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- *Analysis of Micropolar Elastic*
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A Blow up Result In The Cauchy Problem For A Semi-Linear Accretive Wave Equation

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Abstracts - We investigate the blow up of the semi - linear wave equation given by $u_{tt} - \Delta u = |u_t|^{p-1} u_t$, and prove that for a given time T>0, there exist always initial data with sufficiently negative initial energy for which the solution blows up in time ≤T.

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AMS Classification : *35 L 45, 35 L 70.*

A BLOW UP RESULT IN THE CAUCHY PROBLEM FOR A SEMI-LINEAR ACCRETIVE WAVE EQUATION

Strictly as per the compliance and regulations of:

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Ch. Messikh

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Keywords : Wave equation, Negative initial energy, blow up, finite time.

I. Introduction

A very rich literature has been done on the semi - linear wave equation

$$u_{tt} - \Delta u = a\,|u_t|^{p-1}\,u_t + b\,|u|^{q-1}\,u, \tag{1}$$

where a and b are real numbers. Some special cases for the coefficients a and b have being considered by many authors:

1) When $a \leq 0$ and $b = 0$, the damping term $|u_t|^{p-1} u_t$ ensures global existence for arbitrary data (See, for instance, Haraux and Zuazua [5]).
2) When $a = 0$ and $b \geq 0$, the source term $|u|^{q-1} u$ is responsible for finite blow up of the global nonexistence of solutions with negative initial energy (See Ball [2]; Kalantarov and Ladyzhenskaya [7]; and, Yordanov and Zhang [12]).
3) When $a \leq 0, b > 0$ and $p > q$ or when $a \leq 0, b > 0$ and $p = 1$, the global solutions (in time) under negative energy condition exist (Georgiev and Todorova [3] and Messaoudi [9]).
4) The case $a > 0$ is more complicated. For instance, a local existenceuniqueness solutions are guaranteed only for small values of p and regular initial data. This is due to the fact that the non linear term $|u_t|^{p-1} u_t$ has bad sign and is not locally Lipschitz continuous on $L^2(\Omega)$, where Ω is a bounded open domain of $\mathbb{R}^n$. This problem was studied by Haraux [4]. He showed that (with $b = 0$ on bounded domain) there is no nontrivial global and bounded solution. He also constructed blow up solutions with arbitrary small initial data. The same problem was considered by Jazar and Kiwan (See [6] and the references therein for the same equation on bounded domain).
5) For the case when $a = a(x,t)$ is a positive function, the author (see Ref. [10]) proved that any strong solution, with $\int u_t dx \geq C$, where C is a positive constant depends only of p, n, and R, blows up in finite time, when $supp(u_0) \cup supp\,(u_1) \subset B_R(0)$ (the ball of radius R).

In this paper, we consider the semi-linear wave equation with $a = 1$ and $b = 0$:

$$\begin{cases} u_{tt} - \Delta u = |u_t|^{p-1}\,u_t \quad (x,t) \in \mathbb{R}^N \times [0,T)\,, \\ u\,(x,0) = u_0\,(x) \in H^1_{loc,\,u}\left(\mathbb{R}^N\right), \\ u_t\,(x,0) = u_1\,(x) \in L^2_{loc,\,u}\left(\mathbb{R}^N\right). \end{cases} \tag{2}$$

and show that given any time $T > 0$, there exist initial data with sufficiently negative energy for which the solution blows up in a time $t^* \leq T$. To achieve this goal, we will follow the same approach of Zaag and Merle [MZ1] by comparing, for our case, the growth u_t and k, where k is a solution of the explosively EDO $k_{tt} = |k_t|^{p-1} k_t$ associated with the equation (2). Unfortunately, the presence of the viscous term $|u_t|^{p-1} u_t$ makes our task more difficult. To overcome this difficulty, we draw attention to the work of Rivera and Fator [11] and rewrite (2) as follows:

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$$\begin{cases} u_{tt} - \int_0^t \Delta u_t(\tau)\,d\tau - \Delta u_0 = |u_t|^{p-1} u_t \quad (x,t) \in \mathbb{R}^N \times [0,T), \\ u(x,0) = u_0(x) \in H^1_{loc,\,u}(\mathbb{R}^N), \\ u_t(x,0) = u_1(x) \in L^2_{loc,\,u}(\mathbb{R}^N). \end{cases} \tag{3}$$

Then, we substitute the following change of variable:

$$v(x,t) = u_t(x,t), \tag{4}$$

in (3) to obtain the integro-differential equation

$$\begin{cases} v_t - \int_0^t \Delta v(\tau)\,d\tau - \Delta u_0(x) = |v|^{p-1} v, \ (x,t) \in \mathbb{R}^N \times [0,T) \\ v(x,0) = u_t(x,0) = u_1(x) =: v_0 \in L^2_{loc,\,u}(\mathbb{R}^N). \end{cases} \tag{5}$$

Now, we introduce $w := u_t/k$, where $k := \kappa (T-t)^{-\beta}$ with $\beta := \frac{1}{p-1}$ and $\kappa := \beta^{\beta}$. Using the following transformation defined by:

For $a \in \mathbb{R}^N$ and $T > 0$

$$z = x - a, \ s = -\log(T-t), \ v(t,x) = \frac{1}{(T'-t)^{\beta}} \theta_{T',\,a}(s,z) \tag{6}$$

and

$$u(x,0) =: \frac{1}{(T')^{\beta+!}} \theta_{a,00} \quad , \quad v(0,z) =: \frac{1}{(T')^{\beta}} \theta(s_0, y) =: \frac{1}{(T')^{\beta}} \theta_{a,0},$$

where $s_0 = -log(T)$. We then see that the function $\theta_a = \theta_{T,a}$ (we write θ for simplicity) satisfies for all $s \geq -Log(T)$ and all $z \in \mathbb{R}^N$

$$g(s)\,\theta_s + \beta g(s)\,\theta - \int_{s_0}^{s} g_2(\tau)\,\Delta\theta d\tau - g(s_0)\,\Delta\theta_{00} = g(s)\,|\theta|^{p-1}\,\theta \tag{7}$$

Where $g(s) = e^{(\beta+1)s}$ and $g(s) = e^{(\beta-1)s}$.

In the new set of variables (s,z), the behavior of u_t as $t \uparrow T$ is equivalent to the behavior of θ as $s \to \infty$. As far as we know, no local existence of solution was given for our problem (2). For this reason, we assume that there exists a set $A \subset R$ for which our problem (2) admits solutions for some $p \in A \subset \mathbb{R}$. In this work, We do not consider the same condition as in [10]. First let us provide the following assumption. H_1 we assume that $\alpha > \max\left(2, \frac{\beta}{2}(\beta+1)\right)$.

Our main result in this paper is:

Theorem 1 : Let be $p \in A \cap \left(1, \frac{N+3}{N-1}\right)$, and assume that the hypothesis H_1 is satisfied and θ a solution for (7) on B such that $E(\theta)(s_0) < 0$ for some $s_0 \in \mathbb{R}$, then θ blows up in $H^1(B) \times L^2(B)$ in time $s^* \leq s$, where B is the unit ball and E is the functional of energy associated to the equation (7). The above theorem implies directly the following blowing - up result for (5).

Proposition 2 : Let $p \in A \cap \left(1, \frac{N+3}{N-1}\right)$, and suppose that the hypothesis H_1 holds and v is a solution of (5) on B as $\Xi_{T,a}(v)(t) = E(\theta_{T,a})(-\log(T-t)) < 0$ for some $0 \leq t \leq T$ and $a \in \mathbb{R}^N$, then v blows up in finite time $T' < T$. The paper is organized as follows. In section 2 we define an associated decreasing energy to equation (7)(see Lemma 3) and in the section 3 we provide proofs for Theorem 1 and Proposition 2.

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II. The Associated Energy

In this section we start first by defining a weighted energy associated to the equation (7) and then, prove the lemma 3 . The wighted energy is given by

$$\begin{aligned} E(s) &= -\frac{\beta}{2}\int_B g(s)\,\rho^\alpha\theta^2 dz+\frac{1}{p+1}\int_B g(s)\,\rho^\alpha\,|\theta|^{p+1}\,dz \\ &+\frac{1}{8}\int_{s_0}^{s}\int_B \rho^\alpha g_2(\tau)\left\{|4\nabla\theta(\tau)-\nabla\theta(s)|^2-|\nabla\theta(s)|^2\right\}dzd\tau \\ &+\alpha\int_{s_0}^{s}\int_B g_2(\tau)\left[(N\rho-2(\alpha-1))\,|z|^2\right]\rho^{\alpha-2}\left\{|\theta(s)-\theta(\tau)|^2-|\theta(s)|^2\right\}dzd\tau \\ &+\alpha\int_{s_0}^{s}\int_B g(\tau)\,\rho^{\alpha-1}\left\{\left[e^{-2\tau}z\nabla\theta(s)-\theta(\tau)\right]^2-\left[e^{-2\tau}z\nabla\theta(s)\right]^2\right\}dzd\tau \\ &-\frac{g(s_0)}{2}\left\{\int_B\rho^\alpha\,|\nabla\theta(s)+\nabla\theta_{00}|^2\,dz-\int_B\rho^\alpha\,|\nabla\theta(s)|^2\,dz\right\} \\ &-\alpha g(s_0)\left\{\int_B\rho^{\alpha-1}\left[\theta(s)-z\nabla\theta_{00}\right]^2 dz-\int_B\rho^{\alpha-1}\left[\theta(s)\right]^2 dz\right\}. \end{aligned} \tag{8}$$

where B denotes the unit ball, α is any number satisfying $\alpha > \max\left(2, \frac{\beta}{2}(\beta+1)\right)$, and $\rho(z) := 1-|z|^2$.

Lemma 3 : The energy $s \to E(s)$ is a decreasing function of $s \geq s_0$. Moreover, we have

$$\begin{aligned} &E(s+1)-E(s) \\ &= -\tfrac{(\beta+1)}{p+1}\int_s^{s+1}\int_B g(s)\,\rho^\alpha\,|\theta(s')|^{p+1}\,dzds' \\ &-\int_s^{s+1}\int_B g(s)\,\rho^\alpha\theta_s^2(s')\,dzds' \\ &-\left[\alpha-\tfrac{\beta}{2}(\beta+1)\right]\int_s^{s+1} g(s')\int_B\rho^\alpha\theta^2(s')\,dzds' \\ &-\alpha\int_s^{s+1}\int_B g(s')\,\rho^{\alpha-1}\,|z|^2\,|\theta(s')|^2\,dzds' \\ &-\int_s^{s+1}\int_B g_2(s')\,\rho^\alpha\,|\nabla\theta(s')|^2\,dzds', \end{aligned} \tag{9}$$

where $\alpha > \max\left(2, \frac{\beta}{2}(\beta+1)\right)$.

Proof. To calculate the derivative of E, we multiply equation (7) by $\rho^\alpha\theta_s$ and integrate the equation over B

$$\begin{aligned} &\tfrac{1}{p+1}\tfrac{d}{ds}\int_B g(s)\,\rho^\alpha\,|\theta|^{p+1}\,dz-\tfrac{(\beta+1)}{p+1}\int_B g(s)\,\rho^\alpha\,|\theta|^{p+1}\,dz \\ &=\int_B g(s)\,\rho^\alpha\theta_s^2 dz+\tfrac{\beta}{2}\tfrac{d}{ds}\int_B g(s)\,\rho^\alpha\theta^2 dz-\tfrac{\beta}{2}(\beta+1)\int_B g(s)\,\rho^\alpha\theta^2 dz \\ &-\int_B\int_{s_0}^{s} g(\tau)\,\Delta\theta(\tau)\,\rho^\alpha\theta_s(s)\,d\tau dz-\,g(s_0)\int_B\rho^\alpha\theta_s\Delta\theta_{00}dz \end{aligned}$$

which is equivalent to

$$\begin{aligned} &\tfrac{\beta}{2}\tfrac{d}{ds}\int_B g(s)\,\rho^\alpha\theta^2 dz-\tfrac{1}{p+1}\tfrac{d}{ds}\int_B g(s)\,\rho^\alpha\,|\theta|^{p+1}\,dz \\ &+\int_B\int_{s_0}^{s} g(\tau)\,\rho^\alpha\nabla\theta(\tau)\,\nabla\theta_s(s)\,d\tau dz-2\alpha\int_B\int_{s_0}^{s} g(\tau)\,\rho^{\alpha-1}z\nabla\theta(\tau)\,\theta_s(s)\,d\tau dz \\ &+g(s_0)\int_B\rho^\alpha\nabla\theta_{00}\nabla\theta_s dz-2\alpha g(s_0)\int_B\rho^{\alpha-1}z\nabla\theta_{00}\theta_s dz \\ &=-\tfrac{(\beta+1)}{p+1}\int_B g(s)\,\rho^\alpha\,|\theta|^{p+1}\,dz-\int_B g(s)\,\rho^\alpha\,[\theta_s]^2\,dz+\tfrac{\beta}{2}(\beta+1)\int_B g(s)\,\rho^\alpha\theta^2 dz. \end{aligned}$$

The last equation can be written as

$$\begin{aligned} &\tfrac{\beta}{2}\tfrac{d}{ds}\int_B g(s)\,\rho^\alpha\theta^2 dz-\tfrac{1}{p+1}\tfrac{d}{ds}\int_B g(s)\,\rho^\alpha\,|\theta|^{p+1}\,dz+I_1+I_2+I_3+I_4 \\ &=-\tfrac{(\beta+1)}{p+1}\int_B g(s)\,\rho^\alpha\,|\theta|^{p+1}\,dz-\int_B g(s)\,\rho^\alpha\,[\theta_s]^2\,dz+\tfrac{\beta}{2}(\beta+1)\int_B g(s)\,\rho^\alpha\theta^2 dz, \end{aligned} \tag{10}$$

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where

$$I_1 = \int_B \int_{s_0}^s g_2(\tau)\,\rho^\alpha \nabla\theta(\tau)\,\nabla\theta_s(s)\,d\tau dz$$
$$= -\tfrac{1}{2}\tfrac{d}{ds}\left\{\int_B \int_{s_0}^s \rho^\alpha g_2(\tau)\left|2\nabla\theta(\tau) - \tfrac{1}{2}\nabla\theta(s)\right|^2 d\tau dz\right\}$$
$$+\tfrac{1}{2}\left\{\int_B \rho^\alpha g_2(s)\left|2\nabla\theta(s) - \tfrac{1}{2}\nabla\theta(s)\right|^2 d\tau dz\right\}$$
$$+\tfrac{1}{8}\tfrac{d}{ds}\int_{s_0}^s g_2(\tau)d\tau \int_B \rho^\alpha \left|\nabla\theta(s)\right|^2 dz$$
$$-\tfrac{1}{8}\, g_2(s)\int_B \rho^\alpha \left|\nabla\theta\right|^2 dz,$$

$$= -\tfrac{1}{2}\tfrac{d}{ds}\left\{\int_B \int_{s_0}^s \rho^\alpha g_2(\tau)\left|2\nabla\theta(\tau) - \tfrac{1}{2}\nabla\theta(s)\right|^2 d\tau dz\right\}$$
$$+\tfrac{1}{8}\tfrac{d}{ds}\int_{s_0}^s g_2(\tau)d\tau \int_B \rho^\alpha \left|\nabla\theta(s)\right|^2 dz$$
$$+\, g_2(s)\int_B \rho^\alpha \left|\nabla\theta\right|^2 dz,$$

$$I_3 = g(s_0)\int_B \rho^\alpha \nabla\theta_{00}\nabla\theta_s dz$$
$$= \tfrac{g(s_0)}{2}\tfrac{d}{ds}\left\{\int_B \rho^\alpha \left|\nabla\theta_{00} + \nabla\theta\right|^2 dz - \int_B \rho^\alpha \left|\nabla\theta\right|^2 dz\right\},$$

$$I_4 = -2\alpha g(s_0)\int_B \rho^{\alpha-1} y\nabla\theta_{00}\theta_s dz$$
$$= \alpha g(s_0)\tfrac{d}{ds}\left\{\int_B \rho^{\alpha-1}\left[z\nabla\theta_{00} - \theta\right]^2 dz - \int_B \rho^{\alpha-1}\left[\theta\right]^2 dz\right\}.$$

$$I_2 = -2\alpha \int_B \int_{s_0}^s g(\tau)\,\rho^{\alpha-1}\left[\nabla\theta(\tau)\, zg(\tau)\,\theta_s(s)\right] d\tau dz$$
$$= 2\alpha \int_B \int_{s_0}^s g_2(\tau)\left[\theta(\tau)\,\nabla\left(z\rho^{\alpha-1}\theta_s(s)\right)\right] d\tau dz$$
$$= 2\alpha \int_B \int_{s_0}^s g_2(\tau)\,\theta(\tau)\,N\rho^{\alpha-1}\theta_s(s)\,d\tau dz$$
$$-4\alpha(\alpha-1)\int_B \int_{s_0}^s g_2(\tau)\,\theta(\tau)\left|z\right|^2 \rho^{\alpha-2}\theta_s(s)\,d\tau dz$$
$$+2\alpha \int_B \int_{s_0}^s g_2(\tau)\,\theta(\tau)\, z\rho^{\alpha-1}\nabla\theta_s(s)\,d\tau dz$$
$$= A_1 + A_2 + A_3$$

And

$$A_1 = 2\alpha N\int_B \int_{s_0}^s g_2(\tau)\left[\theta(\tau)\,\rho^{\alpha-1}\theta_s(s)\right] d\tau dz$$
$$= -\alpha N\tfrac{d}{ds}\int_B \int_{s_0}^s g_2(\tau)\,\rho^{\alpha-1}\left[\theta(\tau) - \theta(s)\right]^2 d\tau dz$$
$$+\alpha N\tfrac{d}{ds}\left\{\int_{s_0}^s g_2(\tau)\,d\tau \int_B \rho^{\alpha-1}\theta^2 dz\right\}$$
$$-\alpha N\int_B g_2(s)\,\rho^{\alpha-1}\theta^2 dz,$$

$$A_2 = -4\alpha(\alpha-1)\int_B \int_{s_0}^s g_2(\tau)\,\theta(\tau)\left(\left|z\right|^2 \rho_s^{\alpha-2}\theta(s)\right) d\tau dz$$
$$= 2\alpha(\alpha-1)\tfrac{d}{ds}\int_B \int_{s_0}^s g_2(\tau)\left|z\right|^2 \rho^{\alpha-2}\left[\theta(\tau) - \theta(s)\right]^2 d\tau dz$$
$$-2\alpha(\alpha-1)\tfrac{d}{ds}\left[\int_{s_0}^s g_2(\tau)\,d\tau \int_B \left|z\right|^2 \rho^{\alpha-2}\theta^2 dz\right]$$
$$+2\alpha(\alpha-1)\int_B g_2(s)\left|z\right|^2 \rho^{\alpha-2}\theta^2 dz,$$

$$A_3 = 2\alpha \int_B \int_{s_0}^s g(\tau)\,\theta(\tau)\, z\rho^{\alpha-1}\nabla\left(e^{-2\tau}\theta\right)_s(s)\,d\tau dz$$
$$= -\alpha\tfrac{d}{ds}\int_B \int_{s_0}^s g(\tau)\,\rho^{\alpha-1}\left[\theta(\tau) - e^{-2\tau}z\nabla\theta(s)\right]^2 d\tau dz$$
$$+\alpha\tfrac{d}{ds}\left\{\int_{s_0}^s e^{-4\tau}g(\tau)\,d\tau \int_B \rho^{\alpha-1}\left(z\nabla\theta\right)^2 dz\right\}$$
$$-\alpha \int_B e^{-4s}g(s)\,\rho^{\alpha-1}\left(z\nabla\theta\right)^2 dz$$
$$+\alpha \int_B g(s)\,\rho^{\alpha-1}\left[\theta(s) - e^{-2s}z\nabla\theta(s)\right]^2 dz$$
$$= -\alpha\tfrac{d}{ds}\int_B \int_{s_0}^s g(\tau)\,\rho^{\alpha-1}\left[\theta(\tau) - e^{-2\tau}z\nabla\theta(s)\right]^2 d\tau dz$$
$$+\alpha\tfrac{d}{ds}\left\{\int_{s_0}^s \int_B e^{-4s}g(\tau)\,d\tau\rho^{\alpha-1}\left(z\nabla\theta\right)^2 dz\right\}$$
$$+\alpha \int_B g(s)\,\rho^{\alpha-1}\left[\theta(s)\right]^2 dz - \alpha \int_B g_2(s)\,\rho^{\alpha-1}z\nabla\left(\theta(s)^2\right) dz$$

$$
\begin{aligned}
&= -\alpha \tfrac{d}{ds} \int_B \int_{s_0}^{s} g(\tau)\, \rho^{\alpha-1} \left[\theta(\tau) - e^{-2\tau} z \nabla \theta(s)\right]^2 d\tau dz \\
&+\alpha \tfrac{d}{ds} \left\{ \int_{s_0}^{s} \int_B e^{-4\tau} g(\tau)\, d\tau \rho^{\alpha-1} (z\nabla\theta)^2 dz \right\} \\
&+\alpha \int_B g(s)\, \rho^{\alpha-1} [\theta(s)]^2 dz + \alpha \int_B g_2(s)\, \nabla.\left(\rho^{\alpha-1} z\right) \theta(s)^2 dz \\
&= -\alpha \tfrac{d}{ds} \int_B \int_{s_0}^{s} g(\tau)\, \rho^{\alpha-1} \left[\theta(\tau) - e^{-2\tau} z \nabla \theta(s)\right]^2 d\tau dz \\
&+\alpha \tfrac{d}{ds} \left\{ \int_{s_0}^{s} \int_B e^{-4s} g(\tau)\, d\tau \rho^{\alpha-1} (z\nabla\theta(\tau))^2 dz \right\} \\
&+\alpha \int_B g(s)\, \rho^{\alpha-1} [\theta(s)]^2 dz + \alpha N \int_B g_2(s)\, \rho^{\alpha-1} [\theta(s)]^2 dz \\
&-2\alpha(\alpha-1) \int_B g_2(s)\, \rho^{\alpha-2} |z|^2 \theta(s)^2 dz.
\end{aligned}
$$

Then

$$
\begin{aligned}
I_2 &= -\alpha N \tfrac{d}{ds} \int_B \int_{s_0}^{s} g_2(\tau)\, \rho^{\alpha-1} [\theta(\tau) - \theta(s)]^2 d\tau dz \\
&+\alpha N \tfrac{d}{ds} \left\{ \int_{s_0}^{s} \int_B g_2(\tau)\, d\tau \rho^{\alpha-1} \theta^2 dz \right\} \\
&+2\alpha(\alpha-1) \tfrac{d}{ds} \int_B \int_{s_0}^{s} g_2(\tau) |z|^2 \rho^{\alpha-2} [\theta(\tau) - \theta(s)]^2 d\tau dz \\
&-2\alpha(\alpha-1) \tfrac{d}{ds} \int_{s_0}^{s} g_2(\tau)\, d\tau \int_B |z|^2 \rho^{\alpha-2} [\theta(s)]^2 dz \\
&-\alpha \tfrac{d}{ds} \int_B \int_{s_0}^{s} g(\tau)\, \rho^{\alpha-1} \left[\theta(\tau) - e^{-2\tau} z \nabla\theta(s)\right]^2 d\tau dz \\
&+\alpha \tfrac{d}{ds} \left\{ \int_{s_0}^{s} \int_B e^{-4\tau} g(\tau)\, d\tau \rho^{\alpha-1} (z\nabla\theta)^2 dz \right\} \\
&+\alpha \int_B g(s)\, \rho^{\alpha-1} [\theta(s)]^2 dz.
\end{aligned}
$$

Substitute $I_0, ..., I_4$ in equation (10) we finally obtain

$$
\begin{aligned}
&\tfrac{d}{ds} E(s) = -\tfrac{(\beta+1)}{p+1} \int_B g(s)\, \rho^{\alpha} |w|^{p+1} dz \\
&-\left(\alpha - \tfrac{\beta}{2}(\beta+1)\right) \int_B g(s)\, \rho^{\alpha} [\theta(s)]^2 dz \\
&-\alpha \int_B g(s)\, \rho^{\alpha-1} |z|^2 \theta^2 dz - \int_B g(s)\, \rho^{\alpha} \theta_s^2 dz \\
&- g_2(s) \int_B \rho^{\alpha} |\nabla\theta|^2 dz.
\end{aligned}
$$

We choose $\alpha > \max\left(2, \frac{\beta}{2}(\beta+1)\right)$. So we deduce (8) . This completes the proof of the lemma.

III. Proof The Main Result

In this section, we prove results of explosion for equation (7) and (5), using the method Georgiev and Todorova.

Proof of Proposition 2 : Suppose that there exist $T > 0$, $0 < t_0 < T$, and $a \in \mathbb{R}^n$ such that $\Xi_{T,a}(v)(t_0) < 0$. Let $s_0 = -\log(T - t_0)$, then $E(w_{T,a})(s_0) < 0$. By applying Theorem 3 (see bellow), we find that the solution θ of (7) blows up in finite time $s^* < \infty$. Since $v(t,x) = \frac{1}{(T-t)^{\beta}} \theta(s,y)$, we deduce that v blows-up in finite time T' such that $s^* = -\log(T - t^*) \geq -\log(T - T')$, so we have $T' \leq T - e^{-s^*} < T$.

Proof of Theorem 1 : Since $E(s_0) < 0$ and $E(s)$ is decreasing and then $E(s) < 0$ for all $s \geq s_0$.

By setting $h(s) = -E(s)$, it follows that $h(s) \geq h(s_0)$ for all $s \geq s_0$.

Consider two different cases:

1. Assume that $h(s)$ is bounded. Then, we deduce that all the right terms in the following equation

$$
\begin{aligned}
&\tfrac{(\beta+1)}{p+1} \int_{s_0}^{s} \int_B g(\tau)\, \rho^{\alpha} |\theta|^{p+1} dz d\tau + \int_{s_0}^{s} \int_B g(\tau)\, \rho^{\alpha} \theta_\tau^2 dz d\tau \\
&+\left(\alpha - \tfrac{\beta}{2}(\beta+1)\right) \int_{s_0}^{s} \int_B g(\tau)\, \rho^{\alpha} \theta^2 dz d\tau \\
&+\alpha \int_{s_0}^{s} \int_B \rho^{\alpha-1} g(\tau) (z\theta)^2 dz d\tau + \tfrac{1}{2} \int_{s_0}^{s} \int_B g_2(\tau)\, \rho^{\alpha} |\nabla\theta|^2 dz d\tau \\
&= h(s) - h(s_0).
\end{aligned}
$$

are bounded. It means that

$\int_{s_0}^{s}\int_B g(\tau)\,\rho^\alpha\theta^2 dzd\tau$, $\int_{s_0}^{s}\int_B g(\tau)\,\rho^\alpha\,|\theta|^{p+1}\,dzd\tau$ and $\int_{s_0}^{s}\int_B g_2(\tau)\,\rho^\alpha\,|\nabla\theta|^2\,dzd\tau$ are bounded for $p<\frac{N+3}{N-1}$.

Now, we introduce the following functional defined by

$$\varphi(s)=(h(s))^{1-\delta}+\varepsilon\int_{s_0}^{s}g(\tau)d\tau\int_B\rho^{\alpha+1}\,|\theta(s)|^2\,dz.$$

where $0<\delta<1$ and ε are positive constants to be determined later.

We note that

$$\begin{aligned}[\varphi(s)]^{\frac{1}{1-\delta}} &= C\left(h(s)+\varepsilon\int_{s_0}^{s}g(\tau)d\tau\int_B\rho^{\alpha+1}\,|\theta(s)|^2\,dz\right)^{\frac{1}{1-\delta}}\\ &\le C\left(h(s)^{\frac{1}{1-\delta}}+\int_{s_0}^{s}g(\tau)d\tau\left(\int_B\rho^{\alpha}\,|\theta(s)|^2\,dz\right)^{\frac{1}{1-\delta}}\right)\\ &\le C\left(1+g(s)\left(\int_B\rho^{\alpha}\,|\theta(s)|^{\frac{2}{1-\delta}}\,dz\right)\right),\end{aligned}$$

we choose δ such that $\frac{2}{1-\delta}\le p+1$ so we have $\delta\le\frac{p-1}{p+1}\in(0,1)$.

The derivative of this functional is given by

$$\begin{aligned}\varphi'(s) &= (1-\delta)(h(s))^{-\delta}h(s)'+2\varepsilon\int_{s_0}^{s}g(\tau)d\tau\int_B\rho^{\alpha+1}\theta(s)\,\theta_s(s)\,dz\\ &\quad+2\varepsilon g(s)\int_B\rho^{\alpha+1}\,|\theta(s)|^2\,dz\\ &\ge(1-\delta)\,M_0^{-1}h(s)'+I_0+2\varepsilon g(s)\int_B\rho^{\alpha+1}\,|\theta(s)|^2\,dz.\end{aligned}\tag{11}$$

because h is bounded.

From (7), it follows that

$$\begin{aligned}I_0 &= \int_{s_0}^{s}g(\tau)d\tau\int_B\rho^{\alpha+1}\theta(s)\,\theta_s(s)\,dz\\ &= -\beta\int_{s_0}^{s}g(\tau)d\tau\int_B\rho^{\alpha+1}\theta^2dz\\ &\quad+\left(\int_{s_0}^{s}g(\tau)d\tau\right)g^{-1}(s)\int_B\rho^{\alpha+1}\theta(s)\left(\int_{s_0}^{s}g_2(\tau)\,\Delta\theta(\tau)\,d\tau\right)dz\\ &\quad+\int_{s_0}^{s}g(\tau)d\tau\left[g(s_0-s)\int_B\rho^{\alpha+1}\theta(s)\,\Delta\theta_{00}dz+\int_B\rho^{\alpha+1}\,|\theta(s)|^{p+1}\,dz\right],\end{aligned}\tag{12}$$

then from the Green's formula we can write

$$I_0=-\beta\int_{s_0}^{s}g(\tau)d\tau\int_B\rho^{\alpha+1}\theta^2dz+I_1+I_2+I_3+I_4+\int_{s_0}^{s}g(\tau)d\tau\int_B\rho^{\alpha+1}\,|\theta(s)|^{p+1}\,dz,$$

Where

$$\begin{aligned}I_1 &= -\left(\int_{s_0}^{s}g(\tau)d\tau\right)g^{-1}(s)\int_B\rho^{\alpha+1}\nabla\theta(s)\int_{s_0}^{s}g_2(\tau)\,\nabla\theta(\tau)\,d\tau dz\\ &\ge -\left|\int_B\rho^{\alpha+1}\nabla\theta(s)\int_{s_0}^{s}g_2(\tau)\,\nabla\theta(\tau)\,d\tau dz\right|\\ &\ge -\left[\sigma_1\,g_2(s)\int_B\rho^{\alpha}\,|\nabla\theta(s)|^2\,dz+\sigma_1^{-1}\int_{s_0}^{s}\int_B g_2(\tau)\,\rho^{\alpha}\,|\nabla\theta(\tau)|^2\,dzd\tau\right].\end{aligned}\tag{13}$$

Using Young's inequality, we obtain

$$\begin{aligned} I_2 &= +2(\alpha+1)\left(\int_{s_0}^{s} g(\tau)d\tau\right) g^{-1}(s)\int_B z\rho^{\alpha}\theta(s)\left(\int_{s_0}^{s} g_2(\tau)\nabla\theta(\tau)\,d\tau\right)dz \qquad (14)\\ &\geq -2(\alpha+1)\left[\sigma_2\, g(s)\int_B \rho^{\alpha}\left|z\theta(s)\right|^2 dz + \sigma_2^{-1}\int_{s_0}^{s}\int_B g_2(\tau)\rho^{\alpha}\left|\nabla\theta(\tau)\right|^2 dzd\tau\right]. \end{aligned}$$

Similarly, we find

$$\begin{aligned} I_3 &= -g(s_0-s)\int_{s_0}^{s} g(\tau)d\tau\int_B \rho^{\alpha+1}\nabla\theta(s)\nabla\theta_{00}dz \qquad (15)\\ &\geq -\int_B \rho^{\alpha}\left[\sigma_3\left|\nabla\theta_{00}\right|^2 + \sigma_3^{-1} g_2(s)\left|\nabla\theta(s)\right|^2\right]dz \end{aligned}$$

and

$$\begin{aligned} I_4 &= 2(\alpha+1)\,g(s_0-s)\int_{s_0}^{s} g(\tau)d\tau\int_B z\rho^{\alpha}\theta(s)\nabla\theta_{00}dz \qquad (16)\\ &\geq -2(\alpha+1)\left[\sigma_4^{-1}\int_B \rho^{\alpha}\left|\nabla\theta_{00}\right|^2 + \sigma_4\; g(s)\int_B \rho^{\alpha}\left(z\theta(s)\right)^2 dz\right]. \end{aligned}$$

Substituting (13)-(16) into (11) we obtain

$$\begin{aligned} \varphi'(s) \geq\;& 2\varepsilon g(s)\int_B \rho^{\alpha+1}\left|\theta(s)\right|^2 dz\\ &+\left[M_1\left(\alpha-\frac{\beta}{2}(\beta+1)\right)-2\beta\varepsilon\right]\int_B g(s)\,\rho^{\alpha}\left[\theta(s)\right]^2 dz\\ &+\left[M_1\alpha-4(\alpha+1)\varepsilon(\sigma_4+\sigma_2)\right]\int_B g(s)\,\rho^{\alpha-1}\left[z\theta(s)\right]^2 dz\\ &+\left[\frac{M_1}{2}-2\varepsilon\left(\sigma_3^{-1}+\sigma_1\right)\right] g_2(s)\int_B \rho^{\alpha}\left|\nabla\theta\right|^2 dz\\ &-2\varepsilon\left[2(\alpha+1)\sigma_4^{=1}+\sigma_3\right]\int_B \rho^{\alpha}\nabla\theta_{00}^2 dz\\ &-2\varepsilon\left(\sigma_1^{-1}+\sigma_2^{-1}\right)\int_{s_0}^{s}\int_B \rho^{\alpha+1} g_2(\tau)\left|\nabla\theta(\tau)\right|^2 dzd\tau\\ &+\frac{(\beta+1)}{p+1}M_1\int_{s_0}^{s}\int_B g(s)\,\rho^{\alpha}\left|\theta\right|^{p+1} dzd\tau \end{aligned}$$

where $M_1 = \frac{(1-\delta)}{M_0}$.

