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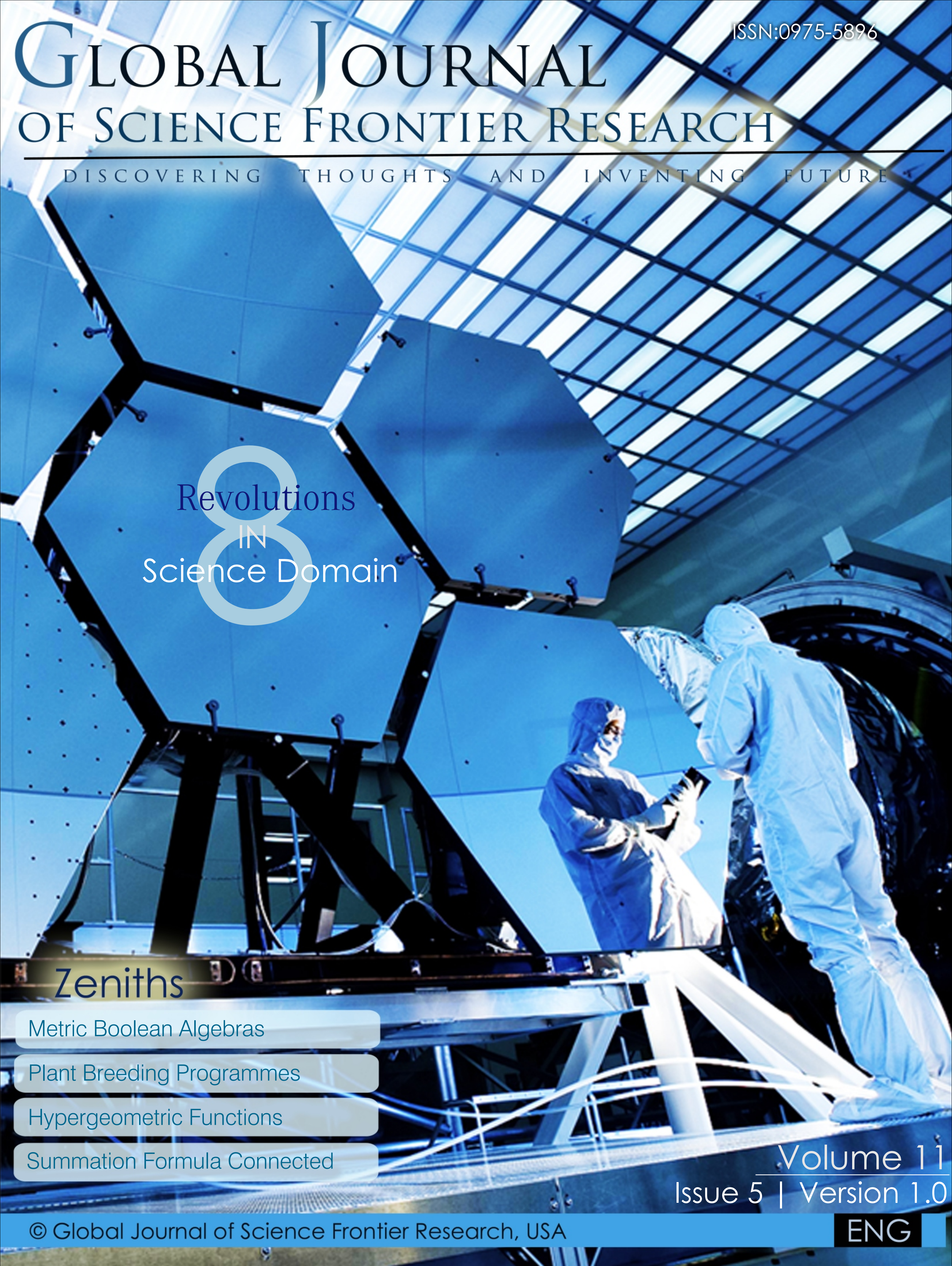
DISCOVERING THOUGHTS AND INVENTING FUTURE



8 Revolutions IN Science Domain

Zeniths

- Metric Boolean Algebras
- Plant Breeding Programmes
- Hypergeometric Functions
- Summation Formula Connected

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Metric Boolean Algebras and an Application To Propositional Logic

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Abstract - Let B be a Boolean algebra and Ω be the set of all homomorphisms from B into D, and μ be a probability measure on Ω . We introduce the concepts of sizes of elements of B and similarity degrees of pairs of elements of B by means of μ , and then define a metric on B . As an application, we propose a kind of approximate reasoning theory for propositional logic.

Keywords : *Boolean algebra, probability measure, size, similarity degree, approximate reasoning, propositional logic.*

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Metric Boolean Algebras and an Application To Propositional Logic

Li FU[α], Guojun Wang[Ω]

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I. Preliminaries

Let L be a distributive lattice and D be the lattice $\{0,1\}$. Assume Ω is the set of all homomorphism from L to D, then $\forall f\in\Omega, f$ is order-preserving, $i.e.,\ a,b\in L, a\le b$ if and only if $f(a)\le f(b)$.
In fact, if $a,b\in L, a\not\le b$, then there exists an $f\in\Omega$, such that $f(a)=1$ and $f(b)=0$ (see [4]). Therefore, if we think of elements of L as functions from Ω to D, $i.e.,\ \forall a\in L$ defining $a:\Omega\to\{0,1\}, a(f)=f(a)(f\in\Omega)$, then the partial order $\le$ or L will have a representation by means of the partial order among function of 2^{Ω}. This fact is also true for Boolean algebras. We use the same symbol Ω to denote the set of all (Boolean) homomorphisms from a Boolean algebra B into D.

Proposition 1 : Suppose that B is a Boolean algebra, $D=\{0,1\}$, and Ω is the set of all homomorphism from L to D, then $a,b\in B, a\le b$ if and only if $\forall f\in\Omega, f(a)\le f(b)$. Throughout this paper, assume that B is a Boolean algebra and a probability measure μ is given on Ω, we will introduce the concepts of sizes of elements of B and similarity degrees between elements pairs of elements of B by means of μ and proposition 1, and finally define a metric ρ on B therefrom. Especially, this can be done the Lindenbaum algebra $B_L=F(S)/\approx$, where $F(S)$ is the set consisting of all logic propositions and $\approx$ is the congruence relation of logically equivalence[6] , and Ω is the set consisting of all valuations of $F(S)$ of which a probability measure can naturally be introduced. Finally, a kind of approximate reasoning theory can be developed on $F(S)$ because there have been distances among propositions.

II. Metric Boolean Algebras

Suppose that μ is a probability measure on Ω

Definition 1 : Define $\tau: B\to[0,1]$ as follows:

$$\tau(a)=\mu(\{f\in\Omega\mid a(f)=1\}),\quad a\in B$$

then $\tau(a)$ is called the *size* of *a* with respect to μ, or briefly , *size* of a if no confusion arises.
The following proposition is obvious.

Proposition 2 : (i) $0\le\tau(a)\le 1, a\in B$;

(ii) $\tau(1_B)=1, \tau(0_B)=0$, where 1_B and 0_B are the greatest element and the least element of B respectively;
(iii) $\tau(a')=1-\tau(a), a\in B$
(iv) If $a\le b$, then $\tau(a)\le\tau(b), a,b\in B$.

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Example 1 : Consider the Boolean algebra $\mathcal{P}(X)$, where X is non-empty finite set , and $\mathcal{P}(X)$ is the power set of X. It is easy to verify that a mapping $f:\mathcal{P}(X)\to D$ is a homomorphism iff there exists an (unique) element x of X such that $f^{-1}(1)=\{A\subset X\mid x\in A\}$. Hence, $|\,\Omega\,|=|\,X\,|$. Assume that μ is the evenly distributed probability measure on Ω, then the size of an element A of $\mathcal{P}(X)$ is the cardinal of A over $|X|$, $i.e., \tau(A)=|A|/|X|$ $(A\in\mathcal{P}(X))$.

Proposition 3 : $\tau(a\vee b)=\tau(a)+\tau(b)-\tau(a\wedge b), a,b\in B$.

Assume that $a\to b=a'\vee b(a,b\in B)$ (see 5), then we have:

Proposition 4 : $\tau(a)\geq\alpha,\tau(a\to b)\geq\beta$, then $\tau(b)\geq\alpha+\beta-1$, where $\alpha,\beta\in[0,1]$.

Proposition 5 : If $\tau(a\to b)\geq\alpha,\tau(b\to c)\geq\beta$, then $\tau(a\to c)\geq\alpha+\beta-1$.

Definition 2 : Define $\eta:B\times B\to[0,1]$ as follows:

$$\eta(a,b)=\tau(a\to b)\wedge(b\to a), a,b\in B.$$

then $\eta(a,b)$ is called the **similarity degree** between a and b, and η the similar relation on B with respect to μ.

Example 2 : Consider the Boolean algebra $\mathcal{P}(X)$ where X is any non-empty set, and μ is any probability measure on Ω. Then

(1) $\eta(A,X-A)=0,(A\in\mathcal{P}(X))$. In fact, $A\to(X-A)=(X-A)\cup(X-A)=X-A,(X-A)\to A=A\cup A=A$

Hence, $\eta(A,X-A)=\tau((X-A)\cap A)=\tau(\emptyset)=0$.

(2) $\eta(A,B)+\eta(A,X-B)=1$. In fact, let $G=((X-A)\cup B)\cap((X-B)\cup A), H=((X-A)\cup(X-B))\cap(B\cup A)$. then it is routine to verify that $G\cup H=X, G\cap H=\emptyset$. Hence, $\eta(A,B)+\eta(A,X-B)=\mu(G)+\mu(H)=1$.

Proposition 6 : $\eta(a,b)=\mu(\{f\in\Omega\mid f(a)=f(b)\}), a,b\in B$.

Proposition 7 : (i) $\eta(a,b)=1$ if and only if $a=b$.
(ii) $\eta(a,a')=0$
(iii) $\eta(a,b)+\eta(a,b')=1$.
(iv) $\eta(a,c)\geq\eta(a,b)+\eta(b,c)-1$.

Proposition 8 : Suppose that $\eta(x,a)\geq\alpha,\eta(y,b)\geq\beta$ then $\eta(x\to y,a\to b)\geq\alpha+\beta-1$.

It follow from proposition 7 that the function ρ defined below is a metric on B.

Definition 3 : Define $\rho:B\times B$ as follows:

$$\rho(a,b)=1-\eta(a,b), a,b\in B.$$

Then ρ is a metric on B and (B,ρ) is called the **metric Boolean algebra** with respect to μ.

Theorem 1 : Let (B,ρ) be the metric Boolean algebra, then

(i) All the operations $',\vee,\wedge,\to$ are uniformly continuous.

(ii) If (B,ρ) is a complete metric space, then B is a $\omega-$complete lattice.

Proof : (i) By proposition 7, it follows that $\rho(a',b')=1-\eta(a',b')=\eta(a',b)=1-\eta(a,b)=\rho(a,b)$. If $\rho(a,b)\leq\varepsilon$, then $\rho(a',b')\leq\varepsilon$, where ε is any given positive number. This proves the continuity of the operation $':B\to B$.

Suppose that $\rho(x,a)\leq\varepsilon,\rho(y,b)\leq\varepsilon$, then $\eta(x,a)>1-\varepsilon,\eta(y,b)>1-\varepsilon$, and it follows proposition 8, having $\eta(x\to y,a\to b)\geq(1-\varepsilon)+(1-\varepsilon)-1=1-2\varepsilon$, hence $\rho(x\to y,a\to b)\leq 2\varepsilon$. This proves that the operation $\to: B\times B\to B$ is uniformly continuous.

Since $a\vee b=a'\to b, a\wedge b=(a'\vee b')'$, hence the operations $\vee$ and $\wedge$ also uniformly continuous.

(ii) Suppose that (B,ρ) is a complete metric space, and $\Delta=\{a_1,a_2,\cdots\}\subset B$. Let $b_n=\vee_{i=1}^n a_i$, then $b_1,b_2,\cdots$ is an increasing sequence and hence $\tau(b_1),\tau(b_2),\cdots$ is an increase sequence in [0,1] and is therefore Cauchy. Note that $b_n\to b_m=b_n'\vee b_m=1_B$, and $b_m'\wedge b_n=0_B$ whenever $b_n\leq b_m$, then $\rho(b_m,b_n)=1-\eta(b_m,b_n)=1-\tau((b_m\to b_n)\wedge(b_n\to b_m))=1-\tau(b_m\to b_n)=1-\tau(b_m'\vee b_n)=1-[\tau(b_m')+\tau(b_n)-\tau(b_m'\wedge b_n)]=1-\tau(b_m')-\tau(b_n)=\tau(b_m)-\tau(b_n)$. hence $b_1,b_2,\cdots$ is a Cauchy sequence in (B,ρ) and converges to an element c of B. Fix an n we have from $b_n=b_n\wedge b_{n+k}$ and the continuity of $\wedge$ that

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$$b_n = lim_{k\to\infty}(b_n \wedge b_{n+k}) = b_n \wedge lim_{k\to\infty} b_{n+k} b_n \wedge c,$$

hence $b_n \le c$ and c is an upper bound of $\Sigma = \{b_1, b_2, \cdots\}$. On the other hand, let e be any upper bound of Σ, then $b_n = b_n \wedge e$ and hence

$$c = lim_{n\to\infty} b_n = lim_{n\to\infty}(b_n \wedge e) = (lim_{n\to\infty} b_n) \wedge e = c \wedge e.$$

This means that $c \le e$, hence $c = sup\,\Sigma$. It is clear that $sup\Delta = sup\Sigma = c$. We can prove that $inf\Delta$ exists in a similar way, hence B is $\omega-$ complete lattice.

III. An Application

Consider the set of all abstract formulas in propositional logic. Let $S = \{p_1, p_2, \cdots\}$ be a countable set and $F(S)$ be free algebra of type $(\neg, \rightarrow)$ generated by S where $\neg$ and $\rightarrow$ are unary and binary operations respectively. Homomorphisms from $F(S)$ to $D = \{0,1\}$ called valuations of $F(S)$, the set is consisting of valuations will be denoted Ω. Assume that A, B are formulas (propositions) of $F(S)$, A and B are logically equivalent if $\forall\nu \in \Omega, \nu(A) = \nu(B)$. It is well known that the logical equivalence relation $\approx$ is a congruence relation on $F(S)$ with respect to $(\neg, \rightarrow)$, and the quotient algebra $F(S)/=\approx$ is a Boolean algebra and called the Lindenbaum algebra[6], and denoted B_L. Let $X = \prod_{n=1}^{\infty} X_n$, where $X_n = \{0,1\}$, and μ_n be the evenly distributed probability measure on X_n, *i.e.*, $\mu(\emptyset) = 0$, $\mu(X_n) = 1$, and $\mu_n(\{0\}) = \mu_n(\{1\}) = \frac{1}{2}, (n = 1, 2, \cdots)$, and let μ be the infinite product of $\mu_1, \mu_2, \cdots$ on X (see[3]). Since $F(S)$ is the free algebra generated by S, a valuation $\nu : F(S) \rightarrow \{0,1\}$ is completely decided by its restriction $\nu|S$. Since $\nu|S = \{\nu(p_1), \nu(p_2), \cdots\}$ can be thought of as a point of X, there is a bijection between Ω and X and hence the measure μ on X can be transplanted on Ω. We use the same symbol μ to denote the measure on Ω, *i.e.*,

$$\mu(\Sigma) = \mu(\{(x_1, x_2, \cdots) \in X \mid \exists\nu \in \Sigma, x_n = \nu(p_n), n = 1, 2, \cdots\})\Sigma \subset \Omega.$$

Assume that $A, B \in F(S)$ and $A \approx B$ and $\nu \in \Omega$, then $\nu(A) = \nu(B)$ hence ν induces a unique Boolean homomorphism $\nu^* : B_L = F(S)/\approx \rightarrow \{0,1\}$ defined by $\nu^*(a) = \nu(A)$, where a is the congruence class containing $A\,(a \in B_L)$. In the following ν^* will be simplified as ν and Ω can be thought of as the set consisting of all Boolean homomorphisms B_L to $\{0,1\}$. Therefore there is a metric ρ on the Lindenbaum algebra B_L and (B_L, ρ) will be called the **metric Lindenbaum algebra** . Denote the congruence class B_L containing A by $[A]$, where $A \in F(S)$ and define $d : F(S) \times F(S) \rightarrow [0,1]$ as follows:

$$d(A,B) = \rho([A],[B]), \quad A, B \in F(S),$$

then d is a pesudo-metric on $F(S)$. Now that there exists the concept of distances among formulas, an approximate reasoning theory can naturally be developed on $F(S)$. We give in the following only a short sketch.

Definition 4 : Let T be the set consisting of all theorems in $F(S)$ (see[2]) and $A \in F(S)$. If $d(A, \mathrm{T}) < \varepsilon$, then A is called a **theorem with error** ε, and denoted $(\varepsilon) \vdash A$. Moreover, let $D(\Gamma)$ be the set of all $\Gamma-$ conclusions, *i.e.*, $D(\Gamma) = \{A \in F(S) \mid \Gamma \vdash A\}$.

If $inf\{H(D(\Gamma), \mathcal{T}) \mid \Gamma \vdash A\} < \varepsilon$, where $H(\Sigma, \Delta)$ is Hausdorff distance between Σ and Δ (Σ, Δ) are non-empty subsets of $F(S)$, for a general definition of the Hausdorff distance between two non-empty, bounded subsets of a metric space without the requirement of closedness see[1], and that of subsets of a pseudo-metric space, see[6]), then A is called $\varepsilon-$ **quasi theorem** and denoted $\vdash (\varepsilon)A$.

Theorem 2 : Suppose that A is any formula of $F(S)$, then

$$(\varepsilon) \vdash A \text{ if and only if } \vdash (\varepsilon)A$$

Definition 5 : Suppose that $\Gamma \subseteq F(S)$, let $Div(\Gamma) = sup\{\rho(A,B) \mid A, B \in D(\Gamma)\}$, where $sup\ \emptyset = 0$ is assumed, then $Div(\Gamma)$ is called **the divergence degree of** *Gamma*. Moreover , let $Dev(\Gamma) = H(D(\Gamma), \mathcal{T})$, then $Dev(\Gamma)$ is called the **deviation** of Γ.

Theorem 3 : Suppose that $\Gamma \subseteq F(S)$, then $Div(\Gamma) = Dev(\Gamma)$.

Proof : Suppose that $Dev(\Gamma) = \alpha$ and $\varepsilon > 0$, then it follows from

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$$H(D(\Gamma),\mathcal{T}) = max(sup\{d(A,\mathcal{T})|A\in D(\Gamma)\}, sup\{d(T,D(\Gamma))|T\in\mathcal{T}\})$$
$$= sup\{d(A,\mathcal{T})|A\in D(\Gamma)\} = \alpha.$$

that there is an $A\in D(\Gamma)$ such that $d(A,\mathcal{T})\geq \alpha-\varepsilon$. Choose any T from $\mathcal{T}$, then $A,T\in D(\Gamma)$ and we have $Div(\Gamma)\geq d(A,\mathcal{T})\geq \alpha-\varepsilon$ Since ε is arbitrary, we have $Div(\Gamma)\geq\alpha = Dev(\Gamma)$.

Conversely, we have

$$Dev(\Gamma) = H(D(\Gamma),\mathcal{T}) = max(sup\{d(A,\mathcal{T})|A\in D(\Gamma)\}, sup\{d(T,\mathcal{T})|T\in D(\Gamma)\})$$
$$= sup\{d(A,\mathcal{T})|A\in D(\Gamma)\} = sup\{1-\tau([A])|A\in D(\Gamma)\}.$$

Moreover, assume that $A,B\in D(\Gamma)$. Note that $[A]\leq[B]\to[A]$, and $[B]\leq[A]\to[B]$, we have $d(A,B) = \rho([A],[B]) = 1-\eta([A],[B]) = 1-\tau(([A]\to[B])\wedge[B]\to[A]))\leq 1-\tau([A])\wedge\tau([B]) = (1-\tau([A]))\vee(1-\tau([A]))$.

Hence , $Div(\Gamma) = sup\{d(A,B)|A,B\in D(\Gamma)\}\leq D(\Gamma)$.

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Review : The Importance of Molecular Markers in Plant Breeding Programmes

By P.M Jonah, L. L. Bello, O. Lucky, A. Midau, S. M. Moruppa
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Abstract - Since the advent of restriction fragment length polymorphism (RFLP) markers, a range of other markers such as Random Amplified Polymorphism DNA (RAPD), amplified fragment length (AFLP), Simple sequence repeats (SSRS), etc has been introduced during the last two decades of the 20th century to fulfill various demands of the last breeding programmes. Ever since their invention, they are being, constantly modified for enhanced utility as a means to solve problems and to bring about automation in the genome analysis, gene tagging, phylogenic analysis and selection of desirable genotypes. It is also evidence that molecular markers (non morphological markers) offer several advantages over the morphological markers (conventional phenotypic markers), as they provide data that can be analyzed objectively; giving new dimension to breeding especially with respect to the time required to developing new improved crop varieties. In terms of scientific progress, the old disciplines of quantitative genetics and plant taxonomy have been revived by the molecular marker approach, which have the immediate application in supportive research for advanced breeding programmes. Therefore, the success of DNA marker technology for bring genetic improvement in crop plants would depend on close interaction between plant breeders and biotechnologists, availability of skilled manpower and substantial financial investment on research.

