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Solution of Fractional Kinetic Equation with Laplace and Fourier Transform

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Abstract - In earlier paper Saxena et al.(2002,2003)[18],[19] derived the solutions of a number of fractional kinetic equations in terms of generalized Mittage-Leffler functions which extended the work of Haubold and Mathai (2000)[5].The objects of present paper is to investigate the solution of fractional diffusion equation involving Mittag-Leffler functions. The method involves simultaneous application of Laplace and Fourier transforms with time and space variable respectively. The results obtained are in a form of H-function.

Keywords : Mittage-Leffler function, Fractional Kinetic Equation, Laplace Transform, Fourier Transform amd H-functions.

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SOLUTION OF FRACTIONAL KINETIC EQUATION WITH LAPLACE AND FOURIER TRANSFORM

Strictly as per the compliance and regulations of :

Solution of Fractional Kinetic Equation with Laplace and Fourier Transform

Satendra Kumar Tripathi [α] & Renu Jain [σ]

Abstract - In earlier paper Saxena et al.(2002,2003)[18],[19] derived the solutions of a number of fractional kinetic equations in terms of generalized Mittage-Leffler functions which extended the work of Haubold and Mathai (2000)[5].The objects of present paper is to investigate the solution of fractional diffusion equation involving Mittag-Leffler functions. The method involves simultaneous application of Laplace and Fourier transforms with time and space variable respectively. The results obtained are in a form of H-function.

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I. Introduction

Fundamental law of physics are written as equations for the time evolution of a quantity X(t), dX(t)/dt=-AX(t), where this could be Maxwell's equation or Schroedinger's equation (If A is limited to linear operators), or it could be Newton's law of motion or Einstein's equations for geodesics (If A may also be a non linear operator). The mathematical solution (for linear operators) is X(t) =X(0)Exp{-At}. The initial value of the quantity at t=0 is given by X(0).

The same exponential behavior referred to above arises if X(t) represents the scalar number density of species at time t that do not interact with each other. If one denote A_p the production rate and A_d the destruction rate, respectively, the number density obey an exponential equation where the coefficient A is equal to the different of A_p-A_d. Subsequently, A_p^{-1} is the average time between production and A_d^{-1} is the average time between destruction. This type of behavior arises frequently in biology, chemistry and physics (Hilfer, 2000; Metzler and Klafter, 2000) [6],[12]. This paper in Section 2 summarizes mathematical result concerning solution of the diffusion equations in section 3 and section 4 respectively, widely distributed in the literature or of very recent origin. These involve the Mittage-Leffler function, H-function and the application of fractional calculus, Fourier transform and Laplace transform to them.

The section 3 and section 4 presented in a closed form solution of a fractional diffusion equation in terms of H-function.

Ref.

6. **Hilfer,R(Ed):2000,** *Application of fractional calculus in physics, World Scientific,Singapore.*

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II. Mathematical Prerequisites

A generalization of the Mittage-Leffler function (Mittage-Leffler, 1903,1905)[9],[10]

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha+1)}, (\alpha \in C, Re(\alpha) > 0) \quad (1)$$

was introduced by wiman(1905)[20] in the general form

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha+\beta)}, (\alpha, \beta \in C, Re(\alpha) > 0) \quad (2)$$

The main result of these functions are available in the handbook of Erdelyi Magnus. Oberhettinger and Tricomi (1955, Section18.1)[4]and the monographs written by Dzherbashyas (1966,1993)[1][2], Prabhakar(1971)[14] introduced a generalization of (2) in the form

$$E_{\alpha,\beta}^{\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n z^n}{\Gamma(n\alpha+\beta)n!}, (\alpha, \beta, \gamma \in C, Re(\alpha) > 0) \quad (3)$$

Where

$$(\gamma)_0 = 1,\ (\gamma)_k = \gamma(\gamma+1)(\gamma+2) \ldots\ldots\ldots\ldots (\gamma+k-1)\ \ (k = 1,2 \ldots\ldots)\ \ \gamma \neq 0 \quad (4)$$

