixed investment as envisaged in column III of table I reveals that in 1981-82 rate was 5.72 percent, it touched the level of 13.20 percent in 1983-84.. The yearly growth rate started to fluctuate in the following years and dipped to 0.76 percent in the year 1991-92 and remained in negative zone for next one year and rose to 7.92 percent in 1993-94 and with some fluctuations in following years, escalated to level of 10.47 percent in 1999-2000. Then again witnessing a sharp dip, rate of growth was glanced in 2000-01 of the order of 0.87 but proved to be finalised at the level of -4.67 percent in 2004-05. Further investigation of the column exhibits a significant CAGR of 9.62 percent in the pre-liberalization period which declined to the level of 5.26 percent in the liberalization period. However, a CAGR of 6.40 percent was observed for the entire period of the study.

*d. Direct Employment*

Perusal of yearly growth rates of employment as contained in column IV of table I exhibits swings of varying magnitude through out the study period. The period 1981-82 observed the rate of growth as 10.83 percent, increased to 16.41 percent in 1983-84 and with fluctuations it became 15.28 percent in 1988-89 and with fluctuations at odd intervals, on the whole declining trend continued to be there till 2004-05 when it became very low of the order of 0.64 percent. Further perusal of the column reveals that the pre-liberalisation period noticed a significant CAGR of 10.52, but the liberalisation period failed to register any growth because of insignificant CAGR of 0.41 percent. However, a CAGR of 5.90 percent was observed for the overall period of the study.

*e. Production*

The annual growth rates of production as sketched in column V of table I reflects an uptrend with volatility at every alternate step. Starting from a yearly growth rate of 4.72 percent in 1981-82 rose to a level of 15.72 percent in 1983-84 and sharp dive appeared when it became 1.95 percent in the very next year. Thereafter, the growth rate after getting variations in the following years, entered the negative zone in 1991-92 and jumped to the level of 13.79 percent in 1994-95. The growth rate started fluctuating again in the following years and reached a level of -0.81 percent in the year 2004-05. The column further reveals that the CAGR for the pre-liberalisation period was found to be 8.05 percent which accelerated to the level of 6.90 percent in the liberalization period. However, a CAGR of 6.69 percent was observed for the overall period of the study. The conclusion that emanates from the above discussion is that the liberalization period failed to register improvement in compound annual growth rates in any of the four variables. In case of number of units, a clear deceleration was recorded while the direct employment disappointed the most also because of very low rather insignificant CAGR. In the sphere of fixed capital investment, the CAGR also declined. Production variable also find decline in growth. Hence the policies of the liberalization remained unable to touch growth of this particular industry in Punjab.

## VI.

## SECTION – II

*a. Productivity Analysis And Profile Of Related Variables*

Productivity depends on the relationship between total output and related inputs such as labour and capital which have been used in production of that output. It is evident that the capacity of the economy to produce goods and services mainly depends on productivity of these factors. Productivity can be enhanced through proper utilization of such resources. It is widely agreed that enhancing productivity is a signal of good health of a system which allows producing at lower cost and makes it competitive.

Table –II depicts the profile of capital intensity, capital output ratio and partial productivities of labour and capital of the small scale non-metallic minerals products industry of Punjab. This table also highlights the compound annual growth rates of capital intensity, capital-output ratio and partial productivities of labour and capital for the pre-liberalization and liberalization period. The detailed column wise explanation of table II is discussed as under:

Insert Table-II

b. *Labour Productivity (AOLR)*

The labour productivity as compiled in column II of table II shows a figure of Rs. 0.0080 crores in 1980-81 which continued to fall to the level of 0.0056 till 1992-93 and remained at the same level for the next year. Thereafter the labour productivity escalated in 1994-95 to touch the level of Rs. 0.0061 crores and in 2003-04, touched the peak level of 0.0089. The labour productivity finally settled at Rs. 0.0087 crores in 2004-05. The column further reveals that the CAGR for the liberalization period (4.29 percent) registered a remarkable improvement over the CAGR of -2.27 percent belonging to the pre-liberalisation period. However, a CAGR of 0.74 percent was observed for the overall period of the study.

c. *Capital Intensity (DOM)*

The profile of annual growth rates of capital intensity as sketched in column III of table II demonstrates that the capital intensity which was Rs. 0.0020 crores in 1980-81 reached a level of Rs. 0.0018 crores in 1990-91 after experiencing minor fluctuations and declined to 0.0016 crores in 1992-93 and kept the level for the next three years and surged to level of 0.0018 crores in 1996-97 and with minor fluctuations, finally settled at a level of Rs. 0.0020 crores in 2004-05. The column further reveals a significant improvement in the CAGR of liberalization period (2.57 percent) from the CAGR of -0.85 percent belonging to the pre-liberalisation period. However, a CAGR of 0.47 percent (insignificant) was noticed in the overall period of the study.

d. *Capital-Output Ratio (Cor)*

The column IV of table II portrays the profile of capital output ratio. Starting from a ratio of 0.25 in 1980-81, continued to increase to the level of 0.30 with marginal fluctuations. Dipped to 0.26 and remaining at the same level for the four years, increased to the level of 0.27 in 1998-99. Showing some fluctuations, ultimately downward march continued till COR settled at a level of 0.22 in 2004-05. Further perusal of the column explains that the CAGR of the pre-liberalisation period which was 1.41 percent declined substantially to reach the level of -1.68 percent in the liberalization period. However, a CAGR of -0.33 percent was observed for overall period of the study.

e. *Capital Productivity (Aocr)*

The column V of table II reflects improvement in capital productivity but also dots fluctuations at odd intervals. Commencing from a capital productivity of 4.06 in 1980-81 fell to the level of 3.36 in 1991-92, accelerated to touch the level of 3.60 in 1992-93, and managed to touch the highest level of 4.48 in 2004-05. The column further reveals that the CAGR which was -1.43 percent for the pre-liberalisation period cheered up

during the liberalization period to reach the level of 1.56 percent. However a CAGR of 0.27 percent was observed for the overall period of the study. From the above discussion it can be safely inferred that the liberalization has encouraged mechanization and technological up gradation in the small scale non-metallic minerals products industry in Punjab. The policies of the liberalization regime have resulted in lower COR and enhancement of factor productivities.

## VII. CONCLUSION AND FINDINGS OF THE STUDY

It is quite evident from the entire discussion that despite the problem of militancy during pre-liberalization period, significant growth rate was observed in all the variables namely number of units, fixed investment and production. But the policies of liberalized regime have resulted in qualitative rather than quantitative growth in the small scale non-metallic minerals products industry in Punjab. Highly significant growth rate was recorded in fixed investment and production, a slow growth was noticed in number of units but insignificant growth was gauged in employment during the liberalization period. However, in the overall period of the study, significant growth rate was registered in the case of all the four variables. Thus, it could safely be inferred from the analysis that the liberalization has resulted in jobless growth as the rate of growth of employment has gone down miserably. The profile of labour and capital productivity reflects that in absolute terms the labour and capital productivity and the capital intensity exhibited significant growth rate capital output ratio recorded negative growth during the liberalization period. The comparative profile of pre-liberalization and liberalization period indicates that during liberalization period, productivities of labour and capital accompanied by capital intensity have improved significantly whereas capital output ratio decelerated.

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Table I

Year to year and Compound Annual Growth Rates ( in percent)

Year	Number of units	Fixed Investment (in Rs.Crore)	Direct Employment (in no.)	Production (in Rs.Crore)
1981-82	12.17	5.72	10.83	4.72
1982-83	14.39	15.43	13.60	10.96
1983-84	15.67	13.20	16.41	15.72
1984-85	10.78	5.93	9.91	1.95
1985-86	8.61	10.93	7.51	8.04
1986-87	6.00	12.61	10.53	7.02
1987-88	7.41	9.73	9.38	14.03
1988-89	9.50	12.96	15.28	8.78
1989-90	7.43	5.82	8.65	10.85
1990-91	4.70	4.71	6.82	2.42
1991-92	4.60	0.76	3.73	-6.24
1992-93	48.94	-3.60	5.52	3.54
1993-94	-26.01	7.92	7.36	6.31
1994-95	4.26	5.91	4.35	13.79
1995-96	2.11	3.47	3.33	2.45
1996-97	4.52	13.60	1.36	14.51
1997-98	0.54	7.66	1.26	7.99
1998-99	1.84	8.99	2.41	4.51
1999-00	2.65	10.47	2.81	11.94
2000-01	2.04	0.87	2.77	2.72
2001-02	1.38	3.69	1.59	8.04
2002-03	0.34	0.29	1.02	3.99
2003-04	0.6	-3.66	0.96	1.59
2004-05	0.64	-4.67	0.64	-0.81
Pre Liberalization	9.1*	9.62*	10.52*	8.05*
Liberalization Period	1.41*	5.26*	0.41**	6.90*
Overall Period	5.06*	6.40*	5.90*	6.69*

\*Significant at 5 percent level of significance.