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REVIEW THE IMPORTANCE OF MOLECULAR MARKERS IN PLANT BREEDING PROGRAMMES

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Review : The Importance of Molecular Markers in Plant Breeding Programmes

P.M Jonah[α], L. L. Bello[β], O. Lucky[β], A. Midau[Ω] S. M. Moruppa[Ω]

Abstract - Since the advent of restriction fragment length polymorphism (RFLP) markers, a range of other markers such as Random Amplified Polymorphism DNA (RAPD), amplified fragment length (AFLP), Simple sequence repeats (SSR_S) etc has been introduced during the last two decades of the 20th century to fulfill various demands of the last breeding programmes. Ever since their invention, they are being, constantly modified for enhanced utility as a means to solve problems and to bring about automation in the genome analysis, gene tagging, phylogenic analysis and selection of desirable genotypes. It is also evidence that molecular markers (non morphological markers) offer several advantages over the morphological markers (conventional phenotypic markers), as they provide data that can be analyzed objectively; giving new dimension to breeding especially with respect to the time required to developing new improved crop varieties. In terms of scientific progress, the old disciplines of quantitative genetics and plant taxonomy have been revived by the molecular marker approach, which have the immediate application in supportive research for advanced breeding programmes. Therefore, the success of DNA marker technology for bring genetic improvement in crop plants would depend on close interaction between plant breeders and biotechnologists, availability of skilled manpower and substantial financial investment on research.

I. Introduction

The theoretical advantages of using genetic markers and the potential value of genetic marker linkage maps and direct selection in plant breeding were first reported about eighty years ago (Crouch and Ortiz 2004). However, it was not until the advent of DNA marker technology in the 1980s, that a large enough number of environmentally insensitive genetic markers generated to adequately follow the inheritance of important agronomic traits and since then DNA marker technology has dramatically enhanced the efficiency of plant breeding. DNA-based molecular markers have acted as versatile tools and have found their own position in various fields like taxonomy, plant breeding, genetic engineering e.t.c (Joshi *et al*, 2011).

A number of breeding companies have in the past two decades to varying degrees started using markers to increase the effectiveness in breeding and to significantly shorten the development time of varieties and therefore plant geneticist consider molecular marker assisted selection a useful additional tool in plant breeding programs to make selection more efficient (Bueren *et al,* 2010; Joshi *et al*, 2011) over the last few decades plant genomics has been studied extensively bring about a revolution in this area, making molecular markers useful for plant genomic analysis, therefore becoming and important tool in this revolution. (Joshi *et al*, 2011).

The most significant breakthrough in agricultural biotechnology is coming from research into the structure of genomes and the genetic mechanisms behind economically important traits. The rapidly progressing discipline of genomics also known as molecular biology, is the provision of information on the identity, location, impact and function of genes affecting such traits which researchers have been identifying, cataloging and mapping single gene markers in many species of higher plants.

Molecular markers include biochemical constituents (e.g. secondary metabolites in plants) and macro-molecules, viz proteins and deoxyribonucleic acid (DNA). Analysis of secondary metabolites is, however restricted to those plants that produce a suitable range of metabolites which can be easily analyzed and which can be distinguished by varieties (Joshi *et al*, 2011). These metabolites which are being use as markers should be ideally neutral to environmental effects or management practices. Hence, amongst the marker molecular markers used, DNA markers are more suitable and ubiquitous to most of the living organisms.

Diversity based on phenotypic and morphological characters, usually varies with environments and evaluation of traits requires growing the plants to full maturity prior to identification, but now the rapid development of biotechnology allows easy analysis of large umber of loci distributed throughout the genome of the plants. Molecular makers have proven to be powerful tools in the assessment of genetic variation and in elucidation of genetic relationships within and among species (Chakravarthi and Naravaneni, 2006).

Molecular markers for classification of genotype are abundant, but unlike morphological traits, markers are not affected by environment (Staub, *et al*, 1997. Collecting DNA marker data to determine whether

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phenotypically similar cultivars are genetically similar would therefore be of great interest in crop breeding programme (Duzyaman, 2005).

Molecular genetics or the use of molecular technique for detecting differences in the DNA of individual plants has many applications of value to crop improvement (Wammanda and Jonah, 2006). The differences are called molecular markers because they are often associated with specific gene and acts as a 'sign posts' to those genes and such markers when very tightly linked to genes of interest, can be use to select indirectly for the desirable allele and this represents the simplest form of marker- assisted selection (MAS). (Hoisington, *et al*, 2002).

Markers can also be used for dissecting polygenic traits into their Mendelian components or Quantitative Trait Loci (QTL) and this increasing understanding of the inheritance and gene action for such traits allows the use of markers – selection procedures (Anderson *et al*, 1993).

The molecular markers are no longer looked upon as simple DNA fingerprinting markers in variability studies or as mere forensic tools, but, they are constantly being modified to enhance their utility and to bring about automation in the process of genome analysis (Joshi *et al*, 2011). The discovery of polymerase chain reaction (PCR) was a land mark in this effort and proved to be a unique process that brought about a new class of DNA profiling marker, which has facilitated the development of marker-based gene tags, map-based cloning of agronomic important genes, variability studies, phylogenetic analysis, synteny mapping, market assisted selection of desirable genotypes e.t.c. DNA markers offer several advantages over traditional phenotypic markers, as they provide data that can be analyzed objective. Therefore, several molecular marker types are available and they each have their advantages and disadvantages (Cadalen *et al*, 1998).

II. Molecular Markers

In the early part of the 20th century, scientist discovered that, Mendelian factor controlling inheritances (genes) are organized in linear order on cytogenetically defined structure called chromosomes. It was shown that, combination of genes can be inherited in a group (i.e. they are linked together because they are close to each other on the same chromosomes. The individual genes flanking within a defined close interval are known as molecular DNA markers. Molecular markers are identifiable DNA sequence, found at specific locations of the genome and associated with the inheritance of a trait or linked gene (FAO, 2004). Thottappilly *et al* (2000), refer to molecular markers as naturally occurring polymorphism which include proteins and nucleic acids that are detectably different. Rapid advances are genome research and molecular biology has led to the use of DNA markers in plant breeding. Target genes in a segregating population can be identified with the assistance of DNA makers so as to accelerate traditional breeding programs (Thottappilly *et al*, 2000). Markers must be polymorphic (i.e. they must exist in different forms so that the chromosome carrying the mutant gene can be distinguished from the chromosome with normal gene by form of the marker it carries. Polymorhism can be detected at three levels: morphological, biochemical or molecular. Recently, the term DNA fingerprinting /profiling is used to describe the combined use of several single locus detection systems and are being used as versatile tools for investigating various aspects of plant genomes. These include characterization of genetic variability, genome fingerprinting, genome mapping, gene localization, analysis of genome evolution, population genetics, taxonomy, plant breeding and diagnostics (Joshi *et al*, 2011) The development of DNA (or molecular markers) has irreversibly change the disciplines of plant genetics and breeding (Collard and Mackill, 2006), According to Joshi *et al* (2011),an ideal DNA makers should however poses the following properties.(i) Highly polymorphism, which is the simultaneous occurrence of a trait at the same population of two or more discontinues variants or genotypes.

(i) Co dominant inheritance- different form of marker should be detected in a diploid organism to allow discrimination of homozygote and heterozygote.
(ii) Frequent occurrence in genome
(iii) Selective neutral behaviour (the DNA sequences of any organism are neutral to environmental conditions or management practices)
(iv) Easy access (availability)
(v) Easy and fast assay
(vi) Reproducible – highly reproducibility and
(vii) Easy exchange of data between laboratories.

It is extremely difficult for a single genetic marker to possess all properties above. Depending on the type of study to be undertaken a marker system can be identified that would fulfill at least a few of the above characteristics.

a) *Types and description of DNA markers*

i. *Non-PCR based genetic markers (Restriction fragment length polymorphism):*

The first and forest molecular markers system called the Restriction Fragment length Polymorphism (RFLP), was developed in early 1980 (Farooq and Azam, 2002). The RFLPs are simply inherited naturally occurring Mendelian characters. Genetic information is stored in the DNA sequence on a chromosome and variation in this sequence is the basis for the genetic diversity within species. Plants are able to replicate their DNA with high accuracy and rapidity, but many mechanisms causing changes (mutation) in the DNA are operative (Joshi *et al*, 2011).This leads to simple or large-base pair changes as a result of inversion, translocation, transpositions or deletion which may occur, resulting in a loss or gain of a recognition sites and in turn lead to restriction fragment of different lengths. This marker was first reported by Botstein *et al*, (1980); in the detection of DNA polymorphism (Agarwal *et al,* 2008).

Genomic restriction fragment of different length between genotypes can be detected on southern blots and by a suitable probe. In this method, DNA is digested with restriction enzyme like EcoR1, which cut the DNA at specific sequences, electrophoresed, blotted on a membrane and probed with a labeled clone. RFLP marker provides a way to directly follow chromosome segments during recombination as they follow Mendelian rules and greatly aid in the construction of genetic maps. When an F_1 plants undergoes meiosis to produce gametes, its chromosomes will undergo recombination by crossing over and this recombination is the basis of conventional genetic mapping and when use, RFLP markers, require hybridization of probe DNA with sampled plant DNA.

III. Polymerase Chain Reaction Based Markers

A decade after the emergence of AFLP, there was another breakthrough which involves the use of PCR in 1990 (Farooq and Azam, 2002).

PCR is an in vitro method of nucleic acid synthesis by which a particular segment of DNA can be specifically replicated (Mullis and Faloona, 1987). The process involves two oligonucleotide primers that flank the DNA fragment of interest and amplification is achieved by a series of repeated cycles of heat denaturation of the DNA, annealing of the primer to their complementary sequences, and extension of the annealed primers with a thermophilic DNA polymerase. Since the extension products themselves are also complementary to primers, successive cycles of amplification essentially double the amount of the target DNA synthesized in the previous cycle and the result is an exponential accumulation of the specific target fragment.

Genomic DNA from two different individual often produces different amplification and a particular fragment generated from one individual but not for other represent DNA polymorphism and can be used as genetic markers. The pattern of amplified bands so obtained could be use for genomic fingerprint (Welsh and McClelland 1990).

a) Randomly - amplified polymorphic DNA marker

The randomly-amplified polymorphic DNA marker (RAPD), detects nucleotide sequence polymorphism in DNA by using a single primer of arbitrary nucleotide sequence (Oligonucleotide primer, mostly ten bases long) (William *et al*, 1991). In this reaction, a single species of primer anneals to the genomic DNA at two different sites on complementary strands of DNA template.

Advantages associated with RAPD analysis include:

(i) Use of small amount of DNA which makes it possible to work with population that is not accessible with RFLP. It is fast and efficient in analysis having high-density genetic mapping as in many plant species such as alfafa (Kiss *et al*, 1993), fabean bean (Torress *et al*, 1993) and apple (Hammat *et al*, 1994)
(ii) Non involvement with radioactive assays (Kiss *et al*, 1993)
(iii) Non – requirement of species specific probe librararies
(iv) Non – involvement in blotting or hybridization.

Limitations of RAPD markers are:

(i) Its polymorphisms are inherited as dominant or recessive characters causing a loss of information relative to markers which show co-dominance.
(ii) Primers are relatively short, a mismatch of even a single nucleotide can often prevent the primer from annealing, hence leads to a loss of band.
(iii) Suffers from problems of repeatability in many systems, especially when transferring between populations or laboratories as is frequently necessary with marker assisted selection programs (Liu *et al*, 1994).

b) Amplified Fragment Length Polymorphism (AFLP)

$AFLP_S$ are fragments of DNA that have been amplified using directed primers from restriction of genomic DNA (Metthes *et al*, 1998). In this approach the sample DNA is enzymatically cut up into small fragments (as with RFLP analysis), but only a fraction of fragments are studied following selective PCR amplification (Liu *et al*, 1994). It is a combination of RFLP and RADP methods.

AFLP technique shares some characteristic with both RFLP and RAPD analysis (Farooq and Azam, 2002) and combines the specifically of restriction analyses with PCR amplification.

AFLP is extremely sensitive technique and the added use of fluorescent primers for automated fragment analysis system and software packages to analyze the biallelic data makes it well suitable for high thorough put analysis.

The major advantages of AFLP techniques (Farooq and Azam, 2002) are: (i) generation of a large number of polymorphism.

(i) No sequence information is required
(ii) The PCR technique is fast with high multiplex ratio which makes the AFLP very attractive choice.

The problems associated with AFLPs are of three types and all are related with practical handling, data generation and analysis. These problems are not unique to AFLP technology but also associated with other markers systems.

An ideal marker should have sufficient variation for the problem under study, be reliable and simple to generate and interpret. Unfortunately, neither AFLP nor other DNA markers exhibit these qualities. Thus a specific technique or techniques selected on the basis of objectives be utilized collectively to achieve the best results (Kharp *et al*, 1997; Harris, 1999).

c) Simple sequence repeat or short tandem repeats (SSRs) or micro satellites

These are ideal genetic markers for detecting differences between and within species of genes of all eukaryotes (Farooq and Azam, 2002).

It consist of tandemly repeated 2-7 base pair units arranged in repeats of mono-, di-, tri-, tetra and penta-nucleotides (A,T, AT, GA, AGG, AAAG etc) with different lengths of repeat motifs. These repeats are widely distributed throughout the plants and animal genomes that display high level of genetic variation based on differences in the number of tandemly repeating units of a locus. The variation in the number of tandemly repeated units results in highly polymorphic banding pattern (Farooq and Azam, 2002) which are detected by PCR, using locus specific flanking region primers where they are known.

Some of the prominent features of these markers are that they are dominant fingerprinting markers and codominant sequence tagged microsatellites (STMS) markers (Joshi *et al*, 2011).

The reproducibility of microsatellites is such that they can be used efficiently by different research laboratories to produce consistent data (Saghai Maroof *et al*, 1994). Locus-specific micro-satellite-based markers have been reported from many plant species such as Lettuce (*Lactuca sativa* L.) (Van de Wiel *et a,l* 1999), barley (*Hordeum vulgare* L.) (Saghai Maroof *et al*, 1994) and rice (*Oryza Sativa* L) (Wu and Tanksley, 1993).

Some other microsatellites based on the same principle include the following:

(i) ***Randomly Amplified Microsatellite Polymorphism (RAMP):*** This is a micro satellite – based marker which show a high degree of allelic polymorphism, but they are labor-intensive (Agarwal and Shrivastava, 2008). On the other hand RAPD markers are inexpensive but exhibit a low degree of polymorphism. To compensate for the weaknesses of these approaches, a technique termed as RAMP was developed (Wu *et al*, 1994). The technique involves a radiolabeled primer consisting of a 5^1 anchor and 3^1 repeats which is used to amplify genomic DNA in the presence or absence of RAPD primers. (Agarwal and Shrivastava, 2008).

(ii) **The Sequence Characterized Amplified Region (SCAR):** The SCARS are PCR-based markers that represent genomic DNA fragments at genetically defined loci that are identified by PCR amplification using sequence specific oligonucleotide primer (McDermoth *et al*, 1994).

(iii) **Simple Primer Amplification Reaction (SPAR):** SPAR uses the single SSR oligonucleotide principles.

(iv) **Sequence – Related Amplified Polymorphism (SRAP):**The aim of SRAP technique (Li and Quiros, 2001) is the amplification of open reading frames (ORFs). It is base on two-primer amplification using the AT- or GC- rich cores to amplify intragenic fragment for polymorphism detection (Agarwal and Shirvastava, 2008).

(v) **Target region amplification polymorphism (TRAP):** The TRAP technique (Hu and Vick, 2003) is a rapid and efficient PCR-based technique, which utilizes bioinformatics tools and expressed sequence tag (EST) database information to generate polymorphic markers, around targeted candidate gene sequences.

IV. Marker Assisted Selection (MAS)

MAS which is sometimes referred to as genomics is a form of biotechnology which uses genetic finger printing techniques to assist plant breeders in matching molecular profile to the physical properties of the variety. It is the identification of DNA sequences located near genes that can be tracked to breed for traits that are difficult to observe (Barloo and Stam, 1999). MAS refer to the use of DNA markers that are tight-linked to target loci as a substitute for or to assist phenotype screening. By determining the allele of a DNA marker, plants that possess particular genes or quantitative trait loci (QTL) may be identified based on their genotype rather than their phenotype.

Collard and Mackill (2006), reported the fundamental advantages of MAS compared to conventional phenotypic selection which are:-

(i) Simpler compared to phenotypic breeding
(ii) Selection may be carried out at breeding stage and single plants may be selected with high reliability.

In this technique, linkages are sought between DNA markers and agronomic important traits such as resistance to pathogens, insects and nematodes, tolerance to biotic stresses, quality parameters and quantitative traits.

MAS is in contrast to genetic engineering which involves the artificial insertion of such individuals genes from one organism into the genetic material of another (typically, but not exclusively from other unrelated species (Wamanda and Jonah, 2006).

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V. Breeding of Polygenic Traits

The utilization of markers can obviously prevent loss of quantitative trait loci (QTL) common with some crops DNA markers and this allow us to unravel the genetic basis of traits expressing continuous phenotypic variations as they are abundant and scattered throughout the genome. By using dense genetic marker maps, the contributions of separate regions of the genome on the trait values can be estimated once the mapping population is sufficiently large. In addition, agronomic important traits like nutritional quality, yield, flower time and durable resistance which appear to follow complex, polygenic inheritance patterns with multiple genes having small effects on the trait value can easily by analyzed using markers. Evidences obtained from various crops indicate that even such complex traits appear to be determined by only a few major factors/genes. (Frary *et al*, 2000 and Thornsberry *et al*, 2001).

VI. Application of Molecular Markers in Plant Genome Analysis and Breeding

Molecular markers have been look upon as a tools for a large number of applications ranging from localization of a gene to improvement of plant varieties by marker-assisted selection, called genome analysis which has generated a vast amount of information and a number of databases are being generated to preserve and popularize it (Joshi *et al*, 2011).

a) *Application of MAS in vegetative propagated crops*

The first generation of DNA markers analysis of vegetative propagated crops at IITA was focused on germplasm characterization, construction of preliminary linkage maps and development of disease diagnostics in plantain/banana, cassava and yam (Ortiz, 2004).

i. *Plantain / Banana*

At IITA plantain improvement was nominated as the model system for developing molecular breeding systems within this Institute (Crouch and Tenkoune, 1999). Parthenocarpy (ability to develop fruit in the absence of seed development) was chosen as an ideal character for the initiation of a marker assisted selection program.

Parthenocarpy seems to be controlled by just a few genes yet a high proportion of current breeding populations are non-parthenocarpic but can not be identified as such until close to harvest. As such MAS for parthenocarpy at seedling stage would have a dramatic influence on breeding efficiency. Plantain and banana current priorities for the molecular breeding of Musa crops focus on the development of appropriate MAS schemes for parthenocarpy,apical dominance/regulated suckering and short cropping cycle. Thereafter, the focus will turn to markers for post-harvest characters and for favorable alleles contributing to heterosis in yield components.

ii. *Cassava*

The development of markers for post-harvest characters and virus resistances appears to warrant the greatest emphasis for cassava breeders. Based on the urgent need, William, (1999) proposed that attention should be focus on the development of DNA markers for tolerance to abiotic stress and for storage characteristics.

iii. *Legumes*

a. *Bambara groundnut*

Amplified fragment length polymorphism (AFLP) was used to assess genetic diversity among 100 selected bambara groundnut (*Vigna subterranea* L Verdc).

The results showed that bambara groundnut landraces from Tanzania form a genetically diverse population. Therefore, AFLP markers can be effectively employed to assess genetic diversity and to measure genetic relationship among accessions (Ntundu *et al*, 2004)

b. *Cowpea*

Cowpea, a legume crop grown in the semi-arid tropics is attached by insect pests. Thus, in cowpea, the development of markers for resistance to thrips, bruchids, maruca and pod borer is considered of great priority. In the long term, markers for resistance to parasitic weed (striga) and markers for genes contributing to drought resistance are considered a high priority intervention (Morales *et al*, 2000).

c. *Soybean*

Tremendous advances in all aspects of the molecular breeding of soybean are being made in advanced laboratories particularly in the USA. These may provide substantial background understanding many of the constraints to soybean cropping in sub-Saharan Africa which are very different. Therefore, a high priority for example, could be the use of marker-assisted breeding for selecting lines with the ability to cause suicidal germination of *Striga hermonthica*, a parasitic weed affecting maize but not soyabean (trap crop). In the longer term, increased nodulation and resistance to pod shattering would be highly important candidates for MAS systems (Ortiz, 2004).

d. *Chickpea*

Ascochyta rabiei (pass) Labri is the most severe fungal disease limiting chickpea production and studies in Syria revealed the occurrence of three pathotypes for *A. rabiei*. A set of micro satellite and RAPD markers were also used which lead to identification of suitable RAPD markers, allowing a more precise determination of the pathotypes. Furthermore, the availability of markers for pathotype I and II allow the monitoring of the pathotype distribution, which gives the recommendation for the planting of suitable chickpea cultivars (Baum, 2003)

IV. *Cereal*

a. *Maize*

Prasanna and Pixley (2010) stress the importance of efforts in meeting the growing demand for maize and provide examples of the recent use of molecular markers with respect to (i) DNA finger printing and genetic diversity analysis of maize germplasm (inbreds and landraces/OPVs), (ii) QTL analysis of important biotic and abiotic stresses and (iii) MAS for maize improvement. Advances in genome analysis led to the identification of numerous DNA markers in maize includes thousand of mapped micro-satellite markers and more recently, single nucleotide polymorphisms (SNPs) and insertion-deletion (INDel) markers. With the SSRs and SNPs, a large number of genes controlling various aspects of plant development, biotic and abiotic stress resistance, quality characters etc, have been cloned and characterized in maize, which are excellent assets for molecular-assisted breeding (Prassana and Pixley, 2011).

