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Evaluation Of Certain Summation Formulae Using Contiguous Relation and Involving Hypergeometric Function

By Salahuddin

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Abstracts : The main object of present paper is to obtain certain summation formulae using the Contiguous relation[1] and derived formula [2].The results are new and has general character.

Keywords : Contiguous relation,Recurrence relation, Gauss second sum-mation theorem .

GJSFR-F Classification : F,MSC NO : 33C05 , 33C20 , 33C60, 33C70

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I. Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{matrix} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{matrix}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1} $$

or

$$ {}_AF_B\left[\begin{matrix} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{matrix}\right] \equiv {}_AF_B\left[\begin{matrix} (a_j)_{j=1}^A & ; & \\ & & z \\ (b_j)_{j=1}^B & ; & \end{matrix}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{2} $$

where the parameters b1, b2, · · · , bB are neither zero nor negative integers and A, B are non-negative integers.

Contiguous Relations are defined by

[Andrews p.367(8), E. D. p.52(19), H.T. F. I p.103(38)]

$$ c\,(1-z)\,{}_2F_1\left[\begin{matrix} a,\ b\ ; & \\ c\ \ ; & z \end{matrix}\right] = c\,{}_2F_1\left[\begin{matrix} a-1,\ b & ; & \\ c & ; & z \end{matrix}\right] - (c-b)\,z\,{}_2F_1\left[\begin{matrix} a,\ b & ; & \\ c+1 & ; & z \end{matrix}\right] \tag{3} $$

[Abramowitz p.558(15.2.18)]

$$ (a-b)\,(1-z)\,{}_2F_1\left[\begin{matrix} a,\ b\ ; & \\ c\ \ ; & z \end{matrix}\right] = (c-b)\,{}_2F_1\left[\begin{matrix} a,\ b-1 & ; & \\ c & ; & z \end{matrix}\right] + (a-c)\,{}_2F_1\left[\begin{matrix} a-1,\ b & ; & \\ c\ ; & & z \end{matrix}\right] \tag{4} $$

Recurrence relation

$$ \Gamma(z+1) = z\,\Gamma(z) \tag{5} $$

Gauss second summation theorem [Prud., 491(7.3.7.5)]

$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+1}{2} & ; & \end{matrix}\right] = \frac{\Gamma(\frac{a+b+1}{2})\,\Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\,\Gamma(\frac{b+1}{2})} \tag{6} $$

$$ = \frac{2^{(b-1)}\,\Gamma(\frac{b}{2})\,\Gamma(\frac{a+b+1}{2})}{\Gamma(b)\,\Gamma(\frac{a+1}{2})} \tag{7} $$

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A new summation formula [2]

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & & ; & 2 \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})}\left\{\frac{(b+a-1)}{(a-1)}\right\} + \frac{2\ \Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\right] \tag{8} $$

II. Main Results of Summation Formulae

For all the results a $\neq$ b

For a < 1 and a > 2

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-2}{2} & & ; & 2 \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-2}{2})}{(a-b)\Gamma(b)}\left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-1}{2})}\left\{\frac{(3a^2-2ab-b^2-2a+2b)}{(a-1)}\right\}+ \right. $$

$$ \left. + \frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-2}{2})}\left\{\frac{(a^2+2ab-3b^2-2a+2b)}{(a-2)}\right\}\right] \tag{9} $$

For a < 1 and a > 3

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-3}{2} & & ; & 2 \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-3}{2})}{(a-b)\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-3}{2})}\left\{\frac{(3a-4a^2+a^3-3b+5a^2b+4b^2-5ab^2-b^3)}{(a-3)(a-1)}\right\}+ \right. $$

$$ \left. + \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-2}{2})}\left\{\frac{(-4a+4a^2+4b-4b^2)}{(a-2)}\right\}\right] \tag{10} $$

For a < 1 and a > 4

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-4}{2} & & ; & 2 \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-4}{2})}{(a-b)\Gamma(b)} \times $$

$$ \times \left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-3}{2})}\left\{\frac{(8a-10a^2+5a^3-8b+4ab+5a^2b+6b^2-9ab^2-b^3)}{(a-3)(a-1)}\right\}+ \right. $$

$$ \left. + \frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-4}{2})}\left\{\frac{(8a-6a^2+a^3-8b-4ab+9a^2b+10b^2-5ab^2-5b^3)}{(a-4)(a-2)}\right\}\right] \tag{11} $$

For a < 1 and a > 5

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-5}{2} & & ; & 2 \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-5}{2})}{(a-b)\Gamma(b)} \times $$

$$ \times \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-5}{2})}\left\{\frac{(-15a+23a^2-9a^3+a^4+15b-21a^2b+14a^3b-23b^2+21ab^2+9b^3-14ab^3-b^4)}{(a-5)(a-3)(a-1)}\right\}+ \right. $$

$$ \left. + \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-4}{2})}\left\{\frac{(26a-16a^2+6a^3-26b+14a^2b+16b^2-14ab^2-6b^3)}{(a-4)(a-2)}\right\}\right] \tag{12} $$

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III. Derivation of Summation Formulae :

Derivation of (9): Substituting c = $\frac{a+b-2}{2}$ and z = $\frac{1}{2}$ in equation (4), we get

$$\left(\frac{a-b}{2}\right) {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b-2}{2} & ; & \end{matrix}\right] = \left(\frac{a-b-2}{2}\right) {}_2F_1\left[\begin{matrix} a,\ b-1\ ; & \frac{1}{2} \\ \frac{a+b-2}{2} & ; \end{matrix}\right] +$$

$$+\left(\frac{a-b+2}{2}\right) {}_2F_1\left[\begin{matrix} a-1,\ b & ; & \frac{1}{2} \\ \frac{a+b-2}{2}; & & \end{matrix}\right]$$

or

$$(a-b)\, {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b-2}{2} & ; & \end{matrix}\right] = (a-b-2)\, {}_2F_1\left[\begin{matrix} a,\ b-1\ ; & \frac{1}{2} \\ \frac{a+b-2}{2}\quad ; & \end{matrix}\right] + (a-b+2)\, {}_2F_1\left[\begin{matrix} a-1,\ b & ; & \frac{1}{2} \\ \frac{a+b-2}{2}; & & \end{matrix}\right]$$

Now using (10), we get

$$L.H.S = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-2}{2})}{\Gamma(b)}\left[\frac{(a-b-2)(b-1)}{2}\left\{\frac{\Gamma(\frac{b-1}{2})}{\Gamma(\frac{a-1}{2})}\left(\frac{(a+b-2)}{(a-1)}\right)+2\,\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})}\right\}\right]+$$

$$+\frac{2^{(b-1)}\ \Gamma(\frac{a+b-2}{2})}{\Gamma(b)}\left[(a-b+2)\left\{\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-2}{2})}\left(\frac{(a+b-2)}{(a-2)}\right)+2\,\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-1}{2})}\right\}\right]$$

$$=\frac{2^{(b-1)}\ \Gamma(\frac{a+b-2}{2})}{\Gamma(b)}\left[(a-b-2)\left\{\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-1}{2})}\left(\frac{(a+b-2)}{(a-1)}\right)+(b-1)\,\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})}\right\}\right]+$$

$$+\frac{2^{(b-1)}\ \Gamma(\frac{a+b-2}{2})}{\Gamma(b)}\left[(a-b+2)\left\{\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-2}{2})}\left(\frac{(a+b-2)}{(a-2)}\right)+2\,\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-1}{2})}\right\}\right]$$

$$=\frac{2^{(b-1)}\ \Gamma(\frac{a+b-2}{2})}{\Gamma(b)}\left[\frac{\Gamma\frac{b+1}{2})}{\Gamma(\frac{a-1}{2})}\left\{\frac{(a+b-2)(a-b-2)}{(a-1)}+2(a-b+2)\right\}+\right.$$

$$\left.+\frac{\Gamma(\frac{b}{2})}{\Gamma\frac{a-2}{2})}\left\{\frac{2\ (a-b-2)(b-1)}{(a-2)}+\frac{(a+b-2)(a-b+2)}{(a-2)}\right\}\right]$$

On simplification, we get the result(9).

On the same way we can prove the other results.

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Economic Analysis of Rural Households Access to Non-Farm Activities in Kwara State, Nigeria

By *Fakayode, S.B, Falola, A, Babatunde, R.O and Adedoyin, J
University of Ilorin. Ilorin. Nigeria.

Abstracts : In the quest to overcome poverty, farm households engage in various non-farm activities. This study therefore examined pluriactive households in Kwara State, Nigeria. It also examines the activities that are non-farm; the reasons why rural households diversify into such activities; and the factors that predispose the farmers to diversify. Primary data was used for the study. Descriptive statistics, diversity index and Kruskal-Wallis test were the analytical tools used for the study. The results of the findings show that most of those who engage in non-farm activities are male, with Trading and Civil service being the major non-farm activities the rural households diversify into. Most rural households diversify in order to improve their standard of living. The diversity of activities increases as the number of activities in the study area increases. The results also reveal that the need to increase income and the small farm sizes of the rural households predispose them most to engage in non-farm activities. It is therefore recommended that rural households should engage in activities that would help them achieve the goal of reducing their state of poverty. Government and development agencies should help provide credit facilities to help the rural households intensify their engagement in these activities.

Keywords : *Poverty, rural households, non-farm activities, diversity index, Kruskal-Wallis test.*

GJSFR-G Classification : *FOR Code : 070106,070108,070301*

ECONOMIC ANALYSIS OF RURAL HOUSEHOLDS ACCESS TO NON-FARM ACTIVITIES IN KWARA STATE, NIGERIA

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Economic Analysis of Rural Households Access to Non-Farm Activities in Kwara State, Nigeria

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Keywords : Poverty, rural households, non-farm activities, diversity index, Kruskal-Wallis test.

I. Introduction

In Nigeria, the alarming increase in the rate of poverty has gone contrary to available natural resources and wealth of the country. The description of Nigeria as a paradox of plenty by the World Bank (1996) has continued to be confirmed by events and official statistics in the country. The paradox is that the poverty level in Nigeria contradicts the country's immense wealth (Obadan, 2002). Among other things, the country is enormously endowed with human, agricultural, petroleum, gas and large untapped solid mineral resources. But rather than record remarkable progress in national socio-economic development, Nigeria retrogressed to become one of the 25 poorest countries at the threshold of twenty-first century whereas she was among the richest 50 in the early - 1970s. The situation has worsened since the late 1990s, to the extent that the country is now considered one of the 20 poorest countries in the world. Over 70percent of her population is classified as poor, with 35 percent living in absolute poverty (IFAD, 2007).Over the years, several efforts have been made to remedy poverty in the economy but these have been ineffective. The increasing poverty incidence, both within and among locations, persisted, in spite of various resources and efforts exerted on poverty-related programme and schemes in the country, thus suggesting that the programmes and schemes were ineffective and ineffectual(Obadan,2002). The extremity of indigenous poverty is influenced by the obvious income-related factors; low-income, income-insecurity, job-insecurity and the lack of education and resources. The causes of poverty vary: these include lack of education, war, natural disasters, political corruption, mental illness and disability which are among the most common causes. Poverty which implies lack of access to necessities varies within the country. It tends to be evenly distributed across the country rather than concentrated in specific geographic areas. However, in some zones, the poverty situation threatens to worsen considerably such as northern area bordering the Niger, which is arid, marginal to agriculture, environmentally damaged and densely populated. It is especially severe in rural areas where social services and infrastructure are limited or non-existent.

The vast majorities of those who live in Nigeria's rural areas are poor and depend on agriculture for food and income (RPPN, 2008). The poorest depend on subsistence living but often grow short of food, particularly during the pre-harvest period. Evidences in Nigeria show that number of those in poverty has continued to increase. For instance, the number of those in poverty has continued to increase from 27 percent in 1980 to 46 percent in 1985. It declined slightly to 42 percent in 1992 and increased very sharply to 67 percent in 1996. By 1999, estimates had it that more than 70 percent of Nigerians lived in poverty (Ogwumike, 2001). As a result of this, goals and objectives were set up by government and policymakers. It can be observed that the primary goal of economic planning in Nigeria is the attainment of rapid increase in the nation's productive capacity with a view to improving the standards of living of the people (Obadan, 2002).The reduction of poverty is the most difficult challenge facing any country in the developing world, where on the average majority of the population are poor. In areas where farming is remunerative, those households with adequate land may earn an acceptable

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income. But where farming cannot fully support household needs, non-farm activities become an increasingly attractive target. Thus, the issue of pluriactivity came into existence. Pluriactivity has been identified as a survival and/or capital accumulation strategy which provides avenues to develop both individual and community well-being (Rupena-Osolink, 1983). Evans and Ilbery (1993) defined pluriactivity as the phenomena of farming in conjunction with other gainful activities, whether on or off farm. In addition to income derived from agriculture, rural households have resolved to engage in other activities aside farming. A large and disparate literature, arising from a variety disciplines, has confirmed that rural people in Nigeria do not normally specialize in livestock, crop or fish production to the total exclusion of other income generating activities, rather, a majority of rural producers have historically diversified their productive activities to encompass a range of other productive areas. Pluriactivity minimizes the risk of specialization (Stark and Levhari, 1982) and also acts as a method of reducing income variances while improving household income (Fuller, 1990; Evans and Ilbery, 1993) and their status (Fuller, 1990). It provides access to information, experience and knowledge, which become the basis for moving into other income generation activities. The advantages obtained through pluriactivity also allow rural households to achieve continued reproduction of the farming business (Evans and Ilbery, 1993). In recent times, changes in agriculture have led to a decline in farm-related jobs and an increase in the stock of land and buildings which are no longer required for agricultural purposes. Farm incomes have also fallen by around 60 percent over the past five years and it is therefore increasingly pertinent for farmers to be able to diversify into other activities in order to supplement their incomes and ensure the survival of their farms. In line with the foregoing, this study appraises the other activities which rural households engage in aside farming using Kwara state as a case study area. The specific objectives of the study were to identify the different non-farm activities farmers engage in; determine the reasons why rural households diversify into non farm activities; measure the diversity of activities in the study area; and identify the factors availing rural households the opportunities to diversify into non farm activities.

II. Methodology and Data Collection

This study was carried out in Kwara State, Nigeria. It is located in the agro-ecological zone of the country. With a population of about 2.37 million (Census,2006), the state is made up of four zones. It has about 260,528 farm families (KWADP, 2006) and about 36,820 hectatres of farmland (FOS, 1995). The state lies between latitudes 7°45'N and 9°30'N and longitudes 2°30'E and 6°35'E. The annual rainfall pattern across the state extends between the months of April and October with minimum temperature ranging from 21.1°C to 25°C while maximum average temperature ranges from 30°C to 35°C. The predominant crops grown are groundnut,shorgum, cassava, yam, cowpea, maize, yam and rice (KWADP, 2006). The data for this study were obtained from primary sources with the use of well-structured questionnaires augumented with personal oral interview. The target population for the study was made up of the rural households in the state. A random sampling procedure was used to select 30 households from each of the four zones giving a total of 120 respondents. Out of the 120 interview schedules, 108 were found useful for analysis. The interview schedule was with the assistance of concerned people of the involved towns and communities.

III. Analytical Techniques

Simple descriptive statistics such as percentages, frequency distribution, mean, mode and ratios were used to describe the socio-economic characteristics; to identify the various non-farm activities; and to determine the reasons the respondents diversify. The Simpson's Diversity Index was used to measure the diversity of activities in the study area. The Diversity Index is expressed as:

$$DI = 1 - \sum^{s} (n_i/N)^2$$

Where, $N = \sum n_i$, which is the total population of all individuals across all activities,

s=the number of activities that are present,

n_i (for i=1 to s) is the number of individuals in the ith activity.

Kruskal-Wallis One-Way ANOVA was used to examine the factors availing rural households the opportunities to diversify. This involved assigning priorities to factors

The equation for estimating the ranks is outlined thus:

$$H = \frac{12}{N+1} \sum \frac{1}{ni} \frac{(Ri\text{-}ni\ (N\text{-}1))}{2} \qquad (2)$$

IV. Results and Discussion

The results of the findings shows that both males and females, young and old, engage in farming and non farming activities. However, the number of males is more than that of females (Table 1). The majority (about four-fifth) of the respondents were males while just about one-fifth were females. This may be due to the fact that the male are usually responsible for the upkeep of the family. This could be in the quest to adequately take up their responsibilities.

Age is an important factor among the socio-economic characteristics of rural households as it determines the effectiveness and competence of labor availability for farm and non-farm activities.

Table 1: Socio-economic Characteristics of the Respondents

Characteristics	Frequency	Percentage
Gender		
Male	87	80.6
Female	21	19.4
Total	108	100
Age		
16-25	12	11
26-35	12	11
36-45	28	25.9
46-55	34	31.5
56-65	18	16.9
>65	4	3.7
Total	108	100
Marital Status		
Single	6	5.6
Married	91	84.3
Widowed	7	6.5
Divorced	1	0.9
Widow	3	2.8
Total	108	100
Education level		
No formal education	28	25.9
Quranic	7	6.5
Primary	22	20.4
Secondary	24	22.2
Tertiary	24	22.2
Adult	3	2.8
Total	108	100
Household size		
1-4	53	49.1
5-8	50	46.3
9-12	4	3.7
>12	1	0.9
Total	108	100
Farming experience		
1-10	20	18.5
11-20	42	38.9
21-30	28	25.9
>30	18	16.7
Total	108	100

Source: Field Survey, 2010.

The modal age group of the respondent was 46.55years while the average age was 44years. About 57% of the respondents were between 36 and 55years of age and this represents the majority. 22.1% of the respondents were those below 36years, while 20.6% were those above 55years. The reason for the low number of both young and aged respondents in both farm and non-farm activities may be because the young and the old are dependants and the age range having the highest percentage are those who have the ability to work effectively.