Now, the first we choose δ such that

$$\delta \leq \min\left(\frac{\left(\alpha-\frac{\beta}{2}(\beta+1)\right)-2\beta\varepsilon M_0}{\left(\alpha-\frac{\beta}{2}(\beta+1)\right)}, \frac{p-1}{p+1}\right).$$

So

$$M_1\left(\alpha-\frac{\beta}{2}(\beta+1)\right)-2\beta\varepsilon \geq 0.$$

After we choose $\sigma_1, \sigma_2, \sigma_3$ and σ_4 such that the following coefficients are Positive

$$\begin{aligned} \left[M_1\alpha-4(\alpha+1)\varepsilon(\sigma_4+\sigma_2)\right] &\geq 0,\\ \left[\frac{M_1}{2}-2\varepsilon\;\left(\sigma_3^{-1}+\sigma_1\right)\int_B \rho^{\alpha}\left|\nabla\theta(\tau)\right|^2 d\tau\right] &\geq 0. \end{aligned}$$

Then

$$\begin{aligned}\varphi'(s) &\geq \frac{(\beta+1)}{p+1}M_1\int_{s_0}^{s}\int_B g(s)\,\rho^{\alpha}\,|\theta|^{p+1}\,dz \\ &\quad -2\varepsilon\left(2(\alpha+1)\sigma_4^{=1}+\sigma_3\right)g(s_0)\int_B\rho^{\alpha+1}\nabla\theta_{00}^2 \\ &\quad -\left(\sigma_1^{-1}+\sigma_2^{-1}\right)2\varepsilon\int_B\int_{s_0}^{s}\rho^{\alpha+1}\,g_2(\tau)\,|\nabla\theta(\tau)|^2\,d\tau dz \\ &\geq C\varphi^{\frac{1}{1-\alpha}}(s)-2\varepsilon\left(2(\alpha+1)\sigma_4^{=1}+\sigma_3\right)\int_B\rho^{\alpha+1}\nabla\theta_{00}^2dz \\ &\quad -\left(\sigma_1^{-1}+\sigma_2^{-1}\right)2\varepsilon\int_B\int_{s_0}^{s}\rho^{\alpha+1}\,g_2(\tau)\,|\nabla\theta(\tau)|^2\,d\tau dz\end{aligned}$$

and as $\int_B\int_{s_0}^{s}\rho^{\alpha+1}g_2(\tau)\,|\nabla\theta(\tau)|^2\,d\tau dz$ is bounded, then we can choose ε small enough such that

$$\begin{aligned}&C\varphi^{\frac{1}{1-\delta}}(s)-2\varepsilon\left[2(\alpha+1)\sigma_4^{=1}+\sigma_3\right]\int_B\rho^{\alpha+1}\nabla\theta_{00}^2dz \\ &-\left(\sigma_1^{-1}+\sigma_2^{-1}\right)2\varepsilon\int_B\int_{s_0}^{s}\rho^{\alpha+1}\,g_2(\tau)\,|\nabla\theta(\tau)|^2\,d\tau dz\geq 0,\end{aligned}$$

This implies that there exists ε' such that

$$\varphi'(s)\geq\varepsilon'\varphi^{\frac{1}{1-\alpha}}(s).$$

So we deduce that

$$\varphi(s)=(h(s))^{1-\alpha}+\varepsilon\int_{s_0}^{s}g(\tau)d\tau\int_B\rho^{\alpha+1}\theta^2(s)\,dz$$

blows-up in finite time s^*, It follows that $\int_{s_0}^{s}g(\tau)d\tau\int_B\rho^{\alpha+1}\theta^2(s)\,dz$ blows up also in finite time because $h(s)$ is bounded. Thus $\|\theta\|_{L^2(B)}$ blows-up also in finite time.

2. We assume that $h(s)$ blows-up in finite time s^* and since

$$h(s)\leq\sup_{s_0\leq s\leq s^*}\left[\|\theta\|_{H^1(B)}+\|\theta_t\|_{L^2(B)}\right],$$

then the solution θ blows - up in finite time

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Formation of Certain Summation Formulae Based On Half Argument Involving Hypergeometric Function

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Strictly as per the compliance and regulations of:

Formation of Certain Summation Formulae Based On Half Argument Involving Hypergeometric Function

Salahuddin[α], M.P. Chaudhary[Ω]

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I. Introduction

Generalized Gaussian hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ccc} a_1,a_2,\cdots,a_A & ; & \\ & & z \\ b_1,b_2,\cdots,b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty}\frac{(a_1)_k(a_2)_k\cdots(a_A)_k z^k}{(b_1)_k(b_2)_k\cdots(b_B)_k k!} $$

or

$$ {}_AF_B\left[\begin{array}{ccc} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right] \equiv {}_AF_B\left[\begin{array}{ccc} (a_j)_{j=1}^{A} & ; & \\ & & z \\ (b_j)_{j=1}^{B} & ; & \end{array}\right] = \sum_{k=0}^{\infty}\frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and A, B are non-negative integers.

Contiguous Relation is defined as follows

[E. D. p.51(10), Andrews p.363(9.16), H.T. F. I p.103(32)]

$$ (a-b)\ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ c & ; & z \end{array}\right] = a\ {}_2F_1\left[\begin{array}{ccc} a+1,\ b & ; & \\ c & ; & z \end{array}\right] - b\ {}_2F_1\left[\begin{array}{ccc} a,\ b+1 & ; & \\ c & ; & z \end{array}\right] \tag{2} $$

Recurrence relation of gamma function is defined as follows

$$ \Gamma(z+1) = z\,\Gamma(z) \tag{3} $$

Legendre duplication formula is defined by

$$ \sqrt{\pi}\,\Gamma(2z) = 2^{(2z-1)}\,\Gamma(z)\,\Gamma\left(z+\frac{1}{2}\right) \tag{4} $$

$$ \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)}\,\Gamma(\frac{b}{2})\,\Gamma(\frac{b+1}{2})}{\Gamma(b)} \tag{5} $$

$$ = \frac{2^{(a-1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\Gamma(a)} \tag{6} $$

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Bailey summation theorem [Prud, p.491(7.3.7.8)]is as follows

$$ {}_2F_1\left[\begin{matrix} a, 1-a & ; & \frac{1}{2} \\ c & ; & \end{matrix}\right] = \frac{\Gamma(\frac{c}{2})\,\Gamma(\frac{c+1}{2})}{\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c+1-a}{2})} = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-1}\,\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c+1-a}{2})} \qquad (7) $$

II. Main Summation Formulae

$$ {}_2F_1\left[\begin{matrix} a \;, & -a-31 & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c+31}} \times \Bigg[\frac{-32(-126777003082329600000)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(20595415066908998400a - 926204391363283776 0a^2 + 1353632653931095440a^3)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(607680617478480a^4 - 99436599786493 20a^5 - 60453402240a^6 + 30640832998545a^7)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(619328526465a^8 - 25924502835a^9 - 1063654515a^{10} - 7639485a^{11} + 177555a^{12})}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(3255a^{13} + 15a^{14} - 2961349321593277440 0c + 331340420273963097 60ac)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(-1060305651380029478 4a^2c + 960201585296727696a^3c + 43315347582256032a^4c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(-599269842622428 8a^5c - 188476928122316a^6c + 10666764589703a^7c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(481387701731a^8c + 46976811a^9c - 259248801a^{10}c - 4024699a^{11}c - 7063a^{12}c + 217a^{13}c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(a^{14}c - 268720397168043110 40c^2 + 219830155796193523 20ac^2)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(-503154046037465856 0a^2c^2 + 241130543916718800a^3c^2 + 26151340987189080a^4c^2)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(-1146075534278100a^5c^2 - 79796202137550a^6c^2 + 575648038980a^7c^2)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(90737193330a^8c^2 + 1232147700a^9c^2 - 12029850a^{10}c^2 - 351540a^{11}c^2 - 1890a^{12}c^2)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(-1335324662646480691 2c^3 + 8172439523536586496ac^3 - 1318114175517939968a^2c^3)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$ + \frac{-32(181141131752426 40a^3c^3 + 6479375107986424a^4c^3 - 45595957626180a^5c^3)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+31}{2})} + $$

$$+\frac{-32(-12824288708390a^{6}c^{3}-151650421356a^{7}c^{3}+6139102074a^{8}c^{3}+144561060a^{9}c^{3})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(488670a^{10}c^{3}-7812a^{11}c^{3}-42a^{12}c^{3}-4190669522667264000c^{4})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(1930407148471353600ac^{4}-211801780361313600a^{2}c^{4}-3250016105076000a^{3}c^{4})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(851154873246000a^{4}c^{4}+12547796499000a^{5}c^{4}-966225468600a^{6}c^{4}-23689890000a^{7}c^{4})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(81396000a^{8}c^{4}+5859000a^{9}c^{4}+37800a^{1}0c^{4}-896210914300549120c^{5})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(309362639107541248ac^{5}-21618987240208448a^{2}c^{5}-933498687245280a^{3}c^{5})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(62914564722480a^{4}c^{5}+2025497521320a^{5}c^{5}-30196918248a^{6}c^{5}-1347049200a^{7}c^{5})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-7230720a^{8}c^{5}+78120a^{9}c^{5}+504a^{1}0c^{5}-136593860006123520c^{6})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(34887267441143040ac^{6}-1362835188496320a^{2}c^{6}-106325086633920a^{3}c^{6})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(2400525998640a^{4}c^{6}+136185033600a^{5}c^{6}+221205600a^{6}c^{6}-34372800a^{7}c^{6}-277200a^{8}c^{6})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-15241536890804224c^{7}+2820561599829248ac^{7}-43480775233984a^{2}c^{7})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-7026484960704a^{3}c^{7}+17197768368a^{4}c^{7}+4832488320a^{5}c^{7}+40122720a^{6}c^{7})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-327360a^{7}c^{7}-2640a^{8}c^{7}-1263644981913600c^{8}+164287692771840ac^{8})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(393170595840a^{2}c^{8}-288230659200a^{3}c^{8}-2365545600a^{4}c^{8}+88387200a^{5}c^{8}+950400a^{6}c^{8})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-78243922831360c^{9}+6836145744384ac^{9}+105021238784a^{2}c^{9}-7241857920a^{3}c^{9})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-99890560a^{4}c^{9}+654720a^{5}c^{9}+7040a^{6}c^{9}-3603584624640c^{10}+198178506240ac^{10})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(4809742080a^{2}c^{10}-102136320a^{3}c^{10}-1647360a^{4}c^{10}-121639970816c^{11})}{\Gamma(\frac{c-a}{2})\;\Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(3800915456ac^{11}+113015552a^2c^{11}-619008a^3c^{11}-9984a^4c^{11}-2921318400c^{12})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-32(43330560ac^{12}+1397760a^2c^{12}-47237120c^{13}+222208ac^{13})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{7168a^2c^{13}-460800c^{14}-2048c^{15})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-140410465978669286400a+128475928177064496000a^2-3378196240605850579 20a^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{1868947329131819726 4a^4+2598574261842425664a^5-139074162017438648a^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-1258772148512519 2a^7+121255987903953a^8+26000045157472a^9+505712970236a^{10}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-7770554736a^{11}-386509018a^{12}-4298336a^{13}-588a^{14}+248a^{15}+a^{16}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{140410466109436723200 0c-361432967541713141760 0ac+196651259616815529984 0a^2c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-325604775785387212800a^3c+197524248877071360 0a^4c+244633073891713536 0a^5c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-6678286574092800a^6c-7792114119087360a^7c-150815024474880a^8c+6741116040960a^9c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{271264492800a^{10}c+1925710080a^{11}c-45615360a^{12}c-833280a^{13}c-3840a^{14}c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{232957039798564945920 0c^2-3283817028961948139520ac^2+1194870967556528406528a^2c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-1203176146210289049 60a^3c^2-473377123991781888 0a^4c^2+760630552772864512a^5c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{22492609658218240a^6c^2-1384355235709312a^7c^2-60829664672896a^8c^2+12467452032a^9c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{33246245760a^{10}c^2+514161536a^{11}c^2+898688a^{12}c^2-27776a^{13}c^2-128a^{14}c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{1655124063020195512320c^3-1566694133878448187904 0ac^3+394025913314442362880a^2c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-210686442786482380 80a^3c^3-208775953671920640 0a^4c^3+99858900085632000a^5c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{6644244654451200a^6c^3 - 53144508533760a^7c^3 - 7728849999360a^8c^3 - 104143334400a^9c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{1032998400a^10c^3 + 29998080a^11c^3 + 161280a^12c^3 + 678738473873018191872c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-459933281420468649984ac^4 + 79793531879829762048a^2c^4 - 13511620269224519680a^3c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-401277002343037440a^4c^4 + 334520106069120 0a^5c^4 + 814270582363520a^6c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{9418598671104a^7c^4 - 394442326656a^8c^4 - 9235242240a^9c^4 - 31167360a^{10}c^4 + 499968a^{11}c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{2688a^{12}c^4 + 181651168683255398400c^5 - 90177345987803873280ac^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{10494783795613040640a^2c^5 + 1396620566791372800a^3c^5 - 42960664547573760a^4c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-6077316426854400a^5c^5 + 49525783050240a^6c^5 + 1204122931200a^7c^5 - 4238438400a^8c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-299980800a^9c^5 - 1935360a^10c^5 + 33878974059312578560c^6 - 12374104894953684992ac^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{909163672031133696a^2c^6 + 377488508417105920a^3c^6 - 2678968046424064a^4c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-849795669596160a^5c^6 + 1300367502336a^6c^6 + 57376327680a^7c^6 + 307722240a^8c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-3333120a^9c^6 - 21504a^{10}c^6 + 4576878391071866880c^7 - 1220508453766103040ac^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{49957863590215680a^2c^7 + 37904436360806400a^3c^7 - 88592200519680a^4c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-49503117312000a^5c^7 - 7765401600a^6c^7 + 1257062400a^7c^7 + 10137600a^8c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{458279130417332224c^8 - 87651604127752192ac^8 + 1431156603582464a^2c^8}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{222069695772672a^3c^8 - 588637361664a^4c^8 - 154388213760a^5c^8 - 1281223680a^6c^8}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{10475520a^7c^8 + 84480a^8c^8 + 34425286950912000c^9 - 4588800697958400ac^9}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-9140546764800a^2c^9 + 8154982809600a^3c^9 + 66583756800a^4c^9 - 2514124800a^5c^9}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-27033600a^6c^9+1946357837332480c^{10}-173223332806656ac^{10}-2635562975232a^2c^{10}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{185101041664a^3c^{10}+2552512512a^4c^{10}-16760832a^5c^{10}-180224a^6c^{10}+82374536724480c^{11}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-4589969080320ac^{11}-111220162560a^2c^{11}+2376990720a^3c^{11}+38338560a^4c^{11}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{2568813019136c^{12}-80962945024ac^{12}-2407022592a^2c^{12}+13205504a^3c^{12}+212992a^4c^{12}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{57252249600c^{13}-853278720ac^{13}-27525120a^2c^{13}+862453760c^{14}-4063232ac^{14}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\left.\frac{-131072a^2c^{14}+7864320c^{15}+32768c^{16}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}\right] \quad (8)$$

$${}_2F_1\left[\begin{matrix} a\ , & -a-32 & ; & \frac{1}{2} \\ c & & ; & 2\end{matrix}\right]=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+32}}\times\left[\frac{-2808209320881060096000a}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+\right.$$

$$+\frac{2663360260009726636800a^2-761012930360984837760a^3+59789981041407736848a^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{3838097925675257664a^5-412887209956538296a^6-17102186004670536a^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{788560364968145a^8+45162707702112a^9+113794887932a^{10}-29121053808a^{11}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-578885594a^{12}-1601376a^{13}+54964a^{14}+504a^{15}+a^{16}+2808209322188734464000c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-7322501051641862553600ac+4167566365404321792000a^2c-776053431621780277248a^3c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{24390885974292556800a^4c+4620781401217787264a^5c-135931294749691584a^6c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-14708322003815968a^7c+10204810088880a^8c+18038108416592a^9c+340115503728a^{10}c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-2388933680a^{11}c-124331760a^{12}c-1064336a^{13}c-624a^{14}c+16a^{15}c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{4659140795971298918400c^2-6716801554917910364160ac^2+2592439908026046253056a^2c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-3118803682234629888000a^3c^2-2396216347714062720a^4c^2+171618747082716384 0a^5c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{97787148174154 56a^6c^2-3554364976514352a^7c^2-72301707344592a^8c^2+1718028618768a^9c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{64981523376a^{10}c^2+424868976a^{11}c^2-4925424a^{12}c^2-63504a^{13}c^2-144a^{14}c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{3310248126040391024640c^3-3233653400134714392576ac^3+878636832223843737600a^2c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-63870969022090296320a^3c^3-3015311036723040000a^4c^3+287807484504014464a^5c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{9078040634562240a^6c^3-324341980864352a^7c^3-13651181869248a^8c^3-29183712096a^9c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{3898762560a^{10}c^3+46316256a^{11}c^3+68544a^{12}c^3-672a^{13}c^3+1357476947746036383744c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-958287782560046014464ac^4+184144118482689985536a^2c^4-6714508274153121024a^3c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-726733557814194304a^4c^4+22135776085684800a^5c^4+1484330483864800a^6c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-4210855387200a^7c^4-991865175840a^8c^4-11279822400a^9c^4+58507680a^{10}c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{1270080a^{11}c^4+3360a^{12}c^4+363302337366510796800c^5-189842774413099925504ac^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{25372391843410427904a^2c^5-1946284385149153 28a^3c^5-89972565214118400a^4c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{234507253939200a^5c^5+114510461788800a^6c^5+1251789931392a^7c^5-29856879360a^8c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-543164160a^9c^5-1330560a^{10}c^5+8064a^11c^5+67757948118625157120c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-26359483031947591680ac^6+2357507403955787776a^2c^6+39185303882199552a^3c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-6546878251691008a^4c^6-93030581685888a^5c^6+4508612110464a^6c^6+90807171840a^7c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-135717120a^8c^6-9313920a^9c^6-29568a^{10}c^6+9153756782143733760c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-2636370051512795136ac^7+1467953244270796800a^2c^7+58427944861214720a^3c^7}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-281415104289792a^4c^7-7799366522112a^5c^7+72776816640a^6c^7+2611361280a^7c^7}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{9630720a^8c^7-42240a^9c^7+916558260834664448c^8-192573448050880512ac^8}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{5772260367971328a^2c^8+400944040915968a^3c^8-6229949628672a^4c^8-296844134400a^5c^8}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-540418560a^6c^8+31933440a^7c^8+126720a^8c^8+68850573901824000c^9}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-10301237301870592ac^9+110259836239872a^2c^9+16439027585024a^3c^9-12727080960a^4c^9}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-5893212160a^5c^9-32778240a^6c^9+112640a^7c^9+3892715674664960c^{10}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-400184633671680ac^{10}-1303326216192a^2c^{10}+416212752384a^3c^{10}+2679998464a^4c^{10}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-55351296a^5c^{10}-292864a^6c^{10}+164749073448960c^{11}-11045373214720ac^{11}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-140164177920a^2c^{11}+6205681664a^3c^{11}+56549376a^4c^{11}-159744a^5c^{11}+5137626038272c^{12}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-207626625024ac^{12}-3753078784a^2c^{12}+46964736a^3c^{12}+372736a^4c^{12}+114504499200c^{13}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-2455240704ac^{13}-47824896a^2c^{13}+114688a^3c^{13}+1724907520c^{14}-15482880ac^{14}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-245760a^2c^{14}+15728640c^{15}-32768ac^{15}+65536c^{16}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{1257627870576709632000 0-20445225825356330342400a+9195864755379017736960a^2}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-1329189482740943051136a^3-856335279231506328 0a^4+108303742957686801 28a^5}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{43378037926115144a^6-37003539895830200a^7-895956191167279a^8+35589243742624a^9}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{1753588267772a^{10}+15507471728a^{11}-396897242a^{12}-9614752a^{13}-64076a^{14}+8a^{15}}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{a^{16}+2978227168006876692480 0c-3354639407009263779840 0ac}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{1079296040520358472908 8a^{2}c-9621666021967235297 28a^{3}c-5244954862633671168 0a^{4}c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{671539632101431974 4a^{5}c+2511208137254044 80a^{6}c-1344154768007216 0a^{7}c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-716603737262160a^{8}c-895551610448a^{9}c+485575752816a^{10}c+9312490544a^{11}c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{21734160a^{12}c-935536a^{13}c-8304a^{14}c-16a^{15}c+276046951819797253324 80c^{2}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-2286291008906176772505 6ac^{2}+528894864869030330572 8a^{2}c^{2}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-244040874762163528704a^{3}c^{2}-31398576827991196032a^{4}c^{2}+1318733262810089280a^{5}c^{2}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{108757286188405536a^{6}c^{2}-653799030000336a^{7}c^{2}-148338641669328a^{8}c^{2}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-2422115790864a^{9}c^{2}+26538983856a^{10}c^{2}+1038392208a^{11}c^{2}+8073744a^{12}c^{2}-1008a^{13}c^{2}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-144a^{1}4c^{2}+141063259243908264099 84c^{3}-879532243143588446208 0ac^{3}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{1441688965310003077120a^{2}c^{3}-159287477759974942 72a^{3}c^{3}-8101198657656397568a^{4}c^{3}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{42563478517256576a^{5}c^{3}+1878936809000416 0a^{6}c^{3}+268960085532512a^{7}c^{3}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-11412872678592a^{8}c^{3}-333793055904a^{9}c^{3}-1474334400a^{10}c^{3}+33678624a^{11}c^{3}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{348096a^{12}c^{3}+672a^{13}c^{3}+458444805853279970918 4c^{4}-216709137977869812531 2ac^{4}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{243008864527168564224a^{2}c^{4}+47736692781570552 32a^{3}c^{4}-1133240400249215104a^{4}c^{4}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-20265188755606080a^{5}c^{4}+1564711799131360a^{6}c^{4}+45980788392000a^{7}c^{4}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-170217764640a^{8}c^{4}-17306520000a^{9}c^{4}-161478240a^{10}c^{4}+20160a^{11}c^{4}+3360a^{12}c^{4}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{102314265171149717504 0c^5 - 365576178424323506176ac^5 + 262392004676723834 88a^2c^5}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{1320185065373554688a^3c^5 - 90806945140032000a^4c^5 - 3442576884341760a^5c^5}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{55119033062016a^6c^5 + 3105691171968a^7c^5 + 21662403840a^8c^5 - 336779520a^9c^5}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-4169088a^{10}c^5 - 8064a^{11}c^5 + 164179858383692103680c^6 - 43864327113745481728ac^6}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{1761239942758438912a^2c^6 + 160703492904553984a^3c^6 - 3805540868512768a^4c^6}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-262533478306176a^5c^6 - 597701064576a^6c^6 + 101531485440a^7c^6 + 1184198400a^8c^6}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-147840a^9c^6 - 29568a^10c^6 + 19490608115873742848c^7 - 3822615444792410112ac^7}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{589659744145612 80a^2c^7 + 117656179044997 12a^3c^7 - 25099911969792a^4c^7}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-11128976272128a^5c^7 - 119769999360a^6c^7 + 1411238400a^7c^7 + 21795840a^8c^7 + 42240a^9c^7}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{1741265002564026368c^8 - 243941753814798336ac^8 - 917210568772608a^2c^8}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{553288264928256a^3c^8 + 5641747676928a^4c^8 - 261080709120a^5c^8 - 4060193280a^6c^8}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{506880a^7c^8 + 126720a^8c^8 + 118054610869944320c^9 - 11359341010485248ac^9}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-209922369175552a^2c^9 + 16662945181696a^3c^9 + 297528535040a^4c^9 - 2822420480a^5c^9}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-58009600a^6c^9 - 112640a^7c^9 + 6078561478246400c^{10} - 379322169081856ac^{10}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-11262679633920a^2c^{10} + 301692971008a^3c^{10} + 7037814784a^4c^{10} - 878592a^5c^{10}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-292864a^6c^{10} + 235981559037952c^{11} - 8739429810176ac^{11} - 336125337600a^2c^{11}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{2668417024a^3c^{11} + 82108416a^4c^{11} + 159744a^5c^{11} + 6790426918912c^{12} - 127990833152ac^{12}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-5971603456a^2c^{12}+745472a^3c^{12}+372736a^4c^{12}+140341411840c^{13}-957874176ac^{13}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$+\frac{-58834944a^2c^{13}-114688a^3c^{13}+1968701440c^{14}-245760ac^{14}-245760a^2c^{14}+16777216c^{15}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}+$$

$$\left.+\ \frac{32768ac^{15}+65536c^{16}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+33}{2})}\right] \tag{9}$$

III. Derivations of The Results(8) to (9)

Derivation of result (8) : putting $b=-a-31, z=\frac{1}{2}$ in known result (2),we get

$$(2a+31)\ {}_2F_1\left[\begin{matrix} a\ ,\ -a-31 & ; \frac{1}{2} \\ c & ; \frac{1}{2}\end{matrix}\right]$$

$$=a\ {}_2F_1\left[\begin{matrix} a+1\ ,\ -a-31 & ; \frac{1}{2} \\ c & ; \frac{1}{2}\end{matrix}\right]+(a+31)\ {}_2F_1\left[\begin{matrix} a\ ,\ -a-30 & ; \frac{1}{2} \\ c & ; \frac{1}{2}\end{matrix}\right]$$

Now using Bailey theorem, we get

$$L.H.S=a\,\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+30}}\times\left[\frac{-46803488615967283200(a+1)+42767159087365735680(a+1)^2}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+\right.$$

$$+\frac{-11270601192070601856(a+1)^3+661015913631944304(a+1)^4}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{76584910384046512(a+1)^5-4417245600019672(a+1)^6-336434811648432(a+1)^7}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{3976287855967(a+1)^8+608379703391(a+1)^9+9950104899(a+1)^{10}-147972013(a+1)^{11}}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-5745971(a+1)^{12}-49203(a+1)^{13}-7(a+1)^{14}+(a+1)^15+46803488703145574400c}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-118859389113316884480(a+1)c+64102962284473595904(a+1)^2c}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-10599521603190073344(a+1)^3c+111032145402626688(a+1)^4c}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{70940427398809792(a+1)^5c-407139854973664(a+1)^6c-199425094414192(a+1)^7c}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-3203075817296(a+1)^8c+144861951696(a+1)^9c+4812296496(a+1)^{10}c}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{27275696(a+1)^{11}c-528752(a+1)^{12}c-6608(a+1)^{13}c-16(a+1)^{14}c}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{7609223030941679616 0c^2 - 105466755123938820096(a+1)c^2}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{37807566181098061824(a+1)^2c^2 - 3801519798063379968(a+1)^3c^2}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-122484531131280000(a+1)^4c^2 + 21183090923693504(a+1)^5c^2}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{525072890414880(a+1)^6c^2 - 33174563270480(a+1)^7c^2 - 1230849813984(a+1)^8c^2}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{1027790064(a+1)^9c^2 + 495596640(a+1)^{10}c^2 + 5892880(a+1)^{11}c^2 + 7392(a+1)^{12}c^2}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-112(a+1)^{13}c^2 + 52634394423692623872c^3 - 48739196162194857984(a+1)c^3}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{+11997982874998750208(a+1)^2c^3 - 647120383330003200(a+1)^3c^3}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-547846663145539 84(a+1)^4c^3 + 2650005405760320(a+1)^5c^3 + 149770214403680(a+1)^6c^3}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1262779162176(a+1)^7c^3 - 137968653984(a+1)^8c^3 - 1507947840(a+1)^9c^3}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{12018720(a+1)^{10}c^3 + 237888(a+1)^{11}c^3 + 672(a+1)^{12}c^3 + 2087013598164418 5600c^4}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-13743447729720811520(a+1)c^4 + 2316611729627405312(a+1)^2c^4}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-42837424766233856(a+1)^3c^4 - 9995837120296320(a+1)^4c^4 + 92690856584640(a+1)^5c^4}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{16695491060640(a+1)^6c^4 + 158053727328(a+1)^7c^4 - 5904024000(a+1)^8c^4}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-105934080(a+1)^9c^4 - 252000(a+1)^{10}c^4 + 2016(a+1)^{11}c^4 + 5359367756720373760c^5}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-2564183483721285632(a+1)c^5 + 287511916583118848(a+1)^2c^5}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{2891094455400960(a+1)^3c^5 - 988409117368320(a+1)^4c^5 - 11460885179520(a+1)^5c^5}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{889877079168(a+1)^6c^5 + 17580998400(a+1)^7c^5 - 50023680(a+1)^8c^5 - 2378880(a+1)^9c^5}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-8064(a+1)^{10}c^5+9506535434197401 60c^6-3312034889384 83712(a+1)c^6}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{232279723845099 52(a+1)^2c^6+825626374939136(a+1)^3c^6-55557808244736(a+1)^4c^6}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1476433967232(a+1)^5c^6+19650247680(a+1)^6c^6+658533120(a+1)^7c^6}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{2472960(a+1)^8c^6-13440(a+1)^9c^6+1208741615884042 24c^7-30349688822075392(a+1)c^7}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{1175475851951104(a+1)^2c^7+75957743533056(a+1)^3c^7-1613337631488(a+1)^4c^7}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-72595015680(a+1)^5c^7-84395520(a+1)^6c^7+9968640(a+1)^7c^7+42240(a+1)^8c^7}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{11246832294297600c^8-1991999707934720(a+1)c^8+30885185565696(a+1)^2c^8}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{3902365108224(a+1)^3c^8-9744641280(a+1)^4c^8-1772770560(a+1)^5c^8}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-10264320(a+1)^6c^8+42240(a+1)^7c^8+772615155220480c^9-93305628573696(a+1)c^9}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-108440735744(a+1)^2c^9+119913953280(a+1)^3c^9+756828160(a+1)^4c^9}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-19937280(a+1)^5c^9-112640(a+1)^6c^9+39124756070400c^{10}-3061463646208(a+1)c^{10}}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-37866008576(a+1)^2c^{10}+2126170112(a+1)^3c^{10}+20410368(a+1)^4c^{10}}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-67584(a+1)^5c^{10}+1441659355136c^{11}-67616333824(a+1)c^{11}-1271508992(a+1)^2c^{11}}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{18849792(a+1)^3c^{11}+159744(a+1)^4c^{11}+37571788800c^{12}-930242560(a+1)c^{12}}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-19222528(a+1)^2c^{12}+53248(a+1)^3c^{12}+656015360c^{13}-6766592(a+1)c^{13}}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-114688(a+1)^2c^{13}+6881280c^{14}-16384(a+1)c^{14}+32768c^{15}}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{20284320493172736 0000-3225131802991139 32800(a+1)+13708117697627970 4320(a+1)^2}{\Gamma(\frac{c+a+32}{2})\,\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-1695844122737276966 4(a+1)^3-6031832149972644 96(a+1)^4}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{146759006054821328(a+1)^5+4133871935431448(a+1)^6-410385710727888(a+1)^7}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-19216954885103(a+1)^8+76930919809(a+1)^9+18767187789(a+1)^{10}}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{359027053(a+1)^{11}+584899(a+1)^{12}-48237(a+1)^{13}-457(a+1)^{14}-(a+1)^{15}}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{4738158914549243904 00c-5142676728180398284 80(a+1)c}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{1527339167906084720 64(a+1)^2c-1043115193547101209 6(a+1)^3c}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-1042530101607866112(a+1)^4c+70205646306447488(a+1)^5c}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{4867596553619456(a+1)^6c-72498049647248(a+1)^7c-9301873967216(a+1)^8c}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-143461234896(a+1)^9c+2457464016(a+1)^{10}c+89007184(a+1)^{11}c+737968(a+1)^{12}c}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-112(a+1)^{13}c-16(a+1)^{14}c+4299526354688689766 40c^2}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-3380049613812787568 64(a+1)c^2+7005902669749217894 4(a+1)^2c^2}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-17592760413197560 32(a+1)^3c^2-479277757288256 00(a+1)^4c^2}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{7123495031408896(a+1)^5c^2+1469730527666880(a+1)^6c^2+17407942640720(a+1)^7c^2}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-1222948011264(a+1)^8c^2-35273078064(a+1)^9c^2-145104960(a+1)^{10}c^2}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{4630640(a+1)^{11}c^2+51072(a+1)^{12}c^2+112(a+1)^{13}c^2+2136519460234369105 92c^3}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-1243571655519657615 36(a+1)c^3+1753268785133792460 8(a+1)^2c^3}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{190507604230245120(a+1)^3c^3-101519794853401984(a+1)^4c^3}{\Gamma(\frac{c+a+32}{2})\;\Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-1414918873379520(a+1)^5c^3+184536822383840(a+1)^6c^3+5294045822016(a+1)^7c^3}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-34417217184(a+1)^8c^3-2670212160(a+1)^9c^3-26567520(a+1)^{10}c^3}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{4032(a+1)^{11}c^3+672(a+1)^{12}c^3+6705071236267622400 0c^4}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-29026509985070018560(a+1)c^4+2631784500033124352(a+1)^2c^4}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{117050823745715456(a+1)^3c^4-11408656870312320(a+1)^4c^4}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-412101963368640(a+1)^5c^4+9483271361760(a+1)^6c^4+501887904672(a+1)^7c^4}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{3510897600(a+1)^8c^4-69457920(a+1)^9c^4-917280(a+1)^{10}c^4-2016(a+1)^{11}c^4}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{14339374628808785920c^5-4586744602902335488(a+1)c^5+238537086089704448(a+1)^2c^5}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{20197806522800640(a+1)^3c^5-657184891672320(a+1)^4c^5-42870132090240(a+1)^5c^5}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-39115880832(a+1)^6c^5+21361670400(a+1)^7c^5+265681920(a+1)^8c^5-40320(a+1)^9c^5}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-8064(a+1)^10c^5+2185501760097976320c^6-508452565484474368(a+1)c^6}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{11307303767053312(a+1)^2c^6+1925277943980544(a+1)^3c^6-9760743879936(a+1)^4c^6}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-2321995498368(a+1)^5c^6-25841195520(a+1)^6c^6+370433280(a+1)^7c^6}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{6101760(a+1)^8c^6+13440(a+1)^9c^6+243864590252867584c^7-40222844632813568(a+1)c^7}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-22289722459136(a+1)^2c^7+113092549008384(a+1)^3c^7+1111218400512(a+1)^4c^7}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-67136593920(a+1)^5c^7-1113361920(a+1)^6c^7+168960(a+1)^7c^7+42240(a+1)^8c^7}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{20218319710617600c^8-2276064016568320(a+1)c^8-40567245886464(a+1)^2c^8}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{4142976178176((a+1)^3c^8+77685822720(a+1)^4c^8-873143040(a+1)^5c^8}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-19134720(a+1)^6c^8-42240(a+1)^7c^8+1251902765301760c^9-90907976146944(a+1)c^9}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-2799334866944(a+1)^2c^9+89515345920(a+1)^3c^9+2226780160(a+1)^4c^9}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-337920(a+1)^5c^9-112640(a+1)^6c^9+57657353994240c^{10}}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-2473121226752(a+1)c^{10}-100757651456(a+1)^2c^{10}+931330048(a+1)^3c^{10}}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{30547968(a+1)^4c^{10}+67584(a+1)^5c^{10}+1946239533056c^{11}-42316292096(a+1)c^{11}}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-2105372672(a+1)^2c^{11}+319488(a+1)^3c^{11}+159744(a+1)^4c^{11}+46741094400c^{12}}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$+\frac{-366878720(a+1)c^{12}-24014848(a+1)^2c^{12}-53248(a+1)^3c^{12}+755793920c^{13}}{\Gamma(\frac{c+a+32}{2})\ \Gamma(\frac{c-a-1}{2})}+$$

$$\left.+\frac{-114688(a+1)c^{13}-114688(a+1)^2c^{13}+7372800c^{14}+16384(a+1)c^{14}+32768c^{15}}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+32}{2})}\right]+$$

$$(a+31)\ \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+30}}\times\left[\frac{-46803488615967283200a+427671590873657335680a^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+30}{2})}+\right.$$

$$+\ \frac{-1127060119207060185\,6a^3+661015913631944304a^4+76584910384046512a^5}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\ \frac{-4417245600019672a^6-336434811648432a^7+3976287855967a^8+608379703391a^9}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\ \frac{9950104899a^{10}-147972013a^{11}-5745971a^{12}-49203a^{13}-7a^{14}+a^{15}}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\ \frac{46803488703145574400c-118859389113316884480ac+64102962284473595904a^2c}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\ \frac{-10599521603190073344a^3c+1110321454026266688a^4c+70940427398809792a^5c}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-4071398549736 64a^6c-199425094414192a^7c-3203075817296a^8c+144861951696a^9c}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\ \frac{4812296496a^{10}c+27275696a^{11}c-528752a^{12}c-6608a^{13}c-16a^{14}c}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{7609223030941679616 0c^2-10546675512393882009 6ac^2+3780756618109806182 4a^2c^2}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-380151979806337996 8a^3c^2-122484531131280000a^4c^2+21183090923693504a^5c^2}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{525072890414880a^6c^2-33174563270480a^7c^2-1230849813984a^8c^2+1027790064a^9c^2}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{495596640a^{10}c^2+5892880a^{11}c^2+7392a^{12}c^2-112a^{13}c^2+5263439442369262387 2c^3}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-4873919616219485798 4ac^3+1199798287499875020 8a^2c^3-64712038333000320 0a^3c^3}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-5478466631455398 4a^4c^3+2650005405760320a^5c^3+149770214403680a^6c^3}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-1262779162176a^7c^3-137968653984a^8c^3-1507947840a^9c^3+12018720a^{10}c^3+237888a^{11}c^3}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{672a^12c^3+2087013598164418560 0c^4-1374344772972081152 0ac^4}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{672a^12c^3+2087013598164418560 0c^4-1374344772972081152 0ac^4}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{231661172962740531 2a^2c^4-42837424766233856a^3c^4-999583712029632 0a^4c^4}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{92690856584640a^5c^4+16695491060640a^6c^4+158053727328a^7c^4-5904024000a^8c^4}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-105934080a^9c^4-252000a^{10}c^4+2016a^{11}c^4+535936775672037376 0c^5}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-256418348372128563 2ac^5+287511916583118848a^2c^5+2891094455400960a^3c^5}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-988409117368320a^4c^5-114608851795 20a^5c^5+889877079168a^6c^5+17580998400a^7c^5}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-50023680a^8c^5-2378880a^9c^5-8064a^{10}c^5+9506535434197401 60c^6}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-331203488938483712ac^6+23227972384509952a^2c^6+825626374939136a^3c^6}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-55557808244736a^4c^6-1476433967232a^5c^6+19650247680a^6c^6+658533120a^7c^6}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+30}{2})}+$$