At present SSRs are the most widely used markers by maize researchers due to their availability in large numbers in the public domain including their simplicity and effectiveness (Maize CrDB; http:www.maizegdb.org). These PCR-based, genetically co-dominate marker are robust, reproducible, hyper variable, abundant, and uniformly dispersed in plant genomes (Powell *et al*, 1996). Also both SSRs and SNPs can be reliably applied on a large scale and therefore offer significant advantages for genetic and breeding purposes.

SSR markers have been successfully used for DNA finger printing and analysis of genetic diversity in china, India, Indonesia and Thailand (Prassana and Pixley, 2010).

Following the first report on QTLs for yield-related traits in maize (Stuber *et al*, 1987), maize researchers worldwide have generated numerous reports of molecular markers tagging genes/QLTs for diverse traits of agronomic and scientific interest (Prasanna and Pixley, 2010).

QLTs for several important traits affecting maize such as plant height, downy mildew resistance, Maize dwarf Mosaic Virus resistance, head smut resistance, drought stress tolerance, water logging, nutrient components under low nitrogen and high-oil content.

Further, significant progress has been made world wide in optimizing MAS for improvement of both qualitative and quantitative inherited traits using maize as a model system. One successful example of MAS for maize development and of particular use is the utilization of opaque 2-specific SSR markers in conversion of maize lines in quality protein maize (QPM) lines with enhanced nutritional quality (Buba *et al*, 2005). A MAS-derived QPM hybrid is the "Vivek QPM hybrid 9," recently released in Almora, India, which was developed through marker-assisted transfer of the 02 gene and phenotypic selection for endosperm modifiers in the parental lines (Buba *et al,* 2005).Using MAS Scientist at IARI have pyramided major genes /QTL_S for resistance to *turicum* leaf blight and *Polysora rust* in five elite Indian lines (Prassana *et al*, 2009b) and these are CM 137, CM138, CM139, CM150 and CM151 which are parents of three single-cross hybrids.

b. *Sorghum*

The development of DNA markers for resistance to pests and diseases in sorghum is receiving great priority e.g. in breeding new populations for striga prone environment (Crouch and Ortiz, 2004). Five genomic regions (QTL) associated with stable striga resistance from resistant line N13 have been identified across a range of 10 field trials in Mali and Kenya and two independent samples of a mapping population involving this resistance source, indicating that the QLT are biological realities.

V. *Vegetable - Okra*

Okra is an important vegetable in India, West Africa, south-east, Asia, USA, Brazil, Australia and Turkey, which provides and important input of vitamin and mineral salts including calcium (IBPGR, 1990).

Omahinmin and Osawaru (2005) reported that high degree of wide morphological variation exist among accession of okra which requires further evidence using molecular markers to clarify. Among wide relatives of okra, *Abelmoschus angulosus* showed complete resistance to yellow vein mosaic virus (YVMV) and powdery mildew disease. *A. ficulneus* and *A. moschatus* accompany a high degree of resistance only to powdery mildew and these germplasm can be potential genetic resources in breeding okra for YVMV and powdery mildew resistance (Samarajeewa and Rathnayaha, 2004). Furthermore, Aladele *et al, (*2008), reported that 93 accessions of okra were assessed for genetic distinctiveness and relationships using RAPD (i.e 75 primers used). 59 showed strong and clear amplification, 7 showed weak amplification, while 9 primers did not show any application.

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Surface Water Quality Status in the part of Bhadravathi Industrial Town, Shimoga District, Karnataka, India

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Abstract - In order to know the surface water pollution in the part of Bhadravathi industrial town, Shimoga District, Karnataka state, the present study has been conducted around Mysore Paper Mill (MPM) solid waste dump site, which may be the source of pollution. Nine surface water samples were collected and analyzed for physico-chemical parameters and heavy metal concentrations. The results indicated that the concentration of Na is high in two samples and very high in three samples. The increase in the concentrations of Na and K in surface water samples may be probably due to the agricultural run-off and effluents discharged from the industries.

Keywords : *Physico-chemical, Bhadravathi, Mysore Paper Mill*

GJSFR Classification : *FOR Code : 960506*

Strictly as per the compliance and regulations of:

Surface Water Quality Status in the part of Bhadravathi Industrial Town, Shimoga District, Karnataka, India

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Abstract - In order to know the surface water pollution in the part of Bhadravathi industrial town, Shimoga District, Karnataka state, the present study has been conducted around Mysore Paper Mill (MPM) solid waste dump site, which may be the source of pollution. Nine surface water samples were collected and analyzed for physico-chemical parameters and heavy metal concentrations. The results indicated that the concentration of Na is high in two samples and very high in three samples. The increase in the concentrations of Na and K in surface water samples may be probably due to the agricultural run-off and effluents discharged from the industries.

Keywords : Physico - chemical, Bhadravathi, Mysore Paper Mill.

I. Introduction

The environmental problems that arise due to industrial activities are water pollution, air pollution, generation of hazardous waste and noise pollution. The nature of emissions and effluents from industries are varied and industry specific. If the effluents discharged from the industries are left on the land for natural evaporation or the solid wastes disposed as landfill or used for agriculture, the pollutants may reach the surface water bodies through runoff and later infiltrates to pollute the ground water. In this regard the present study is concentrated only on the surface water quality assessment in Bhadravathi industrial town of Karnataka, India.

Bhadravathi town is one of the major industrial areas in Karnataka, in which river Bhadra is passing through the heart of the city and receiving sewage and effluents from Mysore Paper Mill (MPM) and Visweswaraiah Iron and Steel (VISL) industries and domestic wastes. Considering these facts the study area has been chosen in a buffer zone of 4.5 Kms from MPM dumpsite (13^0 48'46"N – 13^0 50' 52" N and 75^0 44' 40" E – 75^0 44' 44"E).

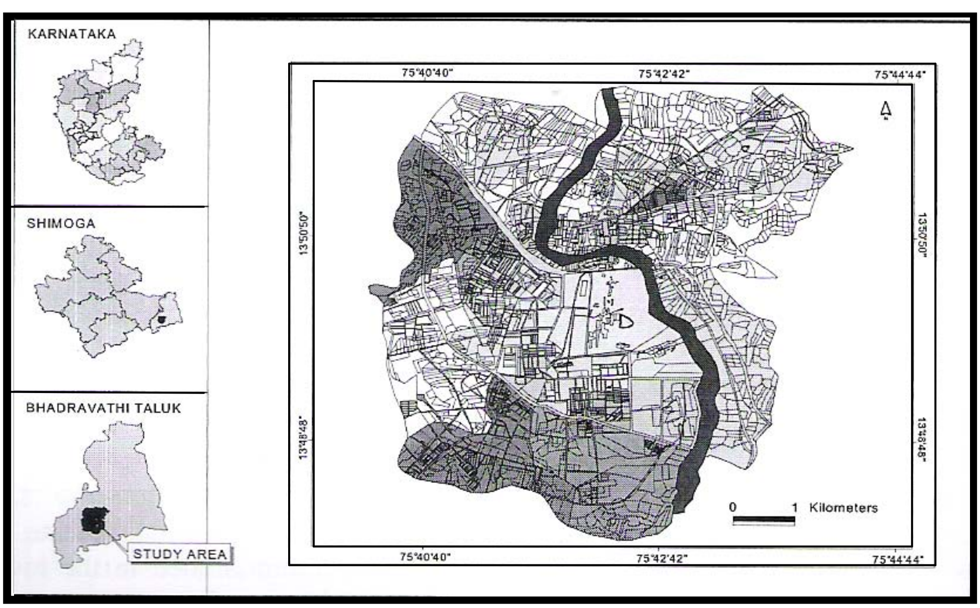

Fig 1 : Map showing the location of the study area

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II. Geology & Soil

Study area consists of ancient rock formation of Achaean age and these are mainly of Dharwar super group and the components of peninsular gneissic complex. The granitic gneisses occurring as scattered out crops, forms the principal exposures of the study area are with minor components of quartz-chlorite schist. These rocks are traversed by three set of joints i.e., N 35^0 W dipping 60^0 E, N 30^0 E dipping 60^0 W & N55^0E dipping almost vertical, respectively for first, second and third set of joints and the major soil type is red loamy.

III. Methodology

The present work is based on nine surface water samples that are collected during the year 2006. Samples were collected were analyzed for pH by Digital pH meter, EC by Conductivity meter, TH, Cl, Mg, Ca by titrimetric method. BOD, COD by Winkler and Reflux condensation methods respectively. TS, TDS, TSS by Evaporation method, SO_4, NO_3, PO_4 by UT-Visible Spectrophotometer, Heavy metals by ICP-AES.

IV. Results and Discussion

pH varies from 6.39 to 8.44 and are with in the permissible limits of 6.5 to 8.5 are prescribed by BIS (1991). The EC values ranges from 238 to 806 and this is within the desirable limit (1500) of BIS except in sample no 4. Concentrations of Cl and TH ranges from 4 to 221 mg/l and 69 to 220 mg/l respectively and these are also well within the permissible limits. Total alkalies expressed as HCO_3 and TDS concentrations of surface water samples of the present study are 120 to 220 mg/l and 155 to 524 mg/l respectively and these are within the permissible limits.

Concentrations of SO_4, NO_3 and PO_4 of the samples are in range of 2.61 to 17.84 mg/l, 3.16 to 23.73 mg/l and 0.93 to 1.94 mg/l respectively and are within the permissible limits. The value of Ca and Mg varies from 17 to 48 mg/l and 6 to 23 mg/l respectively and are well within the permissible limits.

Sodium is present in all natural water as sodium salts, which are high soluble in water. Except in two samples (samples no.3 & 7) concentration of Na is high in sample no 2 & 6 to very high in samples 1, 4 & 5. The concentration of K is within permissible limits excluding sample 4 & 5 in these two samples the levels of K is 45 and 13 respectively. The increase in concentrations of Na and K in the surface water sample no 4 & 5 is probably due to the agricultural run-off and effluents discharged from industries.

BOD of the samples are with in the permissible limits, except sample no 6 & 7, which is 12 and 8 mg/l respectively, this higher values in BOD may be due to sewerage input, which was observed during the field visit.

COD has been found to be more scientific than BOD and it's not much influenced by pH value of water, type of micro-organism, presence of toxic materials, nitrification processes and residual mineral matter (Sharma and Kaur, 1994). The COD values in the study area are slightly higher than the permissible limit of WHO (10 mg/l), except sample no 5. All the above said values are tabulated in Table 1.

a) *Status of Heavy metals in the samples*

The results obtained indicate that the Al value ranges from 0.009 to 0.153 ppm, Fe varies from 0.025 to 0.204 ppm and Mn varies from 0.003 to 0.232 and are well within the permissible limits set by WHO (2005) and BIS (1991). In transitional trace elements Ni and V are detected in all the samples and are varies from 0.001 to 0.003 and 0.001 to 0.005 respectively and are within the permissible limits. Where as Cr and Co have detected in samples no. 1, 2, 5, 6 and sample no 4 respectively, these are within the permissible limits.

Concentrations of Cu, Pb, Zn and Ba in these samples vary from 0.002 to 0.017 ppm, 0.001 to 0.004 ppm, 0.004 to 0.082 ppm and 0.031 to 0.079 ppm respectively and are well within the permissible limits.

b) *Agricultural suitability*

Assessment of surface water samples for agricultural suitability, USSL technique is generally followed for the classification of water for irrigation based on conductivity and Sodium Absorption Ratio (SAR). Sample no 3 and 6 belongs to class 1 of USSL, water of this class are low salinity type and can be used to grow almost all type of crops in all variety of soils. Sample no 1,2 and 7 belongs to class 2 of USSL, water of this class are medium salinity type and can be used to grow plants of moderate salt tolerance. Sample no 4 and 5 belongs to class 3 of USSL, water of this class are high salinity type and can be used to grow salt tolerance plants with special management for salinity control.

Sample no.	pH	EC (μmhos/c)	Cl	TH	HCO_3	TDS	BOD	COD	SO_4	NO_3	PO_4	Ca	Mg	Na	K
1	7.05	566	33	140	200	368	6	20	7.76	23.73	1.94	35	13	35	3
2	7.70	434	21	101	152	282	6	20	4.47	3.16	0.93	25	9	24	5
3	7.13	238	4	69	120	115	5	16	5.80	4.66	Nil	17	6	12	2
4	8.05	3440	249	332	780	2236	6	20	9.53	10.41	Nil	52	48	282	45
5	7.68	806	221	220	200	524	3	8	2.61	12.70	Nil	48	23	113	13
6	8.44	396	5	75	128	257	12	40	4.48	3.71	Nil	19	6	22	8
7	6.93	290	24	90	160	189	8	28	17.84	7.11	Nil	24	7	11	3
8	7.91	306	42	228	101	183	2	16	6.84	4.19	ND	34	11	17	2.3
9	7.84	383	46	148	125	230	2.50	20	7.02	5.02	ND	38	12	21	3.4

Table 1 : Physico-chemical parameters of surface water samples (All the values are in mg/l except pH & EC)

Sample no.	Sample location	Al	Fe	Mn	Cr	V	Co	Ni	Cu	Zn	Ba	Pb
1	Near coconut plantation south gate	32	94	15	2	2	BDL	2	5	82	56	1
2	Pond near the river course	39	36	5	2	2	BDL	3	3	13	35	1
3	Near water works	37	89	13	2	2	BDL	1	2	6	31	1
4	Kalingaeshawara temple	126	48	232	5	5	1	3	5	4	73	4
5	Thimmalapura North	39	27	9	2	2	BDL	2	17	5	79	2
6	Pond water near canal	153	111	12	1	1	BDL	2	5	6	63	1
7	Doddagoppenalli tank	9	204	3	1	1	BDL	1	3	6	36	2
8	Down stream of Gondi canal	ND	14	ND	ND	ND	ND	ND	16	20	ND	NIL
9	Up stream of Gondi canal	ND	16	ND	ND	ND	ND	ND	12	24	ND	NIL

Table 2 : Heavy metal concentrations of surface water samples (values are in ppb)

Sample no.	SAR	Mg Hazard	KR	RSC	CR	%Na	EC (μmhos/cm)	USSL Class	Water type
1	1.28	37.2	0.54	0.006	0.63	44.59	250-750	2	$CaHCO_3$
2	1.06	36.4	0.53	0.004	0.50	46.16	250-750	2	Ca-HCO_3
3	0.006	37.1	0.40	0.003	0.55	39.16	0-250	1	Ca-HCO_3
4	6.77	60.3	1.87	6.07	0.57	76.55	750-2250	3	Na-HCO_3
5	3.33	44.5	1.13	0.00	1.68	63.79	750-2250	3	Na-Cl
6	1.44	33.4	0.66	0.00	0.42	54.05	0-250	1	Na-HCO_3
7	0.005	31.1	0.28	0.003	1.37	31.49	250-750	2	Ca-HCO_3
8	0.647	34.3	0.38	0.00	1.30	30.16	250-750	2	Ca-HCO_3
9	0.752	35.0	0.41	0.00	1.10	32.37	25-750	2	Ca-HCO_3

Table 3 : Irrigation suitability factors in study area

Permissible limits of Sodium Adsorption ratio (SAR), Magnesium hazard (MG-Haz), Kelley's radio (KR), Residual sodium carbonate (RSC), Corrosivity ratio (CR) and Sodium percentage (% Na) are 18-26,<50%,<1, 1.5-2.5 meq/l, <1, 40-60% respectively (Sharma B.K., and Kaur.H 1994).

V. Conclusions

As such there is no significant impact on surface water quality due to disposal of solid waste by MPM as landfill. Na and K concentrations in surface water sample no. 4 & 5 are high and are classified as high salinity type water as per USSL classification, this is probably due to the input of agricultural run-off and effluents discharged from industries and quality of these wastes should be monitored regularly. BOD of 6 and 7 no of samples are higher than the permissible limits. The higher level of BOD in these samples is due to sewerage input and the presence of weeds, which was observed during the field visit.

VI. Acknowledgements

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On Fractional Calculus and Certain Results Involving K_2-Function

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Abstract - In the present paper a new function called K_2 - function, which is an extension of the function defined by Miller and Ross[20], is introduced and studied by the author in terms of some special functions and derived the relations that exists between the K_2- function and the operators of Riemann-Liouville fractional integrals and derivatives.

Keywords and Phrases : fractional calculus, Riemann - Liouville fractional integrals and derivatives.

Mathematics Subject Classification : 26A33, 33C60

ON FRACTIONALCALCULUSANDCERTAINRESULTSINVOLVINGK-FUNCTION

Strictly as per the compliance and regulations of:

On Fractional Calculus and Certain Results Involving K_2 - Function

Kishan Sharma[α], V. S. Dhakar[Ω]

Abstract - In the present paper a new function called K_2- function, which is an extension of the function defined by Miller and Ross[20], is introduced and studied by the author in terms of some special functions and derived the relations that exists between the K_2 - function and the operators of Riemann - Liouville fractional integrals and derivatives.

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I. Introduction and Definitions

Fractional Calculus deals with derivatives and integrals of arbitrary orders. During the last three decades Fractional Calculus has been applied to almost every field of Mathematics like Special Functions etc., Science, Engineering and Technology. Many applications of Fractional Calculus can be found in Turbulence and Fluid Dynamics, Stochastic Dynamical System, Plasma Physics and Controlled Thermonuclear Fusion, Non-linear Control Theory, Image Processing, Non-linear Biological Systems and Astrophysics.

The Mittag-Leffler function has gained importance and popularity during the last one decade due mainly to its applications in the solution of fractional-order differential, integral and difference equations arising in certain problems of mathematical, physical, biological and engineering sciences. This function is introduced and studied by Mittag-Leffler[10,11] in terms of the power series

$$E_\alpha(x)=\sum_{n=0}^{\infty}\frac{x^n}{\Gamma(\alpha n+1)},\quad(\alpha>0) \tag{1.1}$$

A generalization of this series in the following form

$$E_{\alpha,\beta}(x)=\sum_{n=0}^{\infty}\frac{x^n}{\Gamma(\alpha n+\beta)},\quad,(\alpha,\beta>0) \tag{1.2}$$

has been studied by several authors notably by Mittag Leffler[10,11],Wiman[13], Agrawal[15], Humbert and Agrawal[8] and Dzrbashjan[1,2,3]. It is shown in [5] that the function defined by (1.1) and (1.2) are both entire functions of order $\rho=1$ and type $\sigma=1$. A detailed account of the basic properties of these two functions are given in the third volume of Bateman manuscript project[4] and an account of their various properties can be found in [2,12].

The multiindex Mittag-Leffler function is defined by Kiryakova[9] by means of the power series

$$E_{(\frac{1}{\rho i}),(\mu i)}(x)=\sum_{n=0}^{\infty}\varphi_n z^n=\sum_{n=0}^{\infty}\frac{x^n}{\prod_{j=1}^{m}\Gamma(\mu_j+\frac{n}{\rho_j})}$$

Where $m>1$ is an integer, ρ_j and μ_j are arbitrary real numbers.

The multiindex Mittag-Leffler function is an entire function and also gives its asypototic, estimate, order and type see Kiryakova[9].

An interesting generalization of (1.2) is recently introduced by Kilbas and Saigo[16] in terms of a special entire function of the form

$$E_{\alpha,m,l}(x)=\sum_{n=0}^{\infty}c_n x^n, \tag{1.4}$$

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Where $$c_n = \prod_{i=0}^{r-1} \frac{\Gamma[\alpha(im+l)+1]}{\Gamma[\alpha(im+l+1)+1]}, \ (n = 0,1,2,....)$$

and an empty product is to be interpreted as unity. Certain properties of this function associated with fractional integrals and derivatives[12].

