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I. Introduction

Drying is the process of removing the moisture in the product up to certain threshold value by evaporation. In this way, the product can be stored for a long period, since it decreases the water activity of the product, reduces microbiological activity and minimizes physical and chemical changes during storage.

Different drying methods are used in the drying of fruits and vegetables. Air-drying is the most common method in the drying of foodstuffs. The major drawback of air-drying is the longer drying period, low drying rates in the falling rate period, worsening of the taste, colour and nutritional content of the product, higher drying temperature, low energy efficiency and high costs which is not a desirable situation for food industry [1,6,10].

The desire to eliminate this problem, prevent significant quality loss, and achieve fast and effective thermal processing, has resulted in the increase use of other drying heat sources such as microwave and infrared (IR) drying.

Microwave drying is more rapid, more uniform and more highly energy efficient compared to conventional hot air drying and infrared drying [1,2,9]. In recent years, microwave drying has gained popularity as an alternative drying method for a variety of food products such as fruit, vegetable, snack food and dairy product [1,2,3,4,5,6,7,8,9,10]. The usual means of applying microwaves to a drying process is at the end or should be applied in the falling rate period.

The most relevant aspects of drying technology are the mathematical modeling of the process and the experimental setup. The modeling is basically based on the design of a set of equations to describe the system as accurately as possible.

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No information is available on the ohmic drying behavior of tomato in the open literature. Therefore, the aim of this study was to (i) effect of power density on the drying kinetic of date palm, (ii) compare the measured findings obtained during the drying of date palm with the predicted values obtained with Page thin layer drying semi-empirical model, (iii) to calculate the effective moisture diffusivity and activation energy.

II. Materials and Methods

Notes

Date palm, procured from the local market, was used in the present study. They were stored at a temperature of 4 ffi 0.5 ˚C until the drying process. Before the drying experiments, the samples were taken out of the refrigerator and kernel of samples was separated. To determine the initial moisture content, three 30 g of samples were dried in an oven (Memmert UM-400) at 70 ˚C for 3 days. The initial moisture content of date palm was calculated 18ffi1.2 % (w.b.) as an average of the results obtained.

A domestic microwave oven (M945, Samsung Electronics Ins) with maximum output of 1000 W at 2450MHz was used for the drying experiments. The dimensions of the microwave cavity were 327×370×207 mm. The oven has a fan for air flow in drying chamber and cooling of magnetron. The moisture from drying chamber was removed with this fan by passing it through the openings on the right side of the oven wall to the outer atmosphere. The microwave dryer was operated by a control terminal which could control both microwave power level and emission time. Experiments were performed at four initial mass of 20, 30, 40 and 50 g at microwave power of 200 W (or power densities (microwave power/mass) of 9.5, 6.5, 5 and 4 W/g). The moisture losses of samples were recorded at 15 s intervals during the drying process by a digital balance (GF-600, A & D, Japan) and an accuracy of ± 0.001 g.

For measuring the weight of the sample during experimentation, the tray with sample was taken out of the drying chamber, weighed on the digital top pan balance and placed back into the chamber. Drying was carried out until the final moisture content reaches to a level less than 7.5% (w.b.).

It has been accepted that the drying characteristics of biological products in the falling rate period can be described by using Fick's diffusion equation. The following assumptions have been made: moisture is initially uniformly distributed throughout the sample, the thermo-physical properties of the material are constant, shrinkage or deformation of the material during drying is negligible, a spherical shape for sample, the resistance to transfer in medium surrounding the sphere is negligible, heat generation inside the moist sample is negligible, and radiation effects are negligible. General equation mass transfer for sphere shape is:

$$\frac{\partial X}{\partial t} = D_{eff}\left(\frac{\partial^2 X}{\partial r^2} + \frac{2}{r}\frac{\partial X}{\partial r}\right) \tag{1}$$

With the appropriate initial and boundary conditions:

$$X(r,t)|_{t=0} = X_0 \tag{2}$$

$$\left.\frac{\partial X(r,t)}{\partial x}\right|_{r=0} = 0 \tag{3}$$

$$X(R,t)|_{t>0} = X_e \tag{4}$$

The first boundary condition stipulates that the moisture is initially uniformly distributed throughout the product sample. The second implies that the mass transfer is symmetrical with respect to the centre of the product. The third condition states that the surface moisture content of the samples instantaneously reaches equilibrium with the conditions of the surrounding air. The values of X_e are relatively small. Thus third condition can be simplified $X(R,t)|_{t>0} = 0$.