As indicated in Table 1, about 84.3% of the respondents were married, 5.6% are single and about 10.2% are either widow, widower or divorced. The high percentage of married respondents may result from the need for child-bearing in order to have enough labour for the activities.About 74.1% of the respondents had one form of education or the other. This is a reflection of quality of labour. It may be responsible for the high level of innovation on pluriactive activities by the respondents. Only 15.7% did not attend school at all. 20.4% of the respondents had primary education while the modal educational level was 22.2% each for both secondary and tertiary education. A large proportion of the respondents with a family size between 1 and 8 had a percentage of 95.4 and the average family size was 4. This average family size could be the result of the need to bear a few number of children the respondents could adequately cater for. The average number of years of farming experience of the repondents is 23 years. Over 60% of the respondents had been in farming for the past 11 – 30 years. This indicates that the repondents are highly experienced in the cultivation of crops.

V. Non-Farm Activities

Table 2 implies that many of the respondents engaged in trading or civil service in addition to farming. Overall, 65.74% of the respondents engaged in other productive activities besides farming. This indicates that majority of the respondents are pluriactive.

Table 2 : Non-farm activities of the Respondents

Activities	Frequency	Percentage
Trading	26	24.07
Weaving	3	2.78
Grinding	1	0.9
N. Guarding	5	4.62
G. Processing	1	0.9
Chemist	1	0.9
Bricklaying	2	1.9
Civil Service	20	19.0
Drumming	1	0.9
Tailoring	5	4.6
Barbing	2	1.2
Hair Dressing	1	0.9
Carpentry	1	0.9
Pottery	1	0.9
Vulcanizing	1	0.9
Farming Alone	37	34.26
Total	108	100

Source: Field survey, 2010.

Reasons for Diversification

Table 3: Reasons for Diversification

Reason	Frequency	Percentage
Income	23	21.3
Social status	3	2.8
Standard of living	45	63.4
Total	71	87.5

Source: Field Survey, 2010

Table 3 shows that 21.3% of the respondents engaged in non-farming activities in order to increase their income, 2.8% in order to change their social status and 63.4% of the respondents diversify in order to increase their standard of living. 12.5% of the respondents were those that had farming alone as their main occupation.

VI. Diversity Index

Table 4 shows that Trading and Civil Service have higher ratios than other activities. The diversity index increases, that is, diversity of activities increases. This means that the diversity of activities in the study area increases as the number of activities in the study area increases.

Table 4: Diversity Index of Respondents' Activities

Activity	Ratio
Trading	0.4
Weaving	0.04
Grinding	0.01
N. Guarding	0.07
Chemist	0.01
Bricklaying	0.03
Civil Service	0.3
Drumming	0.01
Tailoring	0.07
Barbing	0.03
Hair. Dressing	0.01
Carpentry	0.01
Pottery	0.01
Vulcanizing	0.01

Source: Field Survey, 2010

Table 5 shows the rank of factors according to how they predispose rural households to diversify into non-farm activities. From this, it can be inferred that the need to increase income and the small farm sizes of rural households predisposed the respondents most to engage in non farm activities.

Table 5: Summary of Kruskal-Wallis Test for Prevalence of Factors Predisposing Households to Diversification.

Factors	Mean Rank	Rank
Income	352.50	7
Available Market	280.48	2
Education	297.05	4
Farm Size	336.95	6
Household Size	309.92	5
Age	273.59	1
Marital Status	292.02	3
Chi-Square(X^2)	19.219	
Df	6	
Asymp. Sig.	0.004	

Source: Computer Print-out, 2010.

1-7 Lowest to highest

VII. Conclusion

This study reveals the fact that apart from farming, other activities could be carried out by households which can help increase both income and standard of living of households. Therefore, shifting attention to these activities could assist in the achievement of the goal of poverty reduction in the economy. Based on the findings of the study, it is therefore recommended that rural households should diversify into activities for which they have the certainty that the activities would help in reducing their level of poverty. Policy makers should look for means of improving these activities and make good policies that will promote them without having negative effects on farming. Government and private sectors could also help to provide credit facilities that will help rural households to intensify their engagement in these activities which have the prospects of reducing poverty situation in the economy.

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A Generalization Of Fractional Calculus Involving Ī - Functions On Spaces Fp,u And F 'p,u

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Abstracts : The aim of this paper is to study some properties of two fractional operators defined below involving - functions in space Fp,u and F 'p,u .

Keywords : *Ī -functions, general transform, generalized fractional integration operators.*

GJSFR-F Classification : *MSC: 30C45*

Strictly as per the compliance and regulations of:

A Generalization Of Fractional Calculus Involving $\bar{I}$-Functions On Spaces $F_{p,\mu}$ And $F'_{p,\mu}$

V.B.L. Chaurasia[α], J.C. Arya[Ω]

Abstract : The aim of this paper is to study some properties of two fractional operators defined below involving $\bar{I}$-functions in space $F_{p,\mu}$ and $F'_{p,\mu}$.

$$\left(R^{(e,E),(f,F)}_{(g,G),(h,H)}\,\phi\right)(x) = \frac{1}{x}\int_0^x \bar{I}^{e,f}_{g,h}\left[\frac{t}{x}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,f},(a_j,\alpha_j;1)_{f+1,g}\\(b_j,\beta_j;1)_{1,e},(b_j,\beta_j;B_j)_{e+1,h}\end{matrix}\right.\right]$$

$$\bar{I}^{E,F}_{G,H}\left[\frac{t}{x}\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,F},(a'_j,\alpha'_j;1)_{F+1,G}\\(b'_j,\beta'_j;1)_{1,E},(b'_j,\beta'_j;B'_j)_{E+1,H}\end{matrix}\right.\right]\phi(t)\,dt, \qquad (x>0)$$

$$\left(A^{(e,E),(f,F)}_{(g,G),(h,H)}\,\phi\right)(x) = \frac{1}{x}\int_x^\infty \bar{I}^{e,f}_{g,h}\left[\frac{x}{t}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,f},(a_j,\alpha_j;1)_{f+1,g}\\(b_j,\beta_j;1)_{1,e},(b_j,\beta_j;B_j)_{e+1,h}\end{matrix}\right.\right]$$

$$\bar{I}^{E,F}_{G,H}\left[\frac{x}{t}\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,F},(a'_j,\alpha'_j;1)_{F+1,G}\\(b'_j,\beta'_j;1)_{1,E},(b'_j,\beta'_j;B'_j)_{E+1,H}\end{matrix}\right.\right]\phi(t)\,dt, \qquad (x>0)$$

Keywords : $\bar{I}$ *-functions, general transform, generalized fractional integration operators.*

I. Introduction

Integral transform with Fox's H-function as a kernel were studied by many authors, Shalapakov [4] –[6] and arise special cases of such transforms which generalize classical fractional operators. Transforms having Fox's H-function kernel in L_p $(0,\infty)$ space were considered by Kiryakova [7], [8], Kalla and Kiryakova [2], [3] and in the space $F_{p,\mu}$ and the corresponding space of generalized function $F'_{p,\mu}$ by Raina and Saigo [11], Saigo, Raina and Kilbas [15]. Further, fractional calculus operators with Gauss hypergeometric function $_2F_1$ (a,b;c;z) in $F_{p,\mu}$ and $F'_{p,\mu}$ were studied by Saigo and Glaeske [13], [14].

This paper deals with the fractional integration operators involving $\bar{I}$-function of general kind's composition in space $F_{p,\mu}$ and $F'_{p,\mu}$. The spaces $F_{p,\mu}$ and $F'_{p,\mu}$ developed by McBride [9], [10]. For $1 \le p < \infty$ and $\mu \in C$, $F_{p,\mu}$ the space of functions is defined as

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$$F_{p,\mu} = \left\{ \phi \in C^{\infty}(R_+) : x^k \frac{d^k}{dx^k}(x^{-\mu}\phi) \in L^p(R_+) \text{ for } k \in N_0 \right\} \quad \ldots(1.1)$$

for $1 \le p < \infty$ and

$$F_{\infty,\mu} = \left\{ \phi \in C^{\infty}(R_+) : x^k \frac{d^k}{dx^k}(x^{-\mu}\phi) \to 0 \text{ as } x \to +0 \text{ and } x \to \infty \text{ for } k \in N_0 \right\} \quad \ldots(1.2)$$

for $p = \infty$, where $N_0 = 0,1,2,\ldots$

For each p and $\mu, F_{p,\mu}$, is a complete countable multi-normed space (Frechet space) equipped with the topology generated by the family of semi norms $\{\gamma_k^{p,\mu}\}_{k=0}^{\infty}$ with

$$\gamma_k^{p,\mu}(\phi) = \left\| x^k \frac{d^k}{dx^k}(x^{-\mu}\phi) \right\|_p \quad (k \in N_0) \quad \ldots(1.3)$$

$F'_{p,\mu}$ is the space of continuous linear functionals on $F_{p,\mu}$ equipped with the weak topology. We shall express f and element of $F'_{p,\mu}$ by $< f, \varphi >$ the value of f at a test function $\varphi \in F_{p,\mu}$. For any $f \in F'_{p,\mu}$ we denote x^{σ} f the functional defined by

$$< x^{\sigma} f, \varphi > = < f, x^{\sigma}\varphi >, \quad (\varphi \in F_{p,\mu-\sigma}).$$

We know that the space $F'_{p,\mu}$ is always a complete space.

Now some Lemmas are required to prove our theorems.

Lemma 1. ([9, p.18, Corollary 2.7.]).

The space $C_0^{\infty}(R_+)$ of functions $\varphi \in C^{\infty}(R_+)$ which have compact support is dense in $F_{p,\mu}$ for $1 \le p < \infty$ and $\mu \in C$.

Lemma 2. ([9, p.21, Corollary 2.11.]).

The operator x^{σ} with $\sigma \in C$ defined by

$$(x^{\sigma}\varphi) = x^{\sigma}\varphi(x) \quad \ldots(1.4)$$

is a homeomorphism of $F_{p,\mu}$ onto $F_{p,\mu+\sigma}$ with the inverse $x^{-\sigma}$.

Lemma 3. ([10, p.533, Lemma 6.1.]).

Let the function k(x) be defined almost everywhere on R_+ and

$$\int_0^{\infty} x^{\frac{1}{p} - Re(\mu) - 1} |k(x)| dx < \infty. \quad \ldots(1.5)$$

If T is an integral transform defined by

$$(T\varphi)(x) = (k * \varphi)(x) = \int_0^\infty k\left(\frac{x}{t}\right)\frac{1}{t}\varphi(t)\,dt \qquad \ldots(1.6)$$

then T is a continuous linear mapping from $F_{p,\mu}$ into itself.

Lemma 4. ([9,p.32, Theorem 2.22.]).

For $\sigma \in C$ the operator x^σ is a homeomorphism of $F'_{p,\mu}$ onto $F'_{p,\mu-\sigma}$.

The $\bar{I}$ -function ([12]) is defined and represented by the following Mellin-Barnes type contour integral

$$\bar{I}(z) = \bar{I}^{e,f}_{g,h}\left[z \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,f}, (a_j,\alpha_j;1)_{f+1,g} \\ (b_j,\beta_j;1)_{1,e}, (b_j,\beta_j;B_j)_{e+1,h} \end{matrix} \right.\right]$$

$$= \frac{1}{2\pi i}\int_L \bar{\varphi}(s)\, z^s\, ds, \qquad \ldots(1.7)$$

Where

$$\bar{\varphi}(s) = \frac{\prod_{j=1}^{e}\{\Gamma(b_j - \beta_j s)\}^{B_j} \prod_{j=1}^{f}\{\Gamma(1-a_j+\alpha_j s)\}^{A_j}}{\prod_{j=e+1}^{h}\{\Gamma(1-b_j+\beta_j s)\}^{B_j} \prod_{j=f+1}^{g}\{\Gamma(a_j-\alpha_j s)\}^{A_j}} \qquad \ldots(1.8)$$

Here $z \neq 0$, e,f,g,h are integers satisfying $0 \le e \le h, 0 \le f \le g$; L is a suitable contour in the complex plane. An empty product is to be interpreted as unity. Also $\alpha_j, j=1,\ldots,g; \beta_j, j=1,\ldots,h; A_j, j=1,\ldots,g;$ and $B_j, j=1,\ldots,h$ are positive numbers and $a_j, j=1,\ldots,g$ and $b_j, j=1,\ldots,h$ are complex numbers such that no singularity of $\{\Gamma(b_j-\beta_j s)\}^{B_j}, j=1,\ldots,e$ coincides with any singularity of $\{\Gamma(1-a_j-\alpha_j s)\}^{A_j}$, $j = 1,\ldots,f$.

The series form of $\bar{I}(z)$ is given by

$$\bar{I}(z) = \sum_{r=0}^{\infty}\sum_{k=1}^{e} \frac{\prod_{j=1}^{f}\{\Gamma(1-a_j+\alpha_j\xi_{k,r})\}^{A_j} \prod_{j=1, j\neq k}^{e}\{\Gamma(b_j-\beta_j\xi_{k,r})\}^{B_j}(-1)^r z^{\xi_{k,r}}}{\prod_{j=f+1}^{g}\{\Gamma(a_j-\alpha_j\xi_{k,r})\}^{A_j} \prod_{j=e+1}^{h}\{\Gamma(1-b_j+\beta_j\xi_{k,r})\}^{B_j} r!\beta_k} \qquad \ldots(1.9)$$

$$\text{for } |z| < 1$$

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Where

$$\xi_{k,r} = \frac{b_k + r}{\beta_k}$$

From (1.9), it is clear that

$$\bar{I}(z) \sim z^{\theta}, \text{ Where } \theta = \min_{1 \le j \le e}\left[\operatorname{Re}\left(\frac{b_j}{\beta_j}\right)\right] \text{ for small values of z.} \quad \ldots(1.10)$$

We will mention some properties of $\bar{I}$ -function

$$z^{\sigma}\,\bar{I}^{e,f}_{g,h}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,f},(a_j,\alpha_j;1)_{f+1,g}\\(b_j,\beta_j;1)_{1,e},(b_j,\beta_j;B_j)_{e+1,h}\end{matrix}\right.\right]$$

$$= \bar{I}^{e,f}_{g,h}\left[z\left|\begin{matrix}(a_j+\sigma\alpha_j,\alpha_j;A_j)_{1,f},(a_j+\sigma\alpha_j,\alpha_j;1)_{f+1,g}\\(b_j+\sigma\beta_j,\beta_j;1)_{1,e},(b_j+\sigma\beta_j,\beta_j;B_j)_{e+1,h}\end{matrix}\right.\right], \quad \ldots(1.11)$$

$$\left(M\,\bar{I}^{e,f}_{g,h}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,f},(a_j,\alpha_j;1)_{f+1,g}\\(b_j,\beta_j;1)_{1,e},(b_j,\beta_j;B_j)_{e+1,h}\end{matrix}\right.\right]\right)(s) = \overline{\varphi}(s), \quad \ldots(1.12)$$

$$\left(\min_{1 \le i \le f}\left[\frac{\operatorname{Re}(1-a_i)}{\alpha_i}\right]\right) < \operatorname{Re}(s) < \left(\min_{1 \le j \le f}\left[\frac{\operatorname{Re}(b_j)}{\beta_j}\right]\right)$$

where $\overline{\varphi}(s)$ is given by (1.8)and M is the Mellin transform

$$(M\,\psi)(s) = \int_0^{\infty} \psi(x)\,x^{s-1}\,dx.$$

II. MAIN RESULTS

We write $p' = \frac{p}{p-1}$ for $1 \le p < \infty$

Let θ be defined by (1.10). Then

Theorem 1.A. If $\operatorname{Re}(\mu) > -\theta - \theta' - \frac{1}{p'}$, then the operator $R^{(e,E),(f,F)}_{(g,G),(h,H)}$ is a continuous linear mapping from $F_{p,\mu}$ into itself, and for $\varphi \in F_{p,\mu}$ and $\sigma \in C$ there fold the relation

$$\left(x^{\sigma} R_{(g,G),(h,H)}^{(e,E),(f,F)} \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,f},(a_j,\alpha_j;1)_{f+1,g} \\ (b_j,\beta_j;1)_{1,e},(b_j,\beta_j;B_j)_{e+1,h} \end{matrix} \right] \left[\begin{matrix} (a'_j,\alpha'_j;A'_j)_{1,F},(a'_j,\alpha'_j;1)_{F+1,G} \\ (b'_j,\beta'_j;1)_{1,E},(b'_j,\beta'_j;B'_j)_{E+1,H} \end{matrix} \right] \varphi \right)(x)$$

$$= \left(R_{(g,G),(h,H)}^{(e,E),(f,F)} \left[\begin{matrix} \left(a_j-\frac{\sigma}{2}\alpha_j,\alpha_j;A_j\right)_{1,f},\left(a_j-\frac{\sigma}{2}\alpha_j,\alpha_j;1\right)_{f+1,g} \\ \left(b_j-\frac{\sigma}{2}\beta_j,\beta_j;1\right)_{1,e},\left(b_j-\frac{\sigma}{2}\beta_j,\beta_j;B_j\right)_{e+1,h} \end{matrix} \right] \right.$$

$$\left. \cdot \left[\begin{matrix} \left(a'_j-\frac{\sigma}{2}\alpha'_j,\alpha'_j;A'_j\right)_{1,F},\left(a'_j-\frac{\sigma}{2}\alpha'_j,\alpha'_j;1\right)_{F+1,G} \\ \left(b'_j-\frac{\sigma}{2}\beta'_j,\beta'_j;1\right)_{1,E},\left(b'_j-\frac{\sigma}{2}\beta'_j,\beta'_j;B'_j\right)_{E+1,H} \end{matrix} \right] t^{\sigma}\varphi \right)(x). \qquad \ldots(2.1)$$