\*\*Insignificant at 5 percent level of significance.

Source: Calculated from the data supplied by Directorate of Industries, Punjab.

Note:1. Fixed investment and Production figures are taken on 1993-94 constant prices to compute various growth rates.

2.It is not possible to find change for the first year as it is based on previous year so growth for 1980-81 is not quantifiable.



TABLE II

Profile of Capital Intensity, Capital-Output Ratio and Partial Productivity of Capital and Labour

Year	AOLR (ln Rs.Cr.)	DOM (ln Rs.Cr.)	COR	AOCR
1980-81	0.0080	0.0020	0.25	4.06
1981-82	0.0075	0.0019	0.25	4.02
1982-83	0.0073	0.0019	0.26	3.86
1983-84	0.0073	0.0018	0.25	3.95
1984-85	0.0068	0.0018	0.26	3.80
1985-86	0.0068	0.0018	0.27	3.70
1986-87	0.0066	0.0019	0.28	3.52
1987-88	0.0069	0.0019	0.27	3.65
1988-89	0.0065	0.0018	0.28	3.52
1989-90	0.0066	0.0018	0.27	3.69
1990-91	0.0063	0.0018	0.28	3.61
1991-92	0.0057	0.0017	0.30	3.36
1992-93	0.0056	0.0016	0.28	3.60
1993-94	0.0056	0.0016	0.28	3.55
1994-95	0.0061	0.0016	0.26	3.81
1995-96	0.0060	0.0016	0.26	3.78
1996-97	0.0068	0.0018	0.26	3.81
1997-98	0.0072	0.0019	0.26	3.82
1998-99	0.0074	0.0020	0.27	3.66
1999-00	0.0081	0.0022	0.27	3.71
2000-01	0.0081	0.0021	0.26	3.78
2001-02	0.0086	0.0022	0.25	3.94
2002-03	0.0088	0.0022	0.24	4.08
2003-04	0.0089	0.0021	0.23	4.30
2004-05	0.0087	0.0019	0.22	4.48
<b>CAGRs:-</b>				
Pre-liberalization period	-2.27*	-0.85*	1.41*	-1.43*
Liberalization period	4.29*	2.57*	-1.68*	1.56*
Overall Period	0.74*	0.47*	-0.33**	0.27*

**Source:** Calculated from the data supplied by directorate of industries, Punjab.

Note : \*significant at 5 percent level of significance.

\*\* Insignificant at 5 percent level of significance

**Terms used:**

- a) DOM: Degree of Mechanization (capital intensity):- It is fixed capital at constant prices per employee.
- b) COR: Capital output Ratio: - It is ratio of total fixed capital to total production (both deflated).
- c) AOCR:- Average output capital ratio (Capital Productivity):- It is ratio of total production to total fixed capital (both deflated)
- d) AOLR: - Average Output Labour Ratio (Labour Productivity):- It is total production at constant prices per employee.



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## On Topological Sets and Spaces

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**Abstracts** - In this research paper we are introducing the concept of m-closed set and m-T1/3 space, discussed their properties, relation with other spaces and functions. Also, we would like to discuss the applications of topology in industries through different areas of sciences such as Biology, Chemistry, Physics, Computer Science, Business Economics and Engineering.

**Keywords** : *Alpha - set, Beta – set, generalized alpha-set and generalized alpha-set, DNA replication, electron microscopy, biomathematics, game theory, inorganic species etc.*

**Classification**: *2000 AMS Subject Classifications: 54B23, 54A12, 54C17*



*Strictly as per the compliance and regulations of:*



# On Topological Sets and Spaces

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**Abstract:** In this research paper we are introducing the concept of m-closed set and  $m-T_{1/3}$  spaces, discussed their properties, relation with other spaces and functions. Also, we would like to discuss the applications of topology in industries through different areas of sciences such as Biology, Chemistry, Physics, Computer Science, Business Economics and Engineering.

**Keywords :** Alpha - set, Beta – set, generalized alpha-set and generalized alpha-set, DNA replication, electron microscopy, biomathematics, game theory, inorganic species etc.

## I. INTRODUCTION

Throughout this paper  $(X, \tau)$ ,  $(Y, \tau)$  and  $(Z, \tau)$  represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset  $A$  of a topological space  $(X, \tau)$ ,  $\text{int}(A)$ ,  $\text{cl}(A)$  and  $C(A)$  represents the interior of  $A$ , the closure of  $A$ , and the complement of  $A$  in  $X$  respectively. In present time topology is an important branch of pure mathematics. But it is difficult to fix a date for the starting of topology as a subject in its own right. The first time use of term TOPOLOGY that we know of appeared in title of a book written by J.W. Listing in 1847. Great mathematician Riemann used exclusively the term "Analysis Situs". An important work was done by Poincaré in 1895 when he published his first paper on "Analysis Situs". However, historians of science regard Cantor's research on Fourier series from 1879 to 1884 as the beginning of General Topology. N. Levine<sup>[1]</sup> and M.E. Abd El. Monsef *et al*<sup>[2]</sup> introduced semi-open sets and  $\beta$ -sets

. D. Andrijevic<sup>[3]</sup> used notation semi preopen sets for  $\beta$ -sets. Again N. Levine<sup>[4]</sup> generalized the concept of closed sets to generalized closed sets. P. Bhattacharya and B.K. Lahiri<sup>[5]</sup> generalized the concept of closed sets to semi-generalized closed sets via semi-open sets. N. Biswas<sup>[6]</sup> studied that the complement of a semi-open set is called a semi-closed set. The aim of this paper is to draw a new technique to obtain a new class of sets, called m-closed sets. This class is obtained by generalizing semi-closed sets via semi-generalized open sets. It is shown that the class of m-closed sets properly contains the class of semi-closed sets and is properly

contained in the class of semi-preclosed sets. Also it is shown that the class of m-closed sets is independent from the class of preclosed sets/the class of generalized closed sets/the class of  $g\alpha$ -closed sets/the class of  $\alpha g$ -closed sets. P. Bhattacharya and B.K. Lahiri<sup>[5]</sup>, D.S. Jancovic & K.L. Reilly<sup>[7]</sup> and H. Maki *et al*<sup>[8]</sup> introduced semi- $T_{1/2}$  spaces, semi- $T_D$  and  $\alpha T_{1/2}$ , semi- $T_D$  and semi- $T_{1/2}$  spaces respectively. J. Dontchev<sup>[9&10]</sup> shown that  $\alpha T_{1/2}$  separation axioms are equivalent. R-Devi *et al*<sup>[11]</sup> introduced  $\alpha T_b$  spaces and  $T_b$  spaces respectively. As an application of m-closed sets, we introduced a new class of spaces, namely  $m-T_{1/3}$  spaces. Also characterize  $m-T_{1/3}$  spaces and show that the class of  $m-T_{1/3}$  spaces properly contains the class of semi- $T_{1/2}$  spaces, the class of  $\alpha T_b$  spaces and the class of semi- $T_{1/3}$  spaces.

## II. DEFINITIONS AND NOTATIONS

**Definitions 2.1:** Let  $A$  be a subset of topological space  $X$  then  $A$  is called.

- a generalized closed (*g-closed*) set if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
- A semi-generalized closed (*sg-closed*) set if  $\text{scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $(X, \tau)$ . The complement of a sg-closed set is called a sg-open set.
- A generalized semi-closed (*gs-closed*) set if  $\text{scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
- An  $\alpha$ -generalized closed ( $\alpha g$ -closed) set if  $\alpha \text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open in  $(X, \tau)$ .
- A generalized  $\alpha$ -closed ( $g\alpha$ -closed) set if  $\alpha \text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open in  $(X, \tau)$ .
- A  $g\alpha^{**}$ -closed set if  $\text{cl}(A) \subseteq \text{int}(\text{cl}(U))$ , whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open in  $(X, \tau)$ .

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A  $\delta$ -generalized closed ( $\delta g$ -closed) set if  $\text{cl}\delta(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .

- (g). A *generalized semi-preclosed* (*gsp-closed*) set if  $\text{spcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
- (h). A  $\delta$ -closed set if  $A = \text{cl}\delta(A)$  whenever  $\text{cl}\delta(A) = \{x \in X \mid \text{int}(\text{cl}(U)) \cap A \neq \emptyset, x \in U \text{ and } U \in \tau\}$ .
- (i). A *regular-open* set if  $A = \text{int}(\text{cl}(A))$  and a *regular-closed* set if  $\text{cl}(\text{int}(A)) = A$ .
- (j). A *semi-regular* set if it is both *semi-open* and *semi-closed* in  $(X, \tau)$ .
- (k). A *preopen* set if  $A \subseteq \text{int}(\text{cl}(A))$  and *preclosed* set if  $\text{cl}(\text{int}(A)) \subseteq A$ .
- (l). A *semi-open set* if  $A \subseteq \text{cl}(\text{int}(A))$  and a *semi-closed set* if  $\text{int}(\text{cl}(A)) \subseteq A$ .
- (m). A *semi-preopen* set or  $\beta$ -open set if  $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$  and a *semi-preclosed* set or  $\beta$ -closed set if  $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$ .

**Definitions 2.2:** A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be

- (a). *Pre-semi-open* if  $f(U)$  is semi-open in  $(Y, \sigma)$  for every semi-open set  $U$  of  $(X, \tau)$ .
- (b). *Pre-semi-closed* if  $f(U)$  is semi-closed in  $(Y, \sigma)$  for every semi-closed set  $U$  of  $(X, \tau)$ .
- (c). *Semi-continuous* if  $f^{-1}(V)$  is semi-open in  $(X, \tau)$  for every open set  $V$  of  $(Y, \sigma)$ .
- (d). *Pre-continuous* if  $f^{-1}(V)$  is pre-closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (e). *g-continuous* if  $f^{-1}(V)$  is  $g$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (f).  $\alpha g$ -continuous if  $f^{-1}(V)$  is  $\alpha g$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (g).  $g\alpha$ -continuous if  $f^{-1}(V)$  is  $g\alpha$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (h).  $\alpha$ -continuous if  $f^{-1}(V)$  is  $\alpha$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (i).  $\beta$ -continuous if  $f^{-1}(V)$  is semi-preopen in  $(X, \tau)$  for every open set  $V$  of  $(Y, \sigma)$ .
- (j).  $sg$ -continuous if  $f^{-1}(V)$  is  $sg$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (k).  $gs$ -continuous if  $f^{-1}(V)$  is  $gs$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (l).  $gsp$ -continuous if  $f^{-1}(V)$  is  $gsp$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (m). *Irresolute* if  $f^{-1}(V)$  is semi-open in  $(X, \tau)$  for every semi open set  $V$  of  $(Y, \sigma)$ .
- (n). *sg-irresolute* if  $f^{-1}(V)$  is  $sg$ -closed in  $(X, \tau)$  for every  $sg$ -closed set  $V$  of  $(Y, \sigma)$ .

**Definitions 2.3:** A topological space  $(X, \tau)$  is called a

- (a).  $T_{1/2}$  space if every  $g$ -closed set is closed.
- (b). *Semi- $T_{1/2}$*  space if every  $sg$ -closed set is semi-closed.
- (c). *Semi- $T_D$*  space if every singleton is either open or nowhere dense.
- (d).  $\alpha T_{1/2}$  space if every  $g\alpha^{**}$ -closed set is  $\alpha$ -closed.
- (e).  $\propto T^*i$  space if a  $(X, \tau^a)$  is  $T_i$  where  $i=1, 1/2$ .
- (f).  $\alpha T_b$  space if every  $\alpha g$ -closed set is closed.
- (g).  $T_b$  space if every  $gs$ -closed set is closed.
- (h).  $\alpha T_m$  space if every  $g\alpha^{**}$ -closed set is closed.
- (i). *Feebly- $T_i$*  space if every singleton is either nowhere dense or clopen.
- (j).  $T_{3/4}$  space if every  $\delta g$ -closed set is  $\delta$ -closed.
- (k). *Semi- $T_i$*  space if for any  $x, y \in X$  such that  $x \neq y$ , there exists two semi-open sets  $G$  and  $H$  such that  $x \in G, y \in H$  but  $x \notin H$  and  $y \notin G$ .

### III. M-CLOSED SET AND ITS PROPERTIES

**Definition 3.1:** A subset  $A$  of a topological space  $(X, \tau)$  is called a *m-closed set* if  $\text{scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is a  $g$ -open set of  $(X, \tau)$ .

**Theorem 3.1:** (i).  $m$ -closedness and  $g$ -closedness are independent notions.

(ii).  $m$ -closedness is independent from  $g\alpha$ -closedness,  $\alpha g$ -closedness and preclosedness.

**Proof:** It can be seen by the following examples,

**Example 3.1.1:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$  and  $C = \{c\}$  and  $D = \{a, b\}$ ,  $C$  is a  $m$ -closed set but not even a  $g$ -closed set of  $(X, \tau)$ .  $D$  is  $g$ -closed set but not a  $m$ -closed set of  $(X, \tau)$ .

The following two examples show that  $m$ -closedness is independent from  $g\alpha$ -closedness,  $\alpha g$ -closedness and preclosedness.

**Example 3.1.2:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$  and  $K = \{a\}$ .  $K$  is  $m$ -closed but it is neither a  $g\alpha$ -closed nor a  $\alpha g$ -closed set. Also  $K$  is not preclosed set.

**Example 3.1.3:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$  and  $L = \{b\}$ . Here  $L$  is not a  $m$ -closed set of  $(X, \tau)$ . However  $L$  is a  $g\alpha$ -closed set. Hence it is a  $\alpha g$ -closed set. Also  $L$  is a preclosed set of  $(X, \tau)$ .

**Lemma 3.1:** For a subset  $A$  of a space  $(X, \tau)$ , the following conditions are equivalent.

- (i).  $A$  is pre-open,  $sg$ -open and  $m$ -closed.
- (ii).  $A$  is pre-open,  $sg$ -open and semi-closed.
- (iii).  $A$  is regular open.

The following example shows that a subset  $B$  of a space  $(X, \tau)$  need not be a closed set even though  $B$  is pre-open,  $sg$ -open and a  $Q$ -set.

**Example:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}\}$  and  $B = \{a\}$ . Clearly  $B$  is pre-open,  $sg$ -open and a  $Q$ -set but not a closed set.

**Theorem 3.2:** For a subset  $A$  of a space  $(X, \tau)$ , the following conditions are equivalent.

- (i)  $A$  is clopen.
- (ii)  $A$  is preopen,  $sg$ -open,  $Q$ -set, and  $m$ -closed.

**Proof:** (i)  $\Rightarrow$  (ii): It is obvious.

**Proof:** (ii)  $\Rightarrow$  (i): Since  $A$  is preopen,  $sg$ -open and a  $m$ -closed set of  $(X, \tau)$ , then  $sg$  Lemma 1,  $A$  is a regular open set. This implies  $A$  is open. On the other hand,  $A = \text{int}(\text{cl}(A)) = \text{cl}(\text{int}(A)) \subseteq \text{cl}(A)$ . Since  $A$  is a  $Q$ -set, so  $A$  is closed. Therefore  $A$  is a clopen set of  $(X, \tau)$ .  $\square$

**Corollary 3.1:** The union of two  $m$ -closed sets need not to be  $m$ -closed.

**Proof:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ ,  $A = \{a\}$  and  $B = \{b\}$ . Then both  $A$  and  $B$  are  $m$ -closed but  $A \cup B$  is not a  $m$ -closed set of  $(X, \tau)$ .

#### IV. $M-T_{1/3}$ SPACE AND ITS PROPERTIES

**Definition 4.1:** A space  $(X, \tau)$  is said to be an  $m-T_{1/3}$  space if every  $m$ -closed set in it is semi-closed.

**Theorem 4.1:** For a space  $(X, \tau)$ , the following conditions are equivalent.

- (i).  $(X, \tau)$  is a  $m-T_{1/3}$  space.
- (ii). Every singleton of  $X$  is either  $sg$ -closed or semi-open.
- (iii). Every singleton of  $X$  is either  $sg$ -closed or open.