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$$+\frac{+2472960a^8c^6-13440a^9c^6+120874161588404224c^7-30349688822075392ac^7}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{1175475851951104a^2c^7+759577435330 56a^3c^7-1613337631488a^4c^7-72595015680a^5c^7}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-84395520a^6c^7+9968640a^7c^7+42240a^8c^7+11246832294297600c^8-1991999707934720ac^8}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{30885185565696a^2c^8+3902365108224a^3c^8-9744641280a^4c^8-1772770560a^5c^8}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-10264320a^6c^8+42240a^7c^8+772615155220480c^9-93305628573696ac^9}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-108440735744a^2c^9+119913953280a^3c^9+756828160a^4c^9-19937280a^5c^9-112640a^6c^9}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{39124756070400c^{10}-3061463646208ac^{10}-37866008576a^2c^{10}+2126170112a^3c^{10}}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{20410368a^4c^{10}-67584a^5c^{10}+1441659355136c^{11}-67616333824ac^{11}-1271508992a^2c^{11}}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{18849792a^3c^{11}+159744a^4c^{11}+37571788800c^{12}-930242560ac^{12}-19222528a^2c^{12}}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{53248a^3c^{12}+656015360c^{13}-6766592ac^{13}-114688a^2c^{13}+6881280c^{14}}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+30}{2})}+$$

$$+\frac{-16384ac^{14}+32768c^{15}}{\Gamma(\frac{c+a+30}{2})\,\Gamma(\frac{c-a+1}{2})}+$$

$$+\frac{202843204931727360000-322513180299113932800a+137081176976279704320a^2}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-16958441227372769664a^3-6031832149972644 96a^4+146759006054821328a^5}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{4133871935431448a^6-410385710727888a^7-19216954885103a^8+76930919809a^9}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{18767187789a^{10}+359027053a^{11}+584899a^{12}-48237a^{13}-457a^{14}-a^{15}}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{473815891454924390400c-514267672818039828480ac+152733916790608472064a^2c}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-10431151935471012096a^3c-1042530101607866112a^4c+70205646306447488a^5c}{\Gamma(\frac{c+a+31}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{4867596553619456a^6c - 72498049647248a^7c - 9301873967216a^8c - 143461234896a^9c}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{2457464016a^{10}c + 89007184a^{11}c + 737968a^{12}c - 112a^{13}c - 16a^{14}c}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{4299526354688689766640c^2 - 3380049613812787568 64ac^2 + 700590266974921789 44a^2c^2}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-175927604131975603 2a^3c^2 - 479277775728825600a^4c^2 + 7123495031408896a^5c^2}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{1469730527666880a^6c^2 + 17407942640720a^7c^2 - 1222948011264a^8c^2 - 35273078064a^9c^2}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-145104960a^{10}c^2 + 4630640a^{11}c^2 + 51072a^{12}c^2 + 112a^{13}c^2 + 2136519460234369105 92c^3}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1243571655519657615 36ac^3 + 175326878513379246 08a^2c^3 + 190507604230245120a^3c^3}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-10151979485340198 4a^4c^3 - 1414918873379520a^5c^3 + 184536822383840a^6c^3}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{5294045822016a^7c^3 - 34417217184a^8c^3 - 2670212160a^9c^3 - 26567520a^{10}c^3 + 4032a^{11}c^3}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{672a^{12}c^3 + 6705071236267622400 0c^4 - 2902650998507001856 0ac^4}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{2631784500033124352a^2c^4 + 117050823745715456a^3c^4 - 1140865687031232 0a^4c^4}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-412101963368640a^5c^4 + 9483271361760a^6c^4 + 501887904672a^7c^4 + 3510897600a^8c^4}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-69457920a^9c^4 - 917280a^{10}c^4 - 2016a^{11}c^4 + 1433937462880878592 0c^5}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-4586744602902335488ac^5 + 238537086089704448a^2c^5 + 20197806522800640a^3c^5}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-657184891672320a^4c^5 - 42870132090240a^5c^5 - 39115880832a^6c^5 + 21361670400a^7c^5}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{265681920a^8c^5 - 40320a^9c^5 - 8064a^10c^5 + 218550176009797632 0c^6}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-508452565484474368ac^6 + 11307303767053312a^2c^6 + 1925277943980544a^3c^6}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-9760743879936a^4c^6-2321995498368a^5c^6-25841195520a^6c^6+370433280a^7c^6}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{6101760a^8c^6+13440a^9c^6+243864590252867584c^7-40222844632813568ac^7}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-22289722459136a^2c^7+113092549008384a^3c^7+1111218400512a^4c^7-67136593920a^5c^7}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1113361920a^6c^7+168960a^7c^7+42240a^8c^7+20218319710617600c^8-2276064016568320ac^8}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-40567245886464a^2c^8+4142976178176a^3c^8+77685822720a^4c^8-873143040a^5c^8}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-19134720a^6c^8-42240a^7c^8+1251902765301760c^9-90907976146944ac^9}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-2799334866944a^2c^9+89515345920a^3c^9+2226780160a^4c^9-337920a^5c^9-112640a^6c^9}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{57657353994240c^{10}-2473121226752ac^{10}-100757651456a^2c^{10}+931330048a^3c^{10}}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{30547968a^4c^{10}+67584a^5c^{10}+1946239533056c^{11}-42316292096ac^{11}-2105372672a^2c^{11}}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{319488a^3c^{11}+159744a^4c^{11}+46741094400c^{12}-366878720ac^{12}-24014848a^2c^{12}}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-53248a^3c^{12}+755793920c^{13}-114688ac^{13}-114688a^2c^{13}+7372800c^{14}}{\Gamma(\frac{c+a+31}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\left.\frac{16384ac^{14}+32768c^{15}}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}\right]$$

On simplification , we get

$${}_2F_1\left[\begin{matrix} a\ \ ,\ \ -a-31 & ; & \frac{1}{2} \\ c & ; & \end{matrix}\right]=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+31}}\times\left[\frac{-32(-1267770030823296000)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+\right.$$

$$+\frac{-32(20595415066908998400a-9262043913632837760a^2+1353632653931095440a^3)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(607680617478480a^4-99436599786493 20a^5-60453402240a^6+30640832998545a^7)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(619328526465a^8-25924502835a^9-1063654515a^{10}-7639485a^{11}+177555a^{12})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

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$$+\frac{-32(3255a^{13}+15a^{14}-2961349321593277440 0c+33134042027396309760ac)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-10603056513800294784a^{2}c+9602015852967276 96a^{3}c+433153475822560 32a^{4}c)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-5992698426224288a^{5}c-188476928122316a^{6}c+10666764589703a^{7}c)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(481387701731a^{8}c+46976811a^{9}c-259248801a^{10}c-4024699a^{11}c-7063a^{12}c+217a^{13}c)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(a^{14}c-26872039716804311040c^{2}+21983015579619352320ac^{2})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-5031540460374658560a^{2}c^{2}+241130543916718800a^{3}c^{2}+26151340987189080a^{4}c^{2})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-1146075534278100a^{5}c^{2}-79796202137550a^{6}c^{2}+575648038980a^{7}c^{2})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(90737193330a^{8}c^{2}+1232147700a^{9}c^{2}-12029850a^{10}c^{2}-351540a^{11}c^{2}-1890a^{12}c^{2})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-13353246626464806912c^{3}+8172439523536586496ac^{3}-1318114175517939968a^{2}c^{3})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(18114113175242640a^{3}c^{3}+6479375107986424a^{4}c^{3}-455959576261 80a^{5}c^{3})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-12824288708390a^{6}c^{3}-151650421356a^{7}c^{3}+6139102074a^{8}c^{3}+144561060a^{9}c^{3})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(488670a^{10}c^{3}-7812a^{11}c^{3}-42a^{12}c^{3}-4190669522667264000c^{4})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(1930407148471353600ac^{4}-211801780361313600a^{2}c^{4}-3250016105076000a^{3}c^{4})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(851154873246000a^{4}c^{4}+12547796499000a^{5}c^{4}-966225468600a^{6}c^{4}-23689890000a^{7}c^{4})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(81396000a^{8}c^{4}+5859000a^{9}c^{4}+37800a^{1}0c^{4}-896210914300549120c^{5})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(309362639107541248ac^{5}-216189872402084 48a^{2}c^{5}-933498687245280a^{3}c^{5})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(62914564722480a^{4}c^{5}+2025497521320a^{5}c^{5}-30196918248a^{6}c^{5}-1347049200a^{7}c^{5})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-7230720a^8c^5+78120a^9c^5+504a^10c^5-136593860006123520c^6)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(348872674411143040ac^6-136283518849632 0a^2c^6-106325086633920a^3c^6)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(2400525998640a^4c^6+136185033600a^5c^6+221205600a^6c^6-34372800a^7c^6-277200a^8c^6)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-15241536890804224c^7+2820561599829248ac^7-43480775233984a^2c^7)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-7026484960704a^3c^7+17197768368a^4c^7+4832488320a^5c^7+40122720a^6c^7)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-327360a^7c^7-2640a^8c^7-1263644981913600c^8+164287692771840ac^8)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(393170595840a^2c^8-288230659200a^3c^8-2365545600a^4c^8+88387200a^5c^8+950400a^6c^8)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-78243922831360c^9+6836145744384ac^9+105021238784a^2c^9-7241857920a^3c^9)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-99890560a^4c^9+654720a^5c^9+7040a^6c^9-3603584624640c^{10}+198178506240ac^{10})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(4809742080a^2c^{10}-102136320a^3c^{10}-1647360a^4c^{10}-121639970816c^{11})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(3800915456ac^{11}+113015552a^2c^{11}-619008a^3c^{11}-9984a^4c^{11}-2921318400c^{12})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-32(-32(43330560ac^{12}+1397760a^2c^{12}-47237120c^{13}+222208ac^{13})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{7168a^2c^{13}-460800c^{14}-2048c^{15})}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+31}{2})}+$$

$$+\frac{-140410465978669286 4000a+1284759281770644960000a^2-337819624060585057920a^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{18689473291318197264a^4+2598574261842425664a^5-139074162017438648a^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-1258772148512519 2a^7+121255987903953a^8+26000045157472a^9+505712970236a^{10}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-7770554736a^{11}-386509018a^{12}-4298336a^{13}-588a^{14}+248a^{15}+a^{16}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{14041046610943672320000c - 3614329675417131417600ac + 1966512596168155299840a^2c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-325604775785387212800a^3c + 1975242488770713600a^4c + 2446330738917135360a^5c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-6678286574092800a^6c - 7792114119087360a^7c - 150815024474880a^8c + 6741116040960a^9c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{271264492800a^{10}c + 1925710080a^{11}c - 45615360a^{12}c - 833280a^{13}c - 3840a^{14}c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{2329570397985649459200c^2 - 3283817028961948139520ac^2 + 1194870967556528406528a^2c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-120317614621028904960a^3c^2 - 4733771239917818880a^4c^2 + 760630552772864512a^5c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{22492609658218240a^6c^2 - 1384355235709312a^7c^2 - 60829664672896a^8c^2 + 12467452032a^9c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{33246245760a^{10}c^2 + 514161536a^{11}c^2 + 898688a^{12}c^2 - 27776a^{13}c^2 - 128a^{14}c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{1655124063020195512320c^3 - 1566694133878481879040ac^3 + 394025913314442362880a^2c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-21068644278648238080a^3c^3 - 2087759536719206400a^4c^3 + 99858900085632000a^5c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{6644244654451200a^6c^3 - 53144508533760a^7c^3 - 7728849999360a^8c^3 - 104143334400a^9c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{1032998400a^10c^3 + 29998080a^11c^3 + 161280a^12c^3 + 678738473873018191872c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-459933281420468649984ac^4 + 79793531879829762048a^2c^4 - 1351162026922451968a^3c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-401277002343037440a^4c^4 + 3345201060691200a^5c^4 + 814270582363520a^6c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{9418598671104a^7c^4 - 394442326656a^8c^4 - 9235242240a^9c^4 - 31167360a^{10}c^4 + 499968a^{11}c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{2688a^{12}c^4 + 181651168683255398400c^5 - 90177345987803873280ac^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{10494783795613040640a^2c^5 + 139662056679137280a^3c^5 - 42960664547573760a^4c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

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$$+\frac{-607731642685440a^5c^5+49525783050240a^6c^5+1204122931200a^7c^5-4238438400a^8c^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-299980800a^9c^5-1935360a^10c^5+3387897405931257856c^6-12374104894953684992ac^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{909163672031133696a^2c^6+37748850841710592a^3c^6-2678968046424064a^4c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-84979566959616a^5c^6+1300367502336a^6c^6+57376327680a^7c^6+307722240a^8c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-3333120a^9c^6-21504a^{10}c^6+4576878391071866880c^7-12205084537766103040ac^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{49957863590215680a^2c^7+3790443636080640a^3c^7-88592200519680a^4c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-4950311731200a^5c^7-7765401600a^6c^7+1257062400a^7c^7+10137600a^8c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{458279130417332224c^8-87651604127752192ac^8+1431156603582464a^2c^8}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{222069695772672a^3c^8-588637361664a^4c^8-154388213760a^5c^8-1281223680a^6c^8}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{10475520a^7c^8+84480a^8c^8+34425286950912000c^9-4588800697958400ac^9}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-9140546764800a^2c^9+8154982809600a^3c^9+66583756800a^4c^9-2514124800a^5c^9}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-27033600a^6c^9+1946357837332480c^{10}-173223332806656ac^{10}-2635562975232a^2c^{10}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{185101041664a^3c^{10}+2552512512a^4c^{10}-16760832a^5c^{10}-180224a^6c^{10}+82374536724480c^{11}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-4589969080320ac^{11}-111220162560a^2c^{11}+2376990720a^3c^{11}+38338560a^4c^{11}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{2568813019136c^{12}-80962945024ac^{12}-2407022592a^2c^{12}+13205504a^3c^{12}+212992a^4c^{12}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{57252249600c^{13}-853278720ac^{13}-27525120a^2c^{13}+862453760c^{14}-4063232ac^{14}}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+32}{2})}+$$

$$+\frac{-131072a^2c^{14}+7864320c^{15}+32768c^{16}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+32}{2})}\Bigg]$$

Thus , we prove the result (8).

Similarly, we can prove the result(9).

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Micro-Environmental Change in the Coastal Area of Bangladesh : A Case Study in the Southern Coast at Shitakunda, Chittagong, Bangladesh

By Md. Nymul Islam
Univesity of Chittagong , Bangladesh

Abstracts - Although Foraminiferid are very small members of marine and brackish water fauna they are often present in large numbers and constitute and important element of the meiofauna*. They possess hard parts in the form of tests (or Shells) which on death are preserved in the sediment and are therefore interest to geologists or researchers (Murray,J,W.1979). So, They could be remaining in the sedimentary layer as a proxy data. As because Present is the key to the Past. This research is an attempt to find out the Micro-environmental Change in the coastal Area of Bangladesh accompanying with a new technique, that is population analyze the marine micro faunas (Foraminiferid) in the bottom sediments. Laboratory analysis in the Geography and Environmental Studies, reveal that in the local Foraminiferid assemble zone of Chittagong coast (Shitakunda), they have been nonappearance. This is clear indications that present environmental condition for the sustainable micro fauna sp is not suitable. On the other hand It is indicating the hazardous coastal pollution which influences the sustainability of faunas themendously in this region.

Keywords : *Meiofauna, Foraminiferid.*

GJSFR Classification : *FOR Code : 059999*

MICRO-ENVIRONMENTAL CHANGE IN THE COASTAL AREA OF BANGLADESH A CASE STUDY IN THE SOUTHERN COAST AT SHITAKUNDA, CHITTAGONG, BANGLADESH

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This is clear indications that present environmental condition for the sustainable micro fauna *sp is* not suitable. On the other hand It is indicating the hazardous coastal pollution which influences the sustainability of faunas themendously in this region.

Keywords : Meiofauna, Foraminiferid.

The Foraminiferid are classified as protozoa's because they consist of a single cell which is made up of cytoplasm with one or more nuclei. Foraminiferid are aquatic, mainly marine group of Foraminiferid are also classed as Protozoa. The classification is as follows: Phylum PROTOZOA, Subphylum SARCODINA, Class RHIZOPODEA, Order FORAMINIFERIDA, Suborder TEXTULARIINA, MILIOLINA & ROTALINA (Loeblich and Tappan, 1964). With respect to the marine habitats that Foraminiferid occupy, they can be divided into following species:

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Table 1: Marine habitats of Foraminiferid.

Criteria of Marine environment	Salinity and Environment
Hyoisaline	Salinity < 32 0/00 (Brackish water)
Marginal Marine	Salinity ranges between brackish and marine and include marsh, lagoon and estuarine environments
Normal Marine	Salinity 32-37 0/00 (Open marine)
Hypersaline	Salinity >37 0/00 (Restricted salt water environment).

On the other hand Foraminifera Sp used to identify the water Salinity and Environmental as an Assessment tools. In my study in the local Foraminiferid assemble zone of Chittagong coast (near Sitakunda Kumira), Foraminiferid have not been appearance when bottom sediment analysis in the Laboratory [Environment Lab of Geography and Environmental Studies, University of Chittagong, Bangladesh, It also noticeable that only 30 g. marine sediments sample are analyzing through a microscope at 20 X magnification.]. They might be eliminated or present environmental condition. It may indicate that the water is not suitable for their existence. Or it might be indicating the hazardous coastal pollution that influences the micro-fauna tremendously. It very alarming side that if the meiofunas eliminate from the coastal environment it will be great negative impact on coastal ecological system. As a result we will gradually lose the fish communities as well as biodiversity at the climax.

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Normal Mode Analysis of Micropolar Elastic Medium with Void under Inviscid Fluid

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Abstracts - The present investigation is concerned with the two dimensional problem of micropolar elastic medium with void. Normal mode analysis is used to obtain the expression of components of stresses, displacement components and acoustic pressure of the inviscid fluid. Numerically simulated results are obtained and presented graphically to depict the impact of void for a particular model.

Keywords : *Normal mode analysis, micropolar, void, inviscid fluid, acoustic pressure.*

GJSFR-B Classification : *FOR Code : 091504*

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Aseem Miglani[α], Sachin Kaushal[Ω]

Abstract - The present investigation is concerned with the two dimensional problem of micropolar elastic medium with void. Normal mode analysis is used to obtain the expression of components of stresses, displacement components and acoustic pressure of the inviscid fluid. Numerically simulated results are obtained and presented graphically to depict the impact of void for a particular model.

Keywords : Normal mode analysis, micropolar, void, inviscid fluid, acoustic pressure.

I. Introduction

The micropolar theory of elasticity constructed by Eringen [1] was intended to be applied to such materials and for such problems where the ordinary classical theory of elasticity fails because of microstructure of the material. Also micropolar theory is more appropriate for geological materials like rocks, soil since this theory takes into account the intrinsic rotation and predicts the behavior of material with inner structure. For engineering problem, it can model composites with rigid chopped fibers, elastic solids with rigid inclusion and other industrial materials such as liquid crystal.

The mechanical behavior of solids with voids; solid containing microscopic components cannot be described by classical theory of elasticity. Hence, the theory for granular materials with interstitial voids was presented by Goodman and Cowin [2]. A theory for the behavior of porous solids, in which the skeletal or matrix material is elastic and the interstices are voids of the material, was established by Nunziato and Cowin [3], Cowin and Nunziato [4]. Various author's [5, 6] discussed different problems in micropolar elastic medium with voids. Othman [7] discussed effect of rotation on plane waves in generalized thermoelasticity by using normal mode analysis. Recently, Ezzat and co-author's [8] discussed two dimensional coupled problems in electro-magneto thermoelasticity by using normal mode analysis. The aim of the present problem is to find the components of displacement, stress components, acoustic pressure and volume fraction field in a homogenous isotropic micropolar elastic solid with voids under inviscid liquid by using normal mode analysis.

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II. Basic Equations

Following Eringen [1] and Quintanilla [9], the equation of motion and the constitutive relation in a homogenous isotropic micropolar elastic solid with voids in the absence of body forces, body couples are given as:

$$(\lambda+2\mu+K)\nabla\left(\nabla.\vec{u}\right)-(\mu+K)\nabla\times\nabla\times\vec{u}+K\left(\nabla\times\vec{\phi}\right)+\xi\nabla\psi=\rho\frac{\partial^2\vec{u}}{\partial t^2} \quad (1)$$

$$(\alpha+\beta+\gamma)\nabla\left(\nabla.\vec{\phi}\right)-\gamma\nabla\times\left(\nabla\times\vec{\phi}\right)+K\left(\nabla\times\vec{u}\right)-2K\vec{\phi}+\varsigma\nabla\psi=\rho j\frac{\partial^2\vec{\phi}}{\partial t^2} \quad (2)$$

$$d\nabla^2\psi-\xi\nabla.\vec{u}-\varsigma\nabla.\vec{\phi}-\omega_1^*\frac{\partial\vec{\psi}}{\partial t}-a\vec{\psi}=\rho\chi\frac{\partial^2\psi}{\partial t^2} \quad (3)$$

$$t_{ij}=\lambda\delta_{ij}e_{rr}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\in_{ijk}\phi_k\right)+\xi\psi\delta_{ij} \quad (4)$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+\varsigma\psi\delta_{ij} \quad (5)$$

where λ and μ - Lame's constants, t_{ij} -components of the stress tensor, m_{ij} -components of couple stress tensor, ρ -density, u_i -displacement components, ψ - change in volume fraction, δ_{ij} - Kronecker delta, $\in_{ijk}$ - alternative tensor, ϕ_i microrotation vector, t-time, j - microrotation inertia, K,α,β,γ -material constant, $d,\xi,\varsigma,a,\omega_1^*$ and χ -material constants due to presence of void.

Following Achenbach [10], the field equations can be expressed in terms of velocity potential for inviscid fluid as

$$p = -\bar{\rho}\dot{\varphi}^f$$

$$\left(\nabla^2 - \frac{1}{\alpha^{f^2}}\frac{\partial^2}{\partial t^2}\right)\varphi^f = 0 \quad (6)$$

$$\dot{\vec{u}} = \nabla\varphi^f \quad (7)$$

where $\alpha^{f^2} = \bar{\lambda}/\bar{\rho}$, $\bar{\lambda}$ is the bulk modulus, $\bar{\rho}$ is the density of the liquid, $\dot{\vec{u}}$ is the velocity vector and p is the acoustic pressure in the inviscid fluid,.

For two-dimensional problem, we take

$$\vec{u} = (u_1, 0, u_3), \quad \vec{\phi} = (0, \phi_2, 0) \quad (8)$$

Also, we introduce the non-dimensional quantities defined by the expressions

$$x_i' = \frac{\omega^*}{c_1}x_i, \quad u_i' = \frac{\omega^*}{c_1}u_i, \quad \{\phi_2', \psi'\} = \left(\frac{\rho c_1^2}{K}\right)\{\phi_2, \psi\},$$

$$t' = \omega^* t, \quad \phi^{f'} = \frac{\omega^*}{c_1^2}\phi^f, \quad t_{3i}' = \frac{1}{\mu}t_{3i}, \quad m_{32}' = \frac{\omega^*}{\mu c_1}m_{32}$$

$$p' = \bar{\lambda}p, \dot{\vec{u}}' = \frac{\omega^*}{c_1}\dot{\vec{u}}, \quad \omega^{*2} = \frac{K}{\rho j}, \quad c_1 = \frac{\lambda + 2\mu + K}{\rho},$$

$$i = 1,3 \quad (9)$$

The displacement components u_1 and u_3 are related to the potential functions as,

$$u_1 = \frac{\partial\Phi}{\partial x} - \frac{\partial\Psi}{\partial z}, \quad u_3 = \frac{\partial\Phi}{\partial z} + \frac{\partial\Psi}{\partial x}, \quad (10)$$

Using equations (8), (9) and (10) on equations (1) – (3), (6) (suppressing primes), we get

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\Phi + a_4\psi = 0, \quad (11)$$

$$\left(a_2\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\Psi + a_3\phi_2 = 0 \quad (12)$$

$$\left(a_5\nabla^2 - 2a_7 - \frac{\partial^2}{\partial t^2}\right)\phi_2 - a_6\nabla^2\Psi = 0, \quad (13)$$

$$\left(a_8\nabla^2 - a_{10}\frac{\partial}{dt} - a_{11} - \frac{\partial^2}{dt^2}\right)\psi - a_9\nabla^2\Phi = 0 \quad (14)$$

$$(\nabla^2 - a_{12}\frac{\partial^2}{\partial t^2})\varphi^f = 0 \quad (15)$$

where

$$a_1 = \frac{\lambda+\mu}{\rho c_1^2}, \quad a_2 = \frac{K+\mu}{\rho c_1^2}, \quad a_3 = \frac{K^2}{\rho^2 c_1^4}, \quad a_4 = \frac{\xi K}{\rho^2 c_1^4},$$
$$a_5 = \frac{\gamma}{\rho j c_1^2}, \quad a_6 = \frac{c_1^2}{j\omega^{*2}} \quad a_7 = \frac{K}{\rho j\omega^{*2}}, \quad a_8 = \frac{d}{\chi\rho c_1^2},$$
$$a_9 = \frac{\xi c_1^2}{\chi k\omega^{*2}}, \quad a_{10} = \frac{\omega_1^*}{\chi\rho\omega^*}, \quad a_{11} = \frac{a}{\chi\rho\omega^{*2}}, \quad a_{12} = \frac{c_1^2}{\alpha^{f^2}}$$

III. Normal Mode Analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as following:

$$\left[\Phi, \psi, \phi_2, \Psi, \phi^f\right] = \left[\bar{\Phi}(z), \bar{\psi}(z), \bar{\phi}_2(z), \bar{\Psi}(z), \bar{\phi}^f(z)\right]e^{i(kx-\omega t)} \quad (16)$$

where ω is the complex time constant and k is the wave number in the x- direction. Using equation (16), equations (11)-(15) takes the form

$$\left(D^4 + AD^2 + B\right)\left(\bar{\Phi}, \bar{\psi}\right) = 0 \quad (17)$$

$$\left(D^4 + LD^2 + M\right)\left(\bar{\phi}_2, \bar{\Psi}\right) = 0 \quad (18)$$

$$\left(D^2 + N\right)\bar{\phi}^f = 0 \quad (19)$$

where

$$D = \frac{d}{dz}, \quad N = k^2 + a_{12}\omega^2$$

$$A = \frac{a_5(\omega^2 - 2a_2k^2 - 2a_3a_6k^2) + (\omega^2 - 2a_7)(a_2 + a_3a_6)}{a_5(a_2 + a_3a_6)},$$

$$B=\frac{(\omega^2-2a_7)(\omega^2-a_2k^2-a_3a_6k^2)-a_5k^2(\omega^2-a_2k^2-a_3a_6k^2)}{a_5(a_2+a_3a_6)}$$

$$L=\frac{a_8(\omega^2-k^2)-(a_8k^2+\omega^2+\iota\omega a_{10}+a_{11}+a_4a_9)}{a_8}$$

$$M=\frac{(\omega^2-k^2)(a_8k^2+\omega^2+\iota\omega a_{10}+a_{11}-a_4a_9k^2)}{a_8}$$

The solution of equations (17) and (18) satisfying radiation conditions that $\overline{\Phi},\overline{\psi},\ \overline{\Psi},\ \overline{\phi}_2,\phi^f\ \to 0$ as $x_3\to\infty$ are:

$$\{\overline{\Phi},\overline{\psi}\}=\sum(1,d_i)A_ie^{-m_ix_3}$$

$$\{\overline{\phi}_2,\overline{\Psi}\}=\sum(1,d_j)B_ie^{-m_ix_3},$$

$$\phi^f=Ee^{-m_5x_3}\quad i=1,2 \text{ and } j=1,2 \tag{20}$$

IV. Boundary Conditions

The boundary conditions in this case are:

$$t_{33}-p=-F\ e^{\iota(kx-\omega t)},$$

$$t_{31}=0,$$

$$m_{32}=0,$$

$$\frac{d\psi}{dz}=0,$$

$$\dot{u}_3=u_3^f \text{ at } x_3=0 \tag{21}$$

where F is well defined function.

