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Exponential Chain Ratio and Product type Estimators for Finite Population Mean Under Double Sampling Scheme

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Abstract - In this paper an exponential chain ratio and product type estimators in double sampling have been developed for estimating finite population mean of the study variable when the information on another additional auxiliary character is available along with the main auxiliary character. The bias and mean square error of the proposed estimators have been obtained in two different cases. Theoretical and empirical studies have been done to demonstrate the efficiency of the proposed strategy with respect to the strategies which utilizes the information on one and two auxiliary characteristics.

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Strictly as per the compliance and regulations of :

Exponential Chain Ratio and Product type Estimators for Finite Population Mean Under Double Sampling Scheme

B. K. Singh[α] & Sanjib Choudhury[σ]

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I. Introduction

Information on variables correlated with the main variable under study is popularly known as auxiliary information which may be fruitfully utilized either at planning stage or at design stage or at the information stage to arrive at an improved estimator compared to those, not utilizing auxiliary information. Use of auxiliary information for forming ratio and regression method of estimation were introduced during the 1930's with a comprehensive theory provided by Cochran (1942).

When information on any auxiliary variable x highly correlated with y is readily available on all units of the population, it is well known that ratio and regression estimators provide more efficient estimators of population mean of y, having advance information on population mean $\bar{X}$ of x. However, in many situations of practical importance, the population mean $\bar{X}$ is not known before the start of a survey. In such a situation, the usual thing to do is to estimate it by the sample mean $\bar{x}_1$ based on a preliminary sample of size n_1 of which n is a sub sample $(n < n_1)$. At the most, we use only knowledge of the population mean of another auxiliary character, which is comparatively less correlated to the main characters. That is, if the population mean $\bar{Z}$ of another auxiliary variate Z, closely related to X but compared to X remotely related to Y is known, it is advisable to estimate $\bar{X}$ by $\bar{X} = \bar{x}_1 \bar{Z} / \bar{z}_1$,which would provide better estimate of $\bar{X}$ than $\bar{x}_1$.

Chand (1975) and Sukhatme and Chand (1977) proposed a technique of chaining the available information on auxiliary characteristics with the main characteristics. Kiregyera (1980, 1984) also proposed some chain type ratio and regression estimators based on two auxiliary variates. Further contribution are due to Srivastava and Jhajj

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(1980, 1995), Isaki (1983), Singh and Kataria (1990), Prasad and Singh (1990, 1992), Ahmed *et al.* (2000), Singh and Singh (2001), Al-JPararha and Ahmed (2002), Ahmed *et al.* (2003), Pradhan (2005), Singh *et al.* (2006), Singh *et al.* (2009) and many others.

Let us consider a finite population $U=\{U_1,U_2,...U_N\}$ of size N units and the value of the variables on the i[th] unit U_i, $i=1,2,...,N$, be (y_i,x_i). Let $\bar{Y}=\sum_{i=1}^{N}\frac{y_i}{N}$ and $\bar{X}=\sum_{i=1}^{N}\frac{x_i}{N}$ be the population means of the study variable y and the auxiliary variable x, respectively. For estimating the population mean $\bar{Y}$ of y, a simple random sample of size n is drawn without replacement from the population U. Then the classical ratio and product estimators are defined by

$$\hat{\bar{Y}}_R=\bar{y}\frac{\bar{X}}{\bar{x}},\ \text{if } \bar{x}\neq 0 \quad \text{and} \quad \hat{\bar{Y}}_P=\bar{y}\frac{\bar{x}}{\bar{X}}$$

where $\bar{y}$ and $\bar{x}$ are the sample means of y and x respectively based on a sample of size n out of the population of size N units and $\bar{X}$ is the known population mean of x. With known population mean $\bar{X}$, Bahl and Tuteja (1991) suggested the exponential ratio-type estimator as

$$\hat{\bar{Y}}_{Re}=\bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$$

and the exponential product-type estimator as

$$\hat{\bar{Y}}_{Pe}=\bar{y}\exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right)$$

for the population mean $\bar{Y}$.

If the population mean $\bar{X}$ of the auxiliary variable x is not known before start of the survey, a first-phase sample of size n_1 is drawn from the population, on which only the auxiliary variable x is observed. Then a second phase sample of size n is drawn, on which both study variable y and auxiliary variable x are observed. Let $\bar{x}_1=\sum_{i=1}^{n_1}\frac{x_i}{n_1}$ denotes the sample mean of size n_1 based on the first phase sample and $\bar{y}=\sum_{i=1}^{n}\frac{y_i}{n}$ and $\bar{x}=\sum_{i=1}^{n}\frac{x_i}{n}$ denote the sample means of variables y and x respectively, obtained from the second phase sample of size n. Then the double sampling ratio and product estimators of population mean $\bar{Y}$ are given by

$$\hat{\bar{Y}}_R^d=\bar{y}\frac{\bar{x}_1}{\bar{x}} \quad \text{and} \quad \hat{\bar{Y}}_P^d=\bar{y}\frac{\bar{x}}{\bar{x}_1}.$$

