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Generalization of the Dependent Function in Extenics for Nested Sets with Common Endpoints to 2D-Space, 3D-Space, and Generally to n-D-Space

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Abstract - In this paper we extend Prof. Yang Chunyan and Prof. Cai Wen's dependent function of a point P with respect to two nested sets $X_0 \subset X$, for the case the sets X_0 and X have common ending points, from $1D$ - space to n-D-space. We give several examples in 2D- and 3D-spaces. When computing the dependent function value $k(.)$ of the optimal point O, we take its maximum possible value. Formulas for computing $k(O)$, and the geometrical determination the Critical Zone are also given.

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Generalization of the Dependent Function in Extenics for Nested Sets with Common Endpoints to 2D-Space, 3D-Space, and Generally to n-D-Space

Florentin Smarandache

Abstract - In this paper we extend Prof. Yang Chunyan and Prof. Cai Wen's dependent function of a point *P* with respect to two nested sets $X_0 \subset X$, for the case the sets X_0 and X have common ending points, from *1D* - space to *n-D*-space. We give several examples in *2D*- and *3D*-spaces. When computing the dependent function value *k(.)* of the optimal point *O*, we take its maximum possible value.

Formulas for computing *k(O)*, and the geometrical determination the Critical Zone are also given.

I. Principle of Dependent Function

Principle of Dependent Function of a point *P(x)* with respect to a nest of two sets $X_0 \subset X$, i.e. *the degree of dependence of point P with respect to the nest of the sets* $X_0 \subset X$, is the following.

The dependent function value, *k(x)*, is computed as follows:

– the extension distance between the point P and the larger set's closest frontier, divided by the extension distance between the frontiers of the two sets {both extension distances are taken on the line/geodesic that passes through the point P and the optimal/attracting point O};

– the dependent function value is positive if point P belongs to the larger set, and negative if point P is outside of the larger set.

II. Dependent Function Formula for Nested Sets Having Common Ending Points in 1D-Space

For two nested sets $X_0 \subset X$ from the one-dimensional space of real numbers *R*, with X_0 and X having common endpoints, the **Dependent Function** *K(x)*, which gives the degree of dependence of a point *x* with respect to this pair of included *1D* -intervals, was defined by Yang Chunyan and Cai Wen in [2] as:

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Ref.

2. Yang Chunyan, Cai Wen. Extension Engineering [M]. Beijing: Public Library of Science, 2007.

$$K(x)=\begin{cases}\dfrac{\rho(x,X)}{\rho(x,X)-\rho(x,X_0)} & \rho(x,X)-\rho(x,X_0)\neq 0, x\in X\\ -\rho(x,X_0)+1 & \rho(x,X)-\rho(x,X_0)=0, x\in X_0\\ -\rho(x,X) & \rho(x,X)-\rho(x,X_0)=0, x\notin X_0, x\in X\\ \dfrac{\rho(x,X)}{\rho(x,X)-\rho(x,\hat{X})} & \rho(x,X)-\rho(x,\hat{X})\neq 0, x\in R-X\\ -\rho(x,\hat{X})-1 & \rho(x,X)-\rho(x,\hat{X})=0, x\in R-X\end{cases} \tag{1}$$

where $X_0 = \langle a_0, b_0\rangle$, $X = \langle a, b\rangle$, $\hat{X} = \langle c, d\rangle$, and $X_0 \subset X \subset \hat{X}$.

III. *N-D*-Dependent Function Formula for two Nested Sets Having no Common Ending Points

The extension *n-D*-dependent function *k(.)* of a point *P*, which represents the degree of dependence of the point *P* with respect to the nest of the two sets $X_0 \subset X$, is:

$$k(P)=\frac{\rho(P,BiggerSet)}{\rho(P,BiggerSet)-\rho(P,SmallerSet)}=\frac{\rho_{nD}(P,X)}{\rho_{nD}(P,X)-\rho_{nD}(P,X_0)}=\pm\frac{|PP_2|}{|PP_2|-|PP_1|}=\pm\frac{|PP_2|}{|P_1P_2|} \tag{2}$$

In other words, the extension *n-D*-dependent function *k(.)* of a point *P* is the *n-D*-extension distance between the point *P* and the closest frontier of the larger set *X*, divided by the *n-D*extension distance between the frontiers of the two nested sets *X* and X_0; all these n-Dextension distances are taken along the line (or geodesic) *OP*.