Making use of the equations (4)-(5), (7) and (8) and applying normal mode analysis defined by (16) and substitute the values of $\overline{\Phi},\overline{\psi},\overline{\phi}_2,\overline{\Psi},\phi^f$ from equation (20) in the resulting equations, we obtain the expression for components of displacement, stresses, volume fraction and acoustic pressure as

$$u_3=F_1\left[-\left(m_1\Delta_1e^{-m_1x_3}-m_2\Delta_2e^{-m_2x_3}\right)+ik\left(\Delta_3e^{-m_3x_3}+\Delta_4e^{-m_4x_3}\right)\right], \tag{22}$$

$$u_3^f=F_1m_5\Delta_5e^{-m_5x_3}, \tag{23}$$

$$t_{33}=F_1[\Delta_1s_1e^{-m_1x_3}-\Delta_2s_2e^{-m_2x_3}+\Delta_3s_3e^{-m_3x_3}+\Delta_4s_4e^{-m_4x_3}] \tag{24}$$

$$t_{31}=F_1[\Delta_1s_6e^{-m_1x_3}-\Delta_2s_7e^{-m_2x_3},+\Delta_3s_8e^{-m_3x_3}+\Delta_4s_9e^{-m_4x_3} \tag{25}$$

$$m_{32}=F_1\left[m_3\Delta_3s_3e^{-m_3x_3}+m_4\Delta_4s_4e^{-m_4x_3}\right], \tag{26}$$

$$p=F_1s_5e^{-m_5x_3}, \tag{27}$$

$$\psi=F_1\left[\Delta_1d_1e^{-m_1x_3}-\Delta_2d_2e^{-m_2x_3}\right], \tag{28}$$

where

$$\Delta=\begin{vmatrix} s_1 & s_2 & s_3 & s_4 & s_5\\ s_6 & s_7 & s_8 & s_9 & 0\\ 0 & 0 & m_3 & m_4 & 0\\ m_1d_1 & m_2d_2 & 0 & 0 & 0\\ \iota\omega m_1 & \iota\omega m_2 & \omega k & \omega k & m_5\end{vmatrix}$$

$\Delta_i\ i=1,5$ are obtained by replacing i^{th} column of Δ with $\begin{vmatrix}-F & 0 & 0 & 0 & 0\end{vmatrix}$

where

$$s_i=k^2b_1+b_2m_i+b_3d_i,\ \ s_j=\iota km_j(b_1-b_2),$$

$$s_5=-b_6\iota\omega,\quad \{s_6,s_7\}=(b_4-1)\iota km_i,$$

$$\{s_8,s_9\}=-(m_j^2b_4+b_5d_j+k^2),\ b_1=\frac{\lambda}{\mu},\ \ b_3=\frac{\xi K}{\mu\rho c_1^2}$$

$$b_2=\frac{\lambda+2\mu+K}{\mu},b_4=\frac{\mu+K}{\mu},b_5=\frac{K^2}{\mu\rho c_1^2},b_6=\frac{\overline{\rho}c_1^2}{\lambda^f},$$

$$F_1=\frac{F\,e^{\iota(kx-\omega t)}}{\Delta}\qquad i=1,2\ \ \&\ j=3,4$$

V. Particular Case

1. **Micropolar Elastic Solid:** Neglecting void effects in equations (22)-(28), we obtain the corresponding expression for components of displacement, stresses, and acoustic pressure in micropolar elastic media under inviscid fluid.

2. **In absence of Inviscid liquid:** if $\overline{\rho}\to 0$, then we obtain corresponding expression for micropolar elasticity with void

VI. Numerical Discussion

In order to study, the problem considered in greater details numerically simulated results are computed for a particular model and are presented graphically. For this purpose, we have taken the case of magnesium crystal like material. Following Eringen [11], the physical constants are:

$$\lambda = 9.4\times10^{11}\ dyn\,cm^{-2}, \qquad \mu = 4\times10^{11}\ dyn\ cm^{-2},$$

$$K = 1\times10^{11}\ dyn\ cm^{-2}, \qquad \rho = 1.7\ gm\ cm^{-3},$$

$$\gamma = 0.779\times10^{-4}\ dyn, \qquad j = 0.2\times10^{-15}\ cm^{2},$$

$$\bar{\lambda} = 2.1904\times10^{10}\ dyn\ cm^{-2},\ \bar{\rho} = 1.0\times10^{3}\ gm\,cm^{-3}$$

and the void parameters are

$$d = 3.688\times10^{-4}\ dyn,\ a = 1.475\times10^{11}\ dyn\ cm^{-2},$$

$$\xi = 1.13849\times10^{11}\ dyn\,cm^{-2},\ \omega_1^* = 0.0787\ dyn\ cm^{-2}$$

The computations were carried out for small values of time $t = 0.1$. The numerical results for the stress components (t_{33}, t_{31}, m_{32}), volume fraction field ψ normal velocity u_3^f and acoustic pressure p of inviscid fluid are shown graphically in figures (1)-(6) for different ω i.e. for $\omega = 0.1$ and $\omega = 0.5$ with distance $0 \le x \le 10$. The solid line and dashed line corresponds to Micropolar elastic with void (MEV) for $\omega = 0.1$ and $\omega = 0.5$ respectively, whereas solid line with center symbol 'triangle' and dashed line with center symbol 'circle' corresponds to Micropolar elasticity (ME) for $\omega = 0.1$ and $\omega = 0.5$ respectively.

Figure 1 depicts the variations of t_{33} with distance x. It is noticed that the values of t_{33} for ME at $\omega = 0.1$ and $\omega = 0.5$ are similar in nature in entire range, whereas values of t_{33} for MEV at $\omega = 0.1$ and $\omega = 0.5$ are opposite in nature, which is accounted as void effect.

It is noticed from figure (2), which is plot for t_{31} with distance x that value of t_{31} at $\omega = 0.1$ for MEV increases in range $3 \le x \le 6$ and $9 \le x \le 10$, decreases in remaining range while for ME values of t_{31} at $\omega = 0.1$ decreases in range $0 \le x \le 2$, $5 \le x \le 8$ and vice-versa trends are noticed in remaining range. Whereas values of t_{31} at $\omega = 0.5$ for MEV and ME show similar oscillatory behavior in entire range, magnitude of values for ME are greater as compared to MEV, which reveals the impact of void effect.

The variations of m_{32} with x are noticed in figure 3. It is noticed that values of m_{32} for MEV and ME at $\omega = 0.1$ shows similar behavior in entire range i.e. their values increases and decreases alternately with x, while values of MEV for $\omega = 0.5$ oscillates with greater magnitude as compared to those noticed for MEV and ME for different ω, which clearly shows the impact of complex time constant.

Figure (4) shows the variations of volume fraction ψ with x. It is noticed that the trends of ψ for MEV at $\omega = 0.1$ and $\omega = 0.5$ are opposite in nature in entire range.

The variations of u_3^f are shown in figure (5). It is noticed that values of u_3^f at different ω for MEV and ME increases in range $2 \le x \le 5, 8 \le x \le 10$ and vice-versa trends are noticed in remaining range with significant difference in their magnitude.

Figure (6) depicts the variations of acoustic pressure p with x at $\omega = 0.1$ and $\omega = 0.5$. It is noticed that trends for MEV are opposite in nature as compared to ME for both ω, which is accounted as absence of void effect.

VII. Conclusion

Normal mode technique is employed to solve the problem of micropolar elasticity solid given by Eringen [1] and Quintanilla [9]. From the above discussion, we noticed that presence of void effect shows significant impact on the components of stresses, normal velocity and volume fraction field. Also different values of parameter ω show relevant impact on different calculated parameters in micropolar elasticity and in inviscid fluid.

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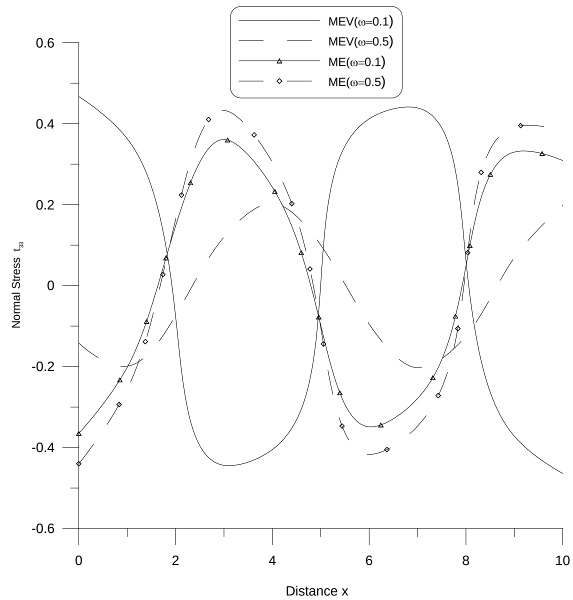

Figure 1 Shows the variations of t_{33} with x

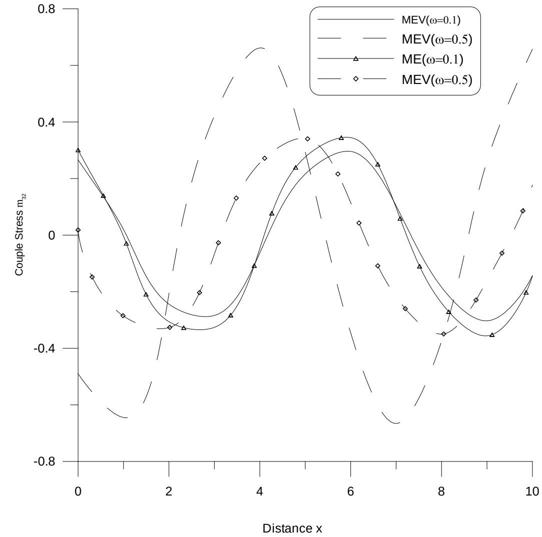

Figure 3 shows the variations of m_{32} with x

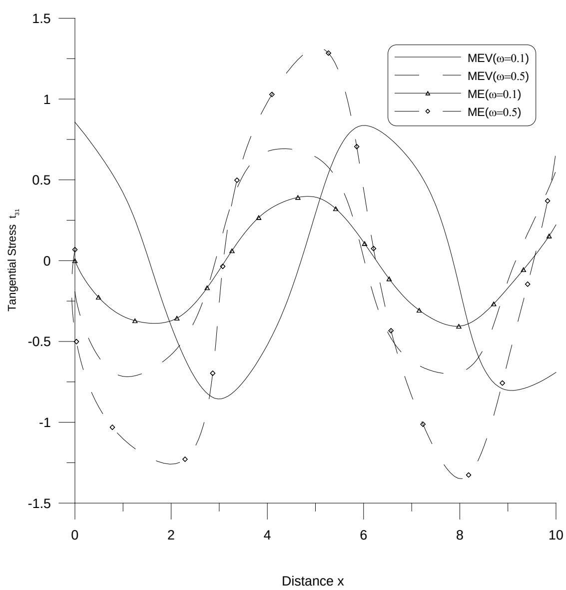

Figure 2 shows the variations of t_{31} with x

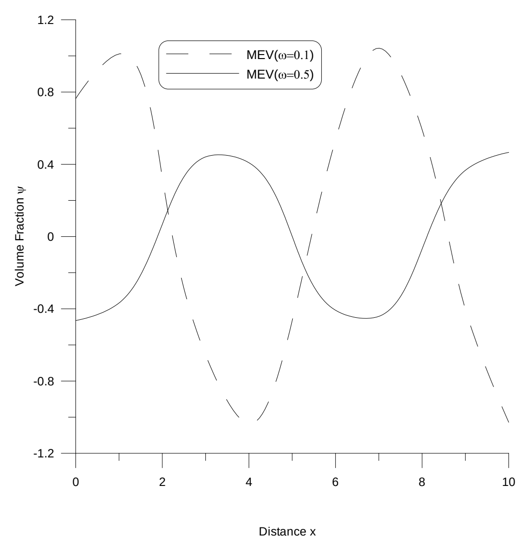

Figure 4 shows the variations of Ψ with x

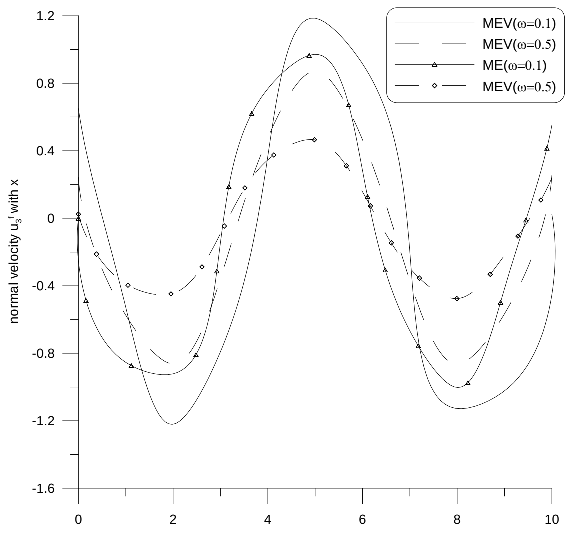

figure 5 shows the variations of u_3^f with x

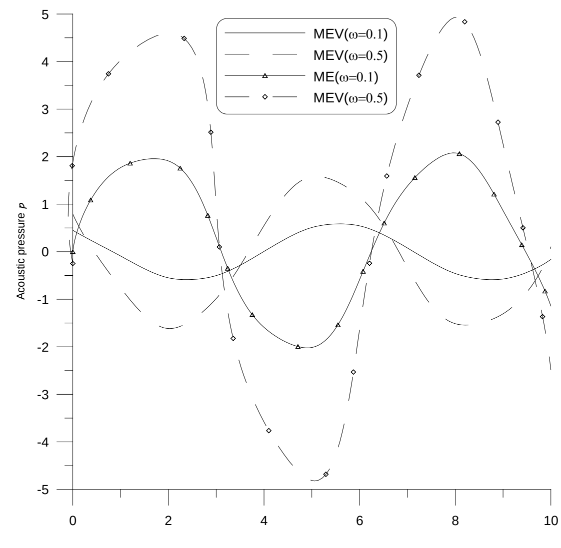

Figure 6 shows the variations of p with x

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An Unitary Unified Quantum Field Theory

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Abstracts - The paper proposes a model of an unitary unified quantum field theory (UUQFT) where the particle is represented as a wave packet. The frequency dispersion equation is chosen so that the packet periodically appears and disappears without changing its form. The envelope of the process is identified with a conventional wave function. Equation of such a field is nonlinear and relativistically invariant. With proper adjustments, they are reduced to Dirac, Schrödinger and Hamilton-Jacobi equations. A number of new experimental effects are predicted both for high and low energies.

Keywords : Wave packet, Dispersion, Unitary quantum theory, Unified theory, Corpuscular-wave dualism, Elementary particle, Unified field, Vacuum fluctuations.

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It is difficult, if not impossible; to avoid the conclusion that only mathematical description expresses all our knowledge about the various aspects of our reality.

- *The opinion extracted from an old Soviet newspaper*

I. Introduction

Over eighty-five years have passed since the field of quantum mechanics emerged. Each day, the experiments being done with huge particle accelerators reveal new details about the design of microcosmic structures, and supercomputers crunch vast quantities of resulting mathematical data. But we have till now no theoretical approach to the determination of the mass spectrum of elementary particles which number reached more than 750, to say nothing of the fact that we do not yet fully understand the strong interactions. The standard quantum theory avoid the physical descriptions of various phenomena in terms of images and movements. There have been many different approaches taken in developing a quantum field theory, but the divergences typically created provoke abundant nightmares for theoretical physicists. Nevertheless, we'll try to classify and formalize these approaches somewhat below.

Let us begin with the common canonical point of view based on the properties of space-time, particles, and the vacuum, on particle interactions, and on mathematical modeling equations. Every postulate of canonical theory may be reduced to the following seven statements (not all of which are without issues):

The Space-Time is four-dimensional, continuous, homogeneous, and isotropic.

The particles and their interactions are local.

There is only one vacuum and it is non-degenerating.

It is a valid proposition in quantum theory that physical values correspond to Hermitian operators and that the physical state corresponds to vectors in Gilbert space with positively determined metrics.

The requirement of relativistic invariance is imposed (four-dimensional rotation with coordinate translation – Poincaré group).

The equations for non-interacting free particles are linear and do not contain derivatives higher than the second order.

Particles' internal characteristics of symmetry are described with the SU2 and SU3 symmetry groups.

The previous statements provide the basis for the construction of the S-matrix, which describes the transformation of one asymptotic state into another and satisfies the conditions of causality and unity. Nevertheless, this approach, which seems mathematically excellent in outward appearance, still leads to divergences. Recent 'normalized' theories, derived to provide a means of avoiding infinities by one technique or another, sometimes end up seeming more like circus tricks.

We shall not criticize such normalized theories here; however, to quote P. A. M. Dirac*:

"…most physicists are completely satisfied with the existing situation. They consider relativistic quantum field theory and electrodynamics to be quite perfect theories and it is not necessary to be anxious about the situation. I should say that I do not like that at all, because according to such 'perfect' theory we have to neglect, without any reason, infinities that appear in the equations. It is just mathematical nonsense. Usually in mathematics the value can be rejected only in the case it were too small, but not because it is infinitely big and someone would like to get rid of it." (*Direction in Physics, New York, 1978)

One can try to solve this problem by looking at it from the other side and forming a theory in such a way that it must not contain divergences at all. However, that way leads to the necessity to reject one or another thesis of the canonical point of view. In canonical theory,

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the appearance of divergences is caused by integrals connected with some of the particle parameters and considered in the whole of space, from zero to infinity, for particles are considered as points. The infinities appear by integration only in the region near zero, i.e., on an infinitesimal scale.

The elimination of divergences might be achieved within the purview of one or more of the following four different parameters or approaches in quantum theory:

The minimal elementary length is introduced and then the integration is carried out not from zero, and therefore all such integrals become finite;

It is considered that space-time is discontinuous, consisting entirely of separate points, whereby such a space-time model corresponds to a crystalline lattice. To get a discontinuous coordinate and time spectrum, time and coordinate operators are introduced (per quantized space-time theory);

Non-linear equations containing derivatives of high order may be used instead of linear equations having only derivatives of the first and second order. Even more desperate measures are sometimes employed, by introducing coordinate systems with indefinite metrics instead of coordinate systems with definite metrics;

It could be assumed that a particle is not a point, and hence a whole series of non-local theories might be derived.

These four parameters approaches have so far not yielded notable results, so another two techniques were subsequently considered: enlargement of the Poincaré group, and generalization of internal symmetry groups.

Let us first discuss the problems connected with the enlargement of the Poincaré group, assuming in accordance with observations of natural phenomena that symmetries of sufficiently high level are realized. There are two such enlargement methods:

The Poincaré group is enlarged up to the conformal group, which includes scale and special conformal transformation in addition to the usual four-dimensional rotation (Lorentz group) and coordinate translations. However, if enlargement of the Poincaré group up to the conformal group is performed, then generators of the same tensor character should be added to the tensor generators of the Poincaré group's $M^{\mu\vartheta}$ (rotation) and P^{μ} -(shifts). Unfortunately, after such enlargement the group multiplets contain either bosons or fermions only; in essence, these multiplets are not mixed. The worst situation is with the basic equation for particles. One can write such a conformal invariant equation only for particles with mass equal to zero. This situation may be improved with a new definition of mass (i.e., the so-called conformal mass is introduced), but thereafter its physical sense of particles becomes positively vague. To get out of a difficult situation in this case, attempts have been made to reject exact conformal invariance; then the mass appears as a result of conformal asymmetry violation. We have the same situation in the case of the SU3 symmetry group. Success has not been achieved by this method.

Generators of the spinor type may be added to the enlarged Poincaré group. Such widening results in a new type of symmetry called 'super-symmetry'. For that purpose, so-called super-space is introduced: an eight-dimensional space where the points are denoted as the common coordinates $x_{\mu}(\mu = 0,1,2,3)$ of space-time and also the anti-commutating spinor θ with four components. In this case, the super-symmetry group may be considered as a transformation group of the newly introduced super-space. The super-symmetry group then includes special super-transformation in addition to four-dimensional rotation and coordinate translations (Poincaré group). Representations (multiplets) Ψ of the super-symmetry group depend both on x^{μ} and $\theta: \Psi(\theta)$ operators. These functions were named super-fields and contain both boson and fermion fields. In other words, super-symmetries, bosons, and fermion fields are mixed. However, within such super-multiplets all particles have equal masses. In addition, this model is far from 'reality', as the physical meaning of super-symmetry is absolutely vague.

Let us now examine the said second approach to eliminating divergences, connected with the generalization of the internal symmetry group. The simplest and most widely used groups of internal symmetry are SU2 and SU3 . There are two such generalizations that have been actively investigated: the chiral group and a group of local calibrating transformations.

The chiral groups are direct products of SU2 and SU3 , yielding SU2 x SU2 and SU3 x SU3 groups. For the construction of a chiral symmetric Lagrangian are used either chiral group multiplets in the form of polynomial functions of the field operators and their derivatives (i.e., linear realization of chiral symmetry), or the Lagrangian is constructed with a small number of fields in the form of non-polynomial functions (for nonlinear realization of the chiral symmetry). In this case, some interesting results have been obtained, but the divergence problem seemingly remained 'infinitely' far from a solution.

With regard to local calibrating transformations, usually standard calibrating transformations do not depend on the coordinates of space-time; in other words, they are global. If we now assume that calibrating transformations are different in different points of the space-time coordinate system, then they may be combined into the local calibrating transformations group. If the Lagrangian is invariant in relation to global calibrating transformations, it is non-invariant in relation to the local calibrating group. Now it is necessary to somehow compensate incipient non-invariance of the global Lagrangian to derive the local invariant Lagrangian from the global invariant. This is done by the introduction of special Yang-Mills fields or compensating fields.

However, only massless vector particles like photons correspond to the Yang-Mills fields. Lack of mass results simply from the calibrating transformation. To obtain particles with non-zero mass, the special mechanism of spontaneous symmetry breaking was proposed. This mechanism is such that, although the Lagrangian remains calibrating-invariant, the overall vacuum average of some fields that are part of the Lagrangian differs from zero, and the vacuum becomes degenerate. But it is impossible to create a substance field by means of Yang-Mills fields, and the former must be separately introduced.

There are several variations in theoretical development of this idea, the most successful being the Glashow-Weinberg-Salam model. According to this model, particles acquire finite mass if the terms responsible for spontaneous symmetry breakdown are added to the Lagrangian, usually by a certain combination of scalar fields (i.e., Higgs mechanism). Unfortunately, even that method has an essential defect, in that divergences still occur. A way was found to eliminate these divergences, but the neutral fields disappeared as well. Nevertheless, that method is considered as the one most propitious, and therefore the special mathematical apparatus based on equations of group renormalization is intensively developed.

Sixty years ago, J.Shwinger calculated the exact value of the anomalous magnetic moment of the electron. It was the remarkable result of modern quantum field theory magnificently confirmed by experimental data. However, in our opinion, his theory did not yield further essential physical correlations. While many mathematicians may deal primarily with quantum field theory, perhaps they are still far from a deep physical understanding of the problem.

As a 'safe' example to illustrate this situation, we will examine the non-linear theory of A.Eddington, M.Born and L.Infeld, which was favorably received and has been incorporated into many quantum theory courses. Normally the authority of these scientists is presumed absolute; however…

The well-known Maxwell-Lorentz equations which describe the location and movement of an electron in a corresponding electro-magnetic field are as follows:

$$rot\boldsymbol{H}=\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}-4\pi\rho\frac{\boldsymbol{v}}{c}, \quad \text{where} \quad div\boldsymbol{E}=4\pi\rho$$

$$rot\boldsymbol{E}+\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t}=0, \quad \text{where} \quad div\boldsymbol{H}=0 .$$

If we consider the electromagnetic field as a 'substance' but not the continuum of charged particles that make up different bodies, and use electrodynamics as a basis for mechanics, then charged particles should be regarded as nodal points of the electromagnetic field. Their location and movement should be governed by the laws of electromagnetic field variations in space and time. Then the only thing that precludes us representing electrons as non-extended particles is the fact that the connected field created by electrons, according to the old concept (or creating them, in accordance with the new one), becomes infinite at their corresponding nodal points. Consequently, their mass as estimated by their electromagnetic energy or momentum becomes infinite also. Thus, to combine the dynamic electromagnetic field theory (as a mechanical properties carrier) with the notion of the electron being non-extended, we should modify the above-mentioned Maxwell-Lorentz equation in such a way that, in spite of charge concentration at nodal points, the electromagnetic field would be finite at an arbitrarily small distance from those points. At median distances from the center of the particle the field should appear 'normal', corresponding to the experimental data. Such a theoretical modification was made in 1922 by A. Eddington and in 1933 by M.Born and L.Infeld.

For this purpose, charge and current densities in the first two Maxwell-Lorentz equations are considered equal to zero over all of space except "special" points intended to be the electron locations. Furthermore, the vectors E and H in the same equations are correspondingly changed:

$$\boldsymbol{D}=\varepsilon\boldsymbol{E}, \quad \boldsymbol{B}=\frac{1}{\mu}\boldsymbol{H}$$

where

$$\varepsilon=\frac{1}{\mu}=\frac{1}{\sqrt{1-\frac{E^2-H^2}{E_0^2}}}$$

Here, $E_0=\frac{e}{r^2}$ represents the maximum possible value of the electric field in the center of the electron and parameter r_0 is considered as the electron's effective radius. The solution of such equations gives the finite electron mass, calculated as total energy of the electric field created by the particle:

$$E=\frac{e}{\sqrt{r_0^4+r^4}}$$

Actually, the electric field at $r \gg r_0$ now behaves in a normal way. However, everything in such a theory, from beginning to end, is fundamentally wrong: In the spherically symmetric case (the only type of event under consideration), the electrostatic intensity ought to be zero in the center of the particle because E is a vector! One can find similar absurdities in numerous modern quantum field theory descriptions, but their authors are still with us.

As for us, we should learn from history, perhaps by considering two rather droll academic episodes connected with distinguished physicist Wolfgang Pauli (which is not generally mentioned in classic scientific literature). It is well known that Louis de Broglie heard crushing criticism from Pauli upon first report of his ideas – but he later received the Nobel Prize for them. [For some time after that incident, de Broglie didn't attend international conferences.] A bit later, Pauli rose in sharp opposition to the publication of the article presented by G.E. Uhlenbek and S.Goudsmit outlining the basic concept of 'spin'. However, this did not prevent him from developing the very same idea and obtaining similar fundamental results, for which he thereafter received the Nobel Prize!

In any event, the mathematical descriptions and exact predictions of numerous very different quantum effects were so impressive that physicists became proud of their quantum science to a point bordering on self-satisfaction and superciliousness. They stopped thinking about physical description of the underlying phenomena and concentrated on the mathematical descriptions only. However, many problems in quantum theory are still far from resolution.

The original idea of Schrödinger was to represent a particle as a wave packet of de Broglie waves. As he wrote in one of his letters, he "was happy for three months" before British mathematician Darwin showed that such packet quickly and steadily dissipates and disappears. So, it turned out that this beautiful and unique idea to represent a particle as a portion of a field is not realizable in the context of wave packets of de Broglie waves. Later, de Broglie tried to save this idea by introducing nonlinearity for the rest of his life, but wasn't able to obtain significant results. It was proved by V.E. Lyamov and L.G. Sapogin in 1968 [10] that every wave packet constructed from de Broglie waves with the spectrum $a(k)$ satisfying the condition of Viner-Pely (the condition for the existence of localized wave packets)

$$\int_{-\infty}^{\infty} \frac{|\ln(a(k))|}{1+k^2} \geq 0$$

Becomes blurred in every case.

There is a school in physics, going back to William Clifford, Albert Einstein, Erwin Schrödinger and Louis de Broglie, where a particle is represented as a cluster or packet of waves in a certain unified field. According to M. Jemer's classification, this is a 'unitary' approach. The essence of this paradigm is clearly expressed by Albert Einstein's own words (back translation): «*We could regard substance as those areas of space where a field is immense. From this point of view, a thrown stone is an area of immense field intensity moving at the stone's speed. In such new physics there would be no place for substance and field, since field would be the only reality . . . and the laws of movement would automatically ensue from the laws of field.*»

However, its realization appeared to be possible only in the context of the Unitary Unified Quantum Field Theory (UUQFT) within last two decades. It is impressive, that the problem of mass spectrum has been reduced to exact analytical solution of a nonlinear integro-differential equation. In UQT the quantization of particles on masses appears as a subtle consequence of a balance between dispersion and nonlinearity, and the particle represents something like a very little water-ball, the contour of which is the density of energy [17, 18, 21-23].

The ideas developed in this paper differ completely from the canonical approach and its previously described versions. Our own approach is non-local, wherein basic theses of standard quantum theory are modified accordingly, and until now no one seems to have investigated such a rearrangement of ideas. With other hand, our approach based on the Unitary Unified Quantum Field Theory- UUQFT has nothing connection with Standard Model of Elementary Particles. In the Standard Model to get good agreements with experiments one has to operate with 19 up to 60 free parameters. It chooses for good agreements with experiments. The UUQFT do not enclose free parameters.

II. Common Approach

To reiterate key basic premises of our Unitary Unified Quantum Field Theory (UUQFT):

According to standard quantum theory, any microparticle is described by a wave function with a probabilistic interpretation that cannot be obtained from the mathematical formalism of non-relativistic quantum theory but is instead only postulated.

The particle is considered as a point, which is "*the source of the field, bun cannot be reduced to the field*". Nothing can really be said about that micro-particle's actual "structure".

This dualism is absolutely not satisfactory as the two substances have been introduced, that is, both the points and the fields. Presence of both points and fields at the same time is not satisfactory from general philosophical positions – "razors of Ockama". Besides that, the presence of the points leads to nonconvergences, which are eliminated by various methods, including the introduction of a re-normalization group that is declined by many mathematicians and physicists, for example, P.A.M. Dirac.

According to UUQFT, such a particle is considered as a bunched field (cluster) or 32-component wave packet of partial waves with linear dispersion [2, 12-25]. Dispersion can be chosen in such a way that the wave packet would be alternately disappear and reappear in movement. The envelope of this process coincides with the quantum mechanical wave function. Such concept helped to construct the relativistic – invariant model of UUQFT. Due to that theory the particle/wave packet, regarded as a function of 4-velocity, is described by partial differential equation in matrix form with 32x32 matrix or by equivalent partial differential system of 32 order. The probabilistic approach to wave function is not postulated, like it was earlier, but strictly results from mathematical formalism of the theory.

Particle mass is replaced in the UUQFT equation system with the integral over the whole volume of the bilinear field combinations, yielding a system of 32 integral-differential equations. In the scalar case the author were able to calculate with 0.3% accuracy the non-dimensional electric charge and the constant of thin structure.

Electric charge quantization emerges as the result of a balance between dispersion and nonlinearity. Since the influence of dispersion is opposite to that of nonlinearity, for certain wave packet types the mutual compensation of these processes is possible. The moving wave packet periodically appears and disappears at the de Broglie wavelength, but retains its form. [A similar phenomenon may correspond to the theoretical case of oscillating solitons, as yet uninvestigated mathematically.]

Micro-particle birth and disintegration mechanisms become readily understood as the reintegrating and splitting-up of partial wave packets. This approach regards all interactions and processes as being simply a result of the mutual diffraction and interference of such wave packets, due to nonlinearity.

The tunneling effect completely loses within UUQFT its mysteriousness. When the particle approaches the potential barrier in such the phase that the amplitude of the wave packet is small, then all the equations become linear and the particle does not even “notice” the barrier, and if the phase corresponds to large packet’s amplitude, then nonlinear interaction begins and the particle can be reflected.

The most important result of our new UUQFT approach is the emergence of a general field basis for the whole of physical science, since the operational description of physical phenomena inherent in standard relativistic quantum theory is so wholly unsatisfying.

The most direct way of eliminating the existing theoretical difficulties in the relativistic interpretation of quantum-mechanical systems lies in the construction of a theory dealing only with a unified field, where are to be observed the quantities and the values that characterize that field at different points in time and space.

There is an impression that during the time since quantum theory was created, no substantial progress has been made in respect to our understanding of that theory. This impression is reinforced by the fact that neither field quantum theory nor the still imperfect theories of elementary particles have made any serious strides in the posing or solution of the following traditional questions:

What are the reasons for the probabilistic interpretation of the wave function, and how can this interpretation be obtained from the mathematical formalism of the theory?

What is really happening to a particle, when we “observe” it during interference experiments (for interference cannot be explained without invoking the particle “splitting-up” concept)?

What is this statement in standard quantum mechanics really saying?: ***“a micro particle described by a point is the source of a field, but cannot be reduced to the field itself”.*** Is it divisible or not? What does it really represent? *Why is* ***all of physics*** *based on two key notions: point-particle as the field source and the field itself?* Can only one field aspect remain, and still be considered as a *physical entity* that is as yet un-analyzable?

There are as yet no answers to these basic questions. “Exorcism” of the complementarity principle is irrelevant because that philosophy was invented *ad hoc*.

Many researchers think that the future of theoretical physics should be based upon a certain single field theory – a unitary approach. In such a theory, particles are represented in the form of field wave clusters or packets. Mass would be purely a field notion, but the movement equations and all ‘physical’ interactions should follow directly from the field equations.

This is a very simple and heretofore unstudied possibility of formulating the unitary quantum theory for a single particle. Here we will deal only with the very general properties inherent in all particles and not touch upon the problems connected with such properties as charge, spin, strangeness, charm etc. After quantum mechanics appeared and was fully developed, a curious situation occurred: half of the founders of the theory clearly spoke out against it! Quoted below are a few of their remarks (back translation):

"The existing quantum picture of material reality is today feebler and more doubtful than it has ever been. We know many interesting details and learn new ones every day. But we are still unable to select from the basic ideas one that could be regarded as certain and used as the foundation for a stable construction. The popular opinion among the scientists proceeds from the fact that the objective picture of reality is impossible in its primary sense (i.e. in terms of images and movements- remark of author). Only very big optimists, among whom I count myself, take it is as philosophic exaltation, as a desperate step in the face of a large crisis. A solution of this crisis will ultimately lead to something better than the existing disorderly set of formulas forming the subject of quantum physics…If we are going to keep the damned quantum jumps I regret that I have dealt with quantum theory at all…" – Erwin Schrödinger.

"The relativistic quantum theory as the foundation of modern science is fit for nothing." – P. A. M. Dirac

"Quantum physics urgently needs new images and ideas, which can appear only in case of a thorough review of its underlying principles." – Louis de Broglie.

Albert Einstein, also, had the following to say:

"Great initial success of the quantum theory could not make me believe in a dice game being the basis of it…I do not believe this principal conception being an appropriate foundation for physics as a whole… Physicists think me an old fool, but I am convinced that the future development of physics will go in another direction than heretofore… I reject the main idea of modern statistical quantum theory… I'm quite sure that the existing statistical character of modern quantum theory should be ascribed to the fact that that theory operates with incomplete descriptions of physical systems only… ."- A. Einstein.

Although today the quantum theory is believed to be essentially correct in describing the phenomena of the micro-world, there is nevertheless experimental evidence—of cold nuclear fusion and mass nuclear transmutations, of anomalous energy sources and perhaps even *perpetual motion*—which contradicts quantum theory.

The trouble with all previous attempts to present a particle as a field wave packet was that such a packet, according to proposed approaches, consisted of de Broglie waves. In our UUQFT approach, the packet consists of partial waves and the de Broglie wave appears as a side product during the movement and evolution of that partial wave packet.

Since we intend to describe physical reality by a continuous field, neither the notion of particles as invariable material points nor the notion of movement can have a fundamental meaning. Only a limited zone of space wherein the quantum field strength or energy density is especially large will be considered as a particle.