For $\gamma = 1$

$$E_{\alpha,\beta}^{1}(z) = E_{\alpha,\beta}(z),$$

For $\gamma = 1, \beta = 1$

$$E_{\alpha,1}^{1}(z) = E_\alpha(z) \quad (5)$$

The Mellin-Barnas integral representation for this function follows from the integral

$$E_{\alpha,\beta}^{\gamma}(z) = \frac{1}{\Gamma(\gamma)} \frac{1}{2\pi\omega} \int_{\Omega} \frac{\Gamma(-\xi)\Gamma(\gamma+\xi)(-z)^{\xi}}{\Gamma(\beta+\xi\alpha)} d\xi \quad (6)$$

where $\omega = (-1)^{1/2}$ The contour Ω is straight line parallel to the imaginary axis at a distance 'c' from the origin and separating the poles of $\Gamma(-\xi)$ at the point $\xi = \nu(\nu = 0,1,2 \ldots.)$ from those of $\Gamma(\gamma+\xi)$ at the points $\xi = -\gamma-\nu(\nu = 0,1,2 \ldots.)$. If we calculate the residues at the poles of $\Gamma(\gamma+\xi)$ at the points $\xi = -\gamma-\nu(\nu = 0,1,2 \ldots.)$ then it gives the analytic continuation formula of this function in the form[2]

$$E_{\alpha,\beta}^{\gamma}(z) = \frac{(-z)^{-\gamma}}{\Gamma(\gamma)} \sum_{\nu=0}^{\infty} \frac{\Gamma(\gamma+\nu)}{\Gamma(\beta-\alpha\gamma-\alpha\nu)} \frac{(-z)^{-\nu}}{\nu!}, |z| > 1 \quad (7)$$

Ref.

9. Mittage-Leffler, G.M.:1903, *Sur la nouvelle fonction* $E_\alpha(x)$, *C.R.Acad.Sci., Paris, (ser.II),* ***137****, 554-558.*

From (7) it follows that for large z its behavior is given by

$$E^{\gamma}_{\alpha,\beta}(z) \sim \mathrm{O}(|z|^{-\gamma}), |z| > 1 \tag{8}$$

The H-function is defined by means of Mellin-Barnes type integral in the following manner (Mathai and Saxena, 1978 p-2)[8]

$$H^{m,n}_{p,q}(z) = H^{m,n}_{p,q}\left[z\left|\begin{matrix}(a_p, A_p)\\(b_q, B_q)\end{matrix}\right.\right] = H^{m,n}_{p,q}\left[z\left|\begin{matrix}(a_1, A_1)\ldots.(a_p, A_p)\\(b_1, B_1)\ldots.(b_q, B_q)\end{matrix}\right.\right]$$

$$= \frac{1}{2\pi i}\int \theta(s) z^{-\xi}\, d\xi \tag{9}$$

$$where\ \theta(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j + B_j\xi)\prod_{j=1}^{n}\Gamma(1 - a_j - A_j\xi)}{\prod_{j=m+1}^{q}\Gamma(1 - b_j - B_j\xi)\prod_{j=n+1}^{p}\Gamma(a_j + A_j\xi)} \tag{10}$$

$$m, n, p, q \in N_0\ with\ 1 \le n \le p, 1 \le m \le q, A_j, B_j \in R_+\ a_j, b_j \in R$$

$$(i = 1,2\ldots..p, j = 1,2\ldots\ldots q)$$

$$A_i(b_j + k) \ne B_j(a_i - l - 1)\ \ (k, l \in N_0\,; i = 1,2\ldots.n, j = 1,2\ldots m) \tag{11}$$

Where we employ the usual notations $N_0 = (0,1,2\ldots)$ $R = (-\infty,\infty)$ $R_+ = (0,\infty)$ and C defines the complex number field. Ω is a suitable contour separating the poles of $\Gamma(b_j + B_j\xi)$ from those of $\Gamma(1 - a_j - A_j\xi)$.