Theorem 1.B

If $\mathrm{Re}(\mu)<\theta+\theta'-\frac{1}{p'}$, then the operator $A_{(g,G),(h,H)}^{(e,E),(f,F)}$ is a continuous linear mapping from $F_{p,\mu}$ into itself, and for $\varphi\in F_{p,\mu}$ and $\sigma\in C$ there hold the relation

$$\left(x^{\sigma} A_{(g,G),(h,H)}^{(e,E),(f,F)} \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,f},(a_j,\alpha_j;1)_{f+1,g} \\ (b_j,\beta_j;1)_{1,e},(b_j,\beta_j;B_j)_{e+1,h} \end{matrix} \right] \left[\begin{matrix} (a'_j,\alpha'_j;A'_j)_{1,F},(a'_j,\alpha'_j;1)_{F+1,G} \\ (b'_j,\beta'_j;1)_{1,E},(b'_j,\beta'_j;B'_j)_{E+1,H} \end{matrix} \right] \varphi \right)(x)$$

$$= \left(A_{(g,G),(h,H)}^{(e,E),(f,F)} \left[\begin{matrix} \left(a_j+\frac{\sigma}{2}\alpha_j,\alpha_j;A_j\right)_{1,f},\left(a_j+\frac{\sigma}{2}\alpha_j,\alpha_j;1\right)_{f+1,g} \\ \left(b_j+\frac{\sigma}{2}\beta_j,\beta_j;1\right)_{1,e},\left(b_j+\frac{\sigma}{2}\beta_j,\beta_j;B_j\right)_{e+1,h} \end{matrix} \right] \right.$$

$$\left. \cdot \left[\begin{matrix} \left(a'_j+\frac{\sigma}{2}\alpha'_j,\alpha'_j;A'_j\right)_{1,F},\left(a'_j+\frac{\sigma}{2}\alpha'_j,\alpha'_j;1\right)_{F+1,G} \\ \left(b'_j+\frac{\sigma}{2}\beta'_j,\beta'_j;1\right)_{1,E},\left(b'_j+\frac{\sigma}{2}\beta'_j,\beta'_j;B'_j\right)_{E+1,H} \end{matrix} \right] t^{\sigma}\varphi \right)(x).. \qquad \ldots(2.2)$$

Proof : To prove the statement we will use the Lemma 3. Here the kernel of $R_{(g,G),(h,H)}^{(e,E),)f,F)}$ is

$$K(v)=\begin{cases} 0\,, & \text{if } 0<v<1, \\ \frac{1}{v}\bar{I}_{g,h}^{e,f}\left(\frac{1}{v}\right)\bar{I}_{G,H}^{E,F}\left(\frac{1}{v}\right), & \text{if } v\geq 1. \end{cases}$$

and

$$\therefore \int_0^{\infty} v^{\frac{1}{p}-\mathrm{Re}(\mu)-1}\,|K(v)|\,dv=\int_1^{\infty} v^{\frac{1}{p}-\mathrm{Re}(\mu)-2}\left|\bar{I}_{g,h}^{e,f}\left(\frac{1}{v}\right)\bar{I}_{G,H}^{E,F}\left(\frac{1}{v}\right)\right|dv.$$

Now let $\frac{1}{v}=x$ so that $-\frac{1}{v^2}dv=dx$ so we get

$$=\int_0^1 x^{Re(\mu)-\frac{1}{p}}\left|\bar{I}_{g,h}^{e,f}(x)\,\bar{I}_{G,H}^{E,F}(x)\right|dx.$$

Here integral is finite always in (0,1] but for x = 0, by checking the asymptotic behavior of the integral,

$$x^{Re(\mu)-\frac{1}{p}}\left|\bar{I}_{g,h}^{e,f}(x)\,\bar{I}_{G,H}^{E,F}(x)\right|\sim x^{Re(\mu)+\theta+\theta'-\frac{1}{p}}$$

therefore, we get

$$\int_0^1 x^{Re(\mu)-\frac{1}{p}}\left|\bar{I}_{g,h}^{e,f}(x)\,\bar{I}_{G,H}^{E,F}(x)\right|dx=\int_0^1 x^{Re(\mu)+\theta+\theta'-\frac{1}{p}}dx$$

and since the integral $\int_0^1 x^{n-1}dx$ exists iff n > 0

$\therefore$ we get $$Re(\mu)+\theta+\theta'-\frac{1}{p}+1>0$$

i.e. $$Re(\mu)>-\theta-\theta'+\frac{1}{p}-1$$

or $$Re(\mu)>-\theta-\theta'-\frac{1}{p'}\ \text{where}\ p'=\frac{p}{p-1}$$

and we observe that (2.3) will converge iff

$$Re(\mu)>-\theta-\theta'-\frac{1}{p'}.$$

Now to prove the equality of (2.1), we take left hand side of (2.1)

$$\left(x^{\sigma}R_{(g,G),(h,H)}^{(e,E),(f,F)}\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,f},(a_j,\alpha_j;1)_{f+1,g}\\(b_j,\beta_j;1)_{1,e},(b_j,\beta_j;B_j)_{e+1,h}\end{matrix}\right]\left[\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,F},(a'_j,\alpha'_j;1)_{F+1,G}\\(b'_j,\beta'_j;1)_{1,E},(b'_j,\beta'_j;B'_j)_{E+1,H}\end{matrix}\right]\varphi\right)(x)$$

$$=\frac{1}{x}\int_0^x x^{\sigma}\,\bar{I}_{g,h}^{e,f}\left[\frac{t}{x}\right]\bar{I}_{G,H}^{E,F}\left[\frac{t}{x}\right]\varphi(t)\,dt$$

$$=\frac{1}{x}\int_0^x\left(\frac{t}{x}\right)^{-\frac{\sigma}{2}}\bar{I}_{g,h}^{e,f}\left[\frac{t}{x}\right]\left(\frac{t}{x}\right)^{-\frac{\sigma}{2}}\bar{I}_{G,H}^{E,F}\left[\frac{t}{x}\right]t^{\sigma}\,\varphi(t)\,dt$$

(multiplying and dividing by t^{σ})

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Now by using property of $\bar{I}$-function, we get

$$= \frac{1}{x}\int_0^x \bar{I}^{e,f}_{g,h}\left[\frac{t}{x}\left|\begin{matrix}\left(a_j-\frac{\sigma}{2}\alpha_j,\alpha_j;A_j\right)_{1,f},\left(a_j-\frac{\sigma}{2}\alpha_j,\alpha_j;1\right)_{f+1,g}\\ \left(b_j-\frac{\sigma}{2}\beta_j,\beta_j;1\right)_{1,e},\left(b_j-\frac{\sigma}{2}\beta_j,\beta_j;B_j\right)_{e+1,h}\end{matrix}\right.\right]$$

$$\cdot\left[\begin{matrix}\left(a'_j-\frac{\sigma}{2}\alpha'_j,\alpha'_j;A'_j\right)_{1,F},\left(a'_j-\frac{\sigma}{2}\alpha'_j,\alpha'_j;1\right)_{F+1,G}\\ \left(b'_j-\frac{\sigma}{2}\beta'_j,\beta'_j;1\right)_{1,E},\left(b'_j-\frac{\sigma}{2}\beta'_j,\beta'_j;B'_j\right)_{E+1,H}\end{matrix}\right] t^{\sigma}\,\varphi(t)\,dt$$

which gives the right hand side of (2.1).

Similarly we can prove Theorem 1.B.

Equation (2.1) and (2.2) are defined when $\mathrm{Re}(\mu)>-\theta-\theta'-\frac{1}{p'}$ and $\mathrm{Re}(\mu)<-\theta-\theta'-\frac{1}{p'}$ respectively.

Theorem 2.A. Let $\varphi\in F_{p,\mu},\mathrm{Re}(\mu)>-\theta-\theta'-\frac{1}{p'}$ and

$$-\min_{1\le j\le e}\left[\frac{\mathrm{Re}(b_j)}{\beta_j}\right]<\mathrm{Re}(s)-1<\min_{1\le i\le e}\left[\frac{1-\mathrm{Re}(a_i)}{\alpha_i}\right],$$

$$-\min_{1\le j\le E}\left[\frac{\mathrm{Re}(b'_j)}{\beta'_j}\right]<\mathrm{Re}(s)-1<\min_{1\le i\le E}\left[\frac{1-\mathrm{Re}(a'_i)}{\alpha'_i}\right]$$

$$\Bigg(M\ R^{(e,E),(f,F)}_{(g,G),(h,H)}\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,f},(a_j,\alpha_j;1)_{f+1,g}\\ (b_j,\beta_j;1)_{1,e},(b_j,\beta_j;B_j)_{e+1,h}\end{matrix}\right]$$

$$\cdot\left[\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,F},(a'_j,\alpha'_j;1)_{F+1,G}\\ (b'_j,\beta'_j;1)_{1,E},(b'_j,\beta'_j;B'_j)_{E+1,H}\end{matrix}\right]\varphi\Bigg)(s)$$

$$=\sum_{r=0}^{\infty}\sum_{k=1}^{e}\frac{\prod_{\substack{j=1\\ j\ne k}}^{E}\{\Gamma(b'_j-\beta'_j\xi'_{k,r})\}^{B'_j}\prod_{j=1}^{e}\{\Gamma(b_j-\beta_j+\beta_j s)-\beta_j\xi_{k,r})\}^{B_j}}{\prod_{j=E+1}^{H}\{\Gamma(1-b'_j+\beta'_j\xi'_{k,r})\}^{B'_j}\prod_{j=e+1}^{h}\{\Gamma[1-(b_j-\beta_j+\beta_j s)+\beta_j\xi_{k,r}]\}^{B_j}}$$

$$\frac{\prod_{j=1}^{F}\{\Gamma(1-a'_j+\alpha'_j\xi'_{k,r})\}^{A'_j}\prod_{j=1}^{f}\{\Gamma[1-(a_j-\alpha_j+\alpha_j s)+\alpha_j\xi_{k,r}]\}^{A_j}(-1)^r}{\prod_{j=F+1}^{G}\{\Gamma(a'_j-\alpha'_j\xi'_{k,r})\}^{A'_j}\prod_{j=f+1}^{g}\{\Gamma[(a_j-\alpha_j+\alpha_j s)-\alpha_j\xi_{k,r}]\}^{A_j}\, r!\beta'_k}(M\varphi)(s).$$

…(2.4)

Theorem 2.B. Let $\varphi\in F_{p,\mu}, \mathrm{Re}(\mu)<\theta+\theta'-\frac{1}{p'}$ and

$$-\min_{1\le i\le e}\left[\frac{1-\mathrm{Re}(a_i)}{\alpha_i}\right]<\mathrm{Re}(s)-1<\min_{1\le j\le e}\left[\frac{\mathrm{Re}(b_j)}{\beta_j}\right],$$

$$-\min_{1\le i\le E}\left[\frac{1-\mathrm{Re}(a'_i)}{\alpha'_i}\right]<\mathrm{Re}(s)-1<\min_{1\le j\le E}\left[\frac{\mathrm{Re}(b'_j)}{\beta'_j}\right]$$

$$\Bigg(M\ A^{(e,E),(f,F)}_{(g,G),(h,H)}\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,f},(a_j,\alpha_j;1)_{f+1,g}\\ (b_j,\beta_j;1)_{1,e},(b_j,\beta_j;B_j)_{e+1,h}\end{matrix}\right.$$

$$\cdot\left.\left[\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,F},(a'_j,\alpha'_j;1)_{F+1,G}\\ (b'_j,\beta'_j;1)_{1,E},(b'_j,\beta'_j;B'_j)_{E+1,H}\end{matrix}\right]\varphi\Bigg)(s)$$

$$=\sum_{r=0}^{\infty}\sum_{k=1}^{E}\frac{\prod_{j=1,j\neq k}^{E}\{\Gamma(b'_j-\beta'_j\xi'_{k,r})\}^{B'_j}\prod_{j=1}^{e}\{\Gamma[(b_j+\beta_j-\beta_j s)-\beta_j\xi_{k,r})]\}^{B_j}}{\prod_{j=E+1}^{H}\{\Gamma(1-b'_j+\beta'_j\xi'_{k,r})\}^{B'_j}\prod_{j=e+1}^{h}\{\Gamma[(1-(b_j+\beta_j-\beta_j s)+\beta_j\xi_{k,r}]\}^{B_j}}\ .$$

$$\frac{\prod_{j=1}^{F}\{\Gamma(1-a'_j+\alpha'_j\xi'_{k,r})\}^{A'_j}\prod_{j=1}^{f}\{\Gamma[1-(a_j+\alpha_j-\alpha_j s)+\alpha_j\xi_{k,r}]\}^{A_j}(-1)^r}{\prod_{j=F+1}^{G}\{\Gamma(a'_j-\alpha'_j\xi'_{k,r})\}^{A'_j}\prod_{j=f+1}^{g}\{\Gamma(a_j+\alpha_j-\alpha_j s)-\alpha_j\xi_{k,r}]\}^{A_j}\, r!\beta'_k}(M\varphi)(s).$$

…(2.5)

Proof. If $\varphi \in C_0^{\infty}(R_+)$, to prove (2.4) and (2.5), we will use Mellin convolution relation

$$(M(\tau_1 * \tau_2))(s) = (M\tau_1)(s)\,(M\tau_2)(s),$$

and the properties of Mellin transform

$$(M\,x^{\lambda}\,\tau)(s) = (M\tau)(s+\lambda),\ \ (\lambda \in C)$$

$$(M\phi)(x^{-1})(s) = (M\phi)(-s)$$

Now left hand side of (2.4)

$$\left(M\,R^{(e,E),(f,F)}_{(g,G),(h,H)}\,\varphi\right)(s) = \left(M\left\{\frac{1}{x}\int_0^x \bar{I}^{e,f}_{g,h}\left[\frac{t}{x}\right]\bar{I}^{E,F}_{G,H}\left[\frac{t}{x}\right]\varphi(t)\,dt\right\}(x)\right)(s)$$

$$= \left(M\left\{\left(\frac{1}{x}\int_0^x \bar{I}^{e,f}_{g,h}\left[\frac{t}{x}\right]\right)\left(\sum_{r=0}^{\infty}\sum_{k=1}^{E}\bar{\eta}^{E,F}_{G,H}\left(\frac{t}{x}\right)^{\xi_{k,r}}\right)\varphi(t)\,dt\right\}(x)\right)(s)$$

where

$$\bar{\eta}^{E,F}_{G,H} = \frac{\prod_{\substack{j=1\\ j\neq k}}^{E}\{\Gamma(b'_j-\beta'_j\xi'_{k,r})\}^{B'_j}\prod_{j=1}^{F}\{\Gamma(1-a'_j+\alpha'_j\xi'_{k,r})\}^{A'_j}(-1)^r}{\prod_{j=E+1}^{H}\{\Gamma(1-b'_j+\beta'_j\xi'_{k,r})\}^{B'_j}\prod_{j=F+1}^{G}\{\Gamma(a'_j-\alpha'_j\xi'_{k,r})\}^{A'_j}\,r!\,\beta'_k}$$

and $\quad \xi'_{k,r} = \dfrac{b'_k + r}{\beta'_k}$

$$\therefore = \sum_{r=0}^{\infty}\sum_{k=1}^{E}\bar{\eta}^{E,F}_{G,H}\left(M\left\{\int_0^x \frac{1}{x}\left(\frac{t}{x}\right)^{\xi_{k,r}}\bar{I}^{e,f}_{g,h}\left[\frac{t}{x}\right]\varphi(t)\,dt\right\}(x)\right)(s)$$

$$= \sum_{r=0}^{\infty}\sum_{k=1}^{E}\bar{\eta}^{E,F}_{G,H}\left(M\left\{\int_0^x \left(\frac{t}{x}\right)^{\xi_{k,r}+1}\bar{I}^{e,f}_{g,h}\left[\frac{t}{x}\right]\frac{\varphi(t)}{t}\,dt\right\}(x)\right)(s)$$

$$= \sum_{r=0}^{\infty}\sum_{k=1}^{E}\bar{\eta}^{E,F}_{G,H}\left(M\left\{\left(\frac{1}{x}\right)^{\xi_{k,r}+1}\bar{I}^{e,f}_{g,h}\left[\frac{1}{x}\right]\right\}\right)(s)\ \ (M\varphi)(s)$$

By using Mellin transform properties and Lemma 3,we get

$$=\sum_{r=0}^{\infty}\sum_{k=1}^{E}\bar{\eta}_{G,H}^{E,F}\left(M\left\{\bar{I}_{g,h}^{e,f}[x]\right\}\right)(\xi_{k,r}+1-s)\,(M\varphi)(s)$$

$$=\sum_{r=0}^{\infty}\sum_{k=1}^{E}\bar{\eta}_{G,H}^{E,F}\frac{\prod_{j=1}^{e}\{\Gamma[b_j-\beta_j(\xi_{k,r}+1-s)]\}^{B_j}\prod_{j=1}^{f}\{\Gamma[1-a_j+\alpha_j(\xi_{k,r}+1-s)]\}^{A_j}}{\prod_{j=e+1}^{h}\{\Gamma[1-b_j+\beta_j(\xi_{k,r}+1-s)]\}^{B_j}\prod_{j=f+1}^{g}\{\Gamma[a_j-\alpha_j(\xi_{h,r}+1-s)]\}^{A_j}}$$

$$.\ (M\varphi)(s).$$

Now by putting the value of $\bar{\eta}_{G,H}^{E,F}$ we get right hand side of (2.4). The relation (2.5) is proved similarly. Relation (2.4) and (2.5) hold for $\varphi\in F_{p,\mu}$ by Lemma 1.