Singh and Vishwakarma (2007) suggested the exponential ratio and product type estimators for $\bar{Y}$ in double sampling respectively, as

$$\hat{\bar{Y}}_{Re}^d=\bar{y}\exp\left(\frac{\bar{x}_1-\bar{x}}{\bar{x}_1+\bar{x}}\right) \quad \text{and} \quad \hat{\bar{Y}}_{Pe}^d=\bar{y}\exp\left(\frac{\bar{x}-\bar{x}_1}{\bar{x}+\bar{x}_1}\right).$$

Notes

Notes

In the present paper, we have proposed an exponential chain ratio and product type estimators in double sampling for estimating finite population mean $\bar{Y}$ using two auxiliary characters.

II. The Proposed Estimator

Let us consider a finite population $U=\{U_1,U_2,...U_N\}$ of size N units. A first phase large sample of size n_1 units is drawn from population U following simple random sampling without replacement (SRSWOR) scheme, while in the second phase; a subsample of size n $(n_1>n)$ is drawn by SRSWOR from either n_1 units or directly from the population U. We assume that $\rho_{yx}>\rho_{yz}>0$.

The exponential chain ratio estimator in double sampling is defined as

$$\hat{\bar{Y}}_{Re}^{dc}=\bar{y}\exp\left(\frac{\bar{x}_1\dfrac{\bar{Z}}{\bar{z}_1}-\bar{x}}{\bar{x}_1\dfrac{\bar{Z}}{\bar{z}_1}+\bar{x}}\right) \tag{1}$$

and exponential chain product estimator in double sampling as

$$\hat{\bar{Y}}_{Pe}^{dc}=\bar{y}\exp\left(\frac{\bar{x}-\bar{x}_1\dfrac{\bar{Z}}{\bar{z}_1}}{\bar{x}+\bar{x}_1\dfrac{\bar{Z}}{\bar{z}_1}}\right) \tag{2}$$

The properties of proposed estimators are obtained for the following two cases.

Case I : When the second phase sample of size n is a subsample of the first phase of size n_1.

Case II : When the second phase sample of size n is drawn independently of the first phase sample of size n_1, Bose (1943).

III. Case I

Bias and Mean Square Error of $\hat{\bar{Y}}_{Re}^{dc}$ and $\hat{\bar{Y}}_{Pe}^{dc}$

To obtain the bias (B) and mean square error (M) of estimators $\hat{\bar{Y}}_{Re}^{dc}$ and $\hat{\bar{Y}}_{Pe}^{dc}$, we write $e_0=(\bar{y}-\bar{Y})/\bar{Y}$, $e_1=(\bar{x}-\bar{X})/\bar{X}$, $e_1'=(\bar{x}_1-\bar{X})/\bar{X}$ and $e_2=(\bar{z}_1-\bar{Z})/\bar{Z}$ such that

$$\left.\begin{aligned}
&E(e_0)=E(e_1)=E(e_1')=E(e_2)=0,\quad E(e_0^2)=\frac{1-f}{n}C_y^2,\quad E(e_1^2)=\frac{1-f}{n}C_x^2,\\
&E(e_1'^2)=\frac{1-f_1}{n_1}C_x^2,\quad E(e_2^2)=\frac{1-f_1}{n_1}C_z^2,\quad E(e_0e_1)=\frac{1-f}{n}C_{yx}C_x^2,\\
&E(e_0e_1')=\frac{1-f_1}{n_1}C_{yx}C_x^2,\quad E(e_0e_2)=\frac{1-f_1}{n_1}C_{yz}C_z^2,\quad E(e_1e_1')=\frac{1-f_1}{n_1}C_x^2,\\
&E(e_1e_2)=\frac{1-f_1}{n_1}C_{xz}C_z^2,\quad E(e_1'e_2)=\frac{1-f_1}{n_1}C_{xz}C_z^2
\end{aligned}\right\} \tag{3}$$

where $f=\dfrac{n}{N}$, $f_1=\dfrac{n_1}{N}$;

$C_y=\dfrac{S_y}{\bar{Y}}$, $C_x=\dfrac{S_x}{\bar{X}}$ and $C_z=\dfrac{S_z}{\bar{Z}}$ are the coefficients of variation of the study variate y, auxiliary variates x and z respectively.

$\rho_{yx}=\dfrac{S_{yx}}{S_xS_y}$, $\rho_{yz}=\dfrac{S_{yz}}{S_yS_z}$ and $\rho_{zx}=\dfrac{S_{zx}}{S_zS_x}$ are the correlation coefficients between y and x, y and z and x and z respectively.