In 1993, Miller and Ross[20] introduced a function as the basis of the solution of fractional order initial value problem. It is defined as the vth integral of the exponential function, that is,

$$E_x[\nu, a] = \frac{d^{-\nu}}{dx^{-\nu}} e^{ax} = x^{\nu} e^{ax} \gamma^*(\nu, ax) = \sum_{n=0}^{\infty} \frac{a^n x^{n+\nu}}{\Gamma(n+\nu+1)}, \nu \in C \tag{1.5}$$

where $\gamma^*(\nu, ax)$ is the incomplete gamma function.

The present paper is organized as follows; In section 2, we give the definition of the K_2 - function and its relations with another special functions, namely Miller-Ross's function,generalization of the Mittag-Leffler function[11] and its generalized form introduced by Prabhakar[20] etc. In section 3, relations that exists between K_2 - function and the operators of Riemann-Liouville fractional calculus are derived.

II. A New Special Function

The K_2 - function introduced by the first author is defined as follows:

$$\underset{(\nu;a)}{\overset{(p;q)}{K_2}}(a_1,\ldots,a_p;b_1,\ldots,b_q;x) = \underset{(\nu;a)}{\overset{(p;q)}{K_2}}(x) = \sum_{n=0}^{\infty} \frac{(a_1)_n\ldots(a_p)_n}{(b_1)_n\ldots(b_q)_n} \frac{a^n x^{n+\nu}}{\Gamma(n+\nu+1)} \tag{2.1}$$

where $\nu \in C$ and $(a_i)_n (i = 1,2,..., p)$ and $(b_j)_n \ (j = 1,2,..., q)$ are the Pochammer symbols.

The series (2.1) is defined when none of the parameters $b_j s$, $j = 1,2,..., q$, is a negative integer or zero. If any numerator parameter a_{jr} is a negative integer or zero, then the series terminates to a polynomial in x. From the ratio test it is evident that the series is convergent for all x if $p > q + 1$. When $p = q + 1$ and $|x| = 1$, the series can converge in some cases. Let $\gamma = \sum_{j=1}^{p} a_j - \sum_{j=1}^{q} b_j$. It can be shown that when $p = q + 1$ the series is absolutely convergent for $|x| = 1$ if $(R(\gamma) < 0$, conditionally convergent for $x = -1$ if $0 \le R(\gamma) < 1$ and divergent for $|x| = 1$ if $1 \le R(\gamma)$.

Special cases :

(i) When there is no upper and lower parameter, we get

$$\underset{(\nu;a)}{\overset{(0;0)}{K_2}}(-;-;x) = \sum_{n=0}^{\infty} \frac{a^n x^{n+\nu}}{\Gamma(n+\nu+1)} \tag{2.2}$$

Which reduces to the function of Miller and Ross[20].

(ii) If we put $a = 1, \nu = 0$ in (2.2), we get

$$\underset{(1;1)}{\overset{(0;0)}{K_2}}(-;-;x) = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(n+1)} \tag{2.3}$$

Which reduces to the Mittag-Leffler function [4] $E_1(x)$ or generalized Mittag - Leffler function [4] $E_{1,1}(x)$ or Exponential function[6] e^x

III. Relations With Riemann - Liouville Fractional Calculus Operators

In this section we derive relations between K_2 - function and the operators of Riemann-Liouville Fractional Calculus.The relations are presented in the form of two theorems as follows:

Theorem 3.1 Let $\alpha > 0, \nu \in C$ and I_x^α be the operator of Riemann-Liouville fractional integral then there holds the relation:

$$I_x^\alpha \, {}_{(\nu;a)}^{(p;q)}K_2(a_1,\ldots,a_p;b_1,\ldots,b_q;x) = \frac{x^{\alpha+\nu}}{\Gamma(\alpha+1)} \, {}_{(\nu;a)}^{(p+1;q+1)}K_2(a_1,\ldots,a_p,1;b_1,\ldots,b_q,\alpha+1;x) \tag{3.1}$$

Proof : Following Section 2 of the book by Samko, Kilbas and Marichev[8], the fractional Riemann – Liouville (R-L) integral operator (For lower limit $a = 0$ w. r. t. variable x) is given by

$$I_x^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt \tag{3.2}$$

By virtue of (3.2) and (2.1), we obtain

$$I_x^\alpha \, {}_{(\nu;a)}^{(p;q)}K_2(a_1,\ldots,a_p;b_1,\ldots,b_q;x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \sum_{n=0}^{\infty} \frac{(a_1)_n\ldots(a_p)_n}{(b_1)_n\ldots(b_q)_n} \frac{a^n t^{n+\nu}}{\Gamma(n+\nu+1)} dt \tag{3.3}$$

Interchanging the order of integration and evaluating the inner integral with the help of Beta function, it gives

$$I_x^\alpha \, {}_{(\nu;a)}^{(p;q)}K_2(a_1,\ldots,a_p;b_1,\ldots,b_q;x) = \frac{x^{\alpha+\nu}}{\Gamma(\alpha+1)} \sum_{n=0}^{\infty} \frac{(a_1)_n\ldots(a_p)_n (1)_n}{(b_1)_n\ldots(b_q)_n (\alpha+1)_n} \frac{a^n x^{n+\nu}}{\Gamma(n+\nu+1)}$$

$$= \frac{x^{\alpha+\nu}}{\Gamma(\alpha+1)} \, {}_{(\nu;a)}^{(p+1;q+1)}K_2(a_1,\ldots,a_p,1;b_1,\ldots,b_q,\alpha+1;x) \tag{3.4}$$

The interchange of the order of integration and summation is permissible under the conditions stated along with the theorem due to convergence of the integrals involved in this process.

This shows that a Riemann-Liouville fractional integral of the K_2 - function is again the K_2 – function with indices $p+1$, $q+1$.

This completes the proof of the theorem (3.1).

Theorem 3.2 Let $\alpha > 0, \nu \in C$ and D_x^α be the operator of Riemann - Liouville fractional derivative then there holds the relation:

$$D_x^\alpha \, {}_{(\nu;a)}^{(p;q)}K_2(a_1,\ldots,a_p;b_1,\ldots,b_q;x) = \frac{x^{-\alpha-\nu}}{\Gamma(1-\alpha)} \, {}_{(\nu;a)}^{(p+1;q+1)}K_2(a_1,\ldots,a_p,1;b_1,\ldots,b_q,1-\alpha;x) \tag{3.5}$$

Proof : Following Section 2 of the book by Samko, Kilbas and Marichev[8], the fractional Riemann- Liouville (R-L) integral operator (For lower limit . $a = 0$ w. r. t. variable x) is given by

$$D_x^\alpha f(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_0^x (x-t)^{n-\alpha-1} f(t)dt \tag{3.6}$$

Where $n = [\alpha] + 1$.

From (2.1) and (3.6) it follows that

$$D_x^{\alpha} \underset{(\nu;a)}{\overset{(p;q)}{K_2}} (a_1,\dots,a_p;b_1,\dots,b_q;x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_0^x (x-t)^{n-\alpha-1} \sum_{n=0}^{\infty} \frac{(a_1)_n \dots (a_p)_n}{(b_1)_n \dots (b_q)_n} \frac{a^n t^{n+\nu}}{\Gamma(n+\nu+1)} dt \quad (3.7)$$

Interchanging the order of integration and evaluating the inner integral with the help of Beta function, it gives

$$D_x^{\alpha} \underset{(\nu;a)}{\overset{(p;q)}{K_2}} (a_1,\dots,a_p;b_1,\dots,b_q;x) = \frac{x^{-\alpha-\nu}}{\Gamma(1-\alpha)} \sum_{n=0}^{\infty} \frac{(a_1)_n \dots (a_p)_n (1)_n}{(b_1)_n \dots (b_q)_n (1-\alpha)_n} \frac{a^n x^{n+\nu}}{\Gamma(n+\nu+1)}$$

$$= \frac{x^{-\alpha-\nu}}{\Gamma(1-\alpha)} \underset{(\nu;a)}{\overset{(p+1;q+1)}{K_2}} (a_1,\dots,a_p,1;b_1,\dots,b_q,1-\alpha;x) \quad (3.8)$$

This shows that a Riemann-Liouville fractional derivative of the K_2 - function is again the K_2 – function with indices $p+1, q+1$.

This completes the proof of the theorem(3.2).

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V. Conclusion

It is expected that some of the results derived in this survey may find applications in the solution of certain fractional order differential and integral equations arising problems of physical sciences and engineering areas.

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Determinant Post Harvest Losses among Tomato Farmers in Imeko-Afon Local Government Area of Ogun State, Nigeria

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Abstract - Food supply can be induced either by increase in production or reduction in loss. Many post harvest losses are direct result of factors such as high field temperatures on crops before harvesting, pests and diseases attack etc, hence increase in the losses after harvest. The study area is Imeko Afon Local Government Area of Ogun State. Purposive sampling technique was employed in selecting 88 respondents and administering the questionnaire, and 88 were used for analyzing the data. Results on socio-economic characteristics revealed that, majority of the farmers (69%) fell into the active workforce and they had farm sizes ranging from 1-5hectares. Larger percentage of the farmers had an education below secondary level (83%).Majority of the tomato farmers had household size greater than 33% and 68% of the farmers had less than 16 years experience in tomato production. About 72% make use of van/pick up in transporting their produce from the farm to the market. No storage facilities were used in the study area to preserve the fruits from rotten after harvesting as at the time of study. The average gross margin with post harvest losses (9,251.41) is less than the average gross margin when no damage occurred in the fruits (72,752.55), thus showing that post harvest losses reduce the mine of farmers in the study area. All the independent variables tested on the dependent variable (Quantity of fruit loss) tested were significant at 5%. The effects of post harvest losses in the study area leads to wastage of the products and tend to frustrate the efforts put into production and their income on the produce.

Keywords : Post - harvest, Income, Gross-Margin, Losses

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DETERMINANT POST HARVEST LOSSES AMONG TOMATO FARMERS IN IMEKO-AFON LOCAL GOVERNMENT AREA OF OGUN STATE, NIGERIA

Strictly as per the compliance and regulations of:

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Ayandiji, A[α], O. R.,Adeniyi[Ω] Omidiji, D.[β]

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Keywords : Post-harvest, Income, Gross-Margin, Losses.

I. Introduction

Fresh fruits and vegetables are very important sources of vitamins that are essential for healthy human diet. The quality and nutritional value of fresh produce is affected by post harvest handling and storage condition. {Sablani et al, 2006}. Vegetables are usually harvested when the plant is fresh and high in moisture and are thus distinguished from field crops, which are harvested at the mature stage for grains, pulses, oil seeds or fibre. This high moisture of vegetable makes their handling, transportation and marketing a special problem particularly in the tropics.

Tomato is a major vegetable crop that has achieved tremendous popularity over the last century. It is grown in practically every country of the world in outdoor fields, greenhouses and net houses. Tomato belongs to the *solanaceae* family. This family also includes other well known species such as potato, tobacco, pepper and egg plant. Tomato has its origin in the South American Andes. {Naika et al, 2005}. The tomato plant is very versatile and the crop can be divided into two categories; fresh market tomatoes, which we are concerned with and processing tomatoes, which are grown only outdoors for the canning industry and mechanically harvested. World production and consumption have grown quite rapidly over the past 25 years. Tomato is one of the most important vegetables worldwide. World Tomato production in 2001 was about 105 million tons of fresh fruit from an estimated 3.9 million hectare. {Naika et al, 2005}. Tomato contributes to a healthy well balanced diet. They are rich in minerals, vitamins, essential amino acids, sugars and dietary fibres. Tomato contains much vitamin B and C, iron and phosphorus. Tomato fruits are consumed fresh in salads or cooked in sauces, soup and meat of fish dishes. They can be processed into purees, juice and ketchup. Canned and dried tomatoes are economically important processed products Tomato has become an important cash and industrial crop in various parts of the world. One of the reasons for this increases is that tomato cultivation is now being moved to places and seasons that are originally unsuitable for its productivity thereby resulting in an increase in the economic importance of the crop.{Bodunde et al 1993}.

Tomato is cultivated almost throughout the country but the areas of high concentration lie in the northern and south-western parts of Nigeria. In southern Nigeria, tomato is cultivated in small holdings under rain fed conditions while in northern Nigeria; it is grown extensively under irrigation.

The deterioration of the product starts during the harvesting operations, because fresh fruits are inherently perishable. The more carefully a product is handled, the slower the deterioration process during subsequent handling operations. The causes of tomato losses included physical damage during handling, and transport, physiological decay, water loss, or sometimes simply because there is a surplus or glut in the market and no buyer can be found (FFCT, 1993).In developing countries like Nigeria, storage, packaging, transport and handling techniques are practically non-existent with

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perishable crops and so, this allows for considerable losses of produce. Thus as more fresh fruits are needed to supply the growing population areas and as more commodities are stored longer to obtain a year round supply, post harvest loss prevention technology measures become paramount. The losses of quality and freshness of the produce could also be due to improper temperature management, drying of the product, mechanical injury, attacks by bacteria and fungi. These losses can therefore lead to decrease in the returns of the farmers. The main objective is to determine the factors leading to post harvest losses among tomato farmers in Imeko-Afon Local Government Area. The specific objectives are:

1) To determine the socio economic characteristics of tomato farmers in the study area.
2) To ascertain the causes of the losses of tomato in the study area.
3) To determine the effect of the losses on the income of tomato farmers in the study area.
4) To examine the adopted preservative methods used by tomato farmers in the study area.

The study area is Imeko Afon Local Government Area of Ogun State. Dominant ethnic groups are the Yewas and the Ketus. Economic activities of the people are mainly farming and trading. Crops grown are maize, cassava, yam, tomatoes, pepper, cocoa, palm e.t.c with which tomato is mostly cultivated by almost every farmer because of its relative profitability compared to others vegetables in the area. Prominent among other occupations is cattle rearing due to the abundance of savannah vegetation in the local government area. Industrial activities in the local government are low and restricted to cottage industries, arts and crafts.

II. Materials and Methods

Purposive sampling technique was employed in administering the questionnaire because the research is targeted at Tomato producers. 100 farmers were selected for the study, 25 farmers from each of the villages. The villages are Imeko, Araromi oloka afon, Owode Afon, Gbanla. However, out of the 100 questionnaires that were administered, 88 was retrieved and used in this analysis since some of the questionnaires were badly filled and did not contain information relevant for the work. Descriptive statistics and regression were used as analytical tools

III. Results and Discussion

Majority of the respondents (51.1%) were male and 48.86% were females, showing that more males are involved in the production of tomato in the study area. This is consistent with the fact that agricultural activities are seen in the western part of Nigeria as labour intensive, and therefore male dominated. Majority of the farmers are below 41 years of age (69.32%). This indicates a good supply of agile workforce in tomato production in the study area. The result of the marital status shows that majority of the farmers are married (86.4%) while 13.64% are either single or divorced. This could have an implication on post harvest losses in tomato production since; married farmers are likely to have access to more family labour especially for harvesting. The time taken to do the harvesting may be longer and in an attempt to rush the works fruits are badly handled due to poor skill in handling as compared with hired labourers. Table 1 also shows that 82.95% of farmers have no formal education, those with secondary education are 15.91% and for post secondary education 1.14% respondents. This statistics shows that majority of the farmers are illiterate. This could be a contributory factor to high post harvest losses in tomato production because only farmers with knowledge to read and write can appreciate and use most of the post harvest technologies available. The result below shows that majority (68.17%) of the farmers have below 16 years experience in tomato production. This could also have an effect on post harvest losses in tomato production. The low years of experience in tomato production might also be responsible for their lack of knowledge and the unavailability of technology of preservation among the farmers. 11.36% of the farmers are with only one person in the household, 30.68% with 2-6 persons in the household, 25% with 7-11 persons in the household and 32.95% household greater than 12 persons.

Table 1: Socio economic Characteristics of respondents

Gender	Frequency	Cumulative frequency	Percentage
Male	45	45	51.14
Female	43	88	48.86
Total	88		100
Age(Years)	Frequency	Cumulative frequency	Percentage
<30	35	35	39.77
31-40	26	61	29.55
41-50	14	75	15.91
>50	13	88	14.77
Total	88		100
Marital Status	Frequency	Cumulative frequency	Percentage
Single	12	12	13.64
Married	76	88	86.36
Total	88		100
Educational Status	Frequency	Cumulative frequency	Percentage
No formal	51	51	57.95
Primary	22	73	25.00
Secondary	14	87	15.91
Post secondary	1	88	1.14
Total	88		100
Household	Frequency	Cumulative	Percentage

size		frequency	
1	10	10	11.36
2-6	27	37	30.68
7-11	22	59	25
>12	29	88	32.95
Total	88		100

Source: field survey, 2010

Majority of the farmers cultivated between 1 and 5 hectares of land and 17.05% cultivated land area between 6 to 10 hectares, 15.9% cultivated between 16 to 20 hectares and 4.55% cultivated 26 to 30 hectares and this shows that small scale farmers prevail in the study area.

Table 2 : Farm size distribution of Tomato Producers (farmers)

Farm size (ha)	Frequency	Cumulative frequency	Percentage
<1	9	9	10.23
1-5	45	54	51.14
6-10	15	69	17.05
11-15	1	70	1.14
16-20	14	84	15.91
21-25	0	84	0.00
26-30	4	88	4.55
Total	88		100

Source: field survey, 2010

Majority of the respondents (72.73%) make use of the van/pick up as a means of transportation, 4.55% respondents employed the use of bicycle, and 22.73% make use of the motorcycle. This may not necessarily affect post harvest losses because the use of van compared to motorcycle and bicycle will reduce the losses, likely to occur in case head loads were used for transportation

Table 3 : Means of Transportation of tomato Producers (farmers)

Transport means	Mean Frequency	Cumulative frequency	Percentage
Head load	0	0	0
Bicycle	4	4	4.55
Motorcycle	20	24	22.73
Van/ Pick-up	64	88	72.73
Others	0	88	0
Total	88		100

Source: field survey, 2010

The use of post harvest technology is very minimal in the study area because only few of the farmers (5 out of 88) use the mini technology of storing such as drying and storing of the produce before taking it to the market to sell. The reason for their lack of preservation knowledge of adequate methods could be as a result of lack of awareness by the extension workers themselves or the farmers on various ways by which they can go about preserving their produce. However, during the course of the study, there was an extension programme by the local government and Agriculture media resources and extension centre (AMREC) from the university of Agriculture for the farmers on ways by which they can preserve their produce to reduce the losses. The various ways as taught by ARMEC include:

1) Making the tomatoes into tomato paste, Tomato ketchup and also tomato juice.
2) Cutting the tomato into slices and drying them.
3) Boiling the tomatoes after which shells are peeled off and rinsed, put in bottles in which there is water and a teaspoon of preservative added to the water and covered.

There is also no method of storage in the area because of unavailability of storage facilities and lack of basic knowledge on the practices. The only way by which some of the farmers stored their produce is by covering it with grasses. However, this could only last for one day before they are taken to the market for sale; a period short enough to address issues involved in market delays

The regression analysis carried out to determine the influence of some factors on the quantity of fruit loss from harvesting to marketing stage, gave an empirical result which was subjected to F-test. The value of the F-statistics was found to be significant at 5%. This implies that all explanatory variables (independent) had a joint impact on the dependent variable. This result is presented in below and also increase in the distance from the farm to the market will increase the quantity of fruit loss this is because the longer the distance of the farm to the market, the longer the time it will take for the produce to get to the market and so, the losses will increase because of the congestion and packaging of the tomato together for a long time. Also increases losses was due to the more the days the fruit spent on the farm after maturity, the more the loss. Increase in the number of baskets harvested also results in increase in the losses because there is no effective method of storage hence the more the quality of harvested produce and the more the spoilage. Also, as the demand for the produce in the market is low during the on-season compared to the supply, the produce that is not sold in the market immediately in fresh form will be lost as a result of there is no storage facility on ground.

Pre harvest working days (PHWD) was not significant. The effect of all the independent variables (Pre harvest working days (man days), harvest working days (man days), Distance from the farm to the market (km), days fruit spent on the farm (days), Age of fruit at harvest (months), Area of land cultivated (hectare), Days fruit spent in the market before getting to the consumer (days) and Number of basket that was harvested) on the dependent variable (Quantity of fruit loss) tested were significant at 5% level of probability with coefficient of determination (R^2) of 0.95.

Linear Function

$Y = 3.95X_1 + 2.66X_2 + 4.34X_3 + 0.53X_4 + 2.69X_5 + 1.47X_6 + 0.69X_7 + 3.40X_8 – 140.39$

{5.33} {45.28} {296.79} {12.35} {34.23} {150.93} + {41.23} + {32.36}

R^2= 0.72, F = 8.07

Semi-log Function

$Y = 2.27 + 0.76X_1 + 0.19X_2 + 0.31X_3 + 0.02X_4 + 0.79X_5 + 0.81X_6 + 0.63X_7 + 0.11X_8$

{0.26} {0.26} {1.47} {0.79} {0.63} {0.44} {0.32} + {0.34}

R^2 = 0.87, F = 8.96

Double-log Function

$Y = 2.25 + 0.12X_1 + 0.19X_2 + 0.21X_3 + 0.31X_4 + 0.22X_5 + 0.42X_6 + 0.19X_7 + 0.73X_8$

S.E.={0.24} {0.08} {0.28} {0.49} {0.26}{0.14} {0.13} {0.33}

R^2= 0.91, F = 9.10

Exponential - log function

$Y= 0.35 - 0.16X_1 + 0.05X_2 + 0.01X_3 + 0.17X_4 + 0.09X_5 + 0.12X_6 + 0.39X_7 + 0.47X_8$

S.E.={0.02} {0.01} {0.22} {0.32} {0.36} {0.17} {0.12} {0.11}

t = 0.92 0.57 0.36 1.03 1.23 0.69 1.23 0.67

R^2= 0.95, F = 9.32

Table 4 below shows the comparison between the gross margin with loss and the Gross margin without loss. The average Gross margin with loss (9,251.41) is less than the average Gross margin without loss (72,251.41). This shows that post harvest losses reduce the income of farmers in Imeko-Afon local Government Area of Ogun State. The percentage loss incurred by the farmers is 87.3%.