Let us conduct the following thought experiment: at the origin of a fixed coordinate system located in an empty space free of other fields, there is a hypothetical immovable observer, past whom a particle moves along the *x* axis at a velocity of $v<<c$. Let us assume that the particle is represented by a wave packet creating a certain hitherto unknown field, and that the observer with the help of a hypothetical microprobe is measuring certain characteristics of the particle's field at different moments in time. This measuring is done on the assumption that the size of the hypothetical energy measuring device is many times less than the size of the particle and that it does not disrupt or influence the field created by this particle.

It is obvious that such an experiment is imaginary and cannot in principle be performed, but it doesn't prevent our imaginary device from being ideologically the simplest possible. In other words, we are interested in how the particle behaves and how it is structured when *"no one is looking at it."* Let the result of measurements at a certain point be function *f(t)*, describing the structure of the wave packet, the size of which is very small compared to the de Broglie wave. Knowing the particle's velocity *v* and the structural function *f(t),* the immovable observer can calculate the "apparent size" of the particle.

Let us assume that inside the corresponding wave packet the linearity of laws is not broken, and that the function *f(t)* satisfies the Dirichlet conditions and can be split into harmonic components which we will call 'partial waves'. In using the complex form of development, we then obtain:

$$f(t) = \sum_{s=-\infty}^{\infty} c_s exp\ (i\omega_s t), \tag{1.1}$$

where coefficients c_s are the amplitudes of the partial harmonics (with the mean value of $C_0 = 0$), and ω_s are the corresponding frequencies. To find the dispersion equation for partial waves, let us use the Rayleigh ratio for the group velocity *v* of the wave packet:

$$v = v_p + k\frac{dv_p}{dk} \tag{1.2}$$

Regarding the wave number *k* of the partial wave as a function of the phase velocity v_p, let us integrate (1.2) with *v=const*, since by the law of inertia the centre of the packet is moving at a constant speed. We will have:

$$k = \frac{C}{|v_p - v|} \tag{1.3}$$

Where *C* is the constant of integration. Integration was made on the assumption that velocity *v* is constant and does not depend on the frequency of the partial waves, which follows from the experimentally derived law of inertia. If we assume that the particle is a wave packet, then its group velocity will be equal to the classical velocity of the particle. Since the particle is moving at a constant speed (inertial) in the absence of external fields, the group velocity of the packet is a constant value independent of the phase velocities of the harmonic components. The unsatisfying form of the dispersion equation (1.3) masks the linear dispersion law, which can be derived from (1.3), by substitution of, $v_p = \frac{\omega_s}{k_s}$ whereby:

$$\omega_s = vk_s \pm C \qquad (1.4)$$

where plus sign corresponds to $v_p > v$ and minus sign corresponds to $v_p < v$. We will now define the integration constant C as follows. Since harmonic components $c_s \exp(i\omega_s t)$ are propagated in the linear medium independently of each other, the behaviour of the wave packet can be presented as a superposition of the harmonic components:

$$c_s \exp(i(\omega_s t - k_s x) + i\phi) \qquad (1.5)$$

Since the wave phase is now defined up to the additive constant, an additional constant ϕ for all partial waves was then introduced. Essentially, this is possible by simple translation of the origin of the coordinates, so the value ϕ can actually be excluded from further consideration. Then, the moving wave packet can be represented as follows:

$$\Phi(x,t) = 2\Re e \sum_1^\infty c_s \exp(i(\omega_s t - k_s x)) \qquad (1.6)$$

Regarding the wave number $k(\omega)$ as a frequency function and substituting (1.4) into (1.6), we obtain:

$$\Phi(x,t) = 2\Re e(\exp(-i(\frac{C}{v}x)) \sum_1^\infty c_s \exp(i\omega_s(t - \frac{x}{v}))) \qquad (1.7)$$

or

$$\Phi(x,t) = \cos(\frac{C}{v}x) f(t - \frac{x}{v}) + \sin(\frac{C}{v}x) f^*(t - \frac{x}{v}),$$

Where function $f^*\left(t - \frac{x}{v}\right)$ describes some additional partial waves with the same frequencies ω_s. Analyzing expression (1.7), we can see that the wave packet $\Phi(x,t)$ in its movement in a "medium" with linear dispersion described by equation (1.4) will disappear and reappear with period $\frac{2\pi v}{C}$ in x and can be regarded as if inscribed in the flat envelope modulating with that period. The state of the wave packet (and of its corresponding particle) in the region where it disappears or its amplitude becomes very small may be thought of as a "phantom state".

Let us find integration constant C. For this, we will require that the wavelength of the monochromatic envelope be equal to the de Broglie wavelength:

$$\lambda_B = \frac{2\pi}{k_B} = \frac{2\pi v}{C} \qquad (1.8)$$

Then, $C = vk_B$, and expression (1.7) will become as follows:

$$\Phi(x,t) = \cos(k_B x) f(t - \frac{x}{v}) + \sin(k_B x) f^*(t - \frac{x}{v}) \cdot \qquad (1.9)$$

The disappearance and reappearance of the particle occurs periodically without change of its apparent dimensions (width and form). It is clear that the dimensions of each packet can be many times less than the de Broglie wavelength. An approximate picture of the behaviour of such a packet in space and time is presented in Fig.1 below, and the results of the mathematical modelling of the scalar Gauss wave packet behaviour in a medium with linear dispersion are presented in Fig.2. The both figures show how such a packet disappears and reappears, changing its sign. Any dispersion without dissipation leaves the packet's energetic spectrum unchanged. When the wave packet moves, only the phase relations between the harmonic components are changed, because dissipation is absent. This concept is based on two postulates:

(1) A particle represents a wave packet with linear field laws. The linear dispersion law follows from the law of inertia, and the particle is regarded as a moving wave packet inscribed in a flat envelope;

(2) The envelope wavelength is equal to the de Broglie wavelength. Nevertheless, any packets of de Broglie waves that are localized enough will be spread over the whole volume, as dispersion of the de Broglie wave $\omega_B = \frac{\hbar k_B^2}{2m}$ differs from linear dispersion. This does not contradict the suggested concept, as the envelope doesn't exist as a real wave and is not included in the set of waves described by eq . (1.5).

Please note that the process of periodicity in the appearance and disappearance of the wave-packet/particle is possible only for very small objects, and that the quantum teleportation of macro-objects being widely discussed today is hardly possible by the principles under discussion here. However, the theoretical possibility of the wave packet spreading in the transverse direction due to diffraction is still a concern. It is in principle that the packet can disperse and not exist as a localized formation. To show that this won't happen, let us put the equation of dispersion into another form. Viz., according to P.Ehrenfest, the

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theoretical envelope velocity of the wave packet equals the classical particle velocity:

$$v = \frac{d\omega}{dk} = \frac{P}{m} \tag{1.10}$$

On the other hand

$$\omega = \frac{E}{\hbar} \quad \text{and} \quad \frac{d\omega}{dk} = \frac{1}{\hbar}\frac{dE}{dk} \tag{1.11}$$

According to classical mechanics, the energy of a free particle is:

$$E = \frac{p^2}{2m} \quad \text{Or} \quad \frac{d\omega}{dk} = \frac{P}{\hbar m}\frac{dP}{dk}$$

Comparing (1.10) and (1.11) we obtain:

$$\frac{P}{\hbar m}\frac{dP}{dk} = \frac{P}{m},$$

And by integrating that differential equation we get

$$P = \hbar k + C .$$

Now, the phase velocity of the waves,

$$v_p = \frac{\hbar \omega_s}{\hbar k_s + C},$$

does not remain a constant value but depends on constant of integration C.

By using another method to determine the velocity phase, the constant of integration may be added to the expression of energy (but this isn't a matter of principle). The choice of the constant of integration C does not influence the results to be obtained in terms of quantum mechanics, and so for simplicity we assume that $C = 0$.

The present conclusion represents a known fact about motion equation invariance as regards gradient calibrating transformation. The same relations for the phase velocity of quasi-particles also hold in solid-state physics, for quasi-particle momentum can be written as a constant divisible by the reciprocal lattice constant.

Let us return to (1.3). The choice of constant C determines the type of dispersion. In the general case, that relation describes the wave set with different k and λ. As we saw previously (and as is true in all inertial coordinate systems), with a certain type of dispersion the envelope of the de Broglie wave process is in a 'space-hold' condition. Putting $v_p = 0$ in eq. (1.3), we obtain

$$C = kv = \frac{mv^2}{\hbar}.$$

Substituting the value for C into this same expression (1.3) and taking into account that $k = \frac{\omega_s}{v_p}$ we will obtain the expression for *subwave* phase velocity:

$$v_p = \frac{\hbar \omega_s}{\pm mv + \frac{\hbar \omega_s}{v}} \tag{1.12}$$

We should note that according to some works in quantum field theory, divergences are in principle eliminated by choice of C.

Above described mathematical construction of a particle contains the submarine reef: there is the theoretical possibility of the wave packet spreading in the transverse direction due to angular diffraction of any wave process. But it turns out that if using the non-linear interpretation of wave transmission theory then the effect of self-refocusing is revealed and this effect ensures the stability of wave packet.

If the theory of wave transmission is linear, then the wave packet will diverge at the angle $\phi = \frac{\lambda}{b}$ (Fig.3a).

Within the non-linear interpretation, one can see that self-focusing is able to compensate transverse diffraction (Fig.3b). For that to occur, the following relationship is necessary:

$$v_p = \frac{c}{n} = \frac{c}{n_0 + n_2 E^2},$$

Where c is light velocity. Then, the peripheral phase fronts bend toward the packet's axis, thus compensating transverse diffraction (as in Fig.3b above). As the wave packet's mass is proportional to the square of its amplitude, relation (1.12) can be rewritten in the following form:

$$v_p = \frac{\hbar\omega_s}{\pm mv + \frac{\hbar\omega_s}{v}} = \frac{c}{\frac{c}{v} \pm \frac{mvc}{\hbar\omega_s}} = \frac{c}{n_0 + n_2 E^2}$$

provided $n_0 \frac{c}{v}$, $n_2 = \pm\frac{vc}{\hbar\omega_s}$, and $m \approx E^2$.

The situation is very similar to soliton process of light spreading when the refraction coefficient of light in medium grows together with amplitude.

As yet we've said nothing about the nature of either the 'medium' or the waves propagating in it. In spite of various modern versions of quantum field theory, and the further development of UUQFT theory is impossible to answer at present the very simple question "what is space-time?" Is it simply the "stage" where performers in the form of a multi-component field are continually appearing and disappearing? Or does the field represent dynamic distortions of the stage itself, so that it's impossible to separate the performers from that stage?

III. The Equation of The Unitary Unified Quantum Field Theory

We will identify described above model of periodically disappearing and reappearing wave packet with a particle. But this model is till now only some mathematical illustration that has no relation with quantum theory. So, we will go also another way and will construct relativistic invariant model, so to say, "manually" and we will derive quantum equation from this model. *It turned out that requirement of relativistic invariance will be satisfied from physical point of view by introduction of own oscillations for immovable wave packet.*

The wave function of a single particle (1.9) was derived on an assumption of non-relativistic velocities, i.e., for v<<c. To obtain its relativistic generalization it is first necessary to make the wave function as a relativistically invariant phase, [2,14,15,20-23] i.e.,

$$\Phi = \exp[-i(Et - \mathbf{P}\mathbf{x})]\mathbf{f}(\mathbf{x} - \mathbf{v}t), \tag{2.1}$$

Where

$$E = \frac{m}{\gamma}; \mathbf{P} = \frac{m\mathbf{v}}{\gamma}; \gamma = \sqrt{1 - v^2}$$

And $\mathbf{f}(\mathbf{x}\text{-}\mathbf{v}t)$ is some structural function (in this paragraph, we use a unit system in which $c = \hbar = 1$). It can be required that structural function $\mathbf{f}(\mathbf{x}\text{-}\mathbf{v}t)$ be scalar and satisfy the Klein-Gordon equation. Then, we will get the following equation for $\mathbf{f}$:

$$(v_i v_k - \delta_{ik})\frac{\partial^2 \mathbf{f}}{\partial \xi_i \partial \xi_k} = 0$$

Here, $\xi_i = x_i - v_i t$; *i,k=1,2,3,* and summarization is obtained by repeated indices as usual. A two-component solution of the Klein-Gordon equation would then appear as follows:

$$\Phi = \exp(-i(Et - \mathbf{P}\mathbf{x}))\begin{pmatrix} \frac{\gamma - 1}{2\gamma}\mathbf{f} - \frac{i}{2m}\mathbf{v}\frac{\partial \mathbf{f}}{\partial \xi} \\ \frac{\gamma + 1}{2\gamma}\mathbf{f} + \frac{i}{2m}\mathbf{v}\frac{\partial \mathbf{f}}{\partial \xi} \end{pmatrix} \tag{2.2}$$

By substituting (2.1) into the Schrödinger equation we may obtain the Laplace equation for structural function as:

$$\nabla_\xi^2 \mathbf{f} = 0,$$

and its solution will enable us to regard the particle as a spherical wave packet "cut into parts" by spherical harmonics.

But such an approach can only serve as a certain illustration, a first approximation based on the assumption of field law linearity. Function **f** described by the Laplace equation will tend to infinity at zero, which is completely unsatisfactory from the physical point of view. Let us do otherwise, and consider just the simplest equations of first and second order, which are satisfied by a one-component relativistic wave function having an arbitrary structural function. These equations have a clearly relativistic form:

$$(u_\mu \frac{\partial}{\partial x_\mu} + im)\Phi = 0 \tag{2.3}$$

$$(u_\mu u_\nu \frac{\partial^2}{\partial x_\mu \partial x_\nu} + m^2)\Phi = 0 \tag{2.4}$$

where: $x_\mu = (\mathbf{x}, it)$; $u_\mu = (\frac{\mathbf{v}}{\gamma}, \frac{i}{\gamma})$ is the particle's four-velocity; and $\mu, \nu =$ 1,2,3,4. It is natural to consider that a particle with an arbitrary spin and mass *m* can be described by a relativistic equation

$$(\Lambda_\mu \frac{\partial}{\partial x_\mu} + m)\Phi = 0 \tag{2.5}$$

Where Φ is an *n*-component column and Λ_μ represents four (n x 4) – matrices (n-rows, 4-column) describing the spin properties of the particle. These matrices are functions of the particle velocity and satisfy relations that are defined by the spin value.

Let us now express particle energy (mass) by means of a field. For Dirac-type equations, neither the character of charge with an integer spin nor charge energy with half-integer spin are defined. In relativistic electrodynamics, according to the Laue theorem, the tensor components of the energy-impulse of the electromagnetic field that is generated by the charge do not form four-vectors, so there is only one method of expressing the particle energy:

$$E = m = \int_V \Phi^+ \Phi d^3 x \tag{2.6}$$

Usually in such cases it is required that the integral (2.6) contain the Green function. However, if we strictly follow the principles of the unitary theory, we should define the particle energy within non-relativistic limits as in expression (2.6).

Let us substitute the invariant relativistic expression $\langle \Phi | \Phi \rangle$ for $\int_V \Phi^+ \Phi d^3 x$, which, for example, equals (O.Costa de Beauregard [3]) for a spin field with a rest mass differing from zero (there are also formulas for the scalar and vector fields):

$$\langle \Phi | \Phi \rangle = \int \{\Phi^* i\gamma_4 \frac{\partial}{\partial t} \hat{\varepsilon} \Phi - \frac{\partial}{\partial t} \Phi^* i\gamma_4 \hat{\varepsilon} \Phi\} dV \tag{2.7}$$

where γ_4 is a Dirac matrix, $\hat{\varepsilon} = +1$ for a particle, and $\hat{\varepsilon} = -1$ for an antiparticle. Then, eq. (2.5) will look as follows:

$$\{\Lambda_\mu \frac{\partial}{\partial x_\mu} + \langle \Phi | \Phi \rangle\}\Phi = 0 \tag{2.8}$$

Or the full relativistic invariant equation for our wave packet is following:

$$i\lambda^{\mu}\frac{\partial\Phi}{\partial x^{\mu}}-\frac{c\Phi}{\hbar}\int\left(\bar{\Phi}\lambda_1 u^{\mu}\frac{\partial\Phi}{\partial x^{\mu}}-u^{\mu}\frac{\partial\bar{\Phi}}{\partial x^{\mu}}\lambda_1\Phi\right)\frac{dV}{\gamma}=0, \tag{2.8a}$$

where Φ is the function of coordinates $x^{\mu}=(ct,x)$, $\mu=0,1,2,3$, describing different characteristics of our wave packet, $u^{\mu}=\left(\frac{1}{\gamma},\frac{v}{\gamma}\right)$ is the four-velocity of the particle, λ_1 is some number matrix and matrices $\lambda^{\mu}(32\times 32)$ satisfy the commutation relations

$$\lambda^{\mu}\lambda^{\nu}+\lambda^{\nu}\lambda^{\mu}=2g^{\mu\nu}I\,,\ \mu,\nu=0,1,2,3,$$

Where $g^{\mu\nu}$ is the metrical tensor.

This nonlinear integro-differential equation is, in our view, fundamental, and must describe all the properties and interactions of particles [12-14, 17-23].The mass spectrum from such equations may be derived after solving stability problems of the Sturm-Liuville type, which will in turn give the particle lifetime. In the theory under consideration, the birth and decay of all particles, and all of their interactions and transformations, are consequences of wave packet splitting and mutual diffraction phenomena due to nonlinearity. The construction of solutions to that problem will plainly require some new mathematical methods. The full proof of relativistically invariant eq.(2.8a) is clumsily, please see [2, 20,21].

Point-like particles may be required to simplify the solution of the preceding eq. (2.8), whereby it reduced to the main equation of nonlinear (W.Heisenberg, [7]) theory written not in operator form but in *c*-numbers. For this it is necessary in eq. (2.5) to substitute $m=\Phi\Phi^{+}$. Then we obtain the following equation:

$$(\Lambda_{\mu}\frac{\partial}{\partial x_{\mu}}+\Phi^{+}\Phi)\Phi=0\,, \tag{2.9}$$

And there was derived an approximate particle mass spectrum with help of this equation.

Let us pass from equation (2.5) to the equation of particle motion in an external electromagnetic field A_{μ} We will therefore make a standard substitution $\frac{\partial}{\partial x_{\mu}}\to\frac{\partial}{\partial x_{\mu}}-ieA_{\mu}$, and eq. (2.5) is transformed as follows:

$$(\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{x}}-iL)\Phi=0 \tag{2.10}$$

Where *L* is a relativistic Lagrangian,

$$L=m\gamma+e\gamma U_{\mu}A_{\mu}\,.$$

If a particle is located in an external electromagnetic field, for example, with vector potential ***A*** and scalar potential φ, then the linear dispersion law is not changed. *L* and **v** will then be certain functions of coordinates and the solution of eq. (2.10) in a general form has the following form:

$$\Phi=\exp(-i\int Ldt)\mathbf{f}\left(\mathbf{x}-\int\mathbf{v}dt\right) \tag{2.11}$$

It is easy to make a standard transition from the relativistic case to the non-relativistic case by using the well-known transformation $\Phi=\Phi e^{-imt}$. Substitution of function (2.11) into the equation (2.10) shows that the equation is satisfied provided *L* is a non-relativistic Lagrangian. Let us now look at the role of the wave function phase, which is the classic action *S* and will enable us to establish a connection between the proposed theory and classical mechanics. Actually, the wave function may be represented in the form below (following Hamilton's principle in classic mechanics):

$$\Phi=\exp(iS)\mathbf{f}(\mathbf{x}-\int\mathbf{v}dt)$$

If we substitute this expression into eq. (2.10), we then obtain an equation for *S*:

$$\frac{\partial S}{dt}+\mathbf{v}\nabla S-L=0 \tag{2.12}$$

In keeping with the requirements of the Hamilton-Jacobi theory, it is necessary to assume that $\mathbf{P}=\nabla S$; then eq. (2.12) will be transformed to the Hamilton-Jacobi equation:

$$\frac{\partial S}{\partial t}+H=0\,,$$

Where $H=\mathbf{P}\mathbf{v}-L$ is the particle's Hamiltonian.

The function *S* can thereby be found, dependent on the particle's coordinates, the physical parameters of the Hamiltonian, and on *q* non-additive integration constants; and then perhaps the problems of motion and dynamics can be solved. The imposed requirement $\mathbf{P}=\nabla S$ implies a transposition to classic mechanics using an optic analogy approximation, whereby the concept of particle trajectory as a beam can be introduced. Such a trajectory will be orthogonal to any given surface of a permanent operation or phase. On the other hand, a quantum object becomes a classical construct after superposition of a large number of wave packets. The case where all wave packets composing an object spread and reintegrate simultaneously despite different velocities and phases is physically impossible. That is why such a combination when averaged out will appear, in general, like a stable and unchanging object moving under the laws of classical mechanics, whereas every elementary object obeys the quantum laws. Note that a *transfer from the unitary quantum theory to classical mechanics is mathematically strict.* In the usual quantum theory, the transfer happens with an imposed condition $\hbar \rightarrow 0$. Mathematically, it is completely unsatisfactory, since $\hbar$ is some physical constant (equal to 1 if given a corresponding units system). We do not remember a single case in mathematics when a similar condition would be imposed in a proof, such as $\pi \rightarrow 1$. Let us consider briefly the hydrogen atom problem. The solution of classical problem of particle movement in the central field allows presenting the wave function (2.1) as follows:

$$\Phi = e^{-iEt} e^{i\int_{r_0}^{r} p_r dr} e^{i\int_{\phi_0}^{\phi} p_\phi d\phi} f(\mathrm{r} - \int_0^t v_r dt; \phi - \int_0^t \dot{\phi} dt)$$

Here, r_0 and ϕ_0 are particle coordinate values (radius and angle correspondingly) at time *t=0.* Stationary orbits appear when the envelope is a standing wave provided:

$$ET = 2\pi n_1 h; \oint p_r dr = 2\pi n_2 h; \oint p_\phi d\phi = 2\pi n_3 h \quad ,$$

Where n_1, n_2, n_3 are integers. These requirements correspond to the terms of Bohr-Sommerfeld quantification.

The process envelope can be identified with the de Broglie wave and in essence the Schrödinger equation describes the envelope of the wave packet's maxima in motion.

In conclusion of this section, let us find matrices Λ_μ. Let us assume that matrices Λ_μ are linear relative to velocity [2, 13, 14, 20, 21]:

$$\Lambda_\mu = \Lambda_{\mu 0} + \Lambda_{\mu\nu} u_\nu \qquad (2.13)$$

Where $\Lambda_{\mu 0}$ x $\Lambda_{\mu\nu}$ are numerical matrices. Let us apply equation (2.5) on the left with operator $\Lambda_\sigma \frac{\partial}{\partial x_\sigma} - m$, obtaining:

$$\{\frac{1}{2}(\Lambda_\mu \Lambda_\sigma + \Lambda_\sigma \Lambda_\mu)\frac{\partial^2}{\partial x_\mu \partial x_\sigma} - m^2\}\Phi = 0 \qquad (2.14)$$

If we require that each component of system (2.14) satisfies the second order equation (2.4), and then

$$\Lambda_\mu \Lambda_\sigma + \Lambda_\sigma \Lambda_\mu = -2u_\mu u_\sigma I \qquad (2.15)$$

Relation (2.15) is satisfied identically if we take ten Hermitian matrices 32x32 as numerical matrices $\Lambda_{\mu\nu}$, satisfying the following commutation relations [2,13,14,20,21]:

$$\Lambda_{\mu\nu}\Lambda_{\sigma\tau} + \Lambda_{\sigma\tau}\Lambda_{\mu\nu} = 2(\delta_{\mu\sigma}\delta_{\nu\tau} - \delta_{\mu\tau}\delta_{\nu\sigma})I \qquad (2.16)$$

Here, indices μ, ν, σ, τ take values 0, 1, 2, 3, 4.

It is interesting to note that if the particle's 4-velocity is assumed to be zero ($u_\mu = 0$) directly in matrix (32x32), then system (2.5) will reduce to eight similar Dirac equations [2, 13, 14, 19-21]. However, this requirement is absolutely unsatisfactory both from the physical and the mathematical points of view. Four-velocity has 4 components, of which three are usual components of the particle velocity along three axes, and they really can tend to zero. But the same cannot be done with the fourth component. Hence, this approach is formally incorrect and requires explanation. In our view, although the Dirac equation describes the hydrogen atom spectrum absolutely correctly, it is not properly a fundamental equation. It has two weak points:

1. The correct magnitude of the velocity operator's proper value is absent. It is known that in any problem of this type the proper value of the velocity operator is always equal to the velocity of light! In fact, Russian physicist and mathematician V.A.Fock regarded this as an essential defect of the Dirac theory;
2. The Klein paradox appears in the solution of the problem of barrier passage, when the number of the particles that pass is bigger than the number of incident particles.

The equations of the Unitary Unified Quantum Field Theory we are proposing are more correct and fundamental. For this reason, a transition from correct fundamental equations to the incompletely accurate Dirac equation needs such a strange requirement as $u_\mu = 0$.

IV. Interpretation of The Unitary Unified Field Theory

a) Non-Relativistic Case

The envelope of the wave function $\Phi(x,t)$ describes a wave packet's field transformation within its motion. There are points at which the packet/particle disappears $\Phi(x,t) = 0$, yet particle energy remains in the form of harmonic components that produce field vacuum fluctuations at some point in space-time. Neither the value nor moment of these fluctuations' appearance nor the background flux at that point depend on the apparent distance to such a vanished particle. This precept does not violate the principles of relativity, however, in that the apparent background does not transfer any information. Our real 'world' continuum consists of an enormous quantity of particles moving with different velocities. Partial waves of the postulated vanishing particles create real vacuum fluctuations that change in a very random way. Certain particles randomly appear in such a system, owing to the harmonic component energy of other vanished particles. The number of such "dependant particles" changes, though; they suddenly appear and vanish forever, as the probability of their reappearance is negligibly small, and so *we do expect that all particles are indebted to each other for their existence. Yet, if some particles are disappearing within an object, other particles are arising at the same moment in that object due to the contribution of those vanishing particles' harmonic components –and vice versa. The simultaneous presence of all of the particles within one discrete macroscopic object is unreal. Some constituent particles vanish within the object while others appear. In general, a mass object is extant overall, but is not instantaneously substantive and merely a 'false' image.* ***All Universes is mathematical focus.*** *It is clear that the number of particles according to such a theory is inconstant and all their ongoing processes are random*, and their probability analysis will remain always on the agenda of future research.

In reality, the hypothetical measurements considered before are impossible, because all measuring instruments are macroscopic. Since the sensor of any such device is an unstable-threshold macro-system, only macroscopic events will be detected, such as fog drops in a Wilson chamber, blackening of photo-emulsion film, photo-effects, and the formation of ions in a Geiger counter. Within macro-devices of any type, the sensor's atomic nuclei and electron shells are in close proximity, creating a stable system which is far from being able to take on all arbitrary energy configurations that might be imagined. The nature of that stable condition allows for only a series of numerous but always-discrete states, and the transition from one state to another is a quantum jump. This is why absorption and radiation of energy in atomic systems takes place by quanta, and is a consequence of subatomic structure. In other words, quantization appears because of the arising of bound states, with 'substance' being the richest collection of an enormous number of bound states. However, *it is known that free particles may vary their energy continuously*. However, this does not mean that while passing from one quantum-mechanical system to another, the quantum or particle remains as something invariable and indivisible. Particle energy can be split up and changed due to vacuum and external field fluctuations, but the measuring conditions of our devices are such that we are able to detect quite definite and discrete particles only.The wave packet/particle exhibits periodicity following our UUQT approach, and the mass of a moving particle such as a proton changes from its maximal value to zero and back again – running the series of intermediate values corresponding to the masses of mesons. For example, it might be said that the proton takes, during some intervals of time, the form of a π-meson. This phenomenon is confirmed by numerous experiments, which are explained in classical quantum theory in another way: The proton is permanently surrounded by a cloud of π-mesons, an explanation which is in essence equivalent to our model. Thus the developing point of view results in the conclusion that *relation $E = \hbar\omega$ is fulfilled at the atomic level only.* Thus the particles may exist (after fragmentation on the mirror) with similar frequency ω_B, but with different wave amplitudes f , and so with different probabilities to be detected. One of the particles being split up at the mirror or grid may be detected in a few points at once. The other particle may disappear completely, making its contribution in vacuum fluctuations without any marks.

Following P. Dirac, the photon may interfere only on its own and so the translucent mirror splits it into two parts. According to standard quantum theory, the photon is not able to split with frequency conservation, so it is assumed that two separate photons may interfere under the condition that they belong to one mode, which occurs in the case of the translucent mirror. However, according to UUQFT, photons are constantly splitting at the translucent mirror with frequency conservation, but the probability to detect such splitting photons is reduced.

An uncertainty relation results from the fact that energy and impulse are not fixed values, but periodically change due to the appearance and disappearance of the particle. That question is examined in detail in sect. 7. Due to the statistical measuring laws, it is impossible to measure energy and impulse by macro-devices exactly because of principal and not-foreseen vacuum fluctuations. On the other hand, for the hypothetical researcher the centre of the wave packet has exact coordinates, impulse, and energy at the given moment of time. However, neither we nor the hypothetical observer are able to predict exactly its value at the following moment. Moreover, we (macro-researchers) do not have even a method of accurate measuring, for the process of macro-devices measuring is statistical.

The presence of vacuum fluctuations makes microcosm laws for each researcher statistical in principle. The exact prediction of the events requires the knowledge of the vacuum fluctuation's exact value in any point and at any moment of time. This is impossible, for it requires the information about behaviour and structure of all various wave packets within the Universe and also the possibility to control their motion.

W. Heisenberg [7] wrote (back translation) : "*If we would like to know the reason why α - particles are* *emitting at an exact moment we must, apparently, know all microscopic states of the whole world we also belong to, and that is, obviously, impossible.*"

This is why the conclusion that Laplace determinism is lost within the modern and future physics of microcosm shall be considered ultimate. The same point of view about the reason of the arising of probability approach in quantum mechanics was expressed by (R. Feynman [6], back translation, 1965) : *"There is almost no doubt that it (probability-author) results from the necessity to intensify the effect of single atomic events up to the level detectable with the help of big systems."*

It is good to remember the deep and remarkable words of J. Maxwell: "*The calculation of probabilities is just the true logic of our world.*"

The most impressive demonstration of the random chaotic nature of all quantum processes can be seen at the start of a nuclear reactor. Chaos of micro-effects at a low level of average power results in enormously huge fluctuations of chain reactions, which exceed to a considerable extent the average level. Atomic chaos manifestations always exasperate the participants and sometimes create a threatening impression of the processes' uncontrollability with all following consequences. However, cadmium rod removal precipitates smoother fluctuations.

The envelope of partial waves appearing in the result of wave packet linear transformations and also in the result of it splitting and fragmentation satisfies the C. Huygens principle. This explains the way it is possible to connect the formally moving particle and plane monochromatic de Broglie wave as it spreads in the line of motion and also all the wave properties of particles (such as interference and diffraction).

For example, let the wave packet run up to the system with two slots. Each of the wave packet harmonic components interferes at these slots. There would be an interference pattern of each harmonic component at the screen (since harmonic components amplitudes are extremely small, it may be not possible to see it). However, above this interference pattern the other interference patterns of an infinite large number of the other harmonic components are superimposed. The general composition results in the long run interference pattern of the de Broglie wave envelope.

For the total reversibility of quantum processes, it is necessary while exchanging $+t$ for $-t$ not only to reproduce the amplitude and form of the packet at $+t$, but also to restore the background fluctuation. The quantum mechanics equations permit formal exchange $+t$ for $-t$ under the condition of simultaneous exchange Φ for Φ^*, i.e. formal reversibility (the amplitude and form of the packet reproduction). Actually, such reversibility does not exist in nature even for the hypothetical observer, as for reproduction of the former vacuum fluctuations the reversibility of all processes in the Universe is required, and that is impossible. However, one is able to think that in terms of Unitary Quantum Theory the reversibility has a statistic character (single processes may be reversible with define probability). Introduced function Φ has a strictly monochromatic character, but does not exist as a real plane running wave. Although this function corresponds to the particle's energy, other notions may also agree with it: "Waves of probability", "informational field", and "waves of knowledge". As stipulated by (A. Alexandrov et al., [1]) a wave function has sense for a separate system, but we can pick it out only by the way of numerous similar experiments and after averaging, though the hypothetical researcher is able to measure this wave function for one particle. It is interesting that the envelope remains fixed within all inertial coordinates systems (only the wave length is changed).

Function Φ may also be connected with wave function Ψ of quantum mechanics describing the plane wave moving in the space. However, the value Φ^2 differs from $\Psi\Psi^*$ not only by presence of frequent oscillations. With Φ^2 the particle's energy is connected, but with $\Psi\Psi^*$ only the probabilities connect.In standard quantum theory all is not so easy. When comparing mathematical expressions for the density matrix in quantum mechanics and the correlation function of random classical wave field, then we find them quite similar, although they describe absolutely different physical objects. In the simplest cases the wave function relates to a single particle and has any sense in the presence of the particle only. Wave function has no sense in those areas where particle is absent. More formally, according to quantum theory, physical values can be obtained in the result of either one or other operators' acts on wave function. Then the average values may be computed by averaging with some weight. That is why notions of absolute phases and amplitudes have no physical sense and may be selected arbitrary for usability only. Large relative changes of the amplitude in far situated points do not result in physical values changes if the wave function gradient is being transformed slightly. So Ψ^2 have a probability distribution sense but not the sense of real wave motion density as it were in the case of classic fields.