$S_y^2=\dfrac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2$, $S_x^2=\dfrac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2$ and $S_z^2=\dfrac{1}{N-1}\sum_{i=1}^{N}\left(z_i-\bar{Z}\right)^2$ are the population variances of study variate y, auxiliary variates x and z respectively.

$S_{yx}=\dfrac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)\left(x_i-\bar{X}\right)$, $S_{yz}=\dfrac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)\left(z_i-\bar{Z}\right)$ and $S_{xz}=\dfrac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)\left(z_i-\bar{Z}\right)$ are the co-variances between y and x, y and z; and x and z respectively; and

$C_{yx}=\dfrac{\rho_{yx}C_y}{C_x}$, $C_{yz}=\dfrac{\rho_{yz}C_y}{C_z}$ and $C_{xz}=\dfrac{\rho_{xz}C_x}{C_z}$.

Expanding the right hand side of equations (1) and (2), multiplying out and neglecting terms of e's having power greater than two, we have

$$\hat{\bar{Y}}_{Re}^{dc}-\bar{Y}\cong\bar{Y}\left[e_0+\frac{1}{2}\left(e_1'-e_1-e_2-e_1e_2\right)-\frac{1}{8}e_1'^2+\frac{3}{8}\left(e_1^2+e_2^2+2e_1e_2\right)-\frac{1}{4}\left(e_1e_1'+e_2e_1'\right)\right.$$
$$\left.+\frac{1}{2}\left(e_0e_1'-e_0e_1-e_0e_2\right)\right] \tag{4}$$

$$\hat{\bar{Y}}_{Pe}^{dc}-\bar{Y}\cong\bar{Y}\left[e_0+\frac{1}{2}\left(e_1-e_1'+e_2\right)+\frac{1}{2}\left(e_1e_2+e_0e_1-e_0e_1'+e_0e_2\right)-\frac{1}{4}\left(e_1^2+e_2^2-e_1'^2+2e_1e_2\right)\right.$$
$$\left.+\frac{1}{8}\left(e_1^2+e_2^2+e_1'^2+2e_1e_2-2e_1'e_2-2e_1e_1'\right)\right] \tag{5}$$

Therefore, the bias of the estimators $\hat{\bar{Y}}_{Re}^{dc}$ and $\hat{\bar{Y}}_{Pe}^{dc}$ can be obtained by using the results of (3) in equations (4) and (5) are respectively as

$$B\left(\hat{\bar{Y}}_{Re}^{dc}\right)_I=\bar{Y}\left[\frac{3}{8}\left(\frac{1-f^*}{n}C_x^2+\frac{1-f_1}{n_1}C_z^2\right)+\frac{1}{2}\left(\frac{1-f^*}{n}C_{yx}C_x^2-\frac{1-f_1}{n_1}C_{yz}C_z^2\right)\right]$$

and

$$B\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_I=\bar{Y}\left[-\frac{1}{8}\left(\frac{1-f^*}{n}C_x^2+\frac{1-f_1}{n_1}C_z^2\right)+\frac{1}{2}\left(\frac{1-f^*}{n}C_{yx}C_x^2+\frac{1-f_1}{n_1}C_{yz}C_z^2\right)\right]$$

where $f^*=n/n_1$.

Notes

Notes

From equations (4) and (5), we have

$$\hat{\bar{Y}}_{Re}^{dc} - \bar{Y} \cong \bar{Y}\left\{e_0 + \frac{1}{2}\left(e_1' - e_1 - e_2\right)\right\} \tag{6}$$

and

$$\hat{\bar{Y}}_{Pe}^{dc} - \bar{Y} \cong \bar{Y}\left\{e_0 + \frac{1}{2}\left(e_1 - e_1' + e_2\right)\right\} \tag{7}$$

Squaring both sides of equations (6) and (7), taking expectations and using the results of (3), we get the MSE of $\hat{\bar{Y}}_{Re}^{dc}$ and $\hat{\bar{Y}}_{Pe}^{dc}$ to the first degree of approximation as

$$M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_I = \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1}{4}\left(\frac{1-f^*}{n}C_x^2 + \frac{1-f_1}{n_1}C_z^2\right) - \frac{1-f^*}{n}C_{yx}C_x^2 - \frac{1-f_1}{n_1}C_{yz}C_z^2\right] \tag{8}$$

$$M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_I = \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1}{4}\left(\frac{1-f^*}{n}C_x^2 + \frac{1-f_1}{n_1}C_z^2\right) + \frac{1-f^*}{n}C_{yx}C_x^2 + \frac{1-f_1}{n_1}C_{yz}C_z^2\right] \tag{9}$$

and the MSE of the usual unbiased estimator $\bar{y}$ under the SRSWOR scheme is

$$M\left(\bar{y}\right) = \bar{Y}^2\frac{1-f}{n}C_y^2. \tag{10}$$

IV. Efficiency Comparisons

a) Efficiency comparisons of exponential chain ratio estimator in double sampling