Theorem 3. For $\rho\in F_{p,\mu},\eta\in F'_{p,\mu}$ and $,\mathrm{Re}(\mu)<\theta+\theta'-\frac{1}{p'}$ then there holds the formula of integration by parts,

$$\int_0^{\infty}\left(\frac{1}{x}R_{(g,G),\,(h,H)}^{(e,E),(f,F)}\eta\right)(x)\,\rho(x)\,dx$$

$$=\int_0^{\infty}\left(\frac{1}{x}A_{(g,G),(h,H)}^{(e,E),(f,F)}\left[\begin{matrix}(a_j+\alpha_j,\alpha_j;A_j)_{1,f},(a_j+\alpha_j,\alpha_j;1)_{f+1,g}\\(b_j+\beta_j,\beta_j;1)_{1,e},(b_j+\beta_j,\beta_j;B_j)_{e+1,h}\end{matrix}\right]\right.$$

$$\left.\cdot\left[\begin{matrix}(a'_j+\alpha'_j,\alpha'_j;A'_j)_{1,F},(a'_j+\alpha'_j,\alpha'_j;1)_{F+1,G}\\(b'_j+\beta'_j,\beta'_j;1)_{1,E},(b'_j+\beta'_j,\beta'_j;B'_j)_{E+1,H}\end{matrix}\right]\rho\right)(x)\,\eta(x)\,dx. \qquad \ldots(2.6)$$

Proof. Taking left hand side

$$\int_0^{\infty}\left(\frac{1}{x}R_{(g,G),\,(h,H)}^{(e,E),(f,F)}\eta\right)(x)\,\rho(x)\,dx$$

$$=\int_0^{\infty}\frac{1}{x}\left\{\frac{1}{x}\int_0^{x}\bar{I}_{g,h}^{e,f}\left[\frac{t}{x}\right]\bar{I}_{G,H}^{E,F}\left[\frac{t}{x}\right]\eta(t)\,dt\right\}\rho(x)\,dx$$

$$=\int_0^{\infty}\int_0^{x}\bar{I}_{g,h}^{e,f}\left[\frac{t}{x}\right]\bar{I}_{G,H}^{E,F}\left[\frac{t}{x}\right]\eta(t)\frac{\rho(x)}{x^2}dt\,dx$$

Here strip of integration is parallel to t-axis when we convert order of integration we get new strip parallel to x-axis and therefore the above integral converts to

$$= \int_0^{\infty} \int_t^{\infty} \bar{I}_{g,h}^{e,f}\left[\frac{t}{x}\right] \bar{I}_{G,H}^{E,F}\left[\frac{t}{x}\right] \eta(t) \frac{\rho(x)}{x^2} dx\, dt$$

multiplying and dividing with t^2, we get

$$= \int_0^{\infty} \frac{\eta(t)}{t^2} \left\{ \int_t^{\infty} \frac{t}{x} \bar{I}_{g,h}^{e,f}\left[\frac{t}{x}\right] \frac{t}{x} \bar{I}_{G,H}^{E,F}\left[\frac{t}{x}\right] \rho(x)\, dx \right\} dt$$

interchanging x and t mutually, we obtain

$$= \int_0^{\infty} \frac{\eta(x)}{x} \left\{ \frac{1}{x} \int_x^{\infty} \bar{I}_{g,h}^{e,f}\left[\frac{x}{t}\middle| \begin{matrix} (a_j+\alpha_j,\alpha_j;A_j)_{1,f},(a_j+\alpha_j,\alpha_j;1)_{f+1,g} \\ (b_j+\beta_j,\beta_j;1)_{1,e},(b_j+\beta_j,\beta_j;B_j)_{e+1,h} \end{matrix}\right]\right.$$

$$\left. \cdot\, \bar{I}_{G,H}^{E,F}\left[\frac{x}{t}\middle| \begin{matrix} (a'_j+\alpha'_j,\alpha'_j;A'_j)_{1,F},(a'_j+\alpha'_j,\alpha'_j;1)_{F+1,G} \\ (b'_j+\beta'_j,\beta'_j;1)_{1,E},(b'_j+\beta'_j,\beta'_j;B'_j)_{E+1,H} \end{matrix}\right] \rho(t)\, dt \right\} dx$$

$$= \int_0^{\infty} \left(\frac{1}{x} A_{(g,G),(h,H)}^{(e,E),(f,F)} \left[\begin{matrix} (a_j+\alpha_j,\alpha_j;A_j)_{1,f},(a_j+\alpha_j,\alpha_j;1)_{f+1,g} \\ (b_j+\beta_j,\beta_j;1)_{1,e},(b_j+\beta_j,\beta_j;B_j)_{e+1,h} \end{matrix} \right] \right.$$

$$\left. \cdot \left[\begin{matrix} (a'_j+\alpha'_j,\alpha'_j;A'_j)_{1,F},(a'_j+\alpha'_j,\alpha'_j;1)_{F+1,G} \\ (b'_j+\beta'_j,\beta'_j;1)_{1,E},(b'_j+\beta'_j,\beta'_j;B'_j)_{E+1,H} \end{matrix} \right] \rho \right)(x)\, \eta(x)\, dx.$$

In order to show that (2.6) is true for $\rho \in F_{p,\mu}$ and $\eta \in F_{p',-\mu}$, it is sufficient to prove that both sides of (2.6) denotes bounded bilinear functions on $L_{\mu-1}^{p} \times L_{1-\mu}^{p'}$, where $L_{\mu}^{p} = \left\{ \rho : x^{-\mu}\ \rho(x) \in L_p(R_+) \right\}$ with the defined norm

$$\| \rho \|_{p,\mu} = \left(\int_0^{\infty} |x^{-\mu} \rho(x)|^p\, dx \right)^{\frac{1}{p}} \qquad \text{(see [9], [10])}$$

From Holder's inequality and Theorem 1.A, we have

$$\left| \int_0^{\infty} \left(\frac{1}{x} R_{(g,G),(h,H)}^{(e,E),(f,F)}\, \eta \right)(x)\, \rho(x)\, dx \right|$$

$$= \left| \int_0^{\infty} x^{\mu-1} (R_{(g,G),(h,H)}^{(e,E),(f,F)}\, \eta)(x)\, (x^{1-\mu} \rho)(x)\, dx \right|$$

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$$\leq \left\| R_{(g,G),(h,H)}^{(e,E),(f,F)} \eta \right\|_{p',1-\mu} \| \rho \|_{p,\mu-1}$$

$$\leq K \| \eta \|_{p',1-\mu} \| \rho \|_{p,\mu-1}$$

With K being a positive constant. Hence the left hand side of (2.6) denotes a bounded linear function on $\sigma_{\mu-1}^{p} \times \sigma_{1-\mu}^{p'}$, as similarly, does the right hand side of (2.6). Thus the theorem is proved.

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Bivariate Laplace Transform Involving Certain Product of Special Functions

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Bivariate Laplace Transform Involving Certain Product of Special Functions

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Abstract : The object of this paper is to obtain new operational relations between originals and the image for the two dimensional Laplace transforms that involve I-function and the multivariable H-function. The result established in this paper are of general nature and hence encompass several cases of interests.

I. Introduction

The integral equation

$$\phi(p,q) = pq \int_0^{\infty} \int_0^{\infty} e^{-px-qy} f(x,y)\, dx\, dy, \operatorname{Re}(p) > 0, \operatorname{Re}(q) > 0 \qquad \ldots(1.1)$$

represents the classical Laplace transform in two variables and $\phi(p,q)$ and $f(x,y)$ related by (1.1) are said to be operationally related to each other $\phi(p,q)$ is called the image and $f(x,y)$ the original. Symbolically we can write

$$\phi(p,q) \quad f(x,y) \text{ or } f(x,y) \div \phi(p,q) \qquad \ldots(1.2)$$

and the symbol is called operational.

The multivariable H-function has been introduced by Srivastava and Panda [10] and is defined and represented in the following manner (see also [11, p.251]):

$$H^{0,u:(M',N');\ldots;(M^r,N^r)}_{v,w:(P',Q');\ldots;(P^r,Q^r)}\left[\begin{matrix}[(a):A',\ldots,A^r]:[b',B'];\ldots;[b^r:B^r];\\ [(c):C',\ldots,C^r]\ :[d',D'];\ldots;[d^r:D^r];\end{matrix}\ z_1,\ldots,z_r\right]$$

$$= (2\pi\ )^{-r} \int_{L_1} \int_{L_r} U_1(s_1)\ldots U_r(s_r) V(s_1,\ldots,s_r) z_1^{s_1}\ldots z_r^{s_r}\, ds_1\ldots ds_r, i = \sqrt{-1}. \qquad \ldots(1.3)$$

where

$$U_i(s_i) = \prod_{j=1}^{M^{(i)}} \Gamma(d_j^{(i)} - D_j^{(i)} s_i) \prod_{j=1}^{N^{(i)}} \Gamma(1 - b_j^{(i)} + B_j^{(i)} s_i)$$

$$.\left\{ \prod_{j=M^{(i)}+1}^{Q^{(i)}} \Gamma(1 - d_j^{(i)} + D_j^{(i)} s_i) \prod_{j=N^{(i)}+1}^{P^{(i)}} \Gamma(b_j^{(i)} - B_j^{(i)} s_i) \right\}^{-1}. \qquad \ldots(1.4)$$

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$$V(s_1,\dots,s_r)=\prod_{j=1}^{u}\Gamma\left(1-a_j+\sum_{i=1}^{r}A_j^{(i)}s_i\right)$$

$$\cdot\left\{\prod_{j=u+1}^{v}\Gamma\left(a_j-\sum_{i=1}^{r}A_j^{(i)}s_i\right)\prod_{j=1}^{w}\Gamma\left(1-c_j+\sum_{i=1}^{r}C_j^{(i)}s_i\right)\right\}^{-1}. \qquad \dots(1.5)$$

The multiple integral in (1.3) converges absolutely if $|\arg(z_i)|<\Delta_i\left(\frac{\pi}{2}\right), i=1,\dots,r.$

when

$$\Delta_i=-\sum_{j=u+1}^{v}A_j^{(i)}+\sum_{j=1}^{N^{(i)}}B_j^{(i)}-\sum_{j=N^{(i)}+1}^{P^{(i)}}B_j^{(i)}-\sum_{j=1}^{w}C_j^{(i)}+\sum_{j=1}^{M^{(i)}}D_j^{(i)}-\sum_{j=M^{(i)}+1}^{Q^{(i)}}D_j^{(i)}>0,(i=1,\dots,r), \qquad \dots(1.6)$$

For other details of this function see [11]. The series representation of I-function is as follows ([8], p.305-306):

$$I_{u,d}^{m,n}=\bar{I}(z)=\sum_{r'=0}^{\infty}\sum_{h=1}^{m}\frac{\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi_{h,r'})\}^{A_j}}{\prod_{j=n+1}^{u}\{\Gamma(a_j-\alpha_j\xi_{h,r'})\}^{A_j}}$$

$$\cdot\frac{\prod_{j=1, j\neq h}^{m}\{\Gamma(b_j-\beta_j\xi_{h,r'})\}^{B_j}(-1)^{r'}z^{\xi_{h,r'}}}{\prod_{j=m+1}^{d}\{\Gamma(1-b_j+\beta_j\xi_{h,r'})\}^{B_j}r'!\beta_h} \quad \text{for} |z|<1, \qquad \dots(1.7)$$

where

$$\xi_{h,r'}=\frac{b_h+r'}{\beta_h}. \qquad \dots(1.8)$$

For the sake of brevity,

$$\Delta=\sum_{j=1}^{m}B_j\beta_j-\sum_{j=m+1}^{d}B_j\beta_j+\sum_{j=1}^{n}A_j\alpha_j-\sum_{j=n+1}^{u}A_j\alpha_j. \qquad \dots(1.9)$$

$$\left.\begin{aligned}\theta&=\frac{b_j}{\beta_j},\ j=1,\dots,m\\ \phi&=\frac{a_j-1}{\alpha_j},\ j=1,\dots,n\end{aligned}\right\} \qquad \dots(1.10)$$

$$\left.\begin{aligned} \theta_i &= \frac{d_j^{(i)}}{D_j^{(i)}},\ j=1,\ldots,M^{(i)} \\ \phi_i &= \frac{1-b_j^{(i)}}{B_j^{(i)}},\ j=1,\ldots,N^{(i)} \quad i=1,\ldots,r \end{aligned}\right\} \qquad \ldots(1.11)$$

In this paper we shall obtain correspondences, involving a product of I-function and the multivariable H-function, between the original and the image in two variables.

In what follows we shall denote the original variable by x and y and the transformed variable by p and q. The notations employed are those of Ditkin and Prudnikov's [5] operational calculus.

II. Theorem 1.

With $\Delta_i, \Delta, \theta, \phi, \theta_i$ and ϕ_i given by (1.6), (1.9), (1.10) and (1.11) respectively, let

$$\Delta_i > 0, \Delta > 0, |\arg(z_i)| < \Delta_i\left(\frac{\pi}{2}\right), |\arg z| < \Delta\left(\frac{\pi}{2}\right), k>0, h_i>0, i=1,\ldots,r$$

and

$$\text{(i)} \qquad \mathrm{Re}\left(\sigma + k\theta + \sum_{i=1}^{r} h_i\theta_i\right) > 0,$$

$$\text{(ii)} \qquad \mathrm{Re}\left(\rho - \sigma - k\phi - \sum_{i=1}^{r} h_i\phi_i\right) < \frac{3}{4}.$$

Also let $0 \le n \le u$, $0 \le m \le d$, and

$$\text{(iii)} \qquad \mathrm{Re}\,(p) > 0.$$

$$p^{-\frac{1}{2}}(pq)^{\frac{\sigma}{2}-\rho+1} \sum_{r'=0}^{\infty} \sum_{h=1}^{m} \frac{\prod_{j=1}^{n} \{\Gamma(1-a_j+\alpha_j\xi_{h,r'})\}^{A_j}}{\prod_{j=n+1}^{u} \{\Gamma(a_j-\alpha_j\xi_{h,r'})\}^{A_j}}$$

$$\cdot \frac{\prod_{\substack{j=1 \\ j\neq h}}^{m} \{\Gamma(b_j-\beta_j\xi_{h,r'})\}^{B_j} (-1)^{r'} z^{\xi_{h,r'}}}{\prod_{j=m+1}^{d} \{\Gamma(1-b_j+\beta_j\xi_{h,r'})\}^{B_j}\, r'!\,\beta_h} (pq)^{\frac{k\xi_{h,r'}}{2}}$$

$$.\ H^{0,0:(M',N');\ldots;(M^r,N^r)}_{v,w:(P',Q');\ldots;(P^r,Q^r)}\left(z_1(\sqrt{pq})^{h_1},\ldots,z_r(\sqrt{pq})^{h_r}\right)$$

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$$\cdot\ \frac{(4xy)^{\rho-\frac{\sigma}{2}-\frac{1}{2}}}{\sqrt{\pi y}}\sum_{r'=0}^{\infty}\sum_{h=1}^{m}\frac{\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi_{h,r'})\}^{A_j}}{\prod_{j=n+1}^{u}\{\Gamma(a_j-\alpha_j\xi_{h,r'})\}^{A_j}}$$

$$\cdot\ \frac{\prod_{\substack{j=1\\ j\neq h}}^{m}\{\Gamma(b_j-\beta_j\xi_{h,r'})\}^{B_j}(-1)^{r'}z^{\xi_{h,r'}}}{\prod_{j=m+1}^{d}\{\Gamma(1-b_j+\beta_j\xi_{h,r'})\}^{B_j}r'!\beta_h}(4xy)^{\frac{-k\xi_{h,r'}}{2}}$$

$$\cdot\ H^{0,0:(M',N');...;(M^r,N^r)}_{v+1,w:(P',Q');...;(P^r,Q^r)}\left[\begin{matrix}[(a):A',...,A^r],[2\rho-\sigma-k\xi_{h,r'}:h_1,...,h_r]:\\ [(c):C',...,C^r],[.......................................]:\end{matrix}\right.$$

$$\left.\begin{matrix}[b':B'];...;[b^r:B^r];\\ [d':D'];...;[d^r:D^r];\end{matrix}\ z_1(2\sqrt{xy})^{-h_1},...,z_r(2\sqrt{xy})^{-h_r}\right]. \qquad ...(2.1)$$

Proof. The Laplace transform of a product of I-function and multivariable H-function is given by

$$\int_0^{\infty}e^{-pt}t^{\sigma-1}I^{m,n}_{u,d}\left[zt^k\left|\begin{matrix}(a_j,\alpha_j,A_j)_u\\ (b_j,\beta_j,B_j)_d\end{matrix}\right.\right]$$

$$.\,H^{0,0:(M',N');...;(M^r,N^r)}_{v,w:(P',Q');...;(P^r,Q^r)}\left[\begin{matrix}[(a):A',...,A^r],[..................................]: & [b':B'];...;[b^r:B^r];\\ [(c):C',...,C^r],[1-\sigma-k\xi_{h,r'}:h_1,...,h_r]: & [d':D'];...;[d^r:D^r];\end{matrix}\ z_1t^{h_1},...,z_rt^{h_r}\right]dt$$

$$=p^{-\sigma}\sum_{h=1}^{m}\sum_{r'=0}^{\infty}\frac{\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi_{h,r'})\}^{A_j}\prod_{\substack{j=1\\ j\neq h}}^{m}\{\Gamma(b_j-\beta_j\xi_{h,r'})\}^{B_j}(-1)^{r'}z^{\xi_{h,r'}}}{\prod_{j=n+1}^{u}\{\Gamma(a_j-\alpha_j\xi_{h,r'})\}^{A_j}\prod_{j=m+1}^{d}\{\Gamma(1-b_j+\beta_j\xi_{h,r'})\}^{B_j}r'!\beta_h}$$

$$.\ p^{-k\xi_{h,r'}}H^{0,0:(M',N');...;(M^r,N^r)}_{v,w:(P',Q');...;(P^r,Q^r)}(z_1p^{-h_1},...,z_rp^{-h_r}). \qquad ...(2.2)$$

The result in (2.2) can be established by substituting the series (1.7) for I-function and changing the order of integration and summation (which is justified due to absolute convergence of the integral involved in the process under the conditions mentioned), then evaluating the inner integral and using the definition (1.1), we arrive at the required result. On writing $(pq)^{-\frac{1}{2}}$ for p, multiplying both sides of (2.2) by $p^{-\frac{1}{2}}(pq)^{1-\rho}$ and then interpreting it with the help of a known result ([5],p.144, eqn. 3.26),we get

$$\frac{(4xy)^{\frac{\rho}{2}-\frac{1}{4}}}{\sqrt{\pi y}}\int_0^\infty t^{\sigma-\rho-\frac{1}{2}} J_{2\rho-1}\left[(64xyt^2)^{\frac{1}{4}}\right] I^{m,n}_{u,d}\left[zt^k \left|\begin{matrix}(a_j,\alpha_j,A_j)_u\\(b_j,\beta_j,B_j)_d\end{matrix}\right.\right]$$

$$.\,H^{0,0:(M',N');...;(M^r,N^r)}_{v,w+1:(P',Q');...;(P^r,Q^r)}\left[\begin{matrix}[(a):A',...,A^r],[.............................]: & [b':B'];...;[b^r:B^r];\\ [(c):C',...,C^r],[1-\sigma-k\xi_{h,r'}:h_1,...,h_r]: & [d':D'];...;[d^r:D^r];\end{matrix}\; z_1 t^{h_1},...,z_r t^{h_r}\right]dt$$

$$p^{-\frac{1}{2}}(pq)^{\frac{\sigma}{2}-\rho+1}\sum_{h=1}^{m}\sum_{r'=0}^{\infty}\frac{\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi_{h,r'})\}^{A_j}\prod_{j=1,\,j\neq h}^{m}\{\Gamma(b_j-\beta_j\xi_{h,r'})\}^{B_j}(-1)^{r'}z^{\xi_{h,r'}}}{\prod_{j=n+1}^{u}\{\Gamma(a_j-\alpha_j\xi_{h,r'})\}^{A_j}\prod_{j=m+1}^{d}\{\Gamma(1-b_j+\beta_j\xi_{h,r'})\}^{B_j}r'!\beta_h}$$

$$.\,(pq)^{\frac{k\xi_{h,r'}}{2}} H^{0,0:(M',N');...;(M^r,N^r)}_{v,w:(P',Q');...;(P^r,Q^r)}\left(z_1(\sqrt{pq})^{h_1},...,z_r(\sqrt{pq})^{h_r}\right), \qquad ...(2.3)$$

where Re(ρ) > 0.