In contrast to ordinary quantum theory the phase plays quite an essential role according to our approach. For example, if a particle reaches the potential barrier being in phase of completely vanishing ($\Phi(x,t) = 0$), then due to linear character and superposition at small $|\Phi|$ it penetrates the quite narrow barrier without any interactions (Fig.4). At the other hand, if the phase is so that value of $|\Phi(x,t)|$ is maximal, then due to non-linear character interactions

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would began and the particle might be reflected. That idea results in new effect: if there were a chain of periodical (with period *a*), narrow enough (in comparison with λ_B) potential barriers, bombarded with monochrome particles flux, then abnormal tunnelling is to be considered at $\lambda_B = 2a$, that does not exist in standard quantum theory. Mathematically the process of the packet's appearing and vanishing without changing its character is possible as it is shown at Fig.1. It enables formally to understand the fundamental fact of two different amplitude interference rules: for bosons when amplitudes interfere with equal signs and for fermions – with different signs (Fig.4).

b) Relativistic case

Analyzing (2.1) one can see that wave packet Φ contains oscillations term with frequency $\omega_s = \frac{mc^2}{\hbar\gamma}$ that corresponds to Schrödinger vibration. The physical meaning of that very quick oscillating process is the follows: after "Creator" having stirred up "the medium" created wave packet the last began oscillating like membrane or string with frequency ω_s. Within the motion there arising de Broglie vibrations with frequency $\omega_B = \frac{mv^2}{\hbar\gamma}$ due to dispersion. At small energies $\omega_s \gg \omega_B$ and in the presence of quick own oscillations have no influence on experiment and all quantum phenomena result from de Broglie oscillations. The value of frequency ω_B tends to ω_s with growth of energy and resonance phenomenon appears that result in oscillating amplitude increase and in mass growth (Fig. 5). *Thus the well-known graph of particle mass dependence on the velocity approaching to light's velocity constitutes actually a half of usual resonance curve for forced oscillation of harmonic oscillator if energy dissipation is absent.* In the case when $v \to c$, frequency $\omega_B \to \omega_s$, $\gamma \to 0$ beats appear with resonance frequency $\omega_d = \omega_s - \omega_B \approx \frac{mc^2\gamma}{\hbar}$, and particle will obtain absolutely new low-frequency envelop with wave length

$$\Lambda = \frac{h}{mc\gamma} \tag{3.1}$$

This is a new wave. In ultra-relativistic limit case the value of Λ becomes much greater as typical dimension of quantum system it (new wave) interacts with. Now the length of new wave grows with energy contrary to de Broglie wave length slowly decreasing, and particle requires the form of quasi-stationary wave packet moving in accordance with classical laws. That explains the success of hydrodynamics fluid theory concerning with numerous particle birth when the packet having extremely big amplitude is able to split into series of packets with smaller amplitudes. But such splitting processes characterize not only high-energy particles. Something like this takes place at small energies also, but overwhelming majority of arising wave packets are under the barrier and so will not be detected. It would be perfect to examine by experiments at future accelerators the appearance of such new wave with the length growing together with energy. But there is once more sufficiently regretting considerations. Due to our point of view relativistic invariance of equations should be apparently changed for something else. The fact is that classical relativistic relation between energy and impulse

$$E^2 = P^2 + m^2 \tag{3.2}$$

Does not working for extra short intervals of time and small particle's displacement (equal to parts of de Broglie wave length). This relation is the result of averaging. What happens with particle impulse and mass when the packet has been spread all over the Universe? Possibly they go to zero, but particle's energy as integral of all harmonic components squares sum remains constant (no wave dissipation) and the above-mentioned relation breaks. And probably the fundamental equation (3.2) should be written in any other form. But to be sure that equation should be solved first.

V. The Theory of Optimal Detector and Quantum's Measurements

Any 'normal' measurement, in the long run, is based on the interchange of energy and is an irreversible process. That is why the particle interferes in the state of macro-device giving up (or acquiring in the case of devices with inversion) quantum of energy θ. The best measuring instrument would be one wherein the discrete threshold energy θ which characterizes device instability was absolutely minimal. With a hypothetical measurement $\theta = 0$, such that the researcher does not influence the particle with his sensor, then such a device would have 100% effectiveness and could detect any vacuum fluctuations.

The measuring instrument should be so that eventually only its classical characteristics were used for its work; in other words, Planck's constant should not play any role in it after the initiation. Such a device is as much as possible (but not totally) free from statistical effects. Thus in measuring processes particle detectors are those reference frames in what respect according to the quantum theory the system's state is to be determined.

Let us consider the process of particle – macro-device interaction [13-16]. Particle energy periodically changes with frequency ω_B and vacuum fluctuations (additionally changing the energy) are imposed at it in a random way. To detect the particle, the macro-device has to *wait* until particle total energy $|\Phi|^2$ and vacuum fluctuations ε exceeds the operation threshold θ of the device:

$$\varepsilon + |\Phi|^2 \geq \theta \tag{4.1}$$

The energy of vacuum fluctuation ε depends on the total number of the particles in the Universe and is created thanks to the particles disappeared. As far as the contribution of each partial wave in every point is infinitesimal (its distribution law may be any) in accordance with Central Limit Theorem of Alexander Lyapunov the summary background to be formed by tremendous number of particles and their partial waves will have a normal distribution with maximal entropy. The probability P of vacuum fluctuations with the energy more than ε_0 is equal to

$$P = \frac{1}{\sqrt{(2\pi)}\sigma} \int_{\varepsilon_0}^{+\infty} \exp\left(-\frac{\varepsilon^2}{2\sigma^2}\right) d\varepsilon \tag{4.2}$$

And the value σ (dispersion), depending on the particles' number within the Universe is considered in our case as constant. *The theory under consideration requires finiteness of σ, and then finiteness of the Universe.* It is evident from the last formula that the probability of the particle's detecting depends on the sensitivity of the measuring instrument.

Without entering into detail of the interaction between quantum particles with macro instruments, the problem of particle recording or detection can be stated as follows: On a wave packet with value $|\Phi|$ a vacuum fluctuation with value ε is additively imposed. For simplicity, let us regard the problem as single-dimensional and the eigenregion of the field as a segment of the numerical axis. Mark on that axis ***x*** a certain threshold value (Fig. 6)

$$\theta < a = |\Phi|$$

And let the eigenregion of the acting field be $\theta < x < \infty$. The measuring macro instrument distinguishes two situations. If there is a particle, then the value of the field which acts on the instrument is $a + \varepsilon$; if there is no particle, the value is ε. The instrument responds (the particle is recorded) when the value of the acting field exceeds a certain threshold θ, and then θ^2 is the minimal quantum energy for the macro-instrument to respond (sensitivity).

Let us find the probability of error of the instrument. Let the distribution of vacuum fluctuations $W_a(x)$ be the distribution of the sum of the particle field and vacuum fluctuations $W_0(x)$. The conditional probability of failing to detect a particle when this goes through the macro instrument is (it is the case of $\theta = \theta_1$ in Fig.6)

$$p_a(0) = p\{a + \varepsilon < \theta\} = \int_{-\infty}^{\theta} W_a(x)\,dx$$

And the conditional probability of detecting a particle when it is not there is

$$p_0(a) = p\{\varepsilon > \theta\} = \int_{\theta}^{\infty} W_0(x)\,dx$$

Let *p(a)* and *p(0)* be *a priori* the probabilities of particle flight or absence. Then the total probability of error is

$$p_{error} = p(a)\,p_a(0) + p(0)\,p_0(a) = p(a)\int_{-\infty}^{\theta} W_a(x)dx + p(0)\int_{\theta}^{\infty} W_0(x)dx$$

An instrument whose P_{error} is minimal can be viewed as optimal. When the threshold θ is lowered, the instrument sensitivity increases and thus the number of undetected particles is reducing, but the vacuum fluctuations increase the number of false recordings. When the threshold θ is increased, the number of false recordings decreases, but the number of undetected particles increases. It is intuitively clear that, at some value of the threshold θ, the value must go down to minimum (Fig. 6). Let us find that

$$\frac{dp_{error}}{d\theta} = p(a)W_a(\theta) - p(0)W(\theta) = 0$$

Assuming for simplicity that $p(a) = p(0)$, $a = Const$ we have

$$W_a(\theta) = W_0(\theta), \quad W_a(x) = W_0(x - a) \tag{4.3}$$

And

$$W_0(\theta) = W_0(\theta - a)$$

Since $W_0(x)$ is an even function,

$$W_0(\theta) = W_0(a - \theta)$$

Hence

$$\theta = \frac{a}{2} = \frac{|\Phi|}{2}; \qquad \theta^2 = \frac{1}{4}|\Phi|^2.$$

Consequently, for the optimal quantum detector the threshold energy should be one-fourth of the particle energy. Usually this relation does not hold and inequality is true $\theta^2 \ll \frac{1}{4}\Re e^2\Phi$ or the number of false recording is very high. In compliance with relation (4.3) the normalizing condition

$$\int_{-\infty}^{+\infty} W_0(x)dx = 1$$

And by assuming that the flight of the particle or its absence are equiprobable events $P(a) = P(0) = 1/2$ expression (4.3) can be transformed:

$$p_{error}=\frac{1}{2}\left(\int_{-\infty}^{\frac{a}{2}}W_a(x)dx+\int_{\frac{a}{2}}^{+\infty}W_0(x)dx\right)=\int_{\frac{a}{2}}^{\infty}W_0(x)dx=\frac{1}{2}-\int_0^{\frac{a}{2}}W_0(x)dx$$

After introducing a new variable $y=\frac{x}{\sigma}$, where σ is the r.m.s. of vacuum fluctuations, being normally distributed, we obtain

$$p_{error}=\frac{1}{2}-\int_0^{\frac{a}{2\sigma}}V_0(y)dy,$$

$$V_0(y)=\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{y^2}{2}\right].$$

Thence,

$$p_{error}=\frac{1}{2}-\frac{1}{\sqrt{\pi}}\int_0^{\frac{a}{8\sigma}}\exp\left(-z^2dz\right)=\frac{1}{2}\left(1-erf\sqrt{\frac{a^2}{8\sigma^2}}\right)$$

Then the error of the detectors is small and expressed as a fraction of the form $P_{error}=10^{-p}$ where P=0…6 for most existing instruments. Denoting $\rho=\frac{a^2}{\sigma^2}$ we have the probability of detecting the particle, if it exists, in the form

$$P=-\log\frac{1}{2}\left(1-erf\sqrt{\frac{\rho}{8}}\right)=-\log\frac{1}{2}\left(1-erf\frac{\operatorname{Re}\Phi}{\sqrt{8\sigma^2}}\right)$$

This is the interpretation of a wave function in unitary quantum theory. The relation $P(\rho)$ does not make an impression until a plot of $P(\rho)$ is seen which is well approximated, in a wide range as a straight line (Fig. 7). *In ordinary quantum mechanics it is postulated that* $P=\Psi^*\Psi$*, but nothing is said about the kind of detectors that are used for the measurement.* ***In unitary quantum mechanics the statistical interpretation is obtained from the mathematical formalism of the theory.*** *The latter includes the consideration of the problem of the statistical interaction between the particle and the detector and the sensitivity of the latter is accounted for.* Since $\rho\approx|\Phi|^2$ and in the ordinary formulation of quantum mechanics $P=\Psi^*\Psi$ then $|\Phi|^2$ and $\Psi^*\Psi$ are seen to coincide with an accuracy of terms of the second order. This correction can be verified experimentally as deflections that appear in the contrast of interference and diffraction pictures should be visible. The position of maxima and minima in such pictures cannot, of course, be affected. The most enterprising experimentalists who want to see the light at the end of the tunnel will hopefully check this.

We can easily paraphrase A. Einstein's words about "*God playing dice*". Now it is quite evident that God does not play each quantum event creating that or another vacuum fluctuation with only one aim: To force the Geiger counter to detect the particle. It is not so absolutely clear that could God do it at all, because for this He should be able tug at all the threads all over the Universe, after careful consideration, and moreover He would need an Ultra-Super-Computer. Apparently God is a perfect mathematician, for He knows Alexander Lyapunov's Central Limit Theorem. That is why He may have decided to make a simple normal distribution of vacuum fluctuations caused by vanishing particles all over the Universe. Two questions remain, however: Was it God who created that Chaos and how did He manage to do it?

VI. The Connection of UUQFT Equations With a Telegraph Equations

It is known that the current and tension of alternating electric current in pare lines satisfy the telegraph equation that was definitely derived for the first time by Oliver Heaviside from the Maxwell equation. That equation is a relativistic non-invariant which nevertheless lets us see how it corresponds to Quantum Mechanics. The question is that the main relativistic relation between energy, impulse, and mass eq. (3.2) has been still beyond any doubt. Nevertheless, we shall ask ourselves once again about what is happening with that relation at the exact moment when the wave packet disappears being spread over the space. At that moment the particle does not exist as a local formation. This means that in the local sense there is no mass, local impulse, or energy. The particle in that case, within sufficiently small period of time, is essentially non-existent, for it does not interact with anything. Perhaps this is why the relation (3.2) is average and its use at the wavelength level is equal or less than the de Broglie wavelength, which is just illegal. The direct experimental check of that relation at small distances and short intervals is hardly possible today. If the relation (3.2) is declined, then it may result in an additional conservation of energy and impulse refusal; but, as we know, according to the Standard Quantum Theory, that relation may be broken within the limits of uncertainty relation. On the other hand, the Lorenz's transformations have appeared when the transformation properties of Maxwell's equations were analyzing. However electromagnetic waves derived from solutions of Maxwell's equations move all in vacuum with the same velocity, i.e. are not subjected to dispersion and do not possess relativistic invariance. Our partial waves, which form wave packet identified with a particle, possess always the linear dispersion. Under such circumstances, it would be quite freely for author to spread the requirement of relativistic invariance to partial waves. Such requirement has sense in respect only to wave packet's envelope, which appears if we observe a moving wave packet and his disappearance and reappearance. *May be the origin of relativistic invariance* *would be connected in future with the fact that an envelope remains fixed in any reference frames; only the wave's length is changed.*

In the case of periodical vanishing and appearing wave packet (UUQFT new wave function), taking into account mass oscillation, may be rewritten in the form:

$$F(x,t) = \exp\left(i\frac{mv^2}{\hbar}t\right)\left[\varphi(x - vt) + \phi(x + vt)\right], \quad (5.1)$$

Where packets running in both positive and negative directions $\varphi(x,t)$ and $\phi(x,t)$ are totally arbitrary. For function $F(x,t)$ telegraph equation can be written in the form:

$$\frac{\partial^2}{\partial x^2}F(x,t) - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}F(x,t) + 2i\frac{m}{\hbar}\frac{\partial}{\partial t}F(x,t) + \frac{m^2v^2}{\hbar^2}F(x,t) = 0 \quad (5.2)$$

Equations resembling (5.2) may be obtained from Maxwell equations by making a supposition about imaginary resistance of the conductor and using Oliver Heaviside reasoning while deriving from the telegraph equation. However, the equation (5.2) has another solution matching the main idea UUQFT about the appearing and vanishing packet. That solution [20-23] has the following form:

$$F(x,t) = \exp\left(\pm i\frac{mv}{\hbar}x\right)\varphi(x \mp vt) \quad (5.3)$$

where we should take the top or bottom sign. Let us write function (5.1) or (5.3) in the form:

$$F(x,t) = \exp\left(i\frac{mv^2}{\hbar}t\right)\Psi(x,t) \quad (5.4)$$

Or

$$F(x,t) = \exp\left(i\frac{mv}{\hbar}x\right)\Psi(x,t) \quad (5.5)$$

By substituting function (5.5) into the equation (5.2) we get

$$\exp\left(i\frac{mv^2}{\hbar}t\right)\left(v^2\frac{\partial^2}{\partial x^2}\Psi(x,t) - \frac{\partial^2}{\partial t^2}\Psi(x,t)\right) = 0$$

Reducing the exponential function we get the wave equation. So in the new quantum equation (5.2) O. Heaviside conditions are automatically satisfied (absence of distortion in telegraph equation solution).

Let us insert in our equation (5.2) potential *U(x)* in a general way. The velocity of the particle with the energy *E* in a field with potential $U(x)$ may be written as follows:

$$v = \sqrt{\frac{2(E - U(x))}{m}}$$

Substituting it into the equation (5.3) and rejecting imaginary terms, we get:

$$\left[-2\hbar^2E\frac{\partial^2}{\partial x^2} + 2\hbar^2U(x)\frac{\partial^2}{\partial x^2} + \hbar^2m\frac{\partial^2}{\partial t^2} - 4mE^2 + 8mEU(x) - 4mU(x)^2\right]F(x,t) = 0 \quad (5.6)$$

Let us divide variables in the equation (5.6) in accordance with the standard Fourier technique, assuming that

$$F(x,t) = \Psi(x)T(t)$$

After a common substitution in (5.6) and dividing by the product of sought functions we get:

$$\frac{\hbar^2}{\Psi(x)}(U(x) - E)\frac{\partial^2\Psi(x)}{\partial x^2} + \frac{m\hbar^2}{2T(t)}\frac{\partial^2T(t)}{\partial t^2} - 2mE^2 + 2mU(x)(2E - U(x)) = 0 \quad (5.7)$$

After coordinate function $\Psi(x)$ separation and after simple transformations we get the following equation

$$\frac{U(x) - E}{\Psi(x)}\left[2mU(x)\Psi(x) - 2mE\Psi(x) - \hbar^2\frac{\partial^2\Psi(x)}{\partial x^2}\right] = 0$$

And we obtain easily the Schrödinger equation:

$$\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x) = (U(x) - E)\Psi(x)$$

Now substitute function (5.4) into equation (5.2). We obtain

$$\exp\left(i\frac{mv}{\hbar}x\right)\left[-2imv^3\frac{\partial\Psi}{\partial x} - \hbar v^2\frac{\partial^2\Psi}{\partial x^2} + \hbar\frac{\partial^2\Psi}{\partial t^2} - 2imv^2\frac{\partial\Psi}{\partial t}\right] = 0$$

.

By rejecting imaginary terms and reducing we get the wave equation and Heaviside conditions for the absence of distortion are again satisfied. It is curious that while rejecting imaginary terms and requiring $v \to c$,equation (5.2) is automatically transformed into the Klein-Gordon type equation. All the previously mentioned reasoning can be easily generalized for the three-dimensional case.

It is possible to write down (for the invariance-lover) the following two variants of our telegraph equations:

$$\frac{1}{v^2}\frac{\partial^2F(x,t)}{\partial t^2} - \frac{\partial^2F(x,t)}{\partial x^2} + \frac{2imc^2\sqrt{1-\frac{v^2}{c^2}}}{\hbar v}\frac{\partial F(x,t)}{\partial x} + \frac{m^2c^4}{\hbar^2v^2}\left(1-\frac{v^2}{c^2}\right)F(x,t) = 0$$

And

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$$\frac{1}{v^2}\frac{\partial^2 F(x,t)}{\partial t^2}-\frac{\partial^2 F(x,t)}{\partial x^2}-\frac{2imc^2\sqrt{1-\frac{v^2}{c^2}}}{\hbar v^2}\frac{\partial F(x,t)}{\partial t}-\frac{m^2c^4}{\hbar^2 v^2}\left(1-\frac{v^2}{c^2}\right)F(x,t)=0$$

These two equations are satisfied exactly by relativistic invariant solutions in the form of a standard planar quantum-mechanical wave and also in the form of disappearing and appearing any scalar wave-packet, viz.

$$F(x,t)=\exp\left(\frac{i}{\hbar}\frac{mc^2t-mvx}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$F(x,t)=\exp\left(\frac{i}{\hbar}\frac{mc^2t-mvx}{\sqrt{1-\frac{v^2}{c^2}}}\right)\varphi(x-vt)$$

The results obtained are quite amazing. It is well known that nearly any equation of theoretically non-quantum physics can result from Maxwell equations. That is why Ludwig Boltzmann said this about Maxwell equations: *"It is God who inscribed these signs, didn't He?"* Modern science has changed not a semi-point in these equations, and now it appears that even non-relativistic quantum mechanics in the form of the Schrödinger equation may also be extracted from the Maxwell equation. The same can be said about the Klein-Gordon relativistic equation. Moreover, telegraph equation, Schrödinger and Klein-Gordon equations have allowed calculating the spectrum of the masses of the elementary particles without any free parameters [21-23]

VII. The Solution of The Approximate UUQFT Scalar Equation and The Value of The Fine Structure Constant

In papers and books [17-21], the basic equation (2.8) was reduced to the scalar equation for the density of the space charge of the space charge of the bunch, which represents the particles:

$$\frac{1}{c}\frac{\partial\Phi(r,t)}{\partial t}+\frac{\partial\Phi(r,t)}{\partial r}+\frac{4\pi i\Phi(r,t)}{\hbar}\int_0^r\left\{\Phi^*(s,t)\frac{\partial\Phi(s,t)}{\partial t}-\frac{\partial\Phi^*(s,t)}{\partial t}\Phi(s,t)\right\}s^2ds=0 \quad (6.1)$$

We seek the solution in the form

$$\Phi(r,t)=\bar{F}(r)\exp\left[-i(\omega t-kr)\right] \quad (6.2)$$

We get the following system of equations if the condition

$$\omega=kc$$

Is fulfilled:

$$\frac{d\bar{F}(r)}{dr}+\frac{8\pi\omega\bar{F}(r)}{h}\int_0^r s^2\bar{F}^2(s)ds=0, \quad (6.3)$$

Let us suppose

$$x=\frac{r}{R},\quad f(x)=\frac{\bar{F}(r)}{\bar{F}(0)},\quad \bar{F}(0)\neq\infty$$

Equation (6.3) can be expressed in dimensionless form:

$$\frac{d^2\ln f(x)}{dx^2}+Kx^2f^2(x)=0 \quad (6.4)$$

Where

$$K=\frac{8\pi\omega R^4\bar{F}^2(0)}{h}$$

Solving numerically the Cauchy problem for the eq. (6.4), taking the value $K=16\pi=2\cdot2\cdot4\pi$ (where 4π from $dV=4\pi r^2dr$, 2 from integral (6.1) and 2 from charge oscillation) and the initial conditions:

$$f(0)=1,\quad f'(0)=0, \quad (6.5)$$

we obtain the following integral:

$$I_Q=\int_0^\infty x^2f^2(x)dx=8.5137256105758897351\cdot10^{-2}$$

$$I_Q^2=1/137.9623876 \quad (6.6)$$

The quantity I_Q is a dimensionless electrical charge, which is brought to the following dimensional form:

$$Q=\sqrt{\hbar c}I_Q=4.78709\cdot10^{-10}CGSE$$

This value is less than the modern experimental value of the electron's charge by only 0.3%. This is a fairly accurate number for the first theoretical attempt of the charge calculation. Thus it is not unusual to bring out the "corrections" of the J. Schwinger type to the integral (6.6)

$$I_e=I_Q+\frac{I_Q^2}{8\pi}-\frac{I_Q^3}{64\pi^2}=8.54246819177841\cdot10^{-2},$$

Which corresponds to the value of charge e = 4.803 2514 • 10^{-10} *CGSE* and the value of fine-structure constant $\alpha=1/137.0355538109$. *The quantization of the electrical charge and masses seems to be the consequence of the balance between the dispersion and nonlinearity, which determines stable solutions.*

We regret that we have not succeeded in finding an analytical solution of eq. (6.4), but we are able to give a decent approximation. Let us look for a solution of eq. (6.4) in the form

$$f(x)=\text{sech } R(x) \quad (6.7)$$

Substituting eq. (6.7) into eq. (6.4) and taking into account that for small *R* we have:

$$\frac{1}{2}\sinh\ 2R \approx R$$

We obtain

$$\left(RR^{'}\right)^{'} = 16\,\pi x^2\,;\quad R = \sqrt{\frac{8\pi}{3}x^2}\qquad f(x) = \operatorname{sech}\sqrt{\frac{8\pi}{3}x^2}$$

Author notice that *not used any other constants except π for calculation of the fine structure constant integration and it had not introduced itself in an underhand way.*

VIII. The Uncertainty Relation and Principle of Complementarity in UUQFT

As far as many nonsense have been announced concerning the uncertainty relation we would like to give more detailed of their obtaining first by W.Heisenberg then by N. Bohr and of not quite adequate their interpretation. So, Heisenberg derived the uncertainty relation on well-known now way, now called the method of Heisenberg's microscope and based on the analysis of conditions when microparticle's position and motion can be experimentally detected. In principle, the particle's position can be determined by observations of light rays reflected, diffused or emitted by the particle. The particle is considered as a source of light and the results of its observation will be always the diffraction circle with radius equal to the wave length λ of this light rays. So the particle position can be determined with precision of order λ.

The most primitive idea to improve the accuracy of measurements is to use light rays with λ being so small as it is possible. It is possible to use, for example, gamma sources, technical implementation of that idea for the time being is not so important. But at the same time we faces A.Compton effect; in the process of measuring the gamma quantum is scattered by the particle and with it the impulse of the particle will be changed for the value equal $\hbar/_{\lambda}$. It is paradoxical, but, for example, we will get the same result, for example, in the case of atom while being allocated with the help not of scattered light but of light emitted by atom itself. If the light is emitted in the form of quantum $\hbar\omega$, then atom will receive recoil momentum $\hbar/_{\lambda}$, and again the study of atoms position will depend on its velocity changes. In both cases the accuracy of atom position determined with the help of scattered or emitted light equals to the wavelength of the light, and momentum change connected with it will be inversely λ. Increasing the measurements accuracy of particle position, we enlarge the error of definition its momentum. In the result it is impossible to determine the particle momentum at the exact moment of time, when is determined the position of particle since the momentum of particle sharply changes at that very instant. The same considerations would be taken into account at velocity determining also, that resulted in famous Heisenberg relations.

The following philosophical problem appears: is it possible, in principle, to observe any phenomenon without changing it or interfering in it? This problem is no doubt quite old and banal. Anybody agrees that, for example, measuring the electric potential of any object should to change to a certain degree this potential. Any innovations of that measuring apparatus have dealt mainly with tendency to enlarge voltmeter internal resistance and with unachievable idea to make it equal to infinity. Every experimentalist has learned to take into account such non-ideal characteristics of instruments in the process of measuring. And nobody was thrown into confusion with that.

It was proudly announced at the outset of quantum theory that micro-particle does not have at the same moment of time the exact values of co-ordinate and momentum and their values are connected by relation:

$$\Delta x \cdot \Delta p \geq \hbar \qquad (7.1)$$

And that statement and that inequality were called as corresponding to nature of micro-worlds objects and quite not caused by lack of appropriate measuring instruments. But the following question may be put: what will happen if within future decades indirect methods possible to use for measuring purposes will be opened? Who is able to foreseen the future?

Shortly after another relation was derived, viz. between energy and moment of time, when that energy being measured:

$$\Delta E \cdot \Delta t \geq \hbar$$

That relation appeared in great number of books due to intellectual inertia of some author. And only much later the investigators made out that such relation does not exist within strict quantum mechanics as well as the following relation

$$t \cdot \hat{H} - \hat{H} \cdot t = i\hbar$$

Does not exist. On the other hand, the operator relation

$$x \cdot \hat{p}_x - \hat{p}_x \cdot x = i\hbar$$

Exists and results in uncertainty relation for the coordinate and momentum.

N. Bohr have obtained the same relation after manipulating with wave packets of de Broglie waves (creating a particle from these waves packets), but he had *carefully forgotten* that these wave packets were

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spreading. To put it mildly that approach is not quite correct. More over the principle of complementarity offered by Bohr *ad hoc*, forbade the constructing any speculative models of particle's motion. Since that ***the main task of the physics became the search of mathematical expressions to be set in one experimental data to obtain the other by computations (!?)*** According to it, the lack of picture in images and motions within quantum physics is not the object of anxiety. We would rehabilitate the strict standard quantum theory and notice once again that, according to it, the uncertainty relation is obtained as the relation between canonically conjugate additional dynamic variables, and we have nothing to say against. In the essence, the corpuscular – wave dualism became the winner. As we can see now, the uncertainty relation is without any doubts valid but methods used *at first* for its obtaining were not totally adequate.

UUQFT overcomes the situation quite easily. As far as the particle (wave packet) is periodically appearing and vanishing at de Broglie wave length (more precisely, the packet disappears twice, and the probability of its detecting is sufficiently big in maximum region only) the position of such a packet may be detected with error

$$\Delta x \geq \frac{\lambda}{2}$$

And then

$$\Delta x \cdot P \geq \frac{h}{2}.$$

As at measuring of momentum module is inevitable the error $\Delta P = 2P$, then we have following inequality:

$$\Delta x \cdot \Delta P \geq h \qquad (7.1)$$

The statements of standard quantum mechanics that particles do not have a trajectory become more understandable. Of course, there is a lot of truth in those words. First, it is possible to say so about intermittent (dotted) motion of the particle with oscillating charge [19-21]. Second, any packet (particle) is able during its motion to split into few parts. Each of that parts being summed with vacuum fluctuation may results, in principle, in few new particles. Or *visa versa* the broken particle may vanish at all and contribute to general fluctuating chaos of the vacuum. But in any case it is better to have more clear idea of particle concrete motion than operate with generally accepted nowadays-obscure sentence about lack of trajectory. The whole preceding science was based on classical description of objects without taking into consideration material character of the observation process. In other words it was the description of objects or processes "in itself". Quantum science has assigned some limit of such understanding, and although UUQFT allows describing hypothetically the behavior of quantum objects in "images and motions" there is now either above mentioned hypothetical researchers or their hypothetical experimental devices, and we will have to be content with experimental data obtained with the help of macro-devices.The principle of complementarity introduced by N.Bohr cannot be explained so easily as it were in the case of uncertainty relation, because it is a set of some philosophical discourses with marks of previous years fight between materialism (it was also called Marxism-Leninism) and other philosophical trends. We would like just now isolate ourselves from any politics, because author do not sympathize politics and philosophical brawls, and tried never to participate in it. Nevertheless, there are objective laws that will not be changed even author and readers disappear, and politicians declare the collapse of materialism and of the said laws. As UUQFT is able to show many "intimate" sides of quantum behavior and to give the sufficient interpretation of existing quantum processes, the result is quite simple: materialism is gained.

Let us consider rather in more details the principle of complementarity. It is hard to disjoint it from uncertainty relation. Even the origin of its name came from ordinary mechanics, where operators non-commutating with each other correspond to complementary quantities. As we have seen above the uncertainty relation descends from that also. Nevertheless, it is appeared a lot of philosophical explanations which Bohr even had not suspected of. The principle of complementarily can be stated quite popular as follows:

1. A quantum object is extremely complicated formation, not quite easily understood yet, and it's corpuscular and wave characteristics are absolutely unlike and only supplement each other. We can draw rough analogy: maps of Eastern and Western hemispheres, men' photos in full and half face and so on.
2. There are two classes of experimental devices. With the help of ones we can measure the coordinate, the energy and the momentum – the attributes of a particle. With other, while observing the processes of interference or diffraction, one can measure the wavelength. At any measuring (in cases of small energies) particle "is lost" or its parameters change radically in the result of macro situation effect. All that is called as uncontrolled effect that is why it is impossible to measure at the same moment of time corpuscular and wave parameters.
3. We should not ask Nature questions that will not be experimentally answered.
4. It is not necessary to make attempts in constructing the quantum pictures in images and motions as it were within before-quantum science. It is quite enough to be able mathematically to solve and to

analyze different quantum equations and to apply the new rules derived within quantum mechanics. The attitude of Paul Langevin to the last two items was as to something disgusting and he called the principle of complementarity as “intellectual debauchery”.

The other numerous statements are based on variants of uncertainty relation.

There were many physical and philosophical discussions about photon behavior at semitransparent mirror (Fig.9). With the help of complementarity principle it was analyzed in what flux (reflected or penetrated) the photon is located while the interference of penetrated or reflected flux is observed and how it correlate with the number of particles to be appeared in penetrated and reflected fluxes. When the flux of particles falling down on the translucent mirror one after another was observed with big exposition, then the interference picture became visible. It contradicts the fact that the particles was detected either in penetrated or reflected flux, and it is incomprehensible how could the interference picture arise. If the particle remains in reflected flux, then it could not been observed in the passed flux, and it is impossible to understand what and with what would interfere. The observed facts of rare simultaneous signals of two particle counters were explained by random appearance of two photons “nearby”, and one of them has penetrated the mirror and the other – has reflected. There were some reasons due to observations of induced radiation (that is the main principle the lasers based on). There were made quite enough different experimental variations at that matter. We should note that they are do not contradict the ideas developed within UUQFT. Of course not only the processes of splitting cause the phenomena of interference and diffraction. It was shown in [19-21] that even indivisible particle described by equation with oscillating charge while spreading is able to show the behavior having seemingly a wave character. All these processes look very knotty.

N.Bohr has offered well-known interpretation of that phenomenon from the principle of complementarity viewpoint. We shall remind it shortly. The particles’ flow falling down at the mirror is described by wave function (i.e. by the amplitude of probability). The particle after hitting at translucent mirror is, so to say, in a potency state: the particle may belong to penetrate or to reflected flux, it maybe appeared (detected) and maybe not. Namely, that potency is interfering, i.e. possibility of particle’s location here or there. These potential possibilities become actual at the finish of object and device interaction only. And though probabilities are referred to potential- possible, i.e. to non-finished experiment, but statistics based on these probabilities is a statistics of realized interactions, i.e. of finished experiments. But if an experimental device would be created being able to follow the destiny of individual particle and to detect to what flux (penetrated or reflected) the particle belong, then the particle would be absorbed or its parameters would be changed at such a value that we would not be able to speak about its participation in interference process. If this process is studied, then it is impossible without violation of interference process to detect the flux, where the photon is located. Either one thing or another, they cannot exist together.

We should note, - *it is worthy of astonishment that N. Bohr was able to imagine that principle and interpretation, because it turned out that if one follows strictly the prescribed principles and rules, then the right results are obtained and no contradictions arise.* **All paradoxes were eliminated by simple prohibition to think about it!** It stimulated a great philosophical discussion but physicists did not pay attention at. And they were right since that discussion took the form of some talks resulted in nothing, but orthodox quantum interpretation answered every physical question to be asked within new unusual game rules and served as perfect instrument of knowledge. Nevertheless for any thinking researcher the question whether it true raised always. Why we could not even imagine that particle has exact values of momentum and coordinate and follow it dynamics in details? Why we could not study with any indirect methods the concrete sides of particle motion (as it take place in other sciences)? There are appeared also absolutely new philosophical problems about “free will” and even about the existence of particles in connection with probability interpretation of wave function. Religion was also admixed due to A. Eddington.

There was quite solitary the question about the cause of quantum mechanics statistical character. In connection with that the words of A.Einstein are quoted especially frequently about his unbelief in “*God is playing cards*”. There are so many different speculations about that. But the main is that *statistical interpretation does not belong to quantum mechanics instrument and does not result from it but simply postulates.* That is not so within our UUQFT and the probability of phenomena appears due to inner content of this theory, and, as we hope, the question about how “*God is playing cards*” has disappeared for most part of our readers at the moment of reading these words.