(i) *with chain ratio estimator in double sampling (Chand, 1975)*

The MSE of chain ratio estimator in double sampling is

$$M\left(\hat{\bar{Y}}_{R}^{dc}\right)_I = \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1-f^*}{n}C_x^2\left(1-2C_{yx}\right) + \frac{1-f_1}{n_1}C_z^2\left(1-2C_{yz}\right)\right] \tag{11}$$

From equations (8) and (11), we have

$$M\left(\hat{\bar{Y}}_{R}^{dc}\right)_I - M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_I = \bar{Y}^2\left[\frac{1-f^*}{n}C_x^2\left(\frac{3}{4} - C_{yx}\right) + \frac{1-f_1}{n_1}C_z^2\left(\frac{3}{4} - C_{yz}\right)\right]$$

$$> 0, \text{ if } \frac{3}{4} - C_{yx} > 0 \text{ and } \frac{3}{4} - C_{yz} > 0$$

Thus, the proposed estimator $\hat{\bar{Y}}_{Re}^{dc}$ is more efficient than $\hat{\bar{Y}}_{R}^{dc}$ if $\frac{3}{4} - C_{yx} > 0$ and $\frac{3}{4} - C_{yz} > 0$. (12)

(ii) *with chain product estimator in double sampling*

The MSE of chain product estimator in double sampling is

$$M\left(\hat{\bar{Y}}_{P}^{dc}\right)_I = \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1-f^*}{n}C_x^2\left(1+2C_{yx}\right) + \frac{1-f_1}{n_1}C_z^2\left(1+2C_{yz}\right)\right] \tag{13}$$

From equations (8) and (13), we have

$$M\left(\hat{\bar{Y}}_P^{dc}\right)_I - M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_I = 3\bar{Y}^2\left[\frac{1-f^*}{n}C_x^2\left(\frac{1}{4}+C_{yx}\right)+\frac{1-f_1}{n_1}C_z^2\left(\frac{1}{4}+C_{yz}\right)\right]$$

$$>0\text{, if } \frac{1}{4}+C_{yx}>0 \text{ and } \frac{1}{4}+C_{yz}>0\text{.}$$

Thus, the proposed estimator $\hat{\bar{Y}}_{Re}^{dc}$ is better than $\hat{\bar{Y}}_P^{dc}$ if $\frac{1}{4}+C_{yx}>0$ and $\frac{1}{4}+C_{yz}>0$. (14)

(iii) *with sample mean per unit estimator* $\bar{y}$

From equations (8) and (10), we have

$$M\left(\bar{y}\right) - M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_I = \bar{Y}^2\left[\frac{1-f^*}{n}C_x^2\left(-\frac{1}{4}+C_{yx}\right)+\frac{1-f_1}{n_1}C_z^2\left(-\frac{1}{4}+C_{yz}\right)\right]$$

$$>0\text{, if } C_{yx}-\frac{1}{4}>0 \text{ and } C_{yz}-\frac{1}{4}>0\text{.}$$

Thus the proposed estimator $\hat{\bar{Y}}_{Re}^{dc}$ has smaller MSE than sample mean per unit estimator $\bar{y}$ if $C_{yx}-\frac{1}{4}>0$ and $C_{yz}-\frac{1}{4}>0$. (15)

b) Efficiency comparisons of exponential chain product estimator in double sampling

(i) *with chain ratio estimator in double sampling (Chand, 1975)*

From equations (9) and (11), we have

$$M\left(\hat{\bar{Y}}_R^{dc}\right)_I - M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_I = 3\bar{Y}^2\left[\frac{1-f^*}{n}C_x^2\left(\frac{1}{4}-C_{yx}\right)+\frac{1-f_1}{n_1}C_z^2\left(\frac{1}{4}-C_{yz}\right)\right]$$

Thus the proposed estimator $\hat{\bar{Y}}_{Pe}^{dc}$ will dominate over the estimator $\hat{\bar{Y}}_R^{dc}$ if

$\frac{1}{4}-C_{yx}>0$ and $\frac{1}{4}-C_{yz}>0$. (16)

(ii) *with chain product estimator in double sampling*

From equations (9) and (13), we have

$$M\left(\hat{\bar{Y}}_P^{dc}\right)_I - M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_I = \bar{Y}^2\left[\frac{1-f^*}{n}C_x^2\left(\frac{3}{4}+C_{yx}\right)+\frac{1-f_1}{n_1}C_z^2\left(\frac{3}{4}+C_{yz}\right)\right]$$

Thus, the proposed estimator $\hat{\bar{Y}}_{Pe}^{dc}$ is better than $\hat{\bar{Y}}_P^{dc}$ if $\frac{3}{4}+C_{yx}>0$ and $\frac{3}{4}+C_{yz}>0$. (17)

