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An Efficient Class of Ratio-Cum-Dual to Product Estimator of Finite Population Mean in Sample Surveys

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Sanjib Choudhury [α] & Bhupendra Kumar Singh [σ]

Abstract - We consider a class of ratio-cum-dual to product estimator for estimating a finite population mean of the study variate. The bias and mean square error of the proposed estimator have been obtained. The asymptotically optimum estimator (AOE) in this class has also been identified along with its approximate bias and mean square error. Theoretical and empirical studies have been done to demonstrate the superiority of the proposed estimator over the other estimators.

Keywords : Finite population mean; ratio-cum-dual to product estimator; Bias; Mean square error; Efficiency.

I. Introduction

In sample surveys, auxiliary information is used at both selections as well as estimation stages to improve the efficiency of the estimators. The use of auxiliary information at the estimation stage appears to have started with the work of Cochran (1940). When the correlation between study variate and auxiliary variate is positive (high), the ratio method of estimation is used for estimating the population mean. The ratio method is most effective if $\rho C_y / C_x > 1/2$, where C_y, C_x and ρ are coefficient of variation of y, coefficient of variation of x and correlation coefficient between y and x respectively. On the other hand, if the correlation is negative, the product method of estimation is used and this is most effective if $\rho C_y / C_x < -1/2$, suggested by Murthy (1964). Srivenkataramana (1980) first proposed dual to ratio estimator and Bandyopadhyay (1980) proposed dual to product estimator. Singh and Tailor (2005), Singh and Espejo (2003), Tailor and Sharma (2009) worked on ratio-cum-product estimators. Sharma and Tailor (2010), Choudhury and Singh (2012) worked on ratio, dual to ratio and dual to product estimators to estimate the study variable. These motivated authors to propose a new ratio-cum-dual to product estimators for estimating the population mean.

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Consider a finite population $U=(u_1,u_2,...,u_N)$ of size N units. Let y and x denotes the study and auxiliary variates respectively. A sample of size n $(n<N)$ is drawn using simple random sampling without replacement (SRSWOR) to estimate the population mean $\bar{Y}=(1/N)\sum_{i=1}^{N}y_i$ of the study variate y. Let the sample mean $(\bar{x},\ \bar{y})$ are the unbiased estimator of $(\bar{X},\bar{Y})$ based on n observations.

The usual ratio and product estimators for $\bar{Y}$ are

$$\bar{y}_R=\bar{y}\left(\bar{X}/\bar{x}\right)$$

and

$$\bar{y}_P=\bar{y}\left(\bar{x}/\bar{X}\right)\quad \text{respectively,}$$

where $\bar{y}=(1/n)\sum_{i=1}^{n}y_i$ and $\bar{x}=(1/n)\sum_{i=1}^{n}x_i$.

Let $x_i^*=(1+g)\bar{X}-gx_i$, $i=1,2,...,N$, where $g=n/(N-n)$.

Then clearly $\bar{x}^*=(1+g)\bar{X}-g\bar{x}$ is also unbiased estimator for $\bar{X}$ and $Corr\left(\bar{y},\bar{x}^*\right)=-\rho$.

Using the transformation $x_i^*=(1+g)\bar{X}-gx_i$, Srivenkataramana (1980) obtained dual to ratio estimator as

$$\bar{y}_R^*=\bar{y}\left(\bar{x}^*/\bar{X}\right)$$

and Bandyopadhyay (1980) obtained dual to product estimator as

$$\bar{y}_P^*=\bar{y}\left(\bar{X}/\bar{x}^*\right).$$

In this paper, we have proposed a class of ratio-cum-dual to product type estimator for estimating population mean $\bar{Y}$. Numerical illustrations are given in the support of the present study.

II. The Proposed Estimator

For estimating population mean $\bar{Y}$, we propose an estimator as

$$\bar{y}_{RdP}=\bar{y}\left[\alpha\left(\frac{\bar{X}}{\bar{x}}\right)+(1-\alpha)\left(\frac{\bar{X}}{\bar{x}^*}\right)\right] \tag{1}$$

where α is a suitably chosen scalar.