A detailed and comprehensive account of the H-function along with convergence condition is available from Mathai and Saxena (1978)[8]

It follows from (7) that the generalized Mittag-Leffler function

$$E^{\gamma}_{\alpha,\beta}(z) = \frac{1}{\Gamma(\gamma)} H^{1,1}_{1,2}\left[-z\left|\begin{matrix}(1-\gamma, 1)\\(0,1)(1-\beta, \alpha)\end{matrix}\right.\right] (\alpha, \beta, \gamma \in C, Re(\alpha) > 0) \tag{12}$$

Putting $\gamma = 1$ in(12)

$$E_{\alpha,\beta}(z) = H^{1,1}_{1,2}\left[-z\left|\begin{matrix}(0,1)\\(0,1)(1-\beta, \alpha)\end{matrix}\right.\right] \tag{13}$$

If we further take $\beta = 1$ in (13) we get

$$E_{\alpha}(z) = H^{1,1}_{1,2}\left[-z\left|\begin{matrix}(0,1)\\(0,1)(0, \alpha)\end{matrix}\right.\right] \tag{14}$$

From Prudnikov, A.P., Brychkov, Yu.A. and Marichev, O.I (1989,p.355,eq2.25.3.2)[15] and Mathai and Saxena(1978,p.49)[8] it follows that the cosine transform of the H-function is given

Ref.

8. Mathai, A.M and Saxena, R.K.;1978, *The H-function with application in Statistics and other disciplines,Halsted Press[John Wiley and Sons],*

$$\int_0^\infty t^{\rho-1}\cos kt\, H_{p,q}^{m,n}\left[at^\mu \left|\begin{matrix}(a_p,A_p)\\(b_q,B_q)\end{matrix}\right.\right]dt$$

$$=\frac{\pi}{k^\rho}H_{q+1,p+2}^{n+1,m}\left[\frac{k^\mu}{a}\left|\begin{matrix}(1-b_q,B_q)\left(\frac{1}{2}+\frac{\rho}{2},\frac{\mu}{2}\right)\\(\rho,\mu)(1-a_p,A_p)\left(\frac{1}{2}+\frac{\rho}{2},\frac{\mu}{2}\right)\end{matrix}\right.\right] \quad (15)$$

The Riemann-Liouvile fractional integral of order $\upsilon \in C$ is defined by Miller and Ross(1993,p.45;)[11] see also Srivastva and saxena,2001)[17]

$$_0D_t^{-\nu}f(t)=\frac{1}{\Gamma(\nu)}\int_0^t (t-u)^{\nu-1}f(u)du \quad (16)$$

where $Re(\nu)>0$ following Samko, S.G., Kilbas, A. A. and Marichev, O.I. (1993,p.37)[16] we define the fractional derivative for $\alpha>0$ in the form

$$_0D_t^{\alpha}f(t)=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_0^t \frac{f(u)}{(t-u)^{\alpha-n+1}}du, (n=[Re(\alpha)]+1) \quad (17)$$

where $[Re(\alpha)]$means the integral part of $Re(\alpha)$.
In particular, if $0<\alpha<1$

$$_0D_t^{\alpha}f(t)=\frac{d}{dt}\int_0^t \frac{f(u)du}{(t-u)^\alpha} \quad (18)$$

And in $\alpha=n\in N$ then

$$_0D_t^{\alpha}f(t)=D^n f(t) \quad (19)$$

is the usual derivative of n.
From Erdelyi, A., Magnus, W., Oberhettinger, F., and Tricomi, F.G (1954,p.182) [3] we have

$$L\{\,_0D_t^{-\nu}f(t)\}=s^{-\nu}F(s) \quad (20)$$

$$F(s)=L\{f(t);s\}=\int_0^\infty e^{-st}f(t)dt \quad (21)$$

where $Re(s)>0$

The Laplace transform of the fractional derivative is given by Oldham and spanier(1974,p.134,eq 8.1.3;)[13]see also (srivastva and saxena 2001)[17]

$$L\{\,_0D_t^{-\nu}f(t)\}=s^{\alpha}F(s)-\sum_{k=1}^{n}s^{k-1}\,_0D_t^{\alpha-k}f(t)|_{t=0} \quad (22)$$

Ref.