Now evaluating the integral on the left hand side of (2.3) by the process mentioned in (2.2) to obtain the desired result. Hence (2.1) is proved.

III. Special Cases

(I) We have the following result from 2.1) by tacitly giving some values to parameters

$$p^{-\frac{1}{2}}(pq)^{\frac{\sigma}{2}-\rho+1}\sum_{r'=0}^{\infty}\sum_{h=1}^{m}\frac{\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi_{h,r'})\}^{A_j}}{\prod_{j=n+1}^{u}\{\Gamma(a_j-\alpha_j\xi_{h,r'})\}}$$

$$.\,\frac{\prod_{j=1,\,j\neq h}^{m}\{\Gamma(b_j-\beta_j\xi_{h,r'})\}(-1)^{r'}z^{\xi_{h,r'}}}{\prod_{j=m+1}^{d}\{\Gamma(1-b_j+\beta_j\xi_{h,r'})\}^{B_j}r'!\beta_h}(pq)^{\frac{k\xi_{h,r'}}{2}}$$

$$.\,H^{0,0:(M',N');...;(M^r,N^r)}_{v,w:(P',Q');...;(P^r,Q^r)}\left(z_1(\sqrt{pq})^{h_1},...,z_r(\sqrt{pq})^{h_r}\right)$$

$$.\,\frac{(4xy)^{\rho-\frac{\sigma}{2}-\frac{1}{2}}}{\sqrt{\pi y}}\sum_{r'=0}^{\infty}\sum_{h=1}^{m}\frac{\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi_{h,r'})\}^{A_j}}{\prod_{j=n+1}^{u}\{\Gamma(a_j-\alpha_j\xi_{h,r'})\}}$$

$$. \frac{\prod_{\substack{j=1\\ j\neq h}}^{m} \{\Gamma(b_j - \beta_j \xi_{h,r'})\}(-1)^{r'} z^{\xi_{h,r'}}}{\prod_{j=m+1}^{d} \{\Gamma(1-b_j+\beta_j\xi_{h,r'})\}^{B_j} r'!\beta_h} (4xy)^{-\frac{k\xi_{h,r'}}{2}}$$

$$. H^{0,0:(M',N');...;(M^r,N^r)}_{v+1,w:(P',Q');...;(P^r,Q^r)} \left[\begin{matrix} [(a):A',...,A^r],[2\rho-\sigma-k\xi_{h,r'}:h_1,...,h_r]: \\ [(c):C',...,C^r],[..................................]: \end{matrix} \right.$$

(II) Taking $A_j = B_j = 1 = \alpha_j = \beta_j, \forall\ j \text{ and } k = 1$ in (3.1), we get the following result involving M-series [9] and multivariable H-function as follows

$$p^{-\frac{1}{2}}(pq)^{\frac{\sigma}{2}-\rho+1} \sum_{r'=0}^{\infty} \frac{(a_1)_{r'}...(a_u)_{r'} z^{r'}}{(b_1)_{r'}...(b_d)_{r'} \Gamma(\alpha r'+1)} (pq)^{\frac{r'}{2}}$$

$$. H^{0,0:(M',N');...;(M^r,N^r)}_{v,w:(P',Q');...;(P^r,Q^r)} \left(z_1(\sqrt{pq})^{h_1},...,z_r(\sqrt{pq})^{h_r} \right)$$

$$. \frac{(4xy)^{\rho-\frac{\sigma}{2}-\frac{1}{2}}}{\sqrt{\pi y}} \sum_{r'=0}^{\infty} \frac{(a_1)_{r'}...(a_u)_{r'} z^{r'}}{(b_1)_{r'}...(b_d)_{r'} \Gamma(\alpha r'+1)} (4xy)^{-\frac{r'}{2}}$$

$$. H^{0,0:(M',N');...;(M^r,N^r)}_{v+1,w:(P',Q');...;(P^r,Q^r)} \left[\begin{matrix} [(a):A',...,A^r],[2\rho-\sigma-r':h_1,...,h_r]: \\ [(c):C',...,C^r],[............................]: \end{matrix} \right.$$

$$\left. \begin{matrix} [b':B'];...;[b^r:B^r]; \\ [d':D'];...;[d^r:D^r]; \end{matrix} z_1(2\sqrt{xy})^{-h_1},...,z_r(2\sqrt{xy})^{-h_r} \right]. \qquad ...(3.2)$$

(III) Putting $A_j = B_j = 1,$ the result in (2.1) reduces to a known result derived by Chaurasia [1].

$$p^{-\frac{1}{2}}(pq)^{\frac{\sigma}{2}-\rho+1} \sum_{h=1}^{m} \sum_{r'=0}^{\infty} \frac{(-1)^{r'}}{r'!\beta_h} \phi(\xi_{h,r'}) z^{\xi_{h,r'}} (pq)^{\frac{k\xi_{h,r'}}{2}}$$

$$. H^{0,0:(M',N');...;(M^r,N^r)}_{v,w:(P',Q');...;(P^r,Q^r)} \left(z_1(\sqrt{pq})^{h_1},...,z_r(\sqrt{pq})^{h_r} \right)$$

$$. \frac{(4xy)^{\rho-\frac{\sigma}{2}-\frac{1}{2}}}{\sqrt{\pi y}} \sum_{h=1}^{m} \sum_{r'=0}^{\infty} \frac{(-1)^{r'}}{r'!\beta_h} \phi(\xi_{h,r'}) z^{\xi_{h,r'}} (4xy)^{-\frac{k\,\xi_{h,r'}}{2}}$$

$$. H^{0,0:(M',N');...;(M^r,N^r)}_{v+1,w:(P',Q');...;(P^r,Q^r)} \left[\begin{matrix} [(a):A',...,A^r],[2\rho-\sigma-k\xi_{h,r'}:h_1,...,h_r]: \\ [(c):C',...,C^r],[.................................]: \end{matrix} \right.$$

$$\left. \begin{matrix} [b':B'];...;[b^r:B^r]; \\ [d':D'];...;[d^r:D^r]; \end{matrix} z_1(2\sqrt{xy})^{-h_1},...,z_r(2\sqrt{xy})^{-h_r} \right], \qquad ...(3.3)$$

where

$$\phi(\xi_{h,r'}) = \frac{\prod_{\substack{j=1 \\ j\neq h}}^{m} \{\Gamma(b_j-\beta_j\xi_{h,r'})\} \prod_{j=1}^{n} \Gamma(1-a_j+\alpha_j\xi_{h,r'})}{\prod_{j=m+1}^{d} \{\Gamma(1-b_j+\beta_j\xi_{h,r'})\} \prod_{j=1}^{u} \Gamma(a_j-\alpha_j\xi_{h,r'})}. \qquad ...(3.4)$$

(IV) Letting $A_j = B_j = 1$ in eqn. (2.1) with k → 0, we obtain (after a little simplification) the Laplace transform for the multivariable H-function derived by Chaurasia [1].

(V) On taking $A_j = B_j = 1$ in eqn. (2.1), we get a known result of Chaurasia and Patni[2] with $n = 0 = n'$.

(VI) Putting $A_j = B_j = 1$ in eqn (2.1), we find a known result of Chaurasia and Godika [3] with $m_1 = ... = m_R = 0 = M_{r+1} = ... = M_{r'}$.

(VII) On giving suitable value to parameters in our results, we have the results recently obtained by Chaurasia and Lata [4].

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Analysis of Water Quality Using Physico-Chemical Parameters Hosahalli Tank in Shimoga District, Karnataka, India

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Abstracts : This Paper deals with the Physico - chemical Parameters of Hosahalli Water Tank in Shimoga District, Karnataka. Monthly Changes in Physical and Chemical Parameters Such as Water Temperature, Turbidity, Total Dissolved Solids, pH, Dissolved Oxygen, Free Carbon dioxide and Total Hardness, Chlorides, Alkalinity, Phosphate and Nitrates were analyzed for a periods of one year from 1st January 2007 to 31st December 2007. All Parameters were within the permissible limits. The results indicate that the tank is Non-polluted and can be used for Domestic, Irrigation and Fisheries.

Keywords : *Perennial tank; Physico-Chemical Parameters, Monthly variation.*

GJSFR-B Classification : *FOR Code: 960608*

ANALYSIS OF WATER QUALITY USING PHYSICO-CHEMICAL PARAMETERS HOSAHALLI TANK IN SHIMOGA DISTRICT, KARNATAKA, INDIA

Strictly as per the compliance and regulations of:

Analysis of Water Quality Using Physico-Chemical Parameters Hosahalli Tank in Shimoga District, Karnataka, India

Basavaraja Simpi[α], S.M. Hiremath[Ω], KNS Murthy[α], K.N.Chandrashekarappa[α], Anil N Patel[β], E.T.Puttiah[β]

Abstract : This Paper deals with the Physico - chemical Parameters of Hosahalli Water Tank in Shimoga District, Karnataka. Monthly Changes in Physical and Chemical Parameters such as Water Temperature, Turbidity, Total Dissolved Solids, pH, Dissolved Oxygen, Free Carbon dioxide and Total Hardness, Chlorides, Alkalinity, Phosphate and Nitrates were analyzed for a periods of one year from 1st January 2007 to 31st December 2007. All Parameters were within the permissible limits. The results indicate that the tank is Non-polluted and can be used for Domestic, Irrigation and Fisheries.

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I. Introduction

Tank irrigation is one of the oldest and significant sources of irrigation in India and is particularly in south India (Palanisamy, 1998).The tanks occupy vital role in the irrigation as well as local ecosystem in the semi-arid and regions of South India. This perennial tank provides multiple uses like source of drinking water for uncountable rural and urban communities and livestock, fish culture, recharge of ground water, control of floods etc., (Gurunathan, 2006). As water is one of the most important compounds of the ecosystem, but due to increased human population, industrialization, use of fertilizers in the agriculture and man-made activity. The natural aquatic resources are causing heavy and varied pollution in aquatic environment leading to pollute water quality and depletion of aquatic biota. It is therefore necessary that the quality of drinking water should be checked at regular time of interval, because due to use of contaminated drinking water, human population suffers from varied of water borne diseases. It is difficult to understand the biological phenomena fully because the chemistry of water revels much about the metabolism of the ecosystem and explain the general hydro - biological relationship.

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The Present Study involves the Analysis of water described by its Physical, Water Quality in Terms of Physico-chemical parameters of Hosahalli System Tank, Shimoga Dist., Karnataka. It is located in 13^0 52'27"N latitude and 75^0 33'09" E longitude and this tank is having an area of 67.51 acres and the area is having 500 acres of command. This tank water is basically for agriculture, fisheries and partially domestic activities. This system tank is connected to left bank canal of Tunga Anicut Project, the area under the project is in semidry zone, The mean annual rainfall in the study area is 814.90 mm during the period from 1991-2007.

II. Materials and Methods

The Water Samples from Hosahalli Tank were collected from two different stations in the morning hours between 10 to 12 am in Polythene bottle regularly for every month. The Water samples were immediately brought in to Laboratory for the Estimation of various Physico-chemical parameters, like water temperature and pH were recorded at the time of sample collection by using Thermometer and Pocket Digital pH Meter. While other Parameters Such as DO, TDS, Free CO_2, Hardness, Alkalinity, Chlorides, Phosphate and Nitrate were estimated in the Laboratory by using Indian Standard Procedures (Titration method, Atomic Absorption Spectrophotometer (AAS) Thermo M5 Model) (Trivedy and Goel,1986, APHA 1985).

III. Results and Discussion

Month	Temperature in ^{0}C	Turbidity NTU	TDS mg/l	pH
Jan	21	10.25	210.0	8.06
Feb	24	11.61	215.1	8.02
Mar	26.2	14.25	220.2	8.40
Apr	24	8.50	156.0	8.30
May	27	8.00	120.0	8.00
Jun	26	4.50	256.4	7.90
July	24	7.80	225. 0	8.10
Aug	23.5	6.10	120.0	7.99
Sept	24	4.00	130.0	7.80
Oct	24	3.90	165.0	7.50
Nov	22	6.70	210.8	7.90
Dec	20	8.00	168.3	8.06

Table 1: Physical parameters of Hosahalli tank, Shimoga district.

a) *Climate*

The area under the project is in semidry zone, there is a rapid increase in temperature after the month of January, April is the hottest month. The climate of the year is divided into four seasons viz hot season from March to May; South-west monsoon from June to September; Post-monsoon from October to November; winter from December to February with an average wind speed of 4.22 km/hr. The maximum and minimum wind velocity in the tank area was observed in the months of July and May are 7.80 and 0.1 km/hr respectively.

b) *Water Temperature*

Generally, the weather in study area is quite cool, however the water temperature plays an important factor which influences the chemical, bio-chemical characteristics of water body. The maximum temperature of 27^0 C was recorded in May and a minimum of 20^0 C was recorded in month of December in the year 2007. Water Temperature in summer, was high due to low water level, high temperature and clear atmosphere (Salve and Hiware, 2008).

Month	Free CO_2	Dissolved oxygen	Hardness	Alkalinity	Chlorite	Phosphate	Nitrate
Jan	0.7	9.25	78.5	110.0	22.0	0.9	2.20
Feb	0.5	9.00	81.0	115.0	31.0	1.28	2.31
Mar	0.9	13.20	94.0	122.0	29.2	1.85	2.80
Apr	3.7	14.75	142.0	118.0	30.5	2.90	10.1
May	4.5	16.00	136.0	165.0	32.5	1.60	10.5
Jun	8.1	14.25	128.0	130.0	30.0	2.90	9.7
July	8.8	9.30	105.0	115.0	34.0	3.80	8.2
Aug	4.4	8.30	79.0	138.0	27.0	5.75	12.8
Sept	16.7	8.00	97.0	145.0	29.6	0.71	5.40
Oct	10.7	7.75	70.0	120.0	21.0	0.90	4.5
Nov	14.8	7.25	87.0	113.0	23.0	0.16	5.2
Dec	18.0	8.90	89.0	130.0	30.0	4.70	2.1

Table 2 : Chemical parameters of Hosahalli tank, Shimoga district (values are in mg/l)

c) Turbidity

The turbidity of water fluctuates from 3.90 to 14.25 NTU. The maximum value of 14.25 NTU was recorded in the month of March, it may be due to human activities, decrease in the water level and presence of suspended particulate matter and minimum value of 3.90 NTU in the month of October.

d) Total Dissolved Solids

The total dissolved solids fluctuate from 120 mg/l to 256.4 mg/l. the maximum value (256.4 mg/l) was recorded in the month of June. It is due to heavy rainfall and minimum value (120 m/l) in the month of May.

e) pH

pH was alkaline values ranges from 7.5 to 8.4. The maximum pH value (8.4) was recorded in the month of April (summer) and minimum (7.5) in the month of October Most of bio-chemical and chemical reactions are influenced by the pH. The reduced rate of photosynthetic activities reduces the assimilation of carbon dioxide and bicarbonates which are ultimately responsible for increase in pH, the low oxygen values coincided with high temperature during the summer month (Kamble, S. M. et al.,). The factors like air temperature bring about changes the pH of water. The higher pH values observed suggests that carbon dioxide, carbonate-bicarbonate equilibrium is affected more due to change in physico-chemical condition (Karanth, 1987; Tiwari et al., 2009).

f) Dissolved Oxygen

The value of DO fluctuates from 7.25 mg/l to 16 mg/l. The maximum values (16 mg/l) was recorded in the month of May and minimum values (7.25 mg/l) in the month of November. The high DO in summer is due to increase in temperature and duration of bright sunlight has influence on the % of soluble gases (O^2 & CO^2). The long days and intense sunlight during summer seem to accelerate photosynthesis by phytoplankton, utilizing CO_2 and giving off oxygen. This possibly accounts for the greater qualities of O_2 recorded during summer. (Krishnamurthy R., et al, 1990)

g) Free Carbon dioxide

The value of free CO_2 ranges from 0.5 mg/l to 28.6 mg/l. The maximum value (18 mg/l) was recorded in the month of December (winter) and minimum value (0.5mg/l) in the month of February. This may be depends upon alkalinity and hardness of water body. The value of CO_2 was high in December. This could be related to the high rate of decomposition in the warmer months.

h) Hardness

The value of hardness fluctuates from 70 mg/l to 142 mg/l. The maximum value (142 mg/l) was recorded in the month of April (summer) and minimum value (70 mg/l) in the month of October. (Hujare, M. S, 2008): was reported total hardness was high during summer than monsoon and winter. High value of hardness during summer can be attributed to decrease in water volume and increase of rate of evaporation of water.

i) Alkalinity

Total alkalinity ranges from 110 mg/l to 165 mg/l the maximum value (165 mg/l) was recorded in the month of May (summer) and minimum value (110 mg/l) in the month of January (winter). The alkalinity was maximum value in April (summer) due to increase in bi-carbonates in the water.(Hujare, M. S. 2008) also reported similar results that it was maximum in summer and minimum in winter due to high photosynthetic rate.

j) Chlorides

The values of chlorides range from 22 mg/l to 32.5 mg/l. The maximum value (32.5 mg/l) was recorded in the month of May (summer) and minimum value (22 mg/l) in the month of January. In the present study maximum value of chloride reaches in summer (Swarnalatha and Narsing rao, 1990).

k) Phosphate

The value of phosphate fluctuates from 0.71 mg/l to 5.75 mg/l. the maximum value (5.75mg/l) was recorded in the month of August (monsoon) and minimum value in the month of September (winter). The high values of phosphate in August (monsoon) months are mainly due to rain, surface water runoff, agriculture run off; washer man activity could have also contributed to the inorganic phosphate content.

l) Nitrates

The values of nitrate ranges from 2.1 mg/l to 12.8 mg/l. the maximum value (12.8mg/l) was observed in the month of August and minimum (2.10 mg/l) in the month of December.