The author is sure that all additional philosophical quantum-mechanical images of the nature will be crushed down in the nearest future and UUQFT will gain, and the above mentioned problems will surprise future generation as well as now we are amazed at ancient opinions about three elephants and three whales supporting our Earth. It is astonishing but even these quite naïve ideas had relaxed or rather lulled humanity mind during very long time.

IX. Possible Experimental Tests and Results

The developed theory will remain a freak of the imagination if the following effects will not be experimentally confirmed:

1. Let very weak source emits by parallel bunch of *N* particles per 1 sec. If the place in front of it is a gate to be opened during the experiment for short interval $\tau \ll \frac{1}{N}$, then most probably that no one particle will penetrate or they will be able to do it one by one.

Let these particles fall down on the angle 45 degrees at translucent mirror (Fig.9). According to ordinary quantum mechanics the particle will either penetrate the mirror or reflect. In accordance with the point of view described at that article the bunch will split up at the mirror into two, three…of smaller bunches that depends on bunch phase in front of the mirror and on structure of the mirror in given place. In general we will get two non-similar wave packets (under-thresholds particles or particles converted into state of phantoms) with smaller amplitudes. There are no changes of frequency ω in formula $E = \hbar\omega$ (reddening), because all processes are linear, i.e. do not depend on amplitude. Besides the particle energy $|\Phi|^2$ decreases, that results in reducing the probability of its detecting (considerable vacuum fluctuations is necessary, but the probability of it appearance is too small). So some particles should disappear sometimes during process of measuring or visa versa two particles should appear instead of one. The appearance of two particles from one does not contradict to energy conservation law, as far as the energy of under threshold particle may be increased up to the necessary level due to fluctuations.

Note. A lot of experiments have been carried for example (R.H. Brown [4], J. Klauder [8]) and many others) resulted in conclusions that particles always have distinct tendency to reach detectors in correlated pairs (!) That result confirms we said above. Amusingly, that some physicists have invented special devices of coherent state type for explanation of these experiments refuting standard quantum mechanics. Late the experiments with delayed choice were carried out also confirming the developing in our article point of view. The description of those experiments can be found at "Scientific American" magazine under the title "Quantum philosophy". And quite recently the effect of electron division into two electrons (!!!) has been experimentally detected (H.Maris, [11]). *If those results were true, then it would be the most direct confirmation of UUQFT and total disaster for the ordinary quantum theory.* Unfortunately till now nobody has taken into his head to interpret the results of all such experiments in this way, because energy conservation law formally prohibits it. The last is thoroughly checked at very high levels of energy, and since the energy in that case considerably exceeds the energy of vacuum fluctuation, everything is hold true. But at small energies nobody have studied that question directly. We should repeat once again that any result to be obtained at small energies for one definite particle is random; more over the indeterminateness principle gives no opportunity to detect something precisely for separate particles.

2. The coefficient of passing of any coherent particles with small energies ($\lambda_B \cong 0.5A$), through the series of periodical potential barriers (mono-crystal) will be maximal at ($\lambda_B = 2a$), where *a* is the target grid mono-crystal constant (Fig.4). The same, but less weaker effect should appear again at ultra-relativistic energies, when $\Lambda = 2a$. To run such experiments the flux of mono-energetic and synchronous in phase particles is required. It can be obtained by selecting narrow packet of particles reflected from mono-crystal.

3. In connection with the fact that slowly changing part of space-time generates a field, and local hump of that field is a particle periodically disintegrating and appearing, the theory cannot consider processes not satisfying the field laws. Then un-removable vacuum fluctuations really existing will be in such theory non-invariant relative to rotations, transmissions, and space and time reflections and, therefore, conservation laws concerned with them will be non-local and approximate. Such infringements easily arise when particle energy $|\Phi|^2$ is of the same range as dispersion σ of vacuum fluctuations is. Unfortunately, these processes will arise near the threshold and therefore they are difficult for investigation.

4. Since every particle can spontaneously arise from vacuum or vanish with very small probability, all chemical elements are subjected to absolutely new type of nuclear transformations: any element may be transformed into his isotope or into one of his nearest neighbour in periodic table. Upon a time, (E.Rutherford, 1905) pointed it out, and these processes were really discovered in geology, but they have no explanations.

5. At collision of any particles the processes of mutual penetration without any other interaction are to be detected in the case when in the point of collision one of particles or both will spread. It seems, *s* – state of hydrogen atom is a good illustration of that. We should note that the same phenomena have appeared in Bohr-Sommerfeld model (pendulum orbits) too, but were rejected at once by standard quantum theory as quite preposterous.

6. We present any particle as a moving wave packet. From mathematical perspective after Fourier transformation our packet equivalents endless set of flat harmonic waves, which nowhere begin and nowhere

end. If a medium with strong dispersion is placed on the way of these waves [26] or behind, it provides conditions for a particle appearance, and at the same time there is nothing moving (!). Also, in UUQFT there are no any limits for velocities of the particles! In the UUQFT also no any limits for velocities of the particles! But on the other hand, usual using the determination to velocities in the UUQFT not applicable. *Let's to entrust the mathematician!* During the last several years many groups have experimentally confirmed possibility of superluminal light propagation. ***This should be considered as direct experimental proof of UUQFT principle.***

7. The new wave from eq. (3.1) is $\Lambda = 120\text{Å}$ for Stanford-SLAC and easy can be measure.

8. Based on UUQFT calculations with high accuracy of the Mass Spectrum of some elementary particles and Electron charge [17, 18, 21-23].

9. The cold nuclear fusion was prediction by author in 1983 [25].

10. The prediction possibility creation of a new source energy and now had good explanation for very much number strange energy installations [19-21].

11. Find new common approach to the any catalytic reactions [24].

12. Violation of the Bell inequality confirm UUQFT.

In general, the ideas of UUQFT can influence in many aspects of civilization. But still there is a question to be discussed:

But remain next problems: What we will sacrifice if replace an Ordinary Quantum Mechanics by the Unitary Unified Quantum Field Theory Field (UUQFT)?

1. There are not in UUQFT strict principles of superposition. It violated if wave packets are collide.
2. There are not in UUQFT strict close systems and the Conservation Laws for small energies. Remark the Conservation Laws forbid origin Universe.
3. The classical relativistic relation between energy and impulses is valid in UUQFT only after averaging of observed phenomena and Relativistic invariance itself is not "the sacred cow".
4. The Space-Time in UUQFT are non homogenous and non isotropic.
5. The particles and their interaction are not local.
6. The existing Standard Model Quantum Theory of Elementary Particles requires much alteration.

There was observed resembling crushing defeat of physics 50 years ago as "weak interaction" burst, so to say, into physics.

X. Conclusion

It would be appropriate to mention one more statement of one of quantum theory founders (quite disavowing this theory, but almost unknown – why? – among broad scientific community):

"There are many experiments that we are just not able to explain if we don't consider the waves as namely waves exerting its influence upon all region, where they spread, and assume the location of these waves being "possibly here, possibly there according to probabilistic viewpoint". E. Schrödinger, Brit.J.Philos.Sci, vol.3, page 233, section 11, 1952 , (back translation).

In conclusion it would be relevant to mention that Louis de Broglie predicted this discovery: *"Those who say that new interpretation is not necessary I would like to note that new interpretation may have more deep roots and such theory in the long run will be able to explain wave-particle dualism, but that explanation will not be received either from abstract formalism, modern nowadays, or from vague notion of supplementary. But I think that the highest aim of the science is always to understand. The history of the science shows if any time somebody succeeded in deeper understanding of physical phenomena class, new phenomena and applications appeared. Hope that many researchers will study that enthralling question casting aside preconceived opinions and not overestimating the importance of mathematical formalism, whatever beautiful and essential it was, because that may result in loss of deep physical sense of phenomena"* Louis de Broglie, Compt. Rend, 258, 6345, 1964, (back translation).

The offered outline of unitary quantum mechanics for a single particle from the position of unified field is rather simple and obvious from hypothetical observer's point of view. If a hypothetical observer usually can measure the value of the wave function amplitude, we cannot do it at all. We have to be satisfied with its probability interpretation keeping in mind that rather very simple mechanism is hidden behind and this mechanism open the way for explanation of quality transformations of quantum phenomena, and allows to reduce the description of the whole nature to description of some united field, and the continuous transformations of that field show the astonishing variety of phenomena being under observation.

Now the UUQRFT is the new Quantum Image of the World. It is a realized the Unitary Program formulated at first by William Clifford, Louis de Broglie and Erwin Schrödinger and later declared by Albert Einstein. William Clifford (1870) wrote (back translation): "I have no doubts about the following: small parts of space are similar in their nature to irregularities on a surface which, on the average, is flat. The quality of being curved and deformed continuously passes from one part of space to another like the phenomenon that we call the movement of matter, ethereal or corporeal. In the real physical world nothing happens except these variations, which is probably in compliance with the continuity law."

Now we have an abstract base as some unified field only. Any particle is represented as a cluster or a wave packet formatted inside this field. And we have intuitively intelligible explanation of the wave-corpuscular dualism, clearing up mechanisms of tunnel effect, of uncertainty relation, of cold nuclear fusion, of electron's division, of chemical catalysis, photon entanglement, teleportation etc.

In spite of mathematical complexity the Unitary Unified Quantum Field Theory will stop being paradoxical and frank words of Richard Feynman : *"I can easily say that nobody understands quantum mechanics"* will become the property of history.

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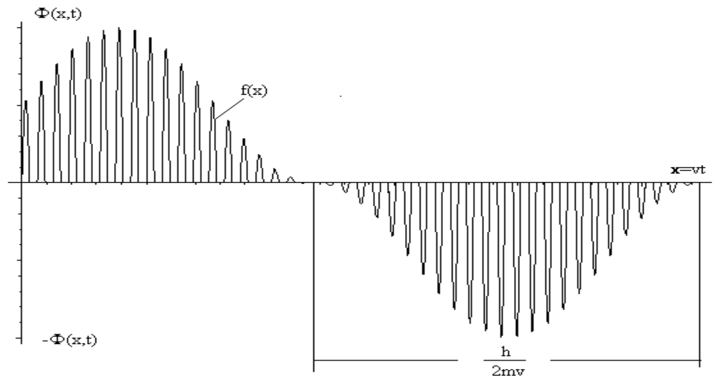


Figure 1: Behaviour of wave packet in linear dispersion medium

(i.e., rather like a series of stroboscopic photographs).

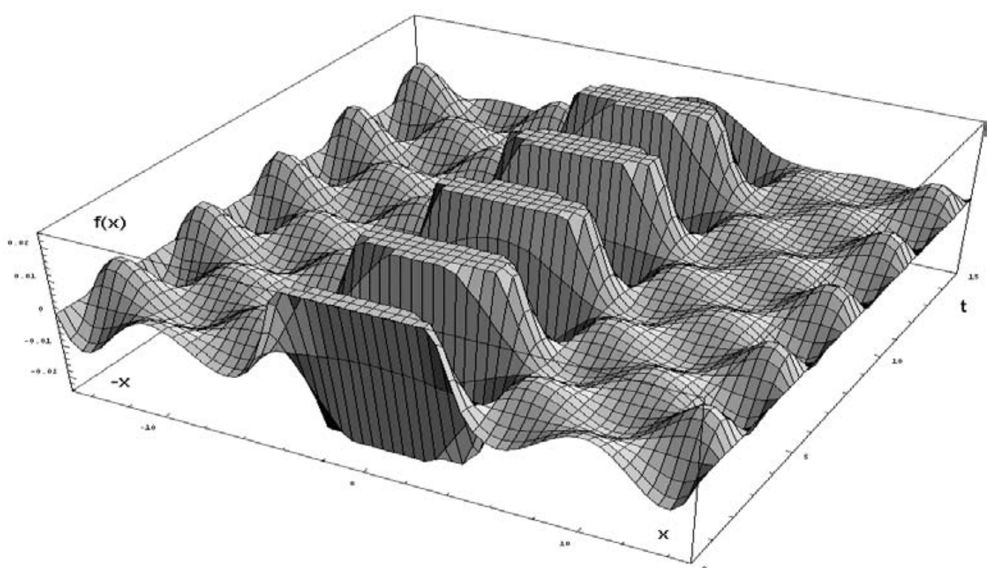


Figure 2 : Mathematical modeling of Gauss packet behaviour

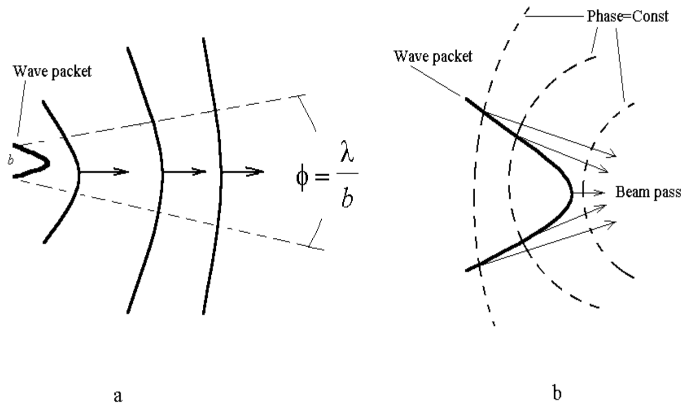


Figure 3 : Wave packet dispersion and refocusing

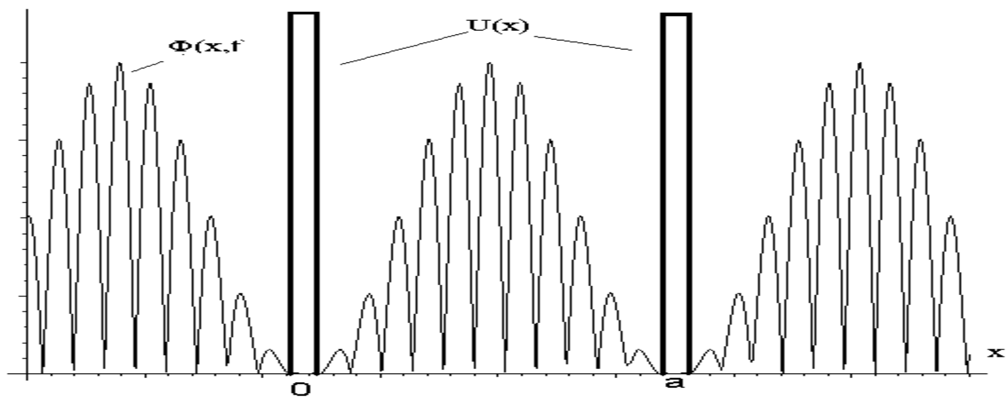
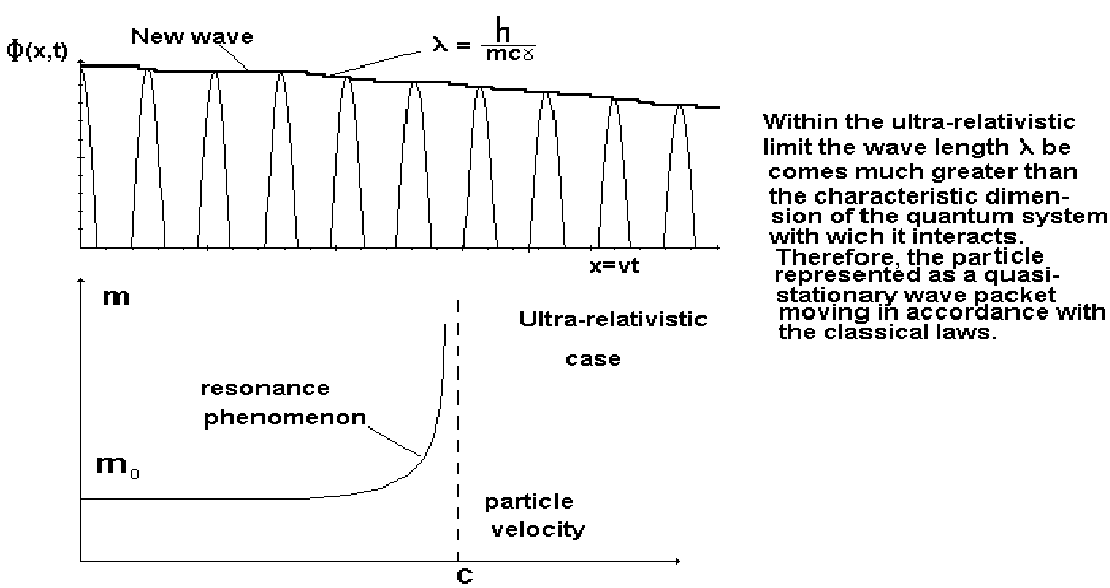


Figure 4 :



$$v \to c, \gamma \to 0, \omega_d = \omega_s - \omega_b \approx \frac{mc^2}{\hbar}$$

Figure 5 :

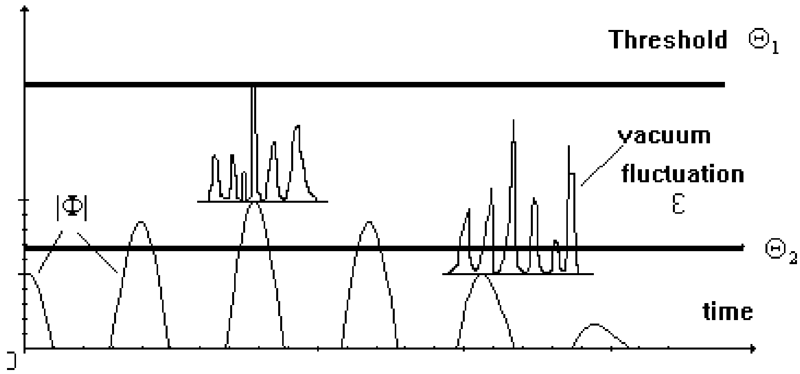


Figure 6 :

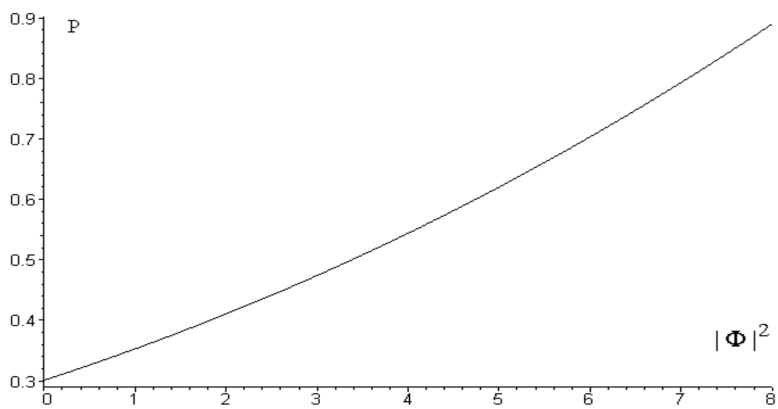


Figure 7 : Probability of regular detection of particle as a function of $|\Phi|^2$

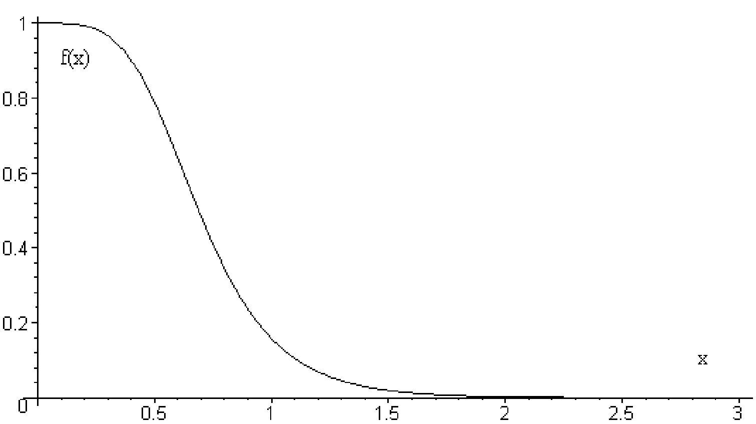


Figure 8 : Shows diagrams for the equation computational solution eq.(6.4).

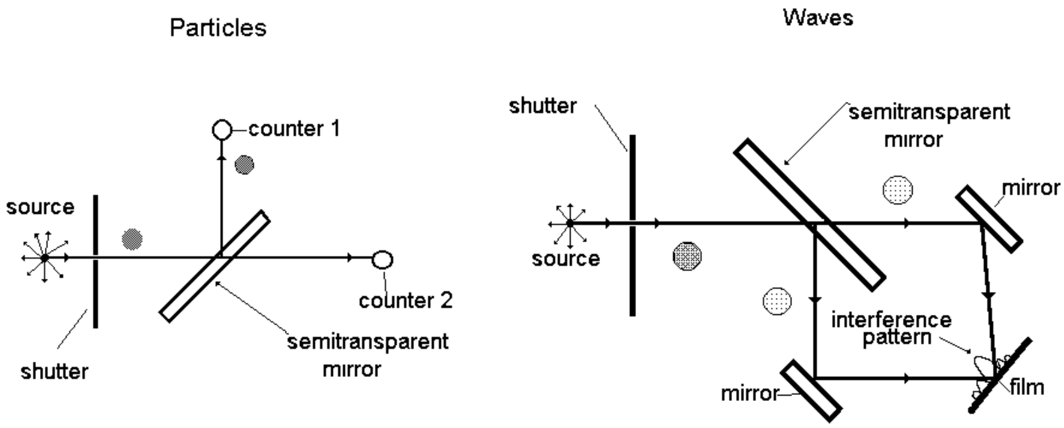


Figure 9 : Experiments with individual photons on semitransparent mirror

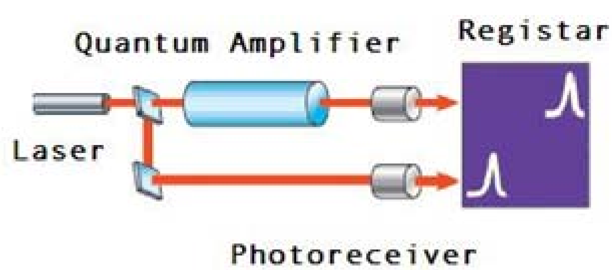


Figure 10 : Experiments of L.Wang - superluminal light propagation.

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Some Definite Integrals of Gradshteyn-Ryzhik and Other Integrals

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Abstracts - In the present paper we evaluate three definite integrals with certain convergence conditions, using Leibnitz rule for differentiation under integral sign and Wallis formula. Some other integrals are also evaluated by means of Leibnitz rule, Kummer's first transformation and reduction formula, series rearrangement techniques under stated convergence conditions.

Keywords and Phrases : *Leibnitz rule for differentiation under the integral sign; Generalized Gaussian Hypergeometric Function; Kamp´e de F´eriet's General Double Hypergeometric Function; Gamma Function ; Kummer's first transformation; Series rearrangement technique.*

2010 Mathematics Subject Classification : *Primary 33B15, 33C20 ; Secondary 33C65*

SOME DEFINITE INTEGRALS OF GRADSHTEYN-RYZHIK AND OTHER INTEGRALS

Strictly as per the compliance and regulations of:

Some Definite Integrals of Gradshteyn-Ryzhik and Other Integrals

M. I. Qureshi[α], Kaleem A. Quraishi[Ω], Ram Pal[α]

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Keywords and Phrases : Leibnitz rule for differentiation under the integral sign; Generalized Gaussian Hypergeometric Function; Kampé de Fériet's General Double Hypergeometric Function; Gamma Function; Kummer's first transformation; Series rearrangement technique.

I. Introduction

The Pochhammer's symbol or Appell's symbol or shifted factorial or rising factorial or generalized factorial function is defined by

$$(b,\ k) = (b)_k = \frac{\Gamma(b+k)}{\Gamma(b)} = \begin{cases} b(b+1)(b+2)\cdots(b+k-1); & \text{if } \ k \in \mathbb{N} \\ 1 \quad ; & \text{if } \ k = 0 \\ k! \quad ; & \text{if } \ b = 1,\ k \in \mathbb{N} \end{cases}$$

Where b is neither zero nor negative integer and the notation Γ stands for Gamma function. Throughout this work we shall employ the following definitions.

Generalized Gaussian Hypergeometric Function

Generalized ordinary hypergeometric function of one variable [4,p.73(2);5,p.42(1)] is defined by

$${}_AF_B\left[\begin{array}{ll} (a_j)_{j=1}^A & ; \\ & \qquad z \\ (b_j)_{j=1}^B & ; \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1.1}$$

Where denominator parameters $b_1, b_2, \ldots, b_B$ are neither zero nor negative integers and A, B are non-negative integers. The symbol $(a_j)_{j=1}^A$ represents the array of A parameters given by $a_1, a_2, \ldots, a_A$ with similar interpretation for others.

Conditions for Convergence of (1.1)

If A $\leq$ B, then series ${}_AF_B$ is always convergent for all finite values of z (real or complex).

If A = B + 1, then series ${}_AF_B$ is convergent for $|z| < 1$.

For more convergence conditions we refer [4,pp.73-74;5,p.43].

Kampé de Fériet's General Double Hypergeometric Function

We recall the definition of general double hypergeometric function of Kampé de Fériet in slightly modified notation of H.M.Srivastava and R.Panda [5,pp.63-64(16,17)]:

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$$F^{A:B;D}_{E:G;H}\left[\begin{array}{ccc}(a_j)_{j=1}^{A}:(b_j)_{j=1}^{B};(d_j)_{j=1}^{D} & ; & \\ & & x,\,y\\ (e_j)_{j=1}^{E}:(g_j)_{j=1}^{G};(h_j)_{j=1}^{H} & ; & \end{array}\right]=\sum_{m,n=0}^{\infty}\frac{((a_A))_{m+n}\,((b_B))_m\,((d_D))_n\,x^m\,y^n}{((e_E))_{m+n}\,((g_G))_m\,((h_H))_n\,m!\,n!} \tag{1.2}$$

Conditions for Convergence of (1.2)

$$\text{(i)}\ \ A+B<E+G+1,\ A+D<E+H+1,\ \ |x|<\infty,\ |y|<\infty,\ \ \text{or}$$

$$\text{(ii)}\ A+B=E+G+1,\ A+D=E+H+1,\ \ \text{and}$$

$$\left\{\begin{array}{ll}|x|^{\frac{1}{(A-E)}}+|y|^{\frac{1}{(A-E)}}<1 & ,\text{if }\ A>E\\ \max\{|x|,|y|\}<1 & ,\text{if }\ A\leq E\end{array}\right\}$$

Leibnitz Rule for Differentiation Under the Integral Sign[3]

If $F(x,\alpha)$ and $\frac{\partial}{\partial\alpha}F(x,\alpha)$ are continuous functions of x and α, then

$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\left\{\int_{\phi(\alpha)}^{\psi(\alpha)}F(x,\alpha)\,\mathrm{d}x\right\}=\int_{\phi(\alpha)}^{\psi(\alpha)}\left\{\frac{\partial}{\partial\alpha}F(x,\alpha)\right\}\mathrm{d}x+F(\psi(\alpha),\alpha)\frac{\mathrm{d}\psi}{\mathrm{d}\alpha}-F(\phi(\alpha),\alpha)\frac{\mathrm{d}\phi}{\mathrm{d}\alpha} \tag{1.3}$$

provided that $\phi(\alpha)$ and $\psi(\alpha)$ possesses continuous first order derivatives with respect to α.

Wallis' Formula

$$\int_0^{\frac{\pi}{2}}\sin^m\theta\,\cos^n\theta\,\mathrm{d}\theta=\frac{\Gamma(\frac{m+1}{2})\,\Gamma(\frac{n+1}{2})}{2\,\Gamma(\frac{m+n+2}{2})};\quad \Re(m)>-1,\ \Re(n)>-1 \tag{1.4}$$

Master Integral

In a paper of Boros and Moll [2,p.972, see also p.974(Th.1)], the following master formula

$$\int_0^{\infty}\left[\frac{x^2}{x^4+2ax^2+1}\right]^r\frac{x^2+1}{x^b+1}\frac{\mathrm{d}x}{x^2}=2^{-\frac{1}{2}-r}(1+a)^{\frac{1}{2}-r}\frac{\sqrt{\pi}\,\Gamma(r-\frac{1}{2})}{\Gamma(r)} \tag{1.5}$$

$$\left(a>-1,\ r>\frac{1}{2}\right)$$

Was used to evaluate a large number of definite integrals.

In the continuation of master integral, we evaluated certain definite integrals in sections 4 and 5.

II. Some Integrals of Gradshteyn and Ryzhik

[1,p.20(4);2,p.974(2.1);3,p.346(3.257]

$$\int_0^{\infty}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}\mathrm{d}x=\frac{\sqrt{\pi}}{2a(c)^{p+\frac{1}{2}}}\frac{\Gamma(p+\frac{1}{2})}{\Gamma(p+1)} \tag{2.1}$$

$$\left(a>0,\ b<0,\ c>0,\ \Re(p)+\frac{1}{2}>0\right)$$

[1,p.20(19);3,p.351(3.276(1))]

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}\mathrm{d}x=\frac{\sqrt{\pi}}{2b(c)^{p+\frac{1}{2}}}\frac{\Gamma(p+\frac{1}{2})}{\Gamma(p+1)} \tag{2.2}$$

$$\left(a<0,\ b>0,\ c>0,\ \Re(p)+\frac{1}{2}>0\right)$$

Under the stated conditions, integrals (2.1) and (2.2) are true. Since these conditions are not given in the table of integrals[3].

[1,p.20(5);3,p.351(3.276(2))]

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}\mathrm{d}x=\frac{\sqrt{\pi}}{(c)^{p+\frac{1}{2}}}\frac{\Gamma(p+\frac{1}{2})}{\Gamma(p+1)};\qquad \Re(p)+\frac{1}{2}>0 \tag{2.3}$$

Under any condition on a, b, c and p, the integral (2.3) is not true.

III. Other Forms of Above Integrals

$$\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}\mathrm{d}x=\frac{\sqrt{\pi}}{2a(4ab+c)^{p+\frac{1}{2}}}\frac{\Gamma(p+\frac{1}{2})}{\Gamma(p+1)} \tag{3.1}$$

$$\left(a>0;\ b\geq 0;\ c+4ab>0;\ \Re(p)+\frac{1}{2}>0\right)$$

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}\mathrm{d}x=\frac{\sqrt{\pi}}{2b\,(4ab+c)^{p+\frac{1}{2}}}\frac{\Gamma(p+\frac{1}{2})}{\Gamma(p+1)} \tag{3.2}$$

$$\left(a\geq 0;\ b>0;\ c+4ab>0;\ \Re(p)+\frac{1}{2}>0\right)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}\mathrm{d}x=\frac{\sqrt{\pi}}{(4ab+c)^{p+\frac{1}{2}}}\frac{\Gamma(p+\frac{1}{2})}{\Gamma(p+1)} \tag{3.3}$$

$$\left(a>0;\ b>0;\ c+4ab>0;\ \Re(p)+\frac{1}{2}>0\right)$$

IV. Proofs of (3.1) - (3.3)

Suppose left hand side of (3.1) is denoted by

$$I(b)=\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}\mathrm{d}x;\quad b\geq 0 \tag{4.1}$$

Therefore

$$I(0)=\int_0^\infty \frac{\mathrm{d}x}{(a^2x^2+c)^{p+1}}=\frac{1}{a(c)^{p+\frac{1}{2}}}\int_0^{\frac{\pi}{2}}\cos^{2p}\theta\,\mathrm{d}\theta=\frac{\sqrt{\pi}}{2a(c)^{p+\frac{1}{2}}}\frac{\Gamma(p+\frac{1}{2})}{\Gamma(p+1)} \tag{4.2}$$

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If we denote left hand side of (3.1) by $I_1^*(a)$, then $I_1^*(0)$ can not be calculated due to the divergent nature of resulting integral.

Differentiate (4.1) with respect to b and apply Leibnitz rule (1.3), we get

$$\frac{\mathrm{d}I}{\mathrm{d}b} = -2(p+1)\int_0^{\infty}\left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-2}\mathrm{d}x$$

$$= -2(p+1)\int_0^{\infty}\left(a+\frac{b}{x^2}\right)\left[\left(ax-\frac{b}{x}\right)^2+(c+4ab)\right]^{-p-2}\mathrm{d}x$$

$$= \frac{-4(p+1)}{(4ab+c)^{\frac{2p+3}{2}}}\int_0^{\frac{\pi}{2}}\cos^{2p+2}\theta\,\mathrm{d}\theta$$

Or

$$\mathrm{d}I = \frac{-\sqrt{\pi}(2p+1)\Gamma(p+\frac{1}{2})}{(4ab+c)^{\frac{2p+3}{2}}\Gamma(p+1)}\,\mathrm{d}b \tag{4.3}$$

Now integrate (4.3), we get

$$I(b) = \frac{\sqrt{\pi}}{2a(4ab+c)^{p+\frac{1}{2}}}\frac{\Gamma(p+\frac{1}{2})}{\Gamma(p+1)} + H \tag{4.4}$$

Where H is constant of integration.

By putting $b = 0$ in (4.4) and in view of the result (4.2), we get $H = 0$, therefore (4.4) reduces to right hand side of (3.1).

Similarly, if we denote the left hand of (3.2) by

$$I(a) = \int_0^{\infty}\frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}\mathrm{d}x;\quad a \geq 0 \tag{4.5}$$

Then

$$I(0) = \int_0^{\infty}\frac{x^{2p}}{(b^2+cx^2)^{p+1}}\,\mathrm{d}x = \frac{\sqrt{\pi}}{2b(c)^{p+\frac{1}{2}}}\frac{\Gamma(p+\frac{1}{2})}{\Gamma(p+1)} \tag{4.6}$$

If we denote left hand side of (3.2) by $I_2^*(b)$, , then I_2^* (0) can not be calculated due to the divergent nature of resulting integral.

Differentiate (4.5) with respect to a and apply Leibnitz rule (1.3), we get

$$\frac{\mathrm{d}I}{\mathrm{d}a} = -2(p+1)\int_0^{\infty}\left(a+\frac{b}{x^2}\right)\left[\left(ax-\frac{b}{x}\right)^2+(4ab+c)\right]^{-p-2}\mathrm{d}x = \frac{-(2p+1)\sqrt{\pi}\Gamma(p+\frac{1}{2})}{(4ab+c)^{\frac{2p+3}{2}}\Gamma(p+1)} \tag{4.7}$$

Now integrate (4.7), we get

$$I(a) = \frac{\sqrt{\pi}\,\Gamma(p+\frac{1}{2})}{2b(4ab+c)^{\frac{2p+1}{2}}\Gamma(p+1)} + G \tag{4.8}$$

Where G is constant of integration.