11. **Miller, K.S and Ross, B.:1993** An introduction the Fractional Calculus and Fractional differential equation, John Wiley and Sons, Newyork.

In this we present solution of the fractional diffusion equation given by (Metzler and Klafter 2000;Jorgenson and Lang,2001)[12][7]

Theorem 1. Consider the fractional diffusion equation

$$\boldsymbol{N(x,t) - N_0 t^{\mu-1} = -c^{\nu}\ {}_0D_t^{-\nu}\ {}_0D_x^{\nu}\, N(x,t)} \tag{23}$$

with initial condition

$$\boldsymbol{{}_0D_t^{\nu-k}\, N(x,t)|_{t=0} = 0 \ \ and\ {}_0D_t^{-\nu-k}\, N(x,t)|_{x=0} = 0, k = 1,2 \dots n} \tag{24}$$

Where $n = [Re(\nu)] + 1; c^{\nu}$ *is diffusion constant then for the solution of (23) is given by*

$$\boldsymbol{N(x,t) = \frac{N_0\Gamma(\mu)}{c^t}\ H_{1,1}^{1,0}\left[\frac{|x|^{\nu}}{(ct)^{\nu}}\middle|\begin{matrix}(\mu+\nu,\nu)\\(1+\nu,\nu)\end{matrix}\right]} \tag{25}$$

Proof- $$N(x,t) - N_0 t^{\mu-1} = -c^{\nu}\ {}_0D_t^{-\nu}\ {}_0D_x^{\nu}\, N(x,t)$$

Apply Laplace and fourier transform with time variable and space vaiable respectively to (23) we get

$$N^*(k,s) - N_0\frac{\Gamma(\mu)}{s^{\mu}} = -c^{\nu}k^{\nu}s^{-\nu}N^*(k,s)$$

$$N^*(k,s)\{1 + ({}^{s}/_{c})^{-\nu}k^{\nu}\} = N_0 s^{-\mu}\Gamma(\mu)$$

$$N^*(k,s) = N_0 s^{-\mu}\Gamma(\mu)\left\{1 + \left({}^{s}/_{kc}\right)^{-\nu}\right\}^{-1}$$

$$= N_0 s^{-\mu}\Gamma(\mu)\sum_{r=0}^{\infty}\frac{(1)_r\left[-\left({}^{s}/_{kc}\right)^{-\nu}\right]^r}{r!}$$

$$= N_0\Gamma(\mu)\sum_{r=0}^{\infty}\frac{(1)_r(kc)^{r\nu}(-1)^r}{r!}s^{-\nu r-\mu}$$

where $N^*(k,s)$ Laplace and Fourier transform of $N(x,t)$
Taking inverse Laplace transform

$$N(k,t) = N_0\Gamma(\mu)\sum_{r=0}^{\infty}(kc)^{r\nu}\,(-1)^r L^{-1}\{s^{-\nu r-\mu}\}$$

Ref.

12. Metzler, R and Klafter,J.:2000 *The random walk's guide to anomalous diffusion: Afractionaldynamics approach, Phys.Rep.339,1-77.*

$$N(k,t) = N_0\Gamma(\mu)\sum_{r=0}^{\infty}(kc)^{r\nu}\,(-1)^r\frac{t^{\mu+r\nu-1}}{\Gamma(r\nu+\mu)}$$

$$= N_0\Gamma(\mu)t^{\mu-1}\,E_{\nu,\mu}(-c^\nu k^\nu t^\nu)$$

which can we expressed in terms of H-function

$$= N_0\Gamma(\mu)t^{\mu-1}H_{1,2}^{1,1}\left[c^\nu k^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(1-\mu,\nu)\end{matrix}\right.\right]$$