Following the numerical procedure, assume a solution of the following form in order to separate the variables:

Notes

$$X(r,t) = F(r) \times G(t) \tag{5}$$

where F is function of r only, and G is function of t only. Combining equations (1), (5) and using the initial and boundary conditions

$$X(r,t) = X_0\left(1 + \frac{2R}{\pi}\sum_{n=0}^{\infty}\left(\frac{(-1)^{n+1}}{n}\frac{1}{r}sin\left(\frac{n\pi r}{R}\right)exp\left(-\frac{D_{eff}\,n^2\pi^2 t}{R^2}\right)\right)\right) \tag{6}$$

The rate of transfer at time t across the surface of the sphere is:

$$4\pi R^2 N_A(t) = -4\pi R^2 D_{eff}\left(\frac{\partial X}{\partial r}\right)_{r=R} \tag{7}$$

with evaluating $(\partial X/\partial r)$ at $r = R$ form equation (6)

$$4\pi R^2 N_A(t) = 8\pi R^2 X_0 D_{eff}\sum_{n=1}^{\infty}\left(\exp\left(-\frac{D_{eff}\,n^2\pi^2 t}{R^2}\right)\right) \tag{8}$$

The total transfer per unit surface up to time t, N'_A is, where:

$$\frac{N'_A}{4\pi R^2} = \int_0^t N_A(t)dt = X_0\,\frac{R}{3}\left(1 - \frac{6}{\pi^2}\sum_{n=0}^{\infty}\left(\frac{1}{n^2}\exp\left(-\frac{D_{eff}n^2\pi^2 t}{R^2}\right)\right)\right) \tag{9}$$

A material balance on the transfer up to time t is:

$$\frac{4\pi R^3}{3}(X_0 - X) = N'_A \tag{10}$$

Where X is the average moisture throughout the sphere at time t. Combining equations (10) and (9):

$$\frac{X_0 - X}{X_0} = 1 - \frac{6}{\pi^2}\sum_{n=0}^{\infty}\left(\frac{1}{n^2}\exp\left(-\frac{D_{eff}n^2\pi^2 t}{R^2}\right)\right) \tag{11}$$

By simplify equation (11):

$$\frac{X}{X_0} = \frac{6}{\pi^2}\sum_{n=0}^{\infty}\left(\frac{1}{n^2}\exp\left(-\frac{D_{eff}\,n^2\pi^2 t}{R^2}\right)\right) \tag{12}$$

The moisture ratio (MR) was calculated using the following equation (13):

$$MR = \frac{X - X_e}{X_0 - X_e} \tag{13}$$

Form third condition; the equation (13) was simplified:

$$MR = \frac{X}{X_0} = \frac{6}{\pi^2}\sum_{n=0}^{\infty}\left(\frac{1}{n^2}\exp\left(-\frac{D_{eff}\,n^2\pi^2 t}{R^2}\right)\right) \tag{14}$$

The diffusion coefficients are typically determined by plotting experimental drying data in terms of ln(MR) versus drying time (t), because the plot gives a straight line with a slope as $\pi^2 D_{eff}/R^2$.

$$\ln(MR) = \ln\left(\frac{6}{\pi^2}\right) - \left(\frac{\pi^2 D_{eff}}{R^2}\right)t \tag{15}$$

The drying rate of date palm was calculated using the following equation:

$$DR = \frac{X_{t+\Delta t} - X_t}{\Delta t} \tag{16}$$

where $X_{t+\Delta t}$ is moisture content at time t+Δt (% d.b.), t is the time (min) and DR is the drying rate (% d.b./min).

Effectively modeling the drying behavior is important for investigation of drying characteristics of bioproduct. In this study, Experimental results of moisture ratio versus drying time was fitted to the semi-theoretical Page model, which are widely used by other workers to describe the kinetics of the drying process. Page's model was defined as follows:

$$MR = \exp(-kt^n) \tag{17}$$

where k is the drying rate constant (1/s) and n is equation constant model.

There are several criteria such as coefficient of determination (R^2) and chi-square (χ^2) are used to determine the quality of the fit. The model is said to be good if R^2 value is high and χ^2 value is low. These parameters are defined as follows:

$$R^2 = 1 - \left(\frac{\sum_{i=1}^{N}\left(MR_{pre,i} - MR_{exp,i}\right)^2}{\sum_{i=1}^{N}\left(MR_{pre,i} - \overline{MR}_{exp}\right)^2}\right) \tag{18}$$

$$\chi^2 = \frac{\sum_{i=1}^{N}\left(MR_{pre,i} - MR_{exp,i}\right)^2}{N - z} \tag{19}$$

where $MR_{pre,i}$ is the ith predicted moisture ratio, $MR_{exp,i}$ is the ith experimental moisture ratio, N is the number of observations and z is the number of constants in drying model.

The dependence of the effective moisture diffusivity on the power density is generally described by the Arrhenius equation:

$$D_{eff} = D_0 \exp\left(-\frac{E_a}{P_d}\right) \tag{20}$$

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where E_a is the activation energy (W/g), P_d is the power density (w/g), and D_0 is the pre-exponential factor (m^2/s).