**Proof:** (i)  $\Rightarrow$  (ii). Let  $x \in X$  and assume that  $\{x\}$  is not a  $sg$ -closed set of  $(X, \tau)$ . Then  $X - \{x\}$  is a  $sg$ -open of  $(X, \tau)$ . So  $x$  is the only  $sg$ -open set containing  $X - \{x\}$ . Therefore  $X - \{x\}$  is a  $m$ -closed set of  $(X, \tau)$ . Since  $(X, \tau)$  is a  $m-T_{1/3}$  space, then  $X - \{x\}$  is a semi-closed set of  $(X, \tau)$ .  $\square$

**Proof:** (iii)  $\Rightarrow$  (i) Let  $A$  be a  $m$ -closed set of  $(X, \tau)$ . Clearly  $A \subseteq \text{scl}(A)$ . Let  $x \in X$ . By supposition,  $\{x\}$  is either  $sg$ -closed or semi-open.  $\square$

**Case I:** Let us suppose that  $\{x\}$  is  $sg$ -closed. By Theorem 1.3.2  $\text{scl}(A) - A$  does not contain any non-empty  $sg$ -closed set. Since  $x \in \text{scl}(A)$ , then  $x \in A$ .

**Case II:** Let us suppose that  $\{x\}$  is a semi-open set. Since  $x \in \text{scl}(A)$ , then  $\{x\} \cap A \neq \emptyset$ . So,  $x \in A$ . Hence in any case  $\text{scl}(A) \subseteq A$ . Therefore  $A = \text{scl}(A)$  or equivalently  $A$  is a semi-closed set of  $(X, \tau)$ . Hence  $(X, \tau)$  is a  $m-T_{1/3}$  space.

**Proof:** (ii)  $\Leftrightarrow$  (iii). It follows from the fact that a singleton is semi-open if it is open. .

#### V. APPLICATIONS OF GENERAL TOPOLOGY

Here we would like to discuss in brief the use of general topology in industries through by some areas of sciences.

##### a) Application in BIOLOGY

In recent years, topologists have developed the discrete geometric language of knots to a fine mathematical art one of the most interesting new scientific application of topology is the use of knot theory in analysis of DNA experiments. One of the important issues in molecular

biology in the 3-dimensional structure of proteins and DNA in solution in the Cell and the relationship between structure and functions. Generally, protein and DNA structures are determined by X-rays crystallization and the manipulation required preparing a specimen for electron microscope. The DNA molecules are long and thin and the packing of DNA molecules in the cell nucleus is very complex. The biological solution to this entanglement problem is the existence of enzymes, which convert DNA from one topological form to another and appear to have a preformed role in the central genetic events of DNA replication, recombination and transcription. The topological approach to enzymology aims to exploit knot theory directly to reveal the secrets of enzyme action. How recent results in 3-dimensional topology have proved to be of use in the description and quantization of the action of cellular enzymes on DNA is best described by D.W.Sumners in his research paper published in 1995.

#### b) *Application in CHEMISTRY*

As a natural continuation of classical knot theory, chemists have been trying to synthesize and measure molecules with topologically interesting structures. The idea of molecules made of linked rings as a realistic possibility, was discussed at least as early as 1912. The most important tools in the topological method of making chemical predictions are known as indices. They derive from algorithms of procedures for converting the topological structure of a molecule into a single characteristic number. For example, an index might involve adding together the total number of rings in a molecule, or a number of atoms that are connected to three or more other atoms. The topological method has found applications beyond the simple prediction of chemical properties. It has the potential to help in modeling the behavior of gases, liquids and solids and of both organic & inorganic species, in developing new anesthetics and psychoactive drugs, in predicting the degree to which various pollutants might spread in the environment and the harm they might do once they spread, in estimating the cancer causing potential of certain chemicals and even in developing in beer with a well balanced taste.

#### c) *Application in Physics*

According to Normal Howes – uniform structures are the most important constructs from the physicist's point of view. The importance of uniform spaces from the physicist's points of view is also well brought out by the proceedings of the Nashville Topological Conference. In fact; topology has intrigued particle physicists for a long time. Recall that Donaldson used the Yang Mills field equations of mathematical physics, themselves generalizations of Maxwell's equations to study in 4 –

space, there by reversing tradition by applying methods from physics to the understanding of topology.

#### d) *Application in Computer Science*

Recent developments in topology are penetrating other fields is best illustrated by the topics discussed at an extra ordinary research conference which was held at Barkley in 1990 in honor of the great topologist Stephen Samale's 60<sup>th</sup> birthday. The proceeding were published with title "Form topology to Computation: Unity and diversity in mathematical sciences" edited by Hirsch, Marsden and Shub. There seems to not many examples of the use of topology in computer science, perhaps because it is not clear how it is related to the fundamental questions. However, in recent years, there have been some interesting results. The problem of the minimal number of conditional statements in an algorithm, to solve a particular problem, seems particularly well suited for the topological approach.

#### e) *Application in Business Economics*

Topology has had tremendous effect on developments in economics. The study of conflicts of interest between individuals is what makes economics interesting and mathematically complex. Indeed we now know that the space of all individual preferences, which define the individual optimization problems, is topologically nontrivial and that is topological complexity is responsible for the impossibility of treating several individual preferences as if they were one. Because it is not possible, in general, to define a single optimization problem. Because of the complexity arising from simultaneous optimization problems, economic differs from physics where many of the fundamental relations derive from a single optimization problem. The attempt to find solutions to conflicts among individual interests led to there different theories about how economics are organized and how they behave.

#### f) *Application in Engineering*

Topology has also found applications in engineering. a problem of great importance to an electric industry, which had failed of solution by its own engineers, has been solved by using methods of set theoretic topology. In particular, Daniel R. Baker has established that topological techniques are used in several robotics applications. Topology has been applied to production and distribution problems as well.

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# The Concept of Heart-Oriented Rhotrix Multiplication

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**Abstracts** - This paper introduces the concept of heart-oriented rhotrix multiplication. A generalized representation of n-dimensional heart-oriented rhotrices viewed from the direction of single array indices and double array row-wise indices is presented. We provided a simplified compact mathematical expression for the two multiplication process presented in the paper. The given methods were implemented based on the sequential algorithm developed.

**Keywords :** *Heart-oriented rhotrix multiplication, Row-wise rhotrix multiplication, Algorithms*

**Classification:** *GJSFR Classification: FOR Code: 010105,010301*



*Strictly as per the compliance and regulations of:*



# The Concept of Heart-Oriented Rhotrix Multiplication

Ezugwu E. Absalom, Sani B., Junaidu B. Sahalu

**Abstract**—This paper introduces the concept of heart-oriented rhotrix multiplication. A generalized representation of  $n$ -dimensional heart-oriented rhotrices viewed from the direction of single array indices and double array row-wise indices is presented. We provided a simplified compact mathematical expression for the two multiplication process presented in the paper. The given methods were implemented based on the sequential algorithm developed.

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## I. INTRODUCTION

The fundamental concept of this paper can be found in [1] which tailored down from the idea of matrix-tertion and matrix-noitrites[2]. The extension of this idea was presented by Ajibade [1] and referred to as rhotrices. In his paper Ajibade presented the initial concept of rhotrix algebra in which he established some interesting relationships between a rhotrix and its hearts. A rhotrix is defined as a mathematical array which is in some way between  $2 \times 2$  matrix and  $3 \times 3$  matrix is given as.

$$R = \left\{ \left\langle \begin{array}{ccccc} & & a & & \\ & b & c & d & \\ e & f & h(R) & h & i \\ & j & k & l & \\ & & m & & \end{array} \right\rangle : a, b, c, \dots, m \in \mathfrak{R} \right\}$$

The above rhotrix is of the fifth dimension.  $h(R)$  is called the heart of the rhotrix. An extension is possible, thereby increasing the dimension which is always odd. The number of entries in an  $n$ -dimensional rhotrix is given by  $\frac{1}{2}(n^2 + 1)$ . Where  $n$  is the dimension of the rhotrix.

## II. HIGH DIMENSIONAL HEART-ORIENTED RHOTRICES

In this paper we present a general idea on heart-oriented rhotrix multiplication and its formal representation for  $n$ -dimensional rhotrix. This new concept of the multiplication of rhotrices gave room for the initial conception of heart-oriented sequential computational implementation which is the basis for this paper.