Table 4 : Gross Margin Analysis

	Total variable cost	Total revenue	Gross margin	Average gross margin
Without loss	1,163,026	7,565,250	6,402,224	72,752.55
With loss	1,163,026	1,977,150	814,124	9,251.41

Source: field survey, 2010

IV. Conclusion

This study has analyzed the determinants of post harvest losses in Tomato production in Imeko-Afon Loacal Government Area of Ogun state. The result indicates that all the identified factors have significant impact on post harvest losses. Therefore, there is a great need to reduce the losses in the study area. The impact of the losses was also noticeable in the income of the farmers with the use of the gross margin analysis. The gross margin with loss as compared with the gross margin without loss shows that losses reduce the income of the farmers considerably.

V. Policy Recommendations

Based on the findings of this study, the following recommendations are made for policy actions to reduce the post harvest losses thereby increasing the standard of living of the tomato producers in Imeko – Afon Local Government Area of Ogun State.

1. There should be good storage facilities to store the produce that are harvested before they are being taken to the farm. This will help to produce the losses that will occur at the farm level.
2. Post harvest technology should be introduced to reduce the losses. However there was an extension Programme by Agriculture Media resources and Extension centre (AMREC) from University of Agriculture Abeokuta (UNAAB) on post harvest technology and preservation techniques and there should be a continuation of the extension programmes in order to encourage the farmers.
3. There should be ready market for the produce. The markets must be well organized and also the road network must be improved in order to aid easy transportation of their produce.
4. Extension services should be rendered to the farmers considering their years of experience in tomato production and also to educate them on the various ways that can be used in preserving their produce from losses.
5. Establishment of Tomato processing factories to add valve to the fruits. For example processing tomato into Tomato Ketchup, Juice and Purees.
6. With the reduction of post harvest losses by 50%, food availability would be increased by 20% without cultivating an additional hectare of land for increasing crop yield.

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On Some Transformations Involving Unit And Quarter Arguments

By Chaudhary Wali Mohd., M.I. Qureshi, Izharul H. Khan
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Abstract - The main object of this paper is to obtain a general theorem on multiple series identity involving bounded sequences. The theorem, in turn, is expressed in terms of Srivastava-Daoust hypergeometric function of three variables. A known result of Joshi and Vyas is deduced as a special case of our reduction formula. Certain results involving unit and quarter arguments associated with hypergeometric polynomials ${}_4F_3$, ${}_5F_4$, ${}_6F_5$, ${}_7F_6$ are also obtained. Further many more known or new results can be obtained by specializing the parameters or the variables or both.

Keywords and Phrases : Multiple bounded sequences, Hypergeometric transformations and polynomials, Saalschütz summation theorem for terminating series ${}_3F_2$, Srivastava - Daoust multiple hypergeometric function.

2010 AMS Subject Classifcation : 33 - Special Functions

ON SOME TRANSFORMATIONS INVOLVING UNIT AND QUARTER ARGUMENTS

Strictly as per the compliance and regulations of:

On Some Transformations Involving Unit And Quarter Arguments

Chaudhary Wali Mohd.[α], M.I. Qureshi [α], Izharul H. Khan[Ω]

Abstract - The main object of this paper is to obtain a general theorem on multiple series identity involving bounded sequences. The theorem, in turn, is expressed in terms of Srivastava- Daoust hypergeometric function of three variables. A known result of Joshi and Vyas is deduced as a special case of our reduction formula. Certain results involving unit and quarter arguments associated with hypergeometric polynomials ${}_4F_3$, ${}_5F_4$, ${}_6F_5$, ${}_7F_6$ are also obtained. Further many more known or new results can be obtained by specializing the parameters or the variables or both.

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I. Introduction

The summation theorem concerned with single Gaussian hypergeometric functions play an important role in the study of transformation and reduction formulae of multiple hypergeometric functions.

In 1836, Kümmer [6] gave a list of quadratic transformations for ${}_2F_1$. Making use of Gauss evaluation of ${}_2F_1$, he was able to calculate some ${}_2F_1$ whose arguments were not unity. In 1881, Goursat [3] used the same technique for the third, fourth and sixth degree transformations of certain ${}_2F_1$.

In the monumental work of Prudnikov et al. [7], we find more information concerning reducible cases of hypergeometric functions. In fact, a number of results of interest do exists and in particular this is true for Clausen's ${}_3F_2$ with argument $\frac{1}{4}$.

In the literature of special functions, many hypergeometric transformations for terminating or infinite series of the type ${}_{q+1}F_q$ involving $+1, -1,$ and $\frac{1}{2}$ arguments with appropriate parametric restrictions, are available. Some evaluation of ${}_3F_2(\frac{1}{4})$ and ${}_3F_2(\frac{3}{4})$ have already been derived by Karlsson [5;176 - 177, p.178(Table 1)] and Prudnikov et al. [7; p.551].

The results and definitions which we need in our subsequent work are as follows:

In the usual notations, the Pochhammer's symbol $(a)_n$ is defined by

$$(a)_n = \begin{cases} \dfrac{\Gamma(a+n)}{\Gamma(a)} & ; \ a \neq 0, -1, -2, \ldots \\ 1 & ; \ n = 0 \\ a(a+1)(a+2)\ldots(a+n-1) & ; \ n = 1, 2, 3\ldots \end{cases} \tag{1.1}$$

If m is either a positive integer or zero, then

$$\frac{(c+1)_m}{(c)_m} = 1 + \frac{m}{c} \tag{1.2}$$

$$\begin{aligned} \frac{(c+2)_m}{(c)_m} &= \frac{(c+1+m)(c+m)}{c(c+1)} \\ &= \frac{c(c+1) + (2c+2)m + m(m-1)}{c(c+1)} \\ &= 1 + \frac{2}{c}m + \frac{m(m-1)}{c(c+1)} \end{aligned} \tag{1.3}$$

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$$
{}_3F_2\left[\begin{array}{ll} a,\, b,\, c+1 & ; \\ & 1 \\ d,\, c & ; \end{array}\right] = \sum_{m=0}^{\infty}\frac{(a)_m\,(b)_m}{(d)_m\,m!}\left[1+\frac{m}{c}\right]
$$

$$
= {}_2F_1\left[\begin{array}{ll} a,\, b & ; \\ & 1 \\ d & ; \end{array}\right] + \frac{ab}{cd}\,{}_2F_1\left[\begin{array}{ll} a+1,\, b+1 & ; \\ & 1 \\ d+1 & ; \end{array}\right]
$$

$$
= \frac{\Gamma(d)\,\Gamma(d-a-b)}{\Gamma(d-a)\,\Gamma(d-b)} + \frac{ab}{c}\,\frac{\Gamma(d)\,\Gamma(d-a-b-1)}{\Gamma(d-a)\,\Gamma(d-b)} \tag{1.4}
$$

Where $\mathrm{Re}(d-a-b) > 1$.

and

$$
{}_3F_2\left[\begin{array}{ll} a,\, b,\, c+2 & ; \\ & 1 \\ d,\, c & ; \end{array}\right] = {}_2F_1\left[\begin{array}{ll} a,\, b & ; \\ & 1 \\ d & ; \end{array}\right] + \frac{2ab}{cd}\,{}_2F_1\left[\begin{array}{ll} a+1,\, b+1 & ; \\ & 1 \\ d+1 & ; \end{array}\right]
$$

$$
+ \frac{a(a+1)b(b+1)}{c(c+1)d(d+1)}\,{}_2F_1\left[\begin{array}{ll} a+2,\, b+2 & ; \\ & 1 \\ d+2 & ; \end{array}\right]
$$

$$
= \frac{\Gamma(d)\,\Gamma(d-a-b)}{\Gamma(d-a)\,\Gamma(d-b)} + \frac{2ab\,\Gamma(d)\,\Gamma(d-a-b-1)}{c\,\Gamma(d-a)\,\Gamma(d-b)} + \frac{a(a+1)b(b+1)\Gamma(d)\Gamma(d-a-b-2)}{c(c+1)\Gamma(d-a)\Gamma(d-b)} \tag{1.5}
$$

Where $\mathrm{Re}(d-a-b) > 2$.

Further, in our study, we shall be using the following identity:

$$
\sum_{m,n,p=0}^{\infty} A(m,n,p) = \sum_{n,p=0}^{\infty}\sum_{m=0}^{p} A(m,n,p-m) \tag{1.6}
$$

Srivastava and Daoust Function: Srivastava and Daoust function [9; p.454, see also 11; p.37(21,22,23) and 12; pp.64-65(18,19,20)] is defined as follows:

$$
\mathrm{F}^{A:B';B'';\ldots;B^{(n)}}_{C:D';D'';\ldots;D^{(n)}}\left(\begin{array}{lll} [(a_A):\theta',\ldots,\theta^{(n)}] & :[(b'_{B'}):\phi'];\ldots;[(b^{(n)}_{B^{(n)}}):\phi^{(n)}] & ; \\ & & z_1, z_2, \ldots, z_n \\ [(c_C):\psi',\ldots,\psi^{(n)}] & :[(d'_{D'}):\delta'];\ldots;[(d^{(n)}_{D^{(n)}}):\delta^{(n)}] & ; \end{array}\right)
$$

$$
= \sum_{m_1,\ldots,m_n=0}^{\infty}\Omega(m_1,\ldots,m_n)\,\frac{z_1^{m_1}\,z_2^{m_2}\ldots z_n^{m_n}}{m_1!\;m_2!\ldots m_n!} \tag{1.7}
$$

Where for convenience,

$$
\Omega(m_1,\ldots,m_n) = \frac{\prod_{j=1}^{A}(a_j)_{m_1\theta'_j+\ldots+m_n\theta^{(n)}_j}\prod_{j=1}^{B'}(b'_j)_{m_1\phi'_j}\cdots\prod_{j=1}^{B^{(n)}}(b^{(n)}_j)_{m_n\phi^{(n)}_j}}{\prod_{j=1}^{C}(c_j)_{m_1\psi'_j+\ldots+m_n\psi^{(n)}_j}\prod_{j=1}^{D'}(d'_j)_{m_1\delta'_j}\cdots\prod_{j=1}^{D^{(n)}}(d^{(n)}_j)_{m_n\delta^{(n)}_j}} \tag{1.8}
$$

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The coefficients

$$\begin{cases} \theta_j^{(k)}, \quad j=1,\ldots,A; \quad \phi_j^{(k)}, \quad j=1,\ldots,B^{(k)}; \ \psi\ _j^{(k)}, \quad j=1,\ldots,C \\ \delta_j^k, \quad j=1,\ldots,D^{(k)}; \quad \text{for all} \quad k\in\{1,2,3,\ldots,n\} \end{cases}$$

are real and positive, and (a_A) abbreviates the array of A parameters $a_1,a_2,\ldots,a_A$, $(b_j{}^{(k)}), j=1,\ldots,B^{(k)}$; for all $k\in\{1,2,\ldots,n\}$, with similar interpretations for (c_C) and $(d\,{}_{D^{(k)}}^{(k)})$, $k=1,\ldots,n$ et cetra.

The convergence conditions of the multiple series occuring in (1.7) is given by Srivastava and Daoust [8 and 10; see also 2].

In present paper, we establish a multiple series identity (2.1). The investigation of this identity is, infact, immediately connected with the transformations of hypergeometric series of two and three variables, which are obtained in Section 3. Further our main result allows a variety of hypergeometric transformation formulas involving unit and quarter arguments which are not available in the literature. For this reason our results (3.4) to (3.9) seem to be of interest.

II. Multiple Series Identity

Theorem : If $\{S_1(\phi n+\theta p)\}$, $\{S_2(\gamma p)\}$ and $\{S_3(\delta n)\}$ are the bounded sequences of real or complex numbers; $m,n,p\in\{0,1,2,\ldots\}$; $\alpha,\beta,\theta,\phi,\gamma$ and δ are real constants; the values of a and b are adjusted in such a way that the parameters $\frac{6a+2b+1}{2}$, $2b$, $3a-b+1$ and $b-3a$ are neither zero nor negative integers, then

$$\sum_{m,n,p=0}^{\infty} \frac{(6a)_{3m+\alpha n+p}}{4^m \left(\frac{6a+2b+1}{2}\right)_{m+\left(\frac{\alpha+\beta}{2}\right)n} (3a-b+1)_{m+\left(\frac{\alpha-\beta}{2}\right)n}} S_1(\theta m+\phi n+\theta p)\, S_2(\gamma m+\gamma p)\, S_3(\delta n)$$

$$\times \frac{(-1)^m\, x^{m+p}\, y^n}{m!\; n!\; p!} = \sum_{n,p=0}^{\infty} \frac{(6a)_{\alpha n+p}\, (2b)_{\beta n+p}\, (b-3a)_{\left(\frac{\beta-\alpha}{2}\right)n-p}}{(2b)_{\beta n-p} \left(\frac{6a+2b+1}{2}\right)_{\left(\frac{\alpha+\beta}{2}\right)n+p} (3a-b+1)_{\left(\frac{\alpha-\beta}{2}\right)n}}$$

$$\times\ S_1(\phi n+\theta p)\, S_2(\gamma p)\, S_3(\delta n)\ \frac{y^n \left(\frac{x}{4}\right)^p}{(b-3a)_{\left(\frac{\beta-\alpha}{2}\right)n}\, n!\, p!} \tag{2.1}$$

Provided that each of the multiple series involved converges absolutely.

Proof. Let the left hand side of the Theorem 2.1 is denoted by T, then

$$T = \sum_{m,n,p=0}^{\infty} S_1(\theta m+\phi n+\theta p)\, S_2(\gamma m+\gamma p)\, S_3(\delta n)$$

$$\times \frac{(-1)^m\, (6a)_{3m+\alpha n+p}\, x^{m+p}\, y^n}{4^m \left(\frac{6a+2b+1}{2}\right)_{m+\left(\frac{\alpha+\beta}{2}\right)n} (3a-b+1)_{m+\left(\frac{\alpha-\beta}{2}\right)n}\, m!\; n!\; p!}.$$

Replacing p by $p-m$, we get

$$T = \sum_{p=0}^{\infty}\sum_{m=0}^{p}\sum_{n=0}^{\infty}(-p)_m\, S_1(\phi n+\theta p)S_2(\gamma p)S_3(\delta n)$$

$$\times \frac{(6a)_{2m+\alpha n+p}\, x^p\, y^n}{(1)_p\, 4^m \left(\frac{6a+2b+1}{2}\right)_{m+\left(\frac{\alpha+\beta}{2}\right)n} (3a-b+1)_{m+\left(\frac{\alpha-\beta}{2}\right)n}\, m!\; n!}$$

$$= \sum_{p,n=0}^{\infty} \frac{S_1(\phi n+\theta p)\, S_2(\gamma p)\, S_3(\delta n)\, (6a)_{\alpha n+p}\, x^p y^n}{p! \left(\frac{6a+2b+1}{2}\right)_{\left(\frac{\alpha+\beta}{2}\right)n} (3a-b+1)_{\left(\frac{\alpha-\beta}{2}\right)n}\; n!}$$

$$\times \sum_{m=0}^{p} \frac{(-p)_m \left(\frac{6a+\alpha n+p}{2}\right)_m \left(\frac{6a+\alpha n+p+1}{2}\right)_m}{\left(\frac{6a+2b+1+\alpha n+\beta n}{2}\right)_m \left(3a-b+1+\frac{\alpha n-\beta n}{2}\right)_m\; m!}$$

$$= \sum_{p,n=0}^{\infty} \frac{S_1(\phi n+\theta p)\, S_2(\gamma p)\, S_3(\delta n)\, (6a)_{\alpha n+p}\, x^p\, y^n}{\left(\frac{6a+2b+1}{2}\right)_{\left(\frac{\alpha+\beta}{2}\right)n} (3a-b+1)_{\left(\frac{\alpha-\beta}{2}\right)n}\, p!\; n!}$$

$$\times {}_3F_2\left[\begin{matrix} -p,\ \frac{6a+\alpha n+p}{2},\ \frac{6a+\alpha n+p+1}{2} & ; \\ \frac{6a+2b+1+\alpha n+\beta n}{2},\ \frac{6a-2b+2+\alpha n-\beta n}{2} & ; \end{matrix}\ 1\right]. \tag{2.2}$$

Now using terminating Saalschütz's summation theorem [1; p.9(2.2.1)]:

$$ {}_3F_2\left[\begin{matrix} -p,\ A,\ B & ; \\ C,\ 1+A+B-C-p & ; \end{matrix}\ 1\right] = \frac{(C-A)_p\,(C-B)_p}{(C)_p\,(C-A-B)_p} \tag{2.3}$$

Where p is a non - negative integer, we get

$$T = \sum_{p,n=0}^{\infty} \frac{S_1(\phi n+\theta p)\, S_2(\gamma p)\, S_3(\delta n)\, (6a)_{\alpha n+p}\, x^p\, y^n}{\left(\frac{6a+2b+1}{2}\right)_{\left(\frac{\alpha+\beta}{2}\right)n} (3a-b+1)_{\left(\frac{\alpha-\beta}{2}\right)n}\, p!\; n!}$$

$$\times \frac{\left(\frac{2b+\beta n-p}{2}\right)_p \left(\frac{2b+\beta n-p+1}{2}\right)_p}{\left(\frac{6a+2b+1+\alpha n+\beta n}{2}\right)_p \left(b-3a-p+\frac{\beta n-\alpha n}{2}\right)_p}$$

$$= \sum_{p,n=0}^{\infty} \frac{S_1(\phi n+\theta p)\, S_2(\gamma p)\, S_3(\delta n)\, (6a)_{\alpha n+p}\, x^p\, y^n}{\left(\frac{6a+2b+1}{2}\right)_{\left(\frac{\alpha+\beta}{2}\right)n} (3a-b+1)_{\left(\frac{\alpha-\beta}{2}\right)n}\, p!\; n!}$$

$$\times \frac{(2b+\beta n-p)_{2p} \left(\frac{6a+2b+1}{2}\right)_{\frac{\alpha n+\beta n}{2}} (b-3a)_{\frac{\beta n-\alpha n-2p}{2}}}{2^{2p} \left(\frac{6a+2b+1}{2}\right)_{\frac{2p+\alpha n+\beta n}{2}} (b-3a)_{\frac{\beta n-\alpha n}{2}}}$$

$$= \sum_{p,n=0}^{\infty} \frac{S_1(\phi n+\theta p)\, S_2(\gamma p)\, S_3(\delta n)\, (6a)_{\alpha n+p}\, x^p\, y^n}{\left(\frac{6a+2b+1}{2}\right)_{\left(\frac{\alpha+\beta}{2}\right)n} (3a-b+1)_{\left(\frac{\alpha-\beta}{2}\right)n}\, p!\; n!}$$

$$\times \frac{(2b)_{\beta n+p}\left(\frac{6a+2b+1}{2}\right)_{\frac{\alpha n+\beta n}{2}}(b-3a)_{\frac{\beta n-\alpha n-2p}{2}}}{2^{2p}(2b)_{\beta n-p}\left(\frac{6a+2b+1}{2}\right)_{\frac{2p+\alpha n+\beta n}{2}}(b-3a)_{\frac{\beta n-\alpha n}{2}}}$$

$$= \sum_{p,n=0}^{\infty} \frac{S_1(\phi n+\theta p)\,S_2(\gamma p)\,S_3(\delta n)\,(6a)_{\alpha n+p}\,x^p\,y^n}{(3a-b+1)_{\frac{\alpha n-\beta n}{2}}\,p!\;n!}$$

$$\times \frac{(2b)_{\beta n+p}\,(b-3a)_{\frac{\beta n-\alpha n-2p}{2}}}{2^{2p}(2b)_{\beta n-p}\left(\frac{6a+2b+1}{2}\right)_{\frac{\alpha n+\beta n+2p}{2}}(b-3a)_{\frac{\beta n-\alpha n}{2}}} \tag{2.4}$$

$$= \sum_{n,p=0}^{\infty} S_1(\phi n+\theta p)\,S_2(\gamma p)\,S_3(\delta n)\,\frac{(6a)_{\alpha n+p}\,y^n\left(\frac{x}{4}\right)^p}{(3a-b+1)_{\frac{\alpha n-\beta n}{2}}\,n!\;p!}$$

$$\times \frac{(2b)_{\beta n+p}\,(b-3a)_{\left(\frac{\beta-\alpha}{2}\right)n-p}}{(2b)_{\beta n-p}\left(\frac{6a+2b+1}{2}\right)_{\left(\frac{\alpha+\beta}{2}\right)n+p}(b-3a)_{\left(\frac{\beta-\alpha}{2}\right)n}}$$

Which is the right hand side of the (2.1).

