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Elastic Scattering of ^{28}Si from Target Nuclei ^{27}Al at Energies 70, 80, 90 and 100 MeV by Strong Absorption Model (SAM)

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Strictly as per the compliance and regulations of :

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I. Introduction

The scattering of p, n, d, τ, ^{3}He and alpha particles in particular, has been playing a very important and vital role in nuclear physics since the very beginning of the subject. The nuclear scattering experiment ascertains many properties of nuclei such as angular momentum, parity, nuclear size, nuclear density etc. Experimental techniques, so far have achieved greater perfection and theoretical interpretation of data has become correspondingly more accurate and detailed.

Nucleus is a complicated, many body problems and a bound system of nucleons, with very short range interaction. Nucleons or other strongly interacting particles can induce a variety of nuclear reactions, whose diversity is due to the individual properties, relative motions, energies of the colliding particles and the target nuclei. Simple and fundamental laws are required in interpreting data to unravel the known properties of the nuclei and this enables us to predict the unknown properties also.

The scattering involving complex nuclei represents a complicated quantum mechanical many-body problem and it is difficult to correlate the experimental data directly with the properties of fundamental nuclear interactions. It is necessary to devise simpler methods (models) which serve as an intermediary between the data and basic nuclear theory. These methods make use of simplifying assumptions by which certain average features of the many-body problem are connected directly with measurable quantities.

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Numerous analysis of the elastic and scattering data of different projectiles, carried out using the SAM formalism by Frahn and Venter [1] during the past several years as available in refs.[2-6] is quite successful in analyzing the scattering data. This model does not suffer from any ambiguities and the model yields a unique set of parameters to describe the experimental data.

In this present work elastic scattering have been analyzed by means of Strong Absorption Model (SAM). All the elastic scattering data are digitized at near barrier energies close to the Coulomb barrier. The analysis of elastic scattering data will help us to determine the parameters like the cut-off or critical angular momentum T, rounding parameter Δ, and the real nuclear phase shift μ. The elastic scattering data have been digitized from different references [7-11]

II. Strong Absorption Model Formalism

a) Strong Absorption Model

Here we introduce the strong absorption model formalism, which is frequently used. The strong absorption generally takes place at medium and high energy projectiles in nuclear reactions for the cases below:

1. Nucleons, mesons and hyperons of E ≥ 100 MeV.
2. Composite particles (deuterons, tritons, helium-3, alpha particles and heavy ions) above the Coulomb barrier.

The depletion of the elastic channels due to the presence of open reaction channels is termed as strong absorption. It is measured by the deviation from the unitarity of the elastic η_l sub-matrix. The condition of the effectiveness for the strong absorption of these partial waves is

$$\eta_l^j \ll 1 \tag{1.1}$$

This condition holds well for some situations in a certain range of orbital angular momentum below a critical value l_0. From this point of view, the scattering is closely identical to diffraction by an opaque obstacle. The relevant approximations concerning such situations are called diffraction models. The description of the diffraction in nuclear processes is more accurate in momentum space than in configuration space as the relation $\Delta L.\Delta\theta \geq \hbar$ is valid in the former. We shall therefore express SAM formalisms in momentum space.

The transition of η_l from zero to unity is a gradual one, extending over a range of l values of width Δ in the vicinity of T, this follows semi-classically from the diffuseness of the nuclear interaction region. Particles, which are moving along classical orbits penetrating the diffuse region, will be only partially absorbed. If Δ is the range of orbital angular momentum that corresponds to the diffuseness d, we obtain Δ for nuclear particles.

$$\Delta = kd \qquad (1.2)$$

and for charged particles

$$\Delta = kd\frac{1-\left(\frac{n}{2kR}\right)}{\left[1-\frac{2n}{kR}\right]^{\frac{1}{2}}} \qquad (1.3)$$

It is possible to give a completely analytical formulation of the parameterized S-matrix model of η_l in l space with or without Coulomb interaction. This can be done by splitting η_l into real and imaginary parts;

$$Re[\eta_l exp(-2i\sigma_l)] = g(t) + \rho\frac{dg}{dt} + \varepsilon[1 - g(t)] \qquad (1.4)$$

$$Im[\eta_l exp(-2i\sigma_l)] = \mu_1\frac{dg(t)}{dt} + \mu_2\frac{d^2g(t)}{dt^2} \qquad (1.5)$$

Here, g's are continuously differentiable function of $^{(T-t)}/_{\Delta}$, whose first derivatives are symmetric and peaked at around T but otherwise arbitrary. Furthermore the function g's are characterized by the cut-off angular momentum $T^{\pm} = \left(L \pm \frac{1}{2}\right)$ and rounding parameter $\Delta^{\pm}$ around $T^{\pm}$ in the l space and possessing the property that their derivatives should have simple Fourier transform; the parameter $\mu^{\pm}$ is associated with the real nuclear phase shift and $\varepsilon^{\pm}$ accounts for any possible transparency of partial waves less than $T^{\pm}$.