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To obtain the bias and mean square error (MSE) of $\bar{y}_{RdP}$ to a first degree of approximation, we write

$$e_0 = \left(\bar{y} - \bar{Y}\right)/\bar{Y} \text{ and } e_1 = \left(\bar{x} - \bar{X}\right)/\bar{X}$$

Such that

$$\left.\begin{array}{l} E(e_0) = E(e_1) = 0, \;\; E(e_0^2) = \dfrac{1-f}{n} C_y^2, \\ E(e_1^2) = \dfrac{1-f}{n} C_x^2, \;\; E(e_0 e_1) = \dfrac{1-f}{n} C C_x^2, \end{array}\right\} \tag{2}$$

where $f = n/N$ is the sampling fraction, $C_y^2 = S_y^2/\bar{Y}^2$, $C_x^2 = S_x^2/\bar{X}^2$, $C = \rho C_y/C_x$ and defined as $\rho = S_{xy}/S_x S_y$, $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2$, $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2$ and $S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_i - \bar{X}\right)$.

Expressing $\bar{y}_{RdP}$ in terms of e's, we obtain

$$\bar{y}_{RdP} = \bar{Y}\left(1+e_0\right)\left\{\alpha\left(1+e_1\right)^{-1} + \left(1-\alpha\right)\left(1-ge_1\right)^{-1}\right\}.$$

We now assume that $|e_1| < 1$ and $|ge_1| < 1$, so that we may expand $\left(1+e_1\right)^{-1}$ and $\left(1-ge_1\right)^{-1}$ as a series in powers of e_1 and ge_1 respectively. Expanding, multiplying out and retaining terms of e's to the second degree, we obtain

$$\bar{y}_{RdP} - \bar{Y} \cong \bar{Y}\left[e_0 + g\left(e_1 + e_0 e_1 + g\, e_1^2\right) + \alpha\left(1+g\right)\left\{-e_1^2 + \left(1-g\right)e_1 - e_0 e_1\right\}\right] \tag{3}$$

Taking the expectation of both sides in equation (3) and using the results of equation (2) we get the bias of $\bar{y}_{RdP}$ as

$$B\left(\bar{y}_{RdP}\right) = \frac{1-f}{n}\bar{Y} C_x^2\left[\left\{g^2 - \alpha\left(g^2 - 1\right)\right\} + C\left\{g - \alpha\left(g+1\right)\right\}\right] \tag{4}$$

The bias, $B\left(\bar{y}_{RdP}\right)$ in (4) is 'zero' if $\alpha = \dfrac{g\left(C+g\right)}{\left(1+g\right)\left(1-g-C\right)}$. Thus the estimator $\bar{y}_{RdP}$ with $\alpha = \dfrac{g\left(C+g\right)}{\left(1+g\right)\left(1-g-C\right)}$ is almost unbiased.

Squaring and taking expectations of both the sides of equation (3) and using the results of equation (2), we obtain the MSE of $\bar{y}_{RdP}$ to the first degree of approximation as

$$M\left(\bar{y}_{RdP}\right) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\left\{g - \alpha\left(1+g\right)\right\}\left\{2C + g - \alpha\left(1+g\right)\right\}\right] \tag{5}$$

which is minimized when

$$\alpha = \frac{1}{1+g}(g+C) = \alpha_{opt.}\text{(say)} \tag{6}$$

Substituting equation (6) in equation (1) yield the 'asymptotically optimum estimator' (AOE) as

$$\bar{y}_{RdP}^{opt.} = \bar{y}\left[\left(\frac{g+C}{1+g}\right)\frac{\bar{X}}{\bar{x}} + \left(\frac{1-C}{1+g}\right)\frac{\bar{x}^*}{\bar{X}}\right]$$

Thus the resulting bias and MSE of $\bar{y}_{PdP}^{opt.}$ respectively as

$$B\left(\bar{y}_{RdP}^{opt.}\right) = \frac{1-f}{n}\bar{Y}C_x^2(1-C)(g+C) \tag{7}$$

and

$$M\left(\bar{y}_{RdP}^{opt.}\right) = \frac{1-f}{n}\bar{Y}^2C_y^2\left(1-\rho^2\right) \tag{8}$$

which is the same as the MSE of the linear regression estimator $\bar{y}_{reg.} = \bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)$, where b_{yx} is the sample regression coefficient of y on x.

From equation (7), we note that the bias of AOE $\bar{y}_{RdP}^{opt.}$ is 'zero' if

either $C=1$ or $C=-g$.

Remark 2.1.

To the first degree of approximation, the proposed strategy $\bar{y}_{RdP}$ under optimality condition (6), is equal to linear regression estimator.