IV. Acknowledgement

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Creation of a Summation Formula Associated To Hypergeometric Function

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Abstracts : The main objective of present paper is to construct of a summation formula allied with the Contiguous relation[1] and Hypergeometric function.

Keywords : Contiguous relation,Recurrence relation, Gauss second sum-mation theorem .

GJSFR-F Classification : 2000 MSC NO: 33C05 , 33C20 , 33C45 , 33C60, 33C70

CREATION OF A SUMMATION FORMULA ASSOCIATED TO HYPERGEOMETRIC FUNCTION

Strictly as per the compliance and regulations of:

Creation of a Summation Formula Associated To Hypergeometric Function

Salahuddin

Abstract : The main objective of present paper is to construct of a summation formula allied with the Contiguous relation[1] and Hypergeometric function.

Keywords : Contiguous relation,Recurrence relation, Gauss second sum-mation theorem .

I. INTRODUCTION

Generalized Gaussian Hypergeometric function of one variable :

$$ {}_AF_B\left[\begin{array}{ccc} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \qquad (1) $$

or

$$ {}_AF_B\left[\begin{array}{ccc} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right] \equiv {}_AF_B\left[\begin{array}{ccc} (a_j)_{j=1}^{A} & ; & \\ & & z \\ (b_j)_{j=1}^{B} & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \qquad (2) $$

where the parameters b1, b2, · · · , bB are neither zero nor negative integers and A, B are non-negative integers.

Contiguous Relations :

[Andrews p.367(8), E. D. p.52(19), H.T. F. I p.103(38)]

$$ c\,(1-z)\,{}_2F_1\left[\begin{array}{ccc} a,\ b\ ; & \\ c\ \ ; & z \end{array}\right] = c\,{}_2F_1\left[\begin{array}{ccc} a-1,\ b & ; & \\ c & ; & z \end{array}\right] - (c-b)\,z\,{}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ c+1 & ; & z \end{array}\right] \qquad (3) $$

[Abramowitz p.558(15.2.19)]

$$ (a-b)\,(1-z)\,{}_2F_1\left[\begin{array}{ccc} a,\ b\ ; & \\ c\ \ ; & z \end{array}\right] = (c-b)\,{}_2F_1\left[\begin{array}{ccc} a,\ b-1 & ; & \\ c & ; & z \end{array}\right] + (a-c)\,{}_2F_1\left[\begin{array}{ccc} a-1,\ b & ; & \\ c\ ; & & z \end{array}\right] \qquad (4) $$

Recurrence relation :

$$ \Gamma(z+1) = z\,\Gamma(z) \qquad (5) $$

Gauss second summation theorem [Prud.,p. 491(7.3.7.5)]

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+1}{2} & ; & \end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \qquad (6) $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \qquad (7) $$

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A new summation formula [Ref.[2], p.337(10)]

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & & ; & \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})}\left\{\frac{(b+a-1)}{(a-1)}\right\} + \frac{2\ \Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\right] \quad (8) $$

II. Main Result of Summation Formula

For the result a ≠ b

For a < 1 and a > 26

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-26}{2} & & ; & \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-26}{2})}{(a-b)\Gamma(b)}\left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-25}{2})}\left\{\frac{(-51011754393600a + 118357504819200a^2)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+\right.\right. $$

$$ +\frac{(-98966710517760a^3 + 50325530075136a^4 - 13499505727488a^5 + 3022190341632a^6)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+ $$

$$ +\frac{(-362148232320a^7 + 43098257088a^8 - 2514923424a^9 + 168298416a^{10} - 4581720a^{11})}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+ $$

$$ +\frac{(167076a^{12} - 1638a^{13} + 27a^{14} + 51011754393600b - 37245403791360ab)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+ $$

$$ +\frac{(-80940037570560a^2b + 116415325569024a^3b - 56842678044672a^4b + 20500735269888a^5b)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+ $$

$$ +\frac{(-3465526377600a^6b + 602603557632a^7b - 45225891936a^8b + 4379847264a^9b)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+ $$

$$ +\frac{(-146597880a^{10}b + 8146944a^{11}b - 96642a^{12}b + 2898a^{13}b - 81112101027840b^2)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+ $$

$$ +\frac{(125436671557632ab^2 - 40136123400192a^2b^2 - 29139543552000a^3b^2 + 28970371269120a^4b^2)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+ $$

$$ +\frac{(-8572247383680a^5b^2 + 2327580541440a^6b^2 - 241499986560a^7b^2 + 33641930160a^8b^2)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+ $$

$$ +\frac{(-1435972200a^9b^2 + 115023480a^{10}b^2 - 1677780a^{11}b^2 + 77805a^{12}b^2 + 54470076530688b^3)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+ $$

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$$+\frac{(-105803449073664ab^3+54798064349184a^2b^3-10427956377600a^3b^3-3542080886400a^4b^3)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(2603552336640a^5b^3-460166232960a^6b^3+98011751040a^7b^3-5699579400a^8b^3)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(650863200a^9b^3-12037740a^{10}b^3+807300a^{11}b^3-20801283170304b^4+39585318651904ab^4)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-30275778276864a^2b^4+7412413123200a^3b^4-959907907200a^4b^4-167925885120a^5b^4)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(95582763360a^6b^4-9286930800a^7b^4+1598971500a^8b^4-39871650a^9b^4+3798795a^{10}b^4)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(5098344323072b^5-10935515816960ab^5+6776317860480a^2b^5-2954903965440a^3b^5)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(375539532480a^4b^5-35771520960a^5b^5-3129724080a^6b^5+1427207040a^7b^5-58641030a^8b^5)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(8351070a^9b^5-854046409216b^6+1652031172736ab^6-1430967960576a^2b^6)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(393494693760a^3b^6-114446656800a^4b^6+7316754480a^5b^6-539607600a^6b^6-18721080a^7b^6)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(7020405a^8b^6+101240723584b^7-222412525312ab^7+131731224192a^2b^7-65902227840a^3b^7)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(8369506800a^4b^7-1772648640a^5b^7+45150840a^6b^7-2674440a^7b^7-8642334272b^8)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(16034040672ab^8-14476664592a^2b^8+3390870600a^3b^8-1141791300a^4b^8+54815670a^5b^8)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-8947575a^6b^8+533428896b^9-1163928480ab^9+585996840a^2b^9-308880000a^3b^9)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(25160850a^4b^9-6216210a^5b^9-235915 68b^{10}+39528632ab^{10}-35902152a^2b^{10})}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(5379660a^3b^{10}-1924065a^4b^{10}+728728b^{11}-1534624ab^{11}+520884a^2b^{11}-278460a^3b^{11})}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-14924b^{12}+19474ab^{12}-17199a^2b^{12}+182b^{13}-350ab^{13}-b^{14})}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}\Bigg\}+$$

$$+\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-26}{2})}\Bigg\{\frac{(-51011754393600a+81112101027840a^2-54470076530688a^3+20801283170304a^4)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-5098344323072a^5+854046409216a^6-101240723584a^7+8642334272a^8-533428896a^9)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(23591568a^{10}-728728a^{11}+14924a^{12}-182a^{13}+a^{14}+51011754393600b)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(37245403791360ab-125436671557632a^2b+105803449073664a^3b-39585318651904a^4b)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(10935515816960a^5b-1652031172736a^6b+222412525312a^7b-16034040672a^8b)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(1163928480a^9b-39528632a^{10}b+1534624a^{11}b-19474a^{12}b+350a^{13}b)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-118357504819200b^2+80940037570560ab^2+40136123400192a^2b^2-54798064349184a^3b^2)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(30275778276864a^4b^2-6776317860480a^5b^2+1430967960576a^6b^2-131731224192a^7b^2)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(14476664592a^8b^2-585996840a^9b^2+35902152a^{10}b^2-520884a^{11}b^2+17199a^{12}b^2)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(98966710517760b^3-116415325569024ab^3+29139543552000a^2b^3+10427956377600a^3b^3)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-7412413123200a^4b^3+2954903965440a^5b^3-393494693760a^6b^3+65902227840a^7b^3)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-3390870600a^8b^3+308880000a^9b^3-5379660a^{10}b^3+278460a^{11}b^3-503255300751136b^4)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(56842678044672ab^4-28970371269120a^2b^4+3542080886400a^3b^4+959907907200a^4b^4)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-375539532480a^5b^4+114446656800a^6b^4-8369506800a^7b^4+1141791300a^8b^4)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-25160850a^9b^4+1924065a^10b^4+134995057274888b^5-20500735269888ab^5)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(8572247383680a^2b^5-2603552336640a^3b^5+167925885120a^4b^5+35771520960a^5b^5)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-7316754480a^6b^5+1772648640a^7b^5-54815670a^8b^5+6216210a^9b^5-30221903416320b^6)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(3465526377600ab^6-2327580541440a^2b^6+460166232960a^3b^6-95582763360a^4b^6)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(3129724080a^5b^6+539607600a^6b^6-45150840a^7b^6+8947575a^8b^6+362148232320b^7)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-602603557632ab^7+241499986560a^2b^7-98011751040a^3b^7+9286930800a^4b^7)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-1427207040a^5b^7+18721080a^6b^7+2674440a^7b^7-430982570888b^8+45225891936ab^8)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-33641930160a^2b^8+5699579400a^3b^8-1598971500a^4b^8+58641030a^5b^8-7020405a^6b^8)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(25149234240b^9-4379847264ab^9+1435972200a^2b^9-650863200a^3b^9+39871650a^4b^9)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-8351070a^5b^9-168298416b^{10}+146597880ab^{10}-115023480a^2b^{10}+12037740a^3b^{10})}{\prod\limits_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-3798795a^4b^{10}+4581720b^{11}-8146944ab^{11}+1677780a^2b^{11}-807300a^3b^{11}-167076b^{12})}{\prod\limits_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$\left.\left.+\frac{(96642ab^{12}-77805a^2b^{12}+1638b^{13}-2898ab^{13}-27b^{14})}{\prod\limits_{\Upsilon=1}^{13}\{a-2\Upsilon\}}\right\}\right] \qquad (9)$$

III. Derivation of Summation Formula

Substituting $c=\frac{a+b-26}{2}$ and $z=\frac{1}{2}$ in equation (4), we get

$$(a-b)\,{}_2F_1\left[\begin{matrix}a,\ b & ; \\ \frac{a+b-26}{2} & ;\end{matrix}\ \frac{1}{2}\right]=(a-b-26)\,{}_2F_1\left[\begin{matrix}a,\ b-1\ ; \\ \frac{a+b-26}{2}\quad ;\end{matrix}\ \frac{1}{2}\right]+(a-b+26)\,{}_2F_1\left[\begin{matrix}a-1,\ b & ; \\ \frac{a+b-26}{2}; & \end{matrix}\ \frac{1}{2}\right]$$

Now involving (8), we get

$$L.H.S=\frac{2^{(b-1)}\ \Gamma(\frac{a+b-26}{2})}{\Gamma(b)}\left[\frac{(a-b-26)}{(a-b+1)}\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-25}{2})}\left\{\frac{(-51011754393600+63421769318400a)}{\prod\limits_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+\right.\right.$$

$$+\frac{(167484063744000a^2-53211710103552a^3+31884367816704a^4-10247133065728a^5)}{\prod\limits_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(2036983501696a^6-277446268736a^7+25686051872a^8-1711613904a^9+76461528a^{10})}{\prod\limits_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-2409212a^{11}+44174a^{12}-493a^{13}+a^{14}+132123855421440b-214509763952640ab)}{\prod\limits_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(69300067000320a^2b+39220418445312a^3b-37409176935936a^4b+13249436039424a^5b)}{\prod\limits_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-2774297373120a^6b+374132468736a^7b-34906601808a^8b+2199314832a^9b)}{\prod\limits_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-96653700a^{10}b+2674152a^{11}b-45981a^{12}b+324a^{13}b-135582177558528b^2)}{\prod\limits_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(247470874140672ab^2 - 132765127372800a^2b^2 + 13141051223040a^3b^2 + 13507644309120a^4b^2)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-6440276531520a^5b^2 + 1464334185600a^6b^2 - 200779225920a^7b^2 + 18441479160a^8b^2)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-1106176500a^9b^2 + 45122220a^{10}b^2 - 1056510a^{11}b^2 + 14625a^{12}b^2 + 752713597700992b^3)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-1455454555517696ab^3 + 927739803449440a^2b^3 - 233071542912000a^3b^3 + 241895784000a^4b^3)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(1433140200960a^5b^3 - 381254852160a^6b^3 + 57692456640a^7b^3 - 4872055500a^8b^3)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(297983400a^9b^3 - 9176310a^1 0b^3 + 215280a^1 1b^3 - 258996274933776b^4 + 51605063198208ab^4)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-353817434131120a^2b^4 + 111880284048000a^3b^4 - 14562357264000a^4b^4 - 48159427680a^5b^4)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(57377502000a^6b^4 - 8314651800a^7b^4 + 844301250a^8b^4 - 34781175a^9b^4 + 1332045a^{10}b^4)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(5952390732288b^5 - 12065729105664ab^5 + 8483867870400a^2b^5 - 2922691799040a^3b^5)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(514262155680a^4b^5 - 33843307680a^5b^5 - 1932676200a^6b^5 + 887599440a^7b^5 - 55986255a^8b^5)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(3749460a^9b^5 - 955287132800b^6 + 1924689738816ab^6 - 1398536380800a^2b^6)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(477591791040a^3b^6 - 93800725200a^4b^6 + 9176632920a^5b^6 - 243374040a^6b^6 - 17383860a^7b^6)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(4345965a^8b^6 + 109883057856b^7 - 224082713088ab^7 + 153166553280a^2b^7)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-56306053440a^3b^7+9910953000a^4b^7-1157560560a^5b^7+52151580a^6b^7-9175763168b^8)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(17856571824ab^8-13073565960a^2b^8+3943660500a^3b^8-833806350a^4b^8+63655605a^5b^8)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-4345965a^6b^8+5570204464b^9-1116930672ab^9+665078700a^2b^9-246675000a^3b^9)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(29354325a^4b^9-3749460a^5b^9-24320296b^{10}+43083612ab^{10}-31068180a^2b^{10})}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(6216210a^3b^{10}-1332045a^4b^{10}+743652b^{11}-1433016ab^{11}+586170a^2b^{11}-215280a^3b^{11})}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-15106b^{12}+20943ab^{12}-14625a^2b^{12}+183b^{13}-324ab^{13}-b^{14})}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}\Bigg\}+$$