Equations (1.4) and (1.5) cover a large variety of structures of η_l in strong absorption situations; the real part changes from finite value at small l value to unity at high l value through some rapid transition in the vicinity of T; the form of the imaginary part is such that the real nuclear phase shifts are relevant only for partial waves in some vicinity of T, except for transparency contribution at lower l values. The first derivative of g(t) is the main term in Im η_l. The higher derivatives in the real and imaginary parts of η_l describe possible asymmetries and other complicated variations in the transition region. For charge particles, η_l is replaced by,

$$\eta_l \exp(-2i\sigma_l)$$

where, σ_l are Coulomb phase-shifts.

b) Coulomb Scattering Angle

The Coulomb scattering angle θ_C is related to cut-off parameter T through the relation

$$\theta_C = 2arctg\left(\frac{n}{T}\right) \qquad (1.6)$$

The angular distribution is divided into two regions:

a) Coulomb region for $\theta \leq \theta_C$ and

b) Diffraction region for $\theta > \theta_C$

c) Total Reaction Cross section

The total reaction cross section can be calculated using the following formulation

$$\sigma_r = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\left(1 - |\eta_l|^2\right) \qquad (1.7)$$

which, for spin zero charged particles becomes,

$$\sigma_r = \frac{\pi T^2}{k^2}\left[1 + 2\frac{\Delta}{T} + \frac{1}{3}\pi^2\left(\frac{\Delta}{T}\right)^2 - \frac{1}{3}\left(\frac{\mu}{\Delta}\right)^2\left(\frac{\Delta}{T}\right)\right] \qquad (1.8)$$

This formula has been used by Frahn and Venter[1] for calculating the value of total reaction cross-section.

III. Method of Analysis

Here, we discuss the method of theoretical analysis of the experimental elastic scattering cross-sections of heavy ions at various projectile energies. The elastic scattering analysis yields unambiguous elastic scattering parameter values.

The method of analysis and the effects of parameter variations on the angular distribution have been given by Rahman et al. [12]. The angular distribution of the elastically scattered particles from a target nucleus is obtained from the relation

$$\sigma(\theta) = |f(\theta)|^2 \qquad (1.9)$$

where $f(\theta)$ is the scattering amplitude. The amplitude can be calculated using the following parameters:

a. The cut-off angular momentum parameter, T

b. The rounding parameter , Δ

c. The real nuclear phase-shift parameters, μ_1 and μ_2

d. The symmetry parameter, ρ and

e. The transparency parameter, ε.

The cut-off angular momentum T is related to the interaction radius R through the semi-classical relation:

$$T = kR\left[1-\left(\frac{2n}{kR}\right)\right]^{1/2} \quad (1.10)$$

The rounding parameter is related to the diffuseness of the nuclear surface through the relation

$$\Delta = kd\left[\left(1-\frac{n}{kR}\right)\left(1-\frac{2n}{kR}\right)^{-1/2}\right] \quad (1.11)$$

where k is the wave number and n is the coulomb parameter respectively.
The total reaction cross-section is given by,

$$\sigma_r = \frac{\pi T^2}{k^2}\left[1+2\frac{\Delta}{T}+\frac{1}{3}\pi^2\left(\frac{\Delta}{T}\right)^2-\frac{1}{3}\left(\frac{\mu}{\Delta}\right)^2\left(\frac{\Delta}{T}\right)\right] \quad (1.12)$$

The frequency of the oscillation in $\sigma(\theta)$ is determined by the parameter T. By increasing T, the whole oscillation pattern moves towards the smaller angles. The parameter Δ controls the ratio of the backward to forward scattering through which the average slope of the angular distribution is fixed. The higher angle regions are mainly affected by an alteration in Δ value and an increase in Δ mainly lowers the maximum keeping the oscillatory pattern unaltered.