III. APPLICATIONS

Suppose the notation (a_A) denotes the sequence of A parameters $a_1, a_2, \ldots, a_A$ and $[(a_A)]_m$ stands for the continued product of "A" Pochhammer's symbols given by $[(a_A)]_m = \prod_{i=1}^{A}(a_i)_m$, with similar interpretation for others.

In Theorem 2.1, setting $\theta = \phi = \gamma = \delta = 1,\; S_1(q) = \frac{[(d_D)]_q}{[(e_E)]_q},\; S_2(q) = \frac{[(g_G)]_q}{[(h_H)]_q}$ and $S_3(q) = \frac{[(k_K)]_q}{[(r_R)]_q}$, We get

$$\sum_{m,n,p=0}^{\infty} \frac{[(d_D)]_{m+n+p}\,[(g_G)]_{m+p}\,[(k_K)]_n}{[(e_E)]_{m+n+p}\,[(h_H)]_{m+p}\,[(r_R)]_n}\,\frac{(6a)_{3m+\alpha n+p}\left(\frac{-x}{4}\right)^m y^n\,x^p}{\left(\frac{6a+2b+1}{2}\right)_{m+\left(\frac{\alpha+\beta}{2}\right)n}(3a-b+1)_{m+\left(\frac{\alpha-\beta}{2}\right)n}\,m!\;n!\;p!}$$

$$= \sum_{n,p=0}^{\infty} \frac{[(d_D)]_{n+p}\,[(k_K)]_n\,[(g_G)]_p}{[(e_E)]_{n+p}\,[(r_R)]_n\,[(h_H)]_p}$$

$$\times \frac{(6a)_{\alpha n+p}\,(2b)_{\beta n+p}\,(b-3a)_{\left(\frac{\beta-\alpha}{2}\right)n-p}\,y^n\left(\frac{x}{4}\right)^p}{(2b)_{\beta n-p}\left(\frac{6a+2b+1}{2}\right)_{\left(\frac{\alpha+\beta}{2}\right)n+p}(3a-b+1)_{\left(\frac{\alpha-\beta}{2}\right)n}\,(b-3a)_{\left(\frac{\beta-\alpha}{2}\right)n}\,n!\;p!}$$

Which can be interpretated in the form of Srivastava and Daoust function as follows:

$$\mathrm{F}^{D+G+1:0;K;0}_{E+H+2:0;R;0}\left(\begin{array}{c}[(d_D):1,1,1],\,[(g_G):1,0,1],\,[6a:3,\,\alpha,1] \qquad :\\ [(e_E):1,1,1],\,[(h_H):1,0,1],\,\left[\frac{6a+2b+1}{2}:1,\frac{\alpha+\beta}{2},0\right],\,\left[3a-b+1:1,\frac{\alpha-\beta}{2},0\right] \;:\end{array}\right.$$

$$\left.\begin{array}{l} \_\,;\ [(k_K):1];\ \_\ \ ; \\ \qquad\qquad\qquad\qquad -\dfrac{x}{4},\ y,\ x \\ \_\,;\ [(r_R):1];\ \_\ \ ; \end{array}\right)$$

$$= \mathrm{F}^{D+3:K;G}_{E+2:R+2;H}\left(\begin{array}{l} [(d_D):1,1],\ [6a:\alpha,1],\ [2b:\beta,1],\ \left[b-3a:\frac{\beta-\alpha}{2},-1\right] \quad : \\ [(e_E):1,1],\ [2b:\beta,-1],\ \left[\frac{6a+2b+1}{2}:\frac{\alpha+\beta}{2},1\right] \quad : \end{array}\right.$$

$$\left.\begin{array}{l} [(k_K):1] \qquad\qquad\qquad\qquad\qquad ;[(g_G):1]\ \ ; \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad y,\ \dfrac{x}{4} \\ [(r_R):1],\ \left[3a-b+1:\frac{\alpha-\beta}{2}\right],\ \left[b-3a:\frac{\beta-\alpha}{2}\right]\ \ ;[(h_H):1]\ \ ; \end{array}\right) \tag{3.1}$$

In (3.1), setting $D = E = G = H = 0$ and using binomial theorem, we get a transformation formula:

$$(1-x)^{-6a}\ \mathrm{F}^{1:0;K}_{2:0;R}\left(\begin{array}{l} [6a:3,\ \alpha] \qquad\qquad\qquad\qquad : \_; \\ \left[\frac{6a+2b+1}{2}:1,\ \frac{\alpha+\beta}{2}\right],\ \left[3a-b+1:1,\ \frac{\alpha-\beta}{2}\right] \quad : \_; \end{array}\right.$$

$$\left.\begin{array}{l} [(k_K):1]\ \ ; \\ \qquad\qquad\qquad \dfrac{-x}{4(1-x)^3},\ \dfrac{y}{(1-x)^{\alpha}} \\ [(r_R):1]\ \ ; \end{array}\right)$$

$$= \mathrm{F}^{3:K;0}_{2:R+2;0}\left(\begin{array}{l} [6a:\alpha,1],\ [2b:\beta,1],\ [b-3a:\frac{\beta-\alpha}{2},-1] \quad : \\ [2b:\beta,-1],\ [\frac{6a+2b+1}{2}:\frac{\alpha+\beta}{2},1] \quad : \end{array}\right.$$

$$\left.\begin{array}{l} [(k_K):1] \qquad\qquad\qquad\qquad\qquad ;\_\ ; \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad y,\ \dfrac{x}{4} \\ [(r_R):1],[3a-b+1:\frac{\alpha-\beta}{2}],\ [b-3a:\frac{\beta-\alpha}{2}]\ \ ;\_\ ; \end{array}\right). \tag{3.2}$$

In (3.1), put $y = 0$ and $D = E = 0$, we get

$$\sum_{m=0}^{\infty}\frac{[(g_G)]_m\,(6a)_{3m}\left(-\frac{x}{4}\right)^m}{[(h_H)]_m\left(\frac{6a+2b+1}{2}\right)_m(3a-b+1)_m\ m!}\ {}_{G+1}F_H\left[\begin{array}{l}(g_G)+m,\ 6a+3m\ \ ; \\ \qquad (h_H)+m \qquad ;\end{array}\ x\right]$$

$$= {}_{G+3}F_{H+2}\left[\begin{array}{l} 6a,\ 2b,\ 1-2b,\ (g_G) \qquad\qquad ; \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ (h_H)\ \ ; \end{array}\ \frac{x}{4}\right]. \tag{3.3}$$

In (3.3), setting $G = H = 1,\ h_1 = h,\ g_1 = -k$ (where k is a non- negative integer) and $x = 1$, we get

$$\sum_{m=0}^{k}\frac{(-k)_m\,(6a)_{3m}\left(-\frac{1}{4}\right)^m}{(h)_m\left(\frac{6a+2b+1}{2}\right)_m(3a-b+1)_m\ m!}\ {}_2F_1\left[\begin{array}{l}-k+m,\ 6a+3m\ \ ; \\ \qquad h+m \qquad ;\end{array}\ 1\right]$$

$$= {}_4F_3\left[\begin{matrix} -k,\ 6a,\ 2b,\ 1-2b & ; \\ & & \frac{1}{4} \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ h & ; \end{matrix}\right] \tag{3.4}$$

Where $\mathrm{Re}(\ h-6a) > 2k.$

Now on using Gauss first summation theorem [1; p.2(1.3.1)], we get

$$\sum_{m=0}^{k} \frac{(-k)_m\,(6a)_{3m}}{\left(\frac{6a+2b+1}{2}\right)_m\,(3a-b+1)_m\ m!}\ \frac{(h-6a)_k\left(\frac{1-h+6a}{2}\right)_m\left(\frac{2-h+6a}{2}\right)_m}{(27)^m\,(h)_k\left(\frac{1-h-k+6a}{3}\right)_m\left(\frac{2-h-k+6a}{3}\right)_m\left(\frac{3-h-k+6a}{3}\right)_m}$$

$$= {}_4F_3\left[\begin{matrix} -k,\ 6a,\ 2b,\ 1-2b & ; \\ & & \frac{1}{4} \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ h & ; \end{matrix}\right]$$

Which on little simplification gives the known transformation of Joshi and Vyas [4; p.1915(4.1)] in the form:

$${}_6F_5\left[\begin{matrix} -k,\ \Delta(3;6a),\Delta(2;1-h+6a) & ; \\ & & 1 \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ \Delta(3;\ 1-h-k+6a) & ; \end{matrix}\right]$$

$$= \frac{(h)_k}{(h-6a)_k}\,{}_4F_3\left[\begin{matrix} -k,\ 6a,\ 2b,\ 1-2b & ; \\ & & \frac{1}{4} \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ h & ; \end{matrix}\right] \tag{3.5}$$

Where k is a non-negative integer and $\mathrm{Re}(h-6a) > 2k$. Also the notation $\Delta(M;b)$ denotes the M parameters $\frac{b}{M}, \frac{b+1}{M}, \frac{b+2}{M}, \ldots, \frac{b+M-1}{M};\ M = 1,2,3,\ldots.$

In (3.3), setting $G = H = 2,\ g_1 = -k,\ g_2 = c+1,\ h_1 = d,\ h_2 = c$ and $x = 1,$ we get

$$\sum_{m=0}^{k} \frac{(-k)_m\,(c+1)_m\,(6a)_{3m}\,(-1)^m}{(d)_m\,(c)_m\left(\frac{6a+2b+1}{2}\right)_m\,(3a-b+1)_m\,4^m\ m!}\,{}_3F_2\left[\begin{matrix} 6a+3m,\ -k+m,\ c+m+1 & ; \\ & & 1 \\ d+m,\ c+m & ; \end{matrix}\right]$$

$$= {}_5F_4\left[\begin{matrix} -k,\ 6a,\ 2b,\ 1-2b,\ c+1 & ; \\ & & \frac{1}{4} \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ c,\ d & ; \end{matrix}\right] \tag{3.6}$$

Where $\mathrm{Re}(d-1-6a) > 2k$ and k is a non-negative integer.

Now using summation theorem (1.4) for ${}_3F_2$ in (3.6), we get

$${}_5F_4\left[\begin{matrix} -k,\ 6a,\ 2b,\ 1-2b,\ c+1 & ; \\ & & \frac{1}{4} \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ c,\ d & ; \end{matrix}\right]$$

$$= \sum_{m=0}^{k} \frac{(-k)_m\,(c+1)_m\,(6a)_{3m}\,(-1)^m\,\Gamma(d+m)\,\Gamma(d-6a+k-3m)}{(d)_m\,(c)_m\,\left(\frac{6a+2b+1}{2}\right)_m\,(3a-b+1)_m\,4^m\,m!\,\Gamma(d+k)\,\Gamma(d-6a-2m)}$$

$$+ \sum_{m=0}^{k} \frac{(-k)_m\,(c+1)_m\,(6a)_{3m}\,(-1)^m\,(6a+3m)\,(-k+m)\,\Gamma(d+m)\,\Gamma(d-6a+k-1-3m)}{(d)_m\,(c)_m\,\left(\frac{6a+2b+1}{2}\right)_m\,(3a-b+1)_m\,4^m\,m!\,(c+m)\,\Gamma(d+k)\,\Gamma(d-6a-2m)}.$$

Further on using algebraic properties of Pochammer's symbols and making little simpli-fication, we get a new hypergeometric transformation formula:

$${}_5F_4\left[\begin{matrix} -k,\ 6a,\ 2b,\ 1-2b,\ c+1 & ; \\ & \frac{1}{4} \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ c,\ d & ; \end{matrix}\right]$$

$$= \frac{(d-6a)_k}{(d)_k}\,{}_7F_6\left[\begin{matrix} -k,\ c+1,\ \Delta(3;\,6a),\ \Delta(2;\,1-d+6a) & ; \\ & 1 \\ c,\ 3a+b+\frac{1}{2},\ 3a-b+1,\ \Delta(3;\,1-d+6a-k) & ; \end{matrix}\right]$$

$$-\frac{6ak(d-6a)_{k-1}}{c(d)_k}\,{}_6F_5\left[\begin{matrix} -k+1,\ \Delta(3;\,6a+1),\ \Delta(2;\,1-d+6a) & ; \\ & 1 \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ \Delta(3;\,2-d+6a-k) & ; \end{matrix}\right] \tag{3.7}$$

Where $\mathrm{Re}(\,d-1-6a) > 2k.$

In (3.3), setting $G = H = 2,\ g_1 = -k,\ g_2 = c+2,\ h_1 = d,\ h_2 = c$ and $x = 1,$ we get

$$\sum_{m=0}^{k} \frac{(-k)_m\,(c+2)_m\,(6a)_{3m}\,(-1)^m}{(d)_m\,(c)_m\,\left(\frac{6a+2b+1}{2}\right)_m\,(3a-b+1)_m\,4^m\,m!}\,{}_3F_2\left[\begin{matrix} 6a+3m,\ -k+m,\ c+m+2 & ; \\ & 1 \\ d+m,\ c+m & ; \end{matrix}\right]$$

$$= {}_5F_4\left[\begin{matrix} -k,\ 6a,\ 2b,\ 1-2b,\ c+2 & ; \\ & \frac{1}{4} \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ c,\ d & ; \end{matrix}\right] \tag{3.8}$$

Where $\mathrm{Re}(\,d-2-6a) > 2k$ and k is a non-negative integer.

Again on making use of summation theorem (1.5) for ${}_3F_2$ in (3.8), we get

$${}_5F_4\left[\begin{matrix} -k,\ 6a,\ 2b,\ 1-2b,\ c+2 & ; \\ & \frac{1}{4} \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ c,\ d & ; \end{matrix}\right] = \sum_{m=0}^{k} \frac{(-k)_m\,(c+2)_m\,(6a)_{3m}\,(-1)^m}{(d)_m\,(c)_m\,\left(\frac{6a+2b+1}{2}\right)_m\,(3a-b+1)_m\,4^m\,m!}$$

$$\times \left\{ \frac{\Gamma(d+m)\,\Gamma(d-6a-3m+k)}{\Gamma(d-6a-2m)\,\Gamma(d+k)} + \frac{2(6a+3m)\,(-k+m)\,\Gamma(d+m)\,\Gamma(d-6a-3m+k-1)}{(c+m)\,\Gamma(d-6a-2m)\,\Gamma(d+k)} \right.$$

$$\left. + \frac{(6a+3m)\,(6a+3m+1)\,(-k+m)\,(-k+m+1)\,\Gamma(d+m)\ \Gamma(d-6a-3m+k-2)}{(c+m)\,(c+m+1)\,\Gamma(d-6a-2m)\,\Gamma(d+k)} \right\}$$

Which on simplification gives:

$$
{}_5F_4\left[\begin{matrix} -k,\ 6a,\ 2b,\ 1-2b,\ c+2 & ; \\ & & \frac{1}{4} \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ c,\ d & ; \end{matrix}\right]
$$

$$
= \frac{(d-6a)_k}{(d)_k}\,{}_7F_6\left[\begin{matrix} -k,\ c+2,\ \Delta(3;\ 6a),\ \Delta(2;\ 1-d+6a) & ; \\ & & 1 \\ c,\ 3a+b+\frac{1}{2},\ 3a-b+1,\ \Delta(3;\ 1-d+6a-k) & ; \end{matrix}\right]
$$

$$
-\frac{12ak(d-6a)_{k-1}}{c(d)_k}\,{}_7F_6\left[\begin{matrix} -k+1,\ c+2,\ \Delta(3;\ 6a+1),\ \Delta(2;\ 1-d+6a) & ; \\ & & 1 \\ c+1,\ 3a+b+\frac{1}{2},\ 3a-b+1,\ \Delta(3;\ 2-d+6a-k) & ; \end{matrix}\right]
$$

$$
+\frac{k(k-1)\,(6a)\,(6a+1)\,(d-6a)_{k-2}}{c(c+1)\,(d)_k}
$$

$$
\times\,{}_6F_5\left[\begin{matrix} -k+2,\ \Delta(3;\ 6a+2),\ \Delta(2;\ 1-d+6a) & ; \\ & & 1 \\ 3a+b+\frac{1}{2},\ 3a-b+1,\ \Delta(3;\ 3-d+6a-k) & ; \end{matrix}\right] \tag{3.9}
$$

Where $\mathrm{Re}(d-2-6a) > 2k$ and k is a non-negative integer.

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Creation of a Summation Formula Connected To Contiguous Relation and Hypergeometric Function

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Abstract - The main aim of present paper is the creation of a summation formula related to Contiguous relation[1] and Hypergeometric function.

Keywords : *Contiguous relation, Recurrence relation, Gauss second summation theorem.*

Classifcation 2000 MSC NO : *33C05 , 33C20 , 33C45 , 33C60, 33C70*

CREATION OF A SUMMATION FORMULA CONNECTED TO CONTIGUOUS RELATION AND HYPERGEOMETRIC FUNCTION

Strictly as per the compliance and regulations of:

Creation of a Summation Formula Connected To Contiguous Relation and Hypergeometric Function

Salahuddin

Abstract - The main aim of present paper is the creation of a summation formula related to Contiguous relation [1] and Hypergeometric function.
Keywords : Contiguous relation, Recurrence relation, Gauss second summation theorem.

I. Introduction

Generalized Gaussian Hypergeometric function of one variable :

$$ {}_AF_B\left[\begin{array}{ccc} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1} $$

Or

$$ {}_AF_B\left[\begin{array}{ccc} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right] \equiv {}_AF_B\left[\begin{array}{ccc} (a_j)_{j=1}^A & ; & \\ & & z \\ (b_j)_{j=1}^B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{2} $$

Where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non - negative integers.