$$+\frac{(a-b-26)(b-1)}{(a-b+1)}\,\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-26}{2})}\Bigg\{\frac{(51011754393600-28138356080640a-27724268961792a^2)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(32602475323392a^3-14448997488640a^4+3688567241216a^5-610937714816a^6)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(69100784896a^7-5451182880a^8+300176448a^9-11317592a^{10}+278512a^{11}-4030a^{12})}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(26a^13-116395514265600b+114522988216320ab-15005106413568a^2b)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-22245090729984a^3b+13643817721344a^4b-3756359681280a^5b+648405662976a^6b)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-71817829632a^7b+5627524032a^8b-286125840a^9b+10316592a^{10}b-204984a^{11}b)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(2574a^{12}b+102010945536000b^2-119270681763840ab^2+44340497510400a^2b^2)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-288101760000a^3b^2-4178311804800a^4b^2+1541357026560a^5b^2-266164456320a^6b^2)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(31589510400a^7b^2-2186379000a^8b^2+115486800a^9b^2-3011580a^{10}b^2+63180a^{11}b^2)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-483476130938888b^3+61214607532032ab^3-28068077798400a^2b^3+5225944742400a^3b^3)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(269493024000a^4b^3-263679978240a^5b^3+62828640000a^6b^3-6397790400a^7b^3)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(519901200a^8b^3-17760600a^9b^3+592020a^{10}b^3+142234813501440b^4-18853937339904ab^4)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(9378156470400a^2b^4-2240652345600a^3b^4+226918036800a^4b^4+19781516160a^5b^4)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-6048126000a^6b^4+965282400a^7b^4-45612450a^8b^4+2466750a^9b^4-2853945391616b^5)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(3737155541760ab^5-1983951601920a^2b^5+504233452800a^3b^5-68857407360a^4b^5)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(3713381280a^5b^5+427281120a^6b^5-42476400a^7b^5+4601610a^8b^5+388525211520b^6)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-535705585920ab^6+264770593920a^2b^6-75069792000a^3b^6+10115758800a^4b^6)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-797297760a^5b^6+18721080a^6b^6+2674440a^7b^6-40218882304b^7+49712345856ab^7)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-27579168000a^2b^7+6330888000a^3b^7-1062655200a^4b^7+63242640a^5b^7-2674440a^6b^7)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(2736575712b^8-3829505472ab^8+1613406600a^2b^8-481150800a^3b^8+44805150a^4b^8)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-4601610a^5b^8-156307008b^9+164152560ab^9-92835600a^2b^9+13813800a^3b^9)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-2466750a^4b^9+5027880b^{10}-7104240ab^{10}+1930500a^2b^{10}-592020a^3b^{10}-156208b^{11})}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(109512ab^{11}-63180a^2b^{11}+1794b^{12}-2574ab^{12}-26b^{13})}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}\Bigg\}\Bigg]+\frac{2^{(b-1)}\,\Gamma(\frac{a+b-26}{2})}{\Gamma(b)}\Bigg[\frac{(a-b+26)}{(a-b-1)}\times$$

$$\times\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-26}{2})}\Bigg\{\frac{(51011754393600-132123855421440a+135582177558528a^2-75271359700992a^3)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(25899627493376a^4-5952390732288a^5+955287132800a^6-109883057856a^7)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(9175763168a^8-557020464a^9+24320296a^{10}-743652a^{11}+15106a^{12}-183a^{13}+a^{14})}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-634217693184000b+2145097639526400ab-2474708741406720a^2b+1455454555176960a^3b)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-516050631982080a^4b+120657291056640a^5b-19246897388160a^6b+2240827130880a^7b)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-178565718240a^8b+11169306720a^9b-430836120a^{10}b+14330160a^{11}b-20943a^{12}b+324a^{13}b)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-1674840637440b^2-6930006700032ab^2+13276512737280a^2b^2-9277398034944a^3b^2)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(3538174341312a^4b^2-848386787040a^5b^2+139853638080a^6b^2-15316655328a^7b^2)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(1307356596a^8b^2-66507870a^9b^2+3106818a^{10}b^2-58617a^{11}b^2+14625a^{12}b^2)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(532117101035520b^3-392204184453120ab^3-131410512230400a^2b^3+233071542912000a^3b^3)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-11188028404800a^4b^3+2922691799040a^5b^3-477591791040a^6b^3+56306053440a^7b^3)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-3943660500a^8b^3+246675000a^9b^3-6216210a^{10}b^3+215280a^{11}b^3-31884367816704b^4)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(37409176935936ab^4-13507644309120a^2b^4-241895784000a^3b^4+1456235726400a^4b^4)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-514262155680a^5b^4+93800725200a^6b^4-9910953000a^7b^4+833806350a^8b^4-29354325a^9b^4)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(1332045a^{10}b^4+10247133065728b^5-13249436039424ab^5+6440276531520a^2b^5)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-1433140200960a^3b^5+48159427680a^4b^5+33843307680a^5b^5-9176632920a^6b^5)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(1157560560a^7b^5-63655605a^8b^5+3749460a^9b^5-2036983501696b^6+2774297373120ab^6)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-1464334185600a^2b^6+381254852160a^3b^6-57377502000a^4b^6+1932676200a^5b^6)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(243374040a^6b^6-52151580a^7b^6+4345965a^8b^6+277446268736b^7-374132468736ab^7)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(200779225920a^2b^7-57692456640a^3b^7+8314651800a^4b^7-887599440a^5b^7+17383860a^6b^7)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-25686051872b^8+34906601808ab^8-18441479160a^2b^8+4872055500a^3b^8-844301250a^4b^8)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(55986255a^5b^8-4345965a^6b^8+1711613904b^9-2199314832ab^9+1106176500a^2b^9)}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-297983400a^3b^9+34781175a^4b^9-3749460a^5b^9-76461528b^{10}+96653700ab^{10})}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(-45122220a^2b^{10}+9176310a^3b^{10}-1332045a^4b^{10}+2409212b^{11}-2674152ab^{11})}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}+$$

$$+\frac{(1056510a^2b^{11}-215280a^3b^{11}-44174b^{12}+45981ab^{12}-14625a^2b^{12}+493b^{13}-324ab^{13}-b^{14})}{\prod_{\Upsilon=1}^{13}\{a-2\Upsilon\}}\Bigg\}+$$

$$+\frac{(a-b+26)}{(a-b-1)}\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-25}{2})}\Bigg\{\frac{(-51011754393600+116395514265600a-102010945536000a^2)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}$$

$$+\frac{(48347613093888a^3-14223481350144a^4+2853945391616a^5-388525211520a^6)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(40218882304a^7-2736575712a^8+156307008a^9-5027880a^{10}+156208a^{11}-1794a^{12})}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(26a^{13}+281383560806640b-114522988216320ab+119270681763840a^2b)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-61214607532032a^3b+18853937339904a^4b-3737155541760a^5b+535705585920a^6b)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-49712345856a^7b+3829505472a^8b-164152560a^9b+7104240a^{10}b-109512a^{11}b)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(2574a^{12}b+277242689617792b^2+15005106413568ab^2-44340497510400a^2b^2)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(28068077798400a^3b^2-9378156470400a^4b^2+1983951601920a^5b^2-264770593920a^6b^2)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(27579168000a^7b^2-1613406600a^8b^2+92835600a^9b^2-1930500a^{10}b^2+63180a^{11}b^2)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(27579168000a^7b^2-1613406600a^8b^2+92835600a^9b^2-1930500a^{10}b^2+63180a^{11}b^2)}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$+\frac{(-326024753233392b^3+22245090729984ab^3+288101760000a^2b^3-5225944742400a^3b^3)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(2240652345600a^4b^3-504233452800a^5b^3+75069792000a^6b^3-6330888000a^7b^3)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(481150800a^8b^3-13813800a^9b^3+592020a^{10}b^3+144489974886640b^4-136438177211344ab^4)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(4178311804800a^2b^4-269493024000a^3b^4-226918036800a^4b^4+68857407360a^5b^4)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(-10115758800a^6b^4+1062655200a^7b^4-44805150a^8b^4+2466750a^9b^4-3688567241216b^5)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(3756359681280ab^5-1541357026560a^2b^5+263679978240a^3b^5-19781516160a^4b^5)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(-3713381280a^5b^5+797297760a^6b^5-63242640a^7b^5+4601610a^8b^5+610937714816b^6)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(-648405662976ab^6+266164456320a^2b^6-62828640000a^3b^6+6048126000a^4b^6)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(-427281120a^5b^6-18721080a^6b^6+2674440a^7b^6-691007784896b^7+71817829632ab^7)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(-31589510400a^2b^7+6397790400a^3b^7-965282400a^4b^7+42476400a^5b^7-2674440a^6b^7)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(5451182880b^8-5627524032ab^8+2186379000a^2b^8-519901200a^3b^8+45612450a^4b^8)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(-4601610a^5b^8-3001764448b^9+286125840ab^9-115486800a^2b^9+17760600a^3b^9)}{\prod_{\Omega=1}^{13}\left\{a-(2\Omega-1)\right\}}+$$

$$+\frac{(-2466750a^4b^9+11317592b^{10}-10316592ab^{10}+3011580a^2b^{10}-592020a^3b^{10}-278512b^{11})}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}+$$

$$\left.\left.+\frac{(204984ab^11-63180a^2b^{11}+4030b^{12}-2574ab^{12}-26b^{13})}{\prod_{\Omega=1}^{13}\{a-(2\Omega-1)\}}\right\}\right]$$

On simplification, we get the result.

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Abstracts : This study aimed at looking into ways of improving the nutritive values of sorghum Stover grown under two conditions: one grown sole and another intercropped with groundnuts. As an abundant source of dry season feeds, the study was conducted to determine its nutritional value and digestibility. Nine Yankasa rams weighing 22±2.5 kg, age between 6 and 12 months were used in this experiment. Rams were completely randomized into three treatments with three rams per treatment. Treatment one was sole sorghum, treatment two intercropped sorghum Stover plus 200 grams groundnuts hulms. Treatment three intercropped Stover plus 400 grams of groundnut hulms. Result of the analysis showed that there was an increase in the rate of digestibility in intercropped Stover fed with supplement. This could be due to high nitrogen content available to ruminal bacteria in the intercropped Stover and supplements, which was not in the sole grown Stover. Based on this result, the experiment recommends the intercropping of sorghum with groundnuts to improve law nutrients value and digestibility. At 200 and 400 grams levels of supplementation there was significant increase at ($P<0.05$) digestibility with accompanied increase in body weighed gain. No decrease in weighed gain was observed in treatment one, although 400 grams supplementation showed the highest weight gain of 5.7 kg.

Keywords : *Rams; Intercropping; Digestibility; Sorghum Stover; Groundnuts.*

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Strictly as per the compliance and regulations of:

Assessment of the Digestibility of Intercropped Sorghum Stover with (Groundnuts) Legume.

Quala, I.D.[α], Yahaya S.M.[β], Midau, A.[Ω], Aji S.B.[α], Muktar Y.M.[β], H.D Nyako.[β]

Abstract : This study aimed at looking into ways of improving the nutritive values of sorghum Stover grown under two conditions: one grown sole and another intercropped with groundnuts. As an abundant source of dry season feeds, the study was conducted to determine its nutritional value and digestibility. Nine Yankasa rams weighing 22±2.5 kg, age between 6 and 12 months were used in this experiment. Rams were completely randomized into three treatments with three rams per treatment. Treatment one was sole sorghum, treatment two intercropped sorghum Stover plus 200 grams groundnuts hulms. Treatment three intercropped Stover plus 400 grams of groundnut hulms. Result of the analysis showed that there was an increase in the rate of digestibility in intercropped Stover fed with supplement. This could be due to high nitrogen content available to ruminal bacteria in the intercropped Stover and supplements, which was not in the sole grown Stover. Based on this result, the experiment recommends the intercropping of sorghum with groundnuts to improve law nutrients value and digestibility. At 200 and 400 grams levels of supplementation there was significant increase at ($P<0.05$) digestibility with accompanied increase in body weighed gain. No decrease in weighed gain was observed in treatment one, although 400 grams supplementation showed the highest weight gain of 5.7 kg.

Keywords : Rams; Intercropping; Digestibility; Sorghum Stover; Groundnuts.

I. Introduction

A major problems faced by livestock industry in Nigeria is inadequate supply of feeds toward the end of the dry season. This period coincides with the time when pastures has either being burnt down, overgrazed or have lost their nutritive values, such that they are reduced to high fibre feed with low digestibility. The consequence is the drastic decline in productivity in terms of weight loss, susceptibility to diseases, reduced productivity and general economic losses (Poppi and Maclenon, 1995). Most farmers in the savannah zone engage in sorghum farming which gives raise to large supply of sorghum Stover; which under proper management and storage can serve as feed during such period of scarcity (Tanko, *et al* 1992). Chopping Stover for preservation can increase the intake rate especially by sheep (Osafo *et al.,* 1974). Digestibility of sorghum Stover is low due to high content of lignin, cellulose, hemicelluloses etc. Nitrogen content of sorghum Stover is also low and for it to be degraded adequately for animal utilization, supplementation is necessary. Supplement should be available and cheap. Cowpea, groundnuts haulms as well as lablab haulms are most readily available and have high nitrogen content (Minson and Wilson, 1980).

Conrad and Hibbs (1968) assert in the work that, nitrogen content of lablab, groundnuts haulms or cowpea have adequate nitrogen available for use of ruminant micro biotic fermentation, which aids digestion. Ammonia is also constituents of these legumes residues, which together contribute nitrogen for protein synthesis by the ruminant microorganism to facilitate ruminal fermentation and digestion (Anderson, 1978). Low level nitrogen in sorghum Stover could be due to stage of maturity at harvest or the state of fertility of soil on which they are grown. Agronomic research have shown that, because legume are capable of fixing soil atmospheric nitrogen by symbiotic fixation mechanism, intercropping sorghum with legume may be a possible way of increasing the nitrogen level of sorghum Stover. Another advantage of this (intercropping) is the increase of leaf area ratio, dry matter accumulation and grain yield. Also an added advantage of intercropping is increased organic matter content (Griggs, 1993).

II. Materials and Methods

Location of study area

The study was conducted at the Federal University of Technology Yola Research and Teaching farm. Federal University of Technology is located in Girei local government area of Adamawa state Nigeria, situated at latitude 7° and 11°E and longitude 11°E and 14°E, at 185.5m above sea level. The ambient temperature ranges between 29.2°C and 30°C. Two distinct seasons exist in the study area, each lasting 6 months of the year; with dry season beginning from November to April while raining season lasts from May through October (Adebayo, 1999).

The experiment was carried out under a shade structure constructed out of wooden posts and fed

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separately. They were at last placed into metabolic cages for sample collection.

III. Experimental Diets

The experimental diets used were sorghum Stover as basal diet produced udder two distinct cropping systems: one grown sole and another grown intercropped with groundnuts: Groundnuts haulms was fed as supplement. The Stover was fed at 600gm twice daily in each case as basal diet. The sole Stover as the control diet was fed to the control animals. On the other hand 600gm 0f sorghum Stover from the intercrop was fed two classes of the experimental animals with graded levels of groundnuts haulms at 200 and 400 gm each; split in each case into two as per daily feeding regime.

IV. Experimental Animals

Nine Yankasa rams of about 6 months of age weighing between 20-24 kg kept under intensive systems were used, each animal was dewormed with 1 bolus each of albendazole and 2-2.5mls of oxytetracycline 200mg/ml intramuscularly to check secondary bacterial infection, the animals were in good health. They were washed and rinsed with Asuntol (Acaricide) against external parasites.

V. Experimental Design

The experiment was conducted using Completely Randomized Design (CRD). Animals were allocated treatment numbers which were T1, T2 and T3 as the control. Treatment 2 were animals fed 600gm of sorghum Stover (sorghum Stover grown sole) supplemented with 200 gm of groundnuts haulms split and fed twice, treatment 3 were also fed 600 gm sorghum Stover (intercropped sorghum Stover) with 400 gm groundnuts haulms as supplement, the fed were given twice daily at 6:30 am and 4:30 pm daily, Clean water was given ad-libitum with mineral salt licks.

VI. Sampling Techniques

The samples of this experiment were feacal and urine samples and the experimental diets. These were analyzed according to AOAC (2000). On assumption of the experiment, this experiment opted for a period of adjustment in which the experimental diets were fed ad-libitum to the animals in preparation for their digestive tract to excrete previous feeds in feaces. These rams were allowed ample time of 3 weeks for adoption physiologically before the experiment commence. At the beginning of the 4th week, measured basal feeds and supplements were fed; 600 gm sorghum Stover only fed to T1 twice; i.e. 300gm in the morning at 6:30am and 300 gm at 4:30 pm. Graded levels of groundnuts haulms and intercropped sorghum Stover was fed to T2 and T3. The Stover was chopped to increase voluntary intake and so residual feed average overall was 150gm. That means about 450 gm of fee control Stover was consumed by T1 and about 750 and 650 respectively for T2 and T3. The data were recorded and noted, consumption to have differed with ambient temperatures. The month of April and late March saw slight decline in intake due to increase in ambient temperatures. The rams were kept outside the metabolic cages until collection period.

Metabolic cages were constructed with care taken to maintain cleanliness and comfort, ensuring that boxes fitted the rams and giving no rooms for turning round in the cages; making sure also that comfort as regards movement, lying down and standing was provided.

To facilitate collection of urine devoid of contamination, a stage of wire mesh through which urine alone can pass was used in constructing the bottom of the cages. The feacal matter was collected in special bags of khaki material fastened by durable shreds which crosses under flank with one on either side over the thurl. They were joined and knotted at the main band that crosses the loin. All feacal matters were collected here and placed into driers to dry at ambient temperature towards oven drying at analysis.

The urine was collected once a day, measured in graduated cylinder and recorded. A sample of each is put into bottles treated with 2% HCL meant to hold the nitrogen content of the urine. Feeding trial and period of adjustment was followed by two weeks of collection period. The collected feaces were labeled according to individual animals; sun dried and stored in labeled polyethene bags. One week collection was done with samples composite separately for separate chemical analysis to be done. This was followed by a straight 14 days collection period. Analysis of this was meant to give comparison for the sake that the first weeks collection would be considered an additional adjustment period. Each 24 hour feacal collection was sundried and weighed; samples were labeled and parceled. The samples were subjected to analysis for crude protein, ether extract, crude fibre, moisture content, ash and minerals, NDF, ADF and NFE.

VII. Statistical Analysis

Data obtained were analyzed using the analysis of variance of the completely randomized design and least significant difference was used to separate the treatment means.

VIII. Result and Discussion

Result from Table 1 shows that dry matter and crude protein was significantly higher ($P<0.05$) in intercropped sorghum.

Table I: Data for dry matter intake

Days	T1	T2	T3
1	40	44	52
2	38	43	50
3	39	45	50
4	37	46	53
5	38	45	54
6	38	43	53
Total	230.0	266.0	312.0
Mean	38.3	44.3	54.0

These differences could be due to nitrogen available to the intercropped sorghum, which was not available to the sole cropped sorghum. There was also significantly ($P<0.05$) high neutral detergent fibre and acid detergent fibre in the sole sorghum, ether extract was slightly higher in the intercropped sorghum compared to the sole cropped sorghum, these is in agreement with the findings of Macdonald (2001).