Ajibade [1], indicated that the dimension of rhotrices can be increased although a rhotrix would always have an odd dimension. He also indicated that a rhotrix  $R_n$  of dimension  $n$  will have  $|R_n|$  entries where  $|R_n| = \frac{1}{2}(n^2 + 1)$ . Let's consider generalizing any given rhotrix  $R_n$  with entries  $a_1, a_2, \dots, a_{\frac{1}{2}(n^2+1)}$ , and we assume that the following holds:

- If we denote the number of entries in a rhotrix by  $N$ , then the middle entry, known as the heart element, can be expressed as  $H = \frac{1}{2}(N + 1)$  from statistical distribution expression for median [3]. In our case, the value of  $H$  indicates the index of the heart entry.
- Similarly if  $N = |R_n|$  then  $N = \frac{1}{2}(n^2 + 1)$ , hence,  $H = \frac{\frac{1}{2}(n^2 + 1) + 1}{2} \equiv \frac{1}{4}[n^2 + 3]$ .

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## III. SINGLE INDICES HEART-ORIENTED RHOTRICES

**Some results:** we can derive the general representation of high dimensional rhotrix by considering a sequence of 3, 5, 7 and 9 dimensional rhotrices as illustrated below:

For a rhotrix of dimension 3, we have  $2 \times 1 + 1 = 3$

For a rhotrix of dimension 5, we have  $2 \times 2 + 1 = 5$

For a rhotrix of dimension 7, we have  $2 \times 3 + 1 = 7$

For a rhotrix of dimension 9, we have  $2 \times 4 + 1 = 9$

...

...

36 For a rhotrix of dimension  $n$ , we have  $2 \times k + 1 = n$

Where  $k$  is the  $k^{th}$  term of the incremental value and  $n$  is the dimension of the rhotrix, and then we have:  
 $k = \frac{1}{2}(n-1)$

- Again, if we denote the direction of the leftmost entry of a rhotrix by  $L$  and the number of entries by  $|R_n| = \frac{1}{2}(n^2 + 1)$  then the rhotrix entry at  $L$  is given by

$$L = \frac{1}{4}[n^2 + 3] - \frac{n-1}{2}$$

- Similarly, the rightmost rhotrix entry denoted by  $R$ , is given by

$$R = \frac{1}{4}[n^2 + 3] + \frac{n-1}{2}$$

- It is important to note that  $L$  and  $R$  denote the leftmost and the rightmost indices of  $a$  and  $b$  respectively in the rhotrix.

This is represented as:

$$R_n = \left( \begin{array}{cccccccc} & & & & a_1 & & & \\ & & & & a_2 & a_3 & a_4 & \\ & & & a_5 & a_6 & a_7 & a_8 & a_9 \\ & & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{\frac{1}{4}(n^2+3)-\frac{n-1}{2}} & \dots & \dots & \dots & \dots & \dots & \dots & a_{\frac{1}{4}(n^2+3)+\frac{n-1}{2}} \\ & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & & a_{\frac{1}{2}(n^2+1)-8} & a_{\frac{1}{2}(n^2+1)-7} & a_{\frac{1}{2}(n^2+1)-6} & a_{\frac{1}{2}(n^2+1)-5} & a_{\frac{1}{2}(n^2+1)-4} & \\ & & & a_{\frac{1}{2}(n^2+1)-3} & a_{\frac{1}{2}(n^2+1)-2} & a_{\frac{1}{2}(n^2+1)-1} & & \\ & & & & a_{\frac{1}{2}(n^2+1)} & & & \end{array} \right)$$

$$Q_n = \left\langle \begin{array}{cccccccc} & & & & b_1 & & & \\ & & & & b_2 & b_3 & b_4 & \\ & & b_5 & b_6 & b_7 & b_8 & b_9 & \\ & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ b_{\frac{1}{4}(n^2+3)-\frac{n-1}{2}} & \dots & \dots & \dots & b_{\frac{1}{4}(n^2+3)} & \dots & \dots & b_{\frac{1}{4}(n^2+3)+\frac{n-1}{2}} \\ & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & & b_{\frac{1}{2}(n^2+1)-8} & b_{\frac{1}{2}(n^2+1)-7} & b_{\frac{1}{2}(n^2+1)-6} & b_{\frac{1}{2}(n^2+1)-5} & b_{\frac{1}{2}(n^2+1)-4} & \\ & & b_{\frac{1}{2}(n^2+1)-3} & b_{\frac{1}{2}(n^2+1)-2} & b_{\frac{1}{2}(n^2+1)-1} & & & \\ & & & b_{\frac{1}{2}(n^2+1)} & & & & \end{array} \right\rangle \quad (2.1)$$

We further simplify equation (2.2) as follow:

since  $R = \frac{\frac{1}{2}(n^2+1)+1}{2} + \frac{n-1}{2}$  then  $R = \frac{\frac{1}{2}(n^2+1)+1+n-1}{2} = \frac{\frac{1}{2}n^2 + \frac{1}{2} + n}{2} = \frac{\frac{1}{2}n^2 + \frac{1}{2} + n}{2}$  which implies that

$$R = \frac{1}{4}n^2 + \frac{1}{4} + \frac{n}{2} = \frac{n^2 + 2n + 1}{4}$$

And hence  $R = \frac{n^2 + 2n + 1}{4}$

We also do the same for L

since  $L = \frac{\frac{1}{2}(n^2+1)+1}{2} - \frac{n-1}{2}$  then  $L = \frac{\frac{1}{2}(n^2+1)+1-n+1}{2} = \frac{\frac{1}{2}n^2 + \frac{1}{2} + 1 - n + 1}{2}$

$$L = \frac{1}{4}n^2 + \frac{5}{4} - \frac{2n}{4} = \frac{n^2 - 2n + 5}{4}$$

Thus,  $L = \frac{n^2 - 2n + 5}{4}$

$$\begin{aligned}
 R_n = & \left\langle \begin{array}{ccccccc} & & & a_1 & & & \\ & & & a_2 & & a_4 & \\ & a_5 & & a_3 & & a_8 & a_9 \\ & \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{a_{n^2-2n+5}}{4} & \dots & \dots & \dots & \dots & \dots & \dots & \frac{a_{n^2+2n+1}}{4} \\ & \dots & \dots & \dots & \dots & \dots & \dots & \\ & & a_{\frac{1}{2}(n^2+1)-8} & a_{\frac{1}{2}(n^2+1)-7} & a_{\frac{1}{2}(n^2+1)-6} & a_{\frac{1}{2}(n^2+1)-5} & a_{\frac{1}{2}(n^2+1)-4} \\ & & a_{\frac{1}{2}(n^2+1)-3} & a_{\frac{1}{2}(n^2+1)-2} & a_{\frac{1}{2}(n^2+1)-1} & & \\ & & & a_{\frac{1}{2}(n^2+1)} & & & \end{array} \right\rangle \\
 Q_n = & \left\langle \begin{array}{ccccccc} & & & b_1 & & & \\ & & & b_2 & & b_4 & \\ & b_5 & & b_3 & & b_8 & b_9 \\ & \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{b_{n^2-2n+5}}{4} & \dots & \dots & \dots & \dots & \dots & \dots & \frac{b_{n^2+2n+1}}{4} \\ & \dots & \dots & \dots & \dots & \dots & \dots & \\ & & b_{\frac{1}{2}(n^2+1)-8} & b_{\frac{1}{2}(n^2+1)-7} & b_{\frac{1}{2}(n^2+1)-6} & b_{\frac{1}{2}(n^2+1)-5} & b_{\frac{1}{2}(n^2+1)-4} \\ & & b_{\frac{1}{2}(n^2+1)-3} & b_{\frac{1}{2}(n^2+1)-2} & b_{\frac{1}{2}(n^2+1)-1} & & \\ & & & b_{\frac{1}{2}(n^2+1)} & & & \end{array} \right\rangle \quad (2.2)
 \end{aligned}$$

Substituting the values  $n = 3$ ,  $n = 5$  and  $n = 7$  in equation (2.2) we have the following rhotrices:

$$R_3 = \left\langle \begin{array}{ccc} & a_1 & \\ a_2 & a_3 & a_4 \\ & a_5 & \end{array} \right\rangle, R_5 = \left\langle \begin{array}{cccc} & a_1 & & \\ & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 & a_9 \\ & a_{10} & a_{11} & a_{12} \\ & & a_{13} & \end{array} \right\rangle \text{ and } R_7 = \left\langle \begin{array}{cccccc} & & a_1 & & & \\ & a_2 & a_3 & a_4 & & \\ & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ & a_{17} & a_{18} & a_{19} & a_{20} & a_{21} \\ & & a_{22} & a_{23} & a_{24} \\ & & & a_{25} & \end{array} \right\rangle$$