The parameter μ mainly affects the minimum and an increase in μ lowers the minimum keeping the angular position and magnitude of the maxima and the whole angular distribution pattern unaltered.

We use a computer program in analyzing scattering phenomena. The program takes the input from one file and produces output to another file. It is desirable that the output of such a program should be in a graphical presentation. The output file is imported onto a graphical program and then resulting graph is plotted.

First we make the three parameters ρ, ε and μ_2 equal to zero, because these parameters have very insignificant effects on the angular distribution for heavy ion projectiles. To determine the SAM parameters, T should be fixed first. The method followed in determining the parameters are:

1. At first we varied T, say we keep the value of T is 30, keeping Δ and μ fixed to a small value, say 0.5 and 0.1 respectively. (For Δ=0, the program will not run, division by zero error will occur).
2. Graphs are plotted simultaneously for various values of T, finally it is varied again with a smaller step size.
3. Since the minima are sharp in general, it is a helpful endeavor to reproduce the positions of minima while fixing T.
4. After having a good fixation of T, then the value of Δ is adjusted, which determines the slope of the angular distribution and whose effect is prominent in the larger angular region.
5. Once the values of T and Δ have been fixed, we vary μ in order to minimize the mean square difference between the experimental and theoretically computed cross-sections.

The mean square difference between experimental and computed cross-section, χ^2 is a measure of how good the fit is. The χ^2 is given by,

$$\chi^2 = \frac{1}{n}\sum\left|\frac{\sigma_{\exp}(\theta)-\sigma_{theo}(\theta)}{\delta\sigma_{exp}(\theta)}\right|^2 \quad (1.13)$$

Here n is the number of data points and other symbols carry the usual meanings.

Finally, all three parameters T, Δ and μ are varied slightly about the obtained values till the best fit parameters are obtained and hence the minimum value of χ^2.

The charge and mass numbers of the projectile and the target, the beam energy, the scattering angles and the corresponding experimental cross-sections and their errors together with the values of the parameters are given in the input of the program. The output gives $\sigma(\theta)$ corresponding to the scattering angle θ with χ^2 for each set of parameters. The interaction radius R, the diffuseness d, standard nuclear radius r_0 and the total reaction cross-section σ_r are computed from the best fit parameters.

IV. Results and Discussions

The differential cross-section for the elastic scattering of ^{28}Si from target nuclei ^{27}Al has been studied on the basis of the Strong Absorption Model formalism (SAM). Data analysis are carried out by a symmetric variation of SAM parameters using the criterion of minimum root square difference between the experimental and theoretical cross-sections.

The result of the SAM analysis rendering the best fit parameter values are summarized in tables 1 and 2. The experimental data along with the theoretically calculated angular distributions are graphically shown in figs.1-4. The quality of fit to the angular distribution throughout the distribution is satisfactory.

Now for further details of the fit quality, the angular distributions data in most of the nuclei are reasonably well reproduced over the angular range covered in the experiment.

a) The Sam Parameters T, Δ And μ

The cut-off angular momentum T and the rounding parameter Δ are respectively given by the expressions (1.10) and (1.11). Their values are shown in the tables 1 and 2. The cut-off angular momentum T increases smoothly with the increase in the incident energy.

The value of Δ is roughly the same for the same target masses for different energies, as for example, the Δ-value assumes 1.0 for the same target mass ^{27}Al for the projectile ^{28}Si for different projectile energies. The rounding parameter Δ controls the ratio of the backward to the forward scattering angle. An increase in Δ mainly affects the cross-sections in the higher angle regions, while the lower angle regions are not affected so much; an increase in Δ value lowers the whole diffraction pattern keeping the oscillatory structure unaltered. The value of real nuclear phase shift μ lies in the domain 0.5≤ μ ≤ 0.7.

b) Interaction Radius R, Surface Diffuseness d and Coulomb Scattering Angle θ_C

The interaction radius R and the surface diffuseness d are respectively given by the semi-classical expressions (1.10) and (1.11). They are presented in tables 1 and 2.

We find from the table 2 that the interaction radius R decreases with increase in beam energy as long as the mass of the projectile and target remain the same. As for example, the interaction radii R for ^{28}Si elastically scattered from ^{27}Al for the projectile energies 70.00 MeV and 80.00 MeV, are 9.97 fm and 9.94 fm respectively.