Contiguous Relations :

[Andrews p.367(8), E. D. p.52(19), H.T. F. I p.103(38)]

$$ c\,(1-z)\,{}_2F_1\left[\begin{array}{ccc} a,\ b\ ; & & \\ c & ; & z \end{array}\right] = c\,{}_2F_1\left[\begin{array}{ccc} a-1,\ b & ; & \\ c & ; & z \end{array}\right] - (c-b)\,z\,{}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ c+1 & ; & z \end{array}\right] \tag{3} $$

[Abramowitz p.558(15.2.19)]

$$ (a-b)\,(1-z)\,{}_2F_1\left[\begin{array}{ccc} a,\ b\ ; & & \\ c & ; & z \end{array}\right] = (c-b)\,{}_2F_1\left[\begin{array}{ccc} a,\ b-1 & ; & \\ c & ; & z \end{array}\right] + (a-c)\,{}_2F_1\left[\begin{array}{ccc} a-1,\ b & ; & \\ c\ ; & & z \end{array}\right] \tag{4} $$

Recurrence relation :

$$ \Gamma(z+1) = z\,\Gamma(z) \tag{5} $$

Gauss second summation theorem [Prud.,p. 491(7.3.7.5)]

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+1}{2} & ; & \end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{6} $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{7} $$

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A new summation formula [Ref.[2], p.337(10)]

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & & ; & \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})}\left\{\frac{(b+a-1)}{(a-1)}\right\} + \frac{2\,\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\right] \quad (8) $$

II. Main Summation Formula

For the main formula a ≠ b

For a < 1 and a > 29

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-29}{2} & & ; & \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-29}{2})}{(a-b)\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-29}{2})}\left\{\frac{(-6190283353629375a)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+\right.\right. $$

$$ +\frac{(144597134843342175a^2 - 131211131429708550a^3 + 65201399543285190a^4)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+ $$

$$ +\frac{(-2046225352864875a^5 + 437602985498315a^6 - 66696220706115a^7 + 7442156684963a^8)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+ $$

$$ +\frac{(-617014151325a^9 + 38205040445a^{10} - 1759562805a^{11} + 59394517a^{12} - 1426425a^{13})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+ $$

$$ +\frac{(23065a^{14} - 225a^{15} + a^{16} + 6190283353629375b - 20224606881433995a^2b)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+ $$

$$ +\frac{(24414112451554866a^3b - 12588514259366505a^4b + 4384293790660180a^5b)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+ $$

$$ +\frac{(-882351615046635a^6b + 148574697339502a^7b - 14777453889915a^8b + 1383157455640a^9b)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+ $$

$$ +\frac{(-71906369145a^{10}b + 3885833198a^{11}b - 100340955a^{12}b + 3024980a^{13}b - 30225a^{14}b)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+ $$

$$ +\frac{(434a^15b - 144597134843342175b^2 + 20224606881433995ab^2 - 11735766636113070a^3b^2)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+ $$

$$+\frac{(9090326128591095a^4b^2 - 2896776266543295a^5b^2 + 757314109245930a^6b^2)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-98832454416660a^7b^2 + 13445576481135a^8b^2 - 847258652835a^9b^2 + 66766662300a^{10}b^2)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-1993919070a^{11}b^2 + 92736345a^{12}b^2 - 1038345a^{13}b^2 + 26970a^{14}b^2 + 13121113142970855b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-24414112451554866ab^3 + 11735766636113070a^2b^3 - 2149637822784855a^4b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(1183582613062230a^5b^3 - 237762013535100a^6b^3 + 48732039707280a^7b^3 - 3991249561815a^8b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(447929438010a^9b^3 - 16234996050a^{10}b^3 + 1085488560a^{11}b^3 - 14199705a^{12}b^3 + 566370a^{13}b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-6520139954328519b^4 + 12588514259366505ab^4 - 9090326128591095a^2b^4)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(2149637822784855a^3b^4 - 154818398535750a^5b^4 + 64459961486850a^6b^4 - 7958826637650a^7b^4)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(1318930698825a^8b^4 - 61618021875a^9b^4 + 5799965925a^{10}b^4 - 92035125a^{11}b^4 + 5259150a^{12}b^4)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(2046225352864875b^5 - 4384293790660180ab^5 + 2896776266543295a^2b^5)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-1183582613062230a^3b^5 + 154818398535750a^4b^5 - 4765553285850a^6b^5)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(1564770434220a^7b^5 - 108515480625a^8b^5 + 14881236900a^9b^5 - 302401125a^{10}b^5)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(24192090a^{11}b^5-4376029854983156b^6+882351615046635ab^6-757314109245930a^2b^6)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(237762013535100a^3b^6-64459961486850a^4b^6+4765553285850a^5b^6-61354862100a^7b^6)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(16380836325a^8b^6-488494125a^9b^6+56448210a^10b^6+66696220706115b^7)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-148574697339502ab^7+98832454416660a^2b^7-48732039707280a^3b^7+7958826637650a^4b^7)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-1564770434220a^5b^7+61354862100a^6b^7-265182525a^8b^7+58929450a^9b^7)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-7442156684963b^8+14777453889915ab^8-13445576481135a^2b^8+3991249561815a^3b^8)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-1318930698825a^4b^8+108515480625a^5b^8-16380836325a^6b^8+265182525a^7b^8)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(6170141513256b^9-1383157455640ab^9+847258652835a^2b^9-447929438010a^3b^9)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(61618021875a^4b^9-14881236900a^5b^9+488494125a^6b^9-58929450a^7b^9-382050404456b^{10})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(71906369145ab^{10}-66766662300a^2b^{10}+16234996050a^3b^{10}-5799965925a^4b^{10})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(302401125a^5b^{10}-56448210a^6b^{10}+1759562805b^{11}-3885833198ab^{11}+1993919070a^2b^{11})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-1085488560a^3b^{11}+92035125a^4b^{11}-24192090a^5b^{11}-59394517b^{12}+100340955ab^{12})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-92736345a^2b^{12}+14199705a^3b^{12}-5259150a^4b^{12}+1426425b^{13}-3024980ab^{13})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(1038345a^2b^{13}-566370a^3b^{13}-23065b^{14}+30225ab^{14}-26970a^2b^{14}+225b^{15})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-434ab^{15}-b^{16})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}\Bigg\}+\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-28}{2})}\Bigg\{\frac{(9929789119274850a-15491642259222720a^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(11638964717574354a^3-4301343369552768a^4+1253747784215818a^5-201126858232640a^6)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(30546110939162a^7-2485955985664a^8+217757164134a^9-9269332800a^{10}+478875254a^{11})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-9962368a^{12}+289198a^{13}-2240a^{14}+30a^{15}-9929789119274850b)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(15555346969469706a^2b-12813960443208960a^3b+6481731067775438a^4b)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-1515408978736896a^5b+337720529526218a^6b-35628106560000a^7b+4390411998314a^8b)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-229181664768a^9b+16800248686a^{10}b-416282880a^{11}b+18201898a^{12}b-166656a^{13}b)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(4030a^{14}b+15491642259222720b^2-15555346969469706ab^2+5635201740688740a^3b^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-2741640041773120a^4b^2+1013554930920866a^5b^2-151523346853376a^6b^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(26800290613336a^7b^2-1790230626240a^8b^2+182599613658a^9b^2-5522995712a^{10}b^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(342148612a^{11}b^2-3739840a^{12}b^2+138446a^{13}b^2-11638964717574354b^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(128139604432089600ab^3-5635201740688740a^2b^3+723609777250674a^4b^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-215556136143360a^5b^3+61365006725256a^6b^3-5693437596672a^7b^3+821169360882a^8b^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-31480247040a^9b^3+2698715292a^{10}b^3-35902464a^{11}b^3+1893294a^{12}b^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(43013433695527680b^4-6481731067775438ab^4+2741640041773120a^2b^4)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-723609777250674a^3b^4+39086251409460a^5b^4-7016427169920a^6b^4+1595208776220a^7b^4)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-83706249600a^8b^4+10049696250a^9b^4-168292800a^{10}b^4+12271350a^{11}b^4)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-1253747784215818b^5+1515408978736896ab^5-1013554930920866a^2b^5)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(215556136143360a^3b^5-39086251409460a^4b^5+943874861580a^6b^5-93850421760a^7b^5)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(17505768510a^8b^5-396998400a^9b^5+40320150a^{10}b^5+2011268582326400b^6)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-337720529526218ab^6+151523346853376a^2b^6-61365006725256a^3b^6+7016427169920a^4b^6)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-943874861580a^5b^6+9848041560a^7b^6-416848320a^8b^6+65132550a^9b^6)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-305461109391620b^7+35628106560000ab^7-26800290613336a^2b^7+5693437596672a^3b^7)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-1595208776220a^4b^7+93850421760a^5b^7-9848041560a^6b^7+35357670a^8b^7)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(24859559856664b^8-4390411998314ab^8+1790230626240a^2b^8-821169360882a^3b^8)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(83706249600a^4b^8-17505768510a^5b^8+416848320a^6b^8-35357670a^7b^8-2177571641346b^9)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(229181664768ab^9-182599613658a^2b^9+31480247040a^3b^9-10049696250a^4b^9)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(396998400a^5b^9-65132550a^6b^9+9269332800b^{10}-16800248686ab^{10}+5522995712a^2b^{10})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-2698715292a^3b^{10}+168292800a^4b^{10}-40320150a^5b^{10}-4788752546b^{11}+416282880ab^{11})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-342148612a^2b^{11}+35902464a^3b^{11}-12271350a^4b^{11}+9962368b^{12}-18201898ab^{12})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(3739840a^2b^{12}-1893294a^3b^{12}-289198b^{13}+166656ab^{13}-138446a^2b^{13}+2240b^{14})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-4030ab^{14}-30b^{15})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}\Bigg\}\Bigg] \tag{9}$$

III. Derivation of Main Summation Formula :

Substituting $c=\frac{a+b-29}{2}$ and $z=\frac{1}{2}$ in equation (4), we get

$$(a-b)\,{}_2F_1\left[\begin{matrix}a,\ b & ; \\ \frac{a+b-29}{2} & ;\end{matrix}\ \frac{1}{2}\right]=(a-b-29)\,{}_2F_1\left[\begin{matrix}a,\ b-1 & ; \\ \frac{a+b-29}{2} & ;\end{matrix}\ \frac{1}{2}\right]+(a-b+29)\,{}_2F_1\left[\begin{matrix}a-1,\ b & ; \\ \frac{a+b-29}{2} & ;\end{matrix}\ \frac{1}{2}\right]$$

Now involving (8), we get

$$L.H.S=\frac{2^{(b-1)}\ \Gamma(\frac{a+b-29}{2})}{\Gamma(b)}\Bigg[\frac{(a-b-29)(b-1)}{(a-b+1)}\ \frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-29}{2})}\Bigg\{\frac{(6190283353629375)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-80559720840359 25a-1616392482200145a^2+6545235516842331a^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-4248216825020805a^4+1462132132021015a^5-320488415412165a^6+48202739351767a^7)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-5162979107715a^8+400775348545a^9-22625638035a^{10}+919973873a^{11}-26244855a^{12})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(498365a^{13}-5655a^{14}+29a^{15}-1445971348434217 5b+24912724982942250ab)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-10303006262294205a^2b-2596215568394844a^3b+3745061321133657a^4b)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-1500877287187498a^5b+352103134363795a^6b-53414874649352a^7b+5770777441211a^8b)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-432319504074a^9b+23960048385a^{10}b-894640604a^{11}b+24237707a^{12}b-370678a^{13}b)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(3625a^{14}b+13121113142970855b^2-25090282486282455ab^2+15211233519617970a^2b^2)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-2788293602201802a^3b^2-805278615559779a^4b^2+592074582718251a^5b^2)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-152005752412020a^6b^2+25039023203844a^7b^2-2577581061327a^8b^2+199541321583a^9b^2)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-9686890590a^{10}b^2+369359718a^{11}b^2-7237269a^{12}b^2+115101a^{13}b^2-6520139954328519b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(13108577078985156ab^3-9036235922107038a^2b^3+2763679802745492a^3b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-236996129977305a^4b^3-81510903588600a^5b^3+35627232576540a^6b^3-5812690315320a^7b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(681267356055a^8b^3-44069719500a^9b^3+2398342050a^{10}b^3-59377500a^{11}b^3+1442025a^{12}b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(2046225352864875b^4-4230153389033277ab^4+3078613199870661a^2b^4)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-1090412569469739a^3b^4+191734678251150a^4b^4-6491930042850a^5b^4)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-3207292744950a^6b^4+904579763850a^7b^4-87179618025a^8b^4+7262379375a^9b^4)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-230865375a^10b^4+8454225a^11b^4-4376029854983l5b^5+909977737682870ab^5)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-690244478052695a^2b^5+254775008026440a^3b^5-53044457970150a^4b^5)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(5586729768900a^5b^5-11357015670a^6b^5-49912543800a^7b^5+9518807025a^8b^5)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-422311050a^9b^5+24582285a^10b^5+666962207061l5b^6-140635945873815ab^6)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(104089464902220a^2b^6-41041143055500a^3b^6+8538331086810a^4b^6-1105546036770a^5b^6)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(67259653020a^6b^6+1325342340a^7b^6-251495685a^8b^6+33266625a^9b^6-7442156684963b^7)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(15316877775928ab^7-11901188994484a^2b^7+4270299001416a^3b^7-1002318344370a^4b^7)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(116087214600a^5b^7-9443799540a^6b^7+269174520a^7b^7+9694845a^8b^7+617014151325b^8)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-1295202140631ab^8+901837698633a^2b^8-364964664267a^3b^8+66788562375a^4b^8)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-9924009525a^5b^8+520009875a^6b^8-25662825a^7b^8-382050404445b^9+75064195830ab^9)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-58140440475a^2b^9+17676240660a^3b^9-4270231875a^4b^9+331665750a^5b^9-31865925a^6b^9)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(17595628805b^{10}-3613820925ab^{10}+2150426850a^2b^{10}-869003850a^3b^{10}+102327225a^4b^{10})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-15737865a^5b^{10}-593945175b^{11}+105400516ab^{11}-80080078a^2b^{11}+15757092a^3b^{11})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-3817125a^4b^{11}+1426425b^{12}-2806167ab^{12}+1132131a^2b^{12}-451269a^3b^{12}-23065b^{13})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(31850ab^{13}-23345a^2b^{13}+225b^{14}-405ab^{14}-b^{15})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}\Bigg\}+\frac{(a-b-29)}{(a-b+1)}\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-28}{2})}\times$$

$$\times\Bigg\{\frac{(9716331072597975-5938876097374545a-5008529359741041a^2+6487628849502951a^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-3119276152843517a^4+859410426286395a^5-158335532807893a^6+20043813222499a^7)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-1854935290971a^8+121930780205a^9-5964037651a^{10}+200533749a^{11}-4855487a^{12})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(69545a^{13}-615a^{14}+a^{15}-2569922351932639 5b+25171279942688730ab)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-2416223309724297a^2b-6479001690355356a^3b+3943894121896461a^4b)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-1188391890161850a^5b+222399378231639a^6b-28728676894728a^7b+2588486627943a^8b)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-169894944570a^9b+7802522013a^{10}b-256988316a^{11}b+5440071a^{12}b-72870a^{13}b)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(405a^{14}b+2718634464859707 9b^2-32032729028145123ab^2+11386302357850434a^2b^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(733537341812478a^3b^2-1752277497289123a^4b^2+633533549962775a^5b^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-125659728215252a^6b^2+16438837192564a^7b^2-1464979112799a^8b^2+93031157115a^9b^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-4033380494a^{10}b^2+122243758a^{11}b^2-2182453a^{12}b^2+23345a^{13}b^2-15714610310684283b^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(20322778632253764ab^3-9399215529829782a^2b^3+1589873109506676a^3b^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(240339705407979a^4b^3-160880872128120a^5b^3+37894714290636a^6b^3-4928138731512a^7b^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(453967376427a^8b^3-25961358540a^9b^3+1124193642a^{10}b^3-25610364a^{11}b^3+451269a^{12}b^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(5701581389114131b^4-7726030369046057ab^4+4013955046526517a^2b^4)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-970795132030695a^3b^4+83600202729150a^4b^4+18789810760710a^5b^4-5875790236470a^6b^4)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(942471670770a^7b^4-74963930625a^8b^4+4806325875a^9b^4-139028175a^{10}b^4+3817125a^{11}b^4)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-1404456293068423b^5+1937418830448486ab^5-1064214040307691a^2b^5)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(294859652506440a^3b^5-41070362418750a^4b^5+1727406831780a^5b^5+557686323810a^6b^5)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-86130323640a^{7}b^{5}+9585440325a^{8}b^{5}-365482650a^{9}b^{5}+15737865a^{10}b^{5})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(2427242938411187b^{6}-3442719767774739ab^{6}+1884434242154040a^{2}b^{6}-561189739042220a^{3}b^{6})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(90366685762500a^{4}b^{6}-737047079370a^{5}b^{6}+12336525180a^{6}b^{6}+6606541620a^{7}b^{6})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-412856325a^{8}b^{6}+31865925a^{9}b^{6}-313699034989039b^{7}+426586105673520ab^{7})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-246526788433320a^{2}b^{7}+69282455115600a^{3}b^{7}-12458094934500a^{4}b^{7}+1144198213200a^{5}b^{7})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-5157657540a^{6}b^{7}+136868400a^{7}b^{7}+25662825a^{8}b^{7}+28817469121330b^{8}-41524663248110ab^{8})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(21347953010010a^{2}b^{8}-6726111344550a^{3}b^{8}+1001199849750a^{4}b^{8}-11324029185a^{5}b^{8})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(483031395a^{6}b^{8}-9694845a^{7}b^{8}-2132066573290b^{9}+2678362437180ab^{9}-1575248177430a^{2}b^{9})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(375080925000a^{3}b^{9}-71943453750a^{4}b^{9}+4685341500a^{5}b^{9}-33266625a^{6}b^{9}+105284024690b^{10})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-152570425930ab^{10}+65177697780a^{2}b^{10}-20970436500a^{3}b^{10}+2009504250a^{4}b^{10})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-24582285a^{5}b^{10}-4571143850b^{11}+4815843240ab^{11}-2858647740a^{2}b^{11}+428649000a^{3}b^{11})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-8454225a^{4}b^{11}+111565610b^{12}-162453070ab^{12}+44075070a^{2}b^{12}-14420250a^{3}b^{12})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-274253b^{13}+191)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}\Bigg]\Bigg\}+\frac{2^{(b-1)}\,\Gamma(\frac{a+b-29}{2})}{\Gamma(b)}\Bigg[\frac{(a-b+29)}{(a-b-1)}\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-28}{2})}\times$$

$$\times\left\{\frac{(-9716331072597975+25699223519326395a-27186344648597079a^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+\right.$$

$$+\frac{(15714610310684283a^3-5701581389114131a^4+1404456293068423a^5-242724293841187a^6)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(31369903498903a^7-2881746912133a^8+213206657329a^9-10528402469a^{10})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(457114385a^{11}-11156561a^{12}+274253a^{13}-2465a^{14}+29a^{15}+5938876097374545b)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-25171279942688730ab+32032729028145123a^2b-20322778632253764a^3b)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(7726030369046057a^4b-1937418830448486a^5b+344271976774739a^6b-42658610567352a^7b)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(4152466324811a^8b-267836243718a^9b+15257042593a^{10}b-481584324a^{11}b+16245307a^{12}b)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-191226a^{13}b+3625a^{14}b+5008529359741041b^2+2416223309724297ab^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-11386302357850434a^2b^2+9399215529829782a^3b^2-4013955046526517a^4b^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(1064214040307691a^5b^2-188443424215404a^6b^2+24652678843332a^7b^2-2134795301001a^8b^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(157524817743a^9b^2-6517769778a^{10}b^2+285864774a^{11}b^2-4407507a^{12}b^2+115101a^{13}b^2)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-6487628849502951b^3+6479001690355356ab^3-733537341812478a^2b^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-1589873109506676a^3b^3+970795132030695a^4b^3-294859652506440a^5b^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(56118973904220a^6b^3-6928245511560a^7b^3+672611134455a^8b^3-37508092500a^9b^3)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(2097043650a^{10}b^3-42864900a^{11}b^3+1442025a^{12}b^3+3119276152843517b^4)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-3943894121896461ab^4+1752277497289123a^2b^4-240339705407979a^3b^4)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-83600202729150a^4b^4+41070362418750a^5b^4-9036668576250a^6b^4+1245809493450a^7b^4)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-100119984975a^8b^4+7194345375a^9b^4-200950425a^{10}b^4+8454225a^{11}b^4)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-859410426286395b^5+1188391890161850ab^5-633533549962775a^2b^5)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(160880872128120a^3b^5-18789810760710a^4b^5-1727406831780a^5b^5+737047079370a^6b^5)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-114419821320a^7b^5+11324029185a^8b^5-468534150a^9b^5+24582285a^{10}b^5)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(158335532807893b^6-222399378231639ab^6+125659728215252a^2b^6-37894714290636a^3b^6)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(5875790236470a^4b^6-557686323810a^5b^6-12336525180a^6b^6+5157657540a^7b^6)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-483031395a^8b^6+33266625a^9b^6-20043813222499b^7+28728676894728ab^7)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-16438837192564a^2b^7+4928138731512a^3b^7-942471670770a^4b^7+86130323640a^5b^7)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-6606541620a^6b^7-13686840a^7b^7+9694845a^8b^7+1854935290971b^8-2588486627943ab^8)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(1464979112799a^2b^8-453967376427a^3b^8+74963930625a^4b^8-9585440325a^5b^8)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(412856325a^6b^8-25662825a^7b^8-121930780205b^9+169894944570ab^9-93031157115a^2b^9)}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(25961358540a^3b^9-4806325875a^4b^9+365482650a^5b^9-31865925a^6b^9+5964037651b^{10})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-7802522013ab^{10}+4033380494a^2b^{10}-1124193642a^3b^{10}+139028175a^4b^{10})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-15737865a^5b^{10}-200533749b^{11}+256988316ab^{11}-122243758a^2b^{11}+25610364a^3b^{11})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\frac{(-3817125a^4b^{11}+4855487b^{12}-5440071ab^{12}+2182453a^2b^{12}-451269a^3b^{12}-69545b^{13})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}+$$

$$+\left.\frac{(72870ab^{13}-23345a^2b^{13}+615b^{14}-405ab^{14}-b^{15})}{\prod_{\Omega=1}^{14}\{a-2\Omega\}}\right\}+\frac{(a-b+29)}{(a-b-1)}\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-29}{2})}\times$$

$$\times\left\{\frac{(-6190283353629375+144597134843421175a-1312111314297085 5a^2)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+\right.$$

$$+\frac{(6520139954328519a^3-2046225352864875a^4+437602985498315a^5-66696220706115a^6)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(7442156684963a^7-617014151325a^8+38205040445a^9-1759562805a^{10}+59394517a^{11})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-1426425a^{12}+23065a^{13}-225a^{14}+a^{15}+80559720840359925b-249127249829422 50ab)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(25090282486282455a^2b-13108577078985156a^3b+4230153389033277a^4b)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-909977737682870a^5b+140635945873815a^6b-15316877775928a^7b+1295202140631a^8b)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-75064195830a^9b+3613820925a^{10}b-105400516a^{11}b+2806167a^{12}b-31850a^{13}b)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(405a^{14}b+16163924822200145b^2+10303006262294205ab^2-15211233519617970a^2b^2)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(9036235922107038a^3b^2-3078613199870661a^4b^2+690244478052695a^5b^2)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-104089464902220a^6b^2+11901188994484a^7b^2-901837698633a^8b^2+58140440475a^9b^2)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-2150426850a^{10}b^2+80080078a^{11}b^2-1132131a^{12}b^2+23345a^{13}b^2-6545235516842331b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(2596215568394844ab^3+2788293602201802a^2b^3-2763679802745492a^3b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(1090412569469739a^4b^3-254775008026440a^5b^3+41041143055500a^6b^3-4270299001416a^7b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(364964664267a^8b^3-17676240660a^9b^3+869003850a^{10}b^3-15757092a^{11}b^3+451269a^{12}b^3)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(4248216825020805b^4-3745061321133657ab^4+805278615559779a^2b^4)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(236996129977305a^3b^4-191734678251150a^4b^4+53044457970150a^5b^4-8538331086810a^6b^4)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(1002318344370a^7b^4-66788562375a^8b^4+4270231875a^9b^4-102327225a^{10}b^4+3817125a^{11}b^4)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-1462132132021015b^5+1500877287187498ab^5-592074582718251a^2b^5)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(81510903588600a^3b^5+6491930042850a^4b^5-5586729768900a^5b^5+1105546036770a^6b^5)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-116087214600a^7b^5+9924009525a^8b^5-331665750a^9b^5+15737865a^{10}b^5)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(3204884154121665b^6-352103134363795ab^6+152005752412020a^2b^6-35627232576540a^3b^6)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(3207292744950a^4b^6+11357015670a^5b^6-67259653020a^6b^6+9443799540a^7b^6)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-520009875a^8b^6+31865925a^9b^6-48202739351767b^7+53414874649352ab^7)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-25039023203844a^2b^7+5812690315320a^3b^7-904579763850a^4b^7+49912543800a^5b^7)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-1325342340a^6b^7-269174520a^7b^7+25662825a^8b^7+5162979107715b^8-5770777441211ab^8)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(2577581061327a^2b^8-681267356055a^3b^8+87179618025a^4b^8-9518807025a^5b^8)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(251495685a^6b^8-9694845a^7b^8-400775348545b^9+432319504074ab^9-199541321583a^2b^9)}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(44069719500a^3b^9-7262379375a^4b^9+422311050a^5b^9-33266625a^6b^9+22625638035b^{10})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-23960048385ab^{10}+9686890590a^2b^{10}-2398342050a^3b^{10}+230865375a^4b^{10})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-24582285a^5b^{10}-919973873b^{11}+894640604ab^{11}-369359718a^2b^{11}+59377500a^3b^{11})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$+\frac{(-8454225a^4b^{11}+262448855b^{12}-24237707ab^{12}+7237269a^2b^{12}-1442025a^3b^{12}-498365b^{13})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}+$$

$$\left.\left.+\frac{(370678ab^{13}-115101a^2b^{13}+5655b^{14}-3625ab^{14}-29b^{15})}{\prod_{\Phi=1}^{15}\{a-(2\Phi-1)\}}\right\}\right]$$

On simplification, we get the main formula.