Table 2: Data for Daily weight gain in kg.

Days	T1	T2	T3
1	0.1	0.1	0.6
2	0.2	0.3	0.7
3	0.2	0.2	1.0
4	0.3	0.3	1.2
5	0.1	0.4	1.2
6	0.3	0.3	1.0
Total	1.20	1.60	5.70
Mean	0.20	0.33	0.95

The result of Table 2 shows that there was a significant ($P<0.05$) effect of supplementation on the dry matter intake. The intake increased initially by feeding 600 gm sorghum Stover with 200 gm of groundnuts haulms as supplement by 33 gm.

The intake further increased when the same amount of sorghum Stover was fed with 400 gm of groundnuts haulms supplement, the increase was significant ($P<0.05$), these was accompanied by increase in daily weight gain and agrees with the findings of Hadler and Horst (1991). Digestibility coefficient given as

DC = <u>Grams of feed intake – Grams in feaces</u> x100

Grams of feed intake

The intake increase was indicated when 600 gm was actually eaten and 450 gm was voided; thus

$$D.C = \frac{600 - 450}{600} \times 100 = 25\%$$

The increase reached 45% and 58%; subsequently which was significant ($P<0.05$). The increase in intake continued with an accompanied increase in body weight gain of 1.6 kg when supplemented with 200 gm, which agrees with the report of Humphrey (1985), in his work on nutrients utilization and weight gain in ruminant animals.

Digestibility tables

Table 3: (Dry matter)

Days	T1	T2
1	14.0	13.0
2	16.1	13.4
3	14.0	14.0
4	13.2	13.5
5	13.0	14.9
6	13.4	14.6
Total	83.7	83.40
Mean	13.95	13.90

Table 4: Neutral Detergent Fibre (NDF)

Days	T1	T2	T3
1	11.0	10.2	10.2
2	11.2	10.5	10.3
3	11.4	11.0	10.4
4	11.5	11.2	11.1
5	12.0	11.6	11.5
6	12.2	12.2	12.0
Total	69.30	66.71	65.70
Mean	11.55	11.12	10.95

Further supplementation with 400 gm increased digestibility to 41.66% initially and up to 70.14% at the end of the experiment when 670 gm was eaten and 200 gm of feaces was voided. There was an increase in feed intake.

Given as

$$D.C = \frac{670 - 200}{670} \times 100 = 70.14\%$$

Table 5: Digestibility of Acid Detergent Fibre

Days	T1	T2	T3
1	5.5	5.4	5.2
2	6.2	5.8	6.0
3	6.5	6.2	6.3
4	6.8	6.5	6.7
5	7.5	6.8	7.5
6	8.2	7.1	8.2
Total	40.72	37.61	39.90
Mean	6.78	6.23	6.65

Table 6: Digestibility of Ether Extract

Days	T1	T2	T3
1	0.20	0.13	0.13
2	0.10	0.15	0.14
3	0.10	0.16	0.15
4	0.20	0.18	0.16
5	0.30	0.19	0.17
6	0.30	0.21	0.20
Total	1.20	1.02	0.95
Mean	0.20	0.17	0.16

Table 7: Digestibility of Total Ash

Days	T1	T2	T3
1	0.29	0.41	0.41
2	0.31	0.50	0.46
3	0.31	0.53	0.49
4	0.35	0.55	0.52
5	0.37	0.58	0.55
6	0.37	0.63	0.58
Total	2.05	3.20	3.01
Mean	0.34	0.53	0.50

This increase in digestibility could be due to nitrogen flush available to the microorganisms in the rumen which was not available in the sole Stover diet as earlier reported by Hendrickson *et al* (1981). The crude protein degradation was significant (P<0.05) and continued to increase with level of supplementation. Crude protein feaces was least into feed supplemented with 400 gm of groundnuts haulm which is rich in crude protein as reported by Maynard (1975).

There was also significant (P<0.05) increase in NDF and ADF digestibility, with the highest in feed supplemented with 400 gm groundnuts haulms. This agrees with Conrad and Hibbs (1968) in their report on effect of legume supplementation on nutrients digestibility. The NDF graded in T2 was about 2.6 higher and about 2.7 in that supplement with 400gm. The ADF graded was also significant (P<0.05) at 200 gm supplement reaching about 3.1 in T2 and 1.2 in T3. This as well can be due to the nitrogen content in Stover taken up from the soil Adeniran and Wilson (1978).

IX. Summary, Conclusion and Recommendation

The result obtained from the analysis of experimental diets, which includes sorghum Stover (sole grown and intercropped), and groundnuts haulms revealed a remarkable variations in nutrients contents. Intercropping groundnuts with sorghum provides nitrogen for uptake by the sorghum which resulted in increased intake and digestibility. Variation in levels of supplementation with groundnuts haulms improved body weighed gain compared to sole fed sorghum Stover. Based on this the use of agronomic practice of intercropping sorghum with legumes could increase productivity and reduce the use of other protein source supplements which are scarce and expensive.

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A Model on Changeover Times with Single Machine Problem

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Abstracts : The aim of this note is to discuss single machine problems with changeover times in connection with general shop problems.

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A Model on Changeover Times with Single Machine Problem

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Abstract : The aim of this note is to discuss single machine problems with changeover times in connection with general shop problems.

I. Introduction

In this paper we consider scheduling problems in which the set I of all jobs or all operations (in connection with shop problems) in partitioned into disjoint sets $I_1, I_2, I_3, \cdots, I_r$ called groups, i.e. $I = I_1 \cup I_2 \cup I_3 \cup \cdots \cup I_r$ and $I_f \cap I_g = \phi$ for $f, g \in \{1, 2, 3, \cdots, r\}, f \neq g.$ Let N_j be the number of jobs in I_j. Furthermore, we have the additional restrictions that for any two jobs (operations) i, j with $i \in I_f$ and $j \in I_g$ to be processed on the same machine M_k job (operation) j cannot be started until S_{fgk} time units after the finish time of job (operation) i or job (operation) j. In a typical application, the groups correspond to different types of jobs (operations) and S_{fgk} may be interpreted as a machine dependent changeover time. During the changeover period, the machine cannot process another job. We assume that $S_{fgk} = 0$ for all $f, g \in \{1, 2, 3, \cdots, r\}$, $k \in \{1, 2, 3, \cdots, m\}$ with $f = g$ and that the triangle inequality holds.

$$S_{fgk} + S_{ghk} \geq S_{fhk} \text{ for all } f, g, h \in \{1, 2, 3, \cdots, r\},\ k \in \{1, 2, 3, \cdots, m\}. \quad (1)$$

Both assumptions are realistic in practice.

The model can also handle set-up times for starting the first operations on the machines. For this purpose, we introduce a dummy operations belonging to a dummy group I_0 as an initial job, then the set-up time for an operation from group I_h starting on M_k can be represented by S_{ohk}.

If we consider single machine problems or if the changeover times are machine independent, we replace S_{fgk} by S_{fg}. If the changeover time do not depend on both group I_f and I_g, but only on the group I_g to which the job to be processed next belongs then we replace S_{fg} by S_g. In the later case, the changeover times are called sequence independent, contrary to the general case in which they are called sequence dependent. If $S_{fg} = S$ for all $f, g = 1, 2, 3, \cdots, r$ then we have constant changeover times.

To indicate problems with changeover times, we add $\beta \in \{S_{fgk}, S_{fg}, S_g, S\}$ to the part of our general classification scheme.

1.1 Single Machine Problems

While problems $1 | S_{fg} | C_{\max}$ and $1 | S_g | L_{\max}$ are NP(Bruno and Downey (1978)), problem $1 | S_g | C_{\max}$ can be solved polynomially by scheduling the jobs group by in any order.

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Next we will present dynamic programming procedures for the problems $1|S_{fg}L_{\max}|S_{fg}|\sum w_iC_i$ and $1|S_{fg}|\sum w_iC_i$ which are due to Monma and Potts [1989]. The following theorem is the basis for these procedures.

II. Main Result

Theorem 1:

a) For problem $1|S_{fg}|\sum w_iC_i$, there exists an optimal schedule where the early jobs within each group are ordered according to nondecreasing due dates.
b) For problem $1|S_{fg}|\sum w_iC_i$, there exists an optimal schedule where the jobs within each group are ordered according to nondecereasing $\frac{p_i}{w_i}$ values.
c) For problem $1|S_{fg}|L_{\max}$, there exists an optimal schedule where the jobs within each group are ordered according to nondecreasing due dates.

Proof :

Consider a schedule of the form D, j, E, i, F, where job i and j are from the same group and D, E, F, represent arbitrary blocks, i.e. partial sequences of jobs. Due to the triangle inequality (1), the total changeover times will not increase if D, j, E, i, F is replaced by D, i, j, E, F. We have to show that if $d_i < d_j$ for early jobs i, j or $\frac{p_i}{w_i} < \frac{p_j}{w_j}$ or $d_i < d_j$ then $\sum \omega_i U_i$ or $\sum \omega_i C_i$ or the $L_{\max}$ value will not increase when moving from the sequence D, j, E, i, F to one of the two sequence D, E, i, j, F or D, i, j, E, F.

a) If we have the objective function $\sum w_i U_i$ then the fact that i is early in D, j, E, i, F implies that i, j are early in D, E, i, j, F because $d_i < d_j$.

b) Consider the objective function $\sum w_i C_i$ and let $\frac{p_i}{w_i} < \frac{p_j}{w_j}$. It is convenient to replace changeover times by set-up jobs with processing time equal to the changeover time and zero weight. Furthermore, we assume that all set ups after j and before i and D, j, E, i, F are included in the partial sequence E. Define

$$p(E) = \sum_{i \in E} p_i \text{ and } w(E) = \sum_{i \in E} w_i$$

then an easy calculation shows that job j and block E can be swapped without increasing the objective value if $\frac{p_j}{w_j} > \frac{p(E)}{w(E)}$. Similarly, i and E can be swapped if $\frac{p(E)}{w(E)} > \frac{p_i}{w_i}$. In the first case we first swap j and E and then j and i without increasing the objective value. This provides the sequence D, E, i, j, F if $\frac{p_j}{w_j} \le \frac{p(E)}{w(E)}$ then $\frac{p_i}{w_i} < \frac{p(E)}{w(E)}$ and we get D, i, j, E, F after two swaps.

c) Consider the objective function $L_{\max}$ and let $d_i < d_j$. Again, changeover times are replaced by set-up jobs with the changeover time and a very large due date. To see the effect of swapping a job j scheduled before a block $E = i_r, i_{r-1}, \cdots, i_1$ we replace E by a job with processing time $p_i, p_{r-1} = p(E)$ and a due date d_i, i_{r-1}, i_i where d_r, i_{r-1}, i_i is calculated by the recursion

$$d_{i\vartheta+l\cdots i_l} = \min\{d_{i\vartheta\cdots i_l}\ d_{i\vartheta+1} + \sum_{k=1}^{\vartheta} p_{ik}\} \text{ for } \vartheta = 1,2,3,\cdots,r-1$$

This follows by induction using the fact that, for two jobs i and j with finish times C_j and $C_i = C_j - p_j$, we have

$$\max\{C_j - d_j, C_j - p_j - d_i\} = C_j - \min\{d_j, d_i + p_j\}$$

Now we can proceed as in part (b). If $d_j > d_E := d_{ir-i_{r-1},1,\cdots i_1}$ then we can swap first j and E and then j and $i.$

For the following dynamic programming algorithms, we assume that the jobs each group are ordered according to theorem 1. We get derive an algorithm for problem $1|S_{fg}\sum w_i C_i$

We define $C(n_1, n_2, \cdots, n_r, t, h)$ to be the minimum cost of a partial schedule containing the first n_j jobs of groups $l_j (j = 1,2,\cdots,r),$ where the last job scheduled comes from group l_h and is completed at time $t,$ we have

$$0 \le n_j \le N_j \text{ for } j = 1,2,\cdots,r$$

and

$$0 \le t \le T := \sum_{i \in l} p_i + \sum_{j=1}^{r} p_j + N_j \max\{S_{f\,j} \,|\, 1 \le f \le r\}$$

The recursion is

$$C(n_1, n_2, \cdots n_r, t, h) = \min\{C(n_1', n_2', \cdots, n_r', t', f) + w_{nh}^h\, t \,|\, 1 \le f \le r\} \quad (2)$$

where $n_j' = n_j$ for $j \ne h, n_h' = n_n - 1,$ $t' = t - p_{n_h}^h - s/h$ and $w_{n_h}^h$ and $p_{n_h}^h$ are the weight and processing time of the n_h-th job in groups $l_h.$

initially, we set $C(0,0,\cdots,0) = 0$ and all other $C-$values to infinity. The optimal schedule cost is found by selecting the smallest value of the form $C(N_1, N_r, t, h)$ for some schedule completion time $0 \le t \le T$ and some group I_h to which the final job belongs.

Because the number of states is bounded by $O(r\,n^r\,T)$ and (2) can be calculated in $O(r)$ steps, we have an $O(r^2\,n^r\,T)-$algorithm

Alternatively, we can replace the state variable t by variables $t_{fh} (f, h = 1,2,\cdots,r)$ representing the number of set-ups from group f to group $h.$ Note that t is readily computed from the state variables using.

$$t = \sum_{f,h=1}^{r} t_{fh} S_{fh} + \sum_{h=1}^{r} \sum_{\upsilon=1}^{n\,h} p_{\upsilon}^{h}$$

The complexity of this version is $O(r^2 n^{r+s})$, where s is the number of different values for changeover times. Note that $s \le r^2$ in general and $s \le r$ in the case of sequence-independent changeover times. If the number of groups is fixed we have a polynomial algorithm.

Similarly, we solve problem $1 \mid S_{fg} \mid L_{\max}$. In this case (2) is replaced by

$$C(n_1, n_2, \cdots n_r, t, h) = \min\{\max\{C(n_1', n_2', \cdots, n_r', t', f), t - d_{nk}^h\} \mid 1 \le f \le r\} \tag{3}$$

The weighted number of late jobs problem is solved differently. By theorem 1(a), we only know an order for family jobs; the algorithm must also determine those jobs that are late. The late jobs may be appended in any order to the schedule of on-time jobs.

We define $C(n_1, n_2, \cdots n_r, t, h)$ to be the minimum weighted number of late jobs for the partial schedule containing the first n_j jobs of group l_j $(j = 1, 2, \cdots, r)$ where the last on-time job comes from group l_h and is completed at time t. We have

$$0 \le n_j \le N_j \text{ for } j = 1, 2, \cdots, r$$

and

$$0 \le t \le T := \min\{\max_{i \in l} d_i \sum_{i \in E} p_i + \sum_{j=1}^{r} N_j \max\{S_{fi} \mid 1 \le f \le r\}\}$$

The recursion is

$$C(n_1, n_2, \cdots n_r, t, h) = \begin{cases} \min\{\min_{1 \le f < r} C(n_1', n_2', \cdots, n_r', t', f) \\ C(n_1', n_2', \cdots, n_r', t, h) + w_{n_h}^h\} \text{ if } t \le d_{n_h}^h \\ C(n_1', n_2', \cdots, n_r', t, h) + w_{n_h}^h \quad \text{if } t \le d_{n_h}^h \end{cases}$$

$$\text{where } n_j' = n_j \text{ for } j \ne h, n_h' = n_n - 1, \quad t' = t - p_{n_h}^h - S_{fh} \tag{4}$$

The initial values are

$$C(n_1, n_2, \cdots, n_r, 0, 0) = \sum_{j=1}^{r} \sum_{\upsilon=1}^{n_j} w_\upsilon^j$$

for $0 \le n_j \le N_j$ where $j = 1, 2, \cdots, r$. All other initial values are set to infinity.

The minimum weighted number of late jobs is found by selecting the smallest value of the form $C(N_1, N_2, \cdots, N_r, t, h)$ for some completion time of on time jobs, where $0 \le t \le T$ and for some group l_h containing the final on-time job.

The complexity is $O(r^2 n^r T)$. Again, it may be desirable to eliminate the state variable t from the recursion. The achieve this, we switch the state variable t with the objective function value as follows.

Define $C(n_1, n_2, \cdots n_r, w, h)$ to be the minimum completion time of on time jobs for a partial schedule containing the first n_j jobs of each group. l_j where the weighted number of late jobs is equal to w, and the last on-time job comes from group l_h. the initial values are $C(n_1, n_2, \cdots n_r, w, 0) = 0$, where $w = \sum_{j=1}^{r} \sum_{\upsilon=1}^{n_j} w_\upsilon^j$ for $0 \le n_j \le N_j (j = 1, 2, \cdots, r)$ and all other values are set to infinity.

The recursion is

$$C(n_1, n_2, \cdots n_r, w, h) = \min\{\min_{1 \le f \le r}\{C(n_1, n_2, \cdots n_r, w, f) + p'|$$

$$C(n_1, n_2, \cdots n_r, w, f) + p' \le d^h_{n_h}\}\ C(n_1', n_2', \cdots n_r', w', h)\}$$

where $n_j' = n_j$ for $j \neq h$, $n_h' = n_h - 1, p' = p^h_{n_h} + S_{fh}$ and $w' = w - w^h_{n_h}$

As in (4), the first term in the minimization chooses the $n_h -$ th job n group I_h to be scheduled on time, if possible, and chooses the previous on-time job from group I_f the second term selects the n_h th job of group I_h to be late.

The minimum weighted number of late jobs are found by selecting the smallest w for which $\min_{0 \le h \le r} C(N_1, N_2, \cdots, N_r, w, h)$ is finite.

The complexity is $O(r^2 n^r W)$, where $W = \sum_{i \in I} w_i$. It reduces to $O(r^2 n^{r+1})$ for the total number for late jobs problem.

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- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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