(iii) *with sample mean per unit estimator* $\bar{y}$

From equations (9) and (10), we have

$$M\left(\bar{y}\right) - M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_I = -\bar{Y}^2\left[\frac{1-f^*}{n}C_x^2\left(\frac{1}{4}+C_{yx}\right)+\frac{1-f_1}{n_1}C_z^2\left(\frac{1}{4}+C_{yz}\right)\right]$$

Thus the proposed estimator $\hat{\bar{Y}}_{Pe}^{dc}$ has smaller MSE than that of the sample mean per unit estimator $\bar{y}$ if $\frac{1}{4}+C_{yx}<0$ and $\frac{1}{4}+C_{yz}<0$. (18)

Notes

V. Case II

Notes

Bias and Mean Square Error of $\hat{\hat{Y}}_{Re}^{dc}$ and $\hat{\hat{Y}}_{Pe}^{dc}$

In case II, we have

$$\left.\begin{aligned} & E(e_0)=E(e_1)=E(e_1')=E(e_2)=0,\quad E(e_0^2)=\frac{1-f}{n}C_y^2, \\ & E(e_1^2)=\frac{1-f}{n}C_x^2,\quad E(e_1'^2)=\frac{1-f_1}{n_1}C_x^2,\quad E(e_2^2)=\frac{1-f_1}{n_1}C_z^2, \\ & E(e_0e_1)=\frac{1-f}{n}C_{yx}C_x^2,\quad E(e_1'e_2)=\frac{1-f_1}{n_1}C_{xz}C_z^2, \\ & E(e_0e_1')=E(e_0e_2)=E(e_1e_1')=E(e_1e_2)=0 \quad . \end{aligned}\right\} \tag{19}$$

Taking expectation in equations (4) and (5) and using the results of equation (19), we get the bias of the estimators $\hat{\hat{Y}}_{Re}^{dc}$ and $\hat{\hat{Y}}_{Pe}^{dc}$ to the first degree of approximation as

$$B\left(\hat{\hat{Y}}_{Re}^{dc}\right)_{II}=\bar{Y}\left[-\frac{1}{8}\frac{1-f_1}{n_1}C_x^2+\frac{3}{8}\left(\frac{1-f}{n}C_x^2+\frac{1-f_1}{n_1}C_z^2\right)-\frac{1}{4}\frac{1-f_1}{n_1}C_{xz}C_z^2-\frac{1}{2}\frac{1-f}{n}C_{yx}C_x^2\right]$$

and

$$B\left(\hat{\hat{Y}}_{Pe}^{dc}\right)_{II}=\bar{Y}\left[\frac{1}{2}\frac{1-f}{n}C_{yx}C_x^2-\frac{1}{8}\left(\frac{1-f}{n}C_x^2+\frac{1-f_1}{n_1}C_z^2\right)+\frac{3}{8}\frac{1-f_1}{n_1}C_x^2-\frac{1}{4}\frac{1-f_1}{n_1}C_{xz}C_z^2\right]$$

Squaring both sides of equations (6) and (7), taking expectations and using the results of (19), we get the MSE of $\hat{\hat{Y}}_{Re}^{dc}$ and $\hat{\hat{Y}}_{Pe}^{dc}$ to the first degree of approximation as

$$M\left(\hat{\hat{Y}}_{Re}^{dc}\right)_{II}=\bar{Y}^2\left[\frac{1-f}{n}C_y^2+\frac{1}{4}\left(f^{**}C_x^2+\frac{1-f_1}{n_1}C_z^2\right)-\frac{1-f}{n}C_{yx}C_x^2-\frac{1}{2}\frac{1-f_1}{n_1}C_{xz}C_z^2\right] \tag{20}$$

and

$$M\left(\hat{\hat{Y}}_{Pe}^{dc}\right)_{II}=\bar{Y}^2\left[\frac{1-f}{n}C_y^2+\frac{1}{4}\left(f^{**}C_x^2+\frac{1-f_1}{n_1}C_z^2\right)+\frac{1-f}{n}C_{yx}C_x^2-\frac{1}{2}\frac{1-f_1}{n_1}C_{xz}C_z^2\right] \tag{21}$$

where $f^{**}=\frac{1-f}{n}+\frac{1-f_1}{n_1}$.