Similarly, from (2.2), we can define the multiplication of any two heart-oriented rhotrices in the following way.

$$R_n \circ Q_n = \left\langle \begin{array}{cccccccc} & & & a_1 & & & & \\ & & & a_2 & a_3 & a_4 & & \\ & & a_5 & a_6 & a_7 & c_8 & a_9 & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{a}{4} \frac{n^2-2n+5}{4} & \dots & \dots & \dots & \frac{a}{4} \frac{1}{4} (n^2+3) & \dots & \dots & \frac{a}{4} \frac{n^2+2n+1}{4} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & \frac{a}{2} \frac{1}{2} (n^2+1)-8 & \frac{a}{2} \frac{1}{2} (n^2+1)-7 & \frac{a}{2} \frac{1}{2} (n^2+1)-6 & \frac{a}{2} \frac{1}{2} (n^2+1)-5 & \frac{a}{2} \frac{1}{2} (n^2+1)-4 & & \\ & & \frac{a}{2} \frac{1}{2} (n^2+1)-3 & \frac{a}{2} \frac{1}{2} (n^2+1)-2 & \frac{a}{2} \frac{1}{2} (n^2+1)-1 & & & \\ & & & \frac{a}{2} \frac{1}{2} (n^2+1) & & & & \end{array} \right\rangle.$$

$$\left\langle \begin{array}{cccccccc} & & & b_1 & & & & \\ & & & b_2 & b_3 & b_4 & & \\ & & b_5 & b_6 & b_7 & b_8 & b_9 & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{b}{4} \frac{n^2-2n+5}{4} & \dots & \dots & \dots & \frac{b}{4} \frac{1}{4} (n^2+3) & \dots & \dots & \frac{b}{4} \frac{n^2+2n+1}{4} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & \frac{b}{2} \frac{1}{2} (n^2+1)-8 & \frac{b}{2} \frac{1}{2} (n^2+1)-7 & \frac{b}{2} \frac{1}{2} (n^2+1)-6 & \frac{b}{2} \frac{1}{2} (n^2+1)-5 & \frac{b}{2} \frac{1}{2} (n^2+1)-4 & & \\ & & \frac{b}{2} \frac{1}{2} (n^2+1)-3 & \frac{b}{2} \frac{1}{2} (n^2+1)-2 & \frac{b}{2} \frac{1}{2} (n^2+1)-1 & & & \\ & & & \frac{b}{2} \frac{1}{2} (n^2+1) & & & & \end{array} \right\rangle$$

$$R_5 \circ Q_5 = \left\langle \begin{array}{cccccccc} & & & c_1 & & & & \\ & & & c_2 & c_3 & c_4 & & \\ & & c_5 & c_6 & c_7 & c_8 & c_9 & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{c}{4} \frac{n^2-2n+5}{4} & \dots & \dots & \dots & \frac{c}{4} \frac{1}{4} (n^2+3) & \dots & \dots & \frac{c}{4} \frac{n^2+2n+1}{4} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & \frac{c}{2} \frac{1}{2} (n^2+1)-8 & \frac{c}{2} \frac{1}{2} (n^2+1)-7 & \frac{c}{2} \frac{1}{2} (n^2+1)-6 & \frac{c}{2} \frac{1}{2} (n^2+1)-5 & \frac{c}{2} \frac{1}{2} (n^2+1)-4 & & \\ & & \frac{c}{2} \frac{1}{2} (n^2+1)-3 & \frac{c}{2} \frac{1}{2} (n^2+1)-2 & \frac{c}{2} \frac{1}{2} (n^2+1)-1 & & & \\ & & & \frac{c}{2} \frac{1}{2} (n^2+1) & & & & \end{array} \right\rangle \quad (2.3)$$



*Definition (heart-oriented multiplication)*

Let  $R_5 = \left\langle \begin{array}{cccc} & a_1 & & \\ & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 & a_9 \\ & a_{10} & a_{11} & a_{12} \\ & & a_{13} & \end{array} \right\rangle$  and  $Q_5 = \left\langle \begin{array}{cccc} & b_1 & & \\ & b_2 & b_3 & b_4 \\ b_5 & b_6 & b_7 & b_8 & b_9 \\ & b_{10} & b_{11} & b_{12} \\ & & b_{13} & \end{array} \right\rangle$  be two rhotrices of dimension 5

with entries from  $\mathbb{R}$ , the set of real numbers. We follow the multiplication (called *heart-oriented multiplication* in this paper) which was first defined in [1] on rhotrices of third dimension as follows:

$$\begin{aligned}
 R_5 \circ Q_5 &= \left\langle \begin{array}{cccc} & a_1 & & \\ & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 & a_9 \\ & a_{10} & a_{11} & a_{12} \\ & & a_{13} & \end{array} \right\rangle \circ \left\langle \begin{array}{cccc} & b_1 & & \\ & b_2 & b_3 & b_4 \\ b_5 & b_6 & b_7 & b_8 & b_9 \\ & b_{10} & b_{11} & b_{12} \\ & & b_{13} & \end{array} \right\rangle = \\
 &= \left\langle \begin{array}{cccc} & a_1 & & \\ & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 & a_9 \\ & a_{10} & a_{11} & a_{12} \\ & & a_{13} & \end{array} \right\rangle + a_7 \circ \left\langle \begin{array}{cccc} & b_1 & & \\ & b_2 & b_3 & b_4 \\ b_5 & b_6 & b_7 & b_8 & b_9 \\ & b_{10} & b_{11} & b_{12} \\ & & b_{13} & \end{array} \right\rangle \\
 &= \left\langle \begin{array}{cccc} & a_1 b_7 + a_7 b_1 & & \\ & a_2 b_7 + a_7 b_2 & a_3 b_7 + a_7 b_3 & a_4 b_7 + a_7 b_4 \\ a_5 b_7 + a_7 b_5 & a_6 b_7 + a_7 b_6 & a_7 b_7 & a_8 b_7 + a_7 b_8 & a_9 b_7 + a_7 b_9 \\ & a_{10} b_7 + a_7 b_{10} & a_{11} b_7 + a_7 b_{11} & a_{12} b_7 + a_7 b_{12} \\ & & a_{13} b_7 + a_7 b_{13} & \end{array} \right\rangle \quad (2.4)
 \end{aligned}$$

This can be expressed in the following way:

$$\begin{aligned}
 c_1 &= a_1 b_7 + a_7 b_1 & c_2 &= a_2 b_7 + a_7 b_2 & c_3 &= a_3 b_7 + a_7 b_3 & c_4 &= a_4 b_7 + a_7 b_4 \\
 c_5 &= a_5 b_7 + a_7 b_5 & c_6 &= a_6 b_7 + a_7 b_6 & c_7 &= a_7 b_7 & c_8 &= a_8 b_7 + a_7 b_8 \\
 c_9 &= a_9 b_7 + a_7 b_9 & c_{10} &= a_{10} b_7 + a_7 b_{10} & c_{11} &= a_{11} b_7 + a_7 b_{11} & c_{12} &= a_{12} b_7 + a_7 b_{12} \\
 c_{13} &= a_{13} b_7 + a_7 b_{13} & & & & & & \quad (2.5)
 \end{aligned}$$

where  $a_7$  and  $b_7$  denote the hearts indices defined by  $\frac{1}{4}[n^2+3]$  of the two rhotrices, the resulting value of  $c_7$  is the product of the two hearts from the two rhotrices. We can now extend this to accommodate rhotrices of arbitrary dimensions. Going by (2.5), we can represent the rhotrix's heart entry as  $a_{\frac{1}{4}[n^2+3]}$  and  $b_{\frac{1}{4}[n^2+3]}$  respectively.

Equation (2.5) can be represented in rhotrix for as:

$$R_5 \circ Q_5 = C = \left\langle \begin{array}{ccccc} & & c_1 & & \\ & c_2 & c_3 & c_4 & \\ c_5 & c_6 & c_7 & c_8 & c_9 \\ & c_{10} & c_{11} & c_{12} & \\ & & c_{13} & & \end{array} \right\rangle \quad (2.6)$$

In general, given two rhotrices  $R_n$  and  $Q_n$  of dimension  $n$ , the entries of the heart-oriented product  $C_n$  of  $R_n$  and  $Q_n$  can be expressed as follows:

$$C_i = b_{\frac{1}{4}[n^2+3]} a_i + a_{\frac{1}{4}[n^2+3]} b_i + (1-\lambda)(a_{\frac{1}{4}[n^2+3]} b_{\frac{1}{4}[n^2+3]}), \text{ for } i = 1, 2, \dots, \frac{1}{2}(n^2 + 1) \quad (2.7)$$

where  $\lambda = 0$  when the index value corresponds to that of the heart and  $\lambda = 1$  when otherwise. This condition is further illustrated in table I and II.