Our study further yields the fact that the values of surface diffuseness parameter d roughly spreads in the range 0.165-0.326 fm.

The value of θ_C given by the expression (1.6) and the value is presented in table 1. The value of θ_C generally decreases with the increase in the beam energy for the same projectiles and target nuclei i.e. the value of θ_C decreases as the value of T increases and vice versa. As for example the value of θ_C is 76.48° for the projectile energy 70 MeV at T value 23 and the value of θ_C is 60.62° for the projectile energy 80 MeV at T value 29.

c) The Total Reaction Cross-Section

The total reaction cross-section σ_r yielded by SAM formalism is given by the equation (1.8). These are shown in table 2. The value of σ_r, in general, increases for the same projectile and the target masses as the projectile energy increases. This may be due to the opening of many reaction channels as the beam energy is increased.

The parameter $\sigma_r/\pi R^2$ is more meaningful than σ_r itself. Its value is of the same order of magnitude (0.2-0.4), which is roughly the same as expected.

Our calculated cross section could not be compared for non availability of any other calculations for cross-sections from any other formalism.

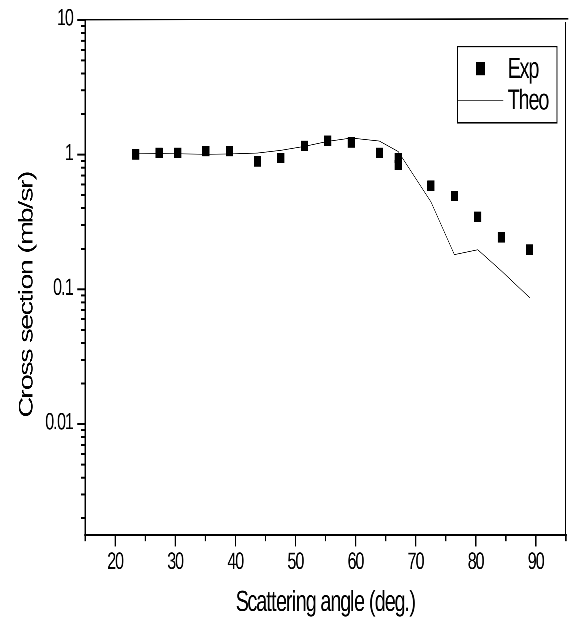

Fig. 1 : SAM analysis for elastic scattering of ^{28}Si from ^{27}Al at energy 70 MeV

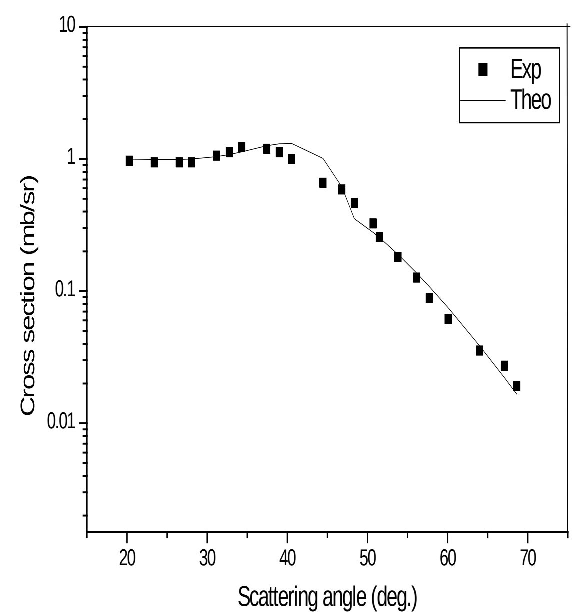

Fig. 2 : SAM analysis for elastic scattering of ^{28}Si from ^{27}Al at energy 90 MeV

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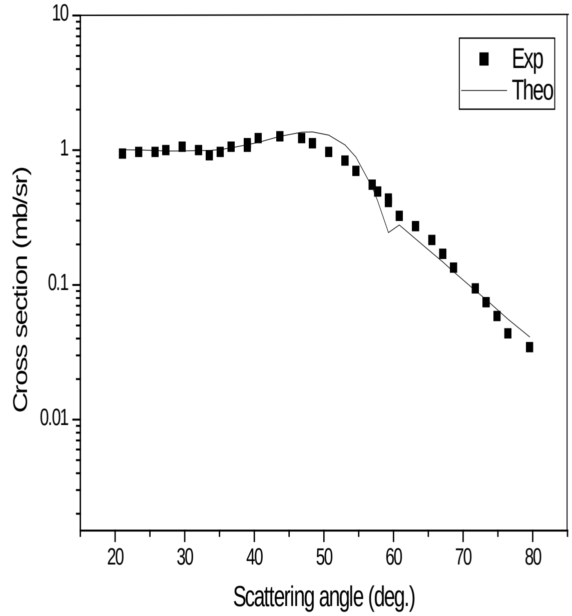