When $a = 0$ in (4.8) and in view of the result (4.6), we get $G = 0$ therefore (4.8) reduces to right hand side of (3.2).

We can not apply Leibnitz rule (1.3) in the left hand side of (3.3).

The left hand side of (3.3) is denoted by

$$I=\int_0^{\infty}\left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}\mathrm{d}x$$

$$=\int_0^{\infty}\left(a+\frac{b}{x^2}\right)\left[\left(ax-\frac{b}{x}\right)^2+(4ab+c)\right]^{-p-1}\mathrm{d}x$$

$$=\frac{2}{(4ab+c)^{p+\frac{1}{2}}}\int_0^{\frac{\pi}{2}}\cos^{2p}\theta\,\mathrm{d}\theta$$

On solving above integral with the help of (1.4), we get the right hand side of (3.3).

Or, if we multiply both sides of (3.1) by a, multiply both sides of (3.2) by b and adding the resulting integrals, we can obtain (3.3).

V. Additional Integrals

Since Pochhammer's symbol is associated with Gamma function and Gamma function is undefined for zero and negative integers, therefore arguments, numerator and denominator parameters are adjusted in such a way that following integrals are completely well defined and meaningful then without any loss of convergence, we have

$$\int_0^{\infty}e^{-ax-bx^2}\,\mathrm{d}x=\frac{\sqrt{\pi}}{2\sqrt{b}}\,e^{\frac{a^2}{4b}}-\frac{a}{2b}\,{}_1F_1\left[\begin{matrix}1\ ; \\ \frac{3}{2}\ ;\end{matrix}\ \frac{a^2}{4b}\right];\ a\geq 0,\ b>0 \tag{5.1}$$

In view of Leibnitz rule (1.3) and Kummer's first transformation [4,p.125(Th.42)] and using same technique,we can derive (5.1).

Using series expansions and hypergeometric forms [4,p.108(1),p.115(2,4)] of Sine,Cosine functions and ordinary Bessel function of first kind,a reduction formula for the product of two ${}_0F_1[4,\text{p.}105(\text{Q.No.}1)]$,interchanging the order of summation and integration, using series rearrangement technique and some algebraic properties of Pochhammer's symbol, we can derive the integrals (5.2)-(5.5) which are convergent for all finite values of parameters.

$$\int_0^{t}\cos(ax)\,\mathrm{J}_{\nu}(bx)\,\mathrm{d}x=\frac{b^{\nu}\,t^{\nu+1}}{2^{\nu}\,\Gamma(\nu+2)}\,F^{1:0;0}_{1:1;1}\left[\begin{matrix}\frac{\nu+1}{2}:\text{——};\text{——}; \\ \frac{\nu+3}{2}:\ \frac{1}{2}\ ;\nu+1\ ;\end{matrix}\ -\frac{a^2\,t^2}{4},\,-\frac{b^2\,t^2}{4}\right] \tag{5.2}$$

where $b\neq a$ and $\nu\neq -1$.

$$\int_0^{t}\sin(ax)\,\mathrm{J}_{\nu}(bx)\,\mathrm{d}x=\frac{a\,b^{\nu}\,t^{\nu+2}}{2^{\nu}\,(\nu+2)\Gamma(\nu+1)}\,F^{1:0;0}_{1:1;1}\left[\begin{matrix}\frac{\nu+2}{2}:\text{——};\text{——}; \\ \frac{\nu+4}{2}:\ \frac{3}{2}\ ;\nu+1\ ;\end{matrix}\ -\frac{a^2\,t^2}{4},\,-\frac{b^2\,t^2}{4}\right] \tag{5.3}$$

where $b\neq a$ and $\nu\neq -2$.

$$\int_0^{t}\cos(ax)\,\mathrm{J}_{\nu}(ax)\,\mathrm{d}x=\frac{a^{\nu}\,t^{\nu+1}}{2^{\nu}\,\Gamma(\nu+2)}\,{}_3F_4\left[\begin{matrix}\frac{\nu+1}{2},\,\frac{2\nu+1}{4},\,\frac{2\nu+3}{4}\ ; \\ \frac{1}{2},\,\nu+1,\,\frac{2\nu+1}{2},\,\frac{\nu+3}{2}\ ;\end{matrix}\ -a^2\,t^2\right] \tag{5.4}$$

where $\nu\neq -1$.

$$\int_0^{t}\sin(ax)\,\mathrm{J}_{\nu}(ax)\,\mathrm{d}x=\frac{a^{\nu+1}\,t^{\nu+2}}{2^{\nu}\,(\nu+2)\,\Gamma(\nu+1)}\,{}_3F_4\left[\begin{matrix}\frac{\nu+2}{2},\,\frac{2\nu+3}{4},\,\frac{2\nu+5}{4}\ ; \\ \frac{3}{2},\,\nu+1,\,\frac{2\nu+3}{2},\,\frac{\nu+4}{2}\ ;\end{matrix}\ -a^2\,t^2\right] \tag{5.5}$$

where $\nu\neq -2$.

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Finite Integrals Pertaining To a Product of Special Functions

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Abstracts - An attempt has been made to establish an integral concerning the product of generalized Lauricella function and two H-function of several complex variables (Srivastava and Panda [4,5]) By giving suitable values to the parameters, the main integral reduces to F function. Mainly we are using the series representation of H-function given by Olkha and Chaurasia [2,3].

Keywords : *Multivariable H-function, H-function in Series form, Lauricella function, Jacobi polynomial, Kampé de Fériet function.*

Mathematics Subject Classification 2000 : *26A33, 44A10, 33C60*

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Finite Integrals Pertaining To a Product of Special Functions

V.B.L. Chaurasia[α], Yudhveer Singh[Ω]

Abstract - An attempt has been made to establish an integral concerning the product of generalized Lauricella function and two H-function of several complex variables (Srivastava and Panda [4,5]) By giving suitable values to the parameters, the main integral reduces to F function. Mainly we are using the series representation of H-function given by Olkha and Chaurasia [2,3].
Keywords : Multivariable H-function, H-function in Series form, Lauricella function, Jacobi polynomial, Kampé de Fériet function.

I. Introduction

The series representation of the H-function of several complex variable studied by Olkha and Chaurasia [2,3] is given as follows:

$$H[z_1,\dots,z_r]=H^{0,\lambda':(u',v');\dots;(u^{(r)},v^{(r)})}_{A',C':[B',D'];\dots;[B^{(r)},D^{(r)}]}\left[\begin{matrix}[(a):\theta',\dots,\theta^{(r)}]:[b':\phi'];\dots;[b^{(r)}:\phi^{(r)}];\\ [(c):\psi',\dots,\psi^{(r)}]:[d':\delta'];\dots;[d^{(r)}:\delta^{(r)}];\end{matrix}\ z_1,\dots,z_r\right]$$

$$=\sum_{m_i=1}^{u^{(i)}}\sum_{n_i=0}^{\infty}\Phi_1\Phi_2\frac{\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}}{\prod_{i=1}^{r}(\delta^{(i)}_{(m_i)}\,n_i!)},\qquad\dots(1.1)$$

Where

$$\Phi_1=\frac{\prod_{j=1}^{\lambda'}\Gamma\left(1-a_j+\sum_{i=1}^{r}\theta_j^{(i)}U_i\right)}{\prod_{j=\lambda'+1}^{A'}\Gamma\left(a_j-\sum_{i=1}^{r}\theta_j^{(i)}U_i\right)\prod_{j=\lambda'+1}^{C'}\Gamma\left(1-c_j+\sum_{i=1}^{r}\psi_j^{(i)}U_i\right)}\qquad\dots(1.2)$$

$$\Phi_2=\frac{\prod_{\substack{j=1\\ j\neq m_i}}^{u^{(i)}}\Gamma(d_j^{(i)}-\delta_j^{(i)}U_i)\prod_{j=1}^{v^{(i)}}\Gamma(1-b_j^{(i)}+\phi_j^{(i)}U_i)}{\prod_{j=u^{(i)}+1}^{D^{(i)}}\Gamma(1-d_j^{(i)}+\delta_j^{(i)}U_i)\prod_{j=v^{(i)}+1}^{B^{(i)}}\Gamma(b_j^{(i)}-\phi_j^{(i)}U_i)}\qquad\dots(1.3)$$

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$$U_i = \frac{d_{m_i}^{(i)} + n_i}{\delta_{m_i}^{(i)}}, \quad i = 1, \ldots, r \qquad \ldots(1.4)$$

which is valid under the following condition

$$\delta_{m_i}^{(i)} [d_j^{(i)} + p_i] \neq \delta_j^{(i)} [d_{m_i}^{(i)} + n_i] \qquad \ldots(1.5)$$

for $j \neq m_i, m_i = 1, \ldots, u^{(i)}; p_i, n_i = 0,1,2,\ldots; z_i \neq 0$

$$\nabla_i = \sum_{j=1}^{A'} \theta_j^{(i)} - \sum_{j=1}^{C'} \psi_j^{(i)} + \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} - \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} < 0, \ \forall \ i = 1, \ldots, r \qquad \ldots(1.6)$$

Srivastava and Panda [5] have introduced the multivariable H-function

$$H[y_1, \ldots, y_R] = H^{0,\lambda:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}_{A,C:[M',N'];\ldots;[M^R,N^R]}$$

$$\left[\begin{matrix} [(g):\gamma',\ldots,\gamma^{(R)}]:[q':n'];\ldots;[q^{(R)}:n^{(R)}]; \\ [(f):\xi',\ldots,\xi^{(R)}]:[p':\varepsilon'];\ldots;[p^{(R)}:\varepsilon^{(R)}]; \end{matrix} \ y_1, \ldots, y_R \right] \qquad \ldots(1.7)$$

$$T_i = \sum_{j=1}^{A} \gamma_j^{(i)} - \sum_{j=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \varepsilon_j^{(i)} \leq 0, \qquad \ldots(1.8)$$

$$\Omega_i = -\sum_{j=\lambda+1}^{A} \gamma_j^{(i)} - \sum_{j=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{\beta^{(i)}} \eta_j^{(i)} - \sum_{j=\beta^{(i)}+1}^{M^{(i)}} \eta_j^{(i)} + \sum_{j=1}^{\alpha^{(i)}} \varepsilon_j^{(i)} - \sum_{j=\alpha^{(i)}+1}^{N^{(i)}} \varepsilon_j^{(i)} > 0, \qquad \ldots(1.9)$$

$$(1.10) \quad |\arg(y_i)| < \frac{T_i \pi}{2}, \ \forall \ i = 1, \ldots, R.$$

II. The Main Integral Transformation

We obtained the following integral transformation for H-function of several complex variables defined by Srivastava and Panda [4] (see also [3])

$$\int_0^1 x^{\rho-1} (1-x)^{\sigma} H[y_1 x^{h_1} (1-x)^{k_1}; \ldots; y_R x^{h_R} (1-x)^{k_R}]$$

$$. \ H\left[z_1 x^{h_1'} (1-x)^{k_1'}; \ldots; z_r x^{h_r'} (1-x)^{k_r'}\right] F^{v:P';\ldots;P^S;0;0}_{\sigma:Q';\ldots;Q^{(S)};1;1}$$

$$\left[\begin{matrix}[\alpha_v]:[a';\ldots;a^{(s)}\gamma,\gamma]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]:[\ldots];[\ldots];z_1',\ldots,z_s',-xt,(1-x)t\\ [\beta_\sigma]:[b';\ldots;b^{(s)},\mu,\mu]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1;1];\end{matrix}\right]dx$$

$$=\sum_{m_i=1}^{u^{(i)}}\sum_{k,n,n_i=0}^{\infty}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}t^n(-n)_k(\alpha+\beta+n+1)_k}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}\left(\delta_{m_i}^{(i)}n_i!\right)(\alpha+1)_n(\beta+1)_n k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$.F^{v:P';\ldots;P^{(s)}}_{\sigma:Q';\ldots;Q^{(s)}}\left[\begin{matrix}[\alpha_v+n\gamma_v]:[a';\ldots;a^{(s)}]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}];\\ [\beta_\sigma+n\mu_\sigma):[b';\ldots;b^{(s)}]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];\end{matrix}\;z_1',\ldots,z_s'\right]$$

$$.H^{0,\lambda+2\quad:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}_{A+2,C+1:[M',N'];\ldots;[M^{(R)},N^{(R)}]}\left[\begin{matrix}\left[1-\rho-\sum_{i=1}^{r}h_i'U_i-k;h_1;\ldots;h_R\right];\left[-\sigma-\sum_{i=1}^{r}k_i'U_i;k_1,\ldots,k_R\right];\\ [(f):\xi',\ldots,\xi^{(R)}];\end{matrix}\right.$$

$$\left.\begin{matrix}[(g):\gamma',\ldots,\gamma^{(R)}];[q':\eta'];\ldots;[q^{(R)}:\eta^{(R)}];\\ \left[-\rho-\sigma-k-\sum_{i=1}^{r}(k_i'+h_i')U_i;(h_1+k_1),\ldots,(h_R+k_R)\right];[p':\varepsilon'];\ldots;[p^{(R)}:\varepsilon^{(R)}];\end{matrix}\;y_1,\ldots,y_R\right],\qquad\ldots(2.1)$$

Where

$$\mathrm{Re}\left[\rho+\sum_{i=1}^{R}h_i\frac{p_j^{(i)}}{\varepsilon_j^{(i)}}+\sum_{\ell=1}^{r}h_\ell'\frac{d_j^\ell}{\delta_j^{(\ell)}}\right]>0,$$

$$\mathrm{Re}\left[\sigma+\sum_{i=1}^{R}\frac{k_ip_j^i}{\varepsilon_j^{(i)}}+\sum_{\ell=1}^{r}k_\ell'\frac{d_j^\ell}{\delta_j^{(\ell)}}\right]>-1,$$

$$j=1,\ldots,\alpha^{(i)},k_i>0,h_i>0,h_\ell',k_\ell'>0,T_i>0,|\arg(y_i)|<\frac{T_i\pi}{2},i=1,\ldots,R,\ell=1,\ldots,r,|t|<1,$$

Where the series on the right hand side is convergent and is given by (1.8).

III. Proof

To prove that (2.1), we start with the following result [6] :

$$F^{v:P';\ldots;P^s;0;0}_{\sigma:Q';\ldots;Q^{(s)};1;1}\left[\begin{matrix}[\alpha_v]:[a';\ldots;a^{(s)},\gamma,\gamma]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]:[\ldots];[\ldots];z_1',\ldots,z_s',-xt,(1-x)t\\ [\beta_\sigma]:[b';\ldots;b^{(s)},\mu,\mu]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1;1]\end{matrix}\right]$$

$$=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\,t^n P_n^{(\alpha,\beta)}(1-2x)}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}(\alpha+1)_n(\beta+1)_n}$$

$$F^{v:P';\ldots;P^{(s)}}_{\sigma:Q';\ldots;Q^{(s)}}\left[\begin{matrix}[(\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}];\\ [\beta_\sigma+n\mu_\sigma):b';\ldots;b^{(s)}]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',\ldots,z_s'\right], \qquad \ldots(3.1)$$

where $\gamma_i\ a_j^{(i)}$; j = 1,…,v and i = 1,…,s $\mu_i\ \beta_j^{(i)}$; j = 1,…, σ and i = 1,…,s; $\rho_i\ \tau_k^{(j)}$; i = 1,…,$P^{(j)}$; j = 1,…,s and k = 1,…,$Q^{(s)}$ are all real and positive, (α_v) stand for the sequence of parameter $\alpha_1,\ldots,\alpha_v$; $(\ell^{(j)})$ stand for the sequence of $P^{(j)}$ parameters $\ell_1^{(j)},\ldots,\ell_{P^{(j)}}^{(j)}$, j = 1,…,s, and also similar interpretation for (β_σ) and $(m^{(J)})$, j = 1,…,s and F denote the generalized Lauricella function of s-complex variables $z_1',\ldots,z_s'$ given by Srivastava and Daoust [8]. Now the proof of main integral transform (2.1) is obtained after multiplying both side (3.1) by

$$x^{\rho-1}(1-x)^{\sigma}\,H[y_1x^{h_1}(1-x)^{k_1},\ldots,y_Rx^{h_R}(1-x)^{k_R}]$$

$$.\ H[z_1x^{h_1'}(1-x)^{k_1'},\ldots,z_rx^{h_r'}(1-x)^{k_r'}]$$

And integrate with respect to x from 0 to 1.

Now we represent the $H[z_1x^{h_1'}(1-x)^{k_1'},\ldots,z_rx^{h_r'}(1-x)^{k_r'}]$ in series form [3], given by (1.1) and interchange the order of integrations and summations. The required formula is obtained by evaluating the innermost integral with the help of result [5].

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma}H\left[y_1x^{h_1}(1-x)^{k_1},\ldots,y_Rx^{h_R}(1-x)^{k_R}\right]P_n^{\alpha,\beta}(1-2x)\,dx$$

$$=\sum_{k=0}^{\infty}\frac{(-n)_k(\alpha+\beta+n+1)_k}{k!(\alpha+1)_k}H^{0,\lambda+2\ \ :(\alpha',\beta');\ldots;(\alpha^R,\beta^R)}_{A+2,C+1:[M',N'];\ldots;[M^R,N^R]}$$

$$\left[\begin{matrix}[1-\rho-k;h_1,...,h_R],[-\sigma;k_1,...,k_R],[(g):\gamma',...,\gamma^{(R)}];[(q'):\eta'];...;[q^{(R)}:\eta^{(R)}]; \\ [(f);\xi',...,\xi^{(R)}],[1-\rho-\sigma-k;(h_1+k_1),...,(h_R+k_R)],[p':\varepsilon'];...;[(p^{(R)}):\varepsilon^{(R)}];\end{matrix}\; y_1,...,y_R\right], \qquad ...(3.2)$$

Where

$$\operatorname{Re}\left(\rho+\sum_{i=1}^{R}\frac{h_i\, p_j^{(i)}}{\varepsilon_j^{(i)}}\right)>0, \operatorname{Re}\left(\sigma+\sum_{i=1}^{R}\frac{k_i\, p_j^{(i)}}{\varepsilon_j^{(i)}}\right)>-1,$$

$j=1,...,\alpha^{(i)}, k_i>0, h_i>0, T_i>0, |\arg(y_i)|<\frac{T_i\pi}{2}, i=1,...,R$ and T_i is given by (1.8).

IV. Special Cases

Taking $h_i \to 0 (i=1 \text{ to } R), h_i' \to 0 (i=1 \text{ to } r)$ in (2.1), we obtain the following result

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma} H\left[y_1(1-x)^{k_1},...,y_R(1-x)^{k_R}\right]$$

$$.\, H\left[z_1(1-x)^{k_1'},...,z_r(1-x)^{k_r'}\right]$$

$$F^{v:P';...;P^{(s)};0;0}_{\sigma:Q';...;Q^{(s)};1;1}\left[\begin{matrix}[\alpha_v]:[a';...;a^{(s)},\gamma,\gamma]:[(\ell'):\rho'];...;[(\ell^{(s)}):\rho^{(s)}]:[...];[...];z_1',...,z_s',-xt,(1-x)t \\ [\beta_\sigma]:[b';...;b^{(s)},\mu,\mu]:[(m'):\tau'];...;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1;1];\end{matrix}\right] dx$$

$$=\sum_{m_i=1}^{u^{(i)}}\sum_{k,n,n_i=0}^{\infty}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}t^n(-n)_k(\alpha+\beta+n+1)_k\Gamma(\rho+k)}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}\left(\delta_{m_i}^{(i)}n_i!\right)(\alpha+1)_n(\beta+1)_n k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$.\, F^{v:P';...;P^{(s)}}_{\sigma:Q';...;Q^{(s)}}\left[\begin{matrix}[\alpha_v+n\gamma_v):a';...;a^s]:[(\ell'):\rho'];...;[(\ell^{(s)}):\rho^{(s)}]; \\ [\beta_\sigma+n\mu_\sigma):b';...;b^{(s)}]:[(m'):\tau'];...;[(m^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',...,z_s'\right]$$

$$.\, H^{0,\lambda+1\;\;:(\alpha',\beta');...;(\alpha^{(R)},\beta^{(R)})}_{A+1,C+1:[M',N'];...;[M^{(R)},N^{(R)}]}\left[\begin{matrix}\left[-\sigma-\sum_{i=1}^{r}k_i'U_i;k_1,...,k_R\right]; \\ [(f):\xi',...,\xi^{(R)}];\end{matrix}\right.$$

$$\left.\begin{matrix} [(g):\gamma',\ldots,\gamma^R];[q':\eta'];\ldots;[q^R:\eta^R]; \\ \left[-\rho-\sigma-k-\sum_{i=1}^{R} k_i' U_i;k_1,\ldots,k_R\right];[p':\varepsilon'];\ldots;[p^{(R)}:\varepsilon^{(R)}]; \end{matrix}\; y_1,\ldots,y_R \right], \qquad \ldots(4.1)$$

Where

$$\operatorname{Re}(\rho)>0, \operatorname{Re}\left(\sigma+\sum_{i=1}^{R}\frac{k_i p_j^{(i)}}{\varepsilon_j^i}+\sum_{\ell=1}^{r} k_\ell' \frac{d_j^{(\ell)}}{\delta_j^{(\ell)}}\right)>-1,$$

$$j=1,\ldots,\alpha^{(i)}, k_i>0, k_\ell'>0, T_i>0, |\arg(y_i)|<\frac{T_i\pi}{2}, i=1,\ldots,R, \ell=1,\ldots,r, |t|<1.$$

(2) Taking $k_i \to 0 (i=1 \text{ to } R), k_i' \to 0 (i=1 \text{ to } r)$ in (2.1), we obtain the following result

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma} H\left[y_1 x^{h_1},\ldots,y_R x^{h_R}\right] H\left[z_1 x^{h_1'},\ldots,z_r x^{h_r'}\right]$$

$$.F^{v:P';\ldots;P^{(s)};0;0}_{\sigma:Q';\ldots;Q^S;1;1}\left[\begin{matrix}[\alpha_v]:[a';\ldots;a^{(s)},\gamma,\gamma]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]:[\ldots];[\ldots];z_1',\ldots,z_s',-xt,(1-x)t \\ [\beta_\sigma]:[b';\ldots;b^{(s)},\mu,\mu]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1;1];\end{matrix}\right] dx$$

$$=\sum_{m_i=1}^{u^{(i)}}\sum_{k,n,n_i=0}^{\infty}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)} t^n (-n)_k(\alpha+\beta+n+1)_k\Gamma(\sigma+1)}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}\left(\delta_{m_i}^{(i)} n_i!\right)(\alpha+1)_n(\beta+1)_n k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$.F^{v:P';\ldots;P^{(s)}}_{\sigma:Q';\ldots;Q^{(s)}}\left[\begin{matrix}[\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]; \\ [\beta_\sigma+n\mu_\sigma):b';\ldots;b^{(s)}]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',\ldots,z_s'\right]$$

$$.H^{0,\lambda+1\;\;:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}_{A+1,C+1:[M',N'];\ldots;[M^{(R)},N^{(R)}]}\left[\begin{matrix}\left[1-\rho-k-\sum_{i=1}^{r} h_i' U_i;h_1,\ldots,h_R\right]; \\ [(f):\xi',\ldots,\xi^{(R)}];\end{matrix}\right.$$

$$\left.\begin{matrix} [(g):\gamma',\ldots,\gamma^R];[q':\eta'];\ldots;[q^{(R)}:\eta^{(R)}]; \\ \left[-\rho-\sigma-k-\sum_{i=1}^{R} h_i' U_i;h_1,\ldots,h_R\right];[p':\varepsilon'];\ldots;[p^{(R)},\varepsilon^{(R)}]; \end{matrix}\; y_1,\ldots,y_R \right], \qquad \ldots(4.2)$$

Where
$$\mathrm{Re}\left(\rho+\sum_{i=1}^{R}\frac{h_i p_j^{(i)}}{\varepsilon_j^{i}}+\sum_{\ell=1}^{r} h'_{\ell}\frac{d_j^{(\ell)}}{\delta_j^{(\ell)}}\right)>0,$$

$$\mathrm{Re}(\sigma)>-1,\ j=1,\ldots,\alpha^{(i)},h_i>0,h'_{\ell}>0,T_i>0,|\arg(y_i)|<\frac{T_i\pi}{2},i=1,\ldots,R,\ell=1,\ldots,r,|t|<1.$$

(3) Reducing the H- function of several complex variables to the generalized Lauricella function [8] by putting $\lambda=A,\alpha^{i}=1,\beta^{(i)}=M^{i},N^{(i)}=N^{(i)}+1,$ $\forall$ i = 1,…,R in (2.1), we get the following result:

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma} H\left[z_1 x^{h'_1}(1-x)^{k'_1};\ldots;z_r x^{h'_r}(1-x)^{k'_r}\right]$$

$$.F^{v:P';\ldots;P^{(s)};0;0}_{\sigma:Q';\ldots;Q^{(s)};1;1}\left[\begin{matrix}[\alpha_v]:[a';\ldots;a^{(s)}\gamma,\gamma]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]:[\ldots];[\ldots];z'_1,\ldots,z'_s-xt,(1-x)t\\ [\beta_\sigma]:[b';\ldots;b^{(s)},\mu,\mu]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1;1];\end{matrix}\right]$$

$$.F^{A:M';\ldots;M^{(R)}}_{C:N';\ldots;N^{(R)}}\left[\begin{matrix}[1-(g):\gamma',\ldots,\gamma^{(R)}]:[1-(q'):\eta'];\ldots;[1-(q^{(R)}):\eta^{(R)}];\\ [1-(f):\xi',\ldots,\xi^{(R)}]:[1-(p'):\varepsilon'];\ldots;[1-(p^{(R)}):\varepsilon^{(R)}];\end{matrix}\right.$$

$$\left.-y_1 x^{h_1}(1-x)^{k_1},\ldots,-y_R x^{h_R}(1-x)^{k_R}\right]dx$$

$$=\sum_{k,n,n_i=0}^{\infty}\sum_{m_i=1}^{u^{(i)}}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}t^{n}(-n)_k(\alpha+\beta+n+1)_k}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}\prod_{i=1}^{r}\left(\delta_{m_i}^{(i)}n_i!\right)(\alpha+1)_n(\beta+1)_n k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$.F^{v:P';\ldots;P^{s}}_{\sigma:Q';\ldots;Q^{s}}\left[\begin{matrix}[\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(\ell'):P'];\ldots;[(\ell^{(s)}):P^{(s)}];\\ [\beta_\sigma+n\mu_\sigma):b';\ldots;b^{s}]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];\end{matrix}\ z'_1,\ldots,z'_s\right]$$

$$.\frac{\Gamma(1+\sigma+\sum_{i=1}^{r}k'_iU_i)\Gamma(\rho+\sum_{i=1}^{r}h'_iU_i+k)}{\Gamma(1+\rho+\sigma+k+\sum_{i=1}^{r}(h'_i+k'_i)U_i)}$$

$$. F^{A+2:M';...;M^{(R)}}_{C+1:N';...;N^{(R)}}\left[\begin{matrix}\left[1+\sigma+\sum_{i=1}^{r} k_i' U_i;k_1,...,k_R\right];\left[\rho+\sum_{i=1}^{r} h_i' U_i+K;h_1,...,h_R\right], \\ [1-(f):\xi';...;\xi^R];\end{matrix}\right.$$

$$\left.\begin{matrix}[1-(g):\gamma',...,\gamma^{(R)}];[1-(q'):\eta'];...;[1\text{-}(q^{(R)}):\eta^{(R)}]; \\ \left[1+\rho+\sigma+k+\sum_{i=1}^{r}(h_i'+k_i')U_i;(h_1+k_1);...;(h_R+k_R)\right];[1-(p'):\varepsilon'];...;[1-(p^{(R)}):\varepsilon^{(R)}];\end{matrix} -y_1,...,-y_R\right] \qquad ...(4.3)$$

Which is valid under the same condition as given in (2.1).

(4) Reducing the Lauricella function to the Kampé de Fériet function [7] by putting i = 1,2 in (4.3) and we get the following result :

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma} H\left[z_1 x^{h_1'}(1-x)^{k_1'};...;z_r x^{h_r'}(1-x)^{k_r'}\right]$$

$$. F^{v:P';...;P^s;0;0}_{\sigma:Q';...;Q^{(s)};1;1}\left[\begin{matrix}[\alpha_v][:a';...;a^{(s)},\gamma,\gamma]:[(\ell'):\rho'];...;[(\ell^{(s)}):\rho^{(s)}]:[...];[...];z_1',...,z_s',-xt,(1-x)t \\ [\beta_\sigma]:[b';...;b^{(s)},\mu,\mu]:[(m'):\tau'];...;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1:1]\end{matrix}\right]$$

$$.S^{A:M':M''}_{C:N':N''}\left[\begin{matrix}[(g):\gamma',\gamma'']:[(q'):\eta'];[(q''):\eta''];\\ [(f):\xi',\xi'']:[(p'):\varepsilon'];[(p''):\varepsilon''];\end{matrix}\; y_1 x^{h_1}(1-x)^{k_1}, y_2 x^{h_2}(1-x)^{k_2}\right]dx$$

$$= \sum_{k,n,n_i=0}^{\infty}\sum_{m_i=1}^{u^{(i)}} \frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)} t^n(-n)_k(\alpha+\beta+n+1)_k}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_\sigma}\prod_{i=1}^{r}\left(\delta^{(i)}_{m_i}\, n_i!\right)(\alpha+1)_n(\beta+1)_n\, k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$.F^{v:P';...;P^{(s)}}_{\sigma:Q';...;Q^{(s)}}\left[\begin{matrix}[\alpha_v+n\gamma_v):a';...;a^{(s)}]:[(\ell'):\rho'];...;[(\ell^{(s)}):\rho^{(s)}]; \\ [\beta_\sigma+n\mu_\sigma):b';...;b^{(s)}]:[(m'):\tau'];...;[(m^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',...,z_s'\right]$$

$$. S^{A+2:M';M''}_{C+1:N';N''}\left[\begin{matrix}\left[1-\rho-\sum_{i=1}^{r} h_i' U_i-k;h_1,h_2\right];\left[-\sigma-\sum_{i=1}^{r} k_i' U_i;k_1,.k_2\right], \\ [(f):\xi';\xi''];\end{matrix}\right.$$

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$$\left.\begin{matrix}[(g):\gamma',\gamma''];[(q'):\eta'];\ [(q''):\eta''];\\ [-\rho-\sigma-k-\sum_{i=1}^{r}(h_i'+k_i')U_i;(h_1+k_1),(h_2+k_2)];[p'):\varepsilon'];[(p''):\varepsilon''];\end{matrix}\ y_1,y_2\right] \quad \ldots(4.4)$$

Which is valid under the same condition as surrounding (2.1).

(5) Reducing the H-function of several complex variables to the product of R mutually independent H- functions by taking $\lambda = A = C = 0$ in (2.1), we get the following result :

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma} H\left[z_1 x^{h_1'}(1-x)^{k_1'};\ldots;z_r x^{k_r'}(1-x)^{h_r'}\right]$$

$$.\ F^{v:P';\ldots;P^{s};0;0}_{\sigma:Q';\ldots;Q^{(s)};1;1}\left[\begin{matrix}[\alpha_v]:[a';\ldots;a^{(s)},\gamma,\gamma]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}]:[\ldots];[\ldots];z_1',\ldots,z_s',-xt,(1-x)t\\ [\beta_\sigma]:[b';\ldots;b^{(s)},\mu,\mu]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];[\alpha+1:1];[\beta+1:1]\end{matrix}\right]$$

$$.\ \prod_{i=1}^{R}\left\{H^{\alpha^{(i)},\beta^{(i)}}_{M^{(i)},N^{(i)}}\left[y_i\, x^{h_i}(1-x)^{k_i}\left|\begin{matrix}[(q^i):\eta^{(i)}]\\ [(p^i):\varepsilon^{(i)}]\end{matrix}\right.\right]\right\}dx$$

$$=\sum_{k,n,n_i=0}^{\infty}\sum_{m_i=1}^{u^{(i)}}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}\, t^n(-n)_k(\alpha+\beta+n+1)_k}{\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_\sigma}\prod_{i=1}^{r}\left(\delta^{(i)}_{m_i}\, n_i!\right)(\alpha+1)_n(\beta+1)_n\, k!(\alpha+1)_k}\Phi_1\Phi_2$$

$$.\ F^{v:P';\ldots;P^{(s)}}_{\sigma:Q';\ldots;Q^{(s)}}\left[\begin{matrix}[\alpha_v+n\gamma_v):a';\ldots;a^{(s)}]:[(\ell'):\rho'];\ldots;[(\ell^{(s)}):\rho^{(s)}];\\ [\beta_\sigma+n\mu_\sigma):b';\ldots;b^{(s)}]:[(m'):\tau'];\ldots;[(m^{(s)}):\tau^{(s)}];\end{matrix}\ z_1',\ldots,z_s'\right]$$

$$.\ H^{0,2:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}_{2,1:[M',N'];\ldots;[M^{R},N^{(R)}]}\left[\begin{matrix}\left[1-\rho-\sum_{i=1}^{r}h_i'U_i-k;h_1;\ldots;h_R\right],\\ \left[[(f):\xi',\ldots,\xi^{(R)}];[-\rho-\sigma-K-\sum_{i=1}^{r}(k_i'+h_i')U_i;(h_1+k_1),\ldots,(h_R+k_R)\right]\end{matrix}\right.$$

$$\left.\begin{matrix}\left[-\sigma-\sum_{i=1}^{r}k_i'U_i;k_1,\ldots,k_R\right];[(q'):\eta'];\ldots;[q^{(R)}:\eta^{(R)}];\\ [(p'):\varepsilon'];\ldots;[p^{(R)}:\varepsilon^{(R)}];\end{matrix}\ y_1,\ldots,y_R\right] \quad \ldots(4.5)$$

which holds under the same condition as for (2.1).

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31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page

- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.
- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.

- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Administration Rules Listed Before Submitting Your Research Paper to Global Journals Inc. (US)

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.

- Do not give permission to anyone else to "PROOFREAD" your manuscript.

- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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