VI. Efficiency Comparisons

a) Efficiency comparisons of exponential chain ratio estimator in double sampling

(i) *with chain ratio estimator in double sampling (Chand, 1975)*

The MSE of chain ratio estimator in double sampling is

$$M\left(\hat{\hat{Y}}_{R}^{dc}\right)_{II}=\bar{Y}^2\left[\frac{1-f}{n}C_y^2+\frac{1-f}{n}C_x^2\left(1-2C_{yx}\right)+\frac{1-f_1}{n_1}C_x^2+\frac{1-f_1}{n_1}C_z^2\left(1-2C_{xz}\right)\right] \tag{22}$$

From equations (20) and (22), we have

$$M\left(\hat{\bar{Y}}_{R}^{dc}\right)_{II}-M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_{II}=\bar{Y}^{2}\left[\frac{1-f}{n}C_{x}^{2}\left(\frac{3}{4}-C_{yx}\right)+\frac{3}{4}\frac{1-f_{1}}{n_{1}}C_{x}^{2}+\frac{3}{4}\frac{1-f_{1}}{n_{1}}C_{z}^{2}\left(1-C_{xz}\right)\right]$$

Therefore, the proposed estimator $\hat{\bar{Y}}_{Re}^{dc}$ is better than $\hat{\bar{Y}}_{R}^{dc}$ if

$\frac{3}{4}-C_{yx}>0$ and $1-C_{xz}>0$. (23)

(ii) *with chain product estimator in double sampling*

The MSE of chain product estimator in double sampling is

$$M\left\{\bar{y}_{P}^{(dc)}\right\}_{II}=\bar{Y}^{2}\left[\frac{1-f}{n}C_{y}^{2}+\frac{1-f}{n}C_{x}^{2}\left(1+2C_{yx}\right)+\frac{1-f_{1}}{n_{1}}C_{x}^{2}+\frac{1-f_{1}}{n_{1}}C_{z}^{2}\left(1-2C_{xz}\right)\right] \quad (24)$$

From equations (20) and (24), we have

$$M\left(\hat{\bar{Y}}_{P}^{dc}\right)_{II}-M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_{II}=\bar{Y}^{2}\left[\frac{3}{4}f^{**}C_{x}^{2}+3\frac{1-f}{n}C_{yx}C_{x}^{2}+\frac{3}{4}\frac{1-f_{1}}{n_{1}}C_{z}^{2}\left(1-C_{xz}\right)\right]$$

which is positive if $C_{yx}>0$ and $1-C_{xz}>0$.

i.e.,the estimator $\hat{\bar{Y}}_{Re}^{dc}$ is more efficient than $\bar{y}_{P}^{(dc)}$ if $C_{yx}>0$ and $1-C_{xz}>0$. (25)

(iii) *with sample mean per unit estimator $\bar{y}$*

From equations (10) and (20), we have

$$M\left(\bar{y}\right)-M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_{II}=\bar{Y}^{2}\left[-\frac{1}{4}\left(f^{**}C_{x}^{2}+\frac{1-f_{1}}{n_{1}}C_{z}^{2}\right)+\frac{1-f}{n}C_{yx}C_{x}^{2}+\frac{1}{2}\frac{1-f_{1}}{n_{1}}C_{xz}C_{z}^{2}\right]$$

Therefore, the estimator $\hat{\bar{Y}}_{Re}^{dc}$ is better than $\bar{y}$ if

$$\frac{1-f}{n}C_{yx}C_{x}^{2}+\frac{1}{2}\frac{1-f_{1}}{n_{1}}C_{xz}C_{z}^{2}-\frac{1}{4}\left(f^{**}C_{x}^{2}+\frac{1-f_{1}}{n_{1}}C_{z}^{2}\right)>0. \quad (26)$$

b) Efficiency comparisons of exponential chain product estimator in double sampling

(i) *with chain ratio estimator in double sampling (Chand, 1975)*

From equations (21) and (22), we have

$$M\left(\hat{\bar{Y}}_{R}^{dc}\right)_{II}-M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_{II}=3\bar{Y}^{2}\left[\frac{1-f}{n}C_{x}^{2}\left(\frac{1}{4}-C_{yx}\right)+\frac{1}{4}\frac{1-f_{1}}{n_{1}}C_{x}^{2}+\frac{1}{4}\frac{1-f_{1}}{n_{1}}C_{z}^{2}\left(1-2C_{xz}\right)\right]$$

Therefore, the estimator $\hat{\bar{Y}}_{Pe}^{dc}$ is more efficient than the estimator $\hat{\bar{Y}}_{R}^{dc}$ if

$\frac{1}{4}-C_{yx}>0$ and $\frac{1}{2}-C_{xz}>0$. (27)

(ii) *with chain Product estimator in double sampling*

From equations (21) and (24), it is found that the estimator $\hat{\bar{Y}}_{Pe}^{dc}$ will dominate over the estimator $\hat{\bar{Y}}_{P}^{dc}$ if

$$M\left(\hat{\bar{Y}}_{P}^{dc}\right)_{II}-M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_{II}=\bar{Y}^{2}\left[\frac{1-f}{n}C_{x}^{2}\left(\frac{3}{4}+C_{yx}\right)+\frac{3}{4}\frac{1-f_{1}}{n_{1}}C_{x}^{2}+\frac{3}{4}\frac{1-f_{1}}{n_{1}}C_{z}^{2}\left(1-2C_{xz}\right)\right]$$

$$>0 \text{ i.e., if } \frac{3}{4}+C_{yx}>0 \text{ and } \frac{1}{2}-C_{xz}>0. \tag{28}$$