Alternatively, suppose that we now represent the  $n$ -dimensional rhotrix in equation (2.1) by  $R_n = \langle a_i, a_h \rangle$  where of course  $a_i$  and  $a_h$  represents the  $a_i$  entries and its heart respectively, with  $i = 1, 2, 3, \dots, |R_n|$  and  $h = 3, 7, 13, \dots, \frac{1}{4}[n^2 + 3]$ .

Consider an  $i$  dimensional rhotrix, having number of entries  $|R_i| = \frac{1}{2}(i^2 + 1)$ . Let  $\bar{i}$  be the mean of these entries computed as shown in table I:

Table I Setting conditions for  $\lambda$  over a three dimensional rhotrix entries

$i$	$\bar{i}$	$i - \bar{i}$	$ i - \bar{i} $	
1	3	-2	2	$\lambda = 0$
2	3	-1	1	
3	3	0	0	$\lambda = 1$
4	3	1	1	$\lambda = 0$
5	3	2	2	

Table II Setting conditions for  $\lambda$  over a five dimensional rhotrix entries

$i$	$\bar{i}$	$i - \bar{i}$	$ i - \bar{i} $	
1	7	-6	6	$\lambda = 0$
2	7	-5	5	
3	7	-4	4	
4	7	-3	3	
5	7	-2	2	
6	7	-1	1	
7	7	0	0	$\lambda = 1$
8	7	1	1	$\lambda = 0$
9	7	2	2	
10	7	3	3	
11	7	4	4	
12	7	5	5	
13	7	6	6	

Let the heart entry of rhotrix  $Q_n$  denoted by  $b_h$  multiply all the entries of rhotrix  $R_n$  denoted by  $a_i$  and vice versa for the heart entry of the rhotrix  $R_n$  denoted by  $a_h$ . Subsequently, the two corresponding hearts of  $R_n$  and  $Q_n$  multiply each other as a single resulting product. Based on the derived value of  $\lambda$ , we can then define a function on  $\lambda$  as:

$$\lambda = \begin{cases} 0, & |i - \bar{i}| > 0 \\ 1, & |i - \bar{i}| = 0 \end{cases}$$

We define multiplication thus, of any two heart-oriented rhotrices of the same dimension as follow:

$$R_n \circ Q_n = b_h \circ \langle a_i \rangle + a_h \langle b_i \rangle \circ (1 - \lambda)$$

It follows that,

$$C(i) = \begin{cases} b_h \langle a_i \rangle & \text{for } i = h, \lambda = 1 \\ b_h \langle a_i \rangle + a_h \langle b_i \rangle & \text{for } i \neq h, \lambda = 0 \end{cases}$$

Hence, we can thus generalize this as.

$$C(i) = b_h \langle a_i \rangle + a_h \langle b_i \rangle (1 - \lambda) \quad (2.8)$$

Where  $\lambda = 1$  for  $i = h$  and  $\lambda = 0$  for  $i \neq h$

It can also be verified further, though not discussed in this work that multiplication of any two rhotrices is commutative, associative, and distributive over addition. Hence we say that the set R of all rhotrices is a commutative algebra [4].

---

**Algorithm 1: Algorithm for single indexed Multiplication of Rhotrices**

---

**Input:**

$$h = \frac{1}{4}[n^2 + 3]$$

$a[0..k]$

$b[0..k]$

**Output:**

$c[0..k]$

for  $i = 1$  to  $k$

if  $(i = \frac{1}{4}[n^2 + 3])$

$$c[i] \leftarrow a[i] * b[i];$$

else

$$c[i] \leftarrow b[h] \times a[i] + a[h] \times b[i];$$

endif

endfor

#### IV. ROW-WISE DOUBLE INDICES HEART-ORIENTED RHOTRIX MULTIPLICATION

The row-wise rhotrix multiplication tends to put into consideration the position and direction of each entry in the cause of the multiplicative operations. The operation is performed in a row-wise direction indicated by the entry indices  $i$  and  $j$ . The index  $i$  indicate the row entry position while the index  $j$  indicates the row direction. The general representation of the row-wise rhotrices is as depicted in (3.1). It is important to note that division in this case are all integer division, since we are less concerned with the resulting decimal values.

**Definition:** The row of any given rhotrix is an array of entries running diagonally from the top-most left to the bottom rightmost direction of the rhotrix.

$$R_n = \left( \begin{array}{cccccccc} & & & & a_{11} & & & \\ & & & & & a_{21} & a_{12} & \\ & & & a_{31} & a_{32} & a_{22} & a_{13} & \\ & a_{5,1} & a_{4,1} & & & & & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n,1} & \dots & \dots & \dots & a_{\frac{n+1}{2}, \lfloor \frac{n+3}{4} \rfloor} & \dots & \dots & a_{1, \frac{n+1}{2}} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & a_{n, \frac{n+1}{2}-2} & a_{n-1, \frac{n+1}{2}-2} & a_{n-2, \frac{n+1}{2}-1} & a_{n-3, \frac{n+1}{2}-1} & a_{n-4, \frac{n+1}{2}} & & \\ & & a_{n, \frac{n+1}{2}-1} & a_{n-1, \frac{n+1}{2}-1} & a_{n-2, \frac{n+1}{2}} & & & \\ & & & a_{n, \frac{n+1}{2}} & & & & \end{array} \right) \quad (3.1)$$

We use commas in (3.1) to avoid any ambiguity with respect to the array indexes separation. Further simplification of equation 3.1 gives:

$$R_n = \left( \begin{array}{cccccccc} & & & & a_{11} & & & \\ & & & & & a_{21} & a_{12} & \\ & & & a_{31} & a_{32} & a_{22} & a_{13} & \\ & a_{51} & a_{41} & & & & & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n,1} & \dots & \dots & \dots & a_{\frac{n+1}{2}, \lfloor \frac{n+3}{4} \rfloor} & \dots & \dots & a_{1, \frac{n+1}{2}} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & a_{n, \frac{n-3}{2}} & a_{n-1, \frac{n-3}{2}} & a_{n-2, \frac{n-1}{2}} & a_{n-3, \frac{n-1}{2}} & a_{n-4, \frac{n+1}{2}} & & \\ & & a_{n, \frac{n-1}{2}} & a_{n-1, \frac{n-1}{2}} & a_{n-2, \frac{n+1}{2}} & & & \\ & & & a_{n, \frac{n+1}{2}} & & & & \end{array} \right) \quad (3.2)$$

$$R_n \circ Q_n = \left\langle \begin{array}{cccccc} & & & a_{11} & & \\ & & & a_{31} & a_{21} & a_{12} \\ & & a_{51} & a_{41} & a_{32} & a_{22} & a_{13} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & \dots & \dots & \dots & a_{\frac{n+1}{2}, \frac{n+3}{4}} & \dots & \dots & a_{1, \frac{n+1}{2}} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & a_{n, \frac{n-3}{2}} & a_{n-1, \frac{n-3}{2}} & a_{n-2, \frac{n-1}{2}} & a_{n-3, \frac{n-1}{2}} & a_{n-4, \frac{n+1}{2}} & & \\ & & a_{n, \frac{n-1}{2}} & a_{n-1, \frac{n-1}{2}} & a_{n-2, \frac{n+1}{2}} & & & \\ & & & a_{n, \frac{n+1}{2}} & & & & \end{array} \right\rangle \circ \left\langle \begin{array}{cccccc} & & & b_{11} & & \\ & & & b_{31} & b_{21} & b_{12} \\ & & b_{51} & b_{41} & b_{32} & b_{22} & b_{13} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ b_{n1} & \dots & \dots & \dots & b_{\frac{n+1}{2}, \frac{n+3}{4}} & \dots & \dots & b_{1, \frac{n+1}{2}} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & b_{n, \frac{n-3}{2}} & b_{n-1, \frac{n-3}{2}} & b_{n-2, \frac{n-1}{2}} & b_{n-3, \frac{n-1}{2}} & b_{n-4, \frac{n+1}{2}} & & \\ & & b_{n, \frac{n-1}{2}} & b_{n-1, \frac{n-1}{2}} & b_{n-2, \frac{n+1}{2}} & & & \\ & & & b_{n, \frac{n+1}{2}} & & & & \end{array} \right\rangle \quad (3.3)$$

$$C_n = \left\langle \begin{array}{cccccc} & & & c_{11} & & \\ & & & c_{31} & c_{21} & c_{12} \\ & & c_{51} & c_{41} & c_{32} & c_{22} & c_{13} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ c_{n1} & \dots & \dots & \dots & c_{\frac{n+1}{2}, \frac{n+3}{4}} & \dots & \dots & c_{1, \frac{n+1}{2}} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & c_{n, \frac{n+1}{2}-2} & c_{n-1, \frac{n+1}{2}-2} & c_{n-2, \frac{n+1}{2}-1} & c_{n-3, \frac{n+1}{2}-1} & c_{n-4, \frac{n+1}{2}} & & \\ & & c_{n, \frac{n+1}{2}-1} & c_{n-1, \frac{n+1}{2}-1} & c_{n-2, \frac{n+1}{2}} & & & \\ & & & c_{n, \frac{n+1}{2}} & & & & \end{array} \right\rangle \quad (3.4)$$