Fig. 3 : SAM analysis for elastic scattering of ^{28}Si from ^{27}Al at energy 80 MeV

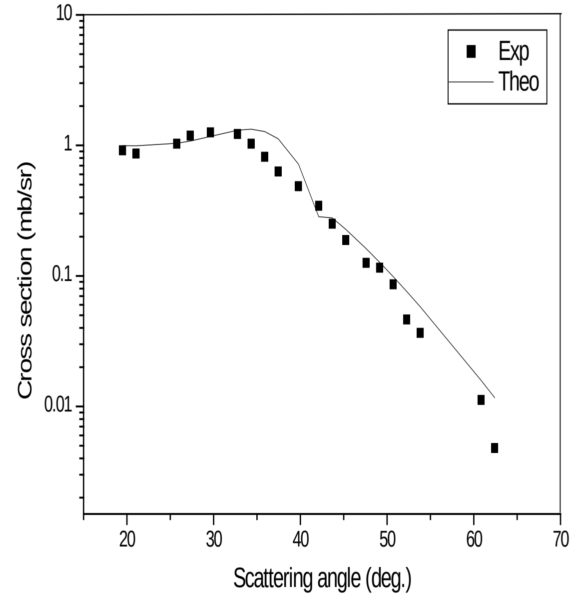

Fig. 4 : SAM analysis for elastic scattering of ^{28}Si from ^{27}Al at energy 100 MeV

Table 1

No	Incident particle + Target nucleus	Beam energy 'E' MeV	T	Δ	μ	$\mu/4\Delta$	θ_C
1	^{28}Si + ^{27}Al	70	23	1	0.5	0.125	76.48
2	^{28}Si + ^{27}Al	80	29	1	0.5	0.125	60.62
3	^{28}Si + ^{27}Al	90	34	2	0.5	0.0625	50.36
4	^{28}Si + ^{27}Al	100	38	2	0.7	0.0875	43.51

Table 2

No	Incident particle + Target nucleus	Beam energy 'E' MeV	r_0	R	d	σ_r	$\sigma_r/\pi R^2$
1	^{28}Si + ^{27}Al	70	1.65	9.97	0.165	2565	0.246
2	^{28}Si + ^{27}Al	80	1.64	9.94	0.169	3522	0.338
3	^{28}Si + ^{27}Al	90	1.64	9.93	0.335	4546	0.438
4	^{28}Si + ^{27}Al	100	1.63	9.87	0.326	5107	0.493

V. Conclusions

The present work was concerned with a study of the elastic scattering of heavy ion ^{28}Si at different energies (70 – 100) MeV from target nuclei ^{27}Al. The motivation was to see to what extent the simple geometrical model can explain the elastic scattering.

The angular distribution have been studied in terms of Strong Absorption Model due to Frahn and Venter[1] and it is evident from these analyses that three parameter SAM formalism provides a reasonable description to elastic scattering of heavy ions. The best fit parameters T, Δ and μ have been obtained. Analysis of the elastic angular distribution have resulted in a consistent set of SAM parameters from which interaction radius R and surface diffuseness d are obtained. The interaction radius increases smoothly as the target mass increases.

It is also observed that R in general decreases with the increases in the beam energy for the same target mass. The surface diffuseness d determined from this work over the incident energy and mass region covered remains roughly the same and agrees with other works [4-6].

The value of the Coulomb scattering angle θ_C generally decreases with the increase in the beam energy for the same projectiles and target nuclei. Coulomb scattering angle θ_C is directly proportional to Coulomb parameter n and related reciprocally with T.

The reaction cross-section σ_r was also calculated from the SAM parameters. The value of σ_r, in general, increases for the same projectile and the target masses as the projectile energy increases.

Finally from this present work we can say that SAM model is a useful, easier, simple method for obtaining various information about nuclear properties. We can also say that an overall good description of the scattering of heavy ions is given by the three parameters of SAM of Frahn and Venter [1].

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