Notes

(iii) *with sample mean per unit estimator* $\bar{y}$

From equations (10) and (21), we have

$$M\left(\bar{y}\right)-M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_{II}=\bar{Y}^{2}\left[-\frac{1}{4}\left(f^{**}C_{x}^{2}+\frac{1-f_{1}}{n_{1}}C_{z}^{2}\right)-\frac{1-f}{n}C_{yx}C_{x}^{2}+\frac{1}{2}\frac{1-f_{1}}{n_{1}}C_{xz}C_{z}^{2}\right]$$

which is positive if

$$\frac{1}{2}\frac{1-f_{1}}{n_{1}}C_{xz}C_{z}^{2}-\frac{1}{4}\left(f^{**}C_{x}^{2}+\frac{1-f_{1}}{n_{1}}C_{z}^{2}\right)-\frac{1-f}{n}C_{yx}C_{x}^{2}>0 \tag{29}$$

Therefore, the estimator $\hat{\bar{Y}}_{Pe}^{dc}$ is more efficient than $\bar{y}$ if the condition (29) is satisfied.

VII. Empirical Study

To examine the merits of the proposed estimators, we have considered four natural population data sets. The sources of populations, nature of the variates y, x and z; and the values of the various parameters are given as.

Population I - Source : Cochran (1977)

Y : Number of 'placebo' children

X : Number of paralytic polio cases in the placebo group

Z : Number of paralytic polio cases in the 'not inoculated' group

$N=34,\ n=10,\ n_1=15,\ \bar{Y}=4.92,\ \bar{X}=2.59,\ \bar{Z}=2.91,\ \rho_{yx}=0.7326,\ \rho_{yz}=0.6430,$

$\rho_{xz}=0.6837,\ C_y^2=1.0248,\ C_x^2=1.5175,\ C_z^2=1.1492.$

Population II - Source: Sukhatme and Chand (1977)

Y : Apple trees of bearing age in 1964

X : Bushels of apples harvested in 1964

Z : Bushels of apples harvested in 1959

$N=200,\ n=20,\ n_1=30,\ \bar{Y}=0.103182\times10^4,\ \bar{X}=0.293458\times10^4,\ \bar{Z}=0.365149\times10^4,$

$\rho_{yx}=0.93,\ \rho_{yz}=0.77,\ \rho_{xz}=0.84,\ C_y^2=2.55280,\ C_x^2=4.02504,\ C_z^2=2.09379.$

Population III - Source: Srivastava *et al.* (1989, Page 3922)

Y : The measurement of weight of children

X : Mid arm circumference of children

Z : Skull circumference of children.

$N=82,\ n=25,\ n_1=43,\ \bar{Y}=5.60\ kg,\ \bar{X}=11.90\ cm,\ \bar{Z}=39.80\ cm,\ \rho_{yx}=0.09,$

$\rho_{yz}=0.12,\ \rho_{xz}=0.86,\ C_y^2=0.0107,\ C_x^2=0.0052,\ C_z^2=0.0008.$

Population IV - Source: Srivastava *et al.* (1989, Page 3922)

Y : The measurement of weight of children

X : Mid arm circumference of children

Z : Skull circumference of children.

$$N = 55,\quad n = 18,\quad n_1 = 30,\quad \bar{Y} = 17.08\ \ kg,\quad \bar{X} = 16.92\ \ cm,\quad \bar{Z} = 50.44\ \ cm,\quad \rho_{yx} = 0.54,$$

$$\rho_{yz} = 0.51,\quad \rho_{xz} = -0.08,\quad C_y^2 = 0.0161,\quad C_x^2 = 0.0049,\quad C_z^2 = 0.0007.$$

To establish the theoretical conditions for efficiencies of proposed estimators obtained in Section 4 and Section 6, empirically, we have conducted empirical studies and these are shown in Table 1 and Table 2.

Table 1 : Empirical study of theoretical conditions explained in Section-4

Population	$M\left(\hat{\bar{Y}}_R^{dc}\right)_I > M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_I$ Condition (12)	$M\left(\hat{\bar{Y}}_P^{dc}\right)_I > M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_I$ Condition (14)	$M(\bar{y}) > M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_I$ Condition (15)	$M\left(\hat{\bar{Y}}_R^{dc}\right)_I > M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_I$ Condition (16)	$M\left(\hat{\bar{Y}}_P^{dc}\right)_I > M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_I$ Condition (17)	$M(\bar{y}) > M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_I$ Condition (18)
I	0.15>0, 0.14>0	0.85>0, 0.86>0	0.35>0, 0.36>0	*, *	1.35>0, 1.36>0	*, *
II	0.01>0, *	0.99>0, 1.10>0	0.49>0, 0.60>0	* , *	1.49>0, 1.60>0	*, *
III	0.62>0, 0.31>0	0.38>0, 0.69>0	*, 0.19>0	0.12>0, *	0.88>0, 1.19>0	*, *
IV	*, *	1.23>0, 2.70>0	0.73>0, 2.20>0	*, *	1.73>0, 3.20>0	*, *