**Definition:** (Row-wise heart-oriented rhotrix multiplication)

**Let**  $a, b, c, \dots, c \in \mathfrak{R}$ ,  $R_n \circ Q_n = a_{11} \times b_{\frac{n+1}{2}, \frac{n+3}{4}} + b_{11} \times a_{\frac{n+1}{2}, \frac{n+3}{4}} + \dots + a_{n, \frac{n+1}{2}} \times b_{\frac{n+1}{2}, \frac{n+3}{4}} + b_{n, \frac{n+1}{2}} \times a_{\frac{n+1}{2}, \frac{n+3}{4}}$ .

Where  $a_{\frac{n+1}{2}, \frac{n+3}{4}}$  and  $b_{\frac{n+1}{2}, \frac{n+3}{4}}$  are the hearts of the two rhotrices. This is illustrated as in (3.5) and (3.6).

$$R_5 \circ Q_5 = \left\langle \begin{array}{cccc} & & a_{11} & \\ & a_{31} & a_{21} & a_{12} \\ a_{51} & a_{41} & a_{32} & a_{22} & a_{13} \\ & a_{52} & a_{42} & a_{33} \\ & & a_{53} & \end{array} \right\rangle \circ \left\langle \begin{array}{cccc} & & b_{11} & \\ & b_{31} & b_{21} & b_{12} \\ b_{51} & b_{41} & b_{32} & b_{22} & b_{13} \\ & b_{52} & b_{42} & b_{33} \\ & & b_{53} & \end{array} \right\rangle \quad (3.5)$$

$$R_5 \circ Q_5 = C_5 = \left\langle \begin{array}{ccccc} & & c_{11} & & \\ & c_{31} & c_{21} & c_{12} & \\ c_{51} & c_{41} & c_{32} & c_{22} & c_{13} \\ & c_{52} & c_{42} & c_{33} & \\ & & c_{53} & & \end{array} \right\rangle \quad (3.6)$$

Similarly, multiplication of the rhotrices in (3.5) is commutative. The product operation is similar to the method given earlier for the single indices multiplication approach. The multiplication process for the two rhotrices in (3.5) can thus be expressed as:

$$b_{32} \circ \left\langle \begin{array}{ccccc} & & a_{11} & & \\ & a_{31} & a_{21} & a_{12} & \\ a_{51} & a_{41} & a_{32} & a_{22} & a_{13} \\ & a_{52} & a_{42} & a_{33} & \\ & & a_{53} & & \end{array} \right\rangle + a_{32} \circ \left\langle \begin{array}{ccccc} & & b_{11} & & \\ & b_{31} & b_{21} & b_{12} & \\ b_{51} & b_{41} & b_{32} & b_{22} & b_{13} \\ & b_{52} & b_{42} & b_{33} & \\ & & b_{53} & & \end{array} \right\rangle =$$

$$\left\langle \begin{array}{ccccc} & & a_{11}b_{32} + b_{11}a_{32} & & \\ & a_{31}b_{32} + b_{31}a_{32} & a_{21}b_{32} + b_{21}a_{32} & a_{12}b_{32} + b_{12}a_{32} & \\ a_{51}b_{32} + b_{51}a_{32} & a_{41}b_{32} + b_{41}a_{32} & a_{32}b_{32} & a_{22}b_{32} + b_{22}a_{32} & a_{13}b_{32} + b_{13}a_{32} \\ & a_{52}b_{32} + b_{52}a_{32} & a_{42}b_{32} + b_{42}a_{32} & a_{33}b_{32} + b_{33}a_{32} & \\ & & a_{53}b_{32} + b_{53}a_{32} & & \end{array} \right\rangle \quad (3.8)$$

$$\begin{aligned} c_{11} &= a_{11}b_{32} + b_{11}a_{32} & c_{12} &= a_{12}b_{32} + b_{12}a_{32} & c_{13} &= a_{13}b_{32} + b_{13}a_{32} & c_{21} &= a_{21}b_{32} + b_{21}a_{32} \\ c_{22} &= a_{22}b_{32} + b_{22}a_{32} & c_{31} &= a_{31}b_{32} + b_{31}a_{32} & c_{32} &= a_{32}b_{32} & c_{33} &= a_{33}b_{32} + b_{33}a_{32} & c_{41} &= a_{41}b_{32} + b_{41}a_{32} \\ c_{42} &= a_{42}b_{32} + b_{42}a_{32} & c_{51} &= a_{51}b_{32} + b_{51}a_{32} & c_{52} &= a_{52}b_{32} + b_{52}a_{32} & c_{53} &= a_{53}b_{32} + b_{53}a_{32} \end{aligned}$$

Where  $a_{32}$  and  $b_{32}$  denote the hearts of the two rhotrices whose indices are defined by  $\frac{n+1}{2}$  and  $\left\lfloor \frac{n+3}{4} \right\rfloor$  respectively. The resulting value of  $c_{32}$  is the product of the two hearts from the two rhotrices. We can equally extend this to accommodate for rhotrix of  $n$ -dimension. Then going by equation (3.8), we can represent the row-wise rhotrix hearts as  $a_{\frac{n+1}{2}, \left\lfloor \frac{n+3}{4} \right\rfloor}$  and  $b_{\frac{n+1}{2}, \left\lfloor \frac{n+3}{4} \right\rfloor}$ , such that:

$$c_{i,j} = a_{\frac{n+1}{2}, \left\lfloor \frac{n+3}{4} \right\rfloor} * a_{i,j} + a_{\frac{n+1}{2}, \left\lfloor \frac{n+3}{4} \right\rfloor} * b_{i,j} (1 - \lambda) \quad (3.9)$$

Where  $\lambda$  denotes a constant value that lies between 1 and 0, such that if the indices of  $a_{ij}$  and  $b_{ij}$  takes the positions of the heart entries, then  $\lambda = 0$  and  $\lambda = 1$ , otherwise.

**Algorithm 2: Heart-oriented Row-wise rhotrix multiplication***Input:*

$$p \leftarrow (n+1)/2$$

$$q \leftarrow (n+3)/4$$

$$a[0 \dots \text{row\_upperbound} , 0 \dots \text{column\_upperbound} ]$$

$$b[0 \dots \text{row\_upperbound} , 0 \dots \text{column\_upperbound} ]$$

*Output:*

$$c[0 \dots \text{row\_upperbound} , 0 \dots \text{column\_upperbound} ]$$

For  $i \leftarrow 0$  to  $\text{row\_upperbound}$

For  $j \leftarrow 0$  to  $\text{column\_upperbound}$

{

if ( $i == p \ \&\& \ j == q$ ) {

$c[i,j] \leftarrow a[i,j] * b[i,j];$

}

else {

$c[i,j] \leftarrow a[i][j] * b[p][q] + b[i,j] * a[p,q];$

}

endfor

## V. CONCLUSION AND FUTURE WORK

In this paper, we have presented some fundamental concept relating to the general ideas and methods from an already predefined multiplicative process for rhotrix multiplication. Our major contributions in this area could be found in the two generalization cases presented and the mathematical expression given, that led to the formulation of our sequential algorithm. The implementation of the algorithms showed that extension of rhotrices to higher dimension is possible as indicated in [1]. What is probably left is to find a suitable application area for rhotrices. We actually believe that in some ways, this paper might serve as a catalyst for future researchers who might develop interest in finding such possible application areas.

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