Now take inverse fourier transformation

$$N(x,t) = \frac{1}{\pi}\int_0^\infty \cos kx\; t^{\mu-1}N_0\Gamma(\mu)H_{1,2}^{1,1}\left[c^\nu k^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(1-\mu,\nu)\end{matrix}\right.\right]dk$$

$$= \frac{t^{\mu-1}N_0\Gamma(\mu)}{\pi}\frac{\pi}{|x|}H_{3,3}^{2,1}\left[\frac{|x|^\nu}{(ct)^\nu}\left|\begin{matrix}(1,1)(\mu,\nu)\left(1,{}^{\nu}\!/_{2}\right)\\(1,1)(1,\nu)\left(1,{}^{\nu}\!/_{2}\right)\end{matrix}\right.\right]$$

Applying a result of Mathai and Saxena (1978, p.4.eq1.2.1) the above expression becomes

$$N(x,t) = \frac{N_0\Gamma(\mu)}{|x|}H_{2,2}^{2,0}\left[\frac{|x|^\nu}{(ct)^\nu}\left|\begin{matrix}(\mu,\nu)\left(1,{}^{\nu}\!/_{2}\right)\\(1,\nu)\left(1,{}^{\nu}\!/_{2}\right)\end{matrix}\right.\right]$$

If we employ the formula Mathai and Saxena (1978,p.4.eq1.2.4)

$$x^\sigma H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_p,A_p)\\(b_q,B_q)\end{matrix}\right.\right] = H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_p+\sigma A_p,A_p)\\(b_q+\sigma B_q,B_q)\end{matrix}\right.\right]$$

$$N(x,t) = \frac{N_0\Gamma(\mu)}{ct}H_{2,2}^{2,0}\left[\frac{|x|^\nu}{(ct)^\nu}\left|\begin{matrix}(\mu+\nu,\nu)\left(1,{}^{\nu}\!/_{2}\right)\\(1+\nu,\nu)\left(1,{}^{\nu}\!/_{2}\right)\end{matrix}\right.\right]$$

$$N(x,t) = \frac{N_0\Gamma(\mu)}{ct}H_{1,1}^{1,0}\left[\frac{|x|^\nu}{(ct)^\nu}\left|\begin{matrix}(\mu+\nu,\nu)\\(1+\nu,\nu)\end{matrix}\right.\right]$$

Theorem 2- Consider the fractional diffusion equation (Metzler and Klafter 2000;Jorgenson and Long,2001)[12][7]

$$_0D_t^\nu\, N(x,t) - E_\nu(-d^\nu t^\nu) = -c^\nu\frac{\partial^2}{\partial x^2}N(x,t) \qquad (26)$$

with initial condition

Ref.

7. **Jorgenson, J. and Lang, S.:2001,** *The ubiquitous heat kernel, in mathematics Unlimited-2001 and Beyond, Eds.B. Engquist and W.Schmid, Springer-Verlag.Berlin and Heidelberg.*

$$ {}_0D_t^{\nu-k}\, N(x,t)|_{t=0} = 0 \quad k = 1,2..n \tag{27}$$

Where $n = [Re(\nu)] + 1; c^\nu$ *is diffusion constant.*
Then for the solution of (26) is given by

Notes

$$\frac{1}{2d^{\nu/2}} \sin\left(d^{\nu/2} x\right) * \frac{1}{(ct)^\nu} H_{1,1}^{1,0}\left[\frac{|x|^2}{(ct)^\nu}\middle| \begin{matrix}(1-\nu/2,\nu)\\(0,2)\end{matrix}\right]$$

$$-\frac{1}{2d^{\nu/2}} \sin\left(d^{\nu/2} x\right) H_{1,2}^{1,1}\left[d^\nu t^\nu \middle| \begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right] \tag{28}$$

Proof-

$${}_0D_t^\nu\, N(x,t) - E_\nu(-d^\nu t^\nu) = -c^\nu \frac{\partial^2}{\partial x^2} N(x,t)$$