Remark 2.2.

For $\alpha=1$, the estimator $\bar{y}_{RdP}$ in equation (1) boils down to the usual ratio estimator $\bar{y}_R$. The bias and MSE of $\bar{y}_R$ can be obtained by putting $\alpha=1$ in equations (4) and (5) respectively as

$$B\left(\bar{y}_R\right) = \frac{1-f}{n}\bar{Y}C_x^2(1-C)$$

$$M\left(\bar{y}_R\right) = \frac{1-f}{n}\bar{Y}^2\left\{C_y^2 + C_x^2(1-2C)\right\} \tag{9}$$

Remark 2.3.

For $\alpha=0$, the estimator $\bar{y}_{RdP}$ in equation (1) boils down to the dual to product estimator $\bar{y}_p^*$, proposed by Bandyopadhyay (1980). The bias and MSE of $\bar{y}_p^*$ can be obtained by putting $\alpha=0$ in equations (4) and (5) respectively as

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$$B\left(\bar{y}_p^*\right)=\frac{1-f}{n}\bar{Y}C_x^2 g\left(g+C\right)$$

and

$$M\left(\bar{y}_p^*\right)=\frac{1-f}{n}\bar{Y}^2\left\{C_y^2+gC_x^2\left(g+2C\right)\right\} \tag{10}$$

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Thus, we see that this study provides unified treatment towards the properties of different estimators.

III. Efficiency Comparisons

a) Comparison of $\bar{y}_{RdP}$

In this section, firstly, we compare MSE of conventional estimators $\bar{y}$, $\bar{y}_R$ and $\bar{y}_P$ with MSE of proposed estimator $\bar{y}_{RdP}$.

The MSE of sample mean $\bar{y}$ under SRSWOR sampling scheme is given by

$$M\left(\bar{y}\right)=\frac{1-f}{n}\bar{Y}^2C_y^2. \tag{11}$$

From equations (5) and (11), it is found that the proposed estimator $\bar{y}_{RdP}$ is more efficient than $\bar{y}$ if

$$\left\{-g+\alpha\left(1+g\right)\right\}\left\{2C+g-\alpha\left(1+g\right)\right\}>0$$

This condition holds if

$$\text{either } \frac{g}{1+g}>\alpha \text{ and } \frac{1}{1+g}\left(2C+g\right)<\alpha,$$

$$\text{or } \frac{g}{1+g}<\alpha \text{ and } \frac{1}{1+g}\left(2C+g\right)>\alpha.$$

Therefore, the range of α for which the proposed estimator $\bar{y}_{RdP}$ is more efficient than $\bar{y}$ is

$$\left[\min\left\{\frac{g}{1+g},\ \frac{1}{1+g}\left(2C+g\right)\right\},\ \max\left\{\frac{g}{1+g},\ \frac{1}{1+g}\left(2C+g\right)\right\}\right].$$

From equations (5) and (9), we note that the estimator $\bar{y}_{RdP}$ has smaller MSE than that of the usual ratio estimator $\bar{y}_R$ if

$$\left\{1+g-\alpha\left(1+g\right)\right\}\left\{1-2C-g+\alpha\left(1+g\right)\right\}>0$$

This condition holds if

$$\text{either } 1>\alpha \text{ and } \frac{1}{1+g}(2C+g-1)<\alpha,$$

$$\text{or } \quad 1<\alpha \text{ and } \frac{1}{1+g}(2C+g-1)>\alpha.$$

Therefore, the range of α for which the proposed estimator $\bar{y}_{RdP}$ is better than $\bar{y}_R$ is

$$\left[\min\left\{1, \frac{1}{1+g}(2C+g-1)\right\}, \max\left\{1, \frac{1}{1+g}(2C+g-1)\right\}\right].$$

To compare the usual product estimator $\bar{y}_P$, we write the bias and MSE of $\bar{y}_P$ to the first degree of approximation respectively as

$$B(\bar{y}_P)=\frac{1-f}{n}\bar{Y}CC_x^2$$

$$M(\bar{y}_P)=\frac{1-f}{n}\bar{Y}^2\left\{C_y^2+C_x^2(1+2C)\right\} \tag{12}$$

We note from equations (5) and (12) that the estimator $\bar{y}_{RdP}$ will dominate over usual product estimator $\bar{y}_P$ if

$$\left\{-(g-1)+\alpha(g+1)\right\}\left\{(2C+1+g)-\alpha(g+1)\right\}>0$$

This condition holds if

$$\text{either } \frac{g-1}{1+g}>\alpha \text{ and } 1+\frac{2C}{1+g}<\alpha$$

$$\text{or } \quad \frac{g-1}{1+g}<\alpha \text{ and } 1+\frac{2C}{1+g}>\alpha.$$