III. Results and Discussion

The variations in moisture content of the date palm as a function of drying time at different temperatures are presented in Fig. 1. It can be seen that the moisture content of the date palm samples decreased with the increase in drying time. Based on these results, the required drying times for the date palm samples to reach a moisture content of 0.28ffi0.2 (% d.b.) in order to obtain safe storage, were found to vary from 150 to 240 s depending on the drying microwave power.

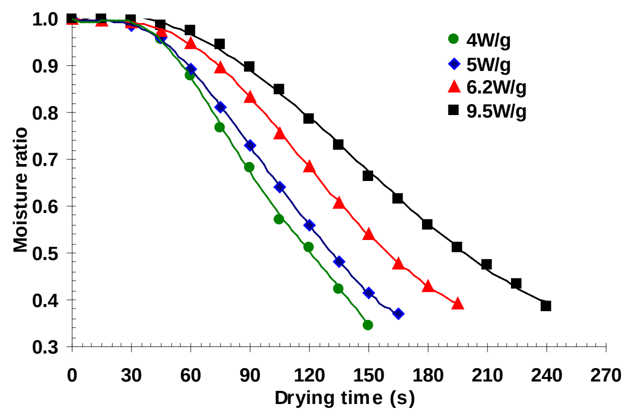

Fig. 1 : Variation of moisture ratio with drying time for the date palm

Validation of the Page model was confirmed by comparing the estimated or predicted moisture ratio at any particular drying condition. The validation of the Page model at different power densities is shown in Fig. 2. The predicted data generally banded around the straight line which showed the suitability of the Page model in describing the microwave drying behavior of the date palm.

Table 1 : Results of statistical analysis on the modeling of moisture content and drying time for the microwave dried date palm

P_d (W/g)	K (1/min)	n	R^2	χ^2
4	0.142	2.221	0.994	0.00038
5	0.126	2.093	0.994	0.00038
6.2	0.078	2.169	0.993	0.00038
9.5	0.052	2.127	0.994	0.00051

The statistical results from page model are summarized in Table 1. The statistical parameter estimations showed that R^2 and χ^2 values were ranged from 0.993 to 0.994 and 0.00038 to 0.0051, respectively. It was determined that the value of the drying rate constant (k) increased with the decrease in the power density.

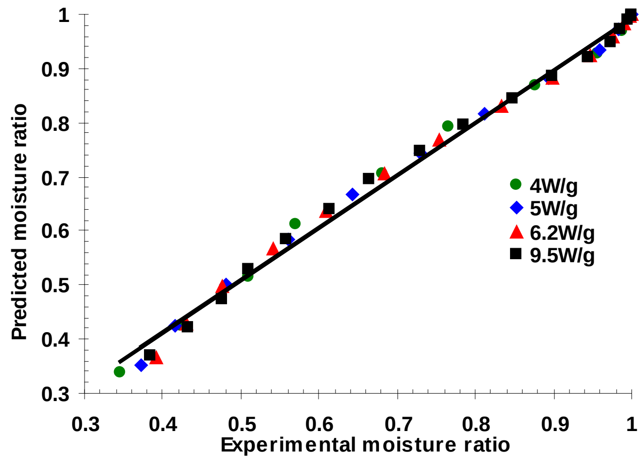

Fig. 2 : Experimental and predicted moisture ratio values for date palm

In order to take into account the effect of power density level on the constants of the Page model, namely, k, n (seen in Table 1), the regression analysis was used to set up the relations between these parameters and the power density level. Thus, the regression equations of these parameters against power density, P_d, (W/g) and the accepted model are as follows:

$$MR = \exp(-kt^n)$$

where,

$$k = 0.8094 P_d^{-1.2282} \qquad R^2 = 0.961 \tag{21}$$

$$n = -0.0189 P_d^3 + 0.374 P_d^2 - 2.3423 P_d + 6.8142 \quad R^2 = 0.961 \tag{22}$$

The drying rate curves for date palm samples dried at different microwave power densities are given in Fig. 3. In general, two distinct periods are identifiable, namely warming up and falling-rate periods. The initial short period coincides with the warming-up stage which corresponds to sample heating and non-isothermal drying conditions due to the low temperature of samples. The drying rates were more after an initial short period of the process probably due to evaporation and moisture from the surface of the date palm and later decreased with decreasing moisture content, for all the drying conditions once the drying process was governed by moisture diffusion. The accelerated drying rates may be attributed to internal heat generation. The absence of a constant drying rate period may be due to the thin layer of product that did not provide a constant supply of water for an applied period of time. Also, some resistance to water movement may exist due to shrinkage of the product on the surface, which reduces the drying rate considerably. The results indicates that mass transfer within the sample was more rapid during higher power density because more heat was generated within the sample creating a large vapor pressure difference between the centre and the surface of the product due to characteristic microwave volumetric heating. Thus, the power density had a crucial effect on the drying rate.