Applying the fourier transform with respect to the space variable *x* and the Laplace transform with respect to the time variable *t.* we get

$$s^\nu N^*(k,s) - \frac{s^{\nu-1}}{s^\nu + d^\nu} = -c^\nu k^2 N^*(k,s)$$

$$\{s^\nu + c^\nu k^2\} N^*(k,s) = \frac{s^{\nu-1}}{s^\nu + d^\nu}$$

$$N^*(k,s) = \frac{s^{\nu-1}}{\{s^\nu + d^\nu\}\{s^\nu + c^\nu k^2\}}$$

$$= \frac{s^{\nu-1}}{c^\nu k^2 - d^\nu}\left[\frac{1}{s^\nu + d^\nu} - \frac{1}{s^\nu + c^\nu k^2}\right].. \tag{29}$$

To invert equation(29).It is convenient to first invert the Laplace transformation and fourier transform.Apply inverse Laplace transform we obtain

$$N(k,t) = \frac{1}{c^\nu k^2 t^\nu}\left[E_\nu(-d^\nu t^\nu) - E_\nu(-c^\nu k^2 t^\nu)\right].. \tag{30}$$

Which can expressed in terms of H-function

$$N(k,t) = \frac{1}{c^\nu k^2 - d^\nu}\left\{H_{1,2}^{1,1}\left[d^\nu t^\nu \middle|\begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right] - H_{1,2}^{1,1}\left[c^\nu k^\nu t^\nu \middle|\begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right]\right\} \tag{31}$$

Invert the fourier transform

$$N(x,t) = \frac{1}{\pi}\int_0^\infty coskx \frac{1}{c^\nu k^2 - d^\nu}\left\{H_{1,2}^{1,1}\left[d^\nu t^\nu \middle| \begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right]dk\right.$$

$$\left. -\frac{1}{\pi}\int_0^\infty coskx \frac{1}{c^\nu k^2 - d^\nu} H_{1,2}^{1,1}\left[c^\nu k^\nu t^\nu \middle| \begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right]dk\right\}$$

$$= -\frac{1}{2d^{\nu/2}}\sin\left(d^{\nu/2}x\right) H_{1,2}^{1,1}\left[d^\nu t^\nu \middle| \begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right] + \frac{1}{2d^{\nu/2}}\sin\left(d^{\nu/2}x\right)$$

$$* \frac{1}{|x|} H_{3,3}^{2,1}\left[\frac{|x|^2}{(ct)^\nu}\middle| \begin{matrix}(1,1)(1,\nu)(1,1)\\(1,2)(1,1)(1,1)\end{matrix}\right]$$

$$= -\frac{1}{2d^{\nu/2}}\sin\left(d^{\nu/2}x\right) H_{1,2}^{1,1}\left[d^\nu t^\nu \middle| \begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right]$$

$$+ \frac{1}{2d^{\nu/2}}\sin\left(d^{\nu/2}x\right) * \frac{1}{(c^\nu t^\nu)^{1/2}} H_{2,2}^{2,0}\left[\frac{|x|^2}{(ct)^\nu}\middle| \begin{matrix}\left(1-\nu/2,\nu\right)\left(1/2,1\right)\\(0,2)\left(1/2,1\right)\end{matrix}\right]$$

$$= \frac{1}{2d^{\nu/2}}\sin\left(d^{\nu/2}x\right) * \frac{1}{(ct)^\nu} H_{1,1}^{1,0}\left[\frac{|x|^2}{(ct)^\nu}\middle| \begin{matrix}\left(1-\nu/2,\nu\right)\\(0,2)\end{matrix}\right]$$

$$- \frac{1}{2d^{\nu/2}}\sin\left(d^{\nu/2}x\right) H_{1,2}^{1,1}\left[d^\nu t^\nu \middle| \begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right]$$

Notes

III. Conclusion

The fractional kinetic equation has been extended to generalized fractional equation (23) and (26). Their respective solutions are given in terms of Mittag-Leffler function and their generalization, which can also be represented as Fox's H-function.

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