Hence, the range of α in which the proposed estimator $\bar{y}_{PdP}$ is better than $\bar{y}_P$ is

$$\left\{\min\left(\frac{g-1}{1+g}, 1+\frac{2C}{1+g}\right), \max\left(\frac{g-1}{1+g}, 1+\frac{2C}{1+g}\right)\right\}.$$

Secondly, comparing the MSE between the proposed estimator and dual to ratio estimator $\bar{y}_R^*$, proposed by Srivenkataramana (1980).

The bias and MSE of $\bar{y}_R^*$ to the first degree of approximation respectively as

$$B\left(\bar{y}_R^*\right)=-\bar{Y}\frac{1-f}{n}gCC_x^2$$

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and

$$M\left(\bar{y}_R^*\right)=\frac{1-f}{n}\bar{Y}^2\left\{C_y^2+gC_x^2\left(g-2C\right)\right\}. \tag{13}$$

From equations (5) and (13), it is found that the proposed estimator $\bar{y}_{RdP}$ will dominate over Srivenkataramana (1980) estimator $\bar{y}_R^*$ if

$$\left\{2g-\alpha\left(g+1\right)\right\}\left\{-2C+\alpha\left(g+1\right)\right\}>0$$

This condition exist if

$$\text{either } \frac{2g}{1+g}>\alpha \text{ and } \frac{2C}{1+g}<\alpha,$$

$$\text{or } \quad \frac{2g}{1+g}<\alpha \text{ and } \frac{2C}{1+g}>\alpha.$$

Therefore, the range of α in which the proposed estimator $\bar{y}_{RdP}$ is more efficient than dual to ratio estimator $\bar{y}_R^*$ is

$$\left\{\min\left(\frac{2g}{1+g},\ \frac{2C}{1+g}\right),\ \max\left(\frac{2g}{1+g},\ \frac{2C}{1+g}\right)\right\}.$$

Lastly, we compare MSE of the proposed estimator $\bar{y}_{RdP}$ with dual to product estimator $\bar{y}_p^*$.

We note from equations (5) and (10) that

$$M\left(\bar{y}_p^*\right)>M\left(\bar{y}_{RdP}\right) \text{ if}$$

$$\alpha\left(1+g\right)\left\{2C+2g-\alpha\left(1+g\right)\right\}>0$$

This condition exist if

$$\text{either } 0<\alpha<\frac{2}{1+g}\left(C+g\right),$$

$$\text{or } \quad \frac{2}{1+g}\left(C+g\right)<\alpha<0.$$

Therefore, the range of α in which the proposed estimator $\bar{y}_{RdP}$ is more efficient than dual to product estimator $\bar{y}_P^*$ is

$$\left[\min\left\{\frac{2\left(C+g\right)}{1+g},\ 0\right\},\ \max\left\{\frac{2\left(C+g\right)}{1+g},\ 0\right\}\right].$$

Thus, it seems from the above results that the proposed estimator $\bar{y}_{RdP}$ may be made better than other estimators by making a suitable choice of the value of α within the respective ranges.

b) Comparison of 'AOE' of $\bar{y}_{RdP}^{opt.}$

From equations (8)-(13), it is found that the 'AOE' $\bar{y}_{RdP}^{opt.}$ is more efficient than the other existing estimators like $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$. Since

$$M\left(\bar{y}\right)-M\left(\bar{y}_{RdP}^{opt.}\right)=\frac{1-f}{n}\bar{Y}^2\rho^2C_y^2>0.$$

$$M\left(\bar{y}_R\right)-M\left(\bar{y}_{RdP}^{opt.}\right)=\frac{1-f}{n}\bar{Y}^2C_x^2\left(1-C\right)^2>0.$$

$$M\left(\bar{y}_P\right)-M\left(\bar{y}_{RdP}^{opt.}\right)=\frac{1-f}{n}\bar{Y}^2C_x^2\left(1+C\right)^2>0.$$

$$M\left(\bar{y}_R^*\right)-M\left(\bar{y}_{RdP}^{opt.}\right)=\frac{1-f}{n}\bar{Y}^2C_x^2\left(C-g\right)^2>0.$$

$$M\left(\bar{y}_p^*\right)-M\left(\bar{y}_{RdP}^{opt.}\right)=\frac{1-f}{n}\bar{Y}^2C_x^2\left(C+g\right)^2>0.$$

Hence, we conclude that the proposed estimator '$\bar{y}_{RdP}$' is the best (in the sense of having optimum MSE).