* Does not satisfy theoretical conditions empirically

Table 2 : Empirical study of theoretical conditions explained in Section-6

Population	$M\left(\hat{\bar{Y}}_R^{dc}\right)_{II} > M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_{II}$ Condition (23)	$M\left(\hat{\bar{Y}}_P^{dc}\right)_{II} > M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_{II}$ Condition (25)	$M(\bar{y}) > M\left(\hat{\bar{Y}}_{Re}^{dc}\right)_{II}$ Condition (26)	$M\left(\hat{\bar{Y}}_R^{dc}\right)_{II} > M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_{II}$ Condition (27)	$M\left(\hat{\bar{Y}}_P^{dc}\right)_{II} > M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_{II}$ Condition (28)	$M(\bar{y}) > M\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_{II}$ Condition (29)
I	0.15>0, 0.21>0	0.21>0, 0.60>0	0.72>0	*, *	1.35, *	*
II	0.01>0, *	*, 0.74>0	85249>0	*, *	1.49>0 *	*
III	0.62>0, *	*, 0.13>0	0.000002>0	0.12>0, *	0.88>0, *	*
IV	*, 1.21 >0	1.21>0, 0.98>0	0.03>0	*, 0.71>0	1.73>0, 0.71>0	*

* Does not satisfy theoretical conditions empirically

Notes

To observe the relative performance of different estimators of $\overline{Y}$, we have computed the percentage relative efficiencies of the proposed estimators $\hat{\hat{\overline{Y}}}{}_{Re}^{dc}$ and $\hat{\hat{\overline{Y}}}{}_{Pe}^{dc}$, chain ratio estimator $\hat{\hat{\overline{Y}}}{}_{R}^{dc}$, product estimator $\hat{\hat{\overline{Y}}}{}_{P}^{dc}$ in double sampling and sample mean per unit estimator $\overline{y}$ with respect to usual unbiased estimator $\overline{y}$ in *Case I* and *Case II* and the findings are presented in Table 3.

Notes

Table 3 : Percentage relative efficiencies of different estimators with respect to $\overline{y}$

Estimator	$\overline{y}$	$\hat{\hat{\overline{Y}}}{}_{R}^{dc}$	$\hat{\hat{\overline{Y}}}{}_{P}^{dc}$	$\hat{\hat{\overline{Y}}}{}_{Re}^{dc}$	$\hat{\hat{\overline{Y}}}{}_{Pe}^{dc}$
Case I					
Population I	100.00	136.91	25.96	184.36	47.55
Population II	100.00	279.93	26.02	247.82	46.58
Population III	100.00	81.92	70.22	97.11	88.38
Population IV	100.00	131.91	61.01	120.57	78.75
Case II					
Population I	100.00	87.63	21.24	141.68	42.15
Population II	100.00	182.67	19.16	220.59	37.90
Population III	100.00	68.82	58.68	91.06	82.82
Population IV	100.00	116.68	48.81	122.79	70.87

VIII. Results and Discussion

We have analyzed the exponential chain ratio and product type estimators in double sampling and obtained its bias and MSE equations in two different cases. The MSEs of the proposed estimators have been compared with the MSEs of classical estimators (ratio, product and sample mean per unit estimator) on a theoretical basis, and conditions have been obtained under which the proposed estimators have smaller MSE than the classical estimators.

Section 4 and section 6 provides the theoretical conditions under which the proposed estimators $\hat{\hat{\overline{Y}}}{}_{Re}^{dc}$ and $\hat{\hat{\overline{Y}}}{}_{Pe}^{dc}$ are more efficient than other estimators. Table 1 and Table 2 establish these theoretical conditions empirically. It shows that almost all theoretical conditions obtained in section 4 and section 6 are satisfied with respect to the population data sets.

From Table 3, it clearly indicates that the proposed estimators $\hat{\hat{\overline{Y}}}{}_{Re}^{dc}$ and $\hat{\hat{\overline{Y}}}{}_{Pe}^{dc}$ are more efficient than the estimators $\hat{\hat{\overline{Y}}}{}_{R}^{dc}$, $\hat{\hat{\overline{Y}}}{}_{P}^{dc}$ and $\overline{y}$ in both the *Cases I* and *II*, except for the data sets of population II and IV in *Case I*, where $\hat{\hat{\overline{Y}}}{}_{R}^{dc}$ is slightly better than $\hat{\hat{\overline{Y}}}{}_{Re}^{dc}$.

Thus, the uses of the proposed estimators are preferable over other estimators.

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Notes