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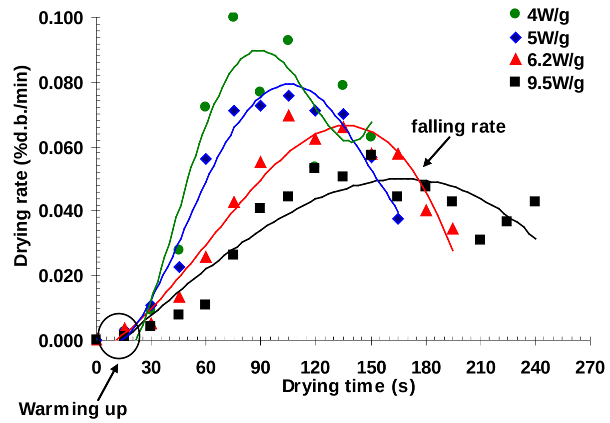

Fig. 3 : Variation of drying rate with drying time for the date palm

The variation in ln (MR) and drying time (t) for different power densities have been plotted in Fig. 4 to obtain the slope S, which can give the effective moisture diffusivity (D_{eff}). The effective diffusivity was calculated using Eq. (15) and is shown in Table. 2. The D_{eff} values of dried samples at power density level of 4–9.5 W/g were varied in the range of 2.72×10^{-6} to 4.73×10^{-6} m^2/s. It can be seen that D_{eff} values increased with increasing power density. When samples were dried at higher power density, increased heating energy would increase the activity of water molecules leading to higher moisture diffusivity.

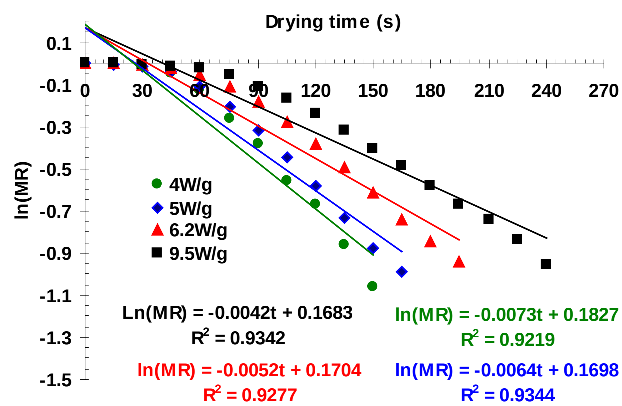

Fig. 4 : Variation in ln (MR) and drying time (in s) for date palm dried at different power densities

Table 2 : Values of effective diffusivity obtained for date palm at different power densities

P_d (W/g)	D_{eff} (m^2/s)
4	2.72×10^{-6}
5	3.37×10^{-6}
6.2	4.15×10^{-6}
9.5	4.73×10^{-6}

The values of effective diffusivity versus $1/P_d$ accurately fit to the exponential model as evident from Fig. 5 with coefficient of determination (R^2) of 0.975. The dependence of the effective diffusivity of date samples on the power density can be represented by the following equation:

$$D_{eff} = 7 \times 10^{-6} \exp\left(-\frac{3.9082}{P_d}\right) \quad R^2 = 0.975 \tag{23}$$

The activation energy for date palm samples was found to be 3.908 W/g.

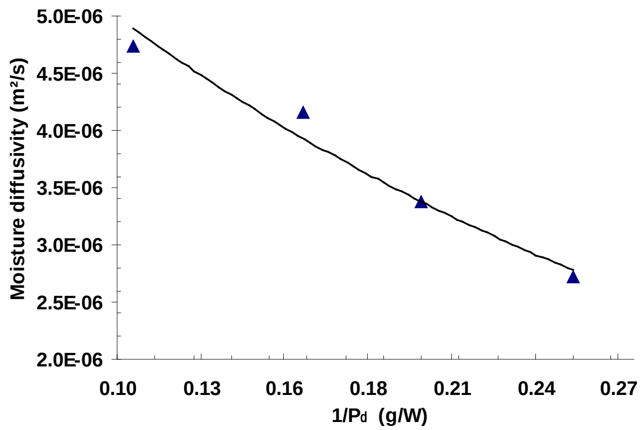

Fig. 5 : Relationship between the values of effective diffusivity and power density

IV. Conclusion

The increase in power density significantly reduced the drying time of the date palm. Drying curves date palm did not show a constant rate-drying period under the experimental employed and showed a warming up rate and falling rate-drying periods. Effective diffusivity varied from 2.72×10^{-6} to 4.73×10^{-6} m^2/s and increased with the power density. An Arrhenius relation with an activation energy value of 3.908 W/g expressed effect of power density on the diffusivity.

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