IV. *N-D*-Dependent Function Formula for two Nested Sets Having Common Ending Points

We generalize the above formulas (1) and (2) to an ***n-D* Dependent Function** of a point $P(x_1, x_2,\ldots, x_n)$ with respect to the nested sets X_0 and *X* having common endpoints, $X_0 \subset X$, from the universe of discourse *U*, in the *n-D*-space:

$$K_{nD}((x_1,x_2,\ldots,x_n))=\begin{cases}\dfrac{\rho_{nD}((x_1,x_2,\ldots,x_n),X)}{\rho_{nD}((x_1,x_2,\ldots,x_n),X)-\rho_{nD}((x_1,x_2,\ldots,x_n),X_0)} & \rho_{nD}((x_1,x_2,\ldots,x_n),X)-\rho_{nD}((x_1,x_2,\ldots,x_n),X_0)\neq 0,(x_1,x_2,\ldots,x_n)\in U\\ -\rho_{nD}((x_1,x_2,\ldots,x_n),X_0)+1 & \rho_{nD}((x_1,x_2,\ldots,x_n),X)-\rho_{nD}((x_1,x_2,\ldots,x_n),X_0)=0,(x_1,x_2,\ldots,x_n)\in X_0\\ -\rho_{nD}((x_1,x_2,\ldots,x_n),X) & \rho_{nD}((x_1,x_2,\ldots,x_n),X)-\rho_{nD}((x_1,x_2,\ldots,x_n),X_0)=0,(x_1,x_2,\ldots,x_n)\in U-X_0\end{cases} \tag{3}$$

V. Example 1 of Nested Rectangles with one Common Side

We have a factory piece whose desired *2D*-dimensions should be *20 cm* × *30 cm*, and acceptable *2D* dimensions *22 cm* × *32 cm*, but the two rectangles have common ending points. We define the extension *2D*-distance, and then we compute the extension *2D*-dependent function. Let's do an extension *2D*-diagram:

Notes

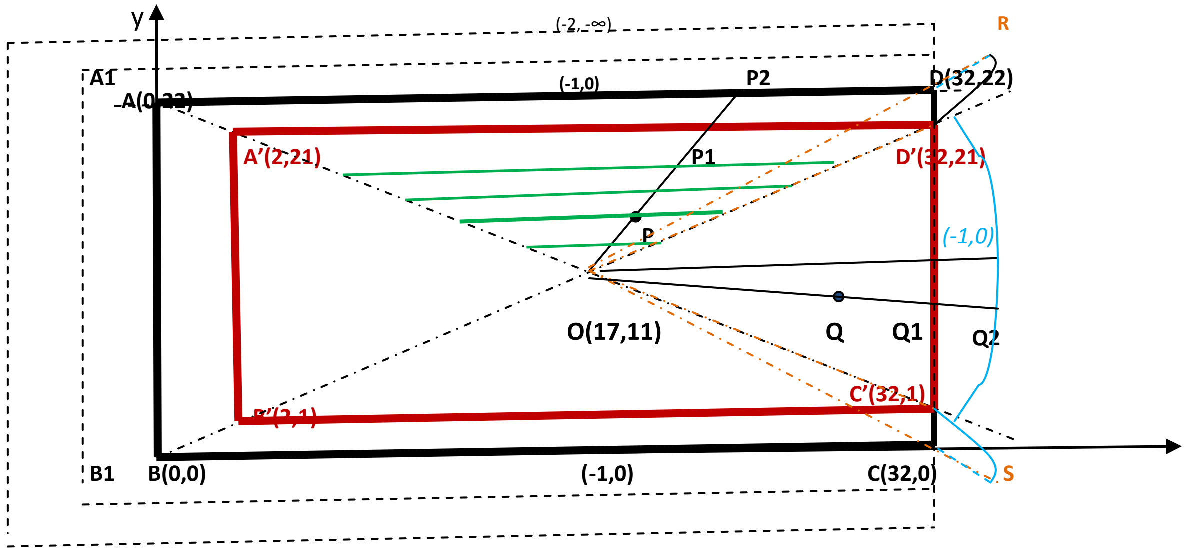

Diagram 1

The Critical Zone in the top, down, and left sides of the Diagram 1 as the same as for the case when the two pink and black rectangles have no common ending points. But on the right-hand side the Critical Zone is delimitated by the a blue curve in the middle and the blue dotted lines in the upper and lower big rectangle's corners.
The dependent function of the points Q, Q_1, Q_2 is respectively:

$$k(Q) = |QQ_1|+1, \text{ and } k(Q_1) = 1 \ (\text{if } Q_1 \in A'B'C'D') \text{ or } 0 \ (\text{if } Q_1 \notin A'B'C'D'), \text{ and } k(Q_2) = -|Q_2Q_1| = -1, \quad (4)$$

where $|MN|$ means the geometrical distance between the points M and N.
The dependent function of point P is normally computing:

$$k(P) = \frac{|PP_2|}{|P_1P_2|}. \quad (5)$$

VI. Example 2 of Nested Rectangles with two Common Sides

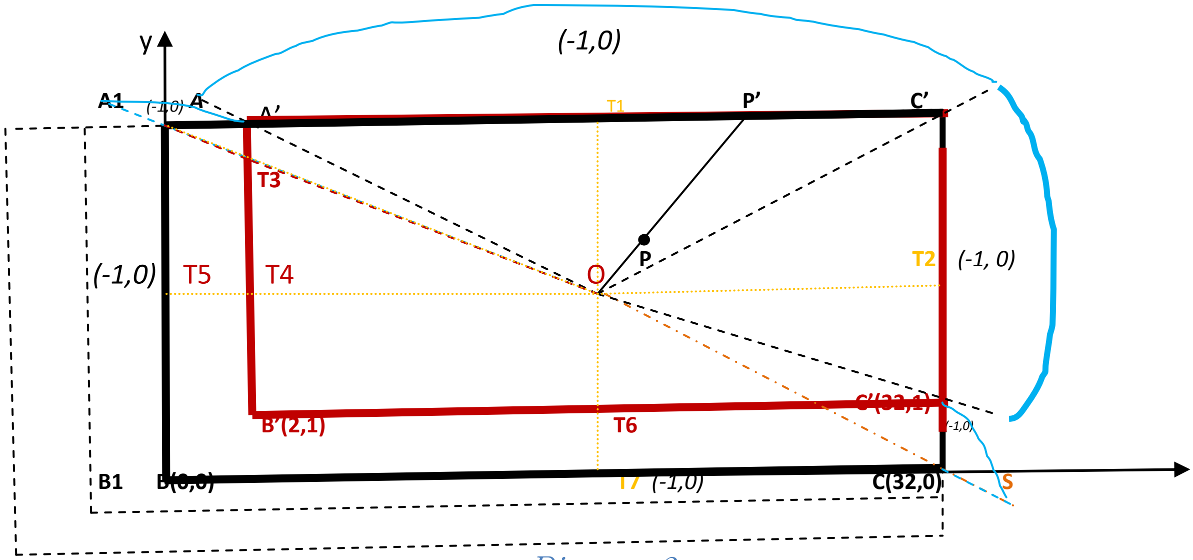

Diagram 2

We observe that the Critical Zone changes dramatically in the places where the common ending points occur, i.e. on the top and respectively left-hand sides. The Critical