IV. Numerical Illustrations

To examine the merits of the constructed estimator over its competitors numerically, we consider eight sets of data. The source of the population, the nature of the variates y and x and the values of the various parameters are listed in Table 1.

To reflect the gain in the efficiency of the proposed estimator $\bar{y}_{RdP}$ over the estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$, the effective ranges along with the optimum value of α are presented in Table 2 with respect to the population data sets.

The percent relative efficiencies (PREs) of the different estimators with respect to usual unbiased estimator $\bar{y}$ computed by the formula

$$PRE\left(\mathrm{E},\bar{y}\right)=\frac{M\left(\bar{y}\right)}{M\left(\mathrm{E}\right)}\times 100$$

where $\mathrm{E}=\bar{y},\ \bar{y}_R,\ \bar{y}_P,\ \bar{y}_R^*,\ \bar{y}_P^*$ and $\bar{y}_{RdP}$ or $\bar{y}_{RdP}^{opt.}$

and are presented in Table 3.

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Table 1 : Description of the populations

Population	Source	Study variate y	Auxiliary variate x	N	n	ρ	C_y	C_x	$\bar{Y}$
1	Steel and Torrie (1960)	Log of leaf burn in secs	Chlorine percentage	30	6	-0.4996	0.7001	0.7493	0.6860
2	Pandey and Dubey (1988)	---	---	20	8	-0.9199	0.3552	0.3943	19.55
3	Kadilar and Cingi (2006) pp. 1054	Level of apple production	Number of apple trees	106	20	0.82	4.18	2.02	15.37
4	Sukhatme and Sukhatme (1970)	Number of villages in the circles.	A circle consisting more than five villages	89	12	0.766	0.604	2.1901	3.36
5	Maddala (1977)	Consumption per capita.	Deflated prices of veal	30	6	-0.6823	0.2278	0.0986	7.6375
6	Murthy (1967)	Output	Fixed capital	80	20	0.9413	0.3542	0.7507	51.8264
7	Murthy (1967)	Output	Number of workers	80	20	0.9150	0.3542	0.9484	51.8264
8	Kadilar and Cingi (2006) pp. 78	---	---	106	20	0.86	5.22	2.1	2212.59

Table 2 : Effective ranges and optimum value of α of $\bar{y}_{RdP}$.

Population	Ranges of α in which the proposed estimator $\bar{y}_{RdP}$ is better than					Optimum value
	$\bar{y}$	$\bar{y}_R$	$\bar{y}_P$	$\bar{y}_R^*$	$\bar{y}_P^*$	$\alpha_{opt.}$
1	(-0.55,0.20)	(-1.35, 1.00)	(-0.60, 0.25)	(-0.75, 0.40)	(-0.35, 0.00)	-0.1734
2	(-0.59, 0.40)	(-1.19, 1.00)	(-0.20, 0.01)	(-0.99, 0.80)	(-0.19, 0.00)	-0.0972
3	(0.19, 2.94)	(1.00, 2.13)	(-0.62, 3.75)	(0.38, 2.75)	(0.00, 3.13)	1.5654
4	(0.13, 0.50)	(-0.36, 1.00)	(-0.73, 1.37)	(0.27, 0.37)	(0.00, 0.64)	0.3176
5	(-2.32, 0.20)	(-3.12, 1.00)	(-1.52, -0.60)	(-2.52, 0.40)	(-2.12, 0.00)	-1.0611
6	(0.25, 0.92)	(0.17, 1.00)	(-0.50, 1.67)	(0.50, 0.67)	(0.00, 1.17)	0.5831
7	(0.25, 0.76)	(0.01, 1.00)	(-0.50, 1.51)	(0.50, 0.51)	(0.00, 1.01)	0.5063
8	(0.19, 3.66)	(1.00, 2.85)	(_0.62, 4.47)	(0.38, 3.47)	(0.00, 3.85)	1.9231

Table 3 : Percentage relative efficiency of $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$, $\bar{y}_P^*$ and $\bar{y}_{RdP}$ or $\bar{y}_{RdP}^{opt.}$ with respect to $\bar{y}$.