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Development of a Summation Formula Related To Hypergeometric Functions

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Abstract - The aim of the present paper is to obtain a summation formula based on half argument related to hypergeometric functions.The result is general in nature and is believed to be new.

Keywords : *Contiguous relation, Gauss second summation theorem.*

AMS Subject Classifications (2010) : *33C05 , 33C20 , 33C70*

DEVELOPMENT OF A SUMMATION FORMULA RELATED TO HYPERGEOMETRIC FUNCTIONS

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Development of a Summation Formula Related To Hypergeometric Functions

Salahuddin[α] , M.P.Chaudhary[Ω]

Abstract - The aim of the present paper is to obtain a summation formula based on half argument related to hypergeometric functions.The result is general in nature and is believed to be new.

Keywords : Contiguous relation, Gauss second summation theorem.

I. Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ccc} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k(a_2)_k\cdots(a_A)_k z^k}{(b_1)_k(b_2)_k\cdots(b_B)_k k!} $$

or

$$ {}_AF_B\left[\begin{array}{ccc} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right] \equiv {}_AF_B\left[\begin{array}{ccc} (a_j)_{j=1}^A & ; & \\ & & z \\ (b_j)_{j=1}^B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1} $$

Where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and A, B are non-negative integers.

Contiguous Relation is defined by [Andrews p.363(9.16), E.D.P.51(10), H.T.F.I p.103(32)]

$$ (a-b)\ {}_2F_1\left[\begin{array}{ccc} a,\ b\ ; & & \\ c\ \ ; & & z \end{array}\right] = a\ {}_2F_1\left[\begin{array}{ccc} a+1,\ b & ; & \\ c & ; & z \end{array}\right] - b\ {}_2F_1\left[\begin{array}{ccc} a,\ b+1 & ; & \\ c\ ; & & z \end{array}\right] \tag{2} $$

Gauss second summation theorem is defined by [Prud., 491(7.3.7.5)]

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ \frac{a+b+1}{2} & ; & \frac{1}{2} \end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{3} $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{4} $$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prud., p.491(7.3.7.3)]

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ \frac{a+b-1}{2} & ; & \frac{1}{2} \end{array}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{5} $$

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form,

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ \frac{a+b-1}{2} & ; & \frac{1}{2} \end{array}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{6} $$

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II. Main Summation Formulae

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b+26}{2} & & ; & 2 \end{matrix}\right] = \frac{2^b\ \Gamma(\frac{a+b+26}{2})}{(a-b)\ \Gamma(b)} \times $$

$$ \times\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})}\left\{\frac{4096(-81749606400a+123436892160a^2-77270003712a^3+26946067456a^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+\right.\right. $$

$$ +\frac{4096(-5887453440a^5+853730240a^6-84401856a^7+5718768a^8-261360a^9+7700a^{10})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+ $$

$$ +\frac{4096(-132a^{11}+a^{12}+81749606400b+367298150400ab-118779801600a^2b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+ $$

$$ +\frac{4096(242509385728a^3b-38595539712a^4b+16452876672a^5b-1303280832a^6b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+ $$

$$ +\frac{4096(217911936a^7b-8342928a^8b+621368a^9b-9108a^{10}b+276a^{11}b+123436892160b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+ $$

$$ +\frac{4096(118779801600ab^2+464899287040a^2b^2-37227551232a^3b^2+76928096320a^4b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+ $$

$$ +\frac{4096(-4716831168a^5b^2+2038636096a^6b^2-67277760a^7b^2+11193732a^8b^2-148764a^9b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+ $$

$$ +\frac{4096(10626a^{10}b^2+77270003712b^3+242509385728ab^3+37227551232a^2b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+ $$

$$ +\frac{4096(125264532736a^3b^3-3669038016a^4b^3+7098446208a^5b^3-181484352a^6b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+ $$

$$ +\frac{4096(74296992a^7b^3-865260a^8b^3+134596a^9b^3+26946067456b^4+38595539712ab^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+ $$

$$ +\frac{4096(76928096320a^2b^4+3669038016a^3b^4+10621924768a^4b^4-124807200a^5b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+ $$

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$$+\frac{4096(219244648a^6b^4 - 1961256a^7b^4 + 735471a^8b^4 + 5887453440b^5 + 16452876672ab^5)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(4716831168a^2b^5 + 7098446208a^3b^5 + 124807200a^4b^5 + 312018000a^5b^5 - 1248072a^6b^5)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(1961256a^7b^5 + 853730240b^6 + 1303280832ab^6 + 2038636096a^2b^6 + 181484352a^3b^6)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(219244648a^4b^6 + 1248072a^5b^6 + 2704156a^6b^6 + 84401856b^7 + 217911936ab^7)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(67277760a^2b^7 + 74296992a^3b^7 + 1961256a^4b^7 + 1961256a^5b^7 + 5718768b^8)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(8342928ab^8 + 11193732a^2b^8 + 865260a^3b^8 + 735471a^4b^8 + 261360b^9 + 621368ab^9)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(148764a^2b^9 + 134596a^3b^9 + 7700b^{10} + 9108ab^{10} + 10626a^2b^{10} + 132b^{11})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{4096(+276ab^{11} + b^{12})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768b(38385745920a + 8277098496a^2 + 11938545664a^3 + 1293154816a^4 + 518927616a^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(29747424a^6 + 4842288a^7 + 138864a^8 + 9944a^9 + 110a^{10} + 3a^{11} + 38385745920b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(50799591424a^2b + 4130196480a^3b + 5067819264a^4b + 257208448a^5b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(92965264a^6b + 2517856a^7b + 370392a^8b + 4048a^9b + 253a^{10}b - 8277098496b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(50799591424ab^2+14021624320a^3b^2+502850656a^4b^2+529633328a^5b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(12452384a^6b^2+3991328a^7b^2+40986a^8b^2+5313a^9b^2+11938545664b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(-4130196480ab^3+14021624320a^2b^3+1206518544a^4b^3+19283936a^5b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(17521952a^6b^3+153824a^7b^3+43263a^8b^3-1293154816b^4+5067819264ab^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(-502850656a^2b^4+1206518544a^3b^4+35778064a^5b^4+208012a^6b^4+163438a^7b^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(5189276166b^5-257208448ab^5+529633328a^2b^5-19283936a^3b^5+35778064a^4b^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(312018a^6b^5-297474246b^6+92965264ab^6-12452384a^2b^6+17521952a^3b^6)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(-208012a^4b^6+312018a^5b^6+4842288b^7-2517856ab^7+3991328a^2b^7)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(-153824a^3b^7+163438a^4b^7-138864b^8+370392ab^8-40986a^2b^8+43263a^3b^8)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768b(9944b^9-4048ab^9+5313a^2b^9-110b^{10}+253ab^{10}+3b^{11})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{32768a(38385745920a-8277098496a^2+11938545664a^3-1293154816a^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(518927616a^5-297474246a^6+4842288a^7-138864a^8+9944a^9-110a^{10}+3a^{11})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(38385745920b+50799591424a^2b-4130196480a^3b+5067819264a^4b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(-257208448a^5b+92965264a^6b-2517856a^7b+370392a^8b-4048a^9b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(253a^{10}b+8277098496b^2+50799591424ab^2+14021624320a^3b^2-502850656a^4b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(529633328a^5b^2-12452384a^6b^2+3991328a^7b^2-40986a^8b^2+5313a^9b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(119385456664b^3+4130196480ab^3+14021624320a^2b^3+1206518544a^4b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(-19283936a^5b^3+17521952a^6b^3-153824a^7b^3+43263a^8b^3+12931548166b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(5067819264ab^4+502850656a^2b^4+1206518544a^3b^4+35778064a^5b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(-208012a^6b^4+163438a^7b^4+5189276166b^5+257208448ab^5+529633328a^2b^5)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(19283936a^3b^5+35778064a^4b^5+312018a^6b^5+297474246b^6+92965264ab^6)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(12452384a^2b^6+17521952a^3b^6+208012a^4b^6+312018a^5b^6+4842288b^7)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(2517856ab^7+3991328a^2b^7+153824a^3b^7+163438a^4b^7+138864b^8+370392ab^8)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768a(40986a^2b^8+43263a^3b^8+9944b^9+4048ab^9+5313a^2b^9+110b^{10}+253ab^{10}+3b^{11})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(81749606400a+123436892160a^2+77270003712a^3+26946067456a^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(5887453440a^5+853730240a^6+84401856a^7+5718768a^8+261360a^9+7700a^{10})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(132a^{11}+a^{12}-81749606400b+367298150400ab+118779801600a^2b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(242509385728a^3b+38595539712a^4b+16452876672a^5b+1303280832a^6b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(217911936a^7b+8342928a^8b+621368a^9b+9108a^{10}b+276a^{11}b+123436892160b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(-118779801600ab^2+464899287040a^2b^2+37227551232a^3b^2+76928096320a^4b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(4716831168a^5b^2+2038636096a^6b^2+67277760a^7b^2+11193732a^8b^2+148764a^9b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(10626a^10b^2-77270003712b^3+242509385728ab^3-37227551232a^2b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(125264532736a^3b^3+3669038016a^4b^3+7098446208a^5b^3+181484352a^6b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(74296992a^7b^3+865260a^8b^3+134596a^9b^3+26946067456b^4-38595539712ab^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(76928096320a^2b^4-3669038016a^3b^4+10621924768a^4b^4+124807200a^5b^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(219244648a^6b^4+1961256a^7b^4+735471a^8b^4-5887453440b^5+16452876672ab^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(-4716831168a^2b^5+7098446208a^3b^5-124807200a^4b^5+312018000a^5b^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(1248072a^6b^5+1961256a^7b^5+853730240b^6-1303280832ab^6+2038636096a^2b^6)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(-181484352a^3b^6+219244648a^4b^6-1248072a^5b^6+2704156a^6b^6-84401856b^7)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(217911936ab^7-67277760a^2b^7+74296992a^3b^7-1961256a^4b^7+1961256a^5b^7)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(5718768b^8-8342928ab^8+11193732a^2b^8-865260a^3b^8+735471a^4b^8-261360b^9)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(621368ab^9-148764a^2b^9+134596a^3b^9+7700b^{10}-9108ab^{10}+10626a^2b^{10})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{4096(-132b^{11}+276ab^{11}+b^{12})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}\Bigg\}\Bigg] \qquad (7)$$

III. Derivation of Summation Formulae (7)

Substituting $c=\frac{a+b+26}{2}$ and $z=\frac{1}{2}$ in equation (2), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix}a,\ b & ; \\ \frac{a+b+26}{2} & ;\end{matrix}\ \frac{1}{2}\right]=a\ {}_2F_1\left[\begin{matrix}a+1,\ b & ; \\ \frac{a+b+26}{2} & ;\end{matrix}\ \frac{1}{2}\right]-b\ {}_2F_1\left[\begin{matrix}a,\ b+1 & ; \\ \frac{a+b+26}{2} & ;\end{matrix}\ \frac{1}{2}\right]$$

Now using Gauss second summation theorem, we get

$$L.H.S=a\,\frac{2^b\ \Gamma(\frac{a+b+26}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+2}{2})}\left\{\frac{2048(-81749606400a+123436892160a^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+\right.\right.$$

$$+\frac{2048(-77270003712a^3+26946067456a^4-5887453440a^5+853730240a^6-84401856a^7)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(5718768a^8-261360a^9+7700a^{10}-132a^{11}+a^12+81749606400b}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(367298150400ab-118779801600a^2b)+242509385728a^3b-38595539712a^4b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(16452876672a^5b-1303280832a^6b+217911936a^7b-8342928a^8b+621368a^9b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(-9108a^10b+276a^11b+123436892160b^2+118779801600ab^2+464899287040a^2b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(-37227551232a^3b^2+76928096320a^4b^2-4716831168a^5b^2+2038636096a^6b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(-67277760a^7b^2+11193732a^8b^2-148764a^9b^2+10626a^{10}b^2+77270003712b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(242509385728ab^3+37227551232a^2b^3+125264532736a^3b^3-3669038016a^4b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(7098446208a^5b^3-181484352a^6b^3+74296992a^7b^3-865260a^8b^3+134596a^9b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(26946067456b^4+38595539712ab^4+76928096320a^2b^4+3669038016a^3b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(10621924768a^4b^4-124807200a^5b^4+219244648a^6b^4-1961256a^7b^4+735471a^8b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(5887453440b^5+16452876672ab^5+4716831168a^2b^5+7098446208a^3b^5)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(124807200a^4b^5+312018000a^5b^5-1248072a^6b^5+1961256a^7b^5+853730240b^6)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(1303280832ab^6+2038636096a^2b^6+181484352a^3b^6+219244648a^4b^6+1248072a^5b^6)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(2704156a^6b^6+84401856b^7+217911936ab^7+67277760a^2b^7+74296992a^3b^7)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(1961256a^4b^7+1961256a^5b^7+5718768b^8+8342928ab^8+11193732a^2b^8)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(865260a^3b^8+735471a^4b^8+261360b^9+621368ab^9+148764a^2b^9+134596a^3b^9)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{2048(7700b^{10}+9108ab^{10}+10626a^2b^{10}+132b^{11}+276ab^{11}+b^{12})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{32768(38385745920a-8277098496a^2+11938545664a^3-1293154816a^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(518927616a^5-29747424a^6+4842288a^7-138864a^8+9944a^9-110a^{10}+3a^{11})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(38385745920b+50799591424a^2b-4130196480a^3b+5067819264a^4b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(-257208448a^5b+92965264a^6b-2517856a^7b+370392a^8b-4048a^9b)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(253a^{10}b+8277098496b^2+50799591424ab^2+14021624320a^3b^2-502850656a^4b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(529633328a^5b^2-12452384a^6b^2+3991328a^7b^2-40986a^8b^2+5313a^9b^2)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(11938545664b^3+4130196480ab^3+14021624320a^2b^3+1206518544a^4b^3)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(-19283936a^5b^3+17521952a^6b^3-153824a^7b^3+43263a^8b^3+1293154816b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(5067819264ab^4+502850656a^2b^4+1206518544a^3b^4+35778064a^5b^4-208012a^6b^4)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(163438a^7b^4+518927616b^5+257208448ab^5+529633328a^2b^5+19283936a^3b^5)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(35778064a^4b^5+312018a^6b^5+297747424b^6+92965264ab^6+12452384a^2b^6)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(17521952a^3b^6+208012a^4b^6+312018a^5b^6+4842288b^7+2517856ab^7)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(3991328a^2b^7+153824a^3b^7+163438a^4b^7+138864b^8+370392ab^8+40986a^2b^8)}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}+$$

$$+\frac{32768(43263a^3b^8+9944b^9+4048ab^9+5313a^2b^9+110b^{10}+253ab^{10}+3b^{11})}{\left(\prod_{\zeta=0}^{11}\{a-b-2\zeta\}\right)\left(\prod_{\eta=1}^{12}\{a-b+2\eta\}\right)}\Bigg\}\Bigg]-$$

$$-b\,\frac{2^{b+1}\,\Gamma(\frac{a+b+26}{2})}{\Gamma(b+1)}\left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\left\{\frac{2048(81749606400a+123436892160a^2+77270003712a^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+\right.\right.$$

$$+\frac{2048(26946067456a^4+5887453440a^5+853730240a^6+84401856a^7+5718768a^8)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(261360a^9+7700a^{10}+132a^{11}+a^{12}-817496064006b+367298150400ab)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(118779801600a^2b+242509385728a^3b+38595539712a^4b+16452876672a^5b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(1303280832a^6b+217911936a^7b+8342928a^8b+621368a^9b+9108a^{10}b+276a^{11}b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(123436892160b^2-118779801600ab^2+464899287040a^2b^2+37227551232a^3b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(76928096320a^4b^2+4716831168a^5b^2+2038636096a^6b^2+67277760a^7b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(11193732a^8b^2+148764a^9b^2+10626a^{10}b^2-77270003712b^3+242509385728ab^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(-37227551232a^2b^3+125264532736a^3b^3+3669038016a^4b^3+7098446208a^5b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(181484352a^6b^3+74296992a^7b^3+865260a^8b^3+134596a^9b^3+26946067456b^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(-38595539712ab^4+76928096320a^2b^4-3669038016a^3b^4+10621924768a^4b^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(124807200a^5b^4+219244648a^6b^4+1961256a^7b^4+735471a^8b^4-5887453440b^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(16452876672ab^5-4716831168a^2b^5+7098446208a^3b^5-124807200a^4b^5)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(312018000a^5b^5+1248072a^6b^5+1961256a^7b^5+8537302400b^6-1303280832ab^6)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(2038636096a^2b^6-181484352a^3b^6+219244648a^4b^6-1248072a^5b^6+2704156a^6b^6)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(-844018856b^7+217911936ab^7-67277760a^2b^7+74296992a^3b^7-1961256a^4b^7)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(1961256a^5b^7+5718768b^8-8342928ab^8+11193732a^2b^8-865260a^3b^8+735471a^4b^8)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(-261360b^9+621368ab^9-148764a^2b^9+134596a^3b^9+7700b^{10}-9108ab^{10})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{2048(10626a^2b^{10}-132b^{11}+276ab^{11}+b^{12})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{32768(38385745920a+8277098496a^2+11938545664a^3+1293154816a^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(518927616a^5+29747424a^6+4842288a^7+138864a^8+9944a^9+110a^{10}+3a^{11})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(38385745920b+50799591424a^2b+4130196480a^3b+5067819264a^4b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(257208448a^5b+92965264a^6b+2517856a^7b+370392a^8b+4048a^9b+253a^{10}b)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(-8277098496b^2+50799591424ab^2+14021624320a^3b^2+502850656a^4b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(529633328a^5b^2+12452384a^6b^2+3991328a^7b^2+40986a^8b^2+5313a^9b^2)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(119385456664b^3-4130196480ab^3+14021624320a^2b^3+1206518544a^4b^3)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(19283936a^5b^3+17521952a^6b^3+153824a^7b^3+43263a^8b^3-1293154816b^4)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(35778064a^4b^5+312018a^6b^5-29747424b^6+92965264ab^6-12452384a^2b^6)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(+17521952a^3b^6-208012a^4b^6+312018a^5b^6+4842288b^7-2517856ab^7)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(3991328a^2b^7-153824a^3b^7+163438a^4b^7-138864b^8+370392ab^8-40986a^2b^8)}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}+$$

$$+\frac{32768(43263a^3b^8+9944b^9-4048ab^9+5313a^2b^9-110b^{10}+253ab^{10}+3b^{11})}{\left(\prod_{\nu=0}^{12}\{a-b-2\nu\}\right)\left(\prod_{\mu=1}^{11}\{a-b+2\mu\}\right)}\Bigg\}\Bigg]$$

On simplification we get the result (7).

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Author Guidelines:

1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript's Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.

- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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