Zone is delimitated by blue curves and lines on the top and respectively left-hand sides. Now, the dependent function of point P is different from the Diagram 1:

$$k(P)=|PP'|+1. \tag{6}$$

The dependent function of the optimal point O should be the maximum possible value. Therefore,

$$k(O) = \max\{|OT_1|+1,\ |OT_2|+1,\ |OP'|+1,\ |OC'|+1,\ \frac{|OT_7|}{|T_6T_7|}, \frac{|OT_5|}{|T_4T_5|}, \frac{|OA|}{|T_3A|}, \text{etc.}\} \tag{7}$$

VII. Example 3 of Nested Circles with one Common Ending Point

Assume the desirable circular factory piece radius is *6 cm* and acceptable is *8 cm*, but they have a common ending point P'.

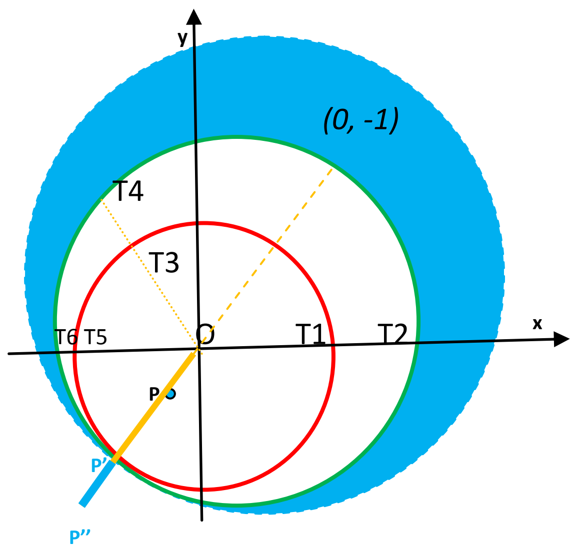

Diagram 3.

The Critical Zone is between the green and blue circles, together with the blue line segment $P''P'$ (this line segment resulted from the fact the P' is a common ending point of the red and green circles).

The dependent function values for the following points are:

$$k(P) = |PP'|+1; \tag{8}$$

k(P') = 1 (if *P'* belongs to the red circle), or *0* (if *P'* does not belong to the red circle); (9)

$$k(P'') = |P''P'|; \tag{10}$$

$$k(O) = \max\{|OP'|+1;\ \frac{|OT_4|}{|T_3T_4|}, \tag{11}$$

where T_3 lies arbitrary on the red circle, but $T_3 \neq P'$, and T_4 lies on the green circle but T_4 belongs to the line (or geodesic) OT_3}.

Notes

VIII. Example 4 of Nested Triangles with one Common Bottom Side

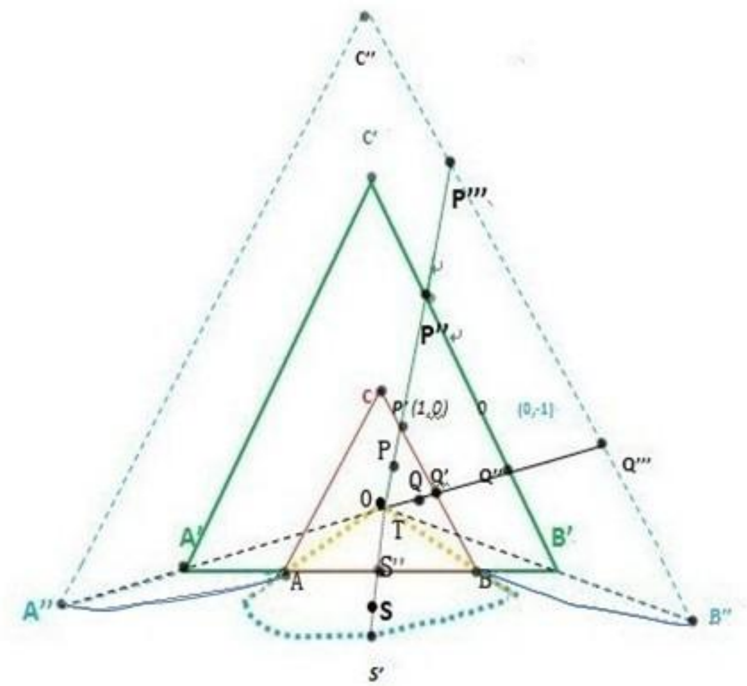

Diagram 4

The Critical Zone is between the green and blue dotted triangle to the left-hand and right-hand sides, while at the bottom side the Critical Zone is delimitated by the blue curve in the middle and the blue small oval triangles *A''AA'* and respectively *B''BB'*. The dependent function values of the following points are given below:

$$k(P)=\frac{|PP''|}{|P'P''|}>1;\ k(P')=1;\ k(P'')=0;\ k(P''')=-1. \quad (12)$$

Similarly:

$$k(Q)=\frac{|QQ''|}{|Q'Q''|}>1;\ k(Q')=1;\ k(Q'')=0;\ k(Q''')=-1. \quad (13)$$

With respect to the bottom common side (where the line segment *AB* lies on line segment *A'B'*) one has:

k(T) = |TS''|+1; k(S'') = 1 (if *S''* belongs to the red triangle *ABC),* or *0* (if *S''* does not belong to the red triangle *ABC); k(S) = |SS''|; k(S') = -1.* (14)

$$k(O)=\max\left\{\max_{S''\in[AB]}(|OS''|+1);\quad \max_{\substack{P'\in[AC]\cup[CB],\,P''\in[A'C']\cup[C'B']\\ P'\in OP''\\ OPP''\ line/geodesic}}\left(\frac{|OP''|}{|P'P''|}\right)\right\}. \quad (15)$$

Notes

IX. Example *5* in *3D*-Space of two Prisms Having a Common Face

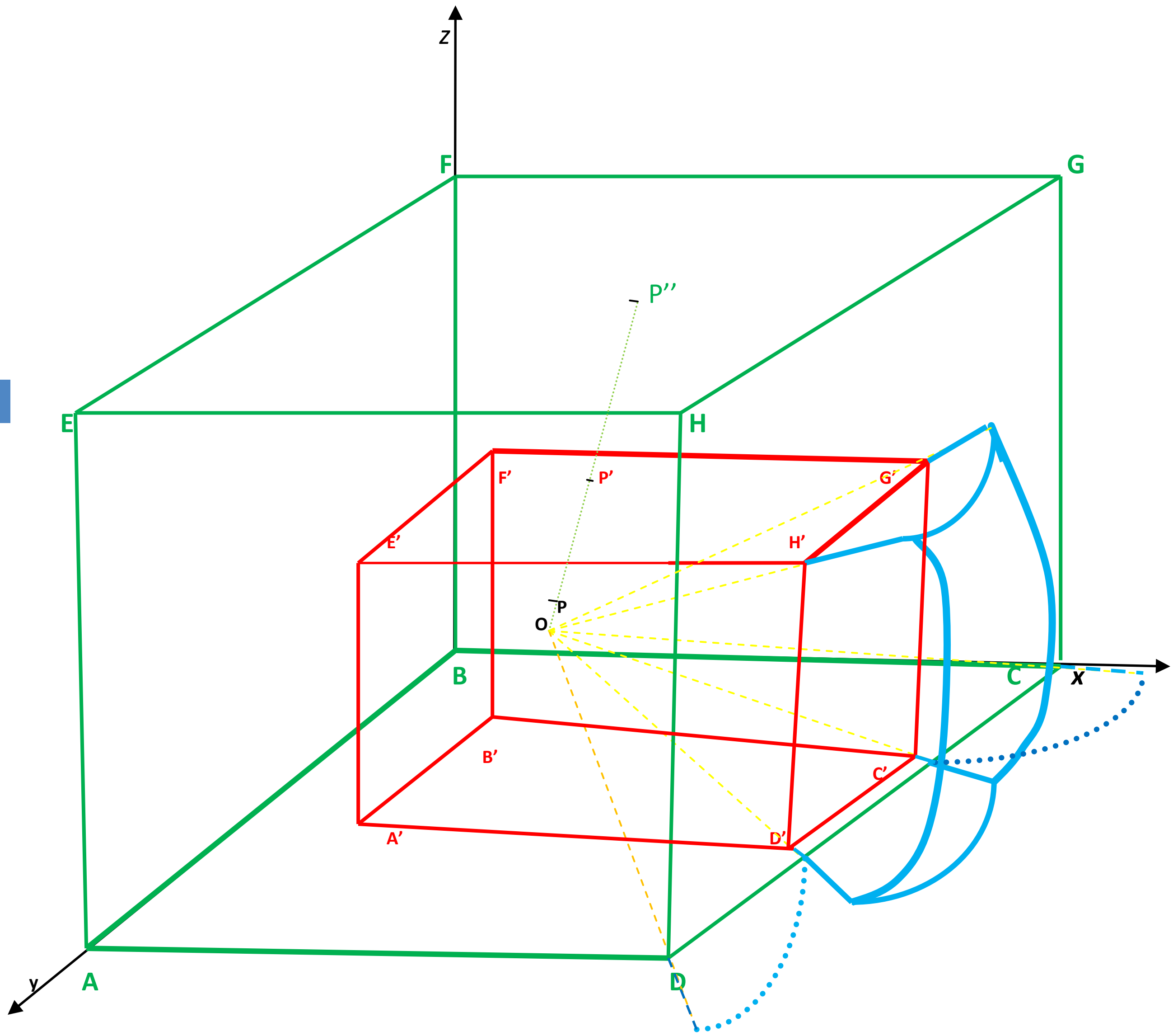

Diagram 5

The Critical Zone (the zone where the extension dependent function takes values between *0* and *-1*) envelopes the larger green prism *ABCDEFGH* at an equal distance from it as the distance between the red prism *A'B'C'D'E'F'G'H'* and the green prism *ABCDEFGH* with respect to the faces *ABCD, ADHE, BCGF, EFGH,* and *ABFE* (because these green faces and their corresponding red faces *A'B'C'D', A'D'H'E', B'C'G'F', E'F'G'H',* and respectively *A'B'F'E'* have no common points).