Population	$\bar{y}$	$\bar{y}_R$	$\bar{y}_P$	$\bar{y}_R^*$	$\bar{y}_P^*$	$\bar{y}_{RdP}$ or $\bar{y}_{RdP}^{opt.}$
1	100.00	†	†	†	124.34	133.26
2	100.00	†	526.50	†	537.23	650.26
3	100.00	226.76	†	120.73	†	305.25
4	100.00	†	†	220.46	†	241.99
5	100.00	†	167.59	†	115.73	187.10
6	100.00	†	†	591.38	†	877.54
7	100.00	†	†	612.44	†	614.34
8	100.00	212.82	†	117.95	†	384.02

†Relative efficiency less than 100%.

V. Conclusion

Table 2 provides the wide ranges along with the optimum value of α for which the proposed estimators $\bar{y}_{RdP}$ or $\bar{y}_{RdP}^{opt.}$ are more efficient than all other estimators considered in this paper. It is also observed from Table 2 that there is a scope for choosing α to obtain better estimators than $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$.

Table 3 shows that there is a substantial gain in efficiency by using proposed estimator $\bar{y}_{RdP}$ (or $\bar{y}_{RdP}^{opt.}$) over $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$. This shows that even if the scalar α deviates from its optimum value$\left(\alpha_{opt.}\right)$, the suggested estimator $\bar{y}_{RdP}$ will yield better estimates then $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$. Thus it is preferred to use the proposed estimators $\bar{y}_{RdP}$ or $\bar{y}_{RdP}^{opt.}$ in practice.

References Références Referencias

1. Bandyopadhyay, S. (1980). Improved ratio and product estimators, Sankhya Series C, 42(2), 45-49.
2. Choudhury, S. and Singh, B. K. (2012). An Efficient Class of Dual to Product-Cum-Dual to Ratio Estimator of Finite Population Mean in Sample Surveys, Global Journal of Science Frontier Research, 12(3), p.25-33.
3. Cochran, W. G. (1940). The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce, The Journal of Agricultural Science, 30(2), 262–275.
4. Kadilar, C. and Cingi, H. (2006). Ratio Estimators for the population variance in simple and stratified random sampling, Applied Mathematics and Computation, 173(2), 1047-1059.
5. Kadilar, C. and Cingi, H. (2006). Improvement in estimating the population mean in simple random sampling, Applied Mathematics Letters, 19, 75-79.
6. Maddala, G. S. (1977). Econometrics, Mcgraw Hills Pub.Co.: New York.
7. Murthy, M. N. (1964). Product method of estimation, Sankhya A, 26, 69-74.

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Notes

8. Murthy, M. N. (1967). Sampling Theory and Methods, Statistical Publishing Society: Calcutta.
9. Pandey, B.N. and Dubey, V. (1988). Modified product estimator using coefficient of variation of auxiliary variable, Assam Stat. Rev., 2, 64-66.
10. Sharma, B. and Tailor, R. (2010). A new ratio-cum-dual to ratio estimator of finite population mean in simple random sampling, Global Journal of Science Frontier Research, 10(1), 27-31.
11. Singh, H. P. and Espejo, M. R. (2003). On linear regression and ratio-product estimation of a finite population mean, The Statistician, 52(1), 59-67.
12. Singh, H. P. and Tailor, R. (2005). Estimation of finite population mean using known correlation coefficient between auxiliary characters, Statistica, Anno LXV(4), 407-418.
13. Srivenkataramana, T. (1980). A dual to ratio estimator in sample surveys, Biometrika, 67(1), 199-204.
14. Steel, R. G. D. and Torrie, J. H. (1960). Principles and Procedures of Statistics, Mc-Graw Hill Book.
15. Sukhatme, P. V. and Sukhatme, B. V. (1970). Sampling Theory of Surveys with Applications, Iowa State University Press, Ames, U. S. A.
16. Tailor, R. and Sharma, B. K. (2009). A modified ratio-cum-product estimator of finite population mean using known coefficient of Variation and coefficient of Kurtosis, Statistics in Transition-new series, 10(1), 15-24.