But the green face *DCGH* contains the red face *D'C'G'H'*, therefore for all their common points (i.e. all points inside of and on the rectangle *D'C'G'H'*) the extension dependent function has wild values. *D'C'G'H'* entirely lies on *DCGH*. The Critical Zone related to the right-hand green face *DCGH* and the red face *D'C'G'H'* is the solid bounded by the blue continuous and dashed curves on the right-hand side.

Notes

In general, let's consider two *n-D* sets, $S_1 \subset S_2$, that have common ending points (on their frontiers). Let's note by C_E their common ending point zone. Then:
The Dependent Function Formula for computing the value of the Optimal Point O is

$$k(O) = max \{ \max_{S'' \in C_E}(|OS''|+1); \quad \max_{\substack{P' \in Fr(S_1 - C_E), P'' \in Fr(S_2 - C_E) \\ P' \in OP'' \\ OPP'' line/geodesic}}(\frac{|OP''|}{|P'P''|}) \}. \quad (16)$$

We can define the Critical Zone in the sides where there are common ending points as:

$$Z_{C1} = \{P(x) | P \in U\text{-}S_2, 0 < d(P,P'') \leq 1, P'' \in Fr(S_1) \cap Fr(S_2) \text{ and } P'' \in OP\}, \quad (17)$$

where *d(P,P'')* is the classical geometrical distance between the points P and P''.
And for the sides which have no common ending points, the Critical Zone is:

$$Z_{C2} = \{P(x) | P \in U\text{-}S_2, 0 < d(P,P'') \leq d(P''P'), \text{ where } P'' \in Fr(S_2) \text{ and } P' \in Fr(S_1) \text{ and } P'' \in OP\}. \quad (18)$$

Whence, the total Critical Zone is: $Z_C = Z_{C1} \cup Z_{C